The Central Bank's Balance Sheet and Treasury Market Disruptions*

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Abstract

This paper studies how Treasury market dynamics depend on adjustments to the central bank balance sheet. We introduce a dynamic model of Treasury bonds with traditional and shadow banks. In the model, both Treasury and repo market disruptions arise as a joint consequence of three frictions: (i) balance sheet costs, (ii) intraday reserves requirements, and (iii) imperfect substitutability between repo and bank deposits. Our model highlights the critical role of both sides of the central bank's balance sheet as well as agents' anticipation of shocks and policy interventions in matching observed market dynamics.

Keywords: Repo, Liquidity Risk, Basis Trade, Shadow Banks, Hedge Funds, Reserves

JEL Classifications: E43, E44, E52, G12

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Central banks have historically played an important role in stabilizing government debt markets.[1] Following a recent series of disruptions in Treasury markets,[2] the Federal Reserve (the Fed) has once more reclaimed prominence in Treasury markets through a combination of direct interventions—such as large-scale asset purchase programs—and indirect support through Treasury funding markets—such as discretionary lending following the repo market disruption in September 2019 and the establishment of a standing repo facility in July 2021. Those developments unfold against a backdrop of stricter bank regulations and a growing involvement of non-bank financial institutions in Treasury markets (Duffie, Geithner, Parkinson, and Stein, 2022).

In this paper, we present a dynamic model of Treasury and repo markets that accounts for those developments and study the role of the balance sheet of the central bank in Treasury markets under actual institutional and regulatory settings. Our model generates endogenous disruptions in Treasury and repo markets as a consequence of the combination of three institutional frictions: (i) banks' balance sheet space is costly for traditional banks (Andersen, Duffie, and Song, 2019); (ii) banks are subject to an intraday reserves requirement that limits repo lending to a portion of available reserves (Copeland, Duffie, and Yang, 2021; d'Avernas, Han, and Vandeweyer, 2023a); and (iii) repo and deposits are imperfect-substitute money-like assets for households (Krishnamurthy and Li, 2022). In equilibrium, unregulated shadow banks hold Treasuries funded with leverage in repo to exploit regulatory arbitrage opportunities. However, these positions are not riskless because they entail liquidity risk stemming from shocks that affect repo markets. When repo rates increase, levered shadow banks face significant losses and may optimally choose to liquidate their Treasury portfolio, despite associated transaction costs. We study how various commonly observed shocks to repo markets—flight-to-deposits, reduced intermediation capacity, tax payments, and Treasury issuances—interact with the central bank's balance sheet to generate disruptions to Treasury markets. We empirically validate the mechanisms of our model using observed price and portfolio flow responses to these shocks.

The main novelty of our approach is that we consider a model of repo and Treasury markets that is both dynamic—agents anticipate shocks and policy—and general equilibrium—every financial asset is a liability of another sector—and thus allows for meaningful analysis of the aggregate impact of dynamic sectoral balance sheet adjustments. By modeling those adjustments explicitly, we can explore at a granular level the economic forces that drive these disruptions

[1]Historical examples include, among others, the Banque de France, which was created by Napoleon I with a mandate to purchase government debt; the Bank of England's extensive purchase of bills of exchange to stabilize gilt markets at the onset of World War I; the Federal Reserve's purchase of short-term Treasury bills from its origination and intervening daily in Treasury bond markets at the start of World War II in 1939, and the Treasury-Federal Reserve Accord in 1951. Additional Treasury purchases from the Federal Reserve took place in 1958, 1961, and 1970 (Garbade and Keane, 2020).

[2]Notable episodes include quarter-end disruptions (Aldasoro, Ehlers, and Eren, 2022; Munyan, 2015); the September 2019 overnight Treasury repo rate surge (Afonso, Cipriani, Copeland, Kovner, La Spada, and Martin, 2020; Avalos, Ehlers, and Eren, 2019; Copeland, Duffie, and Yang, 2021; Correa, Du, and Liao, 2020); the March 2020 Treasury yield increase (He, Nagel, and Song, 2022; Vissing-Jorgensen, 2021); and the September 2022 turmoil in the UK sovereign bond market (Bank of England, 2022).

and inform policy discussions on the optimal design of central bank facilities, discretionary interventions, and the effects of reforming banks' and non-bank financial institutions' regulations.

