(preliminary and incomplete)

Central Counterparty Resolution: The Right Move at The Right Time[1]

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Abstract

We develop a real option model to study the optimal timing of CCP resolution in the context of intertwined market and liquidity risks. When a CCP starts recovery process due to large default losses, the relevant resolution authority faces a trade-off between the option value of waiting and the costs associated with the recovery tools. If the authority steps in too early, it terminates a potentially successful recovery in an irreversible manner. If it steps in too late, the negative externality from the recovery process could be large. The model also sheds light on the dedicated resources to CCP resolution.

Keywords: Central counterparties (CCPs), CCP resolution, Optimal stopping time
JEL Codes: G01, G12, G21, G22

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1 Intoduction

Central clearing of over-the-counter derivatives contracts has grown markedly in terms of market share since the Great Financial Crisis. In large part this has been driven by the G20 commitment that all standardized over-the-counter derivatives contracts be cleared through central counterparties (CCPs). At end-2017, reporting dealers' interest rate positions booked against CCPs totaled \$320 trillion, accounting for about 75% of notional amounts outstanding. In light of the increased importance of CCPs, international standard setting bodies have done a substantial amount of policy work to guard against CCPs becoming the next "too-big-to-fail" entities. This includes requiring CCPs to have comprehensive recovery plans to address all losses resulting from the default of their clearing members (CPMI-IOSCO, 2017)[1].

A CCP recovery plan by itself cannot preclude the possibility that resolution could be required in some extreme circumstances. CCP resolution may be needed at least for two reasons: First, a recovery plan may have, ex post, non-performance risk from clearing members. For example, even if a CCP has the legal capacity to make uncapped cash calls from its clearing members, there is a possibility that one or more clearing members may not be able to honor their cash calls. Second, even if the comprehensive recovery plan allows a CCP to fully recover, it might not be systemically optimal when taking into account the negative externality imposed on the clearing members. In particular, many of the clearing members of CCPs tend to be financial institutions that are of systemic importance to the domestic (or global) financial system. For instance, a CCP with uncapped variation margin gains haircut (VMGH) can reduce the full amount of the variation margins that should be received by the clearing members with in-the-money positions. But if the

[1] The concept of comprehensiveness implies that the CCP should (in theory) be able to cope with any shock, regardless of how extreme the shock turns out to be.

clearing members are using the derivatives contracts to hedge their positions outside of the CCP, haircutting their variation margins may translates to large losses to the clearing members, threatening their viability and financial stability.

Resolution planning is less advanced for CCPs than other financial entities. For banks and insurance companies there is a well-developed framework for resolution, which includes the identification of globally systemically important institutions, and a requirement for additional regulatory and supervisory requirements for these institutions. Most importantly, the globally systemically important banks (G-SIBs) and insurance companies (G-SIICs) are subject to requirements to ensure sufficient loss-absorbing and recapitalization capacity to be available in resolution (eg, TLAC for G-SIBs). The current resolution guidelines for CCPs include a requirement to have in place a crisis management group (CMG) for entities that are considered to be of systemic importance in more than one jurisdiction. There are no other additional regulatory or supervisory requirements for the resolution of these CCPs.

There are on-going policy discussions about the need for adjusting the resolution framework for CCPs. In particular, whether there should be additional (prefunded) financial resources available in the (unlikely) event of a CCP resolution. The unique structure and function of CCPs make it difficult to draw on the already advanced state of discussion for similar questions for bank resolution. CCPs are risk managers and, unlike banks, they don't engage in active market risk-taking. Given this role, a CCP tends to rely almost entirely on its ability to mutualize counterparty credit risk among the members based on a pre-agreed arrangement, i.e., the CCP rule book. In this context, our paper explores two key questions. First, given the unique features of CCP recovery and resolution, when should the resolution authority steps in and resolve a failing CCP? Second, if there is a requirement for CCPs to have additional prefunded financial resources available only in resolution (which would be similar to the TLAC requirements for G-SIBs) to alleviate the loss-absorbing burden of the resolution authority, what would be the impact on the resolution decision?

To explore these questions we outline a model that captures the dynamics between CCP recovery and resolution. The model has three types of agents: clearing members, a CCP and a resolution authority. The CCP has a comprehensive recovery plan in the rule book that specifies the recovery tool: either cash calls or variation margin gains haircut (VMGH).[2] The clearing members all behave in a way that is consistent with the CCP rule book.[3] The CCP has a simplified default waterfall. We don't differentiate initial margin, the CCP's skin-in-the-game, and default fund, since doing so will only complicate the model without providing additional insights. Instead, we assume the CCP has a certain amount of prefunded resources based on value-at-risk measures. In a scenario where a member's default losses exhaust the prefunded resources, the CCP starts a recovery process and resorts to cash calls or VMGH for allocating remaining losses. The resolution authority optimizes the total value of both the CCP and the surviving clearing members. To achieve that, the authority can potentially place a CCP into resolution at any point following the start of the recovery process. However, there are costs involved with the authority's intervention. These costs could reflect inefficiencies of an outside party taking over the CCP in the middle of a crisis, administrative costs related to bridging the critical services of the CCP and/or potential impact on the sentiment of broader markets and the real economy. When the authority resolves the CCP, its equity is used and remaining losses are assumed to be covered by the resolution authority (i.e.,using any financial resources

[2]There is a fast growing literature of CCP loss allocation. Interested readers can refer to Elliott (2013), Singh (2014), Heath et al. (2015), Huang (2016), and Cruz-Lopez and Manning (2017).

[3]In practice, a CCP may not be fully compliant with international standards and thus its recovery plan may not be comprehensive. The clearing members may also have strategic defaults with the CCP and behave differently from the rule book. All these are valid concerns. But to provide a starting point to think of the dynamics between recovery and resolution, we simplify from the reality and focus on the ideal environment specified by the PFMI.

dedicated for resolution and/or public funds).

The resolution authority's decision is modeled as an optimal stopping problem. The key trade-off is as follows: if the authority steps in too early, he loses the option value of waiting and interrupts a potentially successful recovery. If the authority steps in too late, the value of the losses from the recovery process will be sub-optimal. In our framework, the main area of interest is the timing of entry into resolution, which is influenced by a number of model parameters such as the size of remaining CCP equity (and any additional financial resources dedicated for resolution), the intensity of the liquidity shock, and the percentage of surviving members subject to the liquidity shock.

Overall, the model suggests that additional prefunded resources dedicated for use in resolution would make the authority delay triggering the resolution process. This could allow more time for the recovery process to succeed, but also poses the risk that if the CCP is not able to recover despite the additional time, the resulting losses in resolution could impose a larger public sector support.

Our paper contributes to the literature in two ways. To our knowledge, we are the first to explicitly model the dynamics of CCP recovery and resolution. A number of papers have outlined the conceptual and practical issues associated with a CCP resolution. Duffie (2014) reviews possible recovery and resolution plans for insolvent CCPs and argues that CCP resolution should minimize the total expected distress costs of all relevant participants, including clearing members, CCP operators, other market participants and taxpayers who could suffer from spillover effects. Singh and Turing (2018) points out the differences between the recovery and resolution for CCPs and those for banks. They note that the resolution for banks typically means a "lift-out" of the uncontaminated assets from the balance sheet. Meanwhile, CCP resolution typically does not involve this step; to resolve a failing CCP without causing further market stress, they suggest relaxing clearing mandate in the event of CCP failures. Most importantly, CCPs should build reserves and declare publicly their reserves and dividends policy. Neither of these papers develop the conceptual arguments into a theoretical model; our paper fill this void in the literature, and provides a framework to examine and develop the intricate linkages between the recovery and resolution tools for the resolution of a CCP.

In our model, the timing of a CCP's resolution is modeled as an optimal stopping problem widely used in the real option literature. McDonald and Siegel (1986) study the optimal timing of investment in an irreversible project where the cash flows follow continuous stochastic processes. They model the option value of waiting and provide closed-form solutions to it. Dixit (1989) models the option value of both investing a project and liquidating the project jointly. Pindyck (1990) further develops this approach in the context of incremental investment under uncertainty. Finally, Dixit and Pindyck (1994) is the classic textbook that summarizes this strand of literature and further develops the approach in a systemic way. Our work contributes to this literature in two aspects. We apply the optimal stopping techniques to a CCP recovery-resolution setting and in doing so provide closed-form solutions to the optimal resolution timing; this allows us to better understand the drivers behind it. Importantly, we also extend the real option literature by modeling both market risk and funding risk. A CCP's recovery plan allows it to allocate losses to its clearing members in the event of shocks. Therefore, the optimal resolution strategy for a CCP also depends on the financial health of the clearing members. To capture this feature, on top of the usual market risk in the literature, we also model the funding risk related to the recovery tools.

