Endogenous Liquidity and Interdealer Trading in Over-The-Counter Markets*

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Abstract

We develop a model of decentralized asset markets with a tiered trading structure. Dealers, who strategically supply liquidity to traders, are subject to both liquidity and adverse selection costs. Dealers can manage the liquidity cost through interdealer trading. Adverse selection, however, can complicate reallocation as dealers may be reluctant to trade with informed counterparts, as other dealers can acquire information through the accumulation of private information from market-making activities. We show that interdealer trading endogenously arises when the benefits of liquidity management outweigh adverse selection costs, and further show how market liquidity is tightly linked to interdealer liquidity. When adverse selection is too severe, interdealer trading ceases to exist, and markets become segmented. We build on this framework to study how information structure impacts market liquidity. Post-trade information disclosure, by eliminating information externalities, improves market liquidity.

*The views expressed in this paper are those of the authors and do not necessarily reflect the position of the Federal Reserve Bank of New York or the Federal Reserve System.

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1 Introduction

The vast majority of financial transactions occur in decentralized markets. The decentralized nature provides a role for intermediaries to offer liquidity and make markets. These intermediaries are subject to two main sources of risk. First, they must manage liquidity costs associated with large net positions that arise from inventory costs and regulatory compliance. Second, they run the risk of facing informed trades, bringing rise to adverse selection.

In contrast to trading, which is decentralized, post-trade processing, such as clearing is typically centralized because of economies of scale. A centralized entity performing post-trade activities has access to information that would be valuable to intermediaries. Our analysis focuses on how differences in the availability of information in the inter-dealer market impacts market liquidity.

We develop a model of decentralized asset markets with a tiered trading structure. At the center of our model are dealers, who make market for "traders." At the market-making stage, dealers quote a bid-ask spread at which they are willing to purchase or sell the asset to the traders. Traders, who in addition to being better informed about the common value of the asset, derive a private value, decide whether to buy or sell from a dealer.

Dealers who purchase an asset from a trader, referred to as "long dealers", accumulate excess inventory while dealers who sell assets, referred to as "short dealers", would like to replenish their inventory. As a consequence, dealers of opposite positions are naturally incentivized to trade with each other, bringing rise to interdealer trading. At the same time, dealers are concerned about trading with other dealers who may have private information. Indeed, the true value of the assets being traded is not perfectly known by dealers, as trades are dispersed. However, observing the full set of net trades perfectly reveals the value of the asset. Because post-trade activities are centralized, a clearing entity possesses valuable information.

When there is complete segregation of information between trade and post-trade, no dealer is able to access post trade information. When this is the case, we show that: (1) inter-dealer markets can endogenously arise; (2) inter-dealer trading achieves better allocations, even with two-sided asymmetric information problem between dealers; and finally, (3) strategic complementarities and substitutability exist: interdealer liquidity increases individual dealer's incentive to provide liquidity, but also increases incentives to acquire more information than other dealers.

The shift from no information on trades in the market-making stage to full post-trade disclosure strips away informational frictions in interdealer markets and leads to lower spreads.

However, dealers still maintain higher spreads than they would in the solution that maximizes joint dealer welfare. This is because individual dealers do not internalize the positive externality that lowering their individual spreads they post to traders has on other dealers. When a dealer lowers its spread in the market-making stage, it increases the likelihood that it takes on an active position as either a short or long dealer in the interdealer market. This makes it more likely that the dealer it matches with in the interdealer market will be able to successfully unwind its position. An individual dealer, in choosing its spread in the market-making stage, does not take into account the impact of this decision on the future payoff of the other dealer.

Maximizing dealer welfare attains a solution that would be desirable to all dealers if they could commit ex-ante. Hence, it is a solution to a planner's problem that preserves all other assumptions regarding the bargaining protocol, and in particular, dealers' bargaining power. Spreads under the dealer optimal solution are still larger than they would be in the absence of dealer bargaining power. Hence, the solution that maximizes dealer welfare does not capture all possible gains from trade. If we strip away dealers' bargaining power and consider the planners problem that maximizes trader welfare, the bid-ask spread is lower still: It becomes the lowest possible spread that maintains profitability for the dealers. Examination of the social planner's problem under two welfare criteria allows us to hash out the various frictions that are present in our setting, taking as given the trading structure, and it allows us to neatly show how an equilibrium might have differed if we had stronger cooperation between dealers, in an otherwise "decentralized" market.

Our paper contributes to a literature on liquidity provision in decentralized markets. This literature seeks to explain how market liquidity is impacted by search frictions and other aspects of the decentralized trading process (see, for example, Duffie, Gârleanu and Pedersen (2005) and Lagos and Rocheteau (2009)), the interaction of OTC markets and the primary credit market (Arseneau et al. 2017), and policies that reduce informational asymmetries (Cujean and Praz (2016)). Our work contributes to the stream on informational asymmetries and, in particular, focuses on informational asymmetries about the common value of the asset to the dealers that arise endogenously from trading outcomes in the market-making stage, before the interdealer market takes place.

We know of no other papers that address the impact on liquidity provision of policies designed to reduce informational asymmetries in an OTC interdealer market that arise from private OTC trades in the market-making stage. Cujean and Praz (2016) look at private information regarding inventories and examine the impact of a policy to make these inventories public.[1] Cujean and Praz (2016) consider a model with one period OTC trade between investors and do not consider market making activities of dealers. Likewise, previous studies of OTC markets that involve a market-making stage and an interdealer market, such as Duffie et al. (2005), Lagos and Rocheteau (2009), and Dunne, Hau and Moore (2015), assume the interdealer market is competitive.

Our finding that making post-trade information from the market-making stage public before the interdealer market takes place leads to narrower bid-ask spreads and hence increased liquidity in the market-making stage is consistent with empirical studies on market transparency. Bessembinder, Maxwell and Venkataraman (2006), Edwards, Harris and Piwowar (2007), and Bessembinder and Maxwell (2008) examine the introduction of the Transaction Reporting and Compliance Engine (TRACE) for the US corporate bond market in July 2002. Under this program, transaction data related to all trades in publicly issued corporate bonds was made available to the public. These studies all found that the implementation of TRACE led to reductions in bidask spreads and increased liquidity, with some exceptions for thinly traded bonds or very large trades. Benos, Payne and Vasios (2016) examine the impact of the Dodd-Frank trading mandate that required US persons to trade interest rate swaps on Swap Execution Facilities with open limit order books. They found that the introduction of SEF trading led to economically significant improvement in liquidity. Boehmer, Saar and Yu (2005) examined trading on the New York Stock Exchange. They found that effective spreads of trades decline following the introduction of the OpenBook policy in January of 2002 that provided limit-book order information to traders off the exchange floor. Finally, in regards to CDS markets, Loon and Zhong (2016) show that the liquidity improves for index CDS contracts following the introduction real time reporting and public dissemination of OTC swap trades on December 31, 2012.

The remainder of the paper is organized as follows. Section 2 introduces the model. In Section 3, we solve the equilibrium without post-trade information disclosure. Section 4 considers the setting with post-trade information disclosure. In Section 5, we analyze the planner's problem. We conclude in Section 6.

2 Model

Consider a market where an asset is traded bilaterally. There is a measure 1 of dealers, indexed $i \in [0,1]$ and a measure 1 of traders, indexed by $j \in [0,1]$. All agents are risk-neutral.

[1]Formally, they examine variations in a parameter that defines the level of precision of signal on counterparty inventory.

Trading occurs in two stages. In the first stage ("market-making"), dealers and traders are matched at random. Dealers "make markets" by offering bid-ask prices to the traders with which they are matched. In the second stage ("inter-dealer"), dealers are randomly matched with other dealers with whom they have an opportunity to trade. This two-stage structure is intended to capture the tiered trading structure common in decentralized dealer markets.[2]

Market-Making. At $t = 1$, each dealer is matched with one trader. The asset has a common value v to all dealers that equals $\bar{v} + x$ or $\bar{v} - x$ with equal probability. Trader j knows $v_j = v + d_j$, but doesn't know the individual components v and d_j, where d_j is drawn from a uniform distribution with support $[-D, D]$, for some $D > 0$. The magnitude of D captures the dispersion in private value of the asset.

Each dealer makes an ultimatum bid-ask offer $P_i = (P_i^b, P_i^a)$, where P_i^b represents the bid price, as which the trader can sell the asset to the dealer, and P_i^a represents the ask price at which the trader can purchase the asset from the dealer.[3] Given a dealer's set of bid-ask prices P_i, a trader j chooses whether to accept the bid price, accept the ask price, or reject the dealer's offer. Formally, a trader j matched to dealer i chooses an action γ_j, with $\gamma_j \in \{\text{accept } P_i^b, \text{accept } P_i^a, \text{reject}\}$. The action γ_j is chosen to maximize the trader's payoff, which can be written as:

$$\mathbb{1}\{\text{accept } P_i^b\} \cdot (P_i^b - v_j) + \mathbb{1}\{\text{accept } P_i^a\} \cdot (v_j - P_i^a) \geq 0$$

where $\mathbb{1}\{\cdot\}$ is an indicator function for the trader's action. Hence, a trader j chooses the action:

$$\begin{aligned} &\text{accept } P_i^a \text{ if } v_j \geq P_i^a \\ &\text{accept } P_i^b \text{ if } P_i^b \geq v_j \\ &\text{reject otherwise.} \end{aligned}$$

We limit our attention to the case in which dealers offer a symmetric bid-ask spread around $\bar{v}$, such that $P_i = (P_i^b, P_i^a) = (\bar{v} - \delta_i, \bar{v} + \delta_i)$ for some $\delta_i > 0$.[4]

[2] There is considerable empirical evidence that dealer intermediated markets have a tiered structure (For example, see Li and Schürhoff (2014), Afonso, Kovner and Schoar (2013), Craig and Von Peter (2014)). To keep the model tractable, we take this structure as given and focus on the strategic behavior of dealers to endogenize market liquidity. For papers that endogenize the two-stage structure, see Viswanathan and Wang (2004) or Neklyudov (2014).

[3] Empirical studies find that dealers exercise substantial bargaining power (Green, Hollifield and Schürhoff (2006)).

[4] Generalizing the model to allow for asymmetric bid-ask spreads does not yield additional insights.

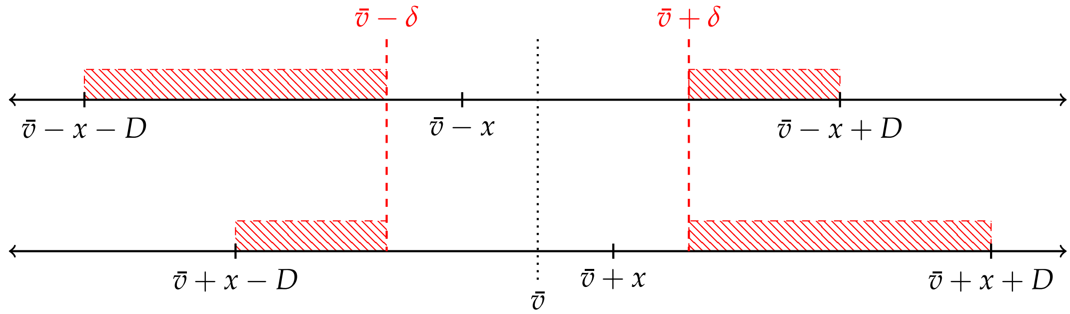

Figure 1: Market-making and the likelihood of trade

In Figure 1, $\bar{v}$ represents a dealer's expected value of v before trading. If the actual value of v is $\bar{v}-x$, then the top line illustrates the distribution of traders. If, instead, the actual value of v is $\bar{v}+x$, then the bottom line illustrates the distribution of traders. The red shaded regions represent the mass of traders who are willing to accept a bid offer (to the left of $\bar{v}-\delta$) or an ask offer (to the right of $\bar{v}+\delta$).

An important insight revealed by Figure 1 is that if $v=\bar{v}-x$, then the likelihood that a trader will accept the dealer's bid price is high compared to the likelihood that a trader would accept the ask price. Conversely, if $v=\bar{v}+x$, then a trader is more likely to accept the dealer's ask price than the bid price.

At the end of $t=1$, dealers who have purchased the asset have a net position of 1 and we refer to them as "long dealers." Dealers who have sold the asset have a net position of -1 and we refer to them as "short dealers." Finally dealer that did not trade have a net position of 0 and we refer to them as "neutral dealers." We use $\theta \in \{l, s, n\}$ to denote the type of the dealer at the end of $t=1$.

Inter-Dealer. At $t=2$ the interdealer market opens. All dealers are randomly bilaterally matched. Within each pair, one dealer is picked at random and allowed to make an ultimatum offer to his or her counterparty. Both dealers have equal probability of being picked. The dealer that makes the ultimatum offer is called the "offering" dealer and the counterpart is the "receiving" dealer.

An offering dealer i of type θ makes an offer $(\sigma_{i,\theta}, P^d_{i,\theta})$, where $\sigma_{i,\theta} \in \{buy, sell, \text{no trade}\}$ indicates the actions that the offering dealer wants, and $P^d_{i,\theta}$ denotes the transaction price.[5] A receiving dealer i who receives offer (σ_{-i}, P^d_{-i}) from dealer $-i$ makes a decision of whether to

[5]The specific form of the interdealer offer, while tractable, is without loss of generality.

accept or reject the offer. Formally, $\gamma_{i,\theta}(\sigma_{-i,\theta}, P^d_{-i,\theta}) \in \{accept, reject\}$.

Post trade. At the end of $t = 2$, after all trade occurs, dealers with a nonzero position face an opportunity cost of $\Delta \in \left(\frac{D}{\sqrt{2}+1}, D\right)$. This cost can be motivated in a number of ways. In this paper, we want to think of Δ as representing the cost of providing collateral to a central counterparty (CCP).

In many over-the-counter (OTC) markets, a CCP helps to reduce counterparty risk between market participants. Over the course of the day, CCP members report their trades to the CCP. At the end of the day, the CCP calculates the net position of each member and asks members to provide contributions to their "default funds" that are proportional to the net positions. In case of default by a CCP member, the default fund protects the CCP and other members.

