Money Talks: Information and Monetary Policy*

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Abstract

We study credible information transmission by a benevolent Central Bank. We consider two possibilities: direct revelation through an announcement, versus indirect information transmission through monetary policy. These two ways of transmitting information have very different consequences. Since the objectives of the Central Bank and those of individual investors are not always aligned, private investors might rationally ignore announcements by the Central Bank. In contrast, information transmission through changes in the interest rate creates a distortion, thus, lending an amount of credibility. This induces the private investors to rationally take into account information revealed through monetary policy.

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1 Introduction

How can a Central Bank effectively communicate its information about economic fundamentals to the private sector? What is the role of monetary policy as a tool for information revelation in the presence of aggregate risk and of potential coordination problems faced by investors? Why do Central Banks typically follow policies that lead to positive average levels of inflation? These questions are highly topical and have a long history in economics. However, decisive answers still elude us. This paper studies monetary policy as a tool for credible information transmission by the Central Bank.[1]

We build a model that contains the following three ingredients: (1) money plays a role in facilitating trade, (2) there is aggregate risk about fundamentals and information about this risk is dispersed, and (3) investment decisions are subject to a coordination problem, an effect emphasized in Keynes (1936), implying that individually optimal decisions might not maximize aggregate welfare. Investors in our economy have heterogeneous expectations about economic fundamentals that affect investment returns, and they suffer a cost if their expectations are wrong. A benevolent Central Bank (CB) has the ability to print money and to provide loans. In addition, the CB has its own information about the true state of the economic fundamentals. We demonstrate that this information cannot always be credibly transmitted to the private sector. In other words, a simple announcement might not be enough, as the CB may prefer to communicate false information if this led to a more socially desirable behavior of investors. We then show that credible information revelation is possible through monetary policy. In order to gain credibility, however, it is *necessary*

[1] There is a large literature on optimal monetary policy in the presence of information frictions. See, for example, Weiss (1980) and Barro and Gordon (1983) for two related early models. Cukierman and Meltzer (1986) emphasize the role of ambiguity for Central Bank policies. Kydland and Prescott (1977) introduce the famous dynamic inconsistency problem. Backus and Drifill (1985) introduce uncertainty about the Central Bank's type (see also King, Lu and Pasten, 2008). Our approach differs in several ways from these papers. Perhaps the most important one is that we concentrate on the role of monetary policy as a *credible* information transmission mechanism in a model that does not rely on reputation building by a long-lived Central Bank. For a recent model that studies credible information transmission by the Central Bank, but without explicitly modelling money or monetary policy, see Moscarini (2007). Ellingsen and Söderström (2001) study the effects of monetary policy on the yield curve and document that when monetary policy reveals information about economic fundamentals, interest rates of all maturities move in the same direction as the policy innovation. In this paper, we provide an explanation why information revelation is done through monetary policy instead of announcements.

that such a policy create a distortion by affecting average inflation. The study of optimal monetary policy in our framework concerns the optimal balancing of the resulting benefits from information revelation against the costs associated with the monetary distortion. More precisely, monetary policy in our model can be thought of as the "translation" of the Central Bank's information, expressed by the corresponding value of the chosen interest rate. Absent the need for information transmission, the benevolent Central Bank would always set the nominal interest rate to zero. Thus, positive nominal interest rates serve as a way for the Central Bank to *credibly* transmit a "message" to the private sector. Since interest rates affect economic fundamentals, this information transmission gains the necessary element of credibility that would be missing in a pure announcement.[2]

Our model is motivated by events that took place in Sweden during the period 2005-2007. These events strongly suggest the need for a Central Bank to supplement its words with deeds. During that period, the Swedish Central bank (the Riksbank) was carefully monitoring the growth of housing prices in Sweden. Concerns about the rapid rise in these prices were repeatedly expressed publicly by the Riksbank. In 2005, they appear in six out of seven press releases that follow monetary policy decisions, as well as in the Financial Stability Report.[3] However, the rise in housing prices continued throughout 2005, reaching 10% in the third quarter of 2005 compared to the same quarter in the previous year.[4] According to the minutes of the Executive Board's monetary policy meeting of December 2005, it was suggested that raising the repo rate by 25 basis points "would also function as a signal that could subdue house price trends and household indebtedness." From January 2006 onwards, the Riksbank started gradually raising interest rates. In the corresponding announcements of monetary policy decisions, this rise was complemented by stressing the concern over housing price developments. According to the minutes of December 2006, while the rate of price

[2]Needless to say, monetary policy in the actual economy serves several purposes. In order to concentrate on its role as a credible communication device, we will abstract from effects related to liquidity provision. For a recent paper that studies credibility of Central Bank policies in a different context see Ennis and Keister (2007).

[3]The Riksbank changed the interest rate only once in 2005: a 0.50 percentage points cut in June.

[4]Moreover, the average annual growth rate of housing prices was about 7.5% between 1996 and 2005. Household borrowing showed a similar average rate of increase in this time period.

increases and borrowing remained high, some slowdown had already taken place.[5] Our interpretation of these events is that the interest rate increases added credibility to simple announcements by the Riksbank, thus, finally changing the behavior of the private sector. This was a task that repeated previous announcements alone had failed to accomplish.

Throughout the paper, we study the efficient and credible ways for a CB to communicate its information to private investors. Motivated by the Swedish experience, we consider two possibilities. First, the CB makes a direct "announcement." For example, one could think of a press statement or an interview delivered, say, by the Chairman of the CB. Alternatively, the CB may indirectly transmit its information through monetary policy; i.e., through varying the nominal interest rate. Our main finding is that these two ways of transmitting information can have very different consequences. Since the objective of the benevolent CB and that of an individual investor in the economy are not directly aligned, the CB might have an incentive to misrepresent its information if this would lead to private investment decisions that improve social welfare. The private investors might, in turn, rationally choose to ignore such announcements. In contrast, credible information transmission through changes in the interest rate is not "cheap talk," as it reduces welfare through creating a distortion associated with a violation of the Friedman rule. We demonstrate that, provided that the costs associated with inflation are sufficiently high, this adds a necessary amount of credibility that induces the private investors to rationally take into account the information provided by the CB. At the same time, social welfare is higher than in the absence of credible information transmission.

How big is the deviation from the Friedman rule needed to induce investors to take the CB's information into account? We show that effective changes in the nominal interest rate need not be large, and credibility may be ensured by deviations of 25 to 50 basis points. This result speaks to an ongoing debate in macroeconomics whether the CB should react to the developments in financial markets which are considered excessive, and which can lead to suboptimal investment decisions. In particular, the effectiveness of monetary policy in affecting investors' choices is often questioned. It is argued that to induce a change in

[5]See http://www.riksbank.com/templates/ItemList.aspx?id=27260 for the Minutes of the Executive Board's monetary policy meetings.

investors' behavior, a corresponding change in the interest rate would have to be very large. This will have a negative impact on the real economy, thus more than outweighing the benefits of improved resource allocation. We explicitly take the costs and benefits of credible information transmission into account, and demonstrate that the associated interest rate changes can be relatively modest.

