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# High and low frequency correlations in global equity markets

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# High and Low Frequency Correlations in Global Equity Markets

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**Systemic risk, bank behaviour and regulation over the business cycle**

**Buenos Aires**
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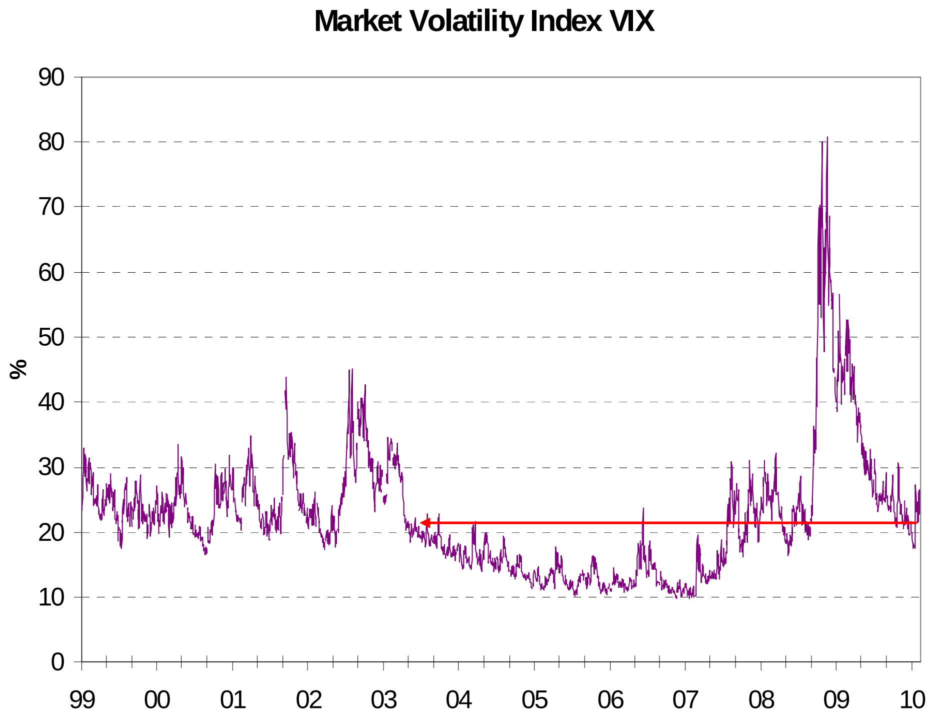
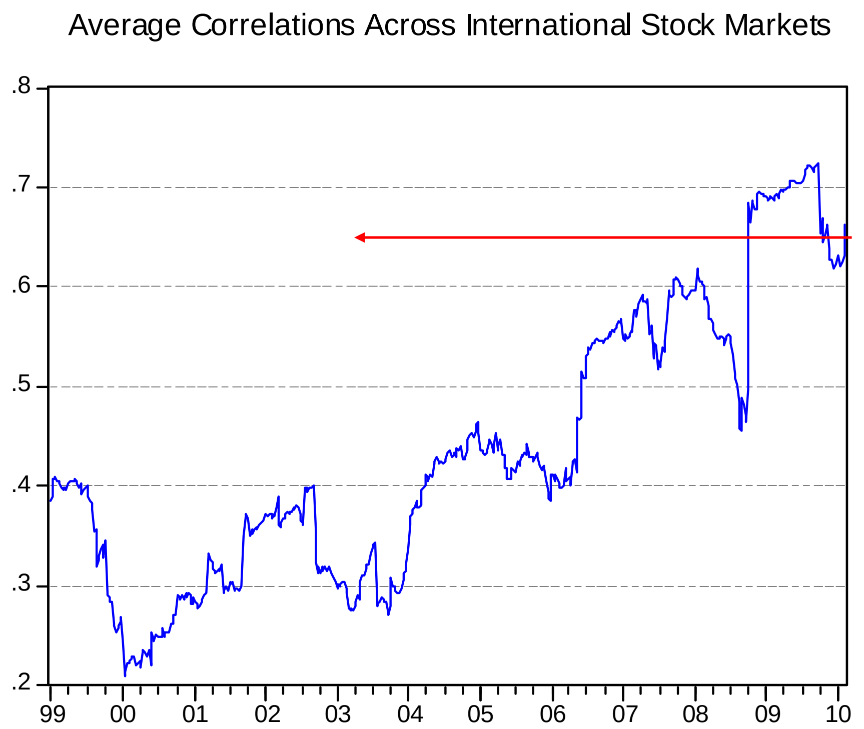
# Market Volatility and Comovements in International Stock Markets




[1/] One-year moving average correlation based on weekly returns (Jan:2000-Feb:2010)

# Implications:

- International asset allocation
  - Changes in the scope of diversification opportunities.
  - Optimal portfolio strategies and hedging demands.
  - Capital flows due to changes in the nature of risk.
- Asset pricing
  - Derivatives based on portfolios of securities.

