

Abstract:

This paper develops a Dynamic Stochastic General Equilibrium (DSGE) model to study how the instability of the banking sector can accelerate and propagate business cycles. The model builds on Bernanke, Gertler and Gilchrist (BGG) (1999), where credit demand friction due to agency costs is considered, but it deviates from BGG in that financial intermediaries have to share aggregate risk with entrepreneurs, and therefore bear uncertainty in their loan portfolios. Once a loan contract based on two parties' expectation of the economic situation in the future has been signed, an unexpected negative shock will lead to higher ex-post loan default rates, and the bank's capital faces larger write-offs because of unexpected loan losses. Under the Basel bank capital regulation regime (in which banks have to hold a minimum capital-to-asset ratio), banks will face have difficulty raising capital in the next period because households perceive a higher default probability, i.e., their capital level will fall below the regulatory threshold. Model simulations show that instability in the banking sector alone can create strong credit supply friction, and have a significant effect on accelerating short-run cycles. In the long run, instability of the banking sector implies a lower capital stock in the economy and therefore a lower level of investment and output.

JEL classification: E32, E44, E52

Key words: Bank capital regulation, banking instability, financial friction, business cycle

Non-Technical Summary

Dynamic Stochastic General Equilibrium (DSGE) models have long been criticized for lacking an appropriate way of modeling the financial markets and for thereby failing to explain important regularities of business cycles. Given the historically repetitive waves of financial crises and the current worldwide deep recession triggered by the U.S. subprime mortgage market crisis and the subsequent banking instability, it has become abundantly clear how relevant financial frictions are in business cycle transmission and amplifications. In the literature that focuses on the role of financial frictions, credit demand frictions have been studied the most extensively. Representative work from Bernanke, Gertler and Gilchrist (1999, BGG thereafter) establishes a link between firms's borrowing cost and their net worth, which can generate counter-cyclical external finance premiums and amplify various macro shocks. However, the financial friction coming from the supply side or the vulnerability of the financial intermediary itself has so far received very little attention and has not yet been incorporated into stylized DSGE models. Recent papers have tried to link the financial structure of banks to their lending rate to motivate the role of bank capital (e.g. Markovic (2006), Aguiar and Drumond (2007)) or have modeled the function of banks in a detailed manner (Gerali, Neri, and Signoretti (2007), Christiano, Motto, and Rostagno (2007)). However, they have all avoided the key issue of endogenously deriving uncertainty in banks' loan default rates and the related banking instability, which is then passed on to the macro economy through the credit market.

This paper sheds new light on how the instability of the banking sector can accelerate and propagate business cycles in a general equilibrium setting. The model builds on BGG, where credit demand friction due to agency cost is considered, but it deviates from BGG in that financial intermediaries have to share aggregate risk with entrepreneurs, and therefore bear uncertainty in their loan portfolios. Once a loan contract based on two parties' expectation of the economic situation in the future has been signed, an unexpected negative shock will lead to higher ex-post loan default rates, and the bank's capital faces larger write-offs because of unexpected loan losses. Under the new Basel II bank capital regulation (which requires banks have to hold a minimum capital-to-asset ratio), banks will find it difficult to raise capital in the next period because households perceive a higher bank default probability, i.e., their capital level will fall below the regulatory threshold. The higher cost to banks of raising funds will be passed through to their lending

decision, which will cause aggregate investment and output to contract even further.

Model simulation shows that instability of the banking sector alone can create strong credit supply friction and can have a significant effect on accelerating short-run cycles. Asset prices and investment become much more volatile after the additional bank capital channel is considered. In the long run, instability of the banking sector implies a lower capital stock in the economy and therefore a lower level of investment and output.

This paper also compares the relative contribution of various frictions in shock transmission. Three cases are considered. In the first case, only nominal rigidity and capital adjustment cost are considered. In the second case, credit demand friction, or the financial accelerator effect, is added to the existing frictions. In the third case, credit supply friction, or the bank balance sheet channel, is incorporated. Model simulations show that the bank capital channel is more important than the financial accelerator in amplifying policy shocks. This is consistent with previous findings in the literature that the financial accelerator contributes only marginally to monetary policy transmission. However, the relative importance of the two channels is reversed when a positive technology shock hits the economy, where strong corporate balance sheets play an important role in driving up asset prices and aggregate investment.

Another highlight of the model is that it can replicate a long-established empirical observation hitherto unexplained in a theoretical model: that aggregate lending does not go down immediately following a contractionary monetary policy shock but increases for four to six quarters and then falls. (Christiano, Eichenbaum and Evans (2005)). The mechanism behind this phenomenon is shown to be that firm net worth contracts faster than the asset price in the initial period following a negative policy shock, and that therefore firms have to rely more on external financing. In the following period, however, the speed of adjustment reverses, and aggregate lending shrinks.

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Bank Capital Regulation, Lending Channel and Business Cycles ¹

1 Introduction

The role of financial friction in business cycle propagation has long been ignored in the literature. The main theoretical justification behind the omission is the Modigliani-Miller proposition that, in a frictionless world of complete information and perfect markets, a firm's value is independent of its capital structure. A more specific interpretation of the theorem in the credit market would be: on the demand side, the firm's leverage ratio does not influence its borrowing capacity; on the supply side, banks' lending decisions are not influenced by their capital ratio. It is clear that the world we are living in is far from perfect, yet it is not clear whether the deviation from the perfect world is big enough to make Modigliani-Miller proposition nontrivial.

Earlier work by Bernanke and Gertler (1989) and Bernanke et al. (1999) (hereinafter BGG) starts to look at the role of asymmetric information from the credit demand side. Their model established a link between firms' borrowing costs and net worth. In an economic downturn, firms' leverage ratio increases, causing them to face a higher external finance premium in borrowing because of exacerbation of information asymmetry and thus driving down capital demand. The drop in capital demand reinforces the decline of firms' net worth and the business cycle is propagated. This mechanism is known in the literature as the 'financial accelerator'. Meier and Muller (2006) estimates a model with a BGG-type financial accelerator by matching the impulse-response of a monetary policy shock from a vector autoregression and find that the financial accelerator contributes only marginally to monetary policy transmission. Therefore they argue that little is lost if DSGE models do not incorporate financial accelerator effects. Christensen

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and Dib (2008) bring a BGG-type DSGE model to the data and estimate the model with maximum likelihood; they find that the model without a financial accelerator is statistically rejected in favor of a model with a financial accelerator, but also report that the importance of the financial accelerator for output fluctuation is relatively minor.

One possible reason why DSGE models with a financial accelerator underperform is that they only pay attention to the financial friction from the credit demand side but overlook the frictions coming from the credit supply side. Just as the firm's leverage ratio is important in deciding its borrowing costs, the bank's leverage ratio also determines the cost of raising external capital. Under the strict banking regulation, especially the 8 percent minimum capital ratio required by Basel II, if the bank's capital ratio goes down households will expect a higher probability that the bank will be shut down by the supervisor for not fulfilling the regulatory requirement and will therefore demand a higher return for holding that bank's equity. Higher costs of funding to the bank will be passed through to a higher lending rate, thus driving down aggregate credit. The crisis we are now facing is a textbook example of the importance of bank capital in determining credit supply. The financial turmoil that began in 2007 in the U.S. subprime market, and subsequently spread to the broader credit and funding markets is now developing into a worldwide recession. As the IMF mentioned in its latest Global Financial Stability Report (International Monetary Fund, 2008), this financial turmoil has weakened the capital and funding of large systemically important financial institutions, which then needed to raise capital or cut back assets to cope with the strain. Markovic (2007) extended BGG (1999) to analyze the macroeconomic consequences of changes in the cost of bank capital. He distinguishes three subchannels: the default risk channel, the adjustment cost channel, and the capital loss channel. He finds that those channels have a relatively important effect in the propagation of business cycles. However, the model hampered by many exogenously imposed empirical functional forms, such as banks' default probability and the evolution rule of bank share prices. Aguiar and Drumond (2007) show that the bank capital is more strongly procyclical under Basel II than under Basel I. The propagation comes mainly come from a countercyclical liquidity premium that the bank has to pay for raising equity. Yet in this model it is assumed that the capital requirement is always binding, which means the bank's capital ratio remains constant across cycles, which is a nontrivial assumption to make. Meh and Moran (2007) embed the Holstrom and Tirole (1997) framework in a dynamic general equilibrium model, where no regulatory rule is imposed, and banks hold capital simply for market reasons.