To what extent can we think of monetary policy purely as a channel for credible information transmission? In Appendix 1, we formalize this question and provide a general equivalence result between the set of feasible monetary policies and the information set of the CB. This result implies that the set of feasible monetary policies is homeomorphic to the set describing the possible values of the CB's information. Whenever this result applies, at the cost of imposing a monetary distortion, the CB can use monetary policy in order to credibly transmit its information to the private sector.

The paper proceeds as follows. Section 2 describes the economic environment and considers the full information benchmark, while Section 3 studies the model under private information. A brief Conclusion follows. Appendix 1 discusses our general equivalence result. Appendix 2 contains the proofs of the propositions in the text.

2 The Environment

Our basic model uses the setup developed by Berentsen and Monnet (2008).[6] Time $t = 0, 1, ...,$ is infinite. The economy consists of a continuum of infinite-lived agents. In addition, there is a benevolent Central Bank (CB) which has the ability to make public announcements, to print money, and to make loans to the private sector. The CB serves for one term only and is replaced by a new CB at the end of each period.[7]

[6] They, in turn, build on Lagos and Wright (2005) and on Kiyotaki and Wright (1989). Other choices of a monetary model are also consistent with our findings. Our choice was for a tractable model in which to study information transmission by the Central Bank and where money is essential. Indeed, our model can be easily converted to a cash-in-advance economy.

[7] This feature allows us to abstract from reputation effects, which, as we mentioned before, have been the focus of study in other papers. In the case of a long-lived CB, truthful information transmission can be assured if (sufficiently patient) investors trigger a severe penalty when they discover that the CB has miscommunicated its information. Our analysis focuses on achieving credible truthful communication in the absence of such reputation effects.

Each period is divided into three stages: 0, 1, and 2. There are three goods: an *investment good*, k, a *stage-1-good*, q, and a *stage-2-good*, z. Investment takes place in stage 0 of each period. The market for good q opens in stage 1. In stage 2, investment pays off in units of good z. Future periods are discounted at rate $\beta \in (0,1)$. There is no discounting between stages. We consider each stage in more detail next.

In stage 0 of each period, half of the agents are randomly chosen to be producers (*investors*). Each investor i chooses how much of an investment good, k, to produce. The utility cost to investor i from producing k_i units of the investment good is $c(k_i)$, where $c(\cdot)$ is strictly increasing and strictly convex. All investments mature in stage 2. The return on this investment is uncertain and is given by θ^2 units of good z per unit of k, where θ is a random variable with an improper uniform prior on $(-\infty, +\infty)$ and is *iid* across periods. Nature draws θ at the start of stage 0.

The random variable θ is our way of introducing aggregate risk about the profitability of investment, a feature that can be traced back at least to Keynes (1936). Some (diverse) information regarding this risk is available in the economy. More precisely, when the true state is θ, the CB receives a signal $y = \theta + \eta$, while each investor i receives a signal $x_i = \theta + \varepsilon_i$. These signals are received in stage 0 of each period, prior to the investment decisions, and are *iid* across time.[8] We assume that the noise terms η and ε_i are normally distributed with mean zero and respective precisions given by $\alpha = 1/\sigma_\eta^2$ and $\delta = 1/\sigma_\varepsilon^2$. Moreover, $E(\varepsilon_i \varepsilon_j) = 0$ holds for $i \neq j$. We use $\mathbf{x}$ to denote the profile of private signals across investors. The assumption that investors have some available information about the future profitability of their investments seems natural. In addition, CBs employ large numbers of specialists in order to collect and analyze economic data. The fact that this data is treated with secrecy suggests that CBs consider the information in their possession important, and that they care about how and when this information is revealed to the public. Notice that we do not take a stand on whether the CB or the private investors have more accurate information.

[8] Stages 0 and 1 are interchangeable in what follows. What is important is that the CB's announcement/interest rate choice takes place prior to the investment decision by the private sector. For simplicity, we will assume that only investors (not consumers) receive signals. Morris and Shin (2002) use a similar signalling structure to study the trade-off created by increasing the quality of the information held by the Central Bank.

In stage 1, investors can produce the stage-1-good, q, at a cost $c(q) = -q$. The remaining agents can consume q, deriving utility $u(q)$, where u is increasing, concave, and satisfies the usual Inada conditions. In particular, there is a unique q^* such that $u'(q^*) = 1$. Good q is sold in a competitive market. Traders are anonymous in this market, so money is used in the exchange of q. In stage 1 of each period, agents have access to a lending facility operated by the CB, where they can borrow money at an interest rate $r \geq 0$. The CB keeps track of all such borrowing and loans are settled in stage 2.[9]

Finally, stage-0-investment delivers a return of $\theta^2 k_i$ units of the non-storable stage-2-good, z.[10] This good is traded in a competitive market in stage 2. The utility (disutility) of consumption (production) from z is linear and is denoted by $(-)z$. Stage 2 can be thought of as a settlement stage. Those consumers that borrowed money in order to consume in stage 1 must produce in order to pay off these loans. The investors, who produce in stage 1, will end up consuming in stage 2.[11]

The sequence of events in each stage is summarized in figure 1 below.

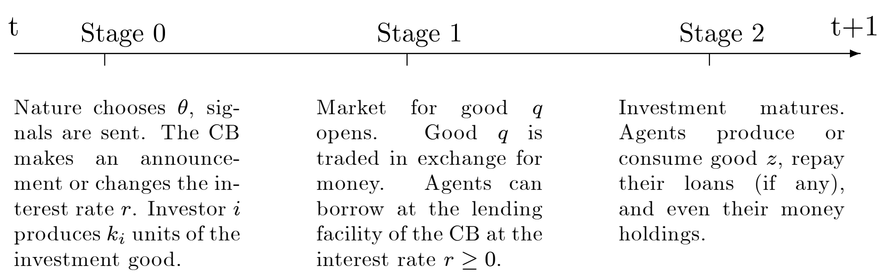

Figure 1: The timing of events within a period

[9] For simplicity, we assume that the CB makes a lump-sum transfer, τ, in the settlement market in order to redistribute any profits made by its lending facility.

[10] Using θ^2 instead of θ guarantees that investment is always positive in our model.

[11] Due to linearity, agents will exit stage 2 with equal money holdings. This dramatically improves tractability in what follows.

