Should Central Banks Issue Digital Currency?*

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Abstract

We study how the introduction of a central bank-issued digital currency affects interest rates, the level of economic activity, and welfare in a model where both central bank money and private bank deposits are used in exchange. Banks in our model are financially constrained and the liquidity premium on bank deposits affects the level of aggregate investment. We study the optimal design of a digital currency in this setting, including whether it should pay interest and how widely it should circulate. We highlight an important policy tradeoff: while a digital currency tends to promote efficiency in exchange, it also crowds out bank deposits, raises banks' funding costs, and decreases investment. Despite these effects, introducing a central bank digital currency often raises welfare.

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1. INTRODUCTION

Recent technological advances have introduced the possibility for central banks to issue a new type of money, often referred to as a digital currency. Traditionally, central banks have issued physical currency, in the form of paper notes and/or coins, and allowed banks and select other institutions to hold deposits at the central bank, often called reserves. A digital currency could combine features of these two existing types of money and, potentially, introduce new features as well. Like physical currency, a central bank-issued digital currency could be made widely available to firms and households in the economy. Like reserves, a digital currency would exist in electronic form, making it easier than physical currency to store and to use in transactions at a distance, and could potentially earn interest. Academics and policy makers have begun discussing a range of issues from the technical design features of a central bank digital currency to political economy concerns.[1] At a fundamental level, however, the macroeconomic implications of such a currency are not well understood.

We study how the introduction of a central bank digital currency affects interest rates, the level of economic activity, and welfare in a model where both central bank money and private bank deposits are used in exchange. We are particularly interested in identifying the extent to which a digital currency issued by the central bank would crowd out private bank deposits and thereby potentially lead to disintermediation in the financial system. We study three distinct forms of digital currency that differ in the types of exchange they can facilitate. We show that when a digital currency competes with bank deposits as a medium of exchange, it tends to raise banks' funding cost and decrease bank-funded investment. At the same time, however, the availability of this new type of money increases production of those goods that can be purchased with it and can potentially increase total output. We derive the optimal design of a digital currency in our setting and provide conditions under which introducing such a currency raises welfare.

We base our analysis on a model in which some form of money is essential for exchange, as in Lagos and Wright (2005) and the subsequent New Monetarist literature.[2] We introduce an investment friction into this environment that creates borrowing constraints, as in Kiyotaki and Moore (1997,2005), Farhi and Tirole (2012) and others. Specifically, bankers in our model

[1] See, for example, Broadbent (2016), Cecchetti and Schoenholtz (2017), Engert and Fung (2017), Fung and Halaburda (2016), and Mersch (2017).

[2] For an overview of this literature, see the survey papers by Williamson and Wright (2010a, 2010b) and Lagos, Rocheteau and Wright (2017), as well as the many references therein.

have access to productive projects but face credit constraints due to limited pledgeability of their returns. Because of these credit constraints, the level of aggregate investment may be inefficiently low. To fund their projects, bankers can issue deposits that serve as a means of payment in decentralized markets. The ability of these deposits to facilitate exchange gives rise to a liquidity premium, which in turn affects banks' funding costs and the level of investment. There is also a central bank that issues physical currency and, potentially, a digital currency.

Two types of meetings occur in decentralized markets. In one type, the seller of goods is only able to verify cash and, therefore, will not accept bank deposits as payment. We interpret these meetings as capturing transactions in which concerns about privacy, fees, and/or a lack of access to the bank-based payment system leads parties to trade using physical currency. In the other type of meeting, the seller of goods is only able to verify bank deposits and, therefore, will not accept cash in exchange. We interpret this second type of meeting as capturing transactions in which the value of trade and/or the distance between the buyer and seller makes the use of physical currency impractical. Having these different types of meetings allows us to study design choices that affect how widely a central bank digital currency would be accepted.

We first study equilibrium in our model in the absence of a digital currency. Agents who will be buyers in the decentralized market acquire a balance of either cash or bank deposits in the previous centralized market. The inflation rate set by the central bank determines the opportunity cost of holding cash and, therefore, the level of production of goods that are purchased with cash. The interest rate on bank deposits is determined jointly by the transactions demand for deposits and the supply of available investment projects. When such projects are relatively scarce, a liquidity premium arises and the equilibrium interest rate on deposits falls below the rate of time preference. This lower interest rate helps overcome the financial friction faced by bankers and raises investment. At the same time, however, it raises the opportunity cost of holding deposits and, therefore, causes a decrease in the production of goods that are purchased with deposits.

Within this framework, we interpret a central bank digital currency as a new, technologically distinct type of outside money. In practice, a central bank could design such a currency in a number of different ways. Would it bear interest, for example? Would its users be required to hold accounts at the central bank, or could they remain anonymous to the authorities, as is the case with physical currency? What fees and other costs would users face? Are any restrictions placed on the size of users' balances or of transactions? We capture different possible designs within our framework by making different assumptions about how the digital currency can be used. We first assume the digital currency is *cash-like* in the sense that it can be used in transactions that currently involve physical currency, but not those that currently involve bank deposits. We think of this case as capturing a design that aims to maintain users' anonymity and minimize usage costs, particularly for smaller transactions. In our second case, the currency is *deposit-like* in the sense that it can be used in transactions that currently involve bank deposits, but not those that currency involve cash. This assumption might be appropriate for a currency that is based on users holding accounts at the central bank, for example. Our final case is a *universal* digital currency that can be easily used in all types of transactions. Our aim in making these assumptions is not to focus on what is or is not technologically feasible, but rather to identify the distinct macroeconomic effects of different broad categories of possible digital currency designs.

In making these design choices, the central bank faces a general problem of aggregate liquidity management. By paying a higher interest rate and/or designing the currency so it can be used in a wider range of transactions, the central bank can increase the supply of publicly-provided (or, outside) liquidity in the economy. A larger supply of public liquidity tends to promote more efficient levels of exchange. However, this outside liquidity also crowds out inside liquidity in the form of bank deposits and thus causes a decrease in bank-financed investment. The optimal design of a digital currency requires balancing these competing effects.

If the digital currency is cash-like, its introduction can increase the production and exchange of goods that are purchased with cash but has no effect on other types of exchange or on investment. If the digital currency is deposit-like, in contrast, its use will tend to crowd out bank deposits, raise the real interest rate on these deposits, and decrease bank-financed investment. The magnitude of this disintermediation effect is increasing in the interest rate paid on the digital currency by the central bank. We derive the optimal interest rate and show that it balances two competing effects: a higher interest rate promotes more efficient levels of exchange but decreases the level of investment.

When the digital currency is universal, the policy tradeoff is similar but the interest rate chosen by the central bank can now affect the level of exchange in all decentralized meetings. We show that the optimal interest rate in the universal case is higher than for an deposit-like currency. Which approach yields the highest welfare depends on parameter values. We derive conditions under which some form of digital currency raises welfare, as well as conditions under which each specific type. In general terms, an deposit-like system tends to be preferred when investment frictions are large and it is valuable for the central bank to be able to set a low interest rate on the digital currency to avoid crowding out too much private investment. When investment frictions are small, in contrast, such crowding out is less of a concern and the central bank will tend to set a higher interest rate on either type of digital currency. In such cases, a universal system is likely to be preferred because it allows a wider range of decentralized transactions to benefit from this higher interest rate.

The literature on digital currencies is small but growing rapidly. Several recent papers discuss the possibility of a central bank digital currency and the many design choices that would need to be made in order to issue one. Bech and Garratt (2017) provides a useful starting point by laying out a taxonomy of types of money and comparing different types of possible digital currencies with existing payment options.[3]

Bordo and Levin (2017) argue that central banks should issue digital currencies and that doing so will raise welfare. They discuss a range of design issues, arguing, for example, that the currency should be based on accounts at the central bank in order to avoid the verification costs associated with decentralized, token-based systems. They also believe the currency should bear interest as a way of promoting efficient exchange. While their recommendations draw on a background of monetary theory, as well as on practical experience, they do not present a model nor any formal justification for their conclusions. In addition, they do not address the question of whether a central bank digital currency will crowd out bank deposits or affect the aggregate level of investment.