# Why do correlations change?

- Some empirical results: correlations change with market conditions and the business cycle (e.g., Erb,Harvey and Viskanta, 1995, Longin and Solnik, 1995, 2001, Ang and Bekaert, 2002)
- More fundamentally, correlations change with news events about fundamentals (Engle, 2009)
  - News about future dividends: country-industry linkages.
  - News about discount factors: macroeconomic news, monetary policy, inflation, interest rates.

# How can a model capture these economic effects?

- Practical issues:
  - The business cycle is only measured at low frequencies.
  - News events are observed at high frequencies.
- Popular approaches to model international correlations:
  - Correlation models based on “high frequency” data are well suited to capture high frequency news effects (e.g., Capiello, Engle and Sheppard, 2003). But not so informative about long-term effects.
  - More aggregated correlation models can approximate low frequency behavior (Ang and Bekaert, 2002, Bekaert, Hodrick, and Zhang, 2008). But not informative about high frequency dynamics. Limited application in forecasting problems.

# Can we have these features in a unified framework?

- Yes, this paper presents an approach to characterize high and low frequency correlation components in international markets.
- Another issue:
  - International data is unsynchronized.
- This paper addresses this problem and allows the use of daily data to characterize term-variation in the correlation structure.
- Model is easy to implement in forecasting applications.
- Highlights further key questions for modeling and forecasting correlations:
  - Are correlations mean-reverting?
  - Do they mean-revert to a constant?

# Factor Models and Correlation Structure

Consider a K factor APT model

$$r_{i,t} - E_{t-1}(r_{i,t}) = \beta_i' F_t + u_{i,t}, \quad F_t = (f_{1,t}, \ldots, f_{K,t})'$$

and the following standard restrictions in a factor structure:

$$(1) \quad E(u_{it} u_{jt}) = 0, \quad \forall i \neq j$$

$$(2) \quad E(f_{k,t} u_{it}) = 0, \quad \forall i, k$$

$$(3) \quad E(f_{k,t} f_{l,t}) = 0, \quad \forall k \neq l$$

If (1), (2) and (3) hold conditionally:

$$\rho_{i,j,t} = \frac{\sum_{k=1}^{K} \beta_{i,k} \beta_{j,k} V_{t\text{-}1}(f_{k,t})}{\sqrt{\left(V_{t\text{-}1}(u_{i,t}) + \sum_{k=1}^{K} \beta_{i,k}^2 V_{t\text{-}1}(f_{k,t})\right)\left(V_{t\text{-}1}(u_{j,t}) + \sum_{k=1}^{K} \beta_{j,k}^2 V_{t\text{-}1}(f_{k,t})\right)}}$$

# High Frequency Covariance

- Relaxing (1) conditionally incorporates the effect of latent unobserved factors.
- Relaxing (2) conditionally incorporates the effect of time varying betas.

$$\beta_{i,k,t} = \frac{\mathrm{cov}_{t-1}(r_{i,t}, f_{k,t})}{V_{t-1}(f_{k,t})} = \beta_{i,k} + \frac{E_{t-1}(u_{i,t} f_{k,t})}{V_{t-1}(f_{k,t})}.$$

- Relaxing (3) allows for dynamic interactions across factors.

