High and low frequency correlations in global equity markets

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Systemic risk, bank behaviour and regulation over the business cycle

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Market Volatility and Comovements in International Stock Markets

Market Volatility Index VIX

Average Correlations Across International Stock Markets

1: One-year moving average correlation based on weekly returns (Jan:2000-Feb:2010)
Implications:

- International asset allocation
  - Changes in the scope of diversification opportunities.
  - Optimal portfolio strategies and hedging demands.
  - Capital flows due to changes in the nature of risk.

- Asset pricing
  - Derivatives based on portfolios of securities.

Why do correlations change?

- Some empirical results: correlations change with market conditions and the business cycle (e.g., Erb, Harvey and Viskanta, 1995, Longin and Solnik, 1995, 2001, Ang and Bekaert, 2002)

- More fundamentally, correlations change with news events about fundamentals (Engle, 2009)
  - News about future dividends: country-industry linkages.
  - News about discount factors: macroeconomic news, monetary policy, inflation, interest rates.
How can a model capture these economic effects?

- Practical issues:
  - The business cycle is only measured at low frequencies.
  - News events are observed at high frequencies.

- Popular approaches to model international correlations:
  - Correlation models based on “high frequency” data are well suited to capture high frequency news effects (e.g., Capiello, Engle and Sheppard, 2003). But not so informative about long-term effects.
  - More aggregated correlation models can approximate low frequency behavior (Ang and Bekaert, 2002, Bekaert, Hodrick, and Zhang, 2008). But not informative about high frequency dynamics. Limited application in forecasting problems.
Can we have these features in a unified framework?

- Yes, this paper presents an approach to characterize high and low frequency correlation components in international markets.
- Another issue:
  - International data is unsynchronized.
- This paper addresses this problem and allows the use of daily data to characterize term-variation in the correlation structure.
- Model is easy to implement in forecasting applications.
- Highlights further key questions for modeling and forecasting correlations:
  - Are correlations mean-reverting?
  - Do they mean-revert to a constant?
Factor Models and Correlation Structure

Consider a K factor APT model

\[ r_{i,t} - E_{t-1}(r_{i,t}) = \beta_i ' F_t + u_{i,t}, \quad F_t = (f_{1,t}, \ldots, f_{K,t})' \]

and the following standard restrictions in a factor structure:

1. \( E(u_{i,t} u_{j,t}) = 0, \quad \forall i \neq j \)
2. \( E(f_{k,t} u_{i,t}) = 0, \quad \forall i, k \)
3. \( E(f_{k,t} f_{l,t}) = 0, \quad \forall k \neq l \)

If (1), (2) and (3) hold conditionally:

\[ \rho_{i,j,t} = \frac{\sum_{k=1}^{K} \beta_{i,k} \beta_{j,k} V_{t-1}(f_{k,t})}{\sqrt{(V_{t-1}(u_{i,t}) + \sum_{k=1}^{K} \beta_{i,k}^2 V_{t-1}(f_{k,t})}(V_{t-1}(u_{j,t}) + \sum_{k=1}^{K} \beta_{j,k}^2 V_{t-1}(f_{k,t}))}} \]
High Frequency Covariance

- Relaxing (1) conditionally incorporates the effect of latent unobserved factors.
- Relaxing (2) conditionally incorporates the effect of time varying betas.
- Relaxing (3) allows for dynamic interactions across factors.

Hence, we can define the high frequency covariance and correlation as:

\[ \beta_{i,k,t} = \frac{\text{cov}_{t-1}(r_{i,t}, f_{k,t})}{V_{t-1}(f_{k,t})} = \beta_{i,k} + \frac{E_{t-1}(u_{i,t}, f_{k,t})}{V_{t-1}(f_{k,t})}. \]

\[ V_{t-1}(r_t) \equiv H_t = B \Sigma_{F,t} B' + B \text{cov}_{t-1}(F_t, u_t') + \text{cov}_{t-1}(u_t, F_t')B' + \Sigma_{u,t} \]

\[ R_t^{HF} = D_t^{-\frac{1}{2}} H_t D_t^{-\frac{1}{2}}, \quad D_t = \text{diag}\{V_{t-1}(r_{1,t}), \ldots, V_{t-1}(r_{N,t})\} \]
Low Frequency Correlation

- Following Engle and Rangel (2008), factors are described by asymmetric spline-GARCH processes:

\[ V_{t-1}(f_{k,t}) = \tau_{f,k,t} g_{f,j,t} \]

\[ g_{f,j,t} \sim \text{Asymmetric GARCH}, \quad \tau_{f,j,t} \sim \text{Exponential Cuadratic Spline} \]

- Also, idiosyncratic terms show asymmetric spline-GARCH dynamics:

\[ V_{t-1}(u_{i,t}) = \tau_{i,t} g_{i,t} \]

\[ g_{i,t} \sim \text{Asymmetric GARCH}, \quad \tau_{i,t} \sim \text{Exponential Cuadratic Spline} \]