Barrdear and Kumhof (2016) introduce a central bank digital currency into a quantitative DSGE model to assess its impact on GDP and to evaluate different monetary policy rules. In their model, the real interest rate is assumed to be increasing in the stock of government bonds held by the public. When the central bank issues digital currency by purchasing government bonds, therefore, this action lowers the real interest rate and tends to thereby raise GDP. In other words, an important part of the effects of issuing a digital currency in their framework come from the asset side of the central bank's balance sheet rather than from the digital currency *per*

[3] See also Ali et at. (2014), Fung and Halaburda (2016) and Engert and Fung (2017).

se. In our model, the assets held by the central bank have no impact on equilibrium outcomes. Nevertheless, a central bank digital currency may still increase economic activity by promoting more efficient levels of production and exchange.

The remainder of the paper is organized as follows. We present the model environment and derive the equilibrium conditions for a general formulation of the type(s) of currency available to agents in Section 2. We analyze equilibrium in a benchmark case without a digital currency in Section 3 and study the effects of introducing different types of digital currency in Section 4. We discuss optimal currency design and offer some concluding remarks in Section 5.

2. THE MODEL

In this section, we describe the physical environment and derive the conditions characterizing an equilibrium of our model for a general formulation of the type(s) of currency available to agents. The subsequent sections then specialize the analysis to study equilibrium allocations and welfare under different monetary regimes.

2.1 The environment

Time is discrete and continues forever. Each period is divided into two subperiods in which economic activity will differ. There is a frictionless centralized market in the first subperiod, while trade is decentralized in the second subperiod. A perishable commodity is produced and consumed in each subperiod. We refer to the commodity produced in the first subperiod as the centralized market (CM) good and to the commodity produced in the second subperiod as the decentralized market (DM) good.

Agents. The economy is populated by three types of agents: buyers, sellers, and bankers. Buyers and sellers are infinitely lived and participate in both markets in each period. They can produce the CM good in the first subperiod using a linear production technology that requires labor as input, and they also have linear utility over CM consumption. In the second subperiod, buyers want to consume but cannot produce, whereas sellers are able to produce but do not want to consume. A buyer is randomly matched with a seller with probability $\alpha \in [0, 1]$ and vice versa, so trade in the decentralized market is bilateral. Each buyer has the period utility function

$$U^b\left(x_t^b, q_t\right) = x_t^b + u\left(q_t\right),$$

where $x_t^b \in \mathbb{R}$ denotes net consumption of the CM good and $q_t \in \mathbb{R}_+$ denotes consumption of the DM good. The function $u : \mathbb{R}_+ \to \mathbb{R}_+$ is increasing, strictly concave, and continuously differentiable, with $u(0) = 0$ and $u'(0) = \infty$. Each seller has the period utility function

$$U^s\left(x_t^s, q_t\right) = x_t^s - w\left(q_t\right),$$

where $x_t^s \in \mathbb{R}$ denotes net consumption of the CM good and $q_t \in \mathbb{R}_+$ denotes production of the DM good. The function $w : \mathbb{R}_+ \to \mathbb{R}_+$ is increasing, convex, and continuously differentiable, with $w(0) = 0$. There is a unit mass each of buyers and sellers, all of whom discount future periods at a common rate $\beta \in (0, 1)$.

Bankers live for two periods, participate only in the centralized market, and consume only in old age. Each period, a new generation of bankers with unit mass is born. Banker j is endowed at birth with an indivisible and nontradable project that requires one unit of the CM good as input and pays off $\gamma_j \in \mathbb{R}_+$ units of the CM good in the following period. Project returns are known in advance and are heterogeneous across bankers; let $G(\gamma)$ denote the distribution of payoffs across the population of bankers. The support of the distribution is $[0, \bar{\gamma}]$ with $\bar{\gamma} > \beta^{-1}$, which implies that some projects are socially efficient to operate but others are not. Bankers have no endowment; they must fund their project by issuing deposits in the centralized market when they are young. The ability to issue deposits is limited by an investment friction: only a fraction $\theta \in (0, 1)$ of the project's return can be pledged to the bank's depositors. This friction will prevent some banks whose project would be profitable at market interest rates from being able to borrow and invest.

Assets and exchange. We assume that buyers and sellers are anonymous (i.e., their identities are unknown and their trading histories are privately observable), which precludes credit in the decentralized market. Because there is no scope for trading future promises in this market, a medium of exchange is essential for decentralized trade. The possible media of exchange in our model are deposits issued by bankers and currency, both physical and digital.

The supply of bank deposits depends on the real interest rate, which determines how many bankers are able to attract funding and operate. The supply of currency is determined by the central bank according to a price-level targeting regime in which the gross inflation rate

$\mu \geq \beta^{-1}$ is assumed to be constant over time. In particular, the central bank stands ready to buy/sell CM goods each period at a predetermined price in either physical or digital currency. By enforcing the same price level target for physical and digital currency, the central bank is effectively offering to convert units of physical currency one-for-one into units of digital currency and vice versa. In this sense, the digital currency in our model is an electronic version of the physical currency and not a distinct item that might trade at a different price.[4] The central bank uses lump-sum taxes/transfers to balance its budget each period.

The extent to which each of these assets can be used in DM exchange depends on the verification technology available to the seller in a particular meeting. We assume that a fraction $\lambda_1 \in (0, 1)$ of sellers is endowed with the technology to recognize physical currency but not deposits. We interpret this assumption as capturing a variety of reasons why cash is used in practice, including concerns about the privacy of the transacting parties and the costs of accessing and using the bank-based payment network. The remaining fraction $\lambda_2 \equiv 1 - \lambda_1$ of sellers is endowed with the technology to recognize bank deposits but not physical currency. The meetings of these sellers correspond to transactions that in practice involve checks, debit cards, or other methods of directly transferring claims on a commercial bank from the buyer to the seller. We view these meetings as at least partially consisting of situations in which the use of cash is impractical, such as large-value purchases, online purchases, etc.[5] We refer to a meeting in which the seller is able to verify physical currency as *type 1* and to a meeting in which the seller can verify bank deposits as *type 2*. The buyer finds out the type of seller she will potentially meet in the next DM before making her portfolio decision in the CM, which implies that she will choose to hold either currency or deposits for transactions purposes, but not both.

When we introduce a digital currency into this environment, a key issue is the type(s) of meetings in which it can be used. We consider three different assumptions about the digital technology that correspond to different ways policy makers might choose to design a digital currency in practice. A *type I* digital currency can only be verified by type 1 sellers and represents

[4] Private digital currencies like bitcoin would be different in this regard, of course. See BIS (2015) for a discussion of the economic implications of private digital currencies and Fernandez-Villaverde and Sanches (2017) for a model of private digital currency issue.

[5] It is straightforward to add a third type of meeting in which both currency and deposits can be verified by the seller. Doing so complicates the presentation without changing the basic insights of our model, as only one of the two forms of payment would typically be used in all such meetings. The important assumption is that each of these assets can be used in some situations where the other cannot.

a design that aims to mimic physical cash as closely as possible. For example, such a design may use cryptography to maintain anonymity of the transacting parties and may minimize the fees and other costs associated with its use. A *type II* digital currency, in contrast, represents a deposit-like design that can be verified only by type 2 sellers. This assumption would likely be appropriate for a digital currency that is based on individuals holding accounts at the central bank and that makes use of the existing payment network. Finally, a *type III* digital currency can be verified by all sellers. This assumption corresponds to a design that would make the digital currency attractive for use in a wide range of transactions. We derive the effects of each of these types of digital currency and then discuss conditions under which each design can be optimal.