Under the strict bank capital framework in force today and the importance of the banking sector in monetary policy transmission, it is probably more relevant to incorporate the banking regulations into the general equilibrium model and study the implication of a policy shock or other macro shocks. At the same time, it is important to demonstrate the interaction between the financial friction coming from both the credit supply and credit demand sides. Therefore, we have chosen BGG (1999) as the starting point for this model. The most important deviation of this model from BGG is to bring uncertainty to the banking sector. In BGG and the aforementioned two papers which build on BGG, the bank can write a state-contingent contract to entrepreneurs, which means the contractual lending loan rate will change according to the economic situation. By making the entrepreneurs take all the aggregate risk, the bank always obtains a risk-free loan portfolio, and the ex-post default rate is exactly equal to the ex-ante default rate. It is obvious how far away removed this assumption is from reality. If banks can could always have a risk-free loan portfolio, and the ex-post default rate never deviated from the ex-ante default rate, we wouldn't have had a financial crisis to begin with. The innovation of this model is to rewrite the financial contract so that the contractual lending rate is based on the agents' expectation of economic conditions in the next period, but once it is signed, the contractual lending rate is fixed. In the next period, if an unexpected negative shock hits the economy, more entrepreneurs than expected by the bank will default, creating unexpected loss in the loan portfolio, which will eat into the bank's capital and push the bank into a bad situation in the equity market because households perceive a higher default probability. The difficulty in raising external capital will cause the bank to be more stringent in extending credit in the following period, with the bank willing to offer credit only at a higher rate. On the other hand, after the negative shock hits the economy, entrepreneurs' net worth will go down and the leverage ratio will go up, which makes them less attractive in the credit market and forces them to pay a higher external finance premium. The shift in credit supply from the bank's side and credit demand from the entrepreneur's side will drive down aggregate investment even further, leading to a deeper recession.

The remainder of the paper is structured as follows. Section 2 presents the model. Section 3 describes the calibration strategy. Section 4 discusses the effect of the bank capital channel on long-run steady states and short-run dynamics. Section 5 concludes.

2 Model

2.1 The Financial Contract

In this section, an optimal financial contract is derived in a partial equilibrium setting, taking the price of capital goods, entrepreneurs' net worth, the cost of deposits and bank capital as given. In the next section, these variables will be endogenously determined in the general equilibrium.

There are two parties to the contract: an entrepreneur with net worth n , and a bank which can raise funds from the household in the form of either deposits or equity and may wish to lend to the entrepreneur.² Both parties are assumed to be risk-neutral. There is a continuum of entrepreneurs, indexed by $i \in (0, 1)$. At the end of period t , entrepreneurs need to purchase capital for use at $t + 1$. The quantity of capital purchased by entrepreneur i is denoted K_{t+1}^i , and the real price paid per unit of capital in period t is q_t . The return on capital is subject to both idiosyncratic risk and aggregate risk. The ex-post gross return to entrepreneur i is $\omega_i R_{t+1}^k$, where ω_i is an idiosyncratic shock to entrepreneur i , and R_{t+1}^k is the ex-post aggregate return on capital. The idiosyncratic shock ω_i is independently (across time and entrepreneurs) distributed with log-normal distribution and a mean of one.

In order to purchase capital, entrepreneurs use their internal funds (net worth) and borrow the rest from a bank. Let N_{t+1}^i denote the net worth of entrepreneur i at the end of period of t ; it then has to borrow the following amount from the bank:

$$L_{t+1}^i = q_t K_{t+1}^i - N_{t+1}^i \quad (1)$$

The banks obtain funds from households in the form of either deposits or equity. Because of regulatory requirements, the bank is obliged to hold some equity; therefore, the opportunity cost of funds is a linear combination of the cost of raising deposits and the cost of raising equity. The exact combination is determined by the bank's capital ratio.

The agency problem is introduced into the model by assuming that ω_i is private information, observed at no cost only by entrepreneur i , while

²The existence of the banking sector in this paper is taken as given. It could also be motivated by assuming that the households need to pay an extremely high cost to monitor the project outcome.

the bank has to pay a monitoring cost to observe it.³ This information asymmetry creates a moral hazard problem in that entrepreneurs may misreport the value of ω_i . As Townsend (1979) and Gale and Hellwig (1985) showed, in this environment the optimal contract between lender and borrower is risky debt. That is, when the idiosyncratic shock exceeds a certain threshold, the entrepreneurs pay a fixed amount R_{t+1}^L ; on the contrary, if the idiosyncratic shock falls below the threshold, entrepreneurs will default, and the bank monitors the result and seizes the entrepreneur's remaining assets. This type of debt structure can motivate the entrepreneurs to always report the true value of ω_i . If $\bar{\omega}$ denotes the threshold value, and R_{t+1}^L the contractual lending rate, the following condition has to hold:

$$\bar{\omega}_{t+1}^i R_{t+1}^k q_t K_{t+1}^i = R_{t+1}^L L_{t+1}^i \quad (2)$$

The contract can thus be characterized by $\{\bar{\omega}, L_{t+1}^i\}$. Gale and Hellwig (1985) only derived the optimal debt structure subject to idiosyncratic shock, where banks can perfectly diversify idiosyncratic risk and hold a risk-free loan portfolio. But in the presence of aggregate risk, it is not yet clear how the risk should be allocated. BGG (1999) assume that entrepreneurs will absorb all of the aggregate risk and that bank hold risk-free loan portfolios. Because, in BGG, banks are agents on behalf of households, they offer state-contingent contracts to entrepreneurs, with the cut-off $\bar{\omega}^i$ and hence the lending rate R_{t+1}^L functions of realized aggregate return on capital R_{t+1}^k . By contrast, because the focus of this paper is on the role of bank capital in the intermediation process, *this model assumes that aggregate risk is shared between banks and entrepreneurs. The financial contract cannot be contingent on the realized capital return but has to be written based on the two parties' expectation of capital return in the next period.*⁴ The implication for this type of risk sharing is that aggregate shock will drive a wedge between the ex-post default rate and the ex-ante rate, and create unexpected losses or gains in banks' loan portfolios.

Let $E_t R_{t+1}^k$ denote the expected capital return at the end of period t ; then the ex-ante loan default and the contractual lending rate have the following relationship:

$$\bar{\omega}_t^{i,a} E_t R_{t+1}^k q_t K_{t+1}^i = R_{t+1}^L L_{t+1}^i \quad (3)$$

³This type of 'costly state verification' has been used in a number of papers, beginning with Townsend (1979).

⁴A state-contingent contract could be prevented by assuming that the state of the economy is not observed by the enforcement of the contract, but only observed at the very end of the period when people form expectations for the next period.

where ω_t^a is the ex-ante loan default threshold. After the financial contract is signed, the expected return to the entrepreneur is given by :

$$\int_{\bar{\omega}^{i,a}}^{\infty} \omega E_t R_{t+1}^k q_t K_{t+1}^i f(\omega) d\omega - (1 - F(\bar{\omega}^a)) \bar{\omega}^a E_t R_{t+1}^k q_t K_{t+1}^i \quad (4)$$

where the first part is the gross return the entrepreneur obtains from capital return and the second part is the loan that it has to pay back. $f(\omega)$ and $F(\bar{\omega}^a)$ are respectively the density function and cumulative distribution function of ω (also the probability of default). Note that if the realization of ω_i is below the threshold value $\bar{\omega}^a$ ⁵, the entrepreneur gets nothing. The bank will monitor the project return and confiscate everything that is left.

The entrepreneur maximizes his expected return subject to the participation constraint of the bank, which is characterized as follows:

$$(1 - F(\bar{\omega}^a)) R_{t+1}^L L_{t+1}^i + (1 - \mu) \int_0^{\bar{\omega}^a} \omega E_t R_{t+1}^k q_t K_{t+1}^i f(\omega) d\omega = R_{t+1}^f L_{t+1}^i \quad (5)$$

There are two parts to the bank's return on the loan portfolio: the loan amount that is paid back by the entrepreneurs, and, in the default case, the acquisition of the firm's remaining assets after paying off the monitoring cost, which is a linear function of capital return. μ is a parameter that captures the degree of monitoring cost or information asymmetry. If μ is set to zero, there will be no information asymmetry between lenders and borrowers; the firm balance sheet channel will no longer exist.

