The Economics of Cryptocurrencies

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Economics of Payments IX

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Disclaimer: The views expressed are those of the authors and do not necessarily reflect the views of the Bank of Canada.

What We Do

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- A ledger of **digital** balances updated in a **decentralized** fashion.

2) We show that cryptocurrencies cannot achieve immediate and final settlement.

- Why? Need to avoid a **double spending problem**.

3) We evaluate the efficiency of a cryptocurrency system.

- **Positive inflation is optimal** while transaction fees should be minimized.
- Currently, **welfare loss in BITCOIN** of 1.4% of consumption, but potentially as low as 0.08%.

Why Cryptocurrency is Special?

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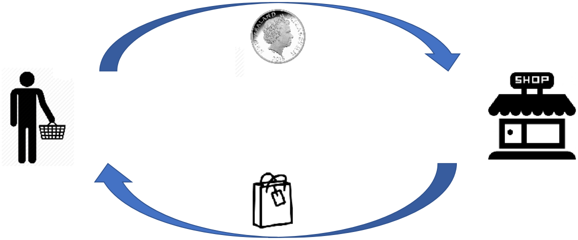
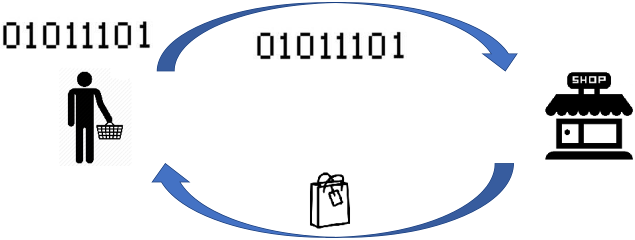
Physical Tokens



Immediate and Final Settlement

Why Cryptocurrency is Special?

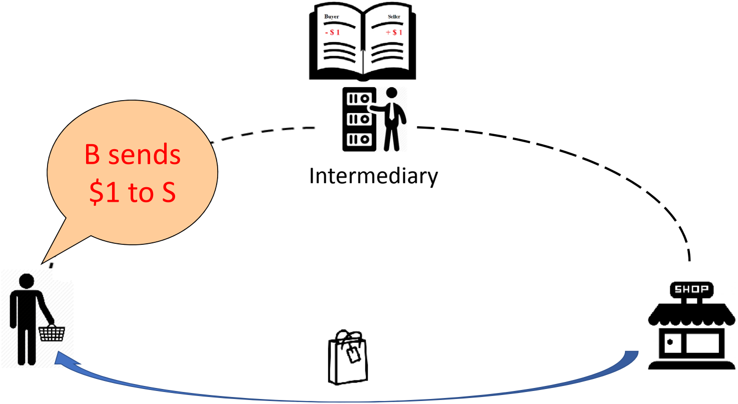
Digital Tokens



Subject to double-spending problems

Why Cryptocurrency is Special?

Digital Currency with Trusted Third Party



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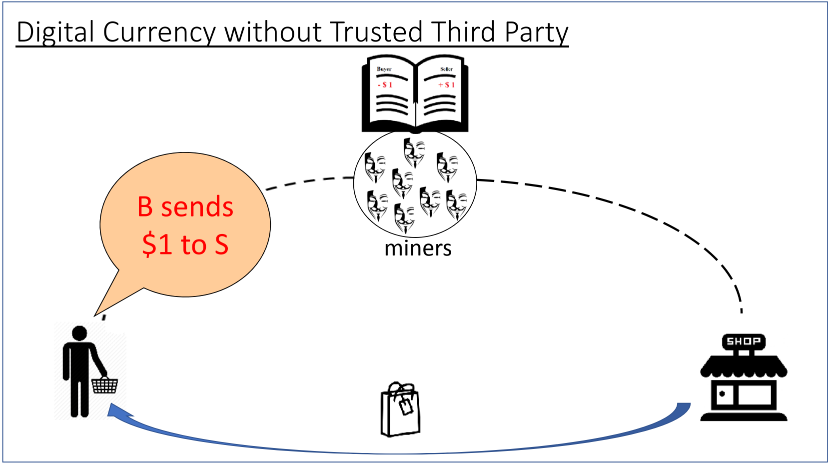

No central authority to keep record

How Cryptocurrency works

1. Consensus Protocol

2. Reward Scheme

3. Confirmation Lags

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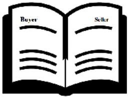
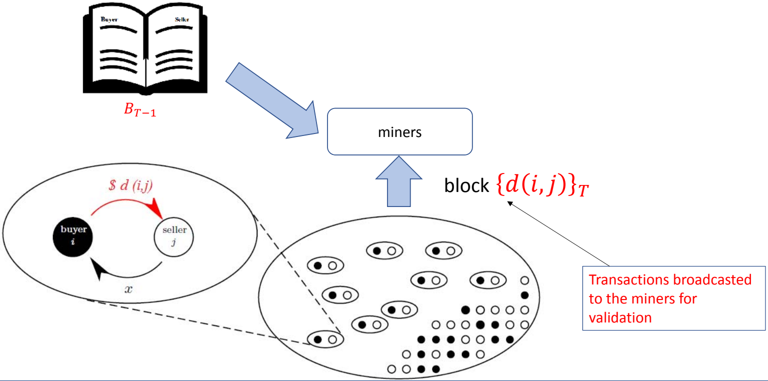
Blockchain