Allocations and welfare. For discussions of optimal policy, we measure welfare using an equal-weighted sum of all agents' utilities. However, as in Williamson (2012), we allow for the possibility that some of the transactions that take place in type 1 meetings, where only currency is used, might have lower social value than private value. For example, a policy maker may want to put less weight on transactions involving illegal activity or foreign use of the domestic currency. Specifically, we follow Williamson (2012) in assuming the a fraction ν of type 1 meetings generate no social value. We can then write aggregate welfare as

$$\sum_{t=0}^{\infty} \beta^t \left\{ x_t^b + x_t^s + x_t + \alpha \left[\lambda_1 \left(1-\nu\right) \left(u\left(q_{1t}\right) - w\left(q_{1t}\right)\right) + \lambda_2 \left(u\left(q_{2t}\right) - w\left(q_{2t}\right)\right)\right] \right\}, \tag{1}$$

where x_t denotes the average CM consumption of old-age bankers. Feasibility of an allocation requires that the net consumption of all agents in the centralized market is no greater than the net output of bankers' investment projects. We focus on allocations characterized by a cutoff value $\hat{\gamma}_t$ above which a banker's project is operated and below which it is not. Feasibility in period t then requires

$$x_t^b + x_t^s + x_t \leq \int_{\hat{\gamma}_{t-1}}^{\bar{\gamma}} \gamma g\left(\gamma\right) d\gamma - \left(1 - G\left(\hat{\gamma}_t\right)\right). \tag{2}$$

The right-hand side of this expression is the output from projects coming to fruition at the current date minus total investment into new projects that will mature the following period. Net consumption of CM goods by all agents can be no larger than this difference.

Given the quasi-linear specification of preferences, the welfare properties of an allocation depend only on the sequences of DM consumption levels $\{q_{1t}, q_{2t}\}$ and of cutoff investment values $\{\hat{\gamma}_t\}$, which determine the total amount of CM consumption available in each period. As equations (1) and (2) make clear, the distribution of CM consumption across agents has no impact on welfare. In the analysis that follows, we summarize an allocation by these three quantities.

2.2 Asset demand

Let ϕ_t denote the goods value of money in the centralized market in period t, so that the real value of M_t dollars can be written as $m_t \equiv \phi_t M_t$. Let i^e denote the net nominal interest rate paid on digital currency by the central bank, which can be either positive or negative. The gross real rate of return on physical currency is then ϕ_{t+1}/ϕ_t and on digital currency is $(1+i^e)\,\phi_{t+1}/\phi_t$. Let $1+r_t$ denote the gross real interest rate on bank deposits. Finally, let $\mathbf{a} \equiv (m, d, e)$ denote an asset portfolio consisting of $m \in \mathbb{R}_+$ units of physical real money balances, $d \in \mathbb{R}_+$ of bank deposits, and $e \in \mathbb{R}_+$ of digital (or "electronic") real money balances, all measured in current CM consumption goods.

We begin the analysis by defining the value function $J_i(\mathbf{a}, t)$ for a buyer entering the centralized market in period t holding portfolio $\mathbf{a}$. The index $i \in \{1, 2\}$ indicates what type of seller she will potentially meet in the next decentralized market. Let $V_i(\mathbf{a}, t)$ denote the value function of this same buyer when she arrives in the decentralized market. Using these two functions, we can write the Bellman equation for this buyer as

$$J_i(\mathbf{a}, t) = \max_{\left(x^b, \mathbf{a}'\right) \in \mathbb{R} \times \mathbb{R}_+^3} \left[x^b + V_i\left(\mathbf{a}', t\right)\right],$$

where the maximization is subject to the budget constraint

$$x^b + \mathbf{p} \cdot \mathbf{a}' = \mathbf{R}_{t-1} \cdot \mathbf{a} + \tau_t.$$

The variable x^b is the buyer's net consumption of the CM good, which can be positive or negative. The price vector $\mathbf{p} \equiv (1, 1, 1)$ measures the cost of acquiring real money balances and deposits in terms of CM goods, while the vector

$$\mathbf{R}_{t-1} = \left(\frac{\phi_t}{\phi_{t-1}}, 1 + r_{t-1}, (1 + i^e)\,\frac{\phi_t}{\phi_{t-1}}\right)$$

measures real returns on assets carried over from the previous period. Finally, τ_t denotes the real value of any lump-sum transfer received by the agent.

The value $V_i(\mathbf{a}', t)$ satisfies

$$V_i\left(\mathbf{a}', t\right) = \alpha\left[u\left(q_i\left(\mathbf{a}', t\right)\right) + \beta J\left(\mathbf{a}' - \mathbf{h}_i\left(\mathbf{a}', t\right), t+1\right)\right] + (1-\alpha)\beta J\left(\mathbf{a}', t+1\right), \tag{3}$$

where $q_i(\mathbf{a}', t) \in \mathbb{R}_+$ denotes the buyer's consumption of the DM good and $\mathbf{h}_i(\mathbf{a}', t) \in \mathbb{R}^3_+$ denotes the payment she makes for this consumption out of her asset holdings $\mathbf{a}'$. The function $J(\mathbf{a}', t)$ in this expression represents the expected value of entering the centralized market before knowing the type of her potential meeting in the following period's decentralized market, that is,

$$J\left(\mathbf{a}', t\right) = \lambda_1 J_1\left(\mathbf{a}', t\right) + \lambda_2 J_2\left(\mathbf{a}', t\right). \tag{4}$$

Throughout the analysis, we assume that the terms of decentralized trade are determined by generalized Nash bargaining. For simplicity, we restrict attention to the case in which the buyer has all the bargaining power. The bargaining problem can then be described as

$$\max_{(q_i, \mathbf{h}_i) \in \mathbb{R}^4_+} \left[u\left(q_i\right) - \beta \times \mathbf{R}_t \cdot \mathbf{h}_i\right]$$

subject to the seller's participation constraint

$$-w\left(q_i\right) + \beta \times \mathbf{R}_t \cdot \mathbf{h}_i \geq 0$$

and the liquidity constraint

$$\mathbf{h}_i \leq \mathbf{f}_i\left(\mathbf{a}\right). \tag{5}$$

The function $\mathbf{f}_i$ enforces the fact that the buyer will only pay with assets her trading partner can verify. If, for example, type 1 sellers can only verify physical currency, then we have $\mathbf{f}_1(\mathbf{a}) = (m, 0, 0)$. If they can verify both physical and digital currency, we have $\mathbf{f}_1(\mathbf{a}) = (m, 0, e)$. In the sections that follow, we impose particular functions $\mathbf{f}_i$ to capture different potential digital currency designs. For now, however, we only impose that type 1 sellers can verify physical currency but not bank deposits and that the reverse holds for type 2 sellers.

The solution to this bargaining problem implies the following schedule for DM output

$$q_i\left(\mathbf{a}, t\right) = \begin{cases} w^{-1}\left(\beta \mathbf{R}_t \cdot \mathbf{f}_i\left(\mathbf{a}\right)\right) & \text{if } \mathbf{R}_t \cdot \mathbf{f}_i\left(\mathbf{a}\right) < \frac{w(q^*)}{\beta} \\ q^* & \text{otherwise} \end{cases} \tag{6}$$

and for payments

$$\mathbf{R}_t \cdot \mathbf{h}_i\left(\mathbf{a}, t\right) = \begin{cases} \mathbf{R}_t \cdot \mathbf{f}_i\left(\mathbf{a}\right) & \text{if } \mathbf{R}_t \cdot \mathbf{f}_i\left(\mathbf{a}\right) < \frac{w(q^*)}{\beta} \\ \frac{w(q^*)}{\beta} & \text{otherwise,} \end{cases}$$

where q^* is the efficient level of DM trade, which satisfies

$$u'(q^*) = w'(q^*).$$

In other words, if the value of the buyer's spendable assets is large enough to induce the seller to produce q^*, the efficient level of trade occurs. If not, the buyer spends all that she can and the seller produces an amount smaller than q^*.