By solving the contract we can derive the following credit demand equation (see Appendix A for details):

$$E_t R_{t+1}^k = S \left(\frac{q_t K_{t+1}^i}{N_{t+1}^i} \right) R_t^f \quad (6)$$

$S' > 0$, where S denotes the external finance premium, which captures the wedge (driven by the existence of monitoring cost) between the cost of finance from the firm's side and the cost of funds from the bank's side. Note that there exists a one-to-one relationship between the leverage ratio and the default threshold or default probability. The more heavily leveraged the firm, the higher its default probability; agency costs go up, and the bank

⁵Since, in equilibrium, the leverage ratios are the same across firms, hence each firm faces the same cut-off value when signing the financial contract, we therefore drop the individual superscripts.

thus charges a higher external finance premium to compensate for it.

After solving for the default threshold in the optimal contract, the contractual lending rate could be solved as

$$R_{t+1}^L = \frac{\bar{\omega}^a E_t R_{t+1}^k q_t K_{t+1}^i}{L_{t+1}^i} \quad (7)$$

Note that in this model the contractual lending rate is *fixed* and the default rate can deviate from the expected value, whereas in BGG the lending rate is *state-contingent*⁶. The bank can raise the default threshold value if a negative shock hits the economy in order to guarantee the household a risk-free return. At the end of period t , entrepreneurs sign the contract with bank, in which a fixed loan amount and a fixed lending rate are provided. In the next session, we will endogenize all the variables that we treat exogenous in solving the optimal contract.

2.2 General Equilibrium

2.2.1 Households

There is a continuum of households in the economy, each indexed by $i \in (0, 1)$. They consume the final good, c_t , invest in bank deposits, d_t , which is risk-free, and bank equity, e_t , supply labor l_t^h and own shares in a monopolistically competitive sector that produces differentiated varieties of goods. The households maximize the utility function:⁷

$$\max E_t \sum_{k=0}^{\infty} \beta^k \left[\ln(c_{t+k}) + \frac{d_{t+k}^{1+\varphi}}{1+\varphi} + \rho \ln(1 - l_{t+k}^h) \right] \quad (8)$$

subject to the sequence of budget constraint:

$$d_{t+1} + e_{t+1} + c_t = w_t l_t + R_t^d d_t + R_t^e (1 - \phi_t) e_t + \Pi_t \quad (9)$$

d_{t+1} and e_{t+1} are deposits and bank equity in real terms. R_{t+1}^d and R_{t+1}^e reflect the gross real return on holding deposit and bank equity, and ϕ_{t+1} is

⁶Note that the very motivation of introducing costly state verification in the literature is to explain the observation that debt contracts are simple rather than state-contingent. The way of risk-sharing adopted in BGG makes the loan contract again state-contingent.

⁷Inserting deposits into the utility function is just a modeling device to capture the bank' liquidity creation function. Model dynamics are robust if we consider a standard utility function with only consumption and leisure.

the default rate on bank capital. l_t is household labor supply, w_t is real wage for household labor, Π_t is dividends received from ownership of retail firms. Following Van den Heuvel (2007), the liquidity services of bank deposits are modeled by assuming that the household has a derived utility function that is increasing in the amount of deposits. The households' optimization problem yields following first-order conditions:

$$U_c(c_t, d_t) = \beta E_t R_{t+1}^e (1 - \phi_{t+1}) U_c(c_{t+1}, d_{t+1}) \quad (10)$$

$$U_c(c_t, d_t) - U_d(c_t, d_t) = \beta E_t R_{t+1}^d U_c(c_{t+1}, d_{t+1}) \quad (11)$$

$$-U_{c,t}/U_{l,t} = w_t \quad (12)$$

Equation (9) shows that households' intertemporal consumption decisions are determined by the default-adjusted return on holding bank equity. Equation (10) implies that the wedge between the return on bank equity and that on deposits is composed of two parts. One is the liquidity premium, as captured in $U_d(c_t, d_t)/U_c(c_t, d_t)$. Since deposits can provide households extra utility in addition to carry a monetary reward, bank equity has to provide a higher return to compete against deposits for households' willingness to hold assets. The second part comes from the default risk of bank capital. Whenever the capital ratio falls below the regulatory threshold, the bank will be shut down and will default on capital return. Since bank default is a steady-state phenomenon, bank equity has to provide a higher return than that on deposits to compensate for default risk.⁸

2.2.2 Entrepreneurs

After signing the financial contract, entrepreneurs combine loans acquired from the bank and their own net worth to purchase capital. They use capital and labor to produce wholesale goods and sell them on a perfect competitive market at a price equal to their nominal marginal cost. The aggregate production function is given by :

$$Y_t = A_t K_t^{\alpha_k} (l_t^h)^{\alpha_h} (l_t^e)^{\alpha_e} (l_t^b)^{\alpha_b} \quad (13)$$

Following BGG (1999), it is assumed that, besides operating firms, entrepreneurs also supply labor services in the general labor market. The same is assumed of bankers. As will be see later, α_e and α_b are calibrated so that

⁸This paper assumes a relationship between households and bankers as delegated monitoring. Therefore, households do not care about the capital structure of banks in their decision.

these two additional labor forces have a negligible effect on the output level and model dynamics.⁹

The optimization problem of production remains standard:

$$z_t = \alpha_k mc_t \frac{Y_t}{K_t} \quad (14)$$

$$w_t^h = \alpha_h mc_t \frac{Y_t}{l_t^h} \quad (15)$$

$$w_t^e = \alpha_e mc_t \frac{Y_t}{l_t^e} \quad (16)$$

$$w_t^b = \alpha_b mc_t \frac{Y_t}{l_t^b} \quad (17)$$

where z_t is the real rental rate of capital and w_t^h, w_t^e and w_t^b are, respectively, the real wage of households, entrepreneurs and bankers. mc_t denotes real marginal cost. The expected return on capital is then:

$$E_t R_{t+1}^k = E_t \left(\frac{z_{t+1} + (1 - \delta)q_{t+1}}{q_t} \right) \quad (18)$$

The accumulation of entrepreneurs' net worth consists of two parts: operating the firms and labor income. It is assumed that, in every period, entrepreneur will die with the probability $1 - \gamma$. This assumption is to make sure that entrepreneurs can never accumulate enough net worth to finance a project without external financing. Those entrepreneurs who die at time t will consume $(1 - \gamma)V_t$. The evolution of aggregate net worth is therefore given by:

$$N_{t+1} = \gamma V_t + w_t^e \quad (19)$$

where V_t represents net return on operating business. It is the difference between gross capital return and loan payment.

$$V_t = \int_{\bar{\omega}^b}^{\infty} \omega R_{t+1}^k q_t K_{t+1} f(\omega) d\omega - (1 - F(\bar{\omega}^b)) R_{t+1}^L L_{t+1}^i \quad (20)$$

⁹The salary that bankers earn from labor supply could be understood as fee income collected from transaction services, a function of financial intermediaries that is not modeled in the paper.

$\bar{\omega}^b$ denotes the ex-post default threshold, which could be derived by the following condition:

$$\bar{\omega}_{t+1}^b = \frac{\bar{\omega}_{t+1}^a E_t R_{t+1}^k}{R_{t+1}^k} \quad (21)$$

From this expression, we can easily see that if ex-post capital return is higher than previously expected, the loan default rate is also lower than expected. Since in the default space the entrepreneur gets nothing, therefore, when more entrepreneurs move to the non-default space, aggregate net worth will go up.

2.2.3 Capital Producers

Capital producers purchase a fraction of final goods from the retailer as investment goods i_t and combine this with existing capital stock to create new capital stock. A quadratic capital adjustment cost is included to motivate a variable price of capital, which contributes to the volatility of firm net worth and bank capital. Capital producers will choose the quantity of investment goods to maximize profit subject to adjustment costs:

$$\max E_t \left[q_t i_t - i_t - \frac{\chi}{2} \left(\frac{i_t}{k_t} - \delta \right)^2 k_t \right] \quad (22)$$

where q_t is the real price of capital. The optimization problem yields the following capital supply curve:

$$q_t = 1 + \chi \left(\frac{i_t}{k_t} - \delta \right) \quad (23)$$

χ is a parameter that captures the degree of capital adjustment cost. The higher χ is, the more volatile the capital price. If χ is set to zero, capital price will be constant at 1. The aggregate capital stock evolves according to:

$$k_{t+1} = i_t + (1 - \delta)k_t \quad (24)$$

where δ is the depreciation rate.