The remainder of the paper is organized as follows. Section 2 presents the model and discusses the assumptions. Section 3 shows the optimal timing of entry into CCP resolution. Section 4 studies the impact of additional resources dedicated to CCP resolution, and section 5 concludes.

2 The model

2.1 Model primitives

The model has three types of agents: clearing members, a CCP and a resolution authority. There is one type of derivatives contract with sufficiently long maturity that is centrally cleared by the CCP. There is a unit mass of buyers and a unit mass of sellers who are clearing members of the CCP.

Clearing members. The buyers are exposed to some real economy risk that is outside of the derivatives market and would like to fully hedge that real risk by entering into long positions on the derivatives contract. The buyers rely on the payments from the derivatives contract to hedge their positions outside of the derivatives market. The sellers are the dealers that make the derivatives market, and hence hold the the short positions on the derivatives contract.

Suppose the buyers have one unit long position and the sellers have one unit short position. When the price of the derivatives contract changes, the buyers and sellers need to exchange variation margin. Without loss of generality, we assume that the price of the derivatives contract changes in such a way that the buyers are in-the-money with the CCP and the sellers are out-of-money with the CCP. Suppose the sellers cannot meet their variation margin calls because of exogenous shocks to their balance sheets. Since the derivatives contract is centrally cleared, the CCP insure the traders against counterparty credit risk. The CCP inherits the defaulting members' portfolios and works to cover the default losses. The CCP has prefunded financial resources but the default losses are so large that the prefunded resources are depleted. The CCP enters into a recovery process, and will seek to mutualize the remaining losses X via tools like cash calls or VMGH (Raykov, 2016).

As the derivatives contract is with long maturity, there is market risk due to the price changes over future periods. To capture this market risk, let X_t the mark-to-market losses at time t. It is modeled as a geometric Brownian motion with initial value X_0 and volatility σ_t:

$$dX_t = \sigma_t X_t dz_t.$$

Furthermore, when the surviving members cannot receive the payment from the derivatives contract in full, they are exposed to liquidity risk, i.e.,that members are not insolvent but do not have liquid funds available at the particular time to meet their payment obligations. Typically, liquidity events result in large losses but happen infrequently, thus they are modeled as a Poisson jump process with intensity λ_t:

$$dN_t = \begin{cases} 0, & 1 - \lambda_t dt; \\ 1, & \lambda_t dt. \end{cases}$$

Each time when a liquidity event happens, a ε proportion of the surviving members are affected. When the surviving members get hit by the liquidity shocks, they undergo stress and cannot meet their obligations specified by the CCP recovery plan (elaborated below). This results in losses of C_t, which is the cost associated with the CCP recovery plan. Let R_t denote the obligations of the surviving clearing members to the CCP during the recovery process, then C_t has an initial value of 0 and follows:

$$dC_t = \varepsilon R_t dN_t.$$

CCP. The CCP has a comprehensive recovery plan in its rule book. The recovery plan specifies that the CCP can allocate the mark-to-market losses X_t to the (surviving) clearing members via recovery tools. Thus, during the recovery process, the CCP's cash inflows R_t come from the surviving members. The CCP's cash outflow X_t depends on the mark-to-market losses of the inherited positions from the defaulting members. The CCP's value is the cumulative net cash inflow:

$$V_t^{CCP} = \int_0^t (R_s - X_s)\, ds \tag{1}$$

During the recovery process, market risk and liquidity risk are intertwined. In particular, the volatility of the cash outflow σ_t and the intensity of liquidity shock arrivals λ_t are large, when the CCP's cash outflow is large relative to the cash inflow. The intuition is as follows. When the CCP's cash outflow is large relative to the cash inflow, the CCP is less likely to recover. Since there is no informational asymmetry in the model, all market participants can observe this development. This, in turn, leads to more uncertainty in the derivatives market, i.e., volatility increases. In addition, market participants are also less willing to provide liquidity to clearing members who have exposure to the stress CCP, leading to a higher intensity of liquidity shocks. Let G_t be the ratio between the CCP's cash outflow and the CCP's cash inflow, then the volatility and the liquidity-shock-arrival intensity can be written as.

$$\sigma_t^2 = \sigma^2 G_t$$

$$\lambda_t = \lambda G_t$$

According to CPMI-IOSCO (2018), most CCPs have either one of these tools or both of them. In the model, we assume that the CCP has one recovery tool, either cash calls or VMGH. When the CCP issues cash calls, the surviving members need to meet the cash calls within a certain period. When the CCP uses VMGH, it means that the CCP can reduce its liability by haircutting the variation margin payment of the in-the-money positions that the surviving members hold.

The surviving members may not be able to honor their commitments that are specified by the recovery tools. As mentioned earlier, when the surviving members get hit by the liquidity shocks, they undergo stress. Although they are in-the-money at the derivatives market, their positions outside of the CCP are out-of-money, which makes these members unable to fulfill their commitments to the CCP unless they incur losses on their balance sheet. Hence, the surviving members that get hit by the liquidity shocks will default at their cash calls. This is the so-called "non-performance" risk of the cash calls.

In the event of VMGH, there is no such non-performance risk (as the CCP has control on the surviving members' variation margins). However, the haircuts will translate into losses for surviving clearing members that hit by the liquidity shocks. Since they don't receive the variation margin payment from the CCP, their out-of-money positions outside of the CCP will weaken their balance sheets. Although there is no non-performance risk associated with VMGH, there is some negative externality on the clearing members originating from VMGH.

Both the non-performance risk in the event of cash calls and the negative externality in the event of VMGH are costs associated with the CCP recovery plan, which is denoted as C_t. Such costs create a conflict between the CCP's interest and the interest of the surviving members: pursuing the CCP recovery plan in full may hurt the financial health of the surviving members, which very often are systemically important financial institutes themselves. For the system as a whole (i.e., taking into account both the CCP's and the surviving members' interests), it could be optimal in some cases to resolve a CCP, even if the CCP has a positive chance to successfully recover. That would require intervention by a resolution authority.

Resolution authority. The resolution authority chooses the timing of its intervention to optimize the total value of both the CCP and the surviving clearing members, which is the sum of the CCP's cumulative net cash inflow and the surviving members' losses due to the recovery process. The total value captures the non-performance risk in the event of cash calls and the negative externality in the event of VMGH. The authority can potentially place a CCP into resolution at any point following the start of the recovery process (and before the successful recovery; elaborated later). When the authority steps in, it seizes the CCP's equity e but also incurs liquidation inefficiency l[4]. Moreover, all uncertainties associated with the recovery plan will be resolved and the remaining losses will be born by the resolution authority. In other words, the authority will bring in additional fund to absorb the losses. In our model we abstract from the discussion of bail-out/bail-in scheme and the magnitude of the associated funding costs.

The resolution authority's decision is modeled as an optimal stopping problem, following the rich literature on real options. A real option is the right - but not the obligation - to make an irreversible decision when the future outcomes are not certain, such as to invest or to liquidate a project under uncertainty. The CCP resolution in out setup is similar to the liquidation decisions in the real options literature, but with some additional parameters to capture the features of the CCP recovery process. The state variables are the CCP's cash inflow, the CCP's cash outflow, and the cost associated with the recovery plan. The cost associated with the recovery plan could be the non-performance risk (cash call) or the negative externality on the surviving members (VMGH).

[4]The liquidation inefficiency could come from different sources such as the operational costs to migrate all the transactions to a bridge CCP.

$$\max_T E\left[\underbrace{\int_0^T (R_t - C_t - X_t)\,dt}_{\text{recovery}} + \underbrace{(e - l + R_T - C_T - X_T)}_{\text{resolution}}\right] := F(R, C, X) \tag{2}$$