Using this solution to the bargaining problem, a buyer's portfolio problem in the centralized market can be written as

$$\max_{\mathbf{a}' \in \mathbb{R}^3_+} \left\{ -\mathbf{p} \cdot \mathbf{a}' + \alpha \left[u\left(q_i\left(\mathbf{a}', t\right)\right) - \beta \mathbf{R}_t \cdot \mathbf{h}_i\left(\mathbf{a}', t\right) \right] + \beta \mathbf{R}_t \cdot \mathbf{a}' \right\}. \tag{7}$$

Recall that the buyer knows the type of seller she will potentially meet in the next DM when making this portfolio choice in the CM. If $i = 1$, the buyer knows the seller will accept physical currency and the slope of the objective function with respect to m' is

$$-1 + \beta \frac{\phi_{t+1}}{\phi_t} \left[\alpha \frac{u'\left(w^{-1}\left(\beta \mathbf{R}_t \cdot \mathbf{f}_1\left(\mathbf{a}'\right)\right)\right)}{w'\left(w^{-1}\left(\beta \mathbf{R}_t \cdot \mathbf{f}_1\left(\mathbf{a}'\right)\right)\right)} + 1 - \alpha \right] \quad \text{for} \quad \mathbf{R}_t \cdot \mathbf{f}_1\left(\mathbf{a}'\right) < \frac{w(q^*)}{\beta}$$

and

$$-1 + \beta \frac{\phi_{t+1}}{\phi_t} \quad \text{for} \quad \mathbf{R}_t \cdot \mathbf{f}_1\left(\mathbf{a}'\right) > \frac{w(q^*)}{\beta}.$$

If $i = 2$, the buyer knows the seller will accept bank deposits and the slope of the objective function with respect to d' is

$$-1 + \beta\left(1 + r_t\right) \left[\alpha \frac{u'\left(w^{-1}\left(\beta \mathbf{R}_t \cdot \mathbf{f}_2\left(\mathbf{a}'\right)\right)\right)}{w'\left(w^{-1}\left(\beta \mathbf{R}_t \cdot \mathbf{f}_2\left(\mathbf{a}'\right)\right)\right)} + 1 - \alpha \right] \quad \text{for} \quad \mathbf{R}_t \cdot \mathbf{f}_2\left(\mathbf{a}'\right) < \frac{w(q^*)}{\beta}$$

and

$$-1 + \beta\left(1 + r_t\right) \quad \text{for} \quad \mathbf{R}_t \cdot \mathbf{f}_2\left(\mathbf{a}'\right) > \frac{w(q^*)}{\beta}.$$

It is helpful to define the function $L : \mathbb{R}_+ \to \mathbb{R}_+$ by

$$L(A) = \begin{cases} \alpha \frac{u'\left(w^{-1}(\beta A)\right)}{w'\left(w^{-1}(\beta A)\right)} + 1 - \alpha & \text{if } \beta A \leq w(q^*) \\ 1 & \text{otherwise} \end{cases} \tag{8}$$

This function measures the expected liquidity benefit of holding one extra unit of spendable assets. If the buyer's current spendable assets are insufficient to purchase the efficient quantity q^*, the increase will allow her to consume more if she is matched in the DM, which occurs with probability α. If she is not matched, or if she already has enough spendable assets to purchase

q^*, she merely holds the extra unit of assets until the following CM. The first-order condition for the real physical currency balances of a buyer who will potentially be in a type 1 match can then be written as

$$L\left(\mathbf{R}_t \cdot \mathbf{f}_1\left(\mathbf{a}_t\right)\right) \leq \frac{\phi_t}{\beta \phi_{t+1}} \tag{9}$$

with equality if $m' > 0$. The first-order condition for the deposits of a buyer who will potentially be in a type 2 match is

$$L\left(\mathbf{R}_t \cdot \mathbf{f}_2\left(\mathbf{a}_t\right)\right) \leq \frac{1}{\beta\left(1+r_t\right)} \tag{10}$$

with equality if $d' > 0$. In addition, only buyers potentially entering type 1 meetings will hold physical currency and only buyers potentially entering type 2 meetings will hold bank deposits.[6] Equations (9) and (10) thus characterize the demand for each of these assets in the period-t CM.

2.3 Asset supply

To derive the supply of deposits, we start by solving the bankers' problem. Given the market interest rate r_t, a banker $\gamma \in [0, \bar{\gamma}]$ born in period t is willing to issue a deposit claim if

$$\gamma - (1 + r_t) \geq 0.$$

Because only a fraction $\theta \in (0, 1)$ of the project's payoff is pledgeable, the banker is subject to the pledgeability restriction

$$1 + r_t \leq \theta\gamma.$$

In other words, the promised repayment cannot exceed the value of the banker's pledgeable future income. Note that $\theta < 1$ implies that this constraint is strictly tighter than the previous one, meaning that some bankers with profitable projects will not be able to raise funds and invest.

Let $\hat{\gamma}_t \in \mathbb{R}_+$ denote the banker whose project's payoff satisfies the pledgeability restriction with equality in period t, given the market interest rate $1 + r_t$. Then, the marginal type $\hat{\gamma}_t$ who will be funded satisfies

$$\hat{\gamma}_t = \frac{1 + r_t}{\theta}. \tag{11}$$

[6] These decisions reflect agents' strict preferences when the real return on an asset is less than β^{-1}. All buyers and sellers are indifferent about holding an asset whose real return equals β^{-1}. In this case, we simplify our notation by assuming, without any loss of generality, that only buyers potentially entering a meeting where an asset is accepted will hold that asset.

This condition simply says that the project's return must exceed the cost of borrowing by a factor $1/\theta$ for the marginal type. Note that the lower the pledgeable portion of the project's payoff, the higher is the return required to satisfy the pledgeability restriction.

Given the marginal type $\hat{\gamma}_t$, the aggregate supply of deposits is

$$1 - G\left(\hat{\gamma}_t\right) = 1 - G\left(\frac{1 + r_t}{\theta}\right). \tag{12}$$

This expression shows that a reduction in the interest rate leads to an increase in investment by allowing a larger number of bankers to issue debt claims. In other words, a lower interest rate mitigates credit rationing in the banking system and increases the supply of deposits.

The supply of both physical and digital currency is set by the central bank following a price-level target rule. We assume the target grows at a constant gross rate $\mu \geq \beta$. Letting $\bar{\phi}_0 \in \mathbb{R}_+$ denote the initial target for the value of money, the target at date t is given by

$$\bar{\phi}_t = \left(\frac{1}{\mu}\right)^t \bar{\phi}_0$$

and we have

$$\frac{\phi_t}{\phi_{t+1}} = \mu \qquad \text{for all } t. \tag{13}$$

The central bank stands ready to exchange units of either physical or digital currency for CM goods at the desired price level each period.[7] Letting $\bar{M}_t \in \mathbb{R}_+$ denote the supply of physical currency and $\bar{E}_t$ the supply of digital currency *per* asset holder, the central bank's budget constraint is

$$\phi_t\left(\bar{M}_t + \bar{E}_t\right) = \phi_t\left(\bar{M}_{t-1} + \left(1 + i^e\right)\bar{E}_{t-1}\right) + \tau_t$$

where the lump-sum tax/transfer τ_t is chosen to balance the budget each period.

2.4 Market clearing

Using the fact that physical currency is only used in type 1 meetings, we can write the market clearing equation as

$$\lambda_1 m_t = \phi_t \bar{M}_t. \tag{14}$$

[7] We could instead take the more standard approach of assuming that the total money supply grows at a constant rate μ. With both physical and digital currency, however, the relative supply of each type of currency is endogenous and the notation becomes more complex. Given that we focus on stationary allocations, the simpler approach we take here is without any loss of generality.

Similarly, the fact that bank deposits are only exchanged in type 2 meetings allows us to write the market-clearing equation for the deposit market as

$$\lambda_2 d_t = 1 - G\left(\frac{1+r_t}{\theta}\right). \tag{15}$$

The demand for digital currency and the form of the market-clearing equation depends on design features that determine in which type(s) of meetings the currency can be used; we analyze three distinct cases below. An equilibrium of the model consists of a sequence of prices $\{r_t, \phi_t\}$, portfolio holdings $\{\mathbf{a}_{1t}, \mathbf{a}_{2t}\}$, and allocations $\{q_{1t}, q_{2t}, \hat{\gamma}_t\}$ satisfying equations (9)-(15) plus the first-order and market-clearing equations for digital currency presented for each case below.

In the next section, we derive the properties of equilibrium in a benchmark model with no digital currency. We then introduce various types of digital currency in Section 4 and analyze the resulting equilibrium allocations and welfare.