2.2.4 Banking Sector

The bank's equity value is accumulated through retained earnings, as shown in the following equation:¹⁰

¹⁰There is a large body of literature in which financial fragility arises purely because of self-fulfilling expectations. This paper focuses on bank instability that is driven by fundamentals.

$$\begin{aligned}
e_{t+1} = & (1 - \phi_t)e_t + R_{t+1}^L L_{t+1}^i (F(\bar{\omega}^a) - F(\bar{\omega}^b)) \\
& + (1 - \mu) \int_0^{\bar{\omega}^b} \omega R_{t+1}^k q_t K_{t+1} f(\omega) d\omega \\
& - (1 - \mu) \int_0^{\bar{\omega}^a} \omega E_t R_{t+1}^k q_t K_{t+1} f(\omega) d\omega + w_t^b
\end{aligned}$$

where ϕ_t is the bank default rate, which will be explained in the bank regulation section. Aggregate bank equity at time $t+1$ consists of three parts: equity from those banks who did not default at time t , unexpected gains or losses in the loan portfolio, and bankers' wages collected by serving in the aggregate production function.

Given the loan portfolio amount on the bank's balance sheet L_t derived from the optimal debt contract, and the amount of bank equity, we can calculate the aggregate bank capital ratio:

$$\Delta_t = \frac{e_t}{L_t} \quad (25)$$

The rest of bank funding

$$d_t = L_t - e_t \quad (26)$$

will be collected from the households in the form of deposits. Therefore, from an aggregate level, the opportunity cost of bank funding is a linear combination of cost of bank equity and cost of deposits, where the proportion of each type of funding varies according to the bank capital ratio.

$$R_{t+1}^f = \Delta_t R_{t+1}^e + (1 - \Delta_t) R_{t+1}^d \quad (27)$$

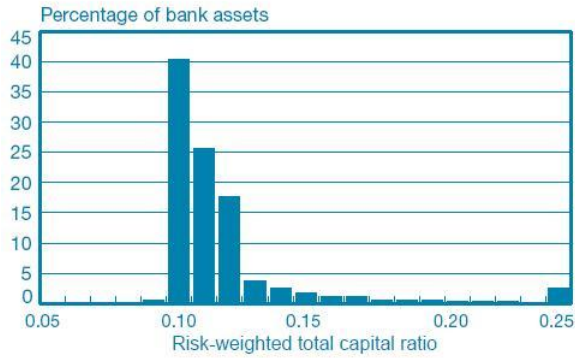
The respective costs of deposits R_{t+1}^d and equity R_{t+1}^e are derived endogenously from households' optimization problem.

Bank regulation In modern banking regulation, capital requirement has become the focal point.¹¹ Given the implicit or explicit government guarantee on bank deposit, bank capital regulation is imposed to curb banks' excessive

¹¹In this paper, bank capital regulation is taken as given, instead of being motivated from a micro perspective. It could be understood to mean that the threshold requirement is set to keep the government or the central bank from having to shoulder the burden of massive bank failures.

risk-taking. In 1987, the Bank for International Settlement (BIS) established the Basel I Accord, which provided a uniform capital standard for all banks in the member countries. Basel I required the ratio of banks' capital to risk-weighted assets to amount to a minimum of 8 percent, with at least 50 percent of it being tier 1 capital. By 1993, nearly all of the world's big banks had satisfied the Basel capital requirement. Many of them have been increasing their capital ratio. Figure 1 presents a histogram of the risk-based total capital ratios of U.S. commercial banks in the fourth quarter of 2000. As we can see from the figure, capital ratios vary across banks, with most of them between 10 and 11 percent, and very few below 10 percent.

Figure 1: Distribution of Bank Capital Ratio of U.S. Banks in 2000:4



Source: Federal Reserve Bank of Chicago

Motivated by this empirical observation, the capital ratio across banks in the model is assumed to have *log-normal* distribution.¹² The mode of the distribution is given by the aggregate capital ratio derived above. $\Delta_{i,t}$ log-normal (Δ_t, σ) . The health of the banking sector as a whole will depend largely on the variation of aggregate capital ratio. With a higher aggregate ratio, the distribution moves rightward, and fewer banks will fall short of the 8 percent threshold and thus default, and vice versa.¹³ The default proba-

¹²This is a shortcut to capture the distribution of the capital ratio among banks, which could be endogenously derived from the bank equity accumulation equation. For simulation purposes, a constant variance is assumed for the distribution.

¹³The conditional distribution of bank capital ratio could be derived endogenously from

bility is given by the cumulative distribution function up to the regulatory threshold:

$$\phi_t = cdf(\Delta_t, \sigma)^{14} \quad (28)$$

The higher the default probability, the more it costs banks to raise equity. Therefore, a low capital ratio today will lead to higher equity costs in the next period. This increase in funding costs will reduce the credit supply and hence also aggregate investment.

Compare this with the BGG setting, in which bank' funding costs are independent of its capital structure. In a downturn, even though large loan losses lead to a weak capital position, funding costs remain the same, as households do not charge a risk premium for the increased banking instability; therefore, there is no acceleration effect of the cycles in the short run.

2.2.5 Retail Sector

The retail sector is introduced into the model to motivate sticky prices. As is standard in the literature, monopolistic competition and Calvo pricing are assumed in this sector. Retailers purchase the wholesale good from entrepreneurs at a price equal to its nominal marginal cost and differentiate them at no cost. They then sell these differentiated retail goods in a monopolistically competitive market. Let $Y_t(i)$ be the quantity of output sold by retailer i , measured in units of wholesale goods, and let $P_t(i)$ be the nominal price. Total final usable goods Y_t are the following composite of retail goods:

$$Y_t = \left[\int_0^1 Y_t(i)^{(\epsilon-1)/\epsilon} di \right]^{\epsilon/(\epsilon-1)} \quad (29)$$

the bank equity accumulation equation. For simplicity, in the simulation only the mean of the distribution is used. As Krussel and Smith (1995) have shown, the behavior of the macroeconomic aggregates can be described almost perfectly using only the mean of the wealth distribution.

¹⁴Since banks that fall below the regulatory threshold cannot make new loans, they exit from the industry. Note that the default case in this model is benign, i.e. banks default because of bad fundamentals. Irrational bank runs caused purely by shifts in people's expectations are not considered here.

with $\epsilon \geq 1$ representing the degree of monopolistic competition. The corresponding price index is given by

$$P_t = \left[\int_0^1 P_t(i)^{1-\epsilon} di \right]^{1/(1-\epsilon)} \quad (30)$$

Following Calvo (1983), in a given period the retailer receives the signal to adjust the price with probability $1 - \theta$ and otherwise has to maintain the previous price. Let $P_t^*(i)$ denote the price set by retailers who are able to change price at t , and $Y_t^*(i)$ the demand given this price. The retailer will thus choose this price to maximize future expected discounted real profits, given by:

$$\max E_t \sum_{k=0}^{\infty} \left[\theta^k \Lambda_{t,k} \Omega_{t+k}(i) / P_{t+k} \right] \quad (31)$$

subject to the demand function

$$Y_{t+k}^*(i) = \left(\frac{P_t^*(i)}{P_{t+k}} \right)^{-\epsilon} Y_{t+k} \quad (32)$$

where the discount rate $\Lambda_{t,k} = \beta^k C_t / C_{t+k}$ (given assumed log utility in consumption) is the household intertemporal marginal rate of substitution, which the retailer takes as given. Ω_{t+k} is nominal profits given by $(P_t^*(i) - MC_{t+k}) Y_{t+k}^*(i)$. The optimization problem yields the following condition:

$$P_t^*(i) = \frac{\theta}{\theta - 1} \frac{E_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,k} MC_{t+k}(i) Y_{t+k}^*(i) / P_{t+k}}{E_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,k} Y_{t+k}(i) / P_{t+k}} \quad (33)$$

Given that the share θ of retailers do not change their price in period t , the aggregate price evolves according to:

$$P_t = [\theta P_{t-1}^{1-\epsilon} + (1 - \theta)(P_t^*)^{1-\epsilon}]^{\frac{1}{1-\epsilon}} \quad (34)$$

Combining the optimal pricing and the evolution of aggregate price and then log-linearizing, we obtain a standard Phillips curve where $\hat{m}c_t$ represents the real marginal cost gap.

$$\beta E_t \pi_{t+1} = \pi_t - (1 - \beta\theta) \frac{1 - \theta}{\theta} \hat{m}c_t \quad (35)$$

2.2.6 Monetary Policy

Following BGG (1999), the model considers a simple rule according to which the central bank adjusts the current nominal interest rate in response to the lagged inflation rate and the lagged interest rate.

$$r_t^n = \rho_r r_{t-1}^n + \rho_\pi \pi_{t-1} + \epsilon_t \quad (36)$$

3 Calibration

In the household utility function, ρ is chosen so that steady-state labor is 0.3. φ is calibrated so that the steady-state liquidity premium is 380 bp on an annual basis. β is calibrated at 0.988. In the aggregate production function, the capital share is 0.33, the share of household labor is 0.66, the share of entrepreneur labor is 0.00956 and the share of banking labor is 0.00044. Capital depreciates at 2.5 percent quarterly. In the retail sector, the degree of monopolistic competition ϵ is calibrated at 6, which implies a steady-state mark-up of 20 percent. The Calvo probability that a firm does not change price in a given period θ is calibrated at 0.75, which implies that prices in the economy are adjusted every four quarters on average. In monetary policy, the autoregressive coefficient is set to 0.65 and the coefficient of lagged inflation 1.2. These calibrations are standard in the literature.