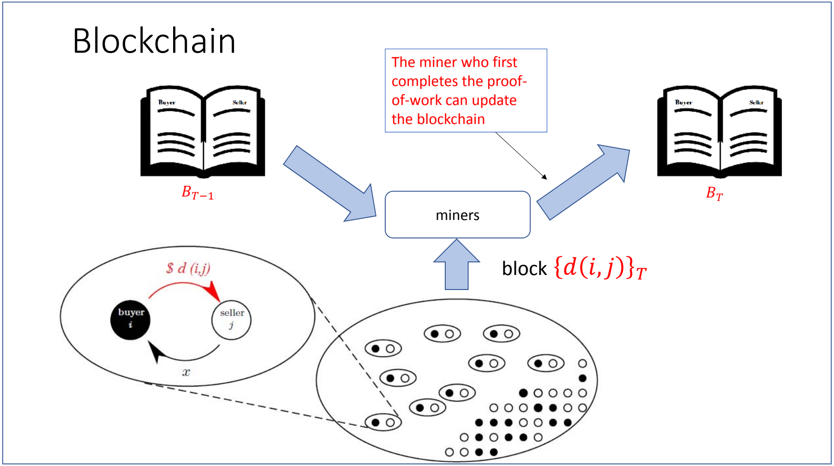
B_{T-1}

A book containing the ledger of all past transactions

Blockchain


Buyer
Seller
B_{T-1}
miners
block $\{d(i,j)\}_T$
$ d (i,j)
buyer
i
seller
j
x
Transactions broadcasted to the miners for validation

Blockchain



The miner who first completes the proof-of-work can update the blockchain
Buyer
Seller
B_{T-1}
B_T
miners
block $\{d(i,j)\}_T$
$\$ d (i,j)$
buyer i
seller j
x

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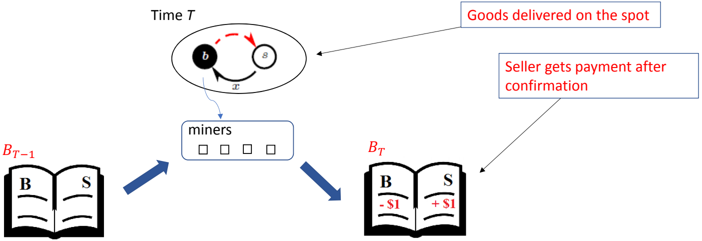
No confirmation lag (N=0)



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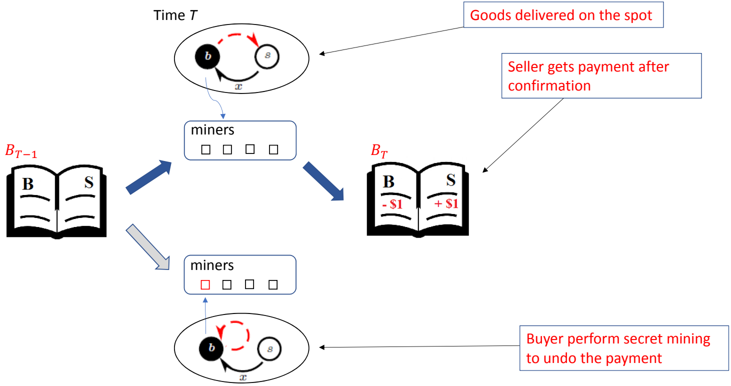

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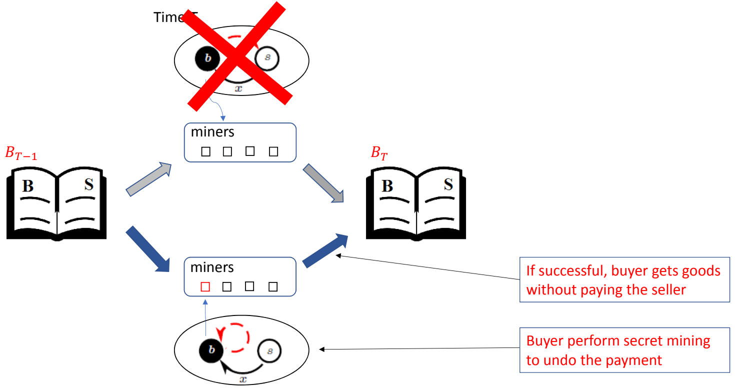
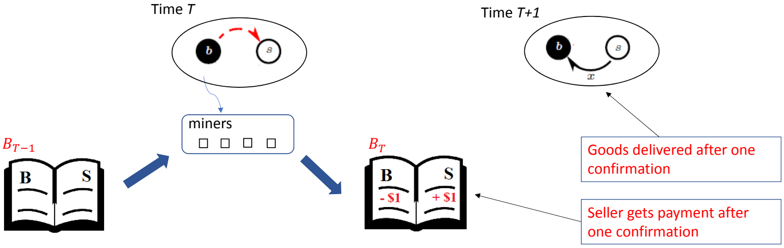