In our model, we assume that dealer are members of the CCP and report their date 1 and date 2 trades to the CCP. At the end of the date 2, the CCP asks for contributions proportional to each dealers net position. Specifically, A dealer with a net position of $x \in \{-2, -1, 0, 1, 2\}$ of the asset must contribute $\Delta|x|$ to the CCP for its default fund.

In addition, any dealer that finishes stage 2 in a long position must unwind this position by selling the asset at a price equal to its true value and any dealer that finishes stage 2 in a short position must cover this position by buying the asset at a price equal to its true value.

Equilibrium. The solution concept is Perfect Bayesian Equilibrium. Given an information structure, an equilibrium consists of dealers' market-making offer strategies δ_i^*, dealers' interdealer market offer strategies $(\sigma^*_{i,\theta}, P^{d*}_{i,\theta})$, and dealers' trade decisions given offers in the interdealer market, traders' trade decisions given offers in the market-making stage, and dealers' and traders' beliefs. We restrict attention to symmetric strategies such that in equilibrium $\delta_i^* = \delta_k^*$ for $\forall i, k$. Formally:

Definition 1. *A Perfect Bayesian Equilibrium is dealers' market-making strategies $\{\delta_i^*\}_i$ and interdealer offer strategies $\{(\sigma^*_{i,\theta}, P^{d*}_{i,\theta})\}_{i;\theta=n,l,s}$, dealers trading strategies conditional on interdealer offers $\{\gamma_i^*(\sigma_\theta, P^d_\theta)\}_i$, traders' trading strategies conditional on bid-ask offers $\{\gamma_j^*(P^b, P^a)\}_j$, and traders' beliefs and dealers' beliefs such that:*

1. *dealer i's market making strategies δ_i^* maximize the dealer's expected profits at $t = 1$, and interdealer offer strategies and $\{(\sigma^*_{i,\theta}, P^{d*}_{i,\theta})\}_{\theta=n,l,s}$ trading strategies $\gamma_i^*(\sigma_\theta, P^d_\theta)$ maximize the dealer's conditional expected payoff at $t = 2$;*

2. *trader j's trading strategy $\gamma_j^*(P^b, P^a)$ maximizes his payoffs at $t = 1$;*

3. *dealers' and traders' beliefs are consistent with Bayes' Rule.*

To sum up, the events of the model unfold as follows:

$t = 1$ Dealers and traders randomly match. Dealer i offer (P_i^b, P_i^a) corresponding to spread δ_i. Traders accept or reject offers. Dealers' positions reported to the CCP.

$t = 2$ Dealers randomly match and trade in interdealer market. Dealers make ultimatum offers, which is accepted or rejected by matched dealers. Dealers' positions reported to the CCP. CCP refunds or demands capital from dealers. Dealers incur liquidity cost.

$t = 3$ CCP settles all positions and returns capital. Payoffs are realized.

3 Laissez-faire OTC market

We first analyze the laissez-faire OTC market.

3.1 Market-Making Strategies

In the market-making stage at $t = 1$, each dealer i offer a bid-ask offer $P_i = (\bar{v} - \delta_i, \bar{v} + \delta_i)$ corresponding to some spread δ_i at which he offers to buy and sell an asset from a trader. In addition to determining profits conditional on trade, a dealer's spread impacts: (1) the likelihood that a trader accepts his offer to trade, and (2) the dealer's posterior belief conditional on a trader accepting his offer. It is useful to make a distinction between these two sets of offers that the dealer can make:

Definition 2 (Market-making strategies)**.** *Dealer i is said to employ:*

- **partially revealing offer** *if he chooses a $\delta_i \in (0, D - x)$;*
- **fully revealing offer** *if he chooses a $\delta_i \geq D - x$.*

Partially revealing offers. To begin, we restrict our attention to partially revealing offers, i.e. when $\delta_i < D - x$. Recall, as outlined in Section 2, that a trader accepts a dealer's bid offer if and only if his valuation v_j is less than $\bar{v} - \delta_i$, and accepts a dealer's ask offer if and only if v_j is greater than $\bar{v} + \delta_i$. It is straightforward to see that a trader is willing to accept at most one of the offers, for any $\delta_i > 0$. The likelihood that dealer i's bid offer $\bar{v} - \delta_i$ is accepted is given by:

$$
\begin{aligned}
& P(v = \bar{v} + x) \cdot P(\bar{v} - \delta_i > v_j | v = \bar{v} + x) + P(v = \bar{v} - x) \cdot P(\bar{v} - \delta_i > v_j | v = \bar{v} - x) && (1) \\
&= \frac{1}{2} \cdot \frac{(\bar{v} - \delta_i) - (\bar{v} + x - D)}{2D} + \frac{1}{2} \cdot \frac{(\bar{v} - \delta_i) - (\bar{v} - x - D)}{2D} && (2) \\
&= \frac{D - \delta_i}{2D}. && (3)
\end{aligned}
$$

Following a similar computation, the likelihood that a dealer i's ask offer $\bar{v} + \delta$ is accepted is given by:

$$\frac{D - \delta_i}{2D}. \tag{4}$$

Note that as δ_i increases, the likelihood that a trader accepts a dealer's offer monotonically decreases. Since a greater spread is associated with a less attractive offer to a trader, fewer traders are willing to accept the dealer's offers.

A trader's valuation v_j is comprised of a common value v and private value d_j. As a result, dealer i, who is initially uninformed about v, revises his beliefs concerning the common value v conditional on an offer being accepted by a trader. This implies that a dealer can directly affect how much he learns from market-making through his bid-ask offer strategy P_i. Specifically, choosing a wider bid-ask spread reduces the probability that the dealer trades, as noted above, but also provides more information about the value of v conditional on a trade. We now formalize this second effect.

We can characterize dealer i's interim beliefs regarding v conditional on successfully trading with a trader. Conditional on dealer i's bid offer $\bar{v} - \delta_i$ being accepted, dealer i's belief on the expected value of v is given by:

$$
\begin{aligned}
& P(v = \bar{v} + x | \bar{v} - \delta_i > v_j) \cdot (\bar{v} + x) + P(v = \bar{v} - x | \bar{v} - \delta_i > v_j) \cdot (\bar{v} - x) && (5) \\
&= \frac{\frac{1}{2}\frac{D-x-\delta_i}{2D}}{\frac{1}{2}\frac{D-x-\delta_i}{2D} + \frac{1}{2}\frac{D+x-\delta_i}{2D}} \cdot (\bar{v} + x) + \frac{\frac{1}{2}\frac{D+x-\delta_i}{2D}}{\frac{1}{2}\frac{D-x-\delta_i}{2D} + \frac{1}{2}\frac{D+x-\delta_i}{2D}} \cdot (\bar{v} - x) && (6) \\
&= \bar{v} - \frac{x}{D - \delta_i} \cdot x && (7)
\end{aligned}
$$

By symmetry, conditional on dealer i's ask offer $\bar{v} + \delta_i$ being accepted, dealer i's belief on the expected value of v is given by:

$$\bar{v} + \frac{x}{D - \delta_i} \cdot x \tag{8}$$

First, consider how the dealer, when employing partially revealing offers, affects the revision of his beliefs regarding v conditional on trading. Since $\frac{x}{D-\delta_i}$ increases in δ_i, a higher δ_i leads to a greater downward revision on the expected value of v, conditional on a bid offer being accepted. Correspondingly, a higher δ_i leads to a greater upward revision in the expected value of v, conditional on a ask offer being accepted.

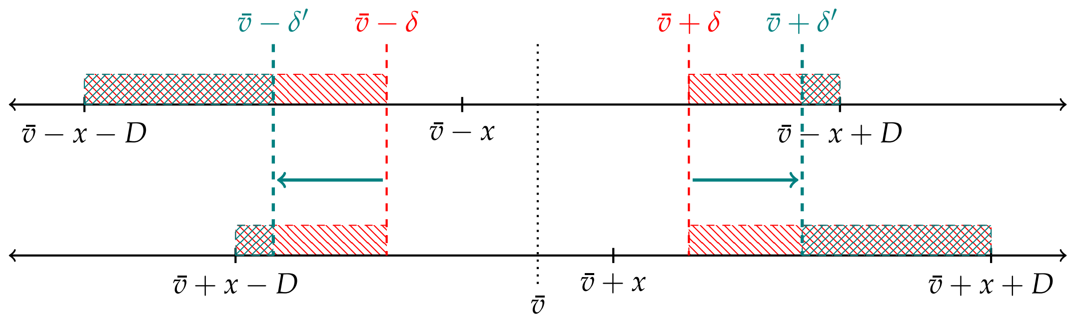

Figure 2: Impact of increasing bid-ask spreads

Each line represents traders' valuations conditional v, where the top pertains to when $v = \bar{v} - x$ and the bottom pertains to when $v = \bar{v} + x$. An increase in the bid-ask spread from δ to δ' corresponds to a smaller likelihood of trade, represented by the teal shaded regions.

This effect is illustrated in example shown in Figure 2. Suppose a trader accepts an ask offer. If the dealer had chosen a tight bid-ask spread (i.e. δ), then Figure 2 suggests that the probability that $v = \bar{v} + x$ is twice as large as the probability that $v = \bar{v} - x$. If, instead, the dealer had chosen a wider bid-ask spread (i.e. δ'), then conditional on trade, the probability that $v = \bar{v} + x$ increases more relative to the probability that $v = \bar{v} - x$. Figure 2 also shows that the probability of any offer being accepts is smaller when the bid-ask spread is wider.

Fully revealing offer. Dealers can also choose to employ a fully revealing offer strategy, in which case $\delta_i \geq D - x$. Given traders' optimal trading strategies, it is straightforward to see that if the dealer sets $\delta_i \geq D - x$, then any accepted offer fully reveals the state of nature. For example, if the state is $v = \bar{v} + x$, then $v_j = v + d_j$ can only be smaller than the bid price $\bar{v} - \delta_i \leq \bar{v} - D + x$ if $\bar{v} + x + d_j \leq \bar{v} - D + x$, which implies $d_j < -D$, which is not possible since $d_l \in [-D, D]$. So a bid offer can only be accepted when the state is $v = \bar{v} - x$. A similar argument shows that an ask offer can only be accepted if the state is $v = \bar{v} + x$. As a consequence, the dealer becomes *perfectly informed* about the common value v through trade.

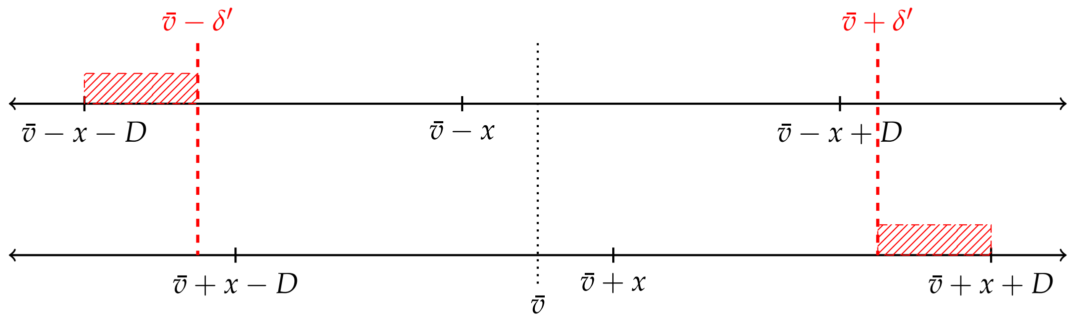

Figure 3: Trading under fully revealing market-making offers

Each line represents traders' valuations conditional v, where the top pertains to when $v = \bar{v} - x$ and the bottom pertains to when $v = \bar{v} + x$. The red shaded regions represent the traders who are willing to accept an offer. For bid-ask spread $\delta > D - x$ a bid and ask offer is accepted by a trader only when $v = \bar{v} - x$ and $v = \bar{v} + x$, respectively.

Figure 3 illustrates the case of a fully-revealing offer. If δ is sufficiently large, bid offers are only accepted if $v = \bar{v} - x$ and ask offers are only accepted if $v = \bar{v} + x$. In other words, conditional on a trade, a dealer knows the state of nature. Of course, the probability of a trade decreases as δ increases.

In the case of a fully revealing offer, the likelihood that dealer i's bid offer $\bar{v} - \delta_i$ is accepted is given by the probability that the state is $v = \bar{v} - x$ multiplied by the probability that $\bar{v} - \delta_i > v_j$ in that state. Since each state of the world occurs with equal probability, this can be written as:

$$\begin{aligned} & P(v = \bar{v} - x) \cdot P(\bar{v} - \delta_i > v_j | v = \bar{v} - x) && (9) \\ & = \frac{1}{2} \cdot \frac{(\bar{v} - \delta_i) - (\bar{v} - x - D)}{2D} && (10) \\ & = \frac{D + x - \delta_i}{4D}. && (11) \end{aligned}$$

Note that (11) is equal to (3) if $\delta_i = D - x$. By symmetry, the likelihood that a dealer i's ask offer $\bar{v} + \delta$ if accepted is given by

$$\frac{D + x - \delta_i}{4D} \tag{12}$$

In general, dealers face a clear tradeoff between acquiring more information through trade, and increasing the likelihood of trade. With partially revealing offers, dealers become better informed through trade but still remain uncertain about the underlying common value v. With fully revealing offers, dealers learn perfectly the underlying state of the world, conditional on a trade, but trade with a lower likelihood.

3.2 Interdealer Markets

Interdealer trading depends on dealers' collective market making strategies, since these strategies determine the share of short, neutral, and long dealers in the interdealer market. In this section we study dealers participation in interdealer markets and how interdealer market liquidity relate to dealers' liquidity provisions at $t = 1$.

Before analyzing the interdealer subgame in which dealers individually select their strategies, it is useful to understand the Pareto efficient interdealer outcome as a benchmark case. In particular, given a set of matches between dealers in the interdealer trading stage, what set of trades are required for the interdealer outcome to be efficient?