In the financial contract, the monitoring cost parameter μ is set to 0.12, following BGG 1999. The probability that entrepreneurs die in a given period $1 - \gamma$ is set to 0.019. The variance of idiosyncratic productivity is set to 0.265. These parameterizations lead to the following steady-state values: a capital-to-net worth ratio of 2 (leverage ratio of 0.5), an annual loan default rate of 2.56 percent, and an annual external finance premium of 180 bp. In the distribution of the bank capital ratio, the steady-state ratio is calibrated at 10 percent and the variance of the distribution is set to match a steady-state bank default rate of 1 percent.

As usual, the aggregate productivity shock follows an AR (1) process, with a coefficient of 0.9 and a standard deviation of 0.0056. Another parameter important for the model dynamics is the capital adjustment parameter χ . Estimates by Chirinko (1993) based on aggregate data show a value of 2, which was then used in King and Wolman (1996). Therefore, in this model, χ is calibrated at 2.

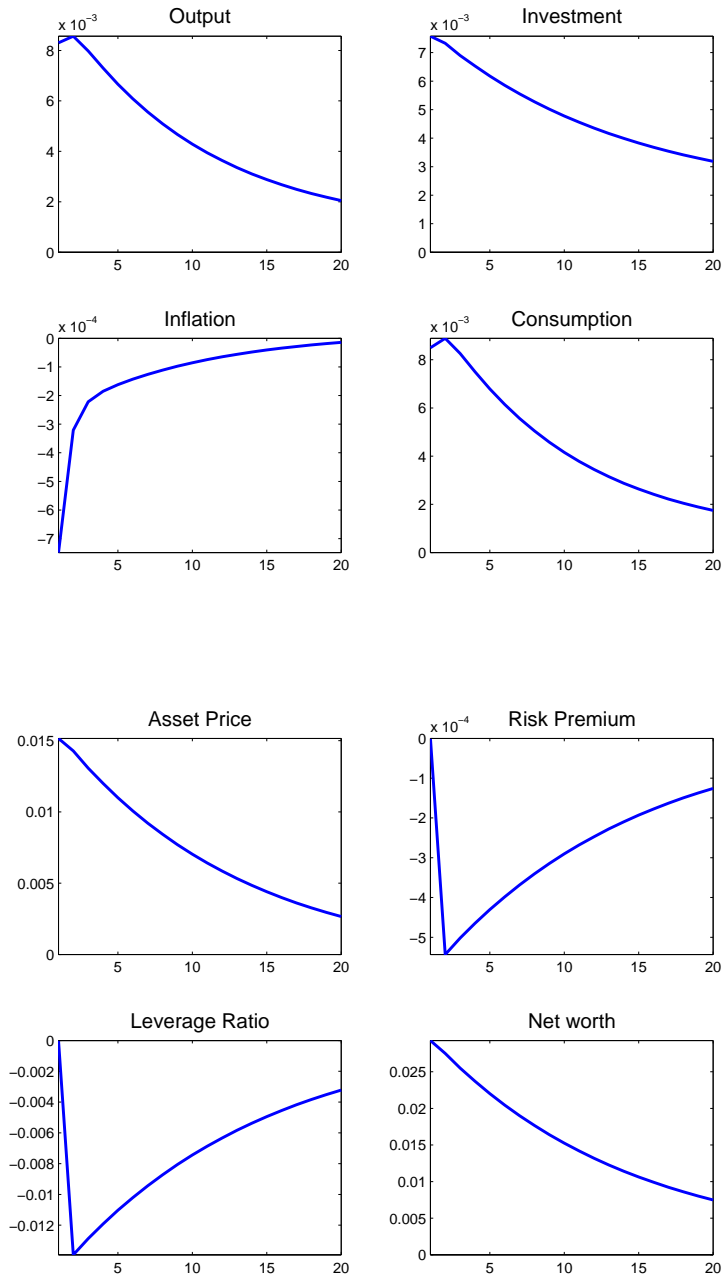
4 Simulation

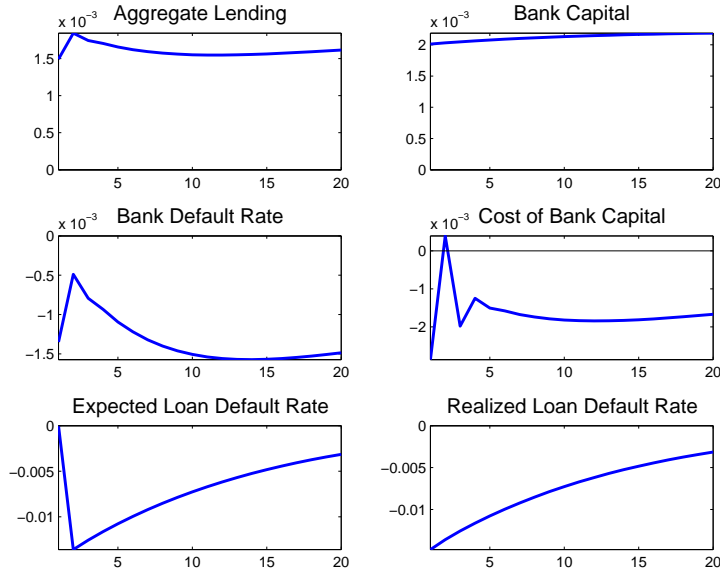
In the simulation, we consider two shocks: a technology shock and a monetary policy shock. First, the impulse responses to shocks are analyzed; then the model is compared with a model where the only financial friction comes from the credit supply side and with a baseline model with no financial friction. The marginal contributions of the bank capital channel to the long-run steady state and short-run dynamics are studied.

4.1 Technology shock

After a positive technology shock, the realized capital return is higher than expected. This means the ex-post loan default rate is lower than the ex-ante rate; the unexpected gain from the loan portfolio will strengthen banks' capital and increase banks' capital ratio. Given the improvement in banks' balance sheets, households expect a lower bank default rate in the next period and are therefore willing to hold bank capital at lower rates of return. The reduction in the cost of funding from the banks' side will push up credit supply and drive up investment in equilibrium. On the other hand, after a positive technology shock, firms' net worth increases and leverage ratios go down, causing them to face lower agency costs in the credit market and enabling them to obtain loans at lower external finance premiums. The positive reaction from both the credit supply and credit demand side drive up aggregate lending to a large extent, which implies an investment boom. This raises output, consumption, and asset prices. The marginal cost of production falls after productivity increases; therefore, inflation goes down.