One confirmation lag (N=1)



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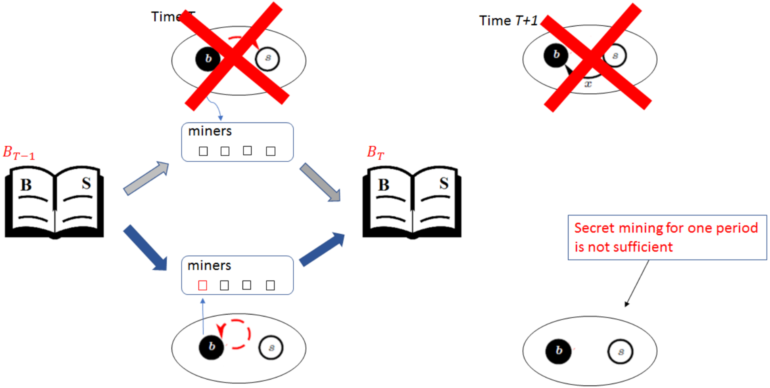

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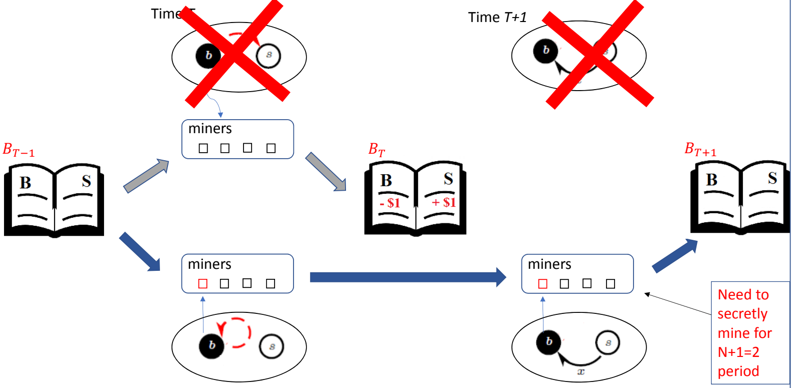

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Questions

Take as given the design of the cryptocurrency system:

1. How well does it function as a payment system?

2. How to optimally set policy parameters?
   e.g. currency growth, transaction fees

3. How best to use it for different types of transactions?
   e.g. retail vs large value

Model

Environment

Based on Lagos and Wright (2005)

Time is discrete: $t = 0, 1, 2, ...$

Three types of agents.

- B buyers
- σB sellers
- M miners

Buyers and seller use balances recorded in a ledger to finance bilateral trade.

Balances in the ledger grow at rate μ and there are transaction fees τ.

Proof-of-Work

M miners compete to update the ledger by solving a costly computational task with a random success rate.

Miner i chooses computer power q_i to maximize profits

$$\rho(q_i)R - q_i\alpha$$

where

- R mining reward in real terms
- α price of computer power
- ρ probability of winning given by

$$\rho(q_i) = \frac{q_i}{\sum_{m=1}^{M} q_m}$$

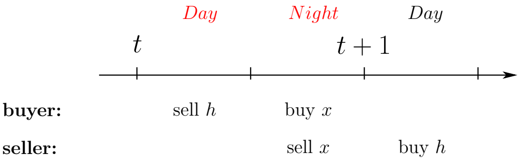
Results:

1) Higher R induces higher mining activities $\sum_{m=1}^{M} q_m = MQ$.

2) As $M \to \infty$, all rents R are dissipated.