The first contribution of our work is to explain several salient developments in the repo and Treasury markets within a single tractable framework and shed light on their underlying economic mechanisms. First, in our model, shadow banks allocate a large portion of their portfolio to Treasuries, taking advantage of low balance sheet costs through high leverage. This mechanism corresponds to the increased participation of hedge funds in Treasury markets since the introduction of Basel III regulation (Barth and Kahn, 2023). Those non-bank market participants exploit the existence of various Treasury bases, which are deviations from the law-of-one-price between Treasuries and exposure to the underlying through derivatives.[3] In our model, as in practice, such a trade is risky because it entails liquidity risk for shadow banks that roll over their repo funding overnight. The equilibrium Treasury basis is, therefore, pinned down by the required compensation for this liquidity risk from shadow banks. This observation explains the puzzling observation that the Treasury basis remains open despite the presence of a largely unconstrained and competitive hedge fund sector.

Second, our model accounts for the behavior of banks acting as "lender-of-next-to-last-resort" (Pozsar, 2019). As documented by Correa, Du, and Liao (2020), banks increase repo lending while running down reserves during recurring dislocations of repo markets such as quarter-ends, tax deadlines, and Treasury issuances. In the model, banks' ability to lend in repo to shadow banks is key to preventing repo rates from deviating from the interest paid on reserves and ensuring that liquidity services are at their optimum. However, because of the existence of the two regulatory frictions, bank repo lending capacity can be impaired and depends on adjustments to the central bank's balance sheet. As a result, shadow banks end up paying a high interest rate on their repo funding and, when the shock is large enough, liquidate their portfolio of Treasuries with significant losses, as seen in September 2019 and March 2020.[4]

Third, our model provides an intuitive interpretation of the observation that Treasury fire sales absorbed most of the shock in March 2020 at the outset of the Covid-19 outbreak, while repo rates spiked to 400 bps in September 2019. In the model, the presence of trading costs implies that the market predominantly impacted depends on agents' expectations regarding the duration of the shock. Specifically, short-lived shocks have a greater impact on the repo market than on the Treasury market because shadow banks are willing to pay a high repo rate for a brief period to avoid incurring the transaction fixed costs associated with liquidating their positions.

[3] An example of such a trade is the Treasury cash-future, which can be exploited through the strategy of financing large Treasury holdings in repo while selling the same Treasury forward in the futures market and thereby hedging duration risk. The cash-future basis was a large and popular trade around 2018-2019 but came down in popularity after the expansion of the Fed balance sheet in March 2020. As the Fed is normalizing its balance sheet, the basis started increasing again in 2023 (Avalos and Sushko, 2023; Barth, Kahn, and Mann, 2023).

[4] These losses were widely reported in the financial press: e.g., "Hedge Funds Hit by Losses in Basis Trade", *The Wall Street Journal*, March 19, 2020, accessible at https://www.wsj.com/articles/hedge-funds-hit-by-losses-in-basis-trade-11584661202.

In contrast, when shadow banks anticipate a long-lasting shock, it becomes optimal to pay the fixed transaction costs once and liquidate their Treasury portfolio rather than face a high repo funding rate over an extended period. Observed dynamics in September 2019 and March 2020 are consistent with this result to the extent that the tax deadline shock of mid-September 2019 was perceived as temporary, while the flight-to-deposit shock caused by the Covid-19 disruption was anticipated to be long-lasting.