Figure 2: Impulse responses to a productivity shock



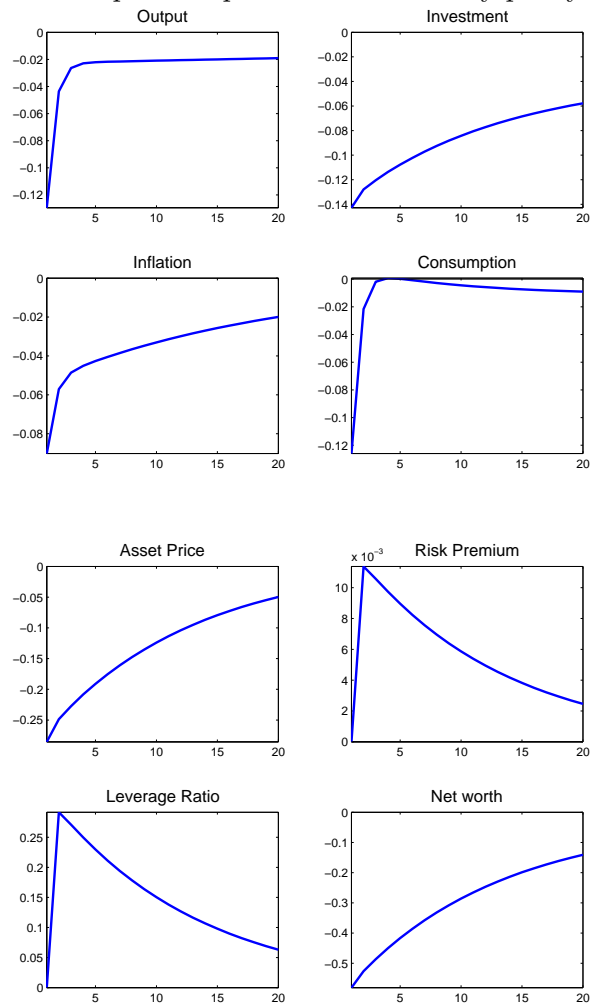


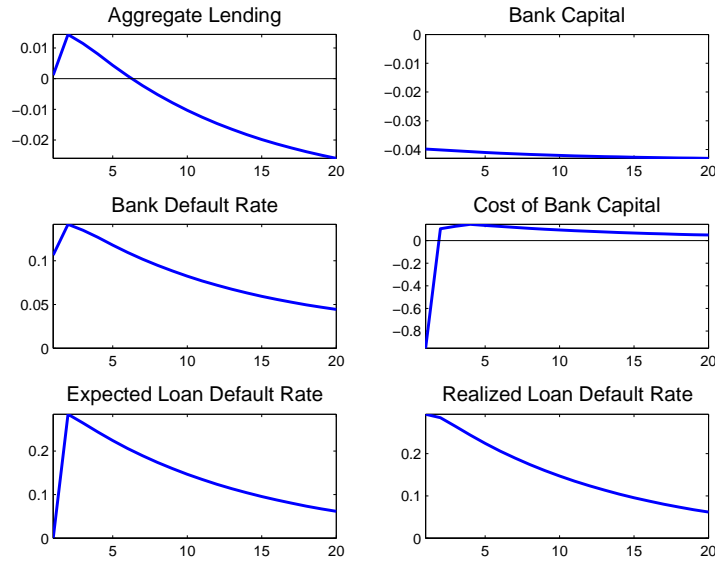
4.2 Monetary policy shock

After a monetary policy tightening, the cost of deposits becomes higher, bank credit supply does down, ex-post loan default rate goes up. The unexpected loss in loan portfolio will write off bank's capital, and decrease bank capital ratio. The deterioration in banks' balance sheets will therefore lead households to demand higher returns for holding bank capital in the next period. The difficulty in raising capital will further depress banks' credit supply and propagate the monetary policy shock. However, if the net worth of entrepreneurs falls below equilibrium, the leverage ratio rises. This makes them look less attractive in the credit market and forces them to pay a higher external finance premium. *Note that, despite the contraction in both credit supply and credit demand, the aggregate lending goes up for about four to six quarters and then goes down.* This kind of loan behavior has been well documented in empirical studies. Christiano, Eichenbaum and Evans (1996) show that 'following a contractionary shock to monetary policy, net funds raised by the business sector increases for roughly a year, after which they then fall'. The reason for the temporary increase in the loan amount is that, after a monetary policy tightening, net worth goes down, as do capital and asset prices. The adjustment speed of capital is low; therefore, the change in aggregate lending depends on the adjustment speed of net worth

and asset prices. Since at the beginning net worth decreases much faster than the asset price, the firm has to borrow more external funds to finance a reduced amount of investment. The rest of dynamics are standard: after interest rates are increased, inflation goes down and so does consumption. Contraction of both output and consumption drives down the output level.

Figure 3: Impulse responses to a monetary policy shock

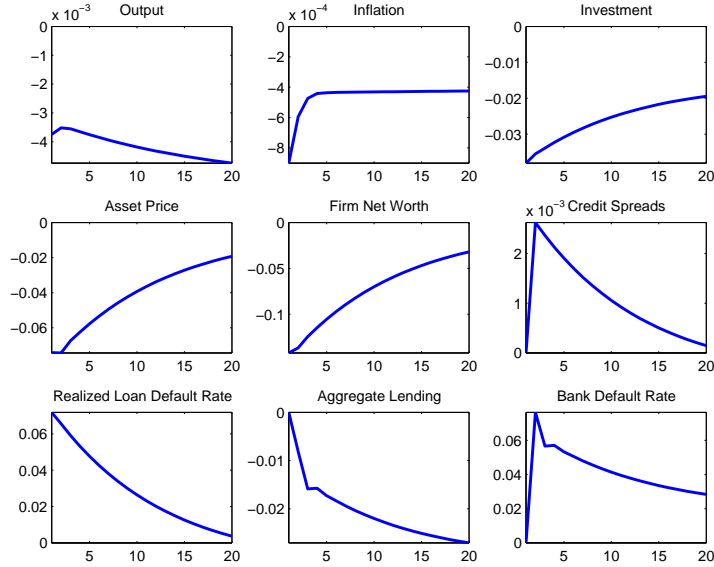




4.3 Financial shock

This section consider the model dynamics after a negative shock to bank capital. Assume there is an exogenous deterioration of bank's balance sheet and therefore a sudden drop of bank capital, possibly due to the burst of an asset price bubble, which leads to larger write-offs of bank equity compared to the case where asset swing is only driven by fundamental as modeled in this paper. From the simulation we can see that, a sudden drop of bank's capital position leads to strong contraction in bank's credit supply. We observe a decrease in aggregate lending and an increase in credit spreads. Tightening of credit market corresponds to dampened aggregate investment, which further deteriorates firm's balance sheet, loan default rate goes up. Weak aggregate demand leads to both low output and inflation.

Figure 4: Impulse responses to a financial shock



4.4 Model Comparison: Marginal Effect of Banking Instability

Next, we compare this model with a model where only the BGG type of financial friction exists as well as with a standard model with no financial friction. The results show that additional instability of the banking sector will reduce aggregate capital stock and the investment level and have an acceleration effect on the short-run dynamics of the model.

4.4.1 Long-run effect

In this model, bank default is an equilibrium phenomenon. That is, those banks whose capital ratio falls below 8 percent will be shut down by regulators. If we consider a model with no regulatory capital requirement, which means the capital ratio remains variable, yet the bank does not default, the following table tell us that the default probability of banks leads to lower output level in the steady state. The reason is that, given the banks' default probability, households will demand a higher return to hold bank capital; the increase in the cost of funding will therefore drive down bank credit supply and therefore investment in equilibrium.

Table 1: Steady states comparison

Variable	Zhang	BGG
Capital	7.1621	7.4116
Investment	0.17905	0.1853
Output	0.86509	0.875
Consumption	0.68604	0.68964

4.4.2 Short-run effect

Figure 4 and Figure 5 compare the relative importance of various frictions in shock transmission. The green line describes impulse responses in a standard DSGE model, where only nominal rigidity and capital adjustment costs are considered. The red line incorporates the additional friction coming from the credit demand side, or the financial accelerator effect. The blue line captures the model dynamics where the bank capital channel is added to the previous frictions.

As we can see from the figures, the bank capital channel has a strong acceleration and propagation effect on both the impulse response to the technology shock and the monetary policy shock.¹⁵ The most significant effect of bank capital is on investment, asset prices, and the external finance premium. The instability in the banking sector introduces extra volatility to these variables. The reason that output does not seem to be so dramatically affected is that consumption, which accounts for 80 percent of output in the model calibration, is not strongly subject to the influence of banking instability. Things would be very different if we consider that households also have to rely on credit to sustain consumption, which is a very relevant case in many countries. As long as households also borrow from banks, banking instability will have a larger effect on the consumption level, and a significant change in output due to bank capital channel along may be expected.

Another observation from figure 4 is that the bank capital channel is more important than the financial accelerator in amplifying policy shocks. This is consistent with previous findings in the literature that the financial accelerator contributes only marginally to monetary policy transmission.

¹⁵Previous literature on the bank capital channel (e.g. Markovic (2006)) can generate the acceleration effect, but not so much the propagation effect, as the marginal contribution of credit supply friction vanishes after around 8 quarters following a policy shock. In this model, bank capital not only accelerates the cycle, but also creates more persistence.