To shed light on this, we consider an arbitrary distribution of long, short, and neutral dealers, μ_l, μ_s, μ_n, where $\mu_l + \mu_s + \mu_n = 1$. In interdealer markets, dealers are randomly bilaterally matched between each other. Consider a trade that occurs between two dealers. First, whatever the price at which a pair of dealers agree to trade, it is effectively a transfer from one dealer to another. Second, if the two dealers do not have opposite positions, then a trade results in a transfer of a nonzero position from one dealer to another. However, the sum of net positions of the two dealers is unchanged. Together, this implies that any trades between dealers with non-offsetting positions do not have implications for efficiency.

The pair of dealers with offsetting positions are the $\mu_l \cdot \mu_s$ mass of long-short dealer matches. Recall, when a dealer fails to offset a nonzero position in interdealer markets, he incurs a liquidity cost Δ per unit of asset. When trade successfully occurs between a long and short dealer, both dealers are able to net their existing position to 0, and each avoid incurring cost Δ. Hence, efficient interdealer trading entails minimizing the total dead-weight loss associated with the nonzero positions held by dealers. Putting this together, it is straightforward to see the efficient interdealer outcome is as follows.

Lemma 1 (Efficient Interdealer Trading)**.** *Suppose the measure of dealer types is given by μ_θ, for $\theta = l, s, n$. An interdealer outcome is Pareto efficient if and only if almost every match between long and short dealers successfully trade, i.e. offsetting trades occur between the $\mu_l \cdot \mu_s$ pair of long-short dealer matches.*

Lemma 1 provides a clear characterization of an efficient interdealer outcome: one in which trade successfully occurs between all long-short dealer matches. To see whether the efficient interdealer outcome is attained under laissez-faire, we proceed by analyzing the main subgame at $t = 2$, conditional on some symmetric market making strategy $\delta_i = \hat{\delta}$ assumed to be employed by dealers at $t = 1$. We start by analyzing the case where dealers used partially revealing offers at the market making stage; $\hat{\delta} < D - x$.[6] Notice that dealers enter interdealer trading with dispersed beliefs regarding v, even if they chose the same market-making spread $\delta_i = \hat{\delta}$. Indeed, as shown in Section 3.1, dealers update their beliefs about v conditional on a trade being accepted.

We assume that dealers do not know the position of the dealers with whom they are matched. Without loss of generality, we use a long dealer as an example to illustrate the strategic considerations in interdealer trading. The dealer's offer must take into account that his counterparty could have a long, short, or neutral position. By making an offer to sell, a long dealer could offset his position, and avoid liquidity cost Δ. However, he may instead prefer to increase his long position if it is more profitable.

First, consider when the long dealer makes an offer to sell, which would offset his position. Suppose that receiving dealers infers that a sell offer is only made a long dealer, i.e. in equilibrium the long dealer separates from other types by signaling through his offer. The reservation prices of a long, neutral, and short dealer who receives an offer to sell is given by:

$$\begin{cases} \bar{v} + \frac{(D-x-\hat{\delta})(D-x-\hat{\delta})-(D+x-\hat{\delta})(D+x-\hat{\delta})}{(D-x-\hat{\delta})(D-x-\hat{\delta})+(D+x-\hat{\delta})(D+x-\hat{\delta})}x - \Delta & \text{for a long dealer} \\ \bar{v} + \frac{(D-x-\hat{\delta})2\hat{\delta}-(D+x-\hat{\delta})2\hat{\delta}}{(D-x-\hat{\delta})2\hat{\delta}+(D+x-\hat{\delta})2\hat{\delta}}x - \Delta & \text{for a neutral dealer} \\ \bar{v} + \frac{(D-x-\hat{\delta})(D+x-\hat{\delta})-(D+x-\hat{\delta})(D-x-\hat{\delta})}{(D-x-\hat{\delta})(D+x-\hat{\delta})+(D+x-\hat{\delta})(D-x-\hat{\delta})}x + \Delta & \text{for a short dealer} \end{cases} \tag{13}$$

$$= \begin{cases} \bar{v} - \frac{2(D-\hat{\delta})x}{(D-\hat{\delta})^2+x^2}x - \Delta & \text{for a long dealer} \\ \bar{v} - \frac{x}{D-\hat{\delta}}x - \Delta & \text{for a neutral dealer} \\ \bar{v} + \Delta & \text{for a short dealer} \end{cases} \tag{14}$$

A dealer's reservation price is comprised of two parts. The first depends on a receiving dealer's belief about the expected value of v *conditional on trade*. Given receiving dealers' beliefs that a sell offer is (correctly) made by a long dealer, they require the reservation price to reflect the expected value of v conditional on that, and their private information.

The second depends on whether the trade provides a netting benefit to the receiving dealer. Note, for any offer received, a dealer incurs an (additional) Δ cost if his net position

[6] As we will show below, when $\hat{\delta} \geq D - x$ then all dealers who trade are on the same side of the market (all are either long or short) and there is no interdealer trading.

increase as a result of accepting the offer. Hence, in the above case, where dealers receive an offer to sell, with the exception of short dealers, a receiving dealer requires an additional Δ subtracted from the price. In the case of short dealers, who gain from netting their -1 position, are willing to pay a premium of Δ.

Note that the reservation price of a dealer strictly decreases in his net position. This reflects two things. In particular, a short dealer's reservation is the lowest, which reflects the netting benefits, and because a short dealer's valuation of the asset is greatest conditional on trade.

The set of reservation prices are three candidate prices at which a long dealer may want to make a sell offer. When a long dealer signals his type by making a sell offer, the counterparty with whom he can make the most profitable trade is a short dealer. Still lowering the price to a dealer type's respective reservation price increases the likelihood of trade. Would doing so be profitable? No.

To see this, consider a long dealer's beliefs on v conditional on trading with each type of dealer:

$$\begin{cases} \bar{v} - \frac{2(D-\hat{\delta})x}{(D-\hat{\delta})^2+x^2}x & \text{for a long dealer} \\ \bar{v} - \frac{x}{D-\hat{\delta}}x & \text{for a neutral dealer} \\ \bar{v} & \text{for a short dealer} \end{cases} \tag{15}$$

Just as a receiving dealer's beliefs adjusted to account for the likelihood that he was matched to a long dealer, a long dealer accounts for the likelihood that he was matched to a particular dealer. As such, conditional on a specific dealer type pair, both parties of an interdealer trade form identical beliefs about v. This reveals a powerful insight: when all dealers are differentially but equally informed, i.e. δ_i is identical, then surplus from trade only occurs when a dealer trades with a counterparty with an opposite position.

More generally, if in equilibrium, separation is to occur through interdealer offers between dealer types, then a dealers' payoff maximizing offer is set at the reservation price of a dealer of a opposite position. In the case of a neutral dealer, no trades yield a positive surplus.

So far, we outline a long dealer's optimal sell offer strategy. Would he instead want to make a buy offer? By making a buy offer, a long dealer increases his net position, which would be associated with an additional Δ liquidity cost. In addition, note that a long dealer's private information works against him – his valuation of v is lower relative to other dealer types. It is straightforward to verify that there does not exist a feasible price at which deviation is profitable. Building on this, we can fully characterize the interdealer subgame given $\delta_i = \hat{\delta} < D - x$ for all i.

Lemma 2 (Interdealer Trading)**.** *Suppose that all dealers execute partially-revealing offers at spread* $\hat{\delta}$ *at* $t = 1$*. Then, in interdealer markets:*

1. *short dealers make offer* $(buy, \bar{v} - \Delta)$ *and only accept offers* $(sell, P^d)$ *for* $P^d \leq \bar{v} + \Delta$*;*
2. *long dealers make offer* $(sell, \bar{v} + \Delta)$ *and only accept offers* (buy, P^d) *for* $P^d \geq \bar{v} - \Delta$*;*
3. *neutral dealers do not make any offers, and reject all offers.*

Interestingly, the price at which dealers make offers is *independent* of $\hat{\delta}$. Even though dealers are asymmetrically informed about each other's type, a trade uniquely identifies the type of the offering and receiving dealers' types. Since successful trades entail matches between dealers of opposite positions, their beliefs ex-post offset each other – conditional on trading, the expected value of v is exactly $\bar{v}$ for both parties.

By Lemma 2, there exists a subgame solution in which interdealer trading occurs in all short to long dealer pairs. Following Lemma 1, this implies that dealers choose to initiate offers that achieve the efficient interdealer outcome:

Corollary 1. *Given* $\hat{\delta} < D - x$*, the efficient interdealer outcome is obtained.*

So far, we took as given that $\hat{\delta} < D - x$. What remains is to characterize interdealer markets when all dealers choose $\hat{\delta} > D - x$.

As highlighted in Section 3.1, when a dealer uses a fully revealing offer in the previous period, he is able to infer the true value of v. When *all* dealers use fully revealing offers, *all* dealers who successfully trade acquire the same position, depending on v. Specifically, all dealers who trade become short or long dealers, if $v = \bar{v} + x$ or $\bar{v} - x$, respectively. In consequence, no offsetting trades can occur in interdealer markets. In other words, there do not exist any interdealer trades that result in positive surplus when $\hat{\delta} > D - x$.

Lemma 3 (No Interdealer Trading)**.** *Suppose that all dealers execute fully-revealing market-making. Then, there do not exist any interdealer trades with positive surplus.*

A direct consequence of Lemma 3 is that if there is a negligible cost of initiating an interdealer trade, no interdealer trades will occur when $\hat{\delta} > D - x$. These results show that interdealer trading occurs if and only if partially revealing market-making strategies are chosen by dealers. In the following section, we investigate when dealers find it incentive compatible to collectively make partially revealing offers in equilibrium.

3.3 Endogenous Interdealer Trading

To fully characterize the equilibrium, we must analyze an individual dealer's decision on δ_i. In particular, an equilibrium requires that some $\delta_i = \delta^*$ maximizes dealer's expected payoff $\Pi_i(\delta_i, \delta^*)$ conditional on all other dealers choose δ^*.

Equilibrium with interdealer trading. To do so, we first analyze the conditions under which there exists an equilibrium with interdealer trading, i.e. when $\delta^* < D - x$. To do so, we take as given interdealer offers and beliefs conditional on all dealers choosing some spread $\hat{\delta} \in (0, D - x)$, and identify the conditions under which there exists some $\hat{\delta}$ such that an individual dealer does not have an incentive to deviate to any $\delta' \neq \hat{\delta}$.

Given the interdealer strategies outlined in Lemma 2, a long (short) dealer only accept offsetting offers at price $P^d \geq \bar{v} - \Delta$ ($\leq \bar{v} + \Delta$). Consider the marginal payoff of an individual dealer i of type θ who deviates to some $\delta_i > \bar{\delta}$. As shown in Equation 7, the dealer becomes more informed than others about v. This implies that conditional on trading at the market making stage, the dealer revises the expected value of v more severely relative to other dealers who offered spread $\hat{\delta}$. As a result, the posterior beliefs conditional on a match with dealer type are also more extreme. To fix ideas, suppose that the dealer assumed a long position. Then, his expectation on v conditional on matching with a short dealer is strictly less than $\bar{v}$. This implies that conditional on receiving an offer $(buy, \bar{v} - \Delta)$, the difference in payoffs between accept and reject is:

$$(\bar{v} - \Delta) - (E_i[v|trade] - \Delta) = \underbrace{\bar{v} - E_i[v|trade]}_{\text{information rent}} > 0, \tag{16}$$

where $E_i[\cdot]$ is used to denote expectations given dealer i's information set. As a receiving dealer, the dealer is not able to retain the surplus from netting positions. However, because he is better informed about v, he extracts an information rent from the offering dealer by accepting an offer to buy that is greater than his reservation price. Similarly, conditional on his matched dealer accepting his offer $(sell, \bar{v} + \Delta)$, he makes a marginal payoff (relative to no trade):

$$\bar{v} + \Delta - (E[v|trade] - \Delta) = \underbrace{\bar{v} - E_i[v|trade]}_{\text{information rent}} + \underbrace{2\Delta}_{\text{gains from netting}} \tag{17}$$

As an offering dealer, in addition to extracting the surplus from netting, he also extracts information rents as well. Notably, $\bar{v} + \Delta$ is the maximum price at which a receiving short dealer accepts a sell offer. This implies that dealer i maximizes his conditional payoff when mimicking the strategy of a long dealer, by offering $(sell, \bar{v}+\Delta)$.

Building on this insight, let an individual dealer i of type θ's expected marginal payoff from interdealer trades be denoted $V_\theta(\delta_i, \hat{\delta})$, conditional on δ_i and other dealers choosing $\hat{\delta}$. Consider a dealer i of type θ's expected marginal payoff from mimicking the interdealer strategies specified in Lemma 2:

1. if $\theta = s$, make offer $(buy, \bar{v}-\Delta)$ and only accept offers $(sell, P^d)$ for $P^d \leq \bar{v}+\Delta$;

2. if $\theta = l$ make offer $(sell, \bar{v}+\Delta)$ and only accept offers (buy, P^d) for $P^d \geq \bar{v}-\Delta$.