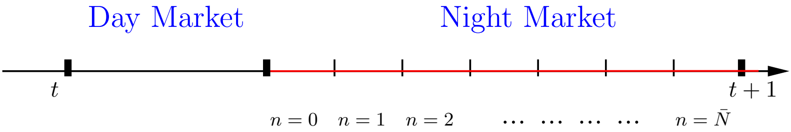
▸ Micro

Trading



- Preferences
  - Buyer: $\varepsilon u(x_t) - h_t$, where $\varepsilon \sim F$
  - Seller: $-c(x_t) + h_t$
- Trading
  - Day: buyer sells h to acquire balances z
  - Night: spends $d \leq z$ to buy x from a seller
  - Next day: the seller uses d to buy h

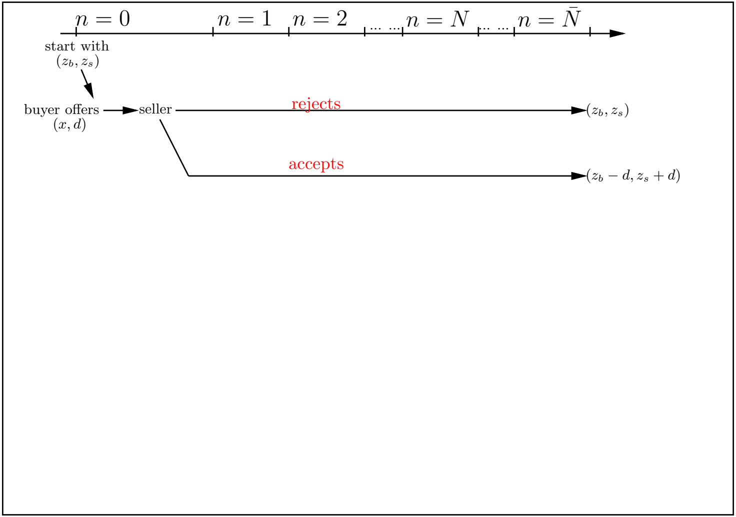
Night Trading



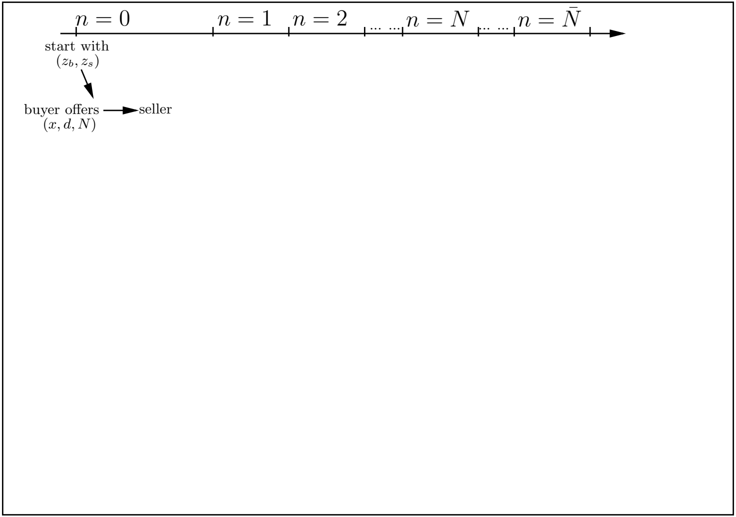
- In session 0, a buyer meets with a seller and makes a take-it-or-leave-it-offer (x, d, N)
  - immediate payment d in real balances
  - x goods to be delivered after confirmations of the payment in N consecutive blocks
- After trade, the buyer can attempt to double spend

Incentives to Double Spend

Transactions in Lagos-Wright



Double-Spending Problem



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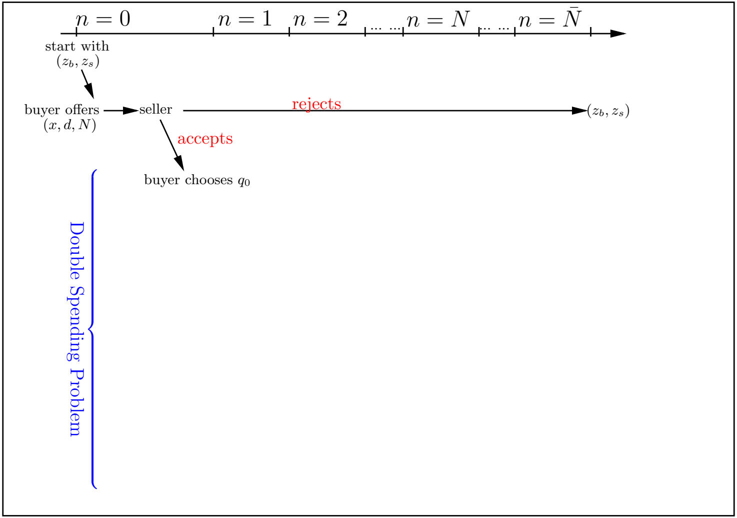