However, the relative importance of the two channels is reversed when a positive technology shock hits the economy, where strong corporate balance sheets play an important role in driving up asset prices and aggregate investment.

Figure 5: Impulse responses to a monetary policy shock

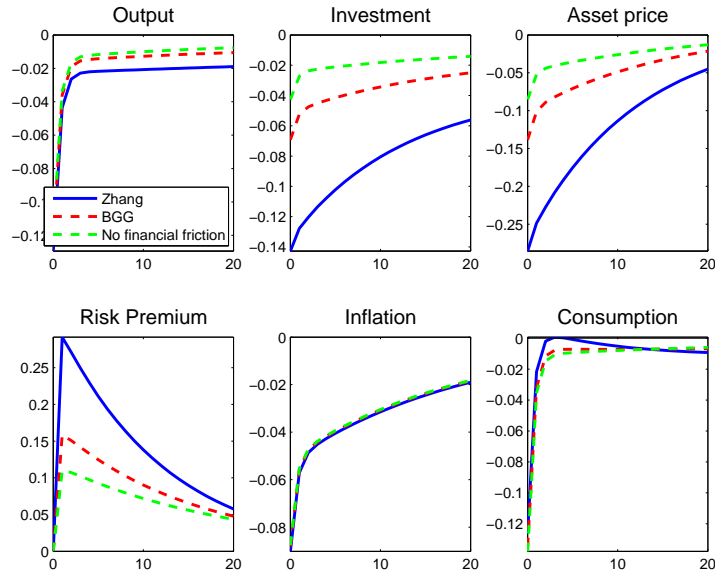
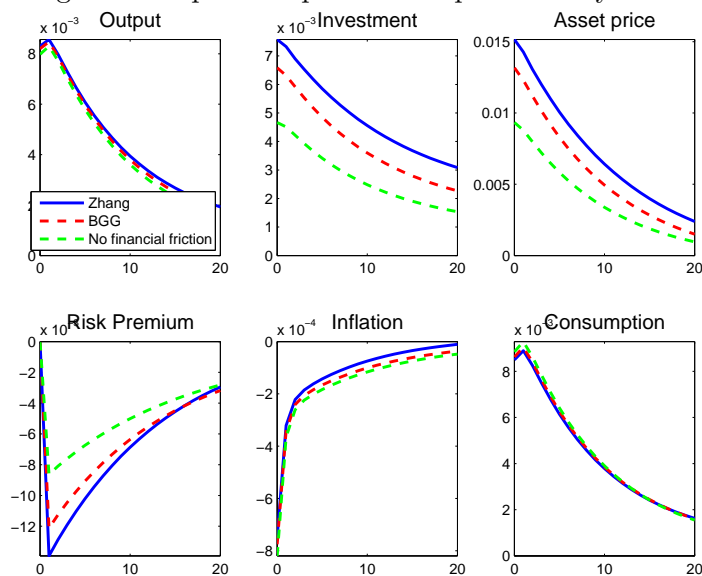


Figure 6: Impulse responses to a productivity shock



5 Conclusion

This paper extends a general equilibrium model with a BGG-type financial accelerator to a model in which financial friction coming from both the credit supply and credit demand sides are considered. Previous strands of the literature (BGG (1999), Carlstrom and Fuerst (1997)) have emphasized credit demand friction under information asymmetry, where borrowers have to pay an external finance premium to compensate lenders for monitoring costs, which are spent to identify project returns in the loan default case. The economic implication for this agency problem is that the more highly leveraged the firm is, the higher the external finance premium it has to pay due to the increased agency costs. In economic downturn, lower net worth of firms leads to higher borrowing costs in the next period, which drives investment down even further and propagates the business cycle. The biggest shortcoming in the previous literature is that it ignores the instability of financial intermediaries and related credit supply friction. In earlier models, banks can diversify idiosyncratic risk and can avoid aggregate risk by writing state-contingent loan contracts, therefore ensuring that the financial intermediary is always in a safe position. The economic crisis in 2008 has made us realize how important the instability of the banking sector is in accelerating business cycles. It is therefore crucial to demonstrate this

significant role of financial intermediaries in the model we use to do policy analysis. In the model, banks can still diversify idiosyncratic risk, but they have to share aggregate risk with the borrowers. The loan contract is written based on two parties' expectation of the economic situation in the next period. When an unexpected negative shock hits the economy, not only does the net worth of firms go down, but banks also face large equity write-offs because of unexpected losses from their loan portfolio. Given the bank capital regulation, where banks have to hold a minimum capital-to-asset ratio, in the next period, banks have to pay more to raise funds from households because they are perceived as having a higher default probability. The difficulty for banks to raise funds themselves and the higher agency costs firms have to pay to obtain credit given an increased leverage ratio will interact and drive the economy much deeper into recession. Model simulation has shown that the instability of the banking sector also creates strong credit supply friction and contributes significantly to accelerating short-run cycles. In the long run, the instability of the banking sector implies a lower capital stock in the economy, and therefore a lower level of investment and output.

In future research, this model could be extended to consumer loans. Since consumption is the major component of output, once the feedback from banking instability to consumption is incorporated, the effect on output will be much more significant compared to the corporate-loans-only case. The model could also be extended to an open economy and study how the instability of a financial intermediary in one country could influence the real sector in the other economy.

Appendix A: The Financial Contract

In the financial contract, the entrepreneurs maximize expected profit subject to the participation constraint of the bank,

$$\max \int_{\bar{\omega}^{i,a}}^{\infty} \omega E_t R_{t+1}^k q_t K_{t+1}^i f(\omega) d\omega - (1 - F(\bar{\omega}^{i,a})) \bar{\omega}^{i,a} E_t R_{t+1}^k q_t K_{t+1}^i$$

subject to

$$(1 - F(\bar{\omega}^{i,a})) R_{t+1}^L L_{t+1}^i + (1 - \mu) \int_0^{\bar{\omega}^{i,a}} \omega E_t R_{t+1}^k q_t K_{t+1}^i f(\omega) d\omega = R_{t+1}^f L_{t+1}^i$$

Recall that

$$\bar{\omega}_t^{i,a} E_t R_{t+1}^k q_t K_{t+1}^i = R_{t+1}^L L_{t+1}^i$$

The key difference in solving the contract compared to BGG is that, in BGG, the expectation operator is outside the brackets, since ω itself is not fixed but instead contingent on R_{t+1}^k :

$$\max E_t \left\{ \int_{\bar{\omega}^{i,a}}^{\infty} \omega R_{t+1}^k q_t K_{t+1}^i f(\omega) d\omega - (1 - F(\bar{\omega}^{i,a})) \bar{\omega}^{i,a} R_{t+1}^k q_t K_{t+1}^i \right\}$$

subject to

$$(1 - F(\bar{\omega}^{i,a})) R_{t+1}^L L_{t+1}^i + (1 - \mu) \int_0^{\bar{\omega}^{i,a}} \omega R_{t+1}^k q_t K_{t+1}^i f(\omega) d\omega = R_{t+1}^f L_{t+1}^i$$

This participation constraint has to hold for each realization of R_{t+1}^k ; therefore, $\bar{\omega}^a$ is a function of R_{t+1}^k . By contrast, in our model the participation constraint only holds for $E_t R_{t+1}^k$ and will break down ex-post if the realization of R_{t+1}^k deviates from expectation.