We attain the following decomposition dealer i's marginal value of interdealer trade:[7]

$$V_\theta(\delta_i, \hat{\delta}) = \underbrace{\left(\sum_v P(v|\theta)P(\text{match with opposite dealer}|v,\theta)\right)(\bar{v} - E_i[v|trade])}_{\text{information rents}} \tag{18}$$

$$+\frac{1}{2}\underbrace{\left(\sum_v P(v|\theta)P(\text{match with opposite dealer}|v,\theta)\right)2\Delta,}_{\text{gains from netting}} \tag{19}$$

where

$$\sum_v P(v|\theta)P(\text{match with opposite dealer}|v,\theta) = \frac{D+x-\delta_i}{2(D-\delta_i)}\frac{D-x-\hat{\delta}}{2D} + \frac{D-x-\delta_i}{2(D-\delta_i)}\frac{D+x-\hat{\delta}}{2D} \tag{20}$$

$$= \frac{D^2-x^2+\delta_i\hat{\delta}-D(\delta_i+\hat{\delta})}{2D(D-\delta_i)} \tag{21}$$

The marginal gains from interdealer trading decompose into two parts: (1) information rents, and (2) gains from netting. Formally,

$$V_\theta(\delta_i, \hat{\delta}) = \underbrace{\frac{D^2-x^2+\delta_i\hat{\delta}-D(\delta_i+\hat{\delta})}{2D(D-\delta_i)}(\bar{v} - E_i[v|trade])}_{\text{information rents}} + \underbrace{\frac{D^2-x^2+\delta_i\hat{\delta}-D(\delta_i+\hat{\delta})}{2D(D-\delta_i)}\Delta}_{\text{gains from netting}} \tag{22}$$

It is easy to verify that for any $\delta_i > \hat{\delta}$, where $\hat{\delta} \in (0, D-x)$, information rents are strictly positive. Dealer i's decision on δ_i exclusively factors into information rent payoff. Since the

[7] Formally, one must verify that interdealer strategies that increase net positions are not profitable. We relegate this and relevant discussion to the proof in the Appendix.

likelihood of matching with a dealer of an opposite position is independent of his choice, V (weakly) increases in δ_i. Adapting Equation 15, the information rents are given by:

$$\bar{v} - \left[\bar{v} + \frac{(D-x-\delta_i)(D+x-\hat{\delta}) - (D+x-\delta_i)(D-x-\hat{\delta})}{(D-x-\delta_i)(D+x-\hat{\delta}) + (D+x-\delta_i)(D-x-\hat{\delta})}x\right] = \frac{\left(\delta_i - \hat{\delta}\right)x}{D^2 - x^2 + \delta_i\hat{\delta} - D(\delta_i + \hat{\delta})}x \tag{23}$$

While deviating to some $\delta_i > \hat{\delta}$ unambiguously increases the marginal interdealer payoff, its overall effect on dealer i's ex-ante payoff is unclear because a wider bid-ask spread lowers the likelihood that a dealer trades at the market-making stage. To understand this, it is useful to decompose dealer i's expected payoff at $t = 1$ conditional on δ_i into two parts. The "autarky" payoff, denoted A in the equation below, is the unconditional payoff obtained through market-making, while "interdealer" payoff, denoted B, corresponds to the marginal payoff from trading in interdealer markets.

$$\Pi_i(\delta_i, \hat{\delta}) = \underbrace{P(\gamma_j(P^b, P^a) = accept|\delta_i) \cdot (\bar{v} + \delta_i - E[v|\delta_i] - \Delta)}_{\equiv A,\ \text{autarky payoff}} + \underbrace{\sum_{\theta} P(\theta_i = \theta|\delta_i) \cdot V_\theta(\delta_i, \hat{\delta})}_{\equiv B,\ \text{interdealer payoff}}, \tag{24}$$

where

$$A = 2 \cdot \frac{D-\delta_i}{2D}\left(\delta_i - \frac{x}{D-\delta_i} \cdot x - \Delta\right), \tag{25}$$

$$B = 2 \cdot \frac{D-\delta_i}{2D}\left(\frac{\left(\delta_i - \hat{\delta}\right)x}{2D(D-\delta_i)}x + \frac{D^2 - x^2 + \delta_i\hat{\delta} - D(\delta_i + \hat{\delta})}{2D(D-\delta_i)}\Delta\right). \tag{26}$$

Each component reveals a tradeoff with respect to δ_i: increasing δ_i unambiguously increases the realized payoff, but unambiguously decreases the likelihood of trade. $\Pi(\delta_i, \hat{\delta})$ is differentiable for $\delta_i \in (\hat{\delta}, D-x)$:

$$\frac{dA}{d\delta_i} = \frac{D - 2\delta_i}{D} + \frac{\Delta}{D}, \tag{27}$$

$$\frac{dB}{d\delta_i} = \frac{x^2}{2D^2} - \frac{D-\hat{\delta}}{2D^2}\Delta. \tag{28}$$

Note that the autarky payoff A increases in δ_i for any $\delta_i < \frac{D+\Delta}{2}$, and decreases in δ_i for $\delta_i > \frac{D+\Delta}{2}$ in $\delta_i \in (0, D-x)$. Interdealer payoff B depends on $\hat{\delta}$ – for small $\hat{\delta}$, gains from netting outweigh the information rents.

Imposing symmetry, we have a unique local solution for the set of $\delta \in (0, D - x)$

$$\delta^{**} = \frac{2D^2 + x^2 + \Delta D}{4D - \Delta}. \tag{29}$$

It remains to show whether this solution is a global maximum. Consider $\delta_i \in (D - x, D + x)$.[8] We can write equation (24) as:

$$\sum_v P(v, \theta_i | \delta_i) \cdot (\delta_i - x - \Delta) + \sum_\theta P(\theta_i = \theta | \delta_i) \cdot \frac{D - x - \hat{\delta}}{2D} (x + \Delta) \tag{30}$$

$$= \frac{D + x - \delta_i}{2D} (\delta_i - x - \Delta) + \frac{D + x - \delta_i}{2D} \frac{D - x - \hat{\delta}}{2D} (x + \Delta) \tag{31}$$

The best response $\delta_i(\hat{\delta})$ for the set of $\delta_i \in (D - x, D + x)$ is:

$$\delta_i(\hat{\delta}) = x + \frac{D + \Delta}{2} - \frac{x + \Delta}{2D} \left(D - x - \hat{\delta} \right) \tag{32}$$

The next Theorem shows that δ^{**} is an equilibrium solution when x is sufficiently small:

Theorem 1 (Equilibrium with Interdealer Markets)**.** *Suppose that $x < x^{trade}$ for some cutoff $x^{trade} > 0$. Then, there exists a symmetric equilibrium with $\delta^{**} \in [0, D - x)$ and interdealer trading.*

The theorem sets out conditions under which dealers offer (P^b, P^a) that are accepted with positive probability under either realization of v and, as a result, also have interdealer trading in equilibrium. Importantly, this requires that x, the common value uncertainty, be sufficiently small. Indeed, dealers face a risk of failing to offload their position in the interdealer market, which brings rise to a winner's curse problem. This risk increases with the amount of uncertainty surrounding the true asset value, as reflected by the magnitude of x. Consider the equilibrium expected profits of a dealer in an equilibrium with interdealer markets, decomposed into three main components:

$$\Pi(\delta) = \underbrace{\delta \cdot \frac{D - \delta}{D}}_{\text{market-making profits}} - \underbrace{x \cdot \frac{x}{D}}_{\text{adverse selection cost}} - \underbrace{\Delta \cdot \frac{D^2 + x^2 - \delta^2}{2D^2}}_{\text{liquidity cost}}$$

The first component, market-making profits, corresponds to the expect profit from a successful trade at the market making stage. The second component, adverse selection cost, captures the

[8] The proof in the Appendix also considers deviations to $\delta_i \in (0, \delta^{**})$.

risk that a bid offer is accepted by a trader when the price the true value of v is low or an ask offer is accepted when the true value of v is high. Finally, the last component, liquidity cost, corresponds to the cost of not being able to offload a position in the interdealer market.

There are several aspects worth noting. First, δ, the width of the bid-ask spread, affects both market-making profits and the liquidity cost, but not the adverse selection cost. δ governs both the likelihood of a trade and the profit margin conditional on a trade. Since higher δ lowers the likelihood of trading (i.e. obtaining a nonzero position), the liquidity cost decreases when δ increases. Partially-revealing market making necessarily exposes dealers to adverse selection, as failing to offload positions in the interdealer market indicates that they are in the excess side of demand.

Equilibrium with market segmentation. So far, we analyzed an equilibrium in which dealers used partially revealing market-making strategies. Recall, as a consequence of Lemma 3, if dealers instead collectively use fully revealing market-making strategies, interdealer trading does not occur.

We proceed by characterizing an equilibrium without interdealer trading, i.e. when all dealers select $\delta_i \in [D - x, D + x)$ in equilibrium. Suppose that all dealers use $\hat{\delta} \in (D - x, D + x)$. Consider an individual dealer i's payoff from choosing some $\delta_i \in (D - x, D + x)$. Since interdealer payoff is zero, the payoff is given by the autarky payoff:

$$\Pi(\delta_i, \hat{\delta} | \delta_i \geq D - x) = \underbrace{\frac{D + x - \delta_i}{2D} (\delta_i - x - \Delta)}_{\text{autarky payoff}} \tag{33}$$

Notably, given $\delta_i \in (D - x, D + x)$, a dealer's payoff is independent of $\hat{\delta}$, since he does not expect to trade with any other dealer in the interdealer stage. Hence, dealer i's best response δ_i simply maximizes his autarky payoff:

$$\delta_i(\hat{\delta}) = x + \frac{D + \Delta}{2}. \tag{34}$$

When D is small (or x is relatively large), dealers may find it more profitable to offer a wide bid-ask spread (i.e. large δ) that perfectly shields them from adverse selection. In this case, they provide liquidity only to those traders with strong private motives to acquire or sell. Given that they only make one-side of the market at any given time, no interdealer trading will occur. As such, the liquidity cost is fully taken into consideration when setting prices at the market-making stage.

It remains to show when an individual dealer i may want to choose some $\delta_i < D - x$ given $\hat{\delta} \in [0, D-x)$. Given expectation of no interdealer trading, the payoff is simply the autarky payoff:

$$\Pi(\delta_i, \hat{\delta} | \delta_i < D - x) = \underbrace{\frac{D - \delta_i}{D}\left(\delta_i - \frac{x^2}{D - \delta_i} - \Delta\right)}_{\text{autarky payoff}} \tag{35}$$

Dealer i maximizes his payoff for $\delta_i = \frac{D+\Delta}{2}$. Note that fully revealing δ_i yields a greater payoff if and only if:

$$\frac{D + x - \left(x + \frac{D+\Delta}{2}\right)}{2D}\left(x + \frac{D+\Delta}{2} - x - \Delta\right) = \frac{1}{2D}\left(\frac{D-\Delta}{2}\right)^2 > \frac{1}{D}\left(\frac{D-\Delta}{2}\right)^2 - \frac{x^2}{D} \tag{36}$$

This condition is satisfied if $x > \frac{D-\Delta}{2\sqrt{2}} = x^{seg}$. This pins down the conditions under which an equilibrium without interdealer trading exists:

Theorem 2 (Equilibrium with Market Segmentation)**.** *Suppose that $x > x^{seg}$. An equilibrium exists in which all dealers offer $(\bar{v} - \delta^*, \bar{v} + \delta^*)$ where $\delta^* = x + \frac{D+\Delta}{2}$ and no interdealer trading occurs.*

When x is greater than threshold x^{seg}, there exists an equilibrium in which all dealers offer fully revealing market-making offers in equilibrium. By doing so, dealers offer trades to those traders who are willing to pay a high premium for liquidity. This is captured by the equilibrium bid-ask spread $\delta^* = x + \frac{D+\Delta}{2}$ which fully insures dealers from the realization of x.

As highlighted in Lemma 3, when all dealers offer fully revealing market-making offers, interdealer trading does not occur. In other words, market segmentation endogenously arises in equilibrium. For $x > x^{seg}$, each dealer, in anticipation of no interdealer trading, selects a wide market-making spread to exclusively maximizes the autarky payoff. In turn, the expectation of no interdealer trading is rationalized by the private incentive of dealers to select a wide market-making spread.

Theorems 1 and 2 lay out how the underlying uncertainty x relate to the nature (or lack of) interdealer trading. While an equilibrium with interdealer trading exists for $x < x^{trade}$, an equilibrium with segmentation exists for $x > x^{seg}$. Combining the two, we characterize set of equilibria over the interval of x:

Theorem 3. *Given the two types of equilibria,*

- *For $x < x^{seg}$, only an equilibrium with interdealer trading exists;*
- *For $x \in (x^{seg}, x^{trade})$, both types of equilibria exist;*
- *For $x > x^{trade}$, only an equilibrium with market segmentation exists.*

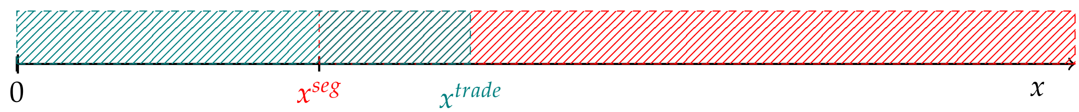

Figure 4: Equilibrium existence over x

The red and teal regions represent the values of x for which an equilibrium with interdealer markets and segmentation exist, respectively.

As illustrated in Figure 4, whether trade actively occurs interdealer markets depends on the value of x. For small values of x (i.e. $x < x^{seg}$), interdealer trading occurs, as the relative gains from . For large values of x (i.e. $x > x^{trade}$), interdealer does not occur in equilibrium. For intermediate values (i.e. $x \in [x^{seg}, x^{trade}]$), whether interdealer trading occurs in equilibrium depends on dealers' beliefs.

The coexistence of an equilibrium with interdealer trading and an equilibrium with segmentation for intermediate values of $x \in [x^{seg}, x^{trade}]$ sheds light on a potential channel through which interdealer markets, and more generally, OTC market liquidity are fragile. In this region, dealers' expectations the existence of interdealer trading pivotally determine their market-making strategies, which validate their beliefs. This self-fulfilling nature of market liquidity reveals a vulnerability of dealer intermediated markets to collective miscoordination by dealers.

In addition, a small uncertainty shock (i.e. increase in x) around x^{trade} may lead to a sudden breakdown in the interdealer markets. For example, interdealer trading, which accounts for 61 percent of all trades in Sterling OTC markets, fell to 2 percent during the Sterling flash crash on October 2016. In a report on the flash crash, the Financial Conduct Authority cites the sharp withdrawal of dealers from interdealer markets as one of the key catalysts of the dramatic illiquidity episode.