2.1 The Full-Information Benchmark

Throughout the paper we explore efficient ways for the CB to communicate its information to private investors. Recall that such information revelation is potentially beneficial for two reasons. First, the CB's signal is informative and this information may lead to investments that are closer to best responses under the true fundamentals.[12] Second, the CB takes into account the social cost of investment, thus, potentially leading to a better allocation through coordinating investors' choices. However, as we shall see, a simple announcement might not be enough, as there are cases where the CB would prefer to communicate false information. Later we will demonstrate that credible information revelation is possible through monetary policy.

Before we study these issues, we introduce the full-information economy as a benchmark. Aggregate period-t welfare, $\mathcal{W}$, is given by

$$\mathcal{W}(k_i,\theta) = \frac{1}{2}\left[u\left(q\right) - q + \theta^2 K - \int_0^1 c\left(k_i\right) di\right], \tag{1}$$

where $K = \int_0^1 k_i di$ is the aggregate production of the investment good in stage 0. The full-information efficient allocation in any given period is given by $(k_i, q) = (K, q^*)$, such that $c'\left(K\right) = \theta^2$.[13] For simplicity, we will concentrate on the special case of a quadratic cost function, $c\left(k_i\right) = \frac{k_i^2}{2}$. Then $K = \theta^2$. As private agents are not readily identifiable during stage 1, some type of record-keeping is needed for transactions to take place. Next, we discuss how full-information efficient allocations can be decentralized through monetary trade. Throughout we assume that money is provided exclusively by the CB. We let M denote the per capita supply of money. The growth rate of money is given by γ. Monetary injections are implemented through a transfer, T, in the settlement stage. In addition, the CB announces its lending rate, r, for next period right before the beginning of stage 0. Since the transfer, τ, equals the CB's profits from its lending facility, the net stock of money grows according to $M_{+1} = M + T$, where T is such that $M_{+1} = \gamma M$. We consider a stationary

[12] See Morris and Shin (2002) for a discussion of why this might not be true for all possible parameter values.

[13] The amount of good z produced in stage 2 is indeterminate.

equilibrium where $\phi M = \phi_+ M_+$, so that $\gamma = \phi/\phi_+$.

We use $W(k, m, l; \theta)$ to denote the discounted lifetime utility of an agent when he enters stage 2 holding k units of the investment good, m units of money, and l units of loans from the CB, given that the realized productivity shock is θ. Let $V(m)$ denote the expected discounted lifetime utility from entering stage 0 with money holdings m. Then, $W(k, m, l; \theta)$ is defined by

$$\begin{aligned} W(k, m, l; \theta) &= \max_{z, m_{+1}} \{-z + \beta E V(m_{+1}; \theta', r_+)\} \\ & \quad s.t. \quad \phi m_{+1} = z + \theta^2 k + \phi m - \phi (1 + r) l + \phi \tau + \phi T, \end{aligned} \tag{2}$$

where z denotes the net production of the general good. The discounted lifetime utility of agents entering stage 1 with m units of money is[14]

$$\begin{aligned} V(m) &= \frac{1}{2} \max_{k_i} \left\{ -c(k_i) + E \left[\max_{q,d} -q + W(k_i, m + pq, 0, \theta) \right] \right\} \\ &+ \frac{1}{2} E \left[\max_{q,l, \text{ s.t. } pq \leq m + l} u(q) + W(0, m - pq + l, l, \theta) \right]. \end{aligned} \tag{3}$$

It is straightforward to demonstrate that, when the true value of θ is publicly observable, the Friedman rule decentralizes the full-information efficient allocation.

3 Information Transmission by the Central Bank

We now turn to the case where the aggregate state of the economy, θ, is unknown. Both the private sector and the CB receive informative signals regarding the true value of θ. The CB maximizes expected period-t welfare given its signal, y, and precision, α. This is given by

$$\mathcal{W}(k_i, K; \theta) = \frac{1}{2} E \left[u(q) - q + \theta^2 K - \int_0^1 c(k_i) \, di \mid \alpha, y \right], \tag{4}$$

[14] The following expression uses the fact that producers never borrow from the CB.

where $K = \int_0^1 k_i di$ is the aggregate production of the investment good. Since c is convex, Jensen's inequality implies

$$\mathcal{W}(k_i, K; \theta) < \frac{1}{2} E\left[u(q) - q + \theta^2 K - c(K) \mid \alpha, y\right]. \tag{5}$$

Hence, the benevolent CB wants to minimize the dispersion in the production of the investment good. Importantly, individual investor optimization does not take into account this effect. Analogously to the full information case, the efficient allocation is given by $(k_i, q) = (K, q^*)$, such that $c'(K) = E(\theta^2 | y, \alpha) = y^2 + \frac{1}{\alpha}$.

Since the CB's signal contains information about the true value of θ, it might be beneficial if this information reaches the private sector. How should the CB transmit its information to the investors? We will consider two possibilities. First, the CB could make a direct "announcement." For example, one could think of a press statement or an interview delivered, say, by the Chairman of the CB. Alternatively, the CB could indirectly transmit its information through monetary policy; i.e., through the interest rate, r. The main finding of our paper is that these two ways of transmitting information can have very different consequences. To see this, recall that the objective of the benevolent CB and that of an individual investor in the economy are not completely aligned. Hence, the CB might have an incentive to misrepresent its information if it believes that this will lead to private investment decisions that improve ex ante social welfare. Realizing this, the private investors might choose to ignore such announcements. In contrast, transmission through changes in the interest rate is not "cheap talk," as it reduces welfare through creating a distortion associated with a violation of the Friedman rule. We will demonstrate that this adds a sufficient amount of "credibility" that can induce the private investors to rationally take into account the information provided by the CB.

3.1 Public Announcements

For simplicity, we assume that the precision of the CB's signal, α, is *iid* across periods and that it can take on two values: $\{\alpha_L, \alpha_H\}$, where $\alpha_L = \alpha_H - \varepsilon$ for some $\varepsilon > 0$.[15] The probability that $\alpha = \alpha_L$ is denoted by π, and the probability that $\alpha = \alpha_H$ by $1 - \pi$. The realization of the CB's signal precision is only observed by the CB. To further simplify the analysis, we will assume that y is publicly observable. In this case, the public (possibly untruthful) announcement concerns only the value of CB's confidence in its information. We denote such announcement by α_a.[16]

Analogously to the full information case, the discounted lifetime utility of an agent entering stage 1 with m units of money is

$$\begin{aligned} V(m|y,\alpha_a) &= \frac{1}{2}\max_{k_i}\left\{-c(k_i) + E\left[\max_{q,d} -q + W\left(k_i, m+pq, 0, \theta\right)|x_i, y, \alpha_a\right]\right\} \\ &+ \frac{1}{2}E\left[\max_{q,l,\text{ s.t. } pq\leq m+l} u\left(q\right) + W\left(0, m-pq+l, l, \theta\right)|y, \alpha_a\right]. \end{aligned} \tag{6}$$

The following result asserts that there are cases such that if the private investors believe the CB's announcement, ex ante social welfare is increased if the CB misrepresents its confidence in the signal.