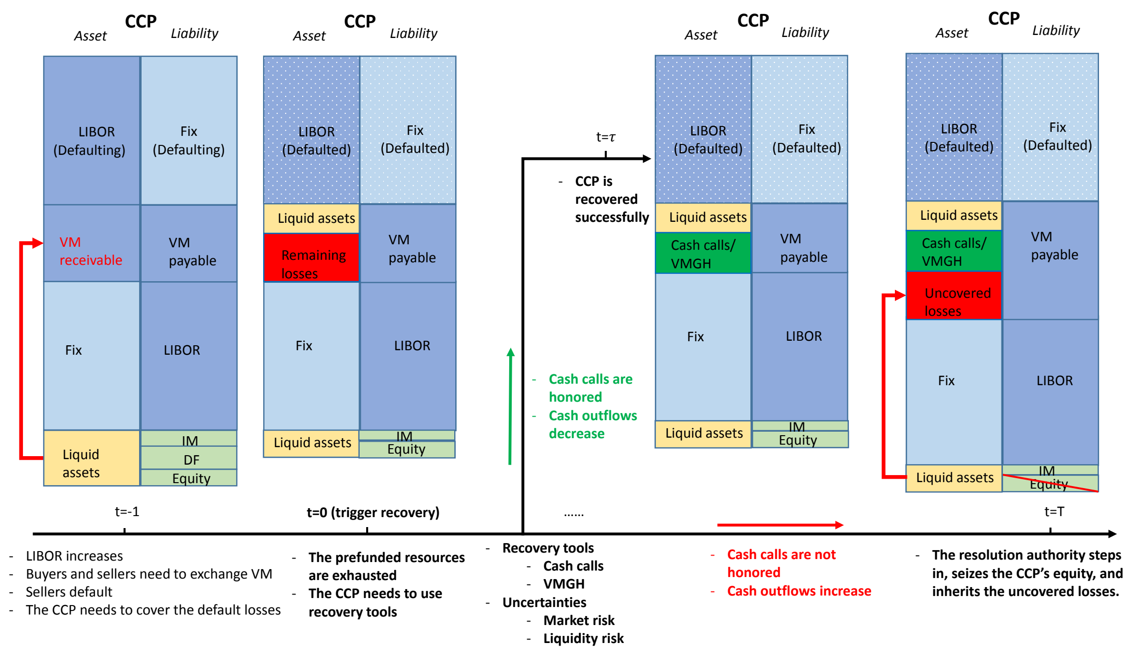
The key trade-off is between the upward uncertainty from the CCP's cash outflow and the increasing costs associated with the recovery process. During the recovery process, the authority faces the non-performance risk or the negative externality from the CCP's recovery plan depending on the tool that it is using. But due to the upward uncertainty in the CCP's cash flow, there is a chance that the CCP can recover successfully .Thus, if the authority steps in too early, he loses the option value of waiting and interrupts a potentially successful recovery. If the authority steps in too late, the non-performance risk and the negative externality from the recovery process will lead to sub-optimal outcome for the surviving members.

The solution to this stopping problem is a set of (interlinked) thresholds for the state variables that separates the resolution region from the recovery region. Let u_t denote the state variables: $\{R_t, C_t, X_t\}$. The optimal stopping regions are separated by threshold u^*, and the optimal timing of entry into resolution T is the first time when u_t reaches u^*. Nonetheless, the CCP may very well have already recover successfully before the resolution is triggered. If the state variables meet the criteria for successful recovery before they hit the resolution thresholds, the CCP is considered to be successfully recovered. Thus, the CCP will be resolved if and only if $T < \tau$, where $\tau(\geq 1)$ is defined by the successful recovery criteria[5].

[5]The restriction that τ has to be as large as 1 merely comes from the continuous setup. Since all the state variables u_t are continuous in the model, the interpretation of u_t is some instantaneous rates at time t. Mapping this to the reality where cash calls and VMGH happen discretely, $t = 1$ is when the cash calls should be fulfilled or when surviving members' variation margins are haircut.

$$\int_0^\tau (R_t - C_t - X_t)\, dt = 0.$$

Timeline. The periods before time 0 are the preparation stage that specify the default scenario. When the price of the derivatives contract changes, the sellers' positions are out-of-money. The sellers cannot meet their margin calls and default on the CCP. The CCP has to take over the sellers' positions and inherits the associated losses. When the losses exhaust the CCP's prefunded resources, the CCP starts the recovery process; this is time period 0. The CCP can use either cash calls or VMGH. Facing market and liquidity risks, the resolution authority can resolve the CCP any moment after time 0. Depending on whether the state variables hit the resolution trigger first or the successful recovery criteria are met first, the CCP is either resolved or successfully recovered. Below is an example of a CCP that clears plain-vanilla interest rate swap (IRS).



3 Optimal timing of entry into CCP resolution

In this section, we solve the optimal stopping problem faced by the resolution authority in the event of cash calls and in the event of VMGH, following the literature in real options (Dixit and Pindyck, 1994). The standard procedure is to write out the Hamilton-Jacob-Bellman (HJB) equation first; then based on the HJB equation, one could find out the differential equations and boundary conditions that characterize the value function. For specific differential equations, there are explicit solutions to figure out the value function and the associated thresholds of the state variables, which enables to spell out the analytical form of the optimal stopping time, i,e, the optimal timing of entry into CCP resolution in our setup.

3.1 Cash calls

When the CCP's recovery tool is cash calls to the surviving members, the CCP will set the size of the initial cash calls equal to the size of the initial mark-to-market losses, i.e., $R_0 = X_0$. The reason is that as X_t is a geometric Brownian motion, which is a martingale process. It means that the best estimate for the future value of X_t is the current value. Hence, the CCP at time 0 expects to recover the losses after one period if all the surviving members honor their commitments for the cash calls.

However, during period 0 and 1, a ε proportion of the surviving members may get hit by a liquidity shock, which arrives as a Poisson process. In the event of the cash calls, these members will undergo financial stress and not be able to meet their cash call obligation to the CCP. Hence, the costs associated with the recovery tool C_t will be passed to the CCP. In other words, when a liquidity event happens, the cash inflow of the CCP from the cash calls will decrease by the size of dC_t, which is the non-performance risk faced by the resolution authority. Given the non-performance risk, the effective cash inflow of the CCP is $R_t - C_t$. Let $\tilde{R}_t$ denote the effective cash inflow, one could have $\tilde{R}_t$ as a Poisson jump process with initial value $\tilde{R}_0 = R_0$:

$$d\tilde{R}_t = -\varepsilon \tilde{R}_t dN_t$$

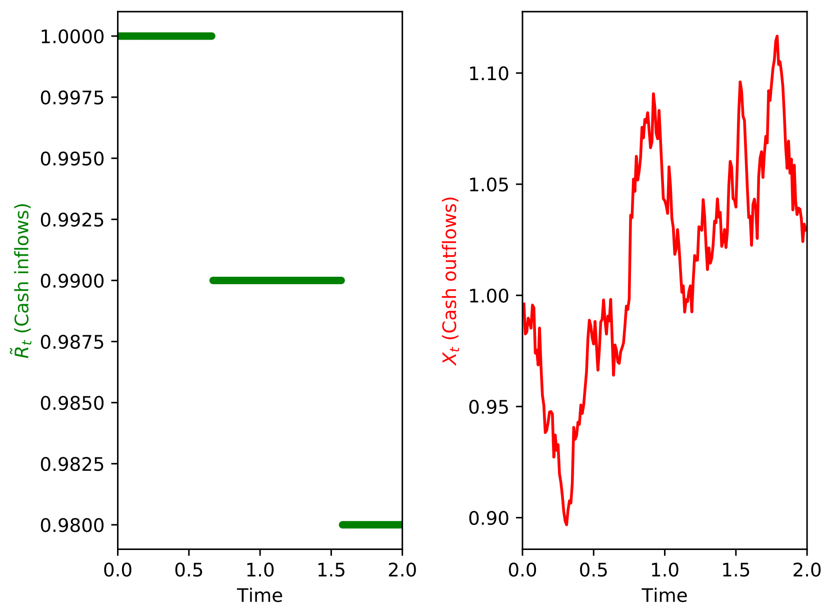
The resolution authority's problem in equation 2 can be written as:

$$\max_T E\left[\int_0^T \left(\tilde{R}_t - X_t\right) dt + \left(e - l + \tilde{R}_T - X_T\right)\right] := F(\tilde{R}, X). \tag{3}$$

$F(\tilde{R}, X)$ denote the value of resolving a CCP in the event of cash calls. Figure 1 visualizes $\tilde{R}_t$ and X_t. The left subplot shows one simulation path of $\tilde{R}_t$, which is the liquidity risk following a Poisson jump process. The right subplot shows one simulation path of X_t, which is the market risk following geometric Brownian motion. The two processes are interlinked because of σ_t and λ_t, both increase in $G_t(=\frac{X_t}{\tilde{R}_t})$.

Figure 1: Liquidity risk and market risk (cash calls)

This figure visualizes the liquidity risk and market risk in the event of cash calls. The left subplot shows the effective cash inflow $\tilde{R}_t$ as a Poisson jump process. In this simulation, there are two jumps, each hit 1% of the surviving members. The right subplot shows the cash outflow X_t as a geometric Brownian motion. In this simulation, the cash outflow of the CCP first reduces before $t = 0.5$ and then increases later. The two processes are interlinked through the governing parameters: jump intensity λ_t for the Poisson process and volatility σ_t for the geometric Brownian motion. Both increase in the ratio between the cash outflow and the effective cash inflow.