3.4 Equilibrium Liquidity Provisions By Dealers

Two aspects can be used to characterize equilibrium liquidity provisions by dealers. First, the equilibrium bid-ask spread captures the liquidity extended to traders by dealers who make markets. Let $\mu(\delta)$ be the measure of traders that accept a dealer's offer at $t = 1$. In an equilibrium with interdealer trading, we get:

$$\mu(\delta^{**}) = \frac{2(D-\Delta)D - x^2}{(4D-\Delta)D}$$

In an equilibrium with market segmentation, μ is given by:

$$\mu(\delta^{*}) = \frac{D-\Delta}{4D}$$

In both cases, μ increases in D and decreases in Δ:

Corollary 2. *Dealer liquidity provision decreases in Δ, (weakly) decreases in x, and increases in D.*

For the interval (x^{seg}, x^{trade}), the two types of equilibria coexist. It is easy to verify that equilibrium interdealer trading vastly improves market liquidity:

Corollary 3. *For $x \in (x^{seg}, x^{trade})$, dealer liquidity provision with interdealer trading is greater than that under market segmentation.*

Finally, while we take as given a matching and bargaining protocol, it is worth discussing how our results relate to the efficiency of decentralized markets relative to other types of market protocols. In our setting, market liquidity and efficiency is greater when private value D is large and uncertainty x is small. The parameter space in which interdealer markets are active is broadly consistent with other studies that study the relative efficiency of centralized and decentralized markets. For instance, Viswanathan and Wang (2004) compares one-shot and sequential auctions and shows that sequential trading is more efficient when customer orders are less informed. Glode and Opp (2017) endogenizes dealer information acquisition and further find that decentralized trading is more efficient relative to an auction when motives to trade are driven by private values.

4 Market Liquidity and Post-Trade Information Disclosure

The analysis in the previous section reveals how dealers' private gains from becoming better informed limited equilibrium interdealer market liquidity, and potentially broke down interdealer trading altogether. Lower interdealer market liquidity in turn lowered dealers' liquidity provision incentives, and ultimately lowered efficiency. This suggests that efficiency can be improved by limiting the private benefits from being better informed in interdealer markets. Does a market solution exist? We demonstrate how market liquidity can be improved upon through post-trade information disclosure.

Post-trade information disclosure. Consider the following extension. Suppose that in the beginning of period 2 and prior to matching between dealers taking place, the CCP publicly discloses to all dealers the set of trades that occurred between traders and dealers in period 1.[9] Specifically, let information regarding a single trade consist of the direction (i.e. buy or sell) and price between an anonymous pair of trader and dealer.

As a precursor, note that observing the outcome of trades is sufficient to perfectly infer the underlying value of x.

Lemma 4. *Suppose that a dealer observes the set of successful trades made at $t = 1$. Then, the dealer perfectly infers the true value of v.*

This implies that the CCP can, in fact, release post-trade information such that all dealers become informed about the true asset value at the beginning of stage 2. Given post-trade information disclosure, consider when all dealers become informed prior to trading at $t = 2$. We characterize the interdealer game conditional on all dealers having chosen some $\hat{\delta} < D - x$:

Lemma 5 (Interdealer Trading under Disclosure)**.** *Suppose that all dealers execute partially-revealing offers at spread $\hat{\delta}$ at $t = 1$. Then, in interdealer markets:*

1. *short dealers make offer $(buy, v - \Delta)$ and only accept offers $(sell, P^d)$ for $P^d \leq v + \Delta$;*
2. *long dealers make offer $(sell, v + \Delta)$ and only accept offers (buy, P^d) for $P^d \geq v - \Delta$;*
3. *neutral dealers do not make any offers, and reject all offers.*

Interdealer trading under disclosure appears remarkably similar to that without disclosure, as outlined in Lemma 2. The key difference are the prices at which dealers transact. Since all dealers are ex-post perfectly informed about v, the price reflects the true value v plus the gains from trade Δ. Trade exclusively occurs between long and short dealers, who gain by offsetting each others' positions. Naturally, this implies that, as in the case without information disclosure, efficient interdealer trading occurs:

Corollary 4. *Efficient interdealer trading occurs under perfect information.*

This implies that for a given $\hat{\delta}$, the set of trades that occur with and without information disclosure *are identical*. As in the case with no information disclosure, given some $\hat{\delta}$, dealers

[9]Note that the CCP is uniquely positioned to execute this disclosure since since all dealers submit trade information to the CCP at the end of period 1.

implement the efficient set of interdealer trades. Dealers reveal their types by making buy (sell) offers when they hold a short (long) position, thereby fully separating. The key difference arises through the prices at which trades occur. Since all dealers become informed in the interim period, offers reflect the true value of v.

The primary effect of full information in interdealer trading is the elimination of strategic behavior aimed at increasing profits by extracting more information (via bigger bid ask spread). Since information is ensured to be available ex-post regardless of liquidity provision, this shuts down incentives to learn more through market-making. To see this, reconsider the decomposition of dealer i's marginal value of interdealer trade. Since the offer fully reflects v, information rents are equal to zero:

$$V_\theta(\delta_i, \hat{\delta}) = \underbrace{0}_{\text{information rents}} + \underbrace{\frac{1}{2}\left(\sum_v P(v|\theta)P(\text{match with opposite dealer}|v,\theta)\right)2\Delta}_{\text{gains from netting}}, \quad (37)$$

By shutting down the information rent, individual dealers' incentives to deviate are weakened. This has two effects. First, all else equal, disclosure *decreases* equilibrium bid-ask spreads. Second, an equilibrium with interdealer trading exists for a larger interval of x. Formally, we characterize an equilibrium with interdealer trading:

Theorem 4 (Perfect Information)**.** *Suppose that $x < x^{trade,disclosure}$. Under disclosure, there exists an equilibrium with interdealer trading, in which dealers offer spread $\delta^{***} = \frac{(2D+\Delta)D}{4D-\Delta}$.*

Most notably, under full information, market liquidity is *enhanced*. This improvement in liquidity is primarily driven by shutting down individual dealer's strategic incentive to marginally increase bid-ask spread in the market making stage, thereby obtaining an information advantage in interdealer trading. Since dealers know that full information about the aggregate state will be available in the interdealer trading stage, regardless of their liquidity provision strategy, individual dealers choose to offer tighter spreads that maximize profits, irrespective of other dealers' strategies:

Corollary 5. *Market liquidity is (weakly) greater under disclosure.*

This points to a channel through which increased disclosure of post-trade information can be used to improve market liquidity. Namely, by eliminating the possibility of asymmetric information between dealers at the interdealer market, all dealers can ex-ante more aggressively offer liquidity to traders without strategic considerations with respect to becoming more informed than future counterparties.

5 The Social Planner's Problem and Free-Riding on Liquidity

In the previous section, we showed how a private solution, in the form of post-trade information disclosure, could improve efficiency by eliminating the information externality. In this section, we analyze the constrained efficient outcome by solving the social planner's problem. This allows us to hash out two other frictions that remain unresolved in a laissez-faire OTC market.

Consider a social planner who can enforce all dealers to select a bid-ask spread δ_i, subject to dealers' participation constraints. We assume that the social planner, as are all dealers, is uninformed about v. First, we characterize the solution to the social planner's problem where only dealer welfare is taken into consideration:

Theorem 5 (Social Planner's Problem: Dealer Welfare)**.** *Suppose that the social planner maximizes dealer welfare. The social planner selects* $\delta^{soc,D} = \frac{D^2}{2D-\Delta}$ *for* $x < x^{soc,D}$*, and* $\delta^* = x + \frac{D+\Delta}{2}$ *otherwise. Furthermore,* $\delta^{soc,D} < \delta^{***}$*, and* $x^{soc,D} \geq x^{trade,disclosure}$*.*

Two things are noteworthy. First, the social planner selects δ independent of x for $x < x^{soc,D}$. As with the case of disclosure, he fully internalizes the cost of information externalities. However, his choice of δ is lower than that under the case of disclosure. This is due to the fact that individual dealers in a laissez-faire environment take as given interdealer market liquidity. Private maximization entails free-riding on market liquidity.

In a laissez-faire environment, dealers' strategic incentives to free-ride on liquidity in turn lower equilibrium interdealer liquidity, and harm dealers' equilibrium payoffs. The social planner, by enforcing a greater liquidity provision at $t = 1$, also improves expected gains from interdealer trade by increasing liquidity, and ultimately increases dealer welfare.

The planner's solution improves dealer welfare by lowering equilibrium bid-ask spread δ, which strictly increases market efficiency as well. Incidentally, lowering δ also increases traders' welfare, as more traders are able to accept dealers' offers. The strict welfare enhancement arises due to the planner internalizing the benefits of greater liquidity in interdealer markets. Even when the potential for information rents are shut down, individual dealers have an incentive to free ride on interdealer market liquidity without "contributing" (by choosing a lower bid-ask spread). Unable to collectively commit to a lower bid-ask spread, under laissez-faire, dealers fail to implement the market-making strategy that ex-ante maximizes their payoff.

As a final note, consider how the solution to the social planner's problem where only dealer welfare is taken into consideration:

Theorem 6 (Social Planner's Problem: Trader Welfare)**.** *The social planner selects some* $\delta^{soc,T} \leq \delta^{soc,D}$ *for* $x < x^{soc,T}$*, and* $\delta^* = x + \Delta$ *otherwise, where* $x^{soc,T} \geq x^{soc,D}$*.*

When the social planner cares exclusively about trader welfare, he minimizes the bid-ask spread δ, subject to dealers' participation constraint. As a consequence, the social planner chooses an equilibrium bid-ask spread that is less than that in Theorem 5, and also selects an equilibrium with interdealer trading for a greater interval of x. The difference in the solutions of Theorem 5 and Theorem 6 is accounted for by dealer market power, which determines δ, impedes on market liquidity.

6 Conclusion

In this paper, we develop a model of decentralized asset markets with a tiered trading structure to study market liquidity in a setting in which dealers face both adverse selection and liquidity costs. First, we show that interdealer trading endogenously arises when the benefits of liquidity management outweigh adverse selection costs, and further show how market liquidity is tightly linked to inter-dealer liquidity. When adverse selection is too severe, inter-dealer trading ceases to exist, and markets become segmented. We build on this framework to study how information structure impacts market liquidity. Equal access to post-trade information supports market liquidity; whether all or no dealers receive that information.

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7 Proofs

7.1 Lemma 1

Proof. Let the measure of dealer types is given by μ_θ, for $\theta = l, s, n$, and by contradiction, suppose that a positive measure of long-short dealer matches fails to trade. Consider any arbitrary pair of long dealer i_1 and short dealer i_2 that fails to trade. We show that there exists a price at which the two dealers can trade such that both dealers a weakly better off, and at least one dealer is strictly better off.

Let the expected value of v conditional on the information set of dealer i_1 and i_2 be $E[v|i_1, i_2]$. Suppose that dealer i_1 sells his asset at price $E[v|i_1, i_2] + \Delta$ to dealer i_2. The dealer i_2's payoff obtained from no trade is $\bar{v} + \delta_{i_2} - E[v|i_1, i_2] - \Delta$ while the payoff obtained from trade is given by $\bar{v} + \delta_{i_2} - (E[v|i_1, i_2] + \Delta)$. Hence, Dealer i_2 is indifferent between trade and no trade. Since dealer i_1 is strictly better off, we obtain a contradiction.

We show that any interdealer outcome in which all long-short dealer matches successfully trade is an efficient outcome. We first show that the interdealer outcome can not be pareto improved by changing the prices at which trade occurs, since doing so only affects the distribution of surplus. Suppose that for some arbitrary pair of long dealer i_1 and short dealer i_2 for which dealer i_1 sells to i_2 at price P. Now, consider when trade instead occurs at some $P' \neq P$. Without loss of generality, suppose that $P' < P$ and both dealers i_1 and i_2 are willing to trade at P' . Then, dealer i_2 is strictly worse off by $P' - P$. Hence, conditional on trade occurring between all long-short pairs, an interdealer outcome is not pareto dominated. Next, we show the interdealer outcome can not be pareto improved by an outcome in which some long-short pairs do not trade. Note, trade occurs between dealers only if their participation condition is satisfied. From our earlier argument, we showed that trade, relative to no trade, strictly improves at least one dealer of the pair. Hence, a no trade outcome cannot improve efficiency. It suffices to show that an interdealer outcome in which trade occurs between matches other than long-short pairs do not affect efficiency. Without generality, consider a deviation from the interdealer outcome in which some dealer i_1 sells to dealer i_2. Note, for any pair that is not long-short, there are no netting benefits. This implies that for any two dealers i_1, i_2, trade results in a shift in the liquidity cost Δ from one dealer to another. Hence, there exists at most one price at which both dealers are not strictly worse off from trade, i.e. efficiency cannot be improved from trade. $\square$

7.2 Lemma 2

Proof. We characterize the interbank market at $t = 2$, taking as given some bid-ask spreads (P^b, P^a) corresponding to spread $\hat{\delta}$ used by dealers in the market making stage at $t = 1$. In the interbank market, three potential types of dealers arise – short dealers, long dealers, and neutral dealers.

First, note that given a symmetric strategy with respect to $\hat{\delta}$, each type of dealer – short, long, and neutral – holds identical beliefs. Given symmetric bid-ask spread strategy $\hat{\delta}$, a nonzero measure of all types of dealers exist since $\hat{\delta} \in (0, D - x)$.

Let $E_\theta[\cdot]$ be used to denote the expected values given beliefs of a dealer of type $\theta = s, n, l$, where $\theta = s$ for a short dealer, $\theta = n$ for a dealer without a trade at $t = 1$, and $\theta = l$ for a long dealer. We can compute the expected value of v conditional on each type of dealers' beliefs as a function of (P^b, P^a):

$$E_s[v] = \frac{\bar{v} - x + D - P^a}{2(\bar{v} + D - P^a)} \cdot (\bar{v} - x) + \frac{\bar{v} + x + D - P^a}{2(\bar{v} + D - P^a)} \cdot (\bar{v} + x) \tag{38}$$

$$E_n[v] = \frac{1}{2} \cdot (\bar{v} - x) + \frac{1}{2} \cdot (\bar{v} + x) \tag{39}$$

$$E_l[v] = \frac{P^b - \bar{v} + x + D}{2(P^b - \bar{v} + D)} \cdot (\bar{v} - x) + \frac{P^b - \bar{v} - x + D}{2(P^b - \bar{v} + D)} \cdot (\bar{v} + x) \tag{40}$$

We can simplify the above equations by substituting the expressions for (P^b, P^a), to get $E_s[v] = \bar{v} + x \cdot \frac{2x}{D-\hat{\delta}}$, $E_n[v] = \bar{v}$, and $E_l[v] = \bar{v} - x \cdot \frac{2x}{D-\hat{\delta}}$. We guess and verify that the interdealer equilibrium strategies of dealers are such that:

- short dealers make offer (buy, P_s^d) only accepted by long dealers;
- long dealers make offer $(sell, P_l^d)$ only accepted by short dealers;
- neutral dealers chooses not to trade, i.e. $\sigma_n =$ no trade.