- The typical low frequency correlation component is:

\[ \bar{\rho}_{i,j,t}^{LF} = \frac{\sum_{k=1}^{K} \beta_{i,k} \beta_{j,k} \tau_{f,k,t} + E(u_{i,t}u_{j,t})}{\sqrt{\left(\tau_{i,t} + \sum_{k=1}^{K} \beta_{i,k}^2 \tau_{f,k,t}\right)\left(\tau_{j,t} + \sum_{k=1}^{K} \beta_{j,k}^2 \tau_{f,k,t}\right)}} \]

- The typical element of \( R_{t}^{HF} \) (high-frequency correlation) mean-reverts toward the LFC.

\[ \rho_{i,j,t}^{HF} \rightarrow \rho_{i,j,t}^{LF} \]
International data and synchronization

- Data: market returns of 43 countries (include developed and emerging markets), returns in US dollars, weekly and daily frequencies from January 1995 to December 2008.
- Three global factors: market returns in America, Europe, and Asia (S&P500, MSCI Europe, and MSCI Asia –Ex Japan).
- Problem: Non-synchronous trading activity biases estimation of loadings and correlations.
- Solution:
  - Use weekly data.
  - “Synchronize” daily observations.
Estimation Results: weekly data (summary)

- Correlation persistence = 0.94, updating correlation parameter = 0.011
- Variance persistence = 0.73, updating variance parameter = 0.08
- Average spline knots: 2.2

Example: High and Low Frequency Correlations of Germany and Japan (From Weekly Data)
Average FSG-DCC Correlations vs Model Free Benchmark

- Get average correlation of each country with respect to all the rest, then average across countries.
- Model-Free Benchmark: sample correlation matrix for every half year in the sample from weekly data (e.g., Bekaert, Hodrick and Zhang (2008)).
Synchronizing Returns

- Synchronized return:
  \[ S_{Et} = r_{Et} - \xi_{t-1} + \xi_t \]

- Estimated synchronized return:
  \[ \hat{S}_{Et} = r_{Et} - \hat{\xi}_{t-1} + \hat{\xi}_t \]

where \( \hat{\xi}_t = E_t(r_{E,t+1} | \{r_{US,t}, r_{E,t}, r_{A,t}\}, \Phi_t) \)

- In a multivariate setup follow Burns et al. (1998)
  \[ r_t = \eta_t + M\eta_{t-1}, \quad V_{t-1}(\eta_t) = H_{\eta,t} \]
  \[ \hat{s}_t = (I + \hat{M})\hat{\eta}_t, \quad V_{t-1}(\hat{s}_t) = (I + \hat{M})\hat{H}_{\eta,t}(I + \hat{M}) \]
Global correlations and current financial turmoil

- Compute average correlations in three subperiods: 08/03/2006 to 08/03/2007, 08/06/2007 to 09/12/2008, 09/15/2008 to 12/15/2008

![Graph showing average correlations in different subperiods.]

- Before the Credit-Crunch (August 2006-August 2007)
  - Average Correlations: 0.34
  - Average Correlations (Low Freq): 0.34

- From the Credit-Crunch to Lehman’s Bankruptcy
  - Average Correlations: 0.48
  - Average Correlations (Low Freq): 0.41

- From Lehman’s Bankruptcy to December 2008
  - Average Correlations: 0.63
  - Average Correlations (Low Freq): 0.43
Distribution of changes on average low frequency correlations across countries

% Change

Average % change

-0.2 -0.1 0 0.1 0.2 0.3 0.4 0.5 0.6 0.7

Peru Colombia New Zealand Philippines Czech Rep. Hungary Japan Korea Poland India Austria Turkey Mexico China UK South Africa Indonesia Chile Denmark France Malaysia Netherlands Thailand Australia Singapore Venezuela Germany Swiss Hong Kong Sweden Greece Taiwan Belgium Brazil Argentina Spain Russia Portugal Norway Ireland Italy Finland Canada
Distribution of changes on average high frequency correlations across countries

% Change

Average % change

Venezuela, New Zealand, Philippines, India, Colombia, Peru, Poland, Taiwan, Portugal, Thailand, Turkey, Hungary, Chile, Malaysia, South Africa, Ireland, Denmark, Indonesia, Australia, Greece, Japan, Czech Rep., Mexico, Singapore, Hong Kong, Korea, Austria, Norway, Switzerland, Finland, UK, Brazil, Spain, Canada, Sweden, Belgium, Argentina, France, Netherlands, Italy, Germany, Russia
Concluding Remarks

- This paper introduces a unified framework to model high and low frequency dynamics of global equity correlations.
- The framework allows us to exploit high frequency available information to measure such international correlations.
- The two term components of global correlations have shown substantial increases on average, especially during the recent financial turmoil.
- However, such effects have not been equally distributed across countries.
  - The effect has been larger in emerging markets.
  - This has important implications for diversification of global risk.