Proposition 1 *Suppose that $\alpha \neq 0$ and $\delta \neq 0$. There exists an $\varepsilon > 0$ such that announcing $\alpha_L = \alpha_H - \varepsilon$ is preferred by the CB to announcing α_H. There is no equilibrium where the CB announces its precision truthfully and where the investors use the CB's announcement.*

The Proof is given in Appendix 2. For an example, let $y = 0.1$, $\delta = 70$, $\alpha_H = 60$, and $\alpha_L = 50$. Note that investors have more precise information than both α_H and α_L-type of the CB. Then, welfare under truth-telling when $\alpha = 60$ is $\mathcal{W}(k_i, K|\alpha = \alpha_a = 60, y = 0.1) = u\left(q^*\right) - q^* + 0.6413 \times 10^{-3}$. On the other hand, if the CB reports $\alpha_a = 50$, its welfare increases to $\mathcal{W}(k_i, K|\alpha = 60, \alpha_a = 50, y = 0.1) = u\left(q^*\right) - q^* + 0.6449 \times 10^{-3}$. Thus, the CB with $\alpha = 60$ prefers to announce $\alpha_a = 50$ and follow the Friedman rule, i.e. set $r\left(\alpha_a\right) = 0$.

[15] The arguments in this paper generalize to the case of any finite number of precision values.

[16] We interpret the public announcement as the result of a public speech, say by the Chairman of the CB. The contents of the announcement then become common knowledge among investors.

In other words, the benevolent CB might have an incentive to misrepresent its confidence in its information. This raises the question of whether it is possible for the CB to use a *costly* method in order to credibly communicate its confidence. We investigate this next.

3.2 Credible Monetary Policy

We continue assuming that the value of the CB's signal, y, is known and that its precision, α, is *iid* across periods. It can take on two values: $\{\alpha_L, \alpha_H\}$, with respective probabilities π and $1 - \pi$. Thus, like before, information transmission concerns the CB's signal precision, α.

The CB chooses an interest rate rule to reveal the precision of its signal, i.e. $r(\alpha)$. In contrast to a press announcement, the choice of r involves a deviation from the Friedman rule. Thus, it creates a distortion in the economy. At the same time, by "inverting" r, private investors can infer the value of α that the CB wishes to communicate. The question is whether, unlike in the case of a pure announcement, information transmission through monetary policy can lead to an equilibrium where the CB reports truthfully and the private sector takes this information into account when making investment decisions.

The equilibrium conditions for individual investors are given by

$$\begin{aligned} c'\left(k_i\left(x_i, y, \alpha_a\right)\right) &= E\left(\theta^2 | x_i, y, \alpha_a\right), \text{and} \\ u'\left(q\left(x_i, y, \alpha_a\right)\right) &= 1 + r\left(\alpha_a\right). \end{aligned} \tag{7}$$

The problem of the CB is to maximize

$$\mathcal{W}(K, \theta) = \frac{1}{2} E\left[u(q) - q + \theta^2 K - \frac{K^2}{2} - \int \frac{\left(k_i - K\right)^2}{2} \mid \alpha, y\right], \tag{8}$$

subject to the equilibrium equations for individual investors. In addition, in order for monetary policy to *credibly* transmit information about the CB's confidence, a set of incentive compatibility constraints must hold. More precisely, we require that

$$\mathcal{W}(K|r(\alpha),\alpha) \geq \mathcal{W}(K|r(\alpha_a),\alpha), \text{ for } \alpha_a \neq \alpha. \tag{9}$$

In words, the resulting welfare must be higher under truthful revelation by the CB. To ease exposition, consider the case where in any given period, the CB with $\alpha = \alpha_H$ prefers to mimic the CB with $\alpha = \alpha_L$, $\alpha_L < \alpha_H$ (this would be the case under the conditions in Proposition 1). Clearly, the incentive compatibility constraint in this case does not bind if $\alpha = \alpha_L$. However, when $\alpha = \alpha_H$, the CB has an incentive to misrepresent its confidence in its information. Thus, the incentive compatibility constraint must bind:

$$\mathcal{W}(K|r(\alpha_H),\alpha_H) = \mathcal{W}(K|r(\alpha_L),\alpha_H). \tag{10}$$

Setting $r(\alpha_H) = 0$, we can obtain the corresponding interest rate, $r(\alpha_L)$, implicitly as the solution to

$$\mathcal{W}(K|0,\alpha_H) = \mathcal{W}(K|r(\alpha_L),\alpha_H). \tag{11}$$

We must also verify that

$$\mathcal{W}(K|r(\alpha_L),\alpha_L) \geq \mathcal{W}(K|0,\alpha_L). \tag{12}$$

In other words, inflating in order to truthfully reveal that $\alpha = \alpha_L$ is preferred by a CB to the alternative of not inflating and announcing $\alpha = \alpha_H$. In addition, when $\alpha = \alpha_L$, the CB must prefer the corresponding outcome to the case where there is no information revelation, hence, no inflationary distortion, and private investors use their priors:

$$\overline{\alpha} = \pi\alpha_L + (1-\pi)\alpha_H.$$

In other words, we need to ensure that the welfare when the CB chooses an interest rate that reveals the true state exceeds the welfare that results if the CB simply follows the Friedman rule and investors use their priors; i.e.,

$$\mathcal{W}(K|r(\alpha_L),\alpha_L) \geq \mathcal{W}(K|r(\overline{\alpha}),\alpha_L), \tag{13}$$

where $r(\overline{\alpha}) = 0$. We have the following.