Our second contribution is to exploit the granularity of our model to provide a precise account of the economic mechanisms that link specific shocks to disruptions in repo and Treasury markets, with implications for stabilization policy. An important insight from the model is that both sides of the central bank's balance sheet are independent determinants of market disruptions. On the asset side, a larger portfolio of Treasury securities held by the central bank diminishes the demand for repo financing from shadow banks, which relaxes the balance sheet cost friction, and subsequently reduces the likelihood of a Treasury disruption. On the liability side, a larger central bank balance sheet with a large supply of reserves enables banks to lend more in repo following a liquidity shock. Consequently, a reduction in the size of the central bank balance sheet exerts simultaneous pressure on both the demand and supply of repos, increasing the likelihood of disruptions. Conversely, net purchases of Treasuries financed through reserves by the central bank can alleviate pressures on repo and Treasury markets, as documented by Vissing-Jorgensen (2021) in March 2020. In addition, our model contributes an important general equilibrium insight into the question of assessing the minimum level of reserves necessary to ensure the stability of repo and Treasury markets (Afonso, Cipriani, Copeland, Kovner, La Spada, and Martin, 2020; Afonso, La Spada, Mertens, and Williams, 2023; Lopez-Salido and Vissing-Jorgensen, 2023): With an intraday liquidity constraint, it needs to be considered in proportion to total Treasuries outstanding.

In addition to the size of the central bank balance sheet, endogenous adjustments to its composition following shocks also have important consequences for Treasury market dynamics. A prevalent case of this mechanism can be observed following an intermediation shock, modeled as the contraction of an unregulated repo dealer balance sheet without balance sheet cost. This type of shock corresponds to the disruptions observed at quarter-ends when foreign dealers window-dress their balance sheets (Munyan, 2015). As observed in the data, contractions by foreign dealers put pressure on domestic dealers, and thereby increase the intermediation spread between the (bilateral) repo rate paid by hedge funds and the (triparty) repo rate received by households. For small shocks, because banks are unconstrained in their repo lending capacity, this adjustment needs to occur through a decrease in the household repo rate, until it reaches the policy rate at the reverse repo facility. The triparty repo rate is then bounded below, and the shock is absorbed with a supply adjustment by swapping reserves into repos within the central bank balance sheet, rather than an increase in the repo rates spread.

The model helps shed light on a subtle general equilibrium arithmetic: Under normal conditions, the reverse repo facility not only provides a floor to the triparty repo rate but also prevents the bilateral repo rate from spiking up. The induced reduction in reserves opens the space on banks' assets required to lend in repo without expanding their balance sheets. This adjustment corresponds to the "reserves-draining" intermediation documented by Correa, Du, and Liao (2020) and is consistent with Diamond, Jiang, and Ma's (2023) finding that large reserves balances crowd out banking lending. An important insight from our model, however, is that such a drawdown of reserves is only stabilizing if banks have enough reserves to fulfill their intraday requirements and settle new repo transactions. Once the intraday reserves constraint binds, any further decrease in reserves restricts bank repo lending capacity and exacerbates the shortage of repo. The bilateral repo rate then spikes above the interest paid on reserves to reflect this scarcity of repo supply, as observed in September 2019.

Our model is also informative regarding several ongoing policy discussions to readjust the post-GFC regulatory apparatus. First, the model addresses the prospect of implementing a standing repo facility, as introduced by the Fed and the Bank of England in 2022. An implication of the model in support of standing facilities is the existence of a "volatility paradox" (Brunnermeier and Sannikov, 2014) in Treasury markets: As the probability of disruption becomes lower (e.g., through the perception of a more interventionist central bank), shadow banks increase their leverage, which results in a higher-intensity shock conditional on a disruption taking place. A well-designed facility that provides systematic intervention may, therefore, be needed to avoid the large disruptions that discretionary interventions may generate.

Furthermore, the model speaks to the design of standing repo facilities and the policy debate as to whether its access should be restricted to banks or available to shadow banks (Duffie, Geithner, Parkinson, and Stein, 2022).[5] Our model shows that a repo facility accessible to banks helps prevent Treasury market disruptions caused by tax deadlines and issuance shocks but not intermediation shocks. In the latter case, the shock impacts repo and Treasury markets by reducing aggregate balance sheet capacities. As a result, a repo facility proves helpful only if directly accessible by shadow banks. In that case, the central bank stabilizes markets by simultaneously borrowing from households at the reverse repo facility and lending to shadow banks at the repo facility. Conversely, because tax deadlines are net repo supply contractions,[6] dealer-banks have spare balance sheet capacity they can mobilize to intermediate repo lending from the central bank to shadow banks. Direct access to shadow banks is therefore not needed to prevent disruptions brought about by tax deadline and Treasury issuance shocks.