Hence, we can define the high frequency covariance and correlation as:

$$V_{t-1}(\mathbf{r}_t) \equiv H_t = B\Sigma_{F,t}B' + B\,\mathrm{cov}_{t-1}(F_t, \mathbf{u}_t') + \mathrm{cov}_{t-1}(\mathbf{u}_t, F_t')B' + \Sigma_{u,t}$$

$$R_t^{HF} = D_t^{-1/2} H_t D_t^{-1/2}, \;\; D_t = diag\{V_{t-1}(r_{1,t}),\ldots,V_{t-1}(r_{N,t})\}$$

# Low Frequency Correlation

- Following Engle and Rangel (2008), factors are described by asymmetric spline-GARCH processes:

(1) $V_{t-1}(f_{k,t}) = \tau_{f,k,t} g_{f,j,t}$

$g_{f,j,t} \sim \textit{Asymmetric GARCH}, \ \tau_{f,j,t} \sim \textit{Exponential Cuadratic Spline}$

- Also, idiosyncratic terms show asymmetric spline-GARCH dynamics:

(2) $V_{t-1}(u_{i,t}) = \tau_{i,t} g_{i,t}$

$g_{i,t} \sim \textit{Asymmetric GARCH}, \ \tau_{i,t} \sim \textit{Exponential Cuadratic Spline}$

- The typical low frequency correlation component is:

(3) $$\bar{\rho}_{i,j,t}^{LF} = \frac{\sum_{k=1}^{K} \beta_{i,k}\beta_{j,k}\tau_{f,k,t} + E(u_{i,t}u_{jt})}{\sqrt{\left(\tau_{i,t} + \sum_{k=1}^{K} \beta_{i,k}^2 \tau_{f,k,t}\right)\left(\tau_{j,t} + \sum_{k=1}^{K} \beta_{j,k}^2 \tau_{f,k,t}\right)}}$$

- The typical element of $R_t^{HF}$ (high-frequency correlation) mean-reverts toward the LFC.

(4) $$\rho_{i,j,t}^{HF} \rightarrow \rho_{i,j,t}^{LF}$$

# International data and synchronization

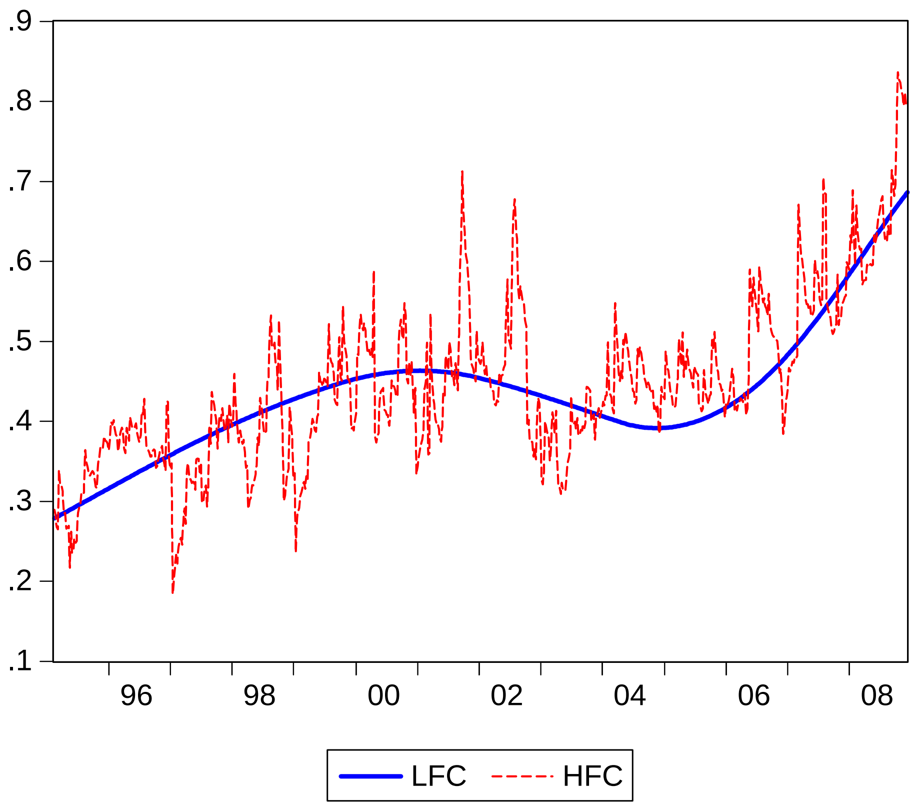
- Data: market returns of 43 countries (include developed and emerging markets), returns in US dollars, weekly and daily frequencies from January 1995 to December 2008.
- Three global factors: market returns in America, Europe, and Asia (S&P500, MSCI Europe, and MSCI Asia –Ex Japan).
- Problem: Non-synchronous trading activity biases estimation of loadings and correlations.
  - Scholes and Williams (1977), Lo and MacKinlay (1990), Burns, Engle, and Mezrich (1998).
- Solution:
  - Use weekly data.
  - “Synchronize” daily observations.