Proposition 2 *Suppose that $\alpha \neq 0$ and $\delta \neq 0$, and let $\pi \to 0$. Then, for ε small and $\alpha_L = \alpha_H - \varepsilon$, there is an equilibrium where the CB communicates its precision truthfully, the investors use the CB's announcement, and $r(\alpha_L) > 0$, $r(\alpha_H) = 0$.*

The proof is given in Appendix 2. For an illustration, fix parameter values to be those in the previous example. Moreover, let $\pi(\alpha_H = 60) = 0.5$. We know that a CB with $\alpha = 60$ has an incentive to report $\alpha_a = 50$. The question is whether a CB with $\alpha = 50$ prefers to costly but credibly communicate the true value of its precision.[17] To prevent a CB with $\alpha = 60$ from communicating the value $\alpha = 50$, the CB must invoke a cost, $\widehat{r} > 0$, where $\widehat{r}$ solves (11). For the parameter values above, $\widehat{r}$ is the solution to

$$u(q(\widehat{r})) - q(\widehat{r}) - u(q^*) + q^* = -0.3619 \times 10^{-5}. \tag{14}$$

Since this difference is negative, such an $\widehat{r}$ exists. To see whether a CB with $\alpha = 50$ chooses credible but costly communication we first check that the welfare under costly communication is higher than the welfare under following the Friedman rule and being taken for an $\alpha = 60$ type:

$$\mathcal{W}(k_i, K|\widehat{r}(\alpha_L), \alpha = \alpha_a = 50) - \mathcal{W}(k_i, K|0, \alpha = 50, \alpha_a = 60) = 0.1325 \times 10^{-4}. \tag{15}$$

Second, we check that welfare under costly communication exceeds welfare under no communication, whereby private investors use their priors on α and the CB sets $r = 0$:

$$\mathcal{W}(k_i, K|\widehat{r}(\alpha_L), \alpha = \alpha_a = 50) - \mathcal{W}(k_i, K|0, \alpha = 50, \alpha_a = \overline{\alpha}) = 0.3709 \times 10^{-5}. \tag{16}$$

Taken together, the two Propositions imply that a public announcement alone may not be effective. In order to transmit information to investors, the CB must violate the Friedman

[17] Note that a CB with $\alpha = 50$ suffers a welfare loss if it cannot reveal its type truthfully and, instead, it is believed to be a CB with $\alpha = 60$: $\mathcal{W}(k_i, K|\alpha = 50, \alpha_a = 60, y = 0.1) = u(q^*) - q^* + 0.8380 \times 10^{-3} < \mathcal{W}(k_i, K|\alpha = \alpha_a = 50, y = 0.1) = u(q^*) - q^* + 0.8549 \times 10^{-3}$.

rule.[18] In fact, incentive constraints put a lower bound on the inflation level that must be tolerated in order for the information transmission to be credible. Otherwise, like a simple announcement, investors will rationally ignore the supplied information, treating it as "cheap talk."[19]

How high is the interest rate increase needed to ensure credible information transmission? This question is related to the ongoing debate about the effectiveness of monetary policy in containing developments in the financial sector, which are considered excessive, and can potentially lead to inefficient investment. If the CB believes that financial imbalances are building up, should it react? One common argument against a monetary policy reaction is that the size of an interest rates adjustment, which would bring about a change in investor's behavior, would have to be so big as to generate too much harm to the real economy. Our approach takes the costs and benefits of credible information transmission explicitly into account. Moreover, we can compute what interest rate change is needed for the CB to affect investment decisions and to improve the resulting equilibrium allocation.

Consider the case of the constant relative risk aversion utility function, i.e. $u(q) = \frac{q^{1-\rho}}{1-\rho}$ where ρ is the coefficient of the relative risk aversion. For $\rho = 1$ (the case of log utility), credible information transmission in our example involves setting the interest rate at $\widehat{r} = 0.27\%$. It is easy to show[20] that as the coefficient of the relative risk aversion increases, credible communication requires larger deviations from the Friedman rule. Still, even for higher (and more empirically plausible) values of the coefficient of the relative risk aversion, like $\rho = 4$, credible signaling involves a deviation from the Friedman rule of just over 50 basis points: $\widehat{r} = 0.54\%$.

To summarize, we have shown that, at least in some cases, monetary policy achieves a

[18] The conditions in the two Propositions above jointly hold for an open set of parameter values. Notice that inflation rates below the Friedman rule are not consistent with the existence of a monetary equilibrium. Thus, in order to communicate its information credibly, the CB *must* choose a positive r, thus, creating a costly distortion in the economy.

[19] We have not searched for general conditions under which a simple public announcement by the CB will suffice. Such conditions would require that truth-telling is a dominant strategy for the CB. A moment's reflection will convince the reader that if an announcement by the CB is sufficient to credibly convey its information, inflationary equilibria cannot exist. The reason is that at any positive inflation level the CB can increase social welfare by further lowering inflation.

[20] By applying the Implicit Function Theorem to equation (14).

credible transmission of information by the CB, while this credibility is absent when the CB communicates its information through a simple announcement. A natural question to ask is whether the CB can always communicate its signal *as well as* its confidence in this information, through monetary policy. In other words, one can ask whether the one-dimensional set of feasible monetary policies is "rich" enough to communicate all possible values of the two-dimensional set consisting of the CB's signal and precision. In Appendix 1, we demonstrate that the answer to this question is affirmative. In other words, in a well defined mathematical sense, monetary policy can in principle be thought of as nothing but a "translation" of a message revealing the CB's information and confidence.

4 Conclusion

We studied a model in which the CB has some information about the true state of the economic fundamentals (not necessarily better than private agents). Since the objective of the benevolent CB and that of individual investors are not directly aligned, the CB might have an incentive to misrepresent its information to improve social welfare. We investigated under what conditions the CB can credibly communicate its information to the private sector. Our main finding identifies monetary policy as a tool that can lead to credible information transmission. While other ways to costly communicate information (e.g. taxes) are possible, CBs around the world are restricted in terms of the policy instruments they can use. A costly monetary distortion can accomplish this information transmission. In other words, inflation adds a sufficient amount of "credibility," thus, inducing private investors to rationally take into account the information provided by the CB.

One might be tempted to think of ways for the CB to communicate information, while avoiding the welfare losses due to inflation.[21] However, *any* such alternative will (by construction) lack credibility in our model. It is precisely the real cost of the monetary distortion that lends credibility and, as a result, makes this channel of information transmission work

[21] Two ideas that have been suggested to us are (1) repeated interest rate announcements by the CB (that might cancel each other's effects), and (2) variations in the timing of the decisions by the CB and by the investors.

in our setup.

Our approach can be used to derive other properties of optimal monetary policy. For example, the model can be used to quantify the size of the monetary policy response needed to induce investors to take the CB's information into account when making their investment decisions. We show that credible information transmission can be achieved with only modest changes in the nominal interest rate.

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5 Appendix 1 - An Equivalence Result

Here we demonstrate how the CB can in principle use monetary policy in order to convey *both* the realized value of its signal and its confidence in the signal through manipulating the interest rate.