Lastly, we make use of the model to understand how reforming the regulatory framework would affect Treasury market dynamics. The model shows that relaxing the regulation of traditional

[5] The financial press has echoed those concerns about the effectiveness of narrow-access facilities. E.g., "What is the new Bank of England short-term lending facility?", Financial Times, October 10, 2022, accessible at https://www.ft.com/content/2aaea9cc-0458-4a12-ad9e-42ed8328d4dc.

[6] Tax deadline shocks, representing money market fund outflows around tax deadlines such as the middle of September, have the potential to trigger Treasury disruptions through a different mechanism: Tax deadlines simultaneously reduce the supply of repos from households while reducing the quantities of reserves available to banks as those are flowing to the Treasury General Account.

banks by removing reserves from leverage ratio calculations alone does not change Treasury market dynamics, unlike removing both reserves and Treasuries. In addition, our model shows that subjecting shadow banks to some leverage constraints, as is currently proposed by the SEC, would help stabilize Treasury markets but at the expense of increasing government steady-state financing costs.

Related Literature Our paper complements the literature on frictions and disruptions in financial markets post-Basel III regulations by providing a comprehensive framework to understand the underlying economic forces at play. Munyan (2015) and Du, Tepper, and Verdelhan (2018) demonstrate that these frictions manifest as quarter-end effects in repo and FX swap rates because foreign dealers reduce balance sheet size to comply with leverage ratio regulations. Andersen, Duffie, and Song (2019) reveal the implications of such regulations on funding value adjustments for major dealers and identify debt overhang costs for shareholders. Correa, Du, and Liao (2020) illustrate how banks engage in "reserves-draining" intermediation to lend in money markets following quarter-end shocks to bypass these constraints. Klingler and Syrstad (2021) provide a comprehensive empirical inquiry across the many factors that influence repo rates. These constraints have also been empirically documented to impact Treasury arbitrage trades, as seen in cash-future basis (Barth and Kahn, 2023); swap spreads (Jermann, 2020); and CIP violations (Du, Tepper, and Verdelhan, 2018). Boyarchenko, Giannone, and Santangelo (2018) show how dealer balance sheet costs affect repo pricing and arbitrage funding for non-banks—a finding that our model rationalizes. He, Nagel, and Song (2022) connect these findings to the extraordinary increase in Treasury yields observed in March 2020 through a preferred habitat model in which dealers incur an increased cost of holding Treasuries when absorbing fire sales from other sectors. Our model highlights the general equilibrium conditions under which those dynamics may or may not occur and economic mechanism underpinning those disruptions. Ma, Xiao, and Zeng (2022) study the role of fire sales from mutual funds in the March 2020 episode. Eisenbach and Phelan (2023) further endogenize Treasury sales in a global game in which investors anticipate dealer balance sheet bottlenecks.