3. EQUILIBRIUM WITH NO DIGITAL CURRENCY

The model with no digital currency provides the benchmark allocation and welfare level against which different types of digital currency will be compared. When there is no digital currency, the functions $\mathbf{f}_i$ in the borrower's liquidity constraint (5) are given by

$$\mathbf{f}_1(\mathbf{a}) = (m, 0, 0) \qquad \text{and} \qquad \mathbf{f}_2(\mathbf{a}) = (0, d, 0).$$

In other words, a buyer can only use her physical currency balances to purchase DM goods in a type 1 meeting and can only use bank deposits in a type 2 meeting. The Inada condition on buyers' utility function then implies that the first-order conditions (9) and (10) for buyers' portfolio choices will hold with equality. Combining these equations with the market-clearing conditions (14) and (15) yield

$$\frac{\mu}{\beta} = L\left(\frac{m_t}{\mu}\right) \tag{16}$$

and

$$\frac{1}{\beta(1+r_t)} = L\left(\left(\frac{1+r_t}{\lambda_2}\right)\left[1 - G\left(\frac{1+r_t}{\theta}\right)\right]\right). \tag{17}$$

The fact that only period-t variables appears in each of these two equations shows that an equilibrium in our model is necessarily stationary.[8] The equations also demonstrate a dichotomy

[8] This result follows from our assumption of a price-level targeting rule, which removes the multiplicity of equilibria and dynamics that are common on monetary models. Our aim here is to analyze the effects of introducing a digital currency taking as given the effectiveness of traditional monetary policy.

between the money and deposit markets in our baseline model. Given the inflation rate μ, equation (16) pins down real money balances independent of the interest rate on deposits. Meanwhile, equation (17) determines the equilibrium interest rate on deposits independent of the inflation rate. We think of our model as capturing long-run phenomena, in which case it is not unreasonable to think that standard monetary policy has a limited effect on real interest rates and the level of investment. Notice that inflation is not neutral here, however. A higher inflation target leads to lower production and consumption in type 1 DM meetings and to lower welfare, as is standard in models of monetary exchange.

We use the superscript N to identify equilibrium values in the model with no digital currency. If the mass of bankers with relatively high-return projects is large enough, the interest rate on deposits will satisfy $\left(1+r^N\right) = \beta^{-1}$ and the production of DM goods in type 2 meetings will equal the efficient level q^*. Because of the pledgeability constraint, however, the level of investment will be inefficiently low: some projects with positive net present value from a social point of view will remain unfunded. If, instead, high-return projects are scarce, a liquidity premium emerges on deposits and $\left(1+r^N\right)$ falls below β^{-1}. Such a change makes DM production in type 2 meetings less efficient, but raises the level of CM investment. If the liquidity premium on deposits is large enough, it is even possible for the level of equilibrium investment to be too high: projects with negative net present value from a social point of view may be funded because the deposits created by the banker are so valuable in exchange.

Figure 1 illustrates the relationship between DM exchange and CM investment in our environment. The upward-sloping (green) curve in panel (a) represents buyers' demand for deposits as a function of the gross return $1+r$, which is derived from the first-order condition (10). Demand is increasing in the return offered by deposits and becomes vertical when the return equals $1/\beta$. The two downward-sloping curves represent the supply of deposits from equation (12) for two different distribution functions G. The dashed (blue) supply curve corresponds to the dashed (blue) distribution function in panel (b), which has a relatively large number of high-productivity projects. In this situation, the equilibrium interest rate equals $1/\beta$ and the equilibrium quantity of deposits is large enough to finance the efficient level of DM trade q^* in type 2 meetings. All projects in the shaded to the right of $\hat{\gamma} = \frac{1}{\theta\beta}$ are then funded in equilibrium. Note, however, that all projects to the right of $\gamma = \frac{1}{\beta}$ are socially productive. Because of the pledgeability constraint, many productive projects remain unfunded in equilibrium.

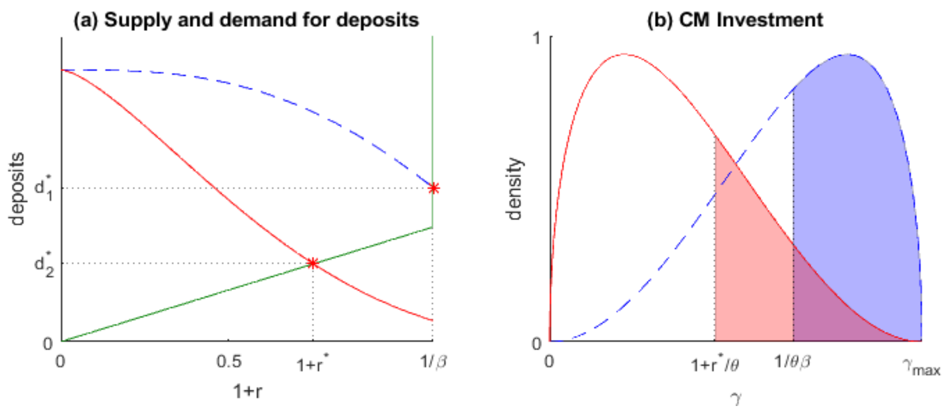

Fig. 1. Equilibrium with no digital currency

Now consider what happens when the distribution changes to the solid (red) curve in panel (b). Because there are fewer high-productivity projects, the deposit supply curve in panel (a) shifts down. As a result, the equilibrium quantity of deposits and interest rate both decrease. Because there are fewer deposits, the quantity of DM good produced and traded in type 2 meetings falls below the efficient level q^*. At the same time, however, the lower interest rate decreases the cutoff value $\hat{\gamma}$ and thereby raises CM investment. This tradeoff between the efficiency of DM exchange and the quantity of CM investment will be central to understanding the macroeconomic effects of introducing a digital currency in the sections that follow.

4. INTRODUCING A CENTRAL BANK DIGITAL CURRENCY

We now introduce a central bank digital currency into the economy. As described above, we view digital currency as a technological innovation that allows the central bank to issue money that can bear either a positive or a negative nominal return and can potentially be used in a different set of situations than physical currency. The macroeconomic effects of this currency will depend in large part on the type(s) of transactions in which it is used. We study three cases in this section: one where digital currency competes only with physical currency, a second where it competes only with deposits, and a third where it competes with both traditional means of payment.

4.1 A cash-like digital currency

We first consider a *type I* digital currency, which can be verified by type 1 sellers but not by type 2 sellers. We think of this case as representing design features for the digital currency that aim to mimic the properties of physical currency as closely as possible. For example, the digital currency may take the form of a token rather than a deposit in an account at the central bank and thereby grant users some degree of anonymity.[9] Alternatively, the central bank may impose a maximum allowable balance or transaction size. The design may also minimize the fees and other costs associated with the currency, particularly for small transactions.

Under this design, the functions $\mathbf{f}_i$ in the borrower's liquidity constraint in equation (5) are given by

$$\mathbf{f}_1(\mathbf{a}) = (m, 0, e) \qquad \text{and} \qquad \mathbf{f}_2(\mathbf{a}) = (0, d, 0).$$

In other words, buyers in a type 1 meeting can use their balances of physical and/or digital currency to make purchases, while buyers in a type 2 meeting can only use bank deposits. Using the function L defined in (8), we can write the first-order condition for a buyer's choice of real digital currency balances e' if she will potentially be in a type 1 meeting as

$$L(\mathbf{R}_t \cdot \mathbf{f}_1(\mathbf{a}_t)) \leq (1 + i^e)\frac{\phi_t}{\beta\phi_{t+1}}, \tag{18}$$

with equality if $e' > 0$. The market-clearing condition for digital currency in this case can be written as

$$\lambda_1 e_t = \phi_t \bar{E}_t.$$

Comparing equation (18) with the first-order condition for physical currency balances in equation (9) shows that the demand for the digital currency will depend critically on the nominal interest rate it earns. If $i^e < 0$, the demand for digital currency will be zero since buyers would strictly prefer to use physical currency.[10] In this case, the equilibrium is unchanged from Section 3 and is again characterized by equations (16) and (17). If $i^e = 0$, buyers are indifferent between using physical and digital currency. In this case, equation (17) is unchanged but equation (16)

[9] Some authors use *pseudosnymity* rather than anonymity to refer to a system in which individuals can use pseudonyms rather than their actual identities to hold and transfer assets.

[10] While some authors have advocated eliminating physical currency to avoid this outcome, we do not consider such a policy here.

becomes

$$\frac{\mu}{\beta} = L\left(\frac{m_t + e_t}{\mu}\right).$$

The paths of $\{m_t\}$ and $\{e_t\}$ are then indeterminate; the equilibrium conditions determine only the sum $(m_t + e_t)$. However, while digital currency may now be used in equilibrium, its introduction still has no effect on equilibrium allocations or on welfare. We summarize these results in the following proposition.