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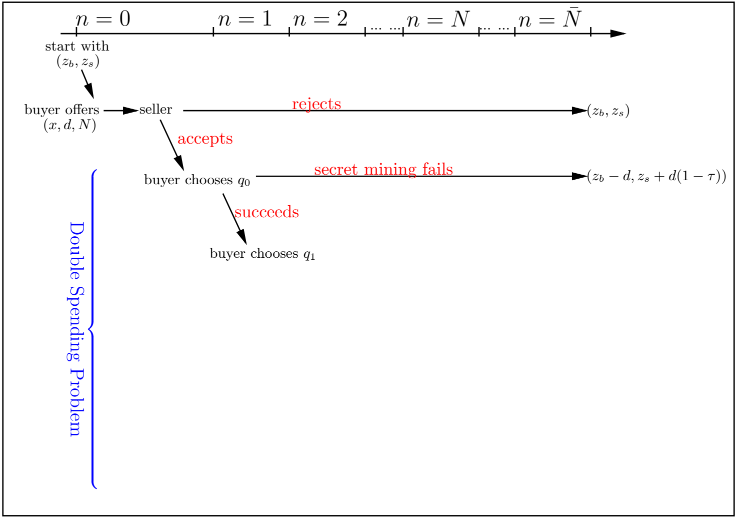

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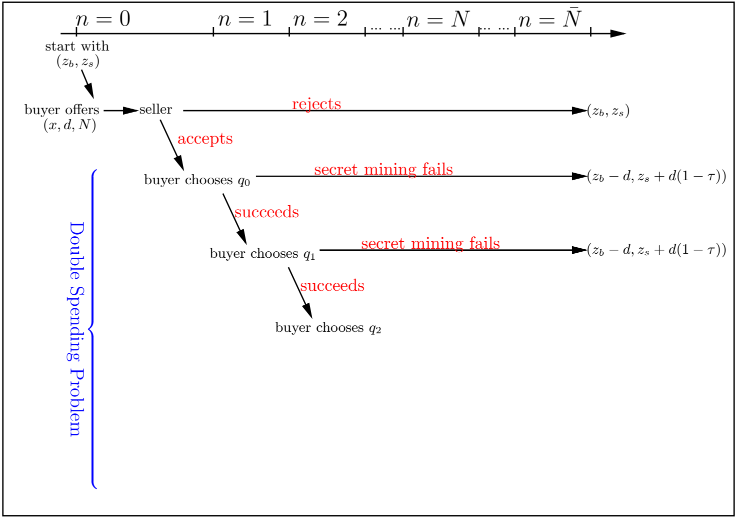

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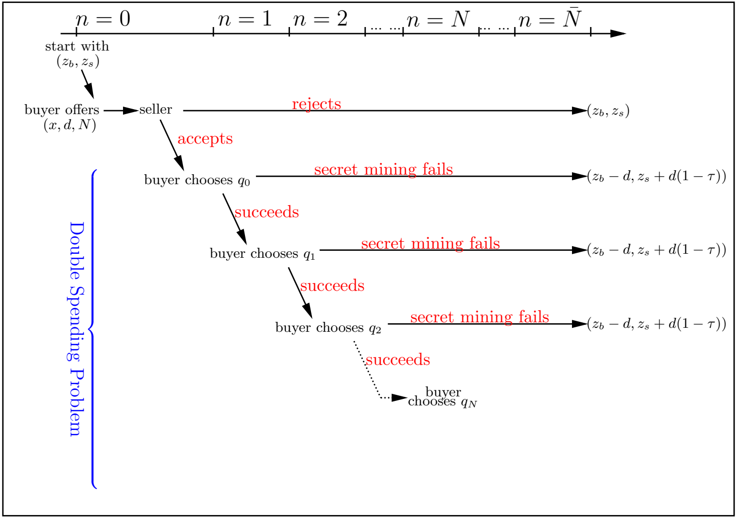

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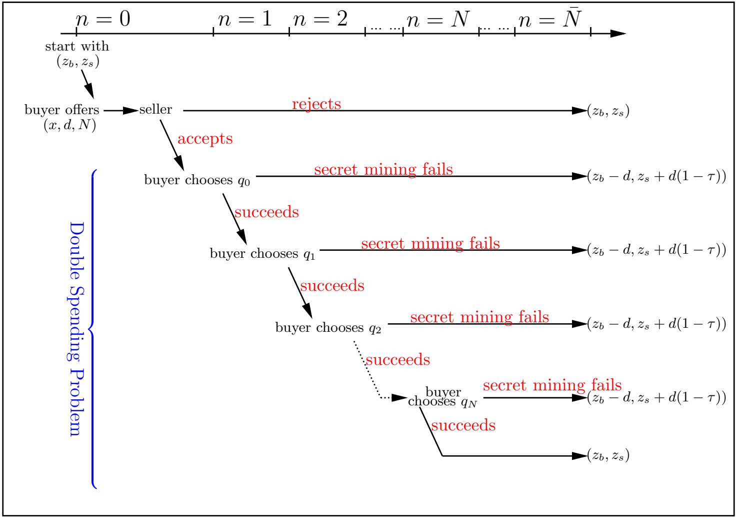