The solution to the optimization problem is a rule to maximize $F(\tilde{R}, X)$, which is a set of jointly determined thresholds for the state variables $(\tilde{R}, X)$ that separate the recovery region from the resolution region. The Hamilton-Jacob-Bellman (HJB) equation to the problem is

$$F(\tilde{R}_t, X_t) = \max\left[\underbrace{\left(\tilde{R}_t - X_t\right)dt + E[F(\tilde{R}_t, X_t) + dF(\tilde{R}_t, X_t)]}_{\text{Recovery}}, \underbrace{\left(e - l + \tilde{R}_T - X_T\right)}_{\text{Resolution}}\right] \tag{4}$$

For some range of values of the state variable $(\tilde{R}, X)$, the maximum on the right hand side of the HJB equation will be achieved by CCP resolution, and for other values of $(\tilde{R}, X)$ it will be achieved by continuing the recovery process. In the continuation region, the first term on the right hand side is the larger one of the two. Hence, one could have

$$\left(\tilde{R}_t - X_t\right)dt + E[dF(\tilde{R}, X)] = 0. \tag{5}$$

In the resolution region, the second term on the right hand side is the larger one of the two, which defines the boundary conditions: value-matching condition and smooth pasting condition. The former one matches the value function to the resolution payoff, while the latter one ensures that the values of $F(\tilde{R}, X)$ and the values of the resolution payoff meet tangentially at the boundary $(\tilde{R}^*, X^*)$.

$$F(\tilde{R}^*, X^*) = e - l + \tilde{R}^* - X^*$$

$$F_{\tilde{R}}(\tilde{R}^*, X^*) = 1, \quad F_X(\tilde{R}^*, X^*) = -1$$

Intuitively, it is optimal to resolve the CCP when $\tilde{R}_t$ is very small or when X_t is very large. Hence, assuming homogeneity of degree one, we could reduce the number of state variables to one. Let $G_t = \frac{X_t}{\tilde{R}_t}$, the value of resolving the CCP can be rewritten as follows:

$$F(\tilde{R}, X) = \tilde{R} f(\frac{X}{\tilde{R}}) = \tilde{R} f(G).$$

Divided by $\tilde{R}_t$ on both sides, HJB equation 5 can be rewritten as follows:

$$(1 - G_t)dt + E[df(G_t)] = 0 \tag{6}$$

To derive the explicit functional form of $f(G_t)$ from the HJB equation,we need to first find out $E[df(G_t)]$ following the generalized Ito's Lemma for Levy processes (see also Merton, 1976; McDonald and Siegel, 1986; Biais et al., 2010). Lemma 1 presents the expected change of the value function $E[df(G_t)]$. The first term on the right hand side comes from the uncertainty of the derivatives market. The second term is from the downside risk of the Poisson jump process.

Lemma 1. *Applying Ito's lemma for Levy processes,*

$$E[df(G_t)] = \underbrace{\frac{1}{2}\sigma^2 G_t^3 f''(G_t)dt}_{\text{Market risk}} + \underbrace{\lambda G_t[(1-\varepsilon)f(G_t/(1-\varepsilon)) - f(G_t)]dt}_{\text{Liquidity risk}}. \tag{7}$$

Proof. See appendix.

Based on lemma 1, HJB equation 6 could be written as a delay differential equation (DDE)[6]:

$$\frac{1}{G_t} - 1 + \frac{1}{2}\sigma^2 G_t^2 f''(G_t) + \lambda[(1-\varepsilon)f(G_t/(1-\varepsilon)) - f(G_t)] = 0.$$

[6]For detail explanation on DDE, interested readers could refer to Kuang (1993) and Balachandran, Kalmár-Nagy, and Gilsinn (2009)

Moreover, for the resolution region, the boundary conditions from HJB equation 5 could be rewritten as following based on the homogeneity of degree one assumption.

$$
\begin{aligned}
&f(G^*) = \frac{k}{(1-\varepsilon)^K} + 1 - G^*, \quad K = 0, 1, 2, ...,\\
&f'(G^*) = -1,
\end{aligned}
$$

where k denote the ratio between $e - l$ and $\tilde{R}_0$, and K denote the number of jumps when the resolution, if there is any, happens. With the DDE and the boundary conditions, the value function $f(G)$ can be characterized in Proposition 1.

Proposition 1. ***Value function (cash calls)***

The value function f is twice continuously differentiable and satisfies the following delay differential equation (DDE):

$$
\frac{1}{G_t} - 1 + \frac{1}{2}\sigma^2 G_t^2 f''(G_t) + \lambda[(1-\varepsilon)f(G_t/(1-\varepsilon)) - f(G_t)] = 0, \tag{8}
$$

together with the boundary conditions:

$$
\begin{aligned}
&f(G^*) = \frac{k}{(1-\varepsilon)^K} + 1 - G^*, \quad K = 0, 1, 2, ...,\\
&f'(G^*) = -1,
\end{aligned}
$$

where G^* is the resolution threshold.

Proof. See appendix.

To derive the explicit form of $f(.)$, proposition 2 solves the value function and presents the optimal timing of entry into resolution G^* in the event of cash calls.

Proposition 2. ***Optimal timing of entry into resolution (cash calls)***

The specific functional form of $f(G)$ could be written as:

$$f(G) = A_1 G^{\beta} + A_2 G^{-1} + A_3 \tag{9}$$

where $A_1 = -\frac{A_2 (G^*)^{-2}+1}{\beta}(G^*)^{1-\beta}, A_2 = \frac{1}{\varepsilon\lambda(2-\varepsilon)-\sigma^2}, A_3 = -\frac{1}{\varepsilon\lambda}$, *and* β *is the negative solution to the following equation:*

$$\frac{1}{2}\sigma^2\beta(\beta-1) + \lambda(1-\varepsilon)^{1-\beta} - \lambda = 0;$$

It is optimal to resolve the CCP when $G_t \geq G^*$ *where*

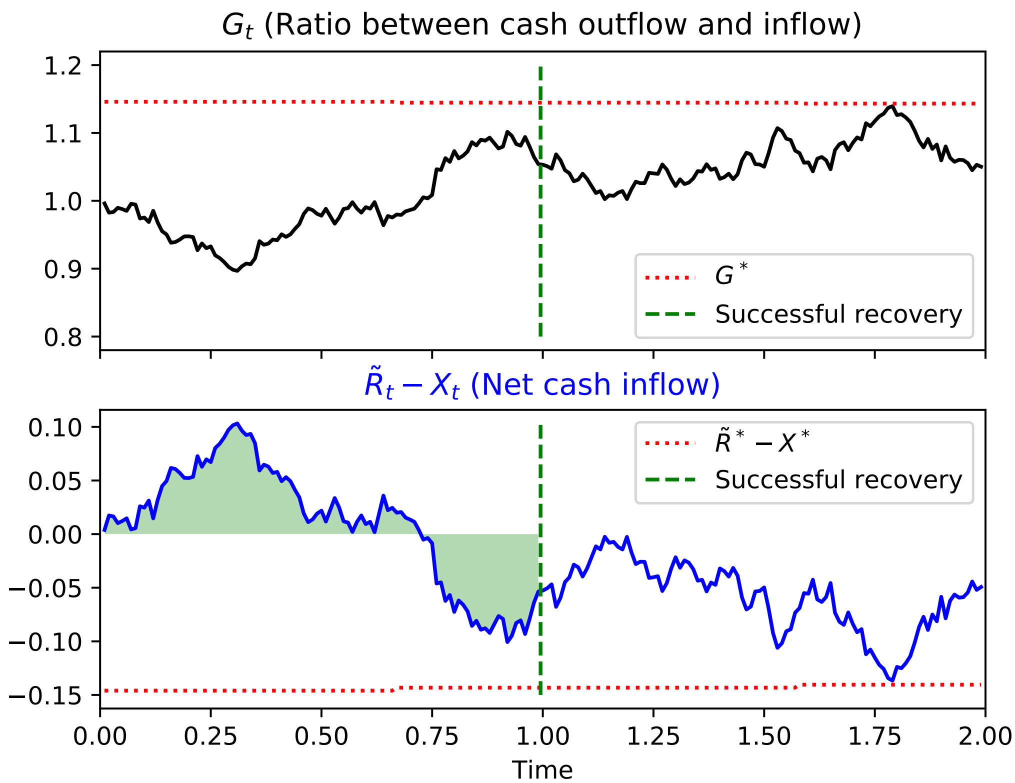
$$G^* = \frac{\frac{\beta}{\beta-1}\left(\frac{1}{\varepsilon\lambda} + \frac{k}{(1-\varepsilon)^K} + 1\right) + \sqrt{\left(\frac{\beta}{\beta-1}\right)^2\left(\frac{1}{\varepsilon\lambda} + \frac{k}{(1-\varepsilon)^K} + 1\right)^2 - \frac{4}{\varepsilon\lambda(2-\varepsilon)-\sigma^2}}}{2} \tag{10}$$

Proof. See appendix.