# Estimation Results: weekly data (summary)

- Correlation persistence = 0.94, updating correlation parameter = 0.011
- Variance persistence = 0.73, updating variance parameter = 0.08
- Average spline knots: 2.2

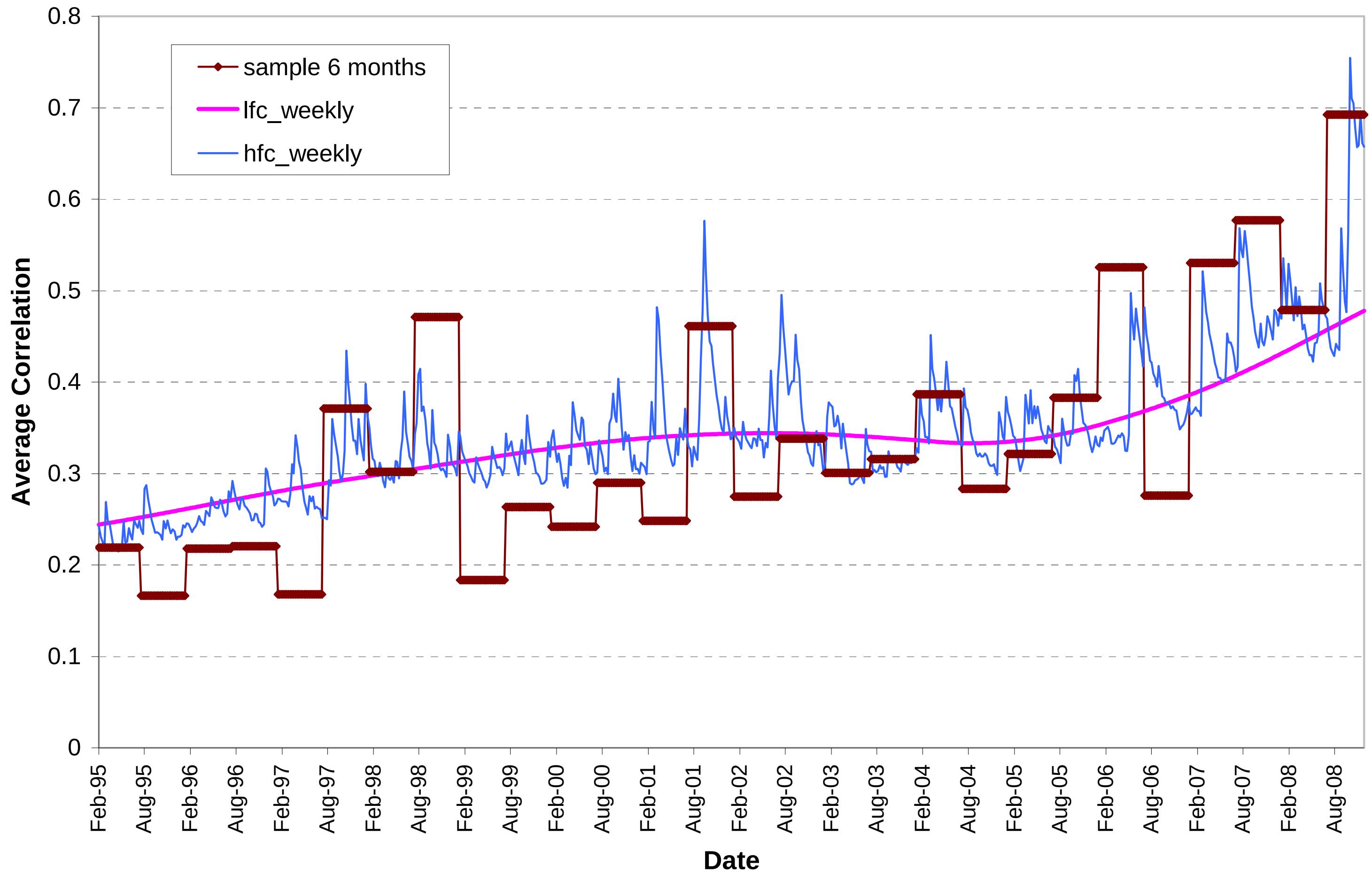
Example:

High and Low Frequency Correlations of Germany and Japan
(From Weekly Data)