In particular, our work is connected to studies that relate the supply of central bank reserves to repo markets through banks' ability to serve as a stabilizing force in repo markets or "lender-of-next-to-last-resort" and effectively increase the financial system's reliance on reserves. Copeland, Duffie, and Yang (2021) and Pozsar (2019) identify potential liquidity concerns related to Treasury settlements and excess balance sheet normalization. Gagnon and Sack (2019) discuss policy options to address these issues, such as a standing repo facility, higher reserve levels, and explicit directives to control the repo rate. In particular, the repo turmoil of September 2019 has been partially attributed to hedge funds' use of repo to finance Treasury holdings by Avalos, Ehlers, and Eren (2019). Afonso, Cipriani, Copeland, Kovner, La Spada, and Martin (2020) provide a detailed account of the event, highlighting the role of reserves and interbank market frictions, while Anbil, Anderson, and Senyuz (2021) emphasize the role of trading relationships. D'Avernas, Han, and Vandeweyer (2023a) and Yang (2022) model the impact of intraday liquidity constraints on money market dislocations, and find that nonlinearities can generate significant spikes in repo rates. Acharya and Rajan (2022) and Acharya, Chauhan, Rajan, and Steffen (2023) identify a ratchet effect on banks' liquidity, which implies that removing reserves during quantitative tightening exposes banks to increased liquidity risk. Consistent with our result whereby reserves can crowd out repo lending, Diamond, Jiang, and Ma (2023) find that large reserve balances may hinder banks' ability to lend. Focusing on the slow-moving dynamics of rates rather than high-frequency shocks, d'Avernas and Vandeweyer (forthcoming) and Anbil, Anderson, Cohen, and Ruprecht (2022) highlight the importance of the demand for liquidity from non-bank financial institutions and reverse repo facilities as a tool to provide this liquidity and provide a floor to triparty repo rates. Complementary to this work, Ma, Eisenschmidt, and Zhang (2024) and Huber (2023) study the role of imperfect competition in explaining a part of repo market spreads with implications for monetary policy.

Our paper further contributes to the literature that assesses the role of a growing non-bank sector that performs liquidity transformation tasks. In particular, the notion that shadow banks' existence (and the negative effect on financial stability) is an unintended consequence of regulatory arbitrage is related to Huang (2018); Luck and Schempp (2014); Moreira and Savov (2017); Plantin (2015); and Begenau and Landvoigt (2021), while the fact that traditional banks benefit from insuring shadow banks against negative shocks is related to Lyonnet and Chretien (2023) and d'Avernas, Vandeweyer, and Darracq-Pariès (2023b).

Our study distinguishes itself from previous literature by providing a detailed account of general equilibrium adjustments to sectoral balance sheets in a dynamic setting, which allows us to study the particular role of the central bank in shaping Treasury market dynamics. This framework enables us to shed light on the mechanisms that connect institutional frictions to market disruptions and offers a unique set of implications absent in prior work. In particular, our framework highlights the important role of both sides of the central bank balance sheet as a stabilizing force in Treasury markets. Despite the complexity of a setting in which anticipation about shocks impacts portfolio allocations, which renders the threshold for binding constraints endogenous, all results are proven analytically.

Our paper is organized as follows: Section 1 discusses recent evolutions to the Fed balance sheet. Section 2 presents our dynamic asset pricing model with financial frictions. Section 3 presents the main theoretical results. Section 4 compares the model predictions to empirical observations across three heterogeneous shocks and discusses policy implications, and Section 5 concludes.