Figure 2 shows G_t and $\tilde{R}_t - X_t$ following the simulation paths in Figure 1. The red dotted lines show the resolution thresholds defined in Proposition 2. The green dashed lines show the timing of successful recovery. At the beginning of the simulation, there are no defaults among the surviving clearing members and the CCP's cash outflow X_t decreases, which brings the ratio between the CCP's cash outflow and inflow below 1 and the net cash inflow above 0. But as X_t increases after $t = 0.25$, the ratio climbs back to 1 and the net cash inflow decreases to 0. At around $t = 0.5$, the first liquidity event happens and a proportion of the surviving members default. At around $t = 0.75$, the ratio gets larger than1 and the net cash inflow becomes negative, since the cash outflow of the CCP increases largely. But fortunately, the ratio G_t hits the resolution trigger after it meets the successful recovery criteria. In other words, in this simulation, the CCP is successfully recovered.

Figure 2: Optimal resolution timing (cash calls)

This figure shows the resolution thresholds and the successful recovery timing based on the same parameters and the same simulation realizations in figure 1. The upper subplot shows the ratio between the CCP cash outflow and the effective cash inflow, i.e., G_t. The lower subplot shows the associated net cash inflow. Obviously, when $G_t < 1$, $\tilde{R}_t - X_t > 0$ and vice versa. The red dotted lines show the resolution thresholds and the green dashed lines show the successful recovery timing.



3.2 VMGH

When the CCP's recovery tool is VMGH, the CCP can haircut the variation margin that should be received by the surviving members. Unlike cash calls, VMGH allows the CCP to directly reduce the CCP's liability to the surviving members. In the event of VMGH, the CCP's instantaneous cash inflow always equals to the CCP's instantaneous cash outflow: $R_t dt = X_t dt$. This means that the ratio between the CCP's cash outflow and the CCP's cash inflow is always 1. Hence, the optimal stopping problem in the event of VMGH will be unaffected by the interlinkage between funding risk and market risk. It also means that the CCP's cash inflow R_t follows a geometric Brownian motion:

$$dR_t = \sigma R_t dz_t.$$

As the CCP can directly reduce its own liability with the surviving members with in-the-money positions in the derivatives market, the surviving members cannot pass their losses to the CCP when they get hit by the liquidity shocks. Hence, the surviving members have to bear the losses. The size of the losses depends on the amount of the variation margin that is haircut by the CCP, which is equal to the CCP's cash inflow. Hence, the surviving members' losses follow a Poisson jump process: $dC_t = \varepsilon R_t dN_t$. The intensity of jump arrivals is λ_t while the jump size is εR_t which follows the geometric Brownian motion. Hence, the jump size is a log-normally distributed random variable with expected value and variance given by[7]:

$$E(\varepsilon R_t) = \varepsilon R_0,$$
$$Var(\varepsilon R_t) = \varepsilon^2 R_0^2 \left(e^{\sigma_t^2 t} - 1\right).$$

The resolution authority optimize the overall value of the financial system (i.e., it takes into account the value of both the CCP's and the surviving members). Hence, if the losses to clearing members are too large, the authority may step in and resolve the CCP, even if the CCP can successfully recover by reducing his own liability. The optimal stopping

[7] R_t is log-normal distributed: $ln(R_t) \sim N(ln(R_0), \sigma^2 t)$.

problem for the resolution authority is

$$\max_{T} E\left[\int_0^T (-C_t)\, dt + (e - l - C_T)\right] := V(C) \tag{11}$$

$V(C)$ denotes the value of resolving a CCP in the event of VMGH. Similar to the event of cash calls, the HJB equation to the problem in the event of VMGH is

$$V(C_t) = \max\left[\underbrace{(-C_t dt + E[V(C_t) + dV(C_t)])}_{\text{Recovery}}, \underbrace{(e - l - C_T)}_{\text{Resolution}}\right] \tag{12}$$

For some range of values of C_t, the maximum on the right hand side of the HJB equation will be achieved by CCP recovery and for other values of C_t it will be achieved by resolving the CCP. In the recovery region, equation 12 will be

$$-C_t dt + E[dV(C_t)] = 0. \tag{13}$$

To solve the HJB equation, we first derive the expected change of the value function. Lemma 2 presents the expected change of the value function $E[dV(C_t)]$. Different from equation 7, equation 14 integrates both market risk $(-\varepsilon R)$ and liquidity risk into one term.

Lemma 2. *Applying Ito's lemma for Levy processes, one could have*

$$E[dV(C_t)] = \lambda E\left[V(C_t - \varepsilon R_t) - V(C_t)\right] dt, \tag{14}$$

Proof. See appendix.

In the resolution region, equation 12 defines the value-matching condition at the boundary C^*[8]:

[8] Since C_t is a jump process, there is no smooth pasting condition.

$$V(C^*) = e - l - C^*. \tag{15}$$

Rewrite $E[dV(C_t)]$ in equation 13, coupled with the value-matching condition, one could characterize the value function $V(C)$. Proposition 3 summarizes the characteristics of functional form for the value function $V(.)$.

Proposition 3. ***Value function (VMGH)***

The value function V is twice continuously differentiable and satisfies the following delay differential equation (DDE):

$$-C_t + \lambda E\left[V(C_t - \varepsilon R_t) - V(C_t)\right] = 0, \tag{16}$$

together with the boundary condition:

$$V(C^*) = e - l - C^*, \quad K = 0, 1, 2, ...,$$

where C^* is the resolution threshold.

Proof. See appendix.

To derive the explicit form of $V(.)$, proposition 4 solves the value function and presents the optimal timing of entry into resolution in the event of VMGH.

Proposition 4. ***Optimal timing of entry into resolution (VMGH)*** *The specific functional form of $V(C)$ could be written as:*

$$V(C) = A_4 C^2 + A_5 C. \tag{17}$$

where $A_4 = -\frac{1}{2\varepsilon\lambda R_0} e^{-\frac{\sigma^2}{2}t}$ and $A_5 = -\frac{1}{2\lambda} e^{\sigma^2 t}$.

It is optimal to resolve the CCP when $C_t \geq C^*$ where

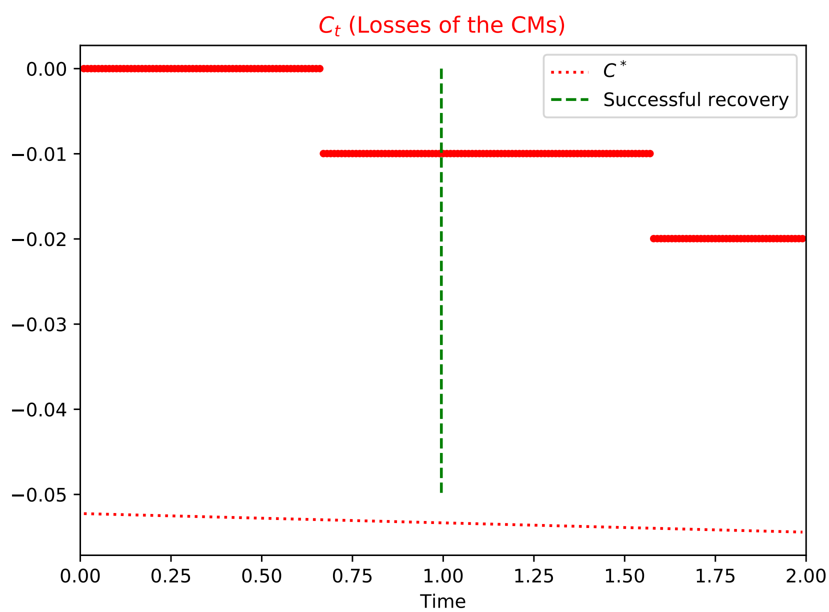
$$C^* = \frac{-(A_5+1) - \sqrt{(A_5+1)^2 - 4A_4(e-l)}}{2A_4} \tag{18}$$

Proof. See appendix.

Figure 3 shows the surviving members' losses $-C_t$. The red dotted line is the threshold that triggers resolution, as defined in Proposition 4. The green dash line is the timing of successful recovery. In this simulation, the resolution threshold is far below the losses born by the surviving CMs, and therefore resolution is not triggered by the authority.

Figure 3: Losses born by the surviving members (VMGH)

This figure shows the losses born by the surviving members, originating from VMGH. The red dotted line shows the resolution threshold and the green dashed line shows the successful recovery timing.