No Double Spending Constraint

For any contract (x, d, N), the expected payoff from a DS attempt is

$$D_0(d, N) = \max_{\{q_n\}_{n=0}^{N}} P\frac{\beta}{\mu}[d + R(1 + N)] - \sum_{n=0}^{N}\left(\prod_{t=0}^{n-1}\frac{q_n}{QM + q_n}\right)\alpha q_n$$

where

$$P = \prod_{n=0}^{N}\left(\frac{q_n}{QM + q_n}\right) \text{ is the prob. of success}$$

$$R = \frac{Z(\mu - 1) + D\tau}{\bar{N} + 1} \text{ are the rewards form mining}$$

Lemma

If $D_0(d, N) = 0$, then the contract (x, d, N) is double-spending proof.

Double-Spending Proof Contracts

Proposition

Suppose $M \to \infty$. A contract (x, d, N) is double-spending proof (i.e. settlement is final) if

$$d < R(N+1)N.$$

Otherwise, the settlement is final only with probability

$$1 - P(d, N) = \frac{N+1}{\sqrt{\frac{d}{R} + (N+1)}}.$$

Results:

- Settlement cannot be both immediate $(N = 0)$ and final $(P = 0)$.
- Rewards help discourage double spending and improve finality.
- There is a trade-off between trade size d, settlement lag N and finality $1 - P$.

Key Trade-off

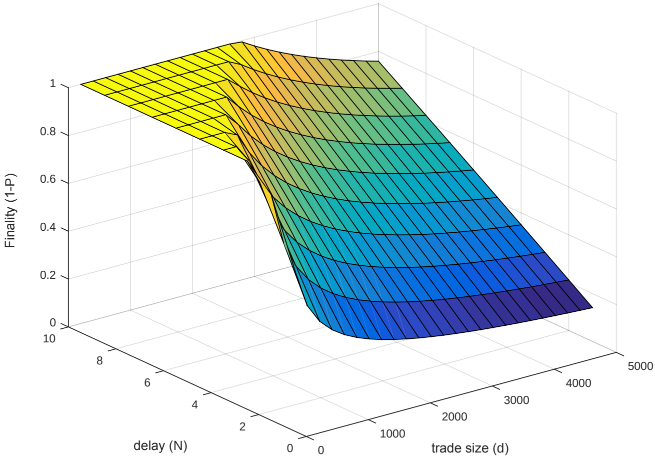

Figure: Trade Size vs. Settlement Lag vs. Finality

Cryptocurrency Equilibrium

Definition

A DS-proof cryptocurrency equilibrium with (μ, τ) and $M \to \infty$ is given by contracts $(x(\varepsilon), d(\varepsilon), N(\varepsilon))$, money demand $z(\varepsilon)$ and a mining choice q such that

1. the contracts satisfy the No-DS-constraint,
2. the money demand and the offer maximizes a buyer's utility,
3. the mining choice maximizes a miner's utility
4. and markets clear.

Theorem

▸ Proof

A DS-proof cryptocurrency equilibrium exists for B sufficiently large.

Optimal Reward Scheme

Define social welfare as

$$\mathcal{W} = \underbrace{B \int [\sigma \delta^{N(\varepsilon)} \varepsilon u(x(\varepsilon)) - x(\varepsilon)] dF_\varepsilon(\varepsilon)}_{\text{trade surplus}} - \underbrace{\frac{\beta}{\mu} R(\bar{N} + 1)}_{\text{mining costs}}$$

Proposition

The optimal reward structure sets transaction fees to zero and only relies on seignorage: $\tau = 0$ *and* $\mu > 1$.

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- ... levying reward costs upfront in terms of inflation allows distortions to be smoothed out across all buyers

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- Implication: long-run zero currency growth is suboptimal

Quantitative Assessment

Calibration – Basic Parameters

	values	targets
β	0.999916	period length = 1 day
δ	0.999999	block time = 10 min
μ	1.00025	money growth (9.6% p.a.)
τ	0.000088	total fees/vol per block
B	6873428	max. # of average-sized transactions
σ	0.0178	vol per day/total BTC
α	1	normalized

Source: 2015 data from Blockchain.info

- We use log utility.
- We use data on the distribution of transactions.
- Confirmation lags cannot be observed directly.

Details

1. Welfare Comparison

Regime	Welfare Cost as % of consumption
Cash (Friedman Rule)	0%
Cash (2% inflation)	0.003%
Bitcoin (benchmark)	1.410%
$\mu - 1 = 9.5\%, \tau = 0.0088\%$	mining cost: \$359.98 millions
Bitcoin (optimal policy)	0.080%
$\mu - 1 = 0.17\%, \tau = 0\%$	mining cost: \$6.9 millions

- Welfare loss is currently very large mainly due to the mining cost.
- ... can be reduced substantially by lowering money growth and setting transaction fees to zero.
- Long-run BTC design will bring money growth to 0 and is, thus, inefficient.