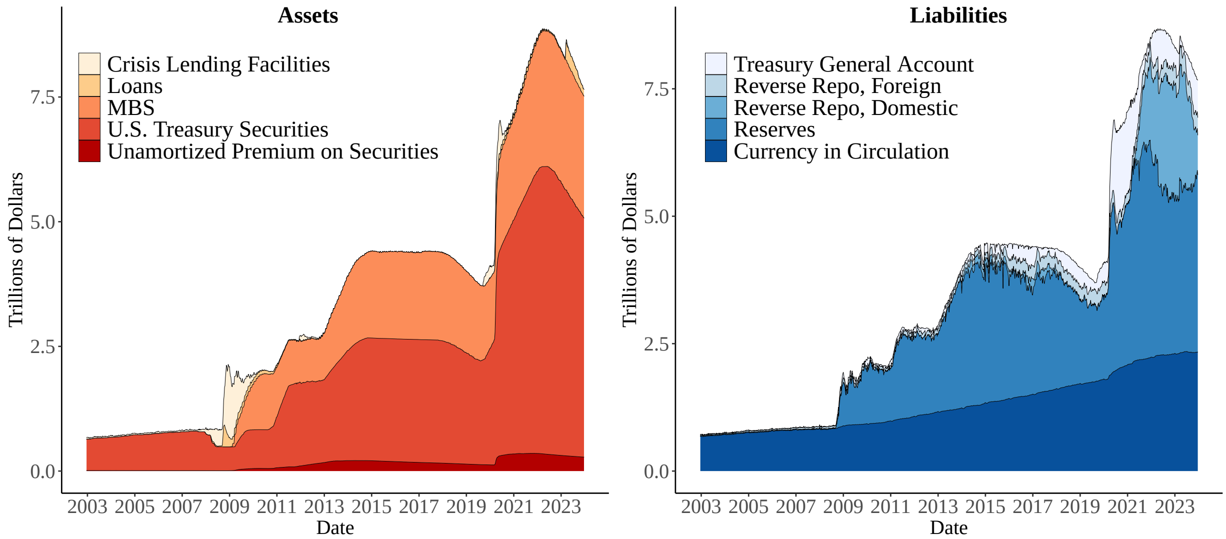

Figure 1: Evolution of the Fed's Balance Sheet. The figure displays the evolution of the Fed's balance sheet between 2003 and 2024. The left panel shows the evolution of the asset side, decomposed into its main categories: crisis lending facilities, loans, mortgage-backed securities, US Treasury securities, and unamortized premium on securities, where crisis lending facilities are composed of liquidity swaps, repo facility balances, term auction credit, and commercial paper funding facility balances. The right panel shows the evolution of the liability side, decomposed into its main categories: Treasury general account, foreign reverse repo facility balances, domestic reverse repo balances, reserves, and currency in circulation. Source: Federal Reserve Economic Data, Federal Reserve Bank of St. Louis.

1 Post-GFC Evolution of the Fed's Balance Sheet

Figure 1 displays both sides of the Federal Reserve System's balance sheet, highlighting a significant uptick in both its size and complexity between 2008 and 2024. Responding to the financial turmoil, in 2008 the Fed started to purchase large amounts of long-term securities—a policy instrument commonly referred to as quantitative easing (QE) or large-scale asset purchase program. As a consequence, the volume of reserves held by banks has increased by a factor of 40, leading to a doubling of the total size of the Fed's balance sheet. In November 2010, the Fed announced the second round of QE and further increased its balance sheet by $600 bn by the end of the second quarter of 2011. The third round of QE was then announced in September 2012, leading to an additional increase in the Fed's balance sheet of over $1.5 tn by 2015. Until October 2017, the Fed kept its balance sheet at a stable size. At this point, it started to "normalize" its balance sheet by not reinvesting a part of its maturing securities and letting its balance sheet reduce. In September 2019—as repo rates suddenly hiked to more than 600 bps—the Fed reversed course and started to increase its balance sheet again, initially by lending in the repo market and eventually through direct purchases of Treasury bills. The Covid-19 crisis, starting in March 2020, led to an additional round of QE and resulted in the Fed doubling its balance sheet once more, mostly through Treasury and agency MBS purchases. The peak size was reached in 2022 at more than $8 tn. From this point, the Fed again started normalizing.

This evolution of the size of the Fed's balance sheet was accompanied by a change in the *composition* of its liabilities. After the 2008 financial crisis, the Fed started offering overnight liquidity services to a growing number of non-bank institutions, which allows for endogenous changes in the supply of reserves or "sterilization" (Pozsar, 2017) given a fixed Fed's balance sheet size. A first key institution impacting the supply of reserves available to banks is the US Treasury, which, since 2008, no longer keeps its cash balances with private banks but rather entirely with the Fed in its Treasury General Account (TGA) (Santoro, 2012). These balances ran as high as \$1.8 tn in Summer 2020 and are highly volatile—particularly during tax payment and debt issuance periods and around quarter-end, year-end, and debt-ceiling events. For instance, in September 2019, TGA balances increased by more than \$150 bn within 2 weeks and thereby removed a similar amount of reserves from the stock available to banks. Second, the Fed allowed some foreign central banks to move their short-term balances to its balance sheet within the Foreign Reverse Repo Facility (FRRP). In September 2019, the FRRP reached a peak of \$300 bn. Third, in 2014, the Fed let money market funds access its balance sheet through the Domestic Reverse Repo Facility (DRRP). The DRRP became an important element in the Fed's monetary policy implementation strategy by creating a floor below which money market funds would not lend to the repo market. The DRRP was heavily used by money market funds, which would deposit an average of \$150 bn with the Fed per day up to 2018, when a large supply of Treasury bills pushed repo rates above the DRRP rate (d'Avernas and Vandeweyer, forthcoming). Since the Covid-19 crisis and the subsequent large increase in the Fed's balance sheet size, the DRRP has reached unprecedented levels, peaking at close to \$2.4 tn between 2022 and 2023. Importantly, all of those new items on the liability side of the Fed's balance sheet are not under its direct control. The Fed decides how much it remunerates the different accounts but does not control the quantities that institutions, including the Treasury, deposit at these facilities.[7] The model we develop in the following sections examines how these adjustments to the Fed's balance sheets turn out to be key variables in post-GFC Treasury market dynamics.