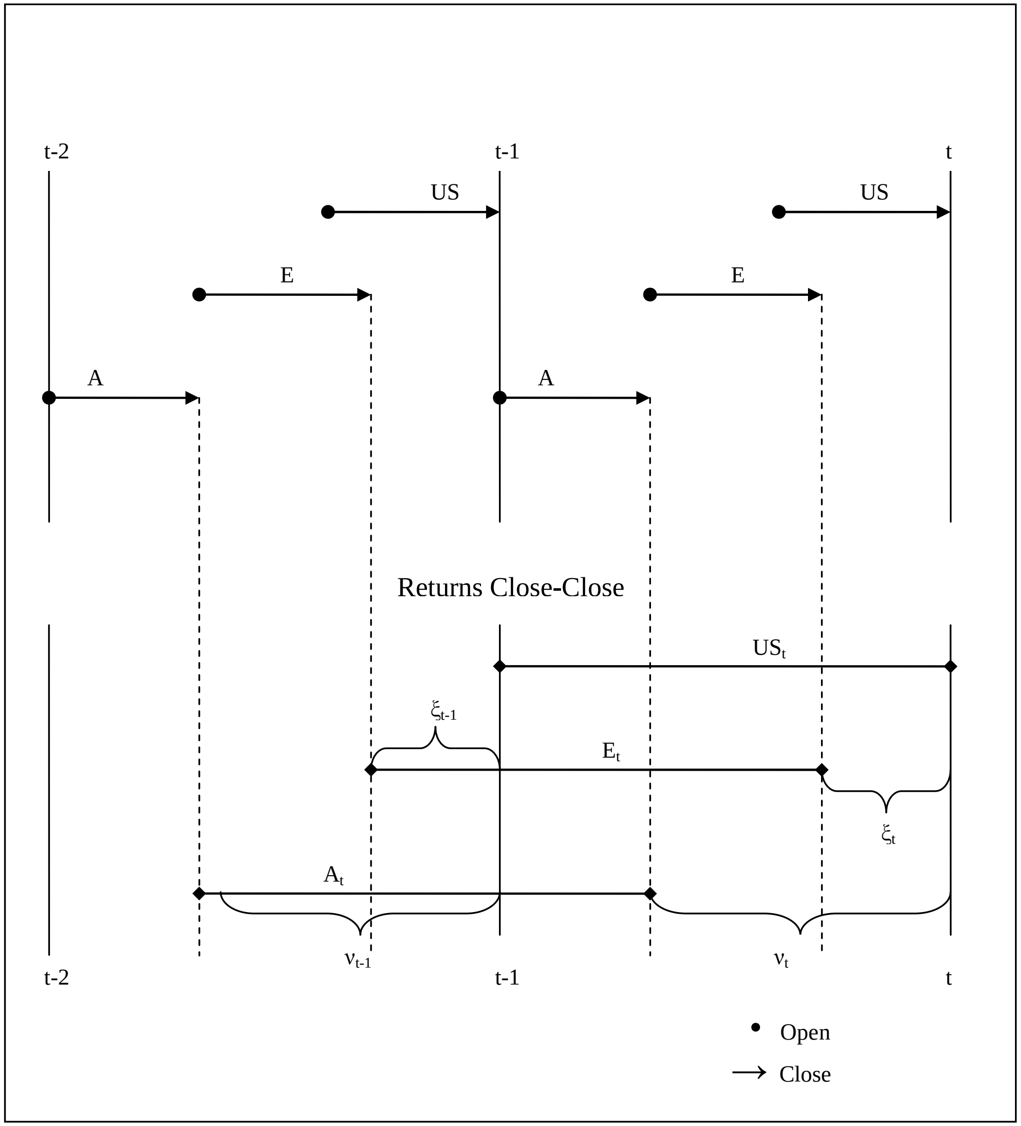


# Average FSG-DCC Correlations vs Model Free Benchmark

- Get average correlation of each country with respect to all the rest, then average across countries.
- Model-Free Benchmark: sample correlation matrix for every half year in the sample from weekly data (e.g., Bekaert, Hodrick and Zhang (2008)).