Define

$$\Gamma(\bar{\omega}^{i,a}) = \int_{\bar{\omega}^{i,a}}^{\infty} \omega f(\omega) d\omega - (1 - F(\bar{\omega}^{i,a})) \bar{\omega}^{i,a} \quad (\text{A-1})$$

$$G(\bar{\omega}^{i,a}) = \int_0^{\bar{\omega}^{i,a}} \omega f(\omega) d\omega \quad (\text{A-2})$$

The financial contract can then be transformed into

$$\max_{K_{t+1}^i, \bar{\omega}^{i,a}} (1 - \Gamma(\bar{\omega}^{i,a})) E_t R_{t+1}^k q_t K_{t+1}^i$$

subject to

$$(\Gamma(\bar{\omega}^{i,a}) - \mu G(\bar{\omega}^{i,a})) E_t R_{t+1}^k q_t K_{t+1}^i = R_{t+1}^f (q_t K_{t+1}^i - N_{t+1}^i)$$

Define external finance premium $s^i = \frac{E_t R_{t+1}^k}{R_{t+1}^f}$ and firm leverage ratio $k^i = \frac{K_{t+1}^i}{N_{t+1}^i}$ and let λ be the Lagrange multiplier on the bank participation constraint. First-order conditions imply that:

$$\lambda = \frac{\Gamma'(\bar{\omega}^{i,a})}{\Gamma'(\bar{\omega}^{i,a}) - \mu G'(\bar{\omega}^{i,a})} \quad (\text{A-3})$$

$$s^i = \frac{\lambda}{1 - \Gamma(\bar{\omega}^{i,a}) + \lambda(\Gamma(\bar{\omega}^{i,a}) - \mu G(\bar{\omega}^{i,a}))} \quad (\text{A-4})$$

Combining first-order conditions with the participation constraint enables us to derive a one-to-one relationship between the external finance premium and the cut-off threshold value, as well as a one-to-one relationship between the leverage ratio and the cut-off threshold value:

$$s^i = s(\bar{\omega}^{i,a}) = \frac{\lambda(\bar{\omega}^{i,a})}{1 - \Gamma(\bar{\omega}^{i,a}) + \lambda(\bar{\omega}^{i,a})(\Gamma(\bar{\omega}^{i,a}) - \mu G(\bar{\omega}^{i,a}))} \quad (\text{A-5})$$

$$k^i = k(\bar{\omega}^{i,a}) = 1 + \frac{\lambda(\Gamma(\bar{\omega}^{i,a}) - \mu G(\bar{\omega}^{i,a}))}{1 - \Gamma(\bar{\omega}^{i,a})} \quad (\text{A-6})$$

Therefore there exists a one-one relationship between the firm leverage ratio and the external finance premium:

$$k^i = \varphi(s^i) \quad (\text{A-7})$$

or $q_t K_{t+1}^i = \varphi(s^i) N_{t+1}^i$. Since the leverage ratio is the same across firms, they pay the same external risk premium s . We can thus easily aggregate this equation, and derive the following:

$$q_t K_{t+1} = \varphi\left(\frac{E_t R_{t+1}^k}{R_{t+1}^f}\right) N_{t+1} \quad (\text{A-8})$$

where K_{t+1} and N_{t+1} represent aggregate capital and firm net worth. We can also rewrite this equation into equation (6) in the paper:

$$E_t R_{t+1}^k = s \left(\frac{q_t K_{t+1}}{N_{t+1}} \right) R_{t+1}^f \quad (\text{A-9})$$

Appendix B: First-Order Conditions

$$U_c(c_t, d_t) = \beta E_t R_{t+1}^e (1 - \phi_{t+1}) U_c(c_{t+1}, d_{t+1}) \quad (\text{B-1})$$

$$U_c(c_t, d_t) - U_d(c_t, d_t) = \beta E_t R_{t+1}^d U_c(c_{t+1}, d_{t+1}) \quad (\text{B-2})$$

$$-U_{c,t}/U_{h,t} = w_t \quad (\text{B-3})$$

$$z_t = \alpha_k m c_t \frac{Y_t}{K_t} \quad (\text{B-4})$$

$$w_t^h = \alpha_h m c_t \frac{Y_t}{h_t} \quad (\text{B-5})$$

$$w_t^e = \alpha_e m c_t \frac{Y_t}{l_t^e} \quad (\text{B-6})$$

$$w_t^b = \alpha_b m c_t \frac{Y_t}{l_t^b} \quad (\text{B-7})$$

$$q_t = 1 + \chi \left(\frac{i_t}{k_t} - \delta \right) \quad (\text{B-8})$$

$$k_{t+1} = i_t + (1 - \delta) k_t \quad (\text{B-9})$$

$$R_{t+1}^k = \frac{z_{t+1} + (1 - \delta) q_{t+1}}{q_t} \quad (\text{B-10})$$

$$E_t R_{t+1}^k = S \left(\frac{q_t K_{t+1}}{N_{t+1}} \right) R_{t+1}^f \quad (\text{B-11})$$

$$R_{t+1}^f = \Delta_t R_{t+1}^e + (1 - \Delta_t) R_{t+1}^d \quad (\text{B-12})$$

$$\bar{\omega}_{t+1}^a E_t R_{t+1}^k q_t K_{t+1} = R_{t+1}^L L_{t+1} \quad (\text{B-13})$$

$$\frac{q_t K_{t+1}}{N_{t+1}} = 1 - s(\bar{\omega}_{t+1}^a) (\Gamma(\bar{\omega}_{t+1}^a) - \mu G(\bar{\omega}_{t+1}^a)) \quad (\text{B-14})$$

$$\bar{\omega}_{t+1}^b = \frac{\bar{\omega}_{t+1}^a E_t R_{t+1}^k}{R_{t+1}^k} \quad (\text{B-15})$$

$$N_{t+1} = \gamma V_t + w_t^e \quad (\text{B-16})$$

$$V_t = \int_{\bar{\omega}^b}^{\infty} \omega R_{t+1}^k q_t K_{t+1} f(\omega) d\omega - (1 - F(\bar{\omega}^b)) R_{t+1}^L L_{t+1}^i \quad (\text{B-17})$$

$$\phi_t = cdf(\Delta_t, \sigma) \quad (\text{B-18})$$

$$P_t^*(i) = \frac{\theta}{\theta - 1} \frac{E_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,k} M C_{t+k}(i) Y_{t+k}^*(i) / P_{t+k}}{E_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,k} Y_{t+k}(i) / P_{t+k}} \quad (\text{B-19})$$

$$\begin{aligned} e_{t+1} = & (1 - \phi_t) e_t + R_{t+1}^L L_{t+1}^i (F(\bar{\omega}^a) - F(\bar{\omega}^b)) \\ & + (1 - \mu) \int_0^{\bar{\omega}^b} \omega R_{t+1}^k q_t K_{t+1} f(\omega) d\omega \\ & - (1 - \mu) \int_0^{\bar{\omega}^a} \omega E_t R_{t+1}^k q_t K_{t+1} f(\omega) d\omega + w_t^b \end{aligned} \quad (\text{B-20})$$

Appendix C: The Steady States

$$R_{ss}^e(1 - \phi_{ss}) = 1/\beta \quad (C-1)$$

$$\frac{c_{ss}^\sigma}{d_{ss}^\sigma} = R_{ss}^c(1 - \phi_{ss}) - R_{ss}^d \quad (C-2)$$

$$\frac{c_{ss}^{-\sigma}}{l_{ss}^\phi} = w_{ss}^h \quad (C-3)$$

$$mc_{ss} = \frac{\theta - 1}{\theta} \quad (C-4)$$

$$z_{ss} = \alpha_k mc_{ss} \frac{y_{ss}}{k_{ss}} \quad (C-5)$$

$$w_{ss}^h = \alpha_h mc_{ss} \frac{y_{ss}}{l_{ss}^h} \quad (C-6)$$

$$w_{ss}^e = \alpha_e mc_{ss} \frac{y_{ss}}{l_{ss}^e} \quad (C-7)$$

$$w_{ss}^b = \alpha_b mc_{ss} \frac{y_{ss}}{l_{ss}^b} \quad (C-8)$$

$$q_{ss} = 1 \quad (C-9)$$

$$i_{ss} = \delta k_{ss} \quad (C-10)$$

$$R_{ss}^k = z_{ss} + 1 - \delta \quad (C-11)$$

$$lev_{ss} = \frac{q_{ss}K_{ss}}{N_{ss}} \quad (C-12)$$

$$R_{ss}^k = S(lev_{ss})R_{ss}^f \quad (C-13)$$

$$R_{ss}^f = \Delta_{ss}R_{ss}^e + (1 - \Delta_{ss})R_{ss}^d \quad (C-14)$$

$$\bar{w}_{ss}^a = \frac{R_{ss}^l L_{ss}}{R_{ss}^k q_{ss} K_{ss}} \quad (C-15)$$

$$\bar{w}_{ss}^b = \bar{w}_{ss}^a \quad (C-16)$$

$$lev_{ss} = 1 - s(\bar{w}_{ss}^a)(\Gamma(\bar{w}_{ss}^a) - \mu G(\bar{w}_{ss}^a)) \quad (C-17)$$

$$N_{ss} = (1 - \gamma)w_{ss}^e \quad (C-18)$$

$$\phi_{ss} = pdf(\Delta_{ss}, \sigma) \quad (C-19)$$

$$e_{ss} = (1 - \phi_{ss})w_{ss}^b \quad (C-20)$$

$$\pi_{ss} = 1 \quad (C-21)$$

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