Correspondingly, let beliefs be such that any buy offer (i.e. $\sigma = buy$) is made by a short dealer and any sell offer (i.e. $\sigma = sell$) by a long dealer. Suppose that this is the case. Recall that for any randomly matched pair of dealers, each dealer gets the opportunity to make an ultimatum offer to buy or sell with equal probability. Then, according to the candidate strategy, a short dealer makes an offer to buy at some price that is (only) accepted by a long dealer, and a long dealer makes an offer to sell at some price that is (only) accepted by a short dealer. This implies that an interdealer trade occurs between a pair of dealers *only* when a short dealer and a long dealer is matched. In addition, it implies that an offer to buy or sell reveals the type of a dealer, and the dealer that makes the offer anticipates that conditional on his offer being accepted, the matched dealer with the opposite net position. The beliefs of a pair of dealers are identical conditional on successfully trading, since:

$$P(v = \bar{v} - x | \text{short dealer matches with long dealer}) \tag{41}$$

$$= \frac{(P^b - \bar{v} + x + D)(\bar{v} - x + D - P^a)}{(P^b - \bar{v} + x + D)(\bar{v} - x + D - P^a) + (P^b - \bar{v} - x + D)(\bar{v} + x + D - P^a)} \tag{42}$$

$$= \frac{1}{2} = P(v = \bar{v} + x | \text{short dealer matches with long dealer}) \tag{43}$$

Given these beliefs, a short dealer (long dealer) makes an offer that are equal to the reservation price of a long dealer (short dealer), which is equal to $\bar{v} - \Delta$ ($\bar{v} + \Delta$).

We must verify that deviations are not profitable. There are three classes of deviations: (1) neutral dealers accepting equilibrium offers from other dealers, (2) neutral dealers making offers to other dealers and (3) long or short dealers making offers at prices other than the proposed equilibrium prices.

Step 1. Show that neutral dealers have no incentive to deviate by accepting an equilibrium offer from the dealer they are matched with in stage 2.

Part 1A. Suppose a neutral dealer receives and offer to buy at price $\bar{v} + \Delta$ from a dealer she is matched with in stage 2. Given our specified equilibrium strategies, she believes the offer is coming from a long dealer and hence her expectation of v, the true value, becomes $E_l[v] = \bar{v} - \frac{x^2}{D - \hat{\delta}} < \bar{v}$. If she accepted the offer to buy, her expected payoff from accepting the offer would be $\bar{v} - \frac{x^2}{D-\hat{\delta}} - \Delta - [\bar{v} + \Delta] = -[\frac{x^2}{D-\hat{\delta}} + 2\Delta] < 0$. So the neutral dealer will not accept an offer to buy at the price $\bar{v} + \Delta$.

Part 1B. Suppose a neutral dealer receives and offer to sell at price $\bar{v} - \Delta$ from a dealer she is matched with in stage 2. Given our specified equilibrium strategies, she believes the offer is coming from a short dealer and hence her expectation of v, the true value, becomes $E_s[v] = \bar{v} + \frac{2x^2}{2(D-\hat{\delta})} > \bar{v}$. If she accepted the offer to sell, her expected payoff from accepting the offer would be $\bar{v} - \Delta - [\bar{v} + \frac{2x^2}{2(D-\hat{\delta})} + \Delta] = -[\frac{x^2}{D-\hat{\delta}} + 2\Delta] < 0$. So the neutral dealer will not accept an offer to sell at the price $\bar{v} - \Delta$.

Step 2. Show that no neutral dealer has an incentive to make a buy or sell offer to a dealer they are matched with in stage 2.

A neutral dealer has no incentive to make an offer to a counterparty. To see this, consider a deviation in which a neutral dealer makes an offer to sell. First consider when a neutral dealer offers to sell at $P' = \bar{v} + \Delta$. Given equilibrium beliefs, the offer is accepted if the neutral dealer is matched to a short dealer, which yields the following payoff:

$$P' - E_n[v|\text{neutral dealer sells to short dealer}] - \Delta \tag{44}$$
$$= \bar{v} - E_n[v|\text{neutral dealer sells to short dealer}] < 0 \tag{45}$$

Since conditional on matching with a short dealer, the conditional expected value of v is greater than $\bar{v}$, the deviation is not profitable. Second, consider when a neutral dealer offers some $P' < \bar{v} + \Delta$, which is only potentially accepted by a short dealer. Given off-equilibrium beliefs, a short dealer's valuation of the asset conditional on receiving an offer form the neutral dealer is equal to $\bar{v}$. Hence, there does not exist any P' such that the neutral dealer makes a positive profit from making an offer. A symmetric arguments holds for deviations by the neutral dealer to make a buying offer.

Step 3. Show that no long or short dealer would want to deviate by making an offer at a price different than the proposed equilibrium price.

Part 3A. Consider a long dealer (who in equilibrium offers to sell at a price $\bar{v} + \Delta$ that is accepted only by short dealer). Recall that we assume dealers have beliefs that are triggered by *any* sell offer, not just an offer at the equilibrium price, and these beliefs are that the dealer making the sell offer is a long dealer. So a deviation to a price $P' > \bar{v} + \Delta$ would be rejected by a short dealer: it would be deemed unprofitable given updated beliefs that the asset's true value is $\bar{v}$. And it would be rejected by a neutral dealer who would have even more pessimistic beliefs about the asset value. What about a price $P'' < \bar{v} + \Delta$? By offering a price less than $\bar{v} + \Delta$ the long dealer would be giving up some surplus when matched with a short dealer. The question is whether she can recoup that when matched with a neutral dealer. However, in order to get a neutral dealer to accept a sell offer she must offer a price of $E_l[v] - \Delta = \bar{v} - \frac{x^2}{D-\hat{\delta}} - \Delta$, but this is exactly the long dealer's expected payoff is if she does not sell the asset in stage 2. So, by offering a price less than $\bar{v} + \Delta$ the long dealer loses surplus when matched with a short dealer and makes no additional surplus when matched with a neutral dealer. So this deviation is not profitable.

Part 3B. Consider a short dealer (who in equilibrium offers to buy at a price $\bar{v} - \Delta$ that is accepted only by long dealer). A similar argument to part 3A shows that no deviation in price is profitable.

□

7.3 Corollary 1

Proof. QED. □

7.4 Lemma 3

Proof. QED. □

7.5 Theorem 1

Proof. We show existence of a symmetric equilibria, in which dealers make markets at $t = 1$ using identical bid-ask spreads $(P^{b**}, P^{a**}) = (\bar{v} - \delta^{**}, \bar{v} + \delta^{**})$ for some $\delta^{**} \in (0, D - x)$ when $x < x^{trade}$ for some cutoff x^{trade}.

The solution is obtained by solving backwards. We proceed in three steps. First, we characterize the optimal interdealer trading strategy of a dealer i who chose some market-making strategy δ_i at $t = 1$, taking as given that all other dealers chose some market-making strategy $\hat{\delta} \in (0, D - x)$ in market making stage at $t = 1$ and follow interdealer trading strategies specified in Lemma 2. Second, by backward induction, we characterize an individual dealer's expected payoff at $t = 1$ who chooses market-making strategy δ_i taking as given that all other dealers choose $\hat{\delta}$. Third, we determine the conditions under which there exists some $\delta^{**} \in (0, D - x)$ such that an individual dealer maximizes his expected payoff by choosing $\delta_i = \delta^{**}$ conditional on all other dealers choosing δ^{**}.

Step 1. We begin by characterizing dealers' strategies interbank market at $t = 2$. Following Lemma 2, consider the following set of candidate equilibrium interdealer strategies of dealers:

1. short dealers make offer $(buy, \bar{v} - \Delta)$ and only accept offers $(sell, P^d)$ for $P^d \leq \bar{v} + \Delta$;
2. long dealers make offer $(sell, \bar{v} + \Delta)$ and only accept offers (buy, P^d) for $P^d \geq \bar{v} - \Delta$;
3. neutral dealers do not make any offers, and reject all offers.

Correspondingly, let dealers' beliefs be such that any buy offer (i.e. $\sigma = buy$) is made by a short dealer and any sell offer (i.e. $\sigma = sell$) by a long dealer. We must verify that given that all other dealers choose some market-making strategy $\hat{\delta} \in (0, D - x)$, an individual dealer i does not find it profitable to deviate to $\delta_i \neq \hat{\delta}$.

First, consider when a dealer selects some $\delta_i > \hat{\delta}$, such that the dealer is better informed than other dealers. Given dealers' beliefs, dealer i's optimal interdealer offer and trading strategies are to mimic other dealers' equilibrium strategies. As an offering dealer, dealer i maximizes conditional profits by offering $(sell, \bar{v} + \Delta)$ and $(buy, \bar{v} - \Delta)$ are the maximum and minimum prices at which a receiving dealer is willing to buy or sell, respectively. Similarly, dealer i's optimal interdealer trading strategies is to mimic other dealers' equilibrium strategies, since conditional on being a short or long dealer, the equilibrium strategy to accept

$(sell, \bar{v}+\Delta)$ as a short dealer, and accept $(buy, \bar{v}-\Delta)$ as a long dealer. Given this, the marginal payoff from interdealer trading $V_\theta(\delta_i, \hat{\delta})$ of dealer i at $t=2$ is given by:

$$V_\theta(\delta_i, \hat{\delta}) = \left(\sum_v P(v|\theta) P(\text{match with opposite dealer}|v,\theta)\right)(\bar{v} - E_i[v|trade]) \tag{46}$$

$$+ \frac{1}{2}\left(\sum_v P(v|\theta) P(\text{match with opposite dealer}|v,\theta)\right) 2\Delta, \tag{47}$$

Here, the probability that dealer i matches with an opposite dealer conditional on becoming a long (or short) dealer is given by:

$$\sum_v P(v|\theta) P(\text{match with opposite dealer}|v,\theta) = \begin{cases} \frac{D+x-\delta_i}{2(D-\delta_i)}\frac{D-x-\hat{\delta}}{2D} + \frac{D-x-\delta_i}{2(D-\delta_i)}\frac{D+x-\hat{\delta}}{2D} & \text{if } \delta_i \leq D-x \\ 1 \cdot \frac{D-x-\hat{\delta}}{2D} + 0 \cdot \frac{D+x-\hat{\delta}}{2D} & \text{if } \delta_i > D-x \end{cases} \tag{48}$$

$$= \begin{cases} \frac{D^2-x^2+\delta_i\hat{\delta}-D(\delta_i+\hat{\delta})}{2D(D-\delta_i)} & \text{if } \delta_i \leq D-x \\ \frac{D-x-\hat{\delta}}{2D} & \text{if } \delta_i > D-x \end{cases} \tag{49}$$

We can explicitly characterize the first component of the marginal payoff in Equation 46:

$$\bar{v} - E[v|trade] = \begin{cases} \bar{v} - \left[\bar{v} + \frac{(D-x-\delta_i)(D+x-\hat{\delta})-(D+x-\delta_i)(D-x-\hat{\delta})}{(D-x-\delta_i)(D+x-\hat{\delta})+(D+x-\delta_i)(D-x-\hat{\delta})}x\right] & \text{if } \delta_i \leq D-x \\ \bar{v} - [\bar{v} - x] & \text{if } \delta_i > D-x \end{cases} \tag{50}$$

$$= \begin{cases} \frac{\left(\delta_i-\hat{\delta}\right)x}{D^2-x^2+\delta_i\hat{\delta}-D(\delta_i+\hat{\delta})}x & \text{if } \delta_i \leq D-x \\ x & \text{if } \delta_i > D-x \end{cases} \tag{51}$$

Pluggin the above in, we find the following expression for $V_\theta(\delta_i, \hat{\delta})$:

$$V_\theta(\delta_i, \hat{\delta}) = \begin{cases} \frac{D^2-x^2+\delta_i\hat{\delta}-D(\delta_i+\hat{\delta})}{2D(D-\delta_i)}\left(\frac{\left(\delta_i-\hat{\delta}\right)x}{D^2-x^2+\delta_i\hat{\delta}-D(\delta_i+\hat{\delta})}x + \Delta\right) & \text{if } \delta_i \leq D-x \\ \frac{D-x-\hat{\delta}}{2D}(x+\Delta) & \text{if } \delta_i > D-x \end{cases} \tag{52}$$

The remaining case is to consider when a dealer selects some $\delta_i < \hat{\delta}$, such that the dealer is less informed than other dealers. In this case, deviations in the interdealer offer strategies are profitable. The optimal trading strategy involves rejecting any equilibrium offer, since given $\delta_i < \hat{\delta}$ implies that the dealer has a lower conditional expected value of v than a dealer that chose $\hat{\delta}$ if a short dealer, and a higher conditional expected value of v than a dealer that chose

$\hat{\delta}$ if a long dealer. Second, mimicking the equilibrium offer strategy is more profitable than no trade if the gains from netting outweigh the losses associated with negative information rents. This holds true if the following inequality holds:

$$\Delta > -\frac{\left(\delta_i - \hat{\delta}\right) x}{D^2 - x^2 + \delta_i \hat{\delta} - D(\delta_i + \hat{\delta})} x \tag{53}$$

where the LHS comes from Equation 51. Now, we can express $V_\theta(\delta_i, \hat{\delta})$ conditional on whether Condition 53 holds or not. If Condition 53 does not hold, then the dealer rejects all offers and does not make any offer. Hence, $V_\theta(\delta_i, \hat{\delta}) = 0$. If Condition 53 holds, then the expression for $V_\theta(\delta_i, \hat{\delta})$ follows from the first case of Equation 52:

$$V_\theta(\delta_i, \hat{\delta}) = \begin{cases} \frac{D^2 - x^2 + \delta_i \hat{\delta} - D(\delta_i + \hat{\delta})}{2D(D - \delta_i)} \left(\frac{\left(\delta_i - \hat{\delta}\right) x}{D^2 - x^2 + \delta_i \hat{\delta} - D(\delta_i + \hat{\delta})} x + \Delta \right) & \text{if Condition 53 holds} \\ 0 & \text{otherwise} \end{cases} \tag{54}$$