# Synchronizing Returns




- Synchronized return:

$$s_{Et} = r_{Et} - \xi_{t-1} + \xi_t$$

- Estimated synchronized return:

$$\hat{s}_{Et} = r_{Et} - \hat{\xi}_{t-1} + \hat{\xi}_t,$$

$$where\ \hat{\xi}_t = E_t(r_{E,t+1} \mid \{r_{US,t}, r_{E,t}, r_{A,t}\}, \Phi_t)$$

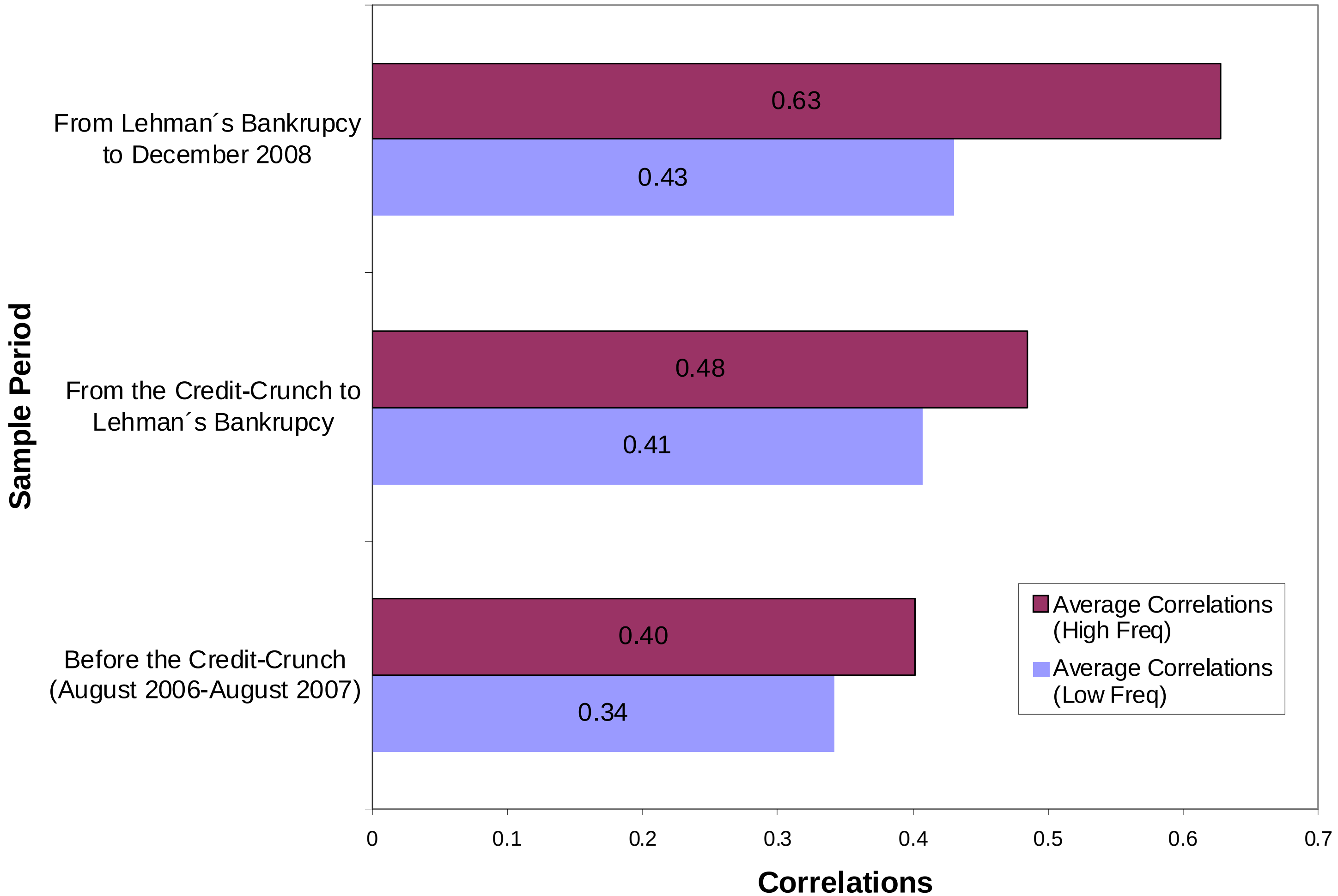
- In a multivariate setup follow Burns et al. (1998)