To study this issue, consider again the case where the precision of the CB's signal, α, can take on two values: $\{\alpha_L, \alpha_H\}$, where $\alpha_L < \alpha_H$. The probability that $\alpha = \alpha_L$ is denoted by π. As before, the CB receives a signal y about the value of θ. The CB also knows the realization of the precision of its signal, α. It can choose to convey the value of y and α to the private sector through monetary policy. This can be accomplished via a rule that takes the following form:

$$r = h(y, \alpha), \tag{17}$$

with $h(y, \alpha) = 0$, if the CB does not reveal its information in state (y, α). Can the CB use monetary policy in order to reveal its information about *both* its signal and that signal's precision? Put differently, is the (expanded) information set of the CB equivalent to the set of feasible monetary policies? We formulate this as a mathematical question. To give an affirmative answer, we need to demonstrate that there exists a homeomorphism between these two sets.

Since the CB needs to signal both y and α to the private sector, it must be the case that $h(y, \alpha_L) \neq h(y', \alpha_H)$ for any y, y'. One candidate policy for $i \neq j \in \{H, L\}$ is defined as follows:

$$h(y, \alpha) = \begin{cases} h(y) \text{ if } \alpha = \alpha_i \\ h(y) + \widehat{r} \text{ if } \alpha = \alpha_j \end{cases}, \tag{18}$$

where $h : \mathbb{R} \to \mathbb{R}$ is a homeomorphism and $\widehat{r}$ stands for the minimum level of the interest rate that makes the corresponding information revelation credible. Indeed, for this policy, we have the following for any $\Delta > \widehat{r} > 0$.

Proposition 3 *Let $h : \mathbb{R} \to (0, \Delta)$ be a homeomorphism where $\Delta > 0$. Then, the function*

$H(y,\alpha):\mathbb{R}\times\{L,H\}\rightarrow(0,\Delta)\cup(\widehat{r},\widehat{r}+\Delta)$ *given by*

$$H(y,\alpha)=\begin{cases} h(y) & \text{if } \alpha=\alpha_L \\ h(y)+\widehat{r} & \text{if } \alpha=\alpha_H \end{cases}, \tag{19}$$

is a homeomorphism.

Proof. We impose the usual topology on $\mathbb{R}$, and the usual subspace topologies on $(0,\Delta)$ and on $(0,\Delta)\cup(\widehat{r},\widehat{r}+\Delta)$. We impose the discrete topology on $\{\alpha_L,\alpha_H\}$. The resulting topology on $\mathbb{R}\times\{\alpha_L,\alpha_H\}$ is the product topology. We need to show that $H(y,\alpha)$ satisfies all the conditions for a homeomorphism.

Claim 1: $H(y,\alpha)$ is injective.

We need to show that $(y_1,\alpha_1)\neq(y_2,\alpha_2)\Rightarrow H(y_1,\alpha_1)\neq H(y_2,\alpha_2)$. Let $(y_1,\alpha_1)\neq(y_2,\alpha_2)$. We need to consider two cases. First, let $\alpha_1=\alpha_L$ and $\alpha_2=\alpha_H$. Then,

$$\begin{aligned} H(y_1,\alpha_1) &= H(y_1,\alpha_L)=h(y_1)<\Delta \\ H(y_2,\alpha_2) &= H(y_2,\alpha_H)=h(y_2)+\widehat{r}>\Delta. \end{aligned} \tag{20}$$

Thus, $H(y_1,\alpha_1)\neq H(y_2,\alpha_2)$. Next, consider the case where $y_1\neq y_2$ and $\alpha_1=\alpha_2$. If $\alpha_1=\alpha_2=\alpha_L$, then $H(y_1,\alpha_1)=H(y_1,\alpha_L)=h(y_1)$, and $H(y_2,\alpha_2)=H(y_2,\alpha_L)=h(y_2)$. Since h is injective, $H(y_1,\alpha_1)=h(y_1)\neq h(y_2)=H(y_2,\alpha_2)$. Thus, in both cases $H(y_1,\alpha_1)\neq H(y_2,\alpha_2)$.

Claim 2: $H(y,\alpha)$ is surjective.

Here we have to show that $\forall z\in(0,\Delta)\cup(\widehat{r},\widehat{r}+\Delta)$, $\exists(y,\alpha)$ such that $H(y,\alpha)=z$. Let $z\in(0,\Delta)\cup(\widehat{r},\widehat{r}+\Delta)$. Then, either $z\in(0,\Delta)$ or $z\in(\widehat{r},\widehat{r}+\Delta)$. If $z\in(0,\Delta)$, since h is surjective, there exists $y\in\mathbb{R}$ such that $h(y)=z$. Thus, $H(y,\alpha_L)=h(y)=z$. Next, suppose that $z\in(\widehat{r},\widehat{r}+\Delta)$. Then, $\widehat{r}<z<\widehat{r}+\Delta$, thus, $0<z-\widehat{r}<\Delta$. Again, since h is surjective, there exists $y\in\mathbb{R}$ such that $h(y)=z-\widehat{r}$. Thus, $H(y,\alpha_H)\equiv h(y)+\widehat{r}=z$. Thus, $H(y,\alpha)$ is surjective.

Claim 3: $H(y,\alpha)$ is continuous.

It suffices to show that the inverse image of every basis (open) set is open in the respective topology. Note that any open set in the range of H is of the form $(a,b)\cap\{(0,\Delta)\cup(\widehat{r},\widehat{r}+\Delta)\}$, with $a, b \in \mathbb{R}$. First, consider any open set, B, such that $B \subset (0, \Delta)$. Then, $h^{-1}(B)$ is open in $\mathbb{R}$, since h is continuous. We have

$$\begin{aligned} & (y, \alpha) \in H^{-1}(B) \\ \Leftrightarrow \quad & H(y, \alpha) \in B \subset (0, \Delta) \\ \Leftrightarrow \quad & \alpha = L, H(y, \alpha) = h(y) \in B \\ \Leftrightarrow \quad & \alpha = \alpha_L, y \in h^{-1}(B) \\ \Leftrightarrow \quad & (y, \alpha) \in h^{-1}(B) \times \{\alpha_L\}. \end{aligned} \tag{21}$$

Thus, $H^{-1}(B) = h^{-1}(B) \times \{\alpha_L\}$ and whenever B is open, we have that $H^{-1}(B)$ is open in the product topology, since $h^{-1}(B)$ is open in $\mathbb{R}$, and $\{\alpha_L\}$ is open in $\{\alpha_L, \alpha_H\}$. Proceeding in the same fashion, we can show that the inverse image of B is open for any open set $B \subset (\widehat{r}, \widehat{r} + \Delta)$, or, a set $B = B_1 \cup B_2$, where $B_1 \subset (0, \Delta)$ and $B_2 \subset (\widehat{r}, \widehat{r} + \Delta)$.

Finally, we have the following.