Proposition 1 *If $i^e \leq 0$, we have $\left(q_1^I, q_2^I, \hat{\gamma}^I\right) = \left(q_1^N, q_2^N, \hat{\gamma}^N\right)$.*

If the digital currency earns a positive interest rate, it will completely replace physical currency and will change the level of trade in type 1 meetings.[11] In this case, equation (16) is replaced with

$$\frac{\mu}{(1+i^e)\,\beta} = L\left(\frac{(1+i^e)}{\mu} e_t\right).$$

Solving this equation for e^I and using (6) to determine the quantity of trade in type 1 meetings yields the following result.

Proposition 2 *If $i^e > 0$, we have $e^I > 0$, $q_1^I > q_1^N$ and $\left(q_2^I, \hat{\gamma}^I\right) = \left(q_2^N, \hat{\gamma}^N\right)$.*

This proposition shows that an interest-bearing digital currency that competes only with physical currency will increase trade in type 1 DM meetings, but will have no effect on investment in the CM or on trade in type 2 meetings. The introduction of such a currency will always raise welfare. However, it is fairly straightforward to show that the equilibrium allocation associated with any choice of $i^e > 0$ can also be achieved without a digital currency by changing the target inflation rate μ. In this sense, introducing a cash-like digital currency does not bring any meaningful benefits in our model.[12]

[11] This result stems from our assumption that there are no meetings in which only physical currency can be used. It would be straightforward to add such meetings to our model. Physical currency would then remain in circulation when $i^e > 0$, but the optimal policy results would be qualitatively unchanged from what we present here.

[12] One could imagine that policy makers might aim to target a particular inflation rate (say, 2%) for reasons that are outside the scope of our model. In such cases, an interest-bearing, cash-like digital currency may be useful for promoting efficient production and exchange while maintaining the desired inflation rate.

4.2 A deposit-like digital currency

Next, we consider a type II digital currency that can only be verified by sellers in type 2 meetings, where bank deposits are already being used. This assumption might represent, for example, a digital currency that is based on accounts at the central bank and in which payments are processed over the existing bank-based network. The functions $\mathbf{f}_i$ in the borrower's liquidity constraint in equation (5) for a type II digital currency are

$$\mathbf{f}_1(\mathbf{a}) = (m, 0, 0) \qquad \text{and} \qquad \mathbf{f}_2(\mathbf{a}) = (0, d, e).$$

These functions state that buyers in a type 1 meeting can only use their physical currency balances to make purchases, while buyers in a type 2 meeting can use their bank deposits and/or digital currency balances. The first-order condition for a buyer's choice of real digital currency balances e' if she is heading into a type 2 meeting is given by

$$L(\mathbf{R}_t \cdot \mathbf{f}_2(\mathbf{a}_t)) \leq (1 + i^e)\frac{\phi_t}{\beta\phi_{t+1}}, \tag{19}$$

with equality if $e' > 0$. The market-clearing condition for digital currency in this case can be written as

$$\lambda_2 e_t = \phi_t \bar{E}_t.$$

Comparing equation (19) with the first-order condition for bank deposits in equation (10) shows that a buyer will choose to hold both bank deposits and digital currency only if they offer the same real return, that is, if

$$1 + r_t = (1 + i^e)\frac{\phi^e_{t+1}}{\phi^e_t}. \tag{20}$$

This condition shows how the central bank's policy choices – the inflation rate and the nominal interest rate on digital currency – will impact the interest rate on bank deposits in this policy regime.

An equilibrium in this case is characterized by equation (16) together with

$$\frac{1}{\beta(1 + r_t)} = L\left((1 + r_t)\left[\frac{1 - G\left(\frac{1+r_t}{\theta}\right)}{1 - \lambda} + e_t\right]\right), \tag{21}$$

and

$$1 + r_t \geq \frac{(1 + i^e)}{\mu}, \tag{22}$$

with equality if $e_t > 0$. Our first result gives a necessary and sufficient condition for a type II digital currency to be held in equilibrium.

Proposition 3 *$e^{II} > 0$ if and only if $1 + i^e > \mu\left(1 + r^N\right)$.*

A type II digital currency is a perfect substitute for bank deposits in transactions. In equilibrium, buyers will only choose to hold and spend the digital currency if it offers a return at least as high as deposits. Of course, the return on deposits is endogenous and depends on the central bank's policy choices. Proposition 3 shows that the equilibrium nominal interest rate on deposits in the baseline economy with no digital currency, $\mu\left(1 + r^N\right)$, is an important threshold. If the central bank sets the nominal interest rate on digital currency, $1 + i^e$, below this threshold, there will be no demand for the new currency and the equilibrium allocation will be the same as without the digital currency. When $1 + i^e$ is above this threshold, in contrast, a positive amount of digital currency is held and the equilibrium allocation changes. Our next result documents the direction of these changes.

Proposition 4 *If $1 + i^e > \mu\left(1 + r^N\right)$, we have $\left(q_2^{II}, \hat{\gamma}^{II}\right) \gg \left(q_2^N, \hat{\gamma}^N\right)$ and $q_1^{II} = q_1^N$. In addition, $r^{II} > r^N$ and $d^{II} < d^N$ hold.*

This result shows that a type II digital currency has multiple effects on the equilibrium allocation. One one hand, by raising the rate of return on assets that can be used as a means of payment in type 2 meetings, the digital currency leads – through buyers' portfolio choice and the outcome of the bargaining process – to higher output being produced in these meetings. On the other hand, however, a type II digital currency tends to crowd out bank deposits, raise the real interest rate on deposits, and decrease CM investment. In other words, a central bank digital currency that competes with bank deposits as a medium of exchange leads to disintermediation in the banking system: buyers shift funds away from private bank deposits into the digital currency. This shift leads to less bank-financed investment. The effect of this shift on welfare depends on the productivity of the marginal investment project; we return to this issue below.

The results in Proposition 4 point to a policy tradeoff the central bank faces when setting the interest rate on a type II digital currency. Raising this interest rate increases DM output and promotes efficient exchange, but decreases investment and output in the CM. The optimal policy choice will balance these competing concerns. Figure 2 illustrates this tradeoff using a numerical example.[13] The upper-left panel of the figure shows how as the gross interest rate

[13] The utility functions for this example are $u(c) = A\frac{c^{1-\sigma}}{1-\sigma}$ with $\sigma = 0.5$ and $A = 0.75$, and $w(q) = q$. The distribution of productivities G follows a beta distribution on $[0, 1.5]$ with parameters $(3, 3)$. The other parameter values are $(\alpha, \beta, \lambda_1, \theta, \mu, \nu) = (1, 0.96, 0.25, 0.85, 1.06, 0)$.

$1+i^e$ is increased, buyers' holdings of real digital currency balances (in red) increase while their holdings of bank deposits (in blue) decrease. The upper-right panel shows the net effect of these changes on the quantity of DM goods produced and consumed in type 2 meetings, q_2^{II}. This quantity is strictly increasing in the central bank's choice of $1+i^e$. The quantity produced and consumed in type 1 meetings is independent of $1+i^e$, as demonstrated in Proposition 4.