$$\mathbf{r}_t = \eta_t + M\eta_{t-1},\ V_{t-1}(\eta_t) = H_{\eta,t}$$

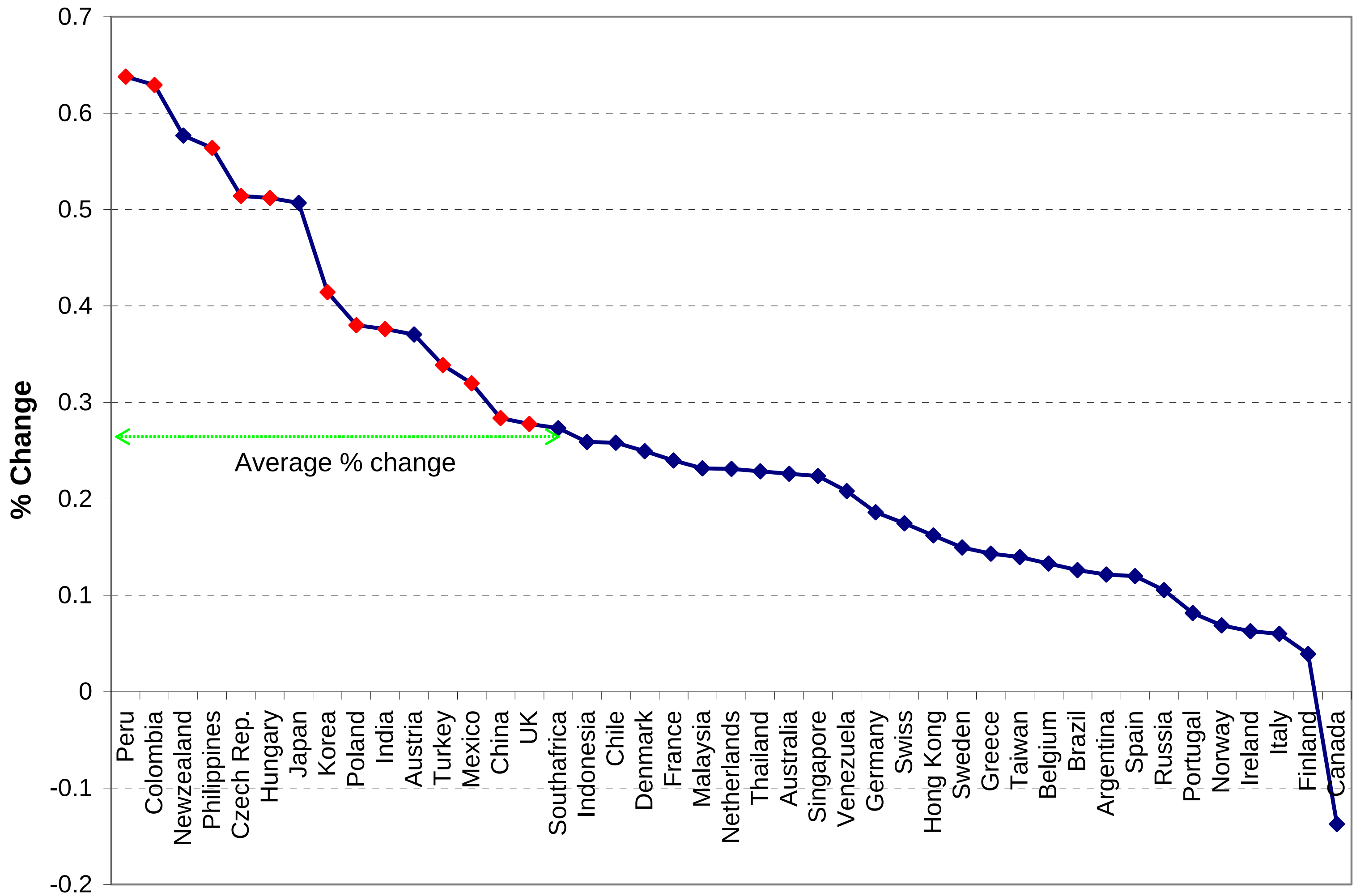
$$\hat{\mathbf{s}}_t = (I + \hat{M})\hat{\eta}_t, \quad V_{t-1}(\hat{\mathbf{s}}_t) = (I + \hat{M})\hat{H}_{\eta,t}(I + \hat{M})$$