4 Additional resources dedicated to CCP resolution

As noted in the introduction, a key current policy discussion is about the need for additional resources dedicated to CCP resolution. The main argument for having these additional resources set aside is that they could alleviate the burden of the resolution authority, and potentially avoid (or minimize) usage of public funds. However, the financing of these resources would incur costs that may disincentify central clearing. Our model does not take a view on the appropriateness/need for additional resources but rather we examine the potential impact of these resources on the optimal resolution timing[9].

4.1 Comparative statics

Suppose the size of the additional resources dedicated to CCP resolution is Δe. In this case, the resources that the resolution authority can get are both the CCP's equity and the additional resources: $e + \Delta e$. Hence, one way to investigate the impact of the additional resources dedicated to CCP resolution could be to study the comparative statics with respect to e.

Before getting to the comparative statics, we note that G^* is a jump-diffusion process through Lemma 3 (see also Hirsa and Neftci, 2013).

Lemma 3. *Applying the multivariate Ito's Lemma, one could have*

$$\frac{dG_t}{G_t} = \sigma_t dz_t + \frac{\varepsilon}{1-\varepsilon} dN_t. \tag{19}$$

[9]For example, in our current model, we assume that the prefunded resources dedicated to CCP resolution is costless and there is no incentive issues around central clearing. However, our results are still meaningful because they would not be weaken without these assumptions.

Proof. See appendix.

To study the comparative statics with respect to e, the key is to find out the impact of increasing e on the resolution thresholds: G^* in the event of cash calls and C^* in the event of VMGH. Proposition 5 summarizes the impact of the additional resources dedicated to CCP resolution.

Proposition 5. ***Comparative statics***

With increasing additional resources dedicated to CCP resolution,

(i) the resolution thresholds are more relaxed,

(ii) the expected time to resolution increases,

(iii) the likelihood of successful recovery increases,

(iv) the losses conditional on resolution increases.

Proof. See appendix.

The intuition is as follows. At each moment, the resolution authority compares the relative value of its two choices: to let the CCP run the recovery plan or to resolve the CCP. This is captured by the HJB equations. Pursuing the first choice implies that the total value of the CCP and the surviving members continues to be exposed to the market and funding uncertainties, while the second choice resolves these uncertainties. As discussed earlier and following the optimality principle in dynamic programming, the former choice specifies the differential equations and determines the form of the value function, while the latter choice specifies the boundary conditions. Additional resources for resolution only have impact on the second choice, and thus only affects the boundary conditions. Under the same uncertainties, if a resolution authority has more additional resources then it faces more "slack" boundary conditions. This translates into a more tolerant resolution authority. Thus, the CCP is more likely to meet the successful recovery criteria before hitting the resolution thresholds. However, because the resolution authority is more tolerant, when resolution is triggered, the losses that the resolution authority will inherit are larger. The implication is that, with one more unit prefunded resources dedicated to resolution, the increase of the total value at resolution is less than one unit. Hence, having more additional resources dedicated to CCP resolution can be viewed as a risky investment: high return (i.e., high likelihood of CCP recovery) is always coupled with high risk (i.e., large losses conditional resolution).

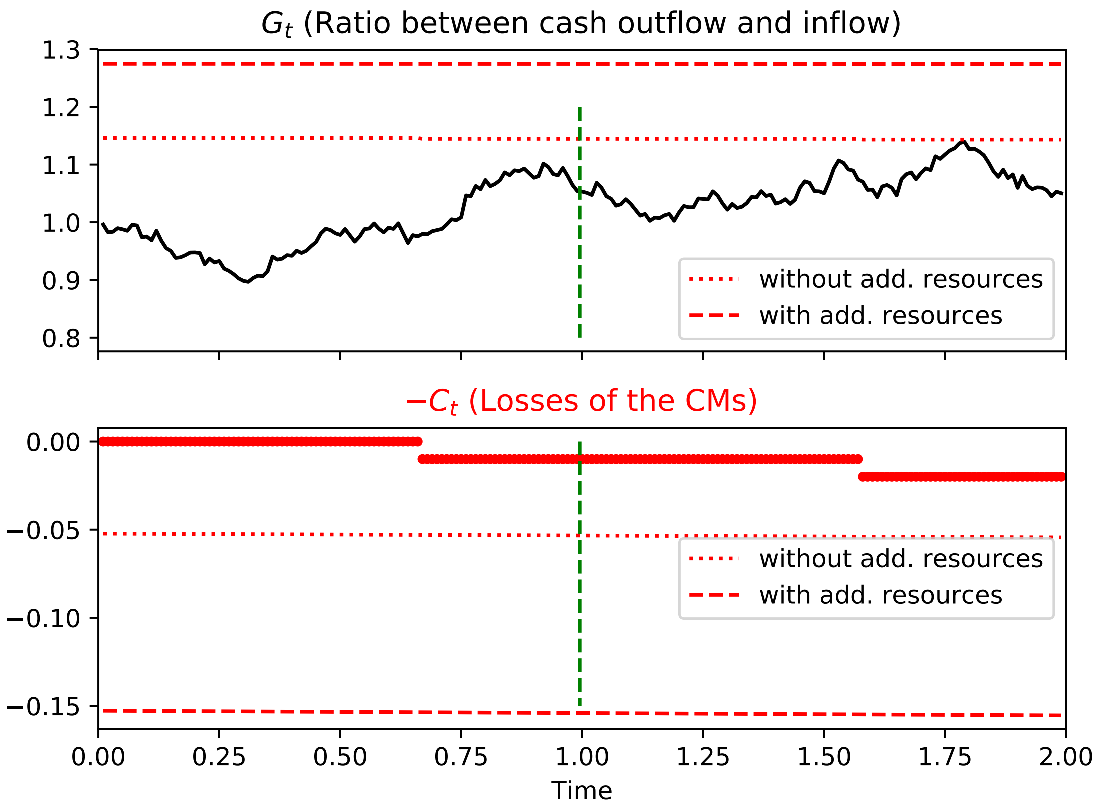
4.2 Numerical example

To illustrate the impact of the additional resources dedicated to CCP resolution, we establish a set of parameters as a base case: $\sigma = 0.1, \varepsilon = 0.1, \lambda = 1, e - l = 1, R_0 = X_0 = 10$. In the base case, we take $\sigma = 0.1$, which means that $ln(X_t)$ has a variance of 1 percent per period. Also, we assume that the liquidity shocks, on expectation, come once per period. When the surviving members are hit by the liquidity shock, 10 percent of them will suffer losses. In the event of cash calls, they will default and pass the costs to the CCP. In the event of VMGH, they will have to bear the costs by themselves. When the authority decides to resolve the CCP, the authority could seize the CCP's equity but also suffer some liquidation inefficiency. The net outcome of these two effects $(e - l)$ is one unit, which is only 10 percent of the initial losses.

Figure 4 shows the impact of additional resources on the resolution threshold in the event of cash calls and VMGH. The numerical results based on the above parameters show that the resolution thresholds are higher for both cash calls and VMGH, i.e.,that the resolution authority is more tolerant when additional resources are available.

Figure 4: Additional resources dedicated to resolution

This figure shows the impact of additional resources dedicated to CCP resolution. The red dashed lines are the resolution thresholds when there are additional resources and the red dotted lines are the resolution thresholds when there are no additional resources. The upper subplot shows the case with cash calls and the lower subplot shows the case of VMGH.



5 Conclusion

We develop a real option model to capture the dynamics between CCP recovery and resolution. There are two important features of the model: (i) the interactions between market risk and liquidity risk during a recovery process, and (ii) the cost associated with the CCP recovery tools on the clearing members. Thus, there is a conflict between the CCP's interest and the clearing members' interests: Pursuing the CCP recovery plan in full can save the CCP but will hurt the clearing members, which calls for a resolution authority that can choose an optimal timing of intervention to maximize both the CCP's value and the clearing members' value.

Based on the model, we also investigate the impact of additional resources dedicated to CCP resolution. It turns out that increasing additional resources dedicated to CCP resolution will lead to more slack resolution thresholds, in both cases of cash calls and VMGH. It means that the resolution authority will be more tolerant, hence with a larger expected time to resolution. In other words, the likelihood that the CCP will recover increases. But, *ex post*, if the resolution thresholds are triggered before the CCP can successfully recover, the losses that born by the resolution authority will be larger.

While our model is a useful guide to understand the dynamics between the recovery and resolution of a CCP, it can be extended in several ways. One possible extension could be to incorporate the market structure of the clearing industry. The clearing industry of different products could be very different. For instance, clearing for equity is rather competitive, while clearing for interest rate derivatives is highly concentrated. It would be interesting to investigate the impact of the market structure on resolution timing. Another possible extension would be to consider the heterogeneity among clearing members as shown in Armakola and Laurent (2015).