Step 2. Now that we have fully characterized a dealer i's optimal interdealer strategy conditional on deviating to $\delta_i \neq \hat{\delta}$, we can backward induct, and fully characterize the expected payoff at $t = 1$ conditional on deviating. The generic $t = 1$ expected payoff of dealer i is given by:

$$\Pi_i(\delta_i, \hat{\delta}) = \underbrace{P(\gamma_j(P^b, P^a) = accept|\delta_i) \cdot (\bar{v} + \delta_i - E[v|\delta_i] - \Delta)}_{\equiv A,\text{ autarky payoff}} + \underbrace{\sum_\theta P(\theta_i = \theta|\delta_i) \cdot V_\theta(\delta_i, \hat{\delta})}_{\equiv B,\text{ interdealer payoff}} \tag{55}$$

Since the autarky payoff A is independent of $\hat{\delta}$ or interdealer payoffs, we can express in conditional on whether $\delta_i \leq D - x$ or $\delta_i > D - x$:

$$A = \begin{cases} 2 \cdot \frac{D - \delta_i}{2D} \left(\delta_i - \frac{x}{D - \delta_i} \cdot x - \Delta \right) & \text{if } \delta_i \leq D - x \\ 2 \cdot \frac{D + x - \delta_i}{4D} \left(\delta_i - x - \Delta \right) & \text{if } \delta_i > D - x \end{cases} \tag{56}$$

As we did in the previous step, we split the analysis into the cases in which $\delta_i > \hat{\delta}$ and $\delta_i < \hat{\delta}$. First, consider when $\delta_i > \hat{\delta}$. Incorporating our earlier expression of $V_\theta(\delta_i, \hat{\delta})$, the interdealer payoff B is given by:

$$B = \begin{cases} 2 \cdot \frac{D-\delta_i}{2D} \left(\frac{\left(\delta_i - \hat{\delta}\right) x}{2D(D-\delta_i)} x + \frac{D^2 - x^2 + \delta_i \hat{\delta} - D(\delta_i + \hat{\delta})}{2D(D-\delta_i)} \Delta \right) & \text{if } \delta_i \leq D - x \\ \frac{D+x-\delta_i}{2D} \frac{D-x-\hat{\delta}}{2D} (x + \Delta) & \text{if } \delta_i > D - x \end{cases} \tag{57}$$

Together, we can express the expected payoff at $t = 1$ for dealer i given some market-making strategy $\delta_i > \hat{\delta}$:

$$\Pi_i(\delta_i, \hat{\delta}) = \begin{cases} \frac{D-\delta_i}{D} \left(\delta_i - \frac{x}{D-\delta_i} \cdot x - \Delta \right) + \frac{D-\delta_i}{D} \left(\frac{\left(\delta_i - \hat{\delta}\right) x}{2D(D-\delta_i)} x + \frac{D^2 - x^2 + \delta_i \hat{\delta} - D(\delta_i + \hat{\delta})}{2D(D-\delta_i)} \Delta \right) & \text{if } \delta_i \leq D - x \\ \frac{D+x-\delta_i}{2D} \left(\delta_i - x - \Delta \right) + \frac{D+x-\delta_i}{2D} \frac{D-x-\hat{\delta}}{2D} (x + \Delta) & \text{if } \delta_i > D - x \end{cases} \tag{58}$$

Second, consider when $\delta_i < \hat{\delta}$. Incorporating our earlier expression of $V_\theta(\delta_i, \hat{\delta})$, the interdealer payoff B is given by:

$$B = \max \left\{ 2 \cdot \frac{D-\delta_i}{2D} \left(\frac{\left(\delta_i - \hat{\delta}\right) x}{2D(D-\delta_i)} x + \frac{D^2 - x^2 + \delta_i \hat{\delta} - D(\delta_i + \hat{\delta})}{2D(D-\delta_i)} \Delta \right), 0 \right\} \tag{59}$$

Together, we can express the expected payoff at $t = 1$ for dealer i given some market-making strategy $\delta_i < \hat{\delta}$:

$$\Pi_i(\delta_i, \hat{\delta}) = \frac{D-\delta_i}{D} \left(\delta_i - \frac{x}{D-\delta_i} \cdot x - \Delta \right) \tag{60}$$

$$+ \max \left\{ 2 \cdot \frac{D-\delta_i}{2D} \left(\frac{\left(\delta_i - \hat{\delta}\right) x}{2D(D-\delta_i)} x + \frac{D^2 - x^2 + \delta_i \hat{\delta} - D(\delta_i + \hat{\delta})}{2D(D-\delta_i)} \Delta \right), 0 \right\}. \tag{61}$$

Step 3. Given that we have a characterization of a dealer i's expected payoff from choosing δ_i, it suffices to determine the conditions under which a symmetric equilibrium with interdealer markets exist, and correspondingly dealers' equilibrium strategies.

We do so by first identifying the local optimal $\delta_i \in (\hat{\delta}, D - x)$. Taking the first order condition of Equation 58 with respect to δ_i yields the following:

$$1 - \frac{2\delta_i}{D} + \frac{x^2}{2D^2} + \Delta \cdot \frac{D + \hat{\delta}}{2D^2} = 0 \tag{62}$$

$$2D^2 + x^2 + \Delta D = 4D\delta_i - \Delta\hat{\delta} \tag{63}$$

$$\frac{2D^2 + x^2 + \Delta D}{4D} + \frac{\Delta}{4D}\hat{\delta} = \delta_i \tag{64}$$

Imposing symmetry by setting $\hat{\delta} = \delta_i$, we obtain:

$$\delta^{**} = \frac{2D^2 + x^2 + \Delta D}{4D - \Delta} \tag{65}$$

In order for Equation 65 to be an equilibrium solution, the expected payoff from $\delta^{**} = \frac{2D^2+x^2+\Delta D}{4D-\Delta}$ must be greater than deviating to any $\delta_i \in (0, D + x)$. Consider any $\delta_i \in (0, \delta^{**})$. First, note that for any δ_i such that Condition 53 holds, δ^{**} is the optimum, since in this case, payoff function over $\delta_i \in (0, D - x)$ is continuous and differentiable. Second, note that if $\frac{\partial B}{\partial \delta_i} < 0$, then $\delta^{**} < \frac{D+\Delta}{2}$, which implies that any $\delta_i < \delta^{**}$ yields a lower expected payoff. This leaves the cases in which $\frac{\partial B}{\partial \delta_i} > 0$ and Condition 53 is violated. Note that

$$\frac{\partial B}{\partial \delta_i} = \frac{x^2}{2D^2} - \frac{D - \delta^{**}}{2D^2}\Delta \tag{66}$$

Organizing the above condition, we get that $\frac{\partial B}{\partial \delta_i} > 0$ holds if and only if:

$$2x^2 > \Delta(D - \Delta) \tag{67}$$

This condition holds as long as $x > \sqrt{\frac{\Delta(D-\Delta}{2}}$. Suppose that it holds. In this case, a dealer i's payoff is:

$$\frac{D - \delta_i}{D}(\delta_i - \Delta) - \frac{x^2}{D} < \frac{D - \delta_i}{D}(\delta_i - \Delta) - \frac{\Delta(D - \Delta)}{2D} \tag{68}$$

$$< \frac{(D - \Delta)^2}{4D} - \frac{\Delta(D - \Delta)}{2D} \tag{69}$$

$$= \frac{D^2 + \Delta(3\Delta - 4D)}{4D} \tag{70}$$

For any $\Delta \in \left(\frac{D}{\sqrt{2}+1}, D\right)$, the above is less than 0. Hence, there does not exist any profitable deviation to some $\delta_i < \delta^{**}$. Next, consider deviations to some $\delta_i \in (D - x, D + x)$. Recall, the expected payoff from choosing $\delta_i \in (D-x, D+x)$, given by Equation 58 is:

$$\frac{D+x-\delta_i}{2D}(\delta_i - x - \Delta) + \frac{D+x-\delta_i}{2D}\frac{D-x-\hat{\delta}}{2D}(x+\Delta) \tag{71}$$

Note that as $x \to 0$, the probability of successfully trading in the market making stage approaches 0, for any $\delta' \in (D-x, D+x)$. As $x \to 0$, for any partially revealing market-making strategy δ', profits approach:

$$\frac{(D-\delta_i)\delta_i}{D} - \Delta \cdot \frac{(D+\delta^{**})(D-\delta_i)}{2D^2} > 0 \tag{72}$$

This implies that there exists some cutoff value x^{trade} such that for x^{trade}, deviation to $\delta_i > D-x$ is not profitable. Hence, we show that there exists a equilibrium with interdealer trading in which dealers employ symmetric market-making offers $\delta^{**} = \frac{2D^2+x^2+\Delta D}{4D-\Delta}$ if $x < x^{trade}$.

□

7.6 Theorem 2

Proof. We show existence of a symmetric equilibrium, in which dealers choose fully revealing market making strategies, i.e. choose market making offers $\delta^* \in (D-x, D+x)$ when $x > x^{one}$ for some cutoff x^{seg}.

The solution is obtained by solving backwards. As a result of Lemma 3, no interdealer trading occurs if all dealers choose some $\delta_i > D-x$, since there are no gains from trade between dealers. Conditional on no interdealer trading, all dealers are indifferent between any set of interdealer strategy. Hence, consider when candidate equilibrium offer strategies are $\sigma_\theta =$ no trade for $\theta = l, n, s$, and dealers' interdealer trading strategies $\delta_\theta = reject$ for $\forall\theta$.

Conditional on no interdealer trading, the marginal interdealer payoff $V_\theta = 0$ for any δ_i. Hence, the expected payoff of dealer i is solely characterized by the autarky payoff from Equation 58 with $\delta_i > D-x$, which can be written as:

$$\frac{D+x-\delta_i}{2D}(\delta_i - x - \Delta). \tag{73}$$

Maximizing with respect to δ_i, we obtain:

$$\delta^* = x + \frac{D+\Delta}{2}. \tag{74}$$

It suffices to show that $x + \frac{D+\Delta}{2} > D-x$ and any deviation to some $\delta_i < D-x$ is not profitable. The first holds when $x > \frac{D-\Delta}{4}$. Next, consider a deviation to some $\delta_i < D - x$. The expected payoff of dealer i is characterized by the autarky payoff from Equation 58 with $\delta_i < D - x$, which is given by:

$$\frac{D-\delta_i}{D}\left(\delta_i - \frac{x^2}{D-\delta_i} - \Delta\right) \tag{75}$$

Which is maximized at $\delta' = \min\left\{D - x, \frac{D+\Delta}{2}\right\}$. A deviation is not profitable as long as:

$$\frac{D+x-\delta^*}{2D}(\delta^* - x - \Delta) > \frac{D-\delta'}{D}\left(\delta' - \frac{x^2}{D-\delta'} - \Delta\right) \tag{76}$$

When $\delta' = D - x$, the deviating payoff is less than that obtained by δ^*, since it is Equation 73 is equal to Equation 73 for $\delta_i = D - x$. Hence, it suffices to check when $\delta' = \frac{D+\Delta}{2}$. Plugging in our expressions for δ^* and δ' be obtain:

$$\frac{D-\Delta}{4D}\left(\frac{D-\Delta}{2}\right) > \frac{D-\Delta}{2D}\left(\frac{D-\Delta}{2}\right) - \frac{x^2}{D} \tag{77}$$

This inequality holds as long as $x^2 > \frac{(D-\Delta)^2}{8}$. Hence, there exists some $x^{seg} \leq \sqrt{\frac{(D-\Delta)^2}{8}}$ such that for $x > x^{seg}$, there exists an equilibrium with segmentation with $\delta^* = x + \frac{D+\Delta}{2}$. $\square$

7.7 Theorem 3

Proof. It suffices to show that for $\Delta \in (\frac{D}{1+2\sqrt{2}}, D)$, $x^{seg} < x^{trade}$. Note that $\Delta > x^{seg}$. We show that for any $\Delta > x^{seg}$, the condition is satisfied.

For $\delta(\delta^{**}) = x + \frac{D+\Delta}{2} - \frac{(x+\Delta)(D-x-\delta^{**})}{2D}$, the condition is given by:

$$\frac{(D-\delta^{**})(\delta^{**}-\Delta)}{D} - \frac{x^2}{D} + \frac{(D+x-\delta^{**})(D-x-\delta^{**})}{2D^2}\Delta \tag{78}$$

$$> \frac{(D+x-\delta)(\delta-x-\Delta)}{2D} + \frac{(D+x-\delta)(D-x-\delta^{**})}{4D^2}(x+\Delta) \tag{79}$$

Since $\Delta > x^{seg}$,

$$\frac{(\delta-\delta^{**})(D-x-\delta^{**})}{2D^2}\Delta > \frac{(D+x-\delta)(\delta-x-\Delta)}{2D} - \frac{(D-\delta^{**})(\delta^{**}-\Delta)}{D} + \frac{x^2}{D} \tag{80}$$

Substituting in $\delta(\delta^{**})$,

$$\frac{(\delta-\delta^{**})(D-x-\delta^{**})}{2D^2}\Delta > \frac{\left(\frac{D-\Delta}{2}\right)^2}{2D} + \frac{\left(\frac{(D-x-\delta^{**})(x+\Delta)}{2D}\right)(-D-x+\delta-x-\Delta)}{2D} - \frac{(D-\delta^{**})(\delta^{**}-\Delta)}{D} + \frac{x^2}{D} \tag{81}$$

Reorganizing the inequality,

$$\frac{(D-\delta^{**})(\delta^{**}-\Delta)}{D} - \frac{x^2}{D} + \frac{(D+2x+\Delta-\delta^{**})(D-x-\delta^{**})}{2D^2}\Delta > \frac{x^2}{D} - \frac{(D-x-\delta^{**})(x+\Delta)}{2D^2} \tag{82}$$

Note, since the RHS is less than payoff conditional on deviating to $\delta = \frac{D+\Delta}{2}$ without interdealer trading, and the LHS is the payoff conditional on δ^{**}, the inequality strictly holds.