Claim 4: $H^{-1}(y, \alpha)$ is continuous.

We must show that $H(B)$ is open for every basis (open) set B. Let $B = (y_1, y_2) \times \{\alpha\}_{\alpha \in \{\alpha_L, \alpha_H\}}$ be an open set in the product topology $\mathbb{R} \times \{\alpha_L, \alpha_H\}$. First, suppose that $\alpha = \alpha_L$; i.e., $B = (y_1, y_2) \times \{\alpha_L\}$. Then,

$$H(B) = H\left((y_1, y_2), \{\alpha_L\}\right) = h\left((y_1, y_2)\right). \tag{22}$$

Thus, $H(B)$ is open in $\mathbb{R}$, since h^{-1} is continuous and $h((y_1, y_2))$ is open. Similarly, if $\alpha = \alpha_H$; i.e., $B = (y_1, y_2) \times \{\alpha_H\}$, we have

$$H(B) = H\left((y_1, y_2) \times \{\alpha_H\}\right) = \{h(y) + \widehat{r} : y \in (y_1, y_2)\}. \tag{23}$$

Let $z = h(y) + \widehat{r}$ and $y \in (y_1, y_2)$. Then, $z - \widehat{r} = h(y) \in h((y_1, y_2))$. Since h^{-1} is continuous, $h((y_1, y_2))$ is open in $\mathbb{R}$. Thus, there exists an open interval (z_1, z_2) such that

$$z - \widehat{r} \in (z_1, z_2) \subset h((y_1, y_2)). \tag{24}$$

This implies that

$$z \in (z_1 + \widehat{r}, z_2 + \widehat{r}) \subset \{h(y) + \widehat{r} : y \in (y_1, y_2)\}. \tag{25}$$

Thus, $H(B) = \{h(y) + \widehat{r} : y \in (y_1, y_2)\}$ is open in $\mathbb{R}$.

We conclude that $H(y, \alpha)$ is a homeomorphism. ■

The proof readily generalizes to a countable set of possible precision values. The above result demonstrate the (topological) equivalence between the set describing the information potentially held by the CB and the set of feasible and credible monetary policies at the CB's disposal. This implies that, at the cost of imposing a distortion associated with some inflation, the CB can use monetary policy in order to reveal both its signal about fundamentals and its confidence in that signal to the private sector.

The figure below plots a candidate monetary policy H as a function of the CB's signal y and its confidence in the signal α. The interest rate path is given by the red line, $h(y)$, if $\alpha = \alpha_L$ and by the blue line, $h(y) + \widehat{r}$, if $\alpha = \alpha_H$. If the CB follows such a policy, it can convey both the realized y and α through manipulating the interest rate.

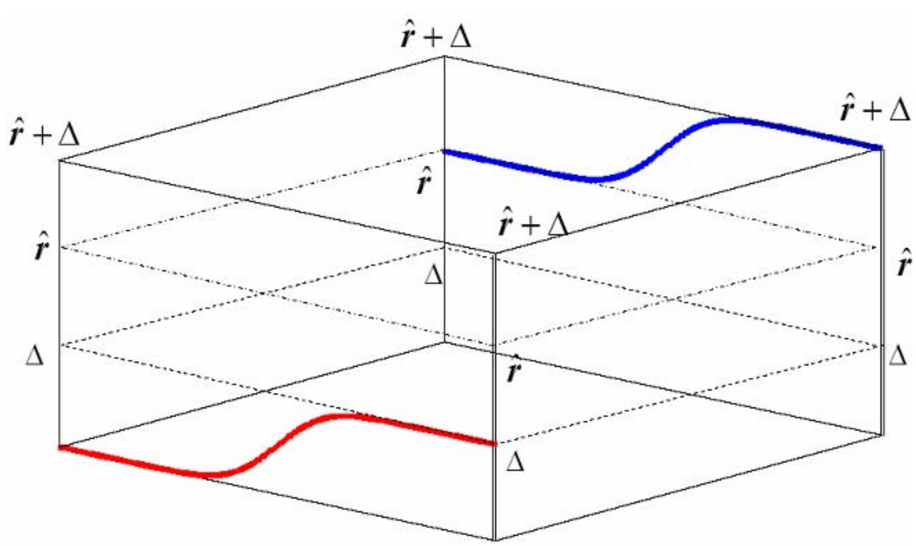

Figure 2: Monetary policy as a homeomorphism

6 Appendix 2 - Proofs

Proof of Proposition 1: When y is known but α is not, expected welfare is given by

$$\begin{aligned}
2\mathcal{W}(k_i,K|\alpha_a,\alpha,y) &= u\left(q_a\right)-q_a+\frac{y^2+\frac{1}{\alpha}}{\left(\alpha_a+\delta\right)^2}\left(\alpha_a^2y^2+\alpha_a+2\delta\right)-\frac{1}{2}\frac{\left(\alpha_a+2\delta\right)^2}{\left(\alpha_a+\delta\right)^4} \\
&+\delta\frac{2\alpha_a y^2\left(y^2+3\frac{1}{\alpha}\right)+\delta\left(y^4+6y^2\frac{1}{\alpha}+3\frac{1}{\alpha^2}\right)}{\left(\alpha_a+\delta\right)^2}-\frac{2\delta y^2}{\left(\alpha_a+\delta\right)^2} \\
&-\frac{1}{2}\alpha_a^2y^2\frac{y^2\left(\alpha_a^2+4\alpha_a\delta+6\delta^2\right)+6\delta^2\frac{1}{\alpha}}{\left(\alpha_a+\delta\right)^4}-\frac{1}{2}\frac{\delta^2\left(4\delta-\alpha\right)}{\alpha\left(\alpha_a+\delta\right)^4} \\
&-\frac{1}{2}\delta^3\frac{\delta\left(y^4+6y^2\frac{1}{\alpha}+3\frac{1}{\alpha^2}\right)+4\alpha_a y^2\left(y^2+3\frac{1}{\alpha}\right)}{\left(\alpha_a+\delta\right)^4} \\
&-\frac{\left(\alpha_a+2\delta\right)\left(y^2\left(\alpha_a+\delta\right)^2+\delta^2\frac{1}{\alpha}\right)}{\left(\alpha_a+\delta\right)^4}. \qquad (26)
\end{aligned}$$

The first-order necessary condition gives

$$\begin{aligned}
&12\alpha\delta^3-\alpha\alpha_a^3-12\delta^3\alpha_a-5\alpha\delta\alpha_a^2-4\alpha\delta^2\alpha_a+5\alpha^2\delta\alpha_a+4\alpha^2\delta^2+\alpha^2\alpha_a^2-6\delta^2\alpha_a^2 \\
&\quad -4y^2\alpha\delta\alpha_a^3-4y^2\alpha\delta^3\alpha_a+4y^2\alpha^2\delta^3-8y^2\alpha\delta^2\alpha_a^2+4y^2\alpha^2\delta\alpha_a^2+8y^2\alpha^2\delta^2\alpha_a=0.
\end{aligned}$$