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Appendix

A Variable summary

Variable	Definition
λ	Poisson jump intensity of liquidity shock arrival
σ	volatility of the CCP's cash outflow
ε	proportion of vulnerable surviving member
e	equity of the CCP that could be seized by the resolution authority
l	liquidation inefficiency when the authority resolves the CCP
k	ratio between $e-l$ and R_0
R_t	cash inflow of the CCP
X_t	cash outflow of the CCP
C_t	cost because of the liquidity shock
G_t	ratio between X_t and R_t

B Proof

Lemma 1.

Proof. First, the (multivariate) Ito's Lemma for Levy process $dX_{i,t} = \underbrace{a_i dt + b_i dz_{i,t}}_{dX^c_{i,t}} + c_i dN_{i,t},\quad i = 1, 2$ is the following:

$$
\begin{aligned}
df(X_{1,t}, X_{2,t}) = {} & \frac{\partial f}{\partial X_{1,t}} dX^c_{1,t} + \frac{\partial f}{\partial X_{2,t}} dX^c_{2,t} + \dots \\
& + \frac{1}{2}\frac{\partial^2 f}{\partial X^2_{1,t}} d\langle X_{1,t}\rangle^c_t + \frac{1}{2}\frac{\partial^2 f}{\partial X^2_{2,t}} d\langle X_{2,t}\rangle^c_t + \frac{\partial^2 f}{\partial X_{1,t}\partial X_{2,t}} d\langle X_{1,t}X_{2,t}\rangle^c_t + \dots \\
& + \left(f(X_{1,t}, X_{2,t}) - f(X_{1,t^-}, X_{2,t^-})\right) dN_t
\end{aligned} \tag{A1}
$$

Given that $d\tilde{R}_t = -\varepsilon \tilde{R}_t dN_t$ and $dX_t = \sigma_t X_t dz_t$, the (multivariate) Ito's Lemma for Levy process leads to

$$
\begin{aligned}
dF(\tilde{R}, X) &= F_X dX_t + \frac{1}{2} F_{XX} d\langle X_t \rangle_t + (F((1-\varepsilon)\tilde{R}, X) - F(\tilde{R}, X))dN_t \\
&= F_X \sigma_t X_t dz_t + \frac{1}{2} F_{XX} \sigma_t^2 X_t^2 dt + (F((1-\varepsilon)\tilde{R}, X) - F(\tilde{R}, X))dN_t
\end{aligned} \tag{A2}
$$

Assuming homogeneous of degree 1, $F(\tilde{R}, X) = \tilde{R} f(G)$. It leads to

$$
F_X = f'(G), \quad F_{XX} = f''(G)/\tilde{R}, \quad F((1-\varepsilon)\tilde{R}, X) = (1-\varepsilon)\tilde{R} f(G/(1-\varepsilon)).
$$

Hence, $dF(\tilde{R}, X) = \tilde{R} df(G)$ where

$$
\begin{aligned}
dF(\tilde{R}, X) &= F_X \sigma_t X_t dz_t + \frac{1}{2} F_{XX} \sigma_t^2 X_t^2 dt + (F((1-\varepsilon)\tilde{R}, X) - F(\tilde{R}, X))dN_t \\
&= f'(G)\sigma_t X_t dz_t + \frac{1}{2} f''(G)\sigma_t^2 \frac{X_t^2}{\tilde{R}} dt + ((1-\varepsilon)\tilde{R} f(G/(1-\varepsilon)) - \tilde{R} f(G))dN_t \\
&= \tilde{R}\left[f'(G)\sigma_t G_t dz_t + \frac{1}{2} f''(G)\sigma_t^2 G_t^2 dt + ((1-\varepsilon) f(G/(1-\varepsilon)) - f(G))dN_t \right]
\end{aligned} \tag{A3}
$$

Dividing equation 7 by $\tilde{R}$ and take expectation, one could have the following because $\sigma_t^2 = \sigma^2 G_t$ and $\lambda_t = \lambda G_t$

$$
\begin{aligned}
E[df(G)] &= \frac{1}{2} f''(G)\sigma_t^2 G_t^2 dt + \lambda_t((1-\varepsilon) f(G/(1-\varepsilon)) - f(G))dt \\
&= \frac{1}{2} f''(G)\sigma^2 G_t^3 dt + \lambda G_t((1-\varepsilon) f(G/(1-\varepsilon)) - f(G))dt.
\end{aligned} \tag{A4}
$$

■

Proposition 1.

Proof.

Plug equation 7 into equation 6 and divide both sides of the equation by G_t, one could have equation 8 which defines how the value function should evolve. The boundary conditions are derived from the value at resolution:

$$F(\tilde{R}_T, X_T) = \tilde{R}_T f(G_T) = e - l + \tilde{R}_T - X_T.$$

Dividing both sides of the equation by $\tilde{R}_T$, one could have

$$f(G_T) = \frac{k}{(1-\varepsilon)^K} + 1 - G_T.$$

Replacing G_T with G^*, one could have the first boundary condition, the value matching condition. Taking the first-order derivative with respect to G^*, one could have the second boundary condition, the smooth pasting condition.

■

Proposition 2.

Proof.

The homogeneous solution to equation 8 is $A_1 G^\beta$ and the specific solution to it is $(A_2 G^{-1} + A_3)$ where β, A_1, A_2, and A_3 are the parameters that will meet the boundary conditions. Hence, the specific functional form of $f(G)$ could be written as:

$$f(G) = A_1 G^\beta + A_2 G^{-1} + A_3 \tag{A5}$$

$$f''(G) = \beta(\beta-1)A_1 G^{\beta-2} + 2A_2 G^{-3}$$

$$f(G/(1-\varepsilon)) = A_1 G^\beta (1-\varepsilon)^{-\beta} + A_2(1-\varepsilon)G^{-1} + A_3$$

Hence, equation 8 can be rewritten as

$$
\begin{aligned}
&\frac{1}{G} - 1 + \frac{1}{2}\beta(\beta-1)\sigma^2 A_1 G^{\beta} + \sigma^2 A_2 G^{-1} \\
&+ \lambda\left[(1-\varepsilon)\left(A_1 G^{\beta}(1-\varepsilon)^{-\beta} + A_2(1-\varepsilon)G^{-1} + A_3\right) - \left(A_1 G^{\beta} + A_2 G^{-1} + A_3\right)\right] &= 0 \\
&A_1 G^{\beta} \underbrace{\left[\frac{1}{2}\beta(\beta-1)\sigma^2 + \lambda(1-\varepsilon)^{1-\beta} - \lambda\right]}_{0} + \frac{1}{G}\underbrace{\left[A_2(\sigma^2 + \lambda(1-\varepsilon)^2 - \lambda) + 1\right]}_{0} - \underbrace{(1+\varepsilon\lambda A_3)}_{0} &= 0
\end{aligned}
\tag{A6}
$$

From the DDE, β is one of the solution to the following equation:

$$
\frac{1}{2}\sigma^2\beta(\beta-1) + \lambda(1-\varepsilon)^{1-\beta} - \lambda = 0;
$$

and $A_2 = \frac{1}{\varepsilon\lambda(2-\varepsilon)-\sigma^2}, A_3 = -\frac{1}{\varepsilon\lambda}$.

Also, when G, the ratio between the CCP's cash outflow and the CCP's cash inflow, goes to infinite, the CCP is very unlikely to recover. Hence, the value of waiting should approach to zero, which means that β should be the negative solution to the above equation.

From the smooth pasting condition, one could have

$$
f'(G^*) = A_1\beta(G^*)^{\beta-1} - A_2(G^*)^{-2} = -1 \tag{A7}
$$

Hence, $A_1 = -\frac{A_2(G^*)^{-2}+1}{\beta}(G^*)^{1-\beta}$. Plug A_1, A_2, A_3 in the value matching condition, one could have

$$
\begin{aligned}
f(G^*) &= A_1(G^*)^{\beta} + A_2(G^*)^{-1} + A_3 \\
&= -\frac{A_2(G^*)^{-2}+1}{\beta}(G^*) + A_2(G^*)^{-1} + A_3 \\
&= \frac{k}{(1-\varepsilon)^K} + 1 - G^*
\end{aligned}
\tag{A8}
$$

Reorganize the equation above, one could have

$$(G^*)^2 + \frac{\beta}{\beta - 1}\left(A_3 - \frac{k}{(1-\varepsilon)^K} - 1\right)G^* + A_2 = 0 \tag{A9}$$

Since G is always positive, G^* should be the positive solution to the above equation, which is

$$G^* = \frac{\frac{\beta}{\beta-1}\left(\frac{1}{\varepsilon\lambda} + \frac{k}{(1-\varepsilon)^K} + 1\right) + \sqrt{\left(\frac{\beta}{\beta-1}\right)^2\left(\frac{1}{\varepsilon\lambda} + \frac{k}{(1-\varepsilon)^K} + 1\right)^2 - \frac{4}{\varepsilon\lambda(2-\varepsilon)-\sigma^2}}}{2} \tag{A10}$$

■

Lemma 2.