This implies that an equilibrium with interdealer trading always exists at threshold x^{seg}. Hence, for $\Delta > \frac{D}{1+2\sqrt{2}}$, there exists a nonempty interval of x for which both types of equilibria exist. □

7.8 Corollary 2

Proof. It is easy to verify that $\mu(\delta^{**})$ and $\mu(\delta^{*})$ increase in D, weakly decrease in x, and decrease in Δ for both one and two-sided market making equilibriums. □

7.9 Corollary 3

Proof. QED. □

7.10 Corollary 4

Proof. QED. □

7.11 Lemma 4

Proof. Given (P^b, P^a), the net demand of the asset between dealers is sufficient to infer v. Hence, observing the vector of positions of dealers perfectly reveals whether $\bar{v}+x$ or $\bar{v}-x$. □

7.12 Corollary 5

Proof. QED. □

7.13 Theorem 4

The proof follows the proof of Theorem 1. While information shared at the clearing stage means that all dealers know the true value of the asset in stage 2, during stage 1 it has an expected value of $\bar{v}$. Hence our symmetric equilibrium prediction for the bid-ask spread announced by all dealers is still $(P^b, P^a) = (\bar{v} - \delta, \bar{v} + \delta)$ for some δ still to be determined.

We guess and verify that the interdealer equilibrium strategy of the dealers are such that:

1. short dealers make the offer (buy, P_s^d), which is only accepted by long dealers
2. long dealers make the offer $(sell, P_l^d)$, which is only accepted by short dealers
3. neutral dealers choose not to trade, i.e., $\sigma_n =$ no trade.

Stage 2 beliefs regarding the type (σ) of dealer making a buy or sell offer are irrelevant in the fully integrated information model since they are not needed to form updated beliefs on the asset's true value (which is common knowledge in Stage 2). Stage 2 offers by short dealers to buy or by long dealers to sell are defined by $P_s^d = v - \Delta$ and $P_l^d = v + \Delta$, where $v \in \{\bar{v} - x, \bar{v} + x\}$ is the (commonly known) true value of the asset.

As in the proof of Theorem 1, we must verify that deviations are not profitable. There are three classes of deviations: (1) neutral dealers accepting equilibrium offers from other dealers, (2) neutral dealers making offers to other dealers and (3) long or short dealers making offers at prices other than the proposed equilibrium prices.

Step 1. Show that neutral dealers have no incentive to deviate by accepting an equilibrium offer from the dealer they are matched with in stage 2.

Part 1A. Suppose a neutral dealer receives and offer to buy at price $v + \Delta$ from a dealer she is matched with in stage 2. If she accepted the offer to buy, her payoff from accepting the offer would be $[v - \Delta] - [v + \Delta] = -2\Delta < 0$. So the neutral dealer will not accept an offer to buy at the price $v + \Delta$.

Part 1B. Suppose a neutral dealer receives and offer to sell at price $v - \Delta$ from a dealer she is matched with in stage 2, where $v \in \{\bar{v} - x, \bar{v} + x\}$ is the true value of the asset. If she accepted the offer to sell, her payoff from accepting the offer would be $[v - \Delta] - [v + \Delta] = -2\Delta < 0$. So the neutral dealer will not accept an offer to sell at the price $v - \Delta$.

Step 2. Show that no neutral dealer has an incentive to make a buy or sell offer to a dealer they are matched with in stage 2.

Part 2A. Suppose a neutral dealer makes an offer to sell at $v + \Delta$. In equilibrium, this is only accepted by a short dealer and in this case the neutral dealer's profit from the deviation is 0 (since she will have to buy at v and pay Δ to cover her short sale).

Suppose a neutral dealer makes an offer to sell at some $P' < v + \Delta$. If she is successful she will have to pay v to obtain the asset and pay Δ. So this deviation is never profitable.

Suppose a neutral dealer makes an offer to sell at some $P'' > v + \Delta$. Accepting such an offer is not beneficial for any type of trader and hence this deviation never ends up being profitable for the neutral dealer.

Part 2B. Suppose a neutral dealer makes an offer to buy at $v - \Delta$. In equilibrium, this is only accepted by a long dealer and in this case the neutral dealer's profit from the deviation is 0 (since she will have to sell at v and pay Δ to unwind her long buy).

Suppose a neutral dealer makes an offer to buy at some $P' > v - \Delta$. If she is successful she will sell at v and pay Δ. So this deviation is never profitable.

Suppose a neutral dealer makes an offer to buy at some $P'' < v - \Delta$. Accepting such an offer is not beneficial for any type of trader and hence this deviation never ends up being profitable for the neutral dealer.

Step 3. Show that no long or short dealer would want to deviate by making an offer at a price different than the proposed equilibrium price.

Part 3A. Consider a long dealer (who in equilibrium offers to sell at a price $v + \Delta$ that is accepted only by short dealer). A deviation to a price $P' > v + \Delta$ would be rejected by a short dealer since $v + \Delta$ is the amount she would have to pay if she failed to buy the asset in stage 2. And it would be rejected by a neutral dealer since her payoff from accepting the offer would be $v - \Delta - P' < 0$ (because if the neutral dealer buys the asset in stage 2 she will sell it at v and have to pay Δ).What about a price $P'' < v + \Delta$? By offering a price less than $v + \Delta$ the long dealer would be giving up some surplus when matched with a short dealer. The question is whether she can recoup that when matched with a neutral dealer. However, in order to get a neutral dealer to accept a sell offer she must offer a price of $v - \Delta$, but this is exactly the long dealer's payoff is if she does not sell the asset in stage 2. So, by offering a price less than $v + \Delta$ the long dealer loses surplus when matched with a short dealer and makes no additional surplus when matched with a neutral dealer. So this deviation is not profitable.

Part 3B. Consider a short dealer (who in equilibrium offers to buy at a price $v - \Delta$ that is accepted only by long dealer).

A deviation to a price $P' < v - \Delta$ would be rejected by a long dealer since $v - \Delta$ is the amount she would receive if she failed to sell the asset in stage 2. And it would be rejected by a neutral dealer since her payoff from accepting the offer would be $P' - [v - \Delta] < 0$ (because if the neutral dealer sells the asset in stage 2 she will have to buy it at v and have to pay Δ). What about a price $P'' > v - \Delta$? By offering a price greater than $v - \Delta$ the short dealer would be giving up some surplus when matched with a long dealer. The question is whether she can recoup that when matched with a neutral dealer. However, in order to get a neutral dealer to accept a buy offer she must offer a price of $v + \Delta$, but this is exactly the long dealer's payoff is if she does not buy the asset in stage 2. So, by offering a price greater than $v + \Delta$ the short dealer loses surplus when matched with a long dealer and makes no additional surplus when matched with a neutral dealer. So this deviation is not profitable.

To complete the proof, we need to pin down the value for δ that defines the stage 1 bid-ask spread (labeled δ^{***} in the statement of the theorem). We begin by writing out the expected profit of a dealer as a function of the proposed equilibrium choice of δ and the deviation δ'.

We start by noting that

$$\Pi_s(\delta',\delta) = P(s-l|s,\delta') * \left[P(v=\bar{v}-x|s-l/s,\delta') * \left(\frac{1}{2}(\bar{v}+\delta'-(\bar{v}-x+\Delta)) + \frac{1}{2}(\bar{v}+\delta'-(\bar{v}-x-\Delta))\right)\right] \tag{83}$$

$$+ P(s-l|s,\delta') * \left[P(v=\bar{v}+x|s-l/s,\delta') * \left(\frac{1}{2}(\bar{v}+\delta'-(\bar{v}+x+\Delta)) + \frac{1}{2}(\bar{v}+\delta'-(\bar{v}+x-\Delta))\right)\right] \tag{84}$$

$$+ P(s-n|s,\delta')(\bar{v}+\delta'-(E[v|s-n/s,\delta']+\Delta)] \tag{85}$$

$$= \left(\frac{D-\delta}{2D} - \frac{x^2}{2D(D-\delta')}\right) \tag{86}$$

$$* \left[\frac{(-\delta'-x+D)(-\delta+x+D)(\delta'+x)+(-\delta'+x+D)(-\delta-x+D)(\delta'-x)}{(-\delta'-x+D)(-\delta+x+D)+(-\delta'+x+D)(-\delta-x+D)}\right] \tag{87}$$

$$+ \left(\frac{D+\delta}{2D} + \frac{x^2}{2D(D-\delta')}\right) * \left(\delta' - x^2\frac{\delta-\delta'+2D}{(D-\delta')(D+\delta)+x^2} - \Delta\right) \tag{88}$$

$$= \left(\frac{D-\delta}{2D} - \frac{x^2}{2D(D-\delta')}\right) * \left[\delta' + x^2\frac{\delta-\delta'}{(D-\delta')(D-\delta)-x^2}\right] \tag{89}$$

$$+ \left(\frac{D+\delta}{2D} + \frac{x^2}{2D(D-\delta')}\right) * \left(\delta' - x^2\frac{\delta-\delta'+2D}{(D-\delta')(D+\delta)+x^2} - \Delta\right) \tag{90}$$

$$= \delta' - \frac{x^2}{D-\delta'} - \left(\frac{D+\delta}{2D} + \frac{x^2}{2D(D-\delta')}\right)\Delta. \tag{91}$$

Profits conditional on being a short dealer or a long dealer are the same. So, the expected profit of a dealer at the beginning of stage 1 is

$$\Pi_i(\delta',\delta) = 2\frac{(D-\delta')}{2D}\left(\delta' - \frac{x^2}{D-\delta'} - \left(\frac{D+\delta}{2D} + \frac{x^2}{2D(D-\delta')}\right)\Delta\right) \tag{92}$$

$$= \frac{(D-\delta')\delta'}{D} - \frac{x^2}{D} - \frac{(D-\delta')(D+\delta)+x^2}{2D^2}\Delta. \tag{93}$$

The first order necessary condition for optimal δ' is

$$2D^2 + D\Delta = 4D\delta' - \delta\Delta \tag{94}$$

Applying symmetry yields

$$\delta^{***} = \frac{2D^2 + D\Delta}{4D-\Delta}. \tag{95}$$

Finally, we show that there exists some threshold $x^{trade,disclosure}$ such that an equilibrium with interdealer trading exists for $x < x^{trade,disclosure}$. First, note that as $x \to D$, is trivially holds that an individual dealer strictly prefers to deviate to some $\delta' > D - x$. This implies that interdealer trading does not occur for $\forall x$. Second, note that the payoff from the candidate equilibrium with δ^{***} yields a strictly greater expected payoff to dealers than δ^{**}. This implies that the x at which an individual dealer is indifferent between δ^{***} and deviating to some $\delta' > D - x$ is greater than x^{trade}. Hence, it follows from the proof of Theorem 1 that there exists some $x^{trade,disclosure} > x^{trade}$ such that an equilibrium with interdealer equilibrium exists under disclosure.

7.14 Theorem 5

Proof. We solve the social planner's problem that maximizes ex-ante dealer welfare by characterizing the optimal δ conditional on $\delta \in (0, D - x)$ and $\delta \geq D - x$, then identifying the conditions under which each solution is the globally payoff maximizing solution.

Given Lemma 2, it suffices to find the payoff maximizing solution conditional on the interdealer profit to dealers:

$$\Pi^D(\delta) = 2 \cdot \frac{D-\delta}{2D}\left(\delta - \left(\frac{D+\delta}{2D} + \frac{x^2}{2D(D-\delta)}\right) \cdot \left[x^2 \cdot \frac{2D}{D^2 - \delta_i^2 + x^2} + \Delta\right]\right) \tag{96}$$

$$= 2 \cdot \frac{D-\delta}{2D}\left(\delta - \left(\frac{D^2 - \delta^2 + x^2}{2D(D-\delta)}\right) \cdot \left[x^2 \cdot \frac{2D}{D^2 - \delta_i^2 + x^2} + \Delta\right]\right) \tag{97}$$

$$= \frac{D-\delta}{D}\left(\delta - \frac{x^2}{D-\delta} - \frac{D^2 - \delta^2 + x^2}{2D(D-\delta)}\Delta\right) \tag{98}$$

$$= \left[\frac{(D-\delta)\delta}{D} - \frac{x^2}{D} - \frac{D^2 - \delta^2 + x^2}{2D^2}\Delta\right] \tag{99}$$

The FOC with respect to δ is given by:

$$\frac{D-2\delta}{D} + \frac{\delta}{D^2}\Delta = 0 \tag{100}$$

$$\frac{D^2}{2D - \Delta} = \delta \tag{101}$$

For segmented markets:

$$\Pi^D(\delta) = 2 \cdot \frac{D + x - \delta}{4D} (\delta - x - \Delta) \tag{102}$$
$$= 2 \cdot \frac{D + x - \delta}{4D} (\delta - x - \Delta) \tag{103}$$

The FOC with respect to δ is given by:

$$\frac{2x + \Delta + D - 2\delta}{2D} = 0 \tag{104}$$
$$2x + \Delta + D - 2\delta = 0 \tag{105}$$
$$x + \frac{D + \Delta}{2} = \delta \tag{106}$$

□

7.15 Theorem 6

Proof. Traders' expected payoff can be written as:

$$\begin{cases} \frac{1}{2}\frac{(D-x-\delta)^2+(D+x-\delta)^2}{2D} & \text{if } \delta < D - x \\ \frac{1}{2}\frac{(D+x-\delta)^2}{2D} & \text{if } \delta \geq D - x. \end{cases} \tag{107}$$

Straightforwardly, we can see that for each case, the δ value that maximizes payoff is 0 and $D - x$, respectively. Hence, it suffices to solve the social planner's problem that maximizes trader welfare by characterizing the minimum δ that satisfies dealers' participation conditions. Note, such δ is less than $\delta^{soc,D}$ for whenever dealer expected payoff is positive. Next, note that since the payoff is strictly greater when $\delta < D - x$ than $\delta > D - x$, $x^{soc,T}$ is set such that for $\delta = D - x^{soc,T}$,

$$\frac{(D-\delta)\delta}{D} - \frac{(x^{soc,T})^2}{D} - \frac{D^2 + (x^{soc,T})^2 - \delta^2}{2D^2}\Delta = 0 \tag{108}$$

It follows directly that $x^{soc,T} > x^{soc,D}$. □