2. Best Usage of Cryptocurrency Technology

	Retail Payments (US Debit cards)	Large Value Payments (Fedwire)
avg transaction size annual volume	\$38.29 59539 millions	\$6552236 135 millions
optimal μ optimal τ confirmation lag		
welfare loss mining cost (per year)		

- DS-proof iff $d < R \cdot N(1 + N)$
  - retail: small trade size, high volume
  - interbank: large trade size, low volume

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avg transaction size	\$38.29	\$6552236
annual volume	59539 millions	135 millions
optimal μ	0.038%	0.53%
optimal τ	0%	0%
confirmation lag	2 mins	12 mins
welfare loss	0.00052%	0.0060%
mining cost (per year)	\$4.33 millions	\$22.10 billions

- DS-proof iff $d < R \cdot N(1 + N)$
  - retail: small trade size, high volume
  - interbank: large trade size, low volume
- retial system incurs a lower welfare loss and mining costs
- ... requires smaller rewards
- ... induces shorter confirmation lags

What to Take Away

1) Owing to its digital nature, a cryptocurrency is fundamentally different from cash.

2) One can understand the economics of such a system well by looking at the incentives to double-spend.

3) BITCOIN is not only really expensive in terms of mining costs, but also inefficient in its long-run design.

4) It provides a more efficient payment system when the volume of transactions is large relative to the individual transaction size.

On-going project: Blockchain for security settlement, cross-border payments, ...

Thanks!

Appendix

Microfoundations for Mining

Investing computing power q_m allows a miner to solve the PoW problem with probability

$$F(t) = 1 - e^{-\mu_m \cdot t}$$

within a time interval t, where $1/\mu_m = D/q(m)$ is the expected time to solve the problem.

Hence, D is the difficulty parameter for the PoW problem.

The first solution among miners, $\min(\tau_1, \ldots, \tau_M)$, is thus also exponentially distributed and the probability for any miner to solve it first is given by

$$\rho_n(q_n) = \frac{q_n}{\sum_{m=1}^{M} q_m}.$$

Back

Oligopolistic Mining Equilibrium

Maximizing profits by miner j yields as a FOC

$$\left(\frac{\sum_{i=1}^{N} q_i - q_j}{\left(\sum_{i=1}^{N} q_i \right)^2} \right) \frac{\beta}{\mu} R = \alpha$$

Imposing symmetry, we obtain for the total mining cost

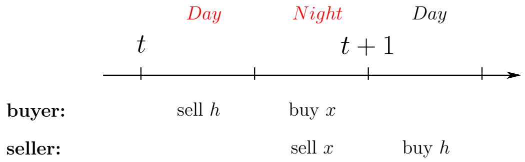
$$C = \alpha M Q = \frac{M-1}{M} \frac{\beta}{\mu} R.$$

For $M \to \infty$ all rents are dissipated and we obtain

$$C = \frac{\beta}{\mu} R$$

Back

Trading



Two markets

- centralized market in day
- decentralized market at night

Preferences

- Buyer: $\varepsilon u(x_t) - h_t$, where $\varepsilon \sim F$
- Seller: $-x_t + h_t$

Trading

- Day: buyer sells h to acquire real balances z
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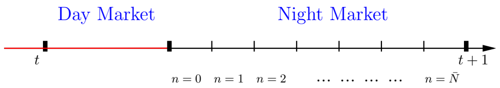
Day Market



The value of a buyer who draws ε is

$$\max_{z',h} -h + V(z';\varepsilon)$$

subject to

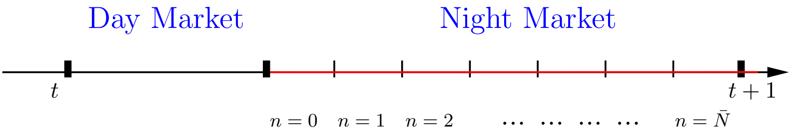
$$h + z \geq z' \geq 0$$

where z' are the real balances carried to the night market.

Assumption:

Transactions can be perfectly monitored and there is full liability so that double spending is not a problem.

Night Market



The night market is divided into $\bar{N}+1$ trading sessions.

- In session 0, a buyer meets with a a seller w.p. σ and makes a take-it-or-leave-it-offer (x, d, N).
- There is immediate payment d in real balances.
- x goods are to be delivered after confirmation of the payment in N consecutive blocks.

The offer (x, d, N) determines whether the buyer has an incentive to double spend or not.

Back

Optimal DS Proof Contracts

At the start of the night market, the buyer with z makes a take-it-or-leave-it offer (x, d, N) to a seller.

The buyer will never carry more real balances than necessary so that $z = d$ and the offer is given by $(x(d), N(d))$.