Proof.

By equation A1, the change of value function could be written as

$$dV(C_t) = (V(C_t - \varepsilon R_t) - V(C_t))\, dN_t.$$

Taking expectation, one could have

$$E[dV(C_t)] = \lambda\,(E[V(C_t - \varepsilon R_t) - V(C_t)])\, dt.$$

Because G_t, the ratio between the CCP's cash outflow and the CCP's cash inflow, is always 1.

■

Proposition 3.

Proof.

Plug in equation 14 into equation 13, one could have the DDE:

$$-C_t + \lambda E\left[V(C_t - \varepsilon R_t) - V(C_t)\right] = 0.$$

The boundary condition is also specified in equation 15.

■

Proposition 4.

Proof.

Observe equation 16, one could guess a possible functional form to solve the DDE is:

$$V(C) = A_4C^2 + A_5C. \tag{A11}$$

With this functional form, one could have

$$\begin{aligned} E[V(C_t - \varepsilon R_t) - V(C_t)] &= E[(A_4(C_t - \varepsilon R_t)^2 + A_5(C_t - \varepsilon R_t)) - (A_4C_t^2 + A_5C_t)] \\ &= E[A_4(\varepsilon R_t)^2 - 2A_4C_t(\varepsilon R_t) - A_5\varepsilon R_t] \\ &= A_4\varepsilon^2 E[R_t^2] - 2A_4\varepsilon E[C_tR_t] - A_5\varepsilon E[R_t] \end{aligned} \tag{A12}$$

Since R_t follows a geometric Brownian motion that is independent from the Poisson jump process, following Dunbar (2016) and Privault (2013), one could have the following:

$$\begin{aligned} E[R_t] &= R_0e^{\frac{\sigma^2}{2}t}; \\ E[R_t^2] &= R_0^2e^{2\sigma^2 t}; \\ E[C_tR_t] &= C_tE[R_t], \\ &= C_tR_0e^{\frac{\sigma^2}{2}t}. \end{aligned} \tag{A13}$$

Plug the above equations into equation A12, it could be rewritten as

$$E[V(C_t - \varepsilon R_t) - V(C_t)] = A_4\varepsilon^2R_0^2e^{2\sigma^2 t} - 2A_4\varepsilon C_tR_0e^{\frac{\sigma^2}{2}t} - A_5\varepsilon R_0e^{\frac{\sigma^2}{2}t} \tag{A14}$$

Hence, DDE in the event of VMGH could be rewritten as

$$
\begin{aligned}
&-C_t + \lambda\left(A_4\varepsilon^2 R_0^2 e^{2\sigma^2 t} - 2A_4\varepsilon C_t R_0 e^{\frac{\sigma^2}{2}t} - A_5\varepsilon R_0 e^{\frac{\sigma^2}{2}t}\right) = 0,\\
&-C_t \underbrace{\left[1 + 2A_4\varepsilon\lambda R_0 e^{\frac{\sigma^2}{2}t}\right]}_{0} + \lambda\underbrace{\left(A_4\varepsilon^2 R_0^2 e^{2\sigma^2 t} - A_5\varepsilon R_0 e^{\frac{\sigma^2}{2}t}\right)}_{0} = 0
\end{aligned}
\tag{A15}
$$

Hence, $A_4 = -\frac{1}{2\varepsilon\lambda R_0} e^{-\frac{\sigma^2}{2}t}$ and $A_5 = -\frac{1}{2\lambda} e^{\sigma^2 t}$.

The optimal timing of entry into resolution in the event of VMGH could be solved by the boundary condition:

$$
\begin{aligned}
V(C^*) &= e - l - C^*,\\
&= A_4(C^*)^2 + A_5 C^*
\end{aligned}
$$

Collecting the terms, one could have C^* as the positive solution to

$$
A_4(C^*)^2 + (A_5 + 1)C^* - (e - l). \tag{A16}
$$

Hence, the explicit form of C^* is

$$
C^* = \frac{-(A_5 + 1) - \sqrt{(A_5 + 1)^2 - 4A_4(e - l)}}{2A_4}. \tag{A17}
$$

■

Lemma 3.

Proof.

First of all, G_t could be rewritten as $g(\tilde{R}, X)$:

$$g(\tilde{R}, X) = \frac{X}{\tilde{R}}.$$

Hence, $g_X = \frac{1}{\tilde{R}}, \quad g_{XX} = 0$. By Equation A1, the change of $g(.)$ could be written as

$$\begin{aligned} dg(\tilde{R}_t, X_t) &= g_X dX_t + \frac{1}{2} g_{XX} d\langle X_t \rangle_t + (g((1-\varepsilon)\tilde{R}, X) - g(\tilde{R}, X))dN_t \\ &= \sigma_t \frac{X_t}{\tilde{R}_t} dz_t + \left(\frac{X}{(1-\varepsilon)\tilde{R}} - \frac{X}{\tilde{R}} \right) dN_t \\ &= \sigma_t G_t dz_t + \frac{\varepsilon}{1-\varepsilon} G_t dN_t. \end{aligned} \tag{A18}$$

■

Proposition 5.

Proof.

First, let's look at the event of cash calls. The resolution threshold G^* should be the positive solution to equation A9, according to the proof in Proposition 2. Let $a(G^*)^2 + bG^* + c = 0$ denote equation A9. Apparently, G^* could be rewritted as

$$G^* = \frac{-b + \sqrt{b^2 - 4ac}}{2a}.$$

The first order derivative of G^* with respect to b is

$$\begin{aligned} \frac{dG^*}{db} &= \frac{-1 + \frac{b}{\sqrt{b^2-4ac}}}{2a}, \\ &< 0. \end{aligned} \tag{A19}$$

The inequality comes from the fact that $b = -\underbrace{\frac{\beta}{\beta - 1}}_{>0} \underbrace{\left(\frac{1}{\varepsilon\lambda} + \frac{k}{(1-\varepsilon)^K} + 1 \right)}_{>0} < 0.$

The first order derivative of b with respect to k is

$$
\begin{aligned}
\frac{db}{dk} &= -\frac{\beta}{\beta-1}\frac{1}{(1-\varepsilon)^K}, \\
&< 0.
\end{aligned} \tag{A20}
$$

Based on equation **??**, one could have the first order derivative of k with respect to e as

$$
\begin{aligned}
\frac{dk}{de} &= \frac{(1-\varepsilon)^K}{\tilde{R}_0}, \\
&> 0.
\end{aligned} \tag{A21}
$$

Hence, the first order derivative of G^* with respect to e is $\frac{dG^*}{db}\frac{db}{dk}\frac{dk}{de} > 0$. It means that when there is increasing additional resources dedicated to CCP resolution, the threshold G^*, the ratio between the CCP's cash outflow and the CCP's cash inflow, is larger.

Following Kou and Wang (2003), the expected time to resolution, i.e., the expected first passage time of a jump diffusion process G_t, increases in the threshold G^*. Hence, the larger threshold in the case of the increasing additional resources dedicated to CCP resolution leads to a larger expected time to resolution, which means that the CCP is more likely to meet the successful recovery criteria before hitting the resolution threshold. However, the losses conditional on resolution is

$$
\tilde{R}_T - X_T = \tilde{R}_T(1-G^*). \tag{A22}
$$

Given everything else unchanged, the larger G^* due to the increasing e give rises to a larger losses conditional on resolution.

Second, in the event of VMGH, the resolution threshold C^* is the positive solution equation A24 in the proof of Proposition 4. The first order derivatives of C^* with respect to e is

$$\frac{dC^*}{de} = \frac{1}{\sqrt{(A_5+1)^2 + 4A_4(e-l)}}, \qquad \text{(A23)}$$
$$> 0.$$

$$A_4(C^*)^2 + (A_5+1)C^* - (e-l). \qquad \text{(A24)}$$

Hence, when there is increasing additional resources dedicated to CCP resolution, the threshold C^* is larger. Given that C_t is a compound Poisson jump process, the larger threshold means a larger expected time to resolution and a higher probability that the CCP will meet the successful recovery criteria before hitting the resolution threshold. Since C^* is the losses conditional on resolution, when there is increasing resources dedicated to CCP resolution, these losses would be larger.

■