# Global correlations and current financial turmoil

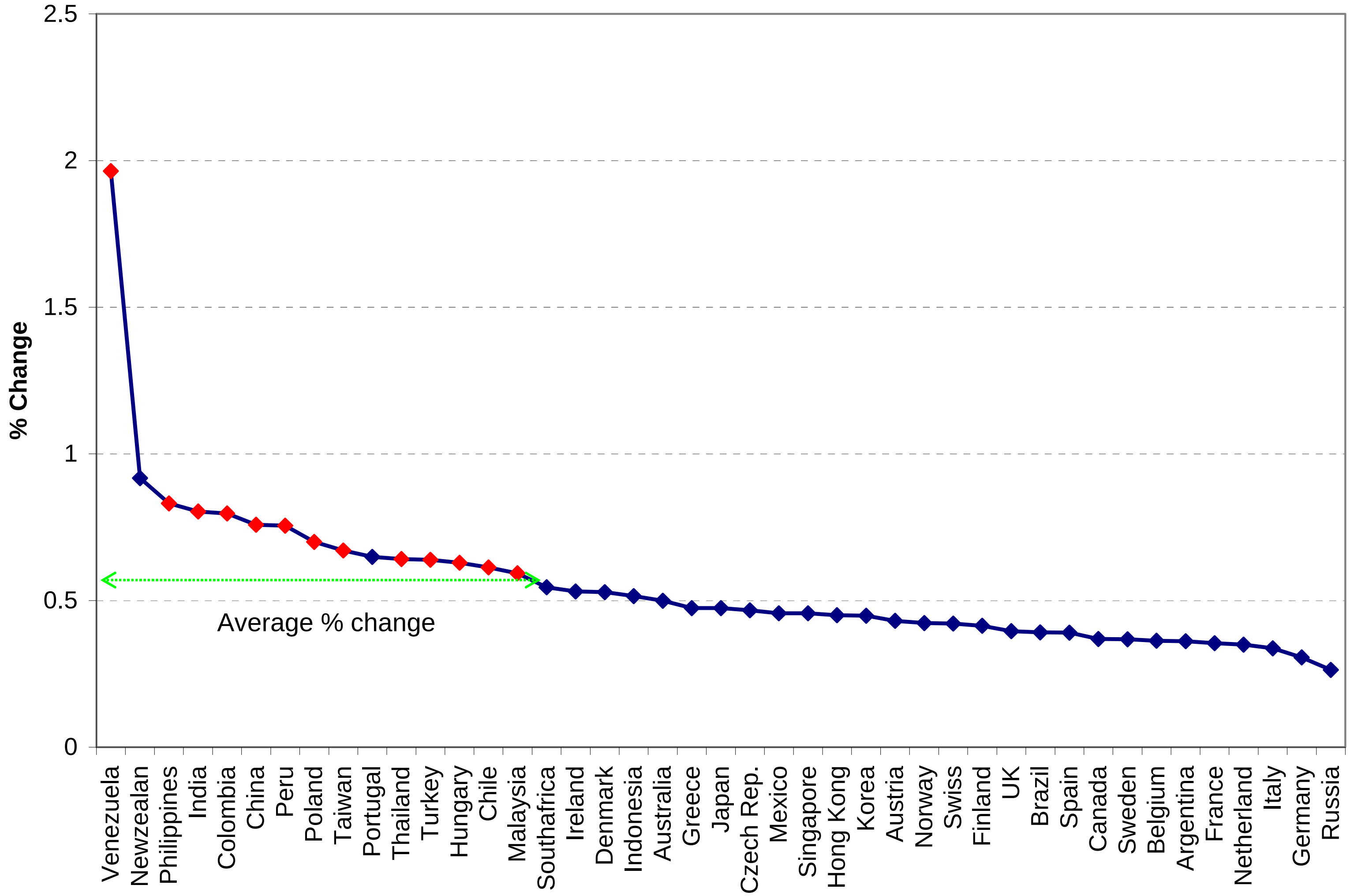
- Compute average correlations in three subperiods: 08/03/2006 to 08/03/2007, 08/06/2007 to 09/12/2008, 09/15/2008 to 12/15/2008



# Distribution of changes on average low frequency correlations across countries



# Distribution of changes on average high frequency correlations across countries



# Concluding Remarks

- This paper introduces a unified framework to model high and low frequency dynamics of global equity correlations.
- The framework allows us to exploit high frequency available information to measure such international correlations.
- The two term components of global correlations have shown substantial increases on average, especially during the recent financial turmoil.
- However, such effects have not been equally distributed across countries.
  - The effect has been larger in emerging markets.
  - This has important implications for diversification of global risk.