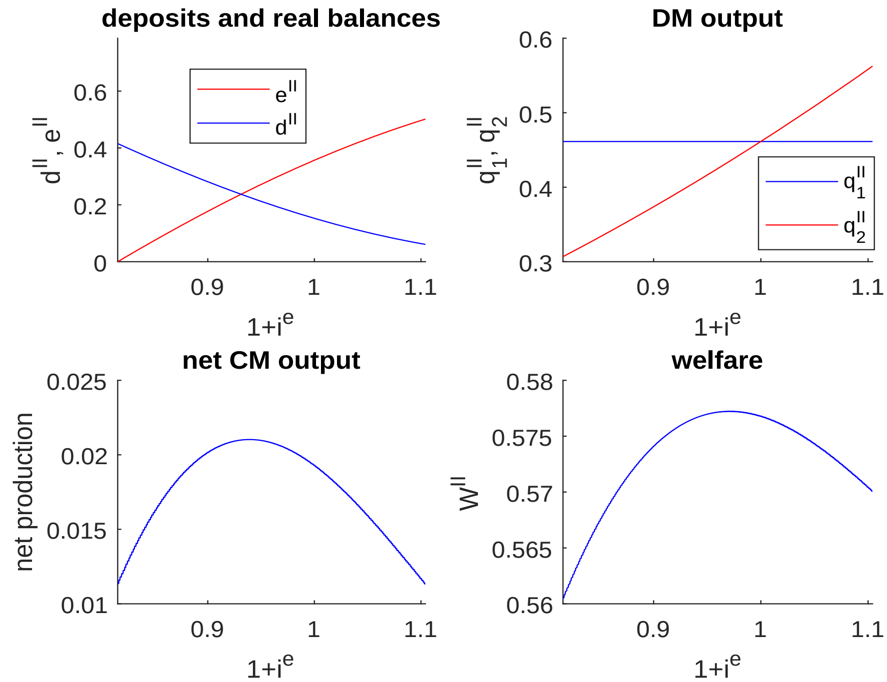

FIG. 2. Equilibrium with a type II digital currency

The bottom-left panel of the figure plots the net output of bankers' investment projects in the CM. Combining equations (11) and (22) shows that banker j's project will be funded if and only if its productivity γ_j is greater than

$$\hat{\gamma}^{II} = \frac{1+i^e}{\mu\theta}.$$

This expression shows that as the interest rate on digital currency $1+i^e$ increases, the cutoff productivity $\hat{\gamma}$ also increases, meaning that fewer projects are funded. In the bottom-left panel of Figure 2, however, net output is initially increasing in $1+i^e$. This fact is an indication that, for low values of $1+i^e$, the liquidity premium on deposits is so large that some projects with negative real returns are being funded. Eventually, as $1+i^e$ increases further, the continued reduction in investment causes net CM output to decrease. The bottom-right panel of the figure plots welfare as measured in equation (1). In the region where net CM production is increasing in $1 + i^e$, welfare will necessarily increase as well.[14] Once net CM production begins to decline, however, there is a tradeoff because further increases in $1 + i^e$ continue to raise the level of DM production and consumption. For this example, the value of $1 + i^e$ that maximizes net CM output is 0.94 and the welfare-maximizing value of $1 + i^e$ is 0.97.

4.3 A universal digital currency

We now examine the effects of introducing a type III digital currency, which can be used in both types of DM meetings. We interpret this type as resulting from design choices that make the digital currency attractive for use in a wide range of transactions. Under this design, the borrower's liquidity constraints in equation (5) become

$$\mathbf{f}_1(\mathbf{a}) = (m, 0, e) \qquad \text{and} \qquad \mathbf{f}_2(\mathbf{a}) = (0, d, e).$$

This specification indicates that a buyer in a type 1 meeting can now pay with any combination of physical and/or digital currency, while a buyer in a type 2 meeting can pay with any combination of digital currency and/or deposits. The first-order condition for a buyer's choice of real digital currency balances e' if she will potentially be in a type 1 meeting is given by equation (18) while the condition for a buyer who will potentially be in a type 2 meeting is equation (19). We denote these holdings by e_{1t} and e_{2t}, respectively. The market-clearing condition for digital currency can then be written as

$$\phi_t \bar{E}_t = \lambda_1 e_{1t} + \lambda_2 e_{2t}.$$

The market-clearing conditions for physical currency and deposits are again given by equations (14) and (15).

If the net nominal interest rate on digital currency is negative ($i^e < 0$), then it will never be held by buyers heading into a type 1 meeting; they will strictly prefer to hold only physical currency. If $i^e = 0$, these buyers will be indifferent between the two types of currency but the introduction of digital currency will have no effect on their total real money balances or on their equilibrium consumption. In both cases, the equilibrium allocation will be the same as with a type II digital currency that cannot be used in these meetings. We record this result in the following proposition.

[14] It is worth pointing out, however, that even in these cases, an increase in $1 + i^e$ does not lead to a Pareto improvement because those bankers whose (low productivity) projects are not funded are made worse off.

Proposition 5 *If $i^e \leq 0$, we have* $\left(q_1^{III}, q_2^{III}, \hat{\gamma}^{III}\right) = \left(q_1^{II}, q_2^{II}, \hat{\gamma}^{II}\right)$.

It follows immediately from this result that equilibrium welfare is the same for both types of digital currency as well. For this reason, we focus our attention in the remainder of this section on the case where $i^e > 0$.

When the digital currency pays a positive nominal interest rate, it will completely replace physical currency in equilibrium as in Section 4.1. In this case, the equilibrium conditions can be written as

$$\frac{\mu}{\beta} = L\left(\frac{(1+i^e)}{\mu} e_{1t}\right), \tag{23}$$

$$\frac{1}{\beta(1+r_t)} = L\left((1+r_t)\left[\frac{1-G\left(\frac{1+r_t}{\theta}\right)}{1-\lambda} + e_{2t}\right]\right), \tag{24}$$

and

$$1 + r_t \geq \frac{(1+i^e)}{\mu} \tag{25}$$

with equality if $e_{2t} > 0$. As in the previous sections, a solution to these equations is necessarily stationary. Our first result gives a sufficient condition for digital currency to be used in both types of DM transactions.

Proposition 6 *If $1 + i^e > \mu\left(1 + r^N\right) \geq 1$, then $e_1^{III} > 0$ and $e_2^{III} > 0$ hold.*

Our next result shows that a type III digital currency can increase the level of exchange in all decentralized meetings. Recall that changes in the nominal interest rate affected output only in one type of meeting in each of the previous cases. Thus, a digital currency designed to serve as a medium of exchange in a wider range of decentralized transactions gives monetary policy a bigger effect on trading activity.

Proposition 7 *If $1 + i^e > \mu\left(1 + r^N\right)$, then $q_1^{III} > q_1^{II} = q_2^N$ and $q_2^{III} = q_2^{II} > q_2^N$. In addition, $\left(d^{III}, r^{III}, \hat{\gamma}^{III}\right) = \left(d^{II}, r^{II}, \hat{\gamma}^{II}\right)$.*

Figure 3 illustrates these results. In the upper left-panel, the behavior of deposits and real digital currency balances by buyers headed into type 2 meetings are the same is in Figure 2. When the nominal interest rate on the digital currency is positive, however, e_1^{III} becomes positive because the digital currency is now used in type 1 meetings as well. The upper-right panel shows that production of the DM good in type 2 meetings is the same as before, but now production in type 1 meetings begins to increase as well when the interest rate on the digital

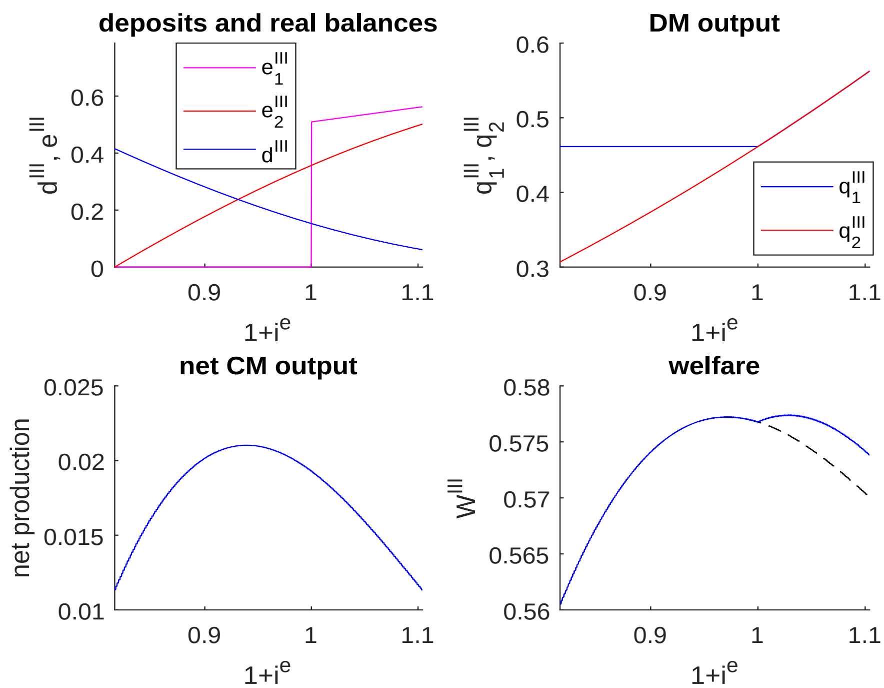

FIG. 3. Equilibrium with a type III digital currency

currency becomes positive. The behavior of net CM output in the lower-left panel is identical to that in Figure 2.