Requiring the offer to be double spending proof the buyer solves

$$\max_{(x,d,N)} -d + V(d;\varepsilon)$$

subject to

$$V(d;\varepsilon) = \sigma\delta^N \varepsilon u(x) + (1-\sigma)\frac{\beta}{\mu}d$$

$$x \leq \frac{\beta}{\mu}d(1-\tau)$$

$$d \leq R(N+1)N$$

Sufficient Condition for DS proof

The optimal contract is DS proof if

$$\sigma\left[\delta\varepsilon_{\max}u'(\bar{x})(1-\tau)\mathcal{E}(x)-1\right]<i$$

where

$$\bar{x}=(1-\tau)2R \qquad \text{is the maximum trade size with } N=1$$

$$\mathcal{E}(x)\leq\frac{3}{4} \qquad \text{is the elasticity of } x \text{ w.r.t. } d \text{ at } N=1$$

The reason is that the tightest constraint to avoid DS is a confirmation lag of $N=1$.

This condition is satisfied when

- the opp. cost of carrying balances is high (i is high)
- the matching friction is high (σ is low)
- the marginal utility is low (ε is low)

Back

Existence Proof

We use Kakutani's Fixed Point Theorem.

Fix (μ, τ). The reward R determines the aggregate money supply $S(R)$ which in turn determines total rewards R'. Hence, we need to find a fixed point for R given aggregate money demand for a correspondence

$$T(R) = \left(\frac{(\mu - 1) + \sigma\tau}{\bar{N} + 1} \right) S(R).$$

Aggregate money demand can be shown to be u.h.c, convex in R which pins down the aggregate transaction fees and, hence, R'.

Furthermore, given B sufficiently large, we can find a lower bound on $R_{\min} > 0$ such that $R > R_{\min}$.

Hence, we can restrict the correspondence to a compact set and show that the correspondence has a closed graph.

Back

Optimal Contracts

We use data on transactions to recover the implied distribution of ε.

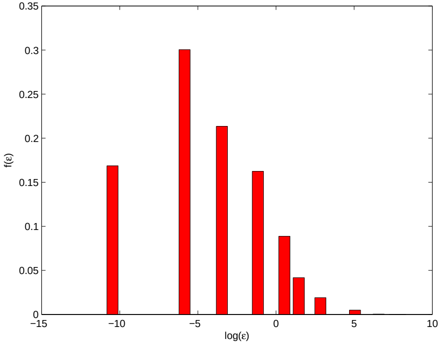


Figure: Implied Distribution of Shocks

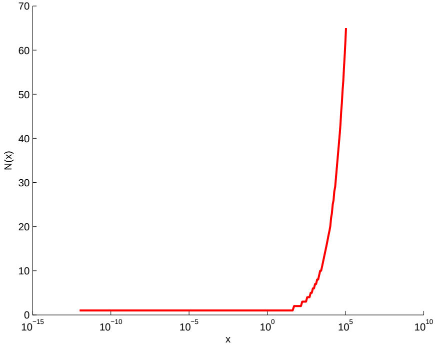


Figure: Optimal Delay

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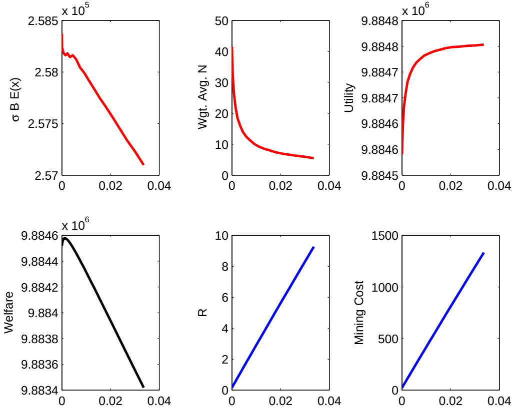
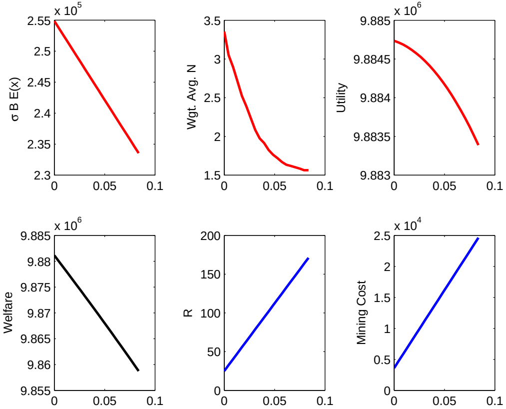
Optimal Design I – Effects of Money Growth Rate



- Higher inflation implies distortions and higher mining costs ...
- .. but positive inflation is optimal due to lower confirmation lags.

Optimal Design II – Effects of Transaction Fees



- Same trade-off ...
- ... but zero transaction costs seem to be optimal given $\mu > 0$.