For $\alpha=\alpha_a$, the above expression simplifies to $-6\delta^2\alpha^2$. Hence, for $\alpha\neq 0$ and $\delta\neq 0$ we have that

$$\frac{\partial}{\partial\alpha_a}\mathcal{W}(k_i,K|\alpha_a,\alpha,y)|_{\alpha=\alpha_a}=-\frac{6\delta^2}{\left(\delta+\alpha\right)^5}<0. \qquad (27)$$

Thus, announcing $\alpha=\alpha_a$ is not optimal. Moreover, since $\frac{\partial}{\partial\alpha_a}\mathcal{W}(k_i,K|\alpha_a,\alpha,y)\mid_{\alpha=\alpha_a}<0$, if the CB believes that investors will use its announcement, it prefers to announce an α_a which is lower than the true α. ■

Proof of Proposition 2: The expected welfare $\mathcal{W}\left(\alpha_a,\alpha,y\right)$ is given above. Then, $\frac{\partial\mathcal{W}(\alpha_a,\alpha,y)}{\partial\alpha_a}$ is given by:

$$\begin{aligned}
&\frac{1}{\left(\delta+\alpha_a\right)^5\alpha^2}\left(12\alpha\delta^3-\alpha\alpha_a^3-12\delta^3\alpha_a-5\alpha\delta\alpha_a^2-4\alpha\delta^2\alpha_a+5\alpha^2\delta\alpha_a+4\alpha^2\delta^2+\alpha^2\alpha_a^2\right. \\
&\quad\left.-6\delta^2\alpha_a^2-4y^2\alpha\delta\alpha_a^3-4y^2\alpha\delta^3\alpha_a+4y^2\alpha^2\delta^3-8y^2\alpha\delta^2\alpha_a^2+4y^2\alpha^2\delta\alpha_a^2+8y^2\alpha^2\delta^2\alpha_a\right).
\end{aligned}$$

In order for the CB to communicate truthfully, we need that $\left|\frac{\partial \mathcal{W}(\alpha_a,\alpha_L,y)}{\partial \alpha_a}\mid_{\alpha_a=\alpha_L}\right| > \left|\frac{\partial \mathcal{W}(\alpha_a,\alpha_H,y)}{\partial \alpha_a}\mid_{\alpha_a=\alpha_L}\right|$, where $\alpha_H = \alpha_L + \varepsilon$. Now, $\frac{\partial \mathcal{W}(\alpha_a,\alpha_L,y)}{\partial \alpha_a}\mid_{\alpha_a=\alpha_L}$ is given by

$$\frac{\partial \mathcal{W}\left(\alpha_a,\alpha_L,y\right)}{\partial \alpha_a}\mid_{\alpha_a=\alpha_L} = -\frac{6\delta^2}{\left(\delta+\alpha_L\right)^5}, \tag{28}$$

while $\frac{\partial \mathcal{W}(\alpha_a,\alpha_H,y)}{\partial \alpha_a}\mid_{\alpha_a=\alpha_L}$ is given by

$$\begin{aligned}\frac{\partial \mathcal{W}\left(\alpha_a,\alpha_H,y\right)}{\partial \alpha_a}\mid_{\alpha_a=\alpha_L} = {} & \frac{1}{\left(\delta+\alpha_L\right)^5\left(\alpha_L+\varepsilon\right)^2}\left(12\left(\alpha_L+\varepsilon\right)\delta^3-\left(\alpha_L+\varepsilon\right)\alpha_L^3\right.\\ & -12\delta^3\alpha_L-5\left(\alpha_L+\varepsilon\right)\delta\alpha_L^2-4\left(\alpha_L+\varepsilon\right)\delta^2\alpha_L+5\left(\alpha_L+\varepsilon\right)^2\delta\alpha_L+4\left(\alpha_L+\varepsilon\right)^2\delta^2\\ & +8y^2\left(\alpha_L+\varepsilon\right)^2\delta^2\alpha_L-6\delta^2\alpha_L^2-4y^2\left(\alpha_L+\varepsilon\right)\delta\alpha_L^3-4y^2\left(\alpha_L+\varepsilon\right)\delta^3\alpha_L\\ & \left.+4y^2\left(\alpha_L+\varepsilon\right)^2\delta^3-8y^2\left(\alpha_L+\varepsilon\right)\delta^2\alpha_L^2+4y^2\left(\alpha_L+\varepsilon\right)^2\delta\alpha_L^2+\left(\alpha_L+\varepsilon\right)^2\alpha_L^2\right),\end{aligned}$$

where we write α_H as $\alpha_L + \varepsilon$. Clearly, for ε sufficiently small, this expression is negative. Thus, checking that $\left|\frac{\partial \mathcal{W}(\alpha_a,\alpha_L,y)}{\partial \alpha_a}\mid_{\alpha_a=\alpha_L}\right| - \left|\frac{\partial \mathcal{W}(\alpha_a,\alpha_H,y)}{\partial \alpha_a}\mid_{\alpha_a=\alpha_L}\right| > 0$ is equivalent to checking that

$$-\frac{\partial \mathcal{W}\left(\alpha_a,\alpha_L,y\right)}{\partial \alpha_a}\mid_{\alpha_a=\alpha_L} + \frac{\partial \mathcal{W}\left(\alpha_a,\alpha_H,y\right)}{\partial \alpha_a}\mid_{\alpha_a=\alpha_L} > 0. \tag{29}$$

The above expression is given by

$$\begin{aligned}&\left(\alpha_L+\varepsilon\right)^{-2}\left(\delta+\alpha_L\right)^{-5}\left(5\delta\varepsilon\alpha_L+12\delta^3+10\delta^2\varepsilon+\alpha_L^3+5\delta\alpha_L^2+16\delta^2\alpha_L\right.\\ &\quad\left.+\varepsilon\alpha_L^2+4y^2\delta^3\varepsilon+4y^2\delta\alpha_L^3+4y^2\delta^3\alpha_L+4y^2\delta\varepsilon\alpha_L^2+8y^2\delta^2\varepsilon\alpha_L+8y^2\delta^2\alpha_L^2\right)\varepsilon.\end{aligned}$$

Note that since $\varepsilon > 0$, this expression is positive. Turning to the investors, since the truthful information revealed by the CB is useful in this case, it is a best response for investors to use this information. ■