The bottom-right panel of Figure 3 shows that the behavior of welfare as $1+i^e$ varies has two distinct regions. When the interest rate on the digital currency is negative, welfare is the same as in Figure 2. As i^e becomes positive, however, welfare is higher with a broad currency than with a deposit-like currency (the latter is indicated by the dashed line in the figure). This pattern arises because the positive interest rate on digital currency leads to an increase in production and consumption of the DM good in type 1 meetings. In other words, if the digital currency pays a positive nominal interest rate, it is better to have a token-based design that allows it to be used in a wider range of transactions. This result is a general property of our model.

Proposition 8 *If* $i^e > 0$, *then* $W^{III} > W^{II}$.

For this example, the welfare function has two local maxima. In one, the digital currency is only used in type 2 meetings and the interest rate corresponds to the optimal rate with a deposit-like currency. At the other local maximum, i^e is strictly positive and the digital currency is used in both types of meetings. The optimal type of digital currency to introduce depends on which of these two local maxima is higher; we address this issue in the next section.

5. OPTIMAL POLICY

Should a central bank issue a digital currency and, if so, how should it be designed? In our model, introducing a digital currency has both positive and negative effects. By raising the rate of return on the means of payment in decentralized transactions, it can move output and consumption in these markets closer to the efficient level. At the same time, however, it raises banks' funding costs by reducing the liquidity premium on deposits. This higher funding cost causes a decrease in bank-financed investment and output. In general, either of these effects can dominate and introducing a central bank digital currency can either raise or lower welfare.

A digital currency does, however, give the central bank a new policy tool: it can choose the nominal interest rate (either positive or negative) that the new currency earns. This interest rate influences both the costs and the benefits described above. If the interest rate is chosen to maximize welfare, then introducing a digital currency of any type cannot decrease welfare, since the interest rate can always be set in a way that minimizes the use of the currency and any associated costs. To make this statement precise, let $\hat{W}^j$ denote welfare under a digital currency of type $j \in \{I, II, III\}$ when the interest rate $1 + i^e$ is chosen to maximize welfare as measured in equation (1). Then we have the following result.

Proposition 9 *We have* $\min\left\{\hat{W}^I, \hat{W}^{II}, \hat{W}^{III}\right\} \geq W^N$.

Figure 4 presents the optimal interest rate and the associated level of welfare for each type of digital currency as the severity of the financial friction θ varies. Panel (a) shows that the optimal interest rate for a type I, cash-like digital currency is independent of θ and equal to $\frac{\mu}{\beta}$ (≈ 1.10 in this example). Note that $\frac{\mu}{\beta}$ corresponds to the gross nominal interest rate on a one-period illiquid bond. Since this type of currency does not influence the level of investment, the best policy is to implement a type of Friedman rule for type 1 transactions regardless of the severity of the investment friction.

For the other two designs, the figure shows that the optimal interest rate is strictly increasing in θ. When θ is small, the investment friction is strong and many bankers with socially-productive projects are unable to raise the required funding. Panel (a) shows that, in these cases, the optimal nominal interest rate i^e on a type II or III currency is negative, which implies that these designs lead to the same outcome. This low interest rate maintains a large liquidity premium on deposits, which enables many bankers to overcome the credit friction and operate their projects.

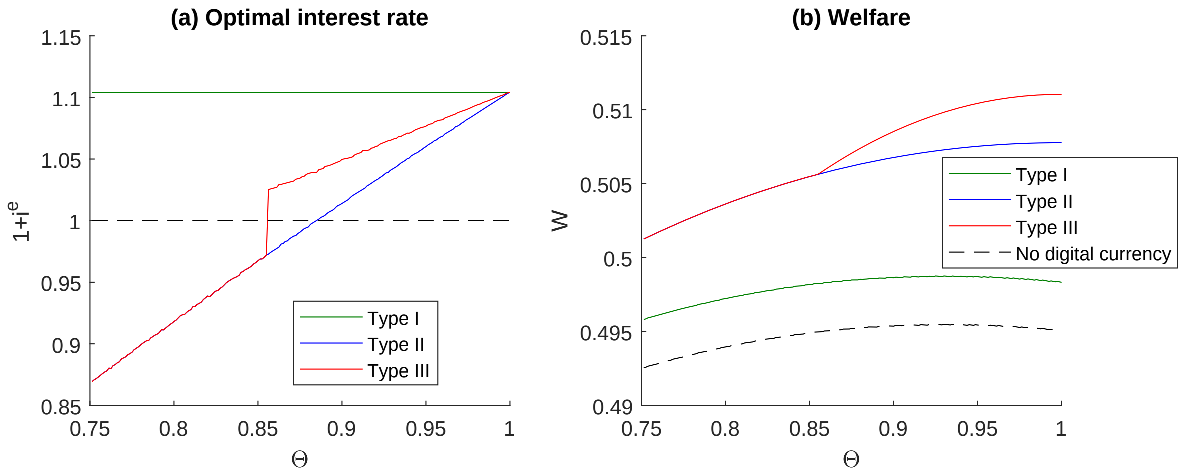

FIG. 4. The optimal interest rate and welfare with each type of digital currency

As θ increases and the investment friction becomes weaker, the gain from setting the interest rate i^e low to maintain a liquidity premium on deposits diminishes. As a result, panel (a) shows that the optimal interest rate i^e increases. When θ is high enough (approximately 0.85 in this example), the optimal interest rate for a type III currency jumps above zero.[15] For this value of θ and higher, the universal design is used in both types of meetings and yields higher welfare than the deposit-like design. When the investment friction is weak, the crowding-out effect of the digital currency on bank deposits is less costly. In these cases, it is better to pay a higher interest rate to encourage more efficient production and exchange in as many DM meetings as possible.

As θ approaches one and the investment friction disappears, the optimal policy approaches $1 + i^e = \frac{\mu}{\beta}$. When $1 + i^e$ is set to this level, digital currency and bank deposits will offer the same return as an illiquid bond in equilibrium, which means there is no liquidity premium. The crowding-out effect of the digital currency on CM investment is still present. However, when there is no investment friction, any liquidity premium on deposits leads to inefficiently high investment from a social point of view. In this case, and only in this case, the optimal policy in our model corresponds to a full implementation of the Friedman rule. Note the fully

[15] This jump corresponds to a shift in the global maximum between the two local maxima shown in the bottom-right panel of Figure 3.

implementing the Friedman rule in our setting requires using a universal design so that the digital currency can be used in both types of DM meetings.

Concluding remarks. The introduction of a central bank digital currency would represent a potentially historic innovation in monetary policy. If households and firms choose to hold and use significant quantities of such a currency, it will lead to a substantial shift in aggregate liquidity, that is, in the types of assets that are used in exchange and that carry a liquidity premium. While there have been much recent discussion of this issue in policy circles, the macroeconomic implications of this shift are not well understood.

Our analysis shows how a fairly standard model in the New Monetarist tradition can generate insight into these issues. In particular, it highlights a potentially-important policy tradeoff: while a digital currency can indeed promote efficient exchange, it leads agents to shift away from bank deposits and thereby raises bank funding costs. By appropriately choosing the interest rate it pays on digital currency, the central bank can balance these competing concerns. When investment frictions are small, the optimal policy is to pay a relatively high interest rate on the digital currency and to design it in a way that it allows it to be used as widely as possible. This approach can be seen as an implementation of the well-known Friedman rule. If, however, frictions prevent a significant number of socially-productive projects from being funded, the optimal policy changes. In these cases, the central bank should choose to pay a lower interest rate on the currency, possibly even a negative rate. Doing so allows bank deposits to carry a liquidity premium and thus helps banks partially overcome the investment friction. These results show how a digital currency can be an important and useful tool for central banks to use in managing aggregate liquidity.

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