2 Model

Let $(\Omega, \mathcal{F}, \mathcal{P})$ be a probability space that satisfies the usual conditions and assume that all stochastic processes are adapted. The economy evolves in continuous time with $t \in [0, \infty)$ and is populated by a representative traditional bank (with a dealer subsidiary), shadow bank, and household, as well as the Treasury and a central bank. To fix ideas about sectorial portfolio holdings in the model, Figure 2 depicts an example of the balance sheets of the different sectors in the economy. The Treasury issues Treasury bonds against future tax liabilities and maintains a balance in the TGA; the central bank holds outstanding Treasury bonds, issues reserves to the banking sector, lends repo at its standing repo facility, and borrows repo at its reverse repo

[7] Although some limits were initially imposed at the reverse repo facility, they have always been increased or lifted to accommodate any uptick in demand.

facility; and households invest their wealth and future tax liabilities in deposits and repos through the domestic and foreign dealers. Traditional banks hold securities, reserves, and Treasury bonds funded by issuing deposits. Traditional banks can also either lend or borrow in repo. Shadow banks hold Treasury bonds leveraged with repo borrowing. In the model, positions are the result of endogenous portfolio optimization under institutional constraints from the various sectors and vary according to shocks.

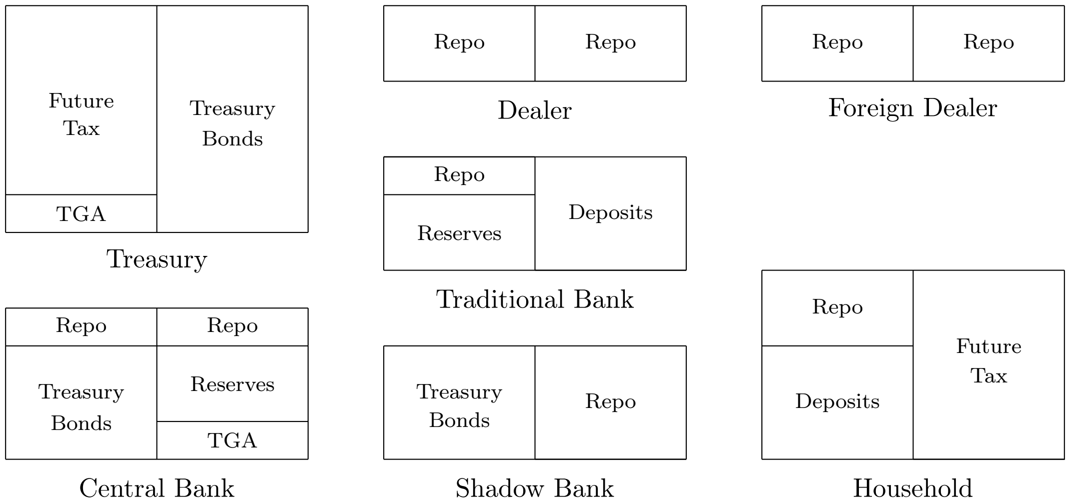

Figure 2: Chart of Sectors' Balance Sheets

2.1 Environment

Preferences Households have risk-neutral preferences over their consumption c_t^h with a time preference ρ and value liquidity services provided by holding repos and deposits:

$$\mathbb{E}_t \left[\int_t^\infty e^{-\rho(u-t)} \left(c_u^h + \beta \log(h(w_u^{h,p}, w_u^{h,d}; \alpha_u)) \right) du \right], \tag{1}$$

where h is a Cobb-Douglas aggregator of deposits and repo portfolio weights $w_t^{h,d}$ and $w_t^{h,p}$,

$$h(w_t^{h,p}, w_t^{h,d}; \alpha_t) = (w_t^d)^{\alpha_t} (w_t^p)^{1-\alpha_t}. \tag{2}$$

The time-varying parameter $0 < \alpha_t < 1$ corresponds to the preference of households for holding deposits relative to repos.

Financial Assets Each asset held by an institution in the economy represents a liability of another institution, and all institutions are owned by households. Additionally, households cannot raise equity, and their consumption can potentially be negative.[8] These assumptions enable us to abstract from several general equilibrium forces that are not the focus of this article, such as sectoral wealth dynamics, consumption-saving decisions, or productive capital misallocation resulting from the balance sheet and transaction costs. Since we do not solve for the consumption-saving problem, the model does not determine the level of rates, but only equilibrium spreads between rates. To establish a benchmark rate, we introduce an illiquid risk-free capital in zero net supply, only accessible to traditional banks, yielding a constant return r^k.

Treasury The Treasury issues bonds against the future tax liabilities of households and is responsible for administering redistributive lump-sum tax policies. The net present value of future tax liabilities must equal the outstanding amount of Treasuries:

$$\tau_t^h + a_t = b_t, \tag{3}$$

where b_t is the quantity of bonds issued, a_t is the size of the TGA account, and τ_t^h is the future tax liability of households.

Central Bank The central bank holds Treasury bonds, $\underline{b}$, financed by reserves held by banks m_t and in the TGA a_t. The underlined notation differentiates the central bank's holdings of Treasury bonds $\underline{b}_t$ from the bonds issued by the Treasury b_t. The central bank can also lend repo at the repo facility, rp_t, and borrow repo at the reverse repo facility, rrp_t. Thus, the balance sheet constraint for the central bank is given by

$$\underline{b}_t + rp_t = m_t + a_t + rrp_t. \tag{4}$$

The central bank determines the interest rates at which the central bank lends at the repo facility and at which it borrows at the reverse repo facility. These rates are denoted by $r^{rp} > r_t^m$ and $r^{rrp} < r_t^m$, respectively. In addition, for simplicity, we assume that the central bank always operates with zero net worth and instantaneously transfers all seigniorage revenues to the Treasury. The budget constraint for the Treasury is therefore given by

$$r_t^b b_t = r_t^\tau + r_t^b \underline{b}_t + r_t^{rp} rp_t - r_t^m m_t - r_t^{rrp} rrp_t, \tag{5}$$

To pay interest on Treasury bonds, the Treasury collects taxes r_t^τ from households and seigniorage revenues rebated from the central bank.

[8]Negative consumption can be interpreted as the negative disutility of providing labor with a linear productivity function.

Dealers and Repo Markets The repo market is assumed to be fully intermediated so that households exclusively invest in repos through dealers rather than directly with traditional or shadow banks.[9] Because of costly balance sheet space (see below), dealers require a spread between their borrowing and lending rates. We denote the rate at which dealers borrow from households as the triparty repo rate r_t^{pt} and the rate at which they lend to shadow banks as the bilateral repo rate r_t^p.[10] The intermediation spread is therefore given by $r_t^{pt} - r_t^p$. To account for fluctuations in intermediation capacities and match quarter-end dynamics, our model includes two types of dealers. The first type is a foreign dealer with a balance sheet size denoted by f_t and treated as a parameter subject to shocks.[11] The second type is a (domestic) dealer-subsidiary, which is part of a traditional bank. Domestic intermediation volumes are therefore an endogenous variable.

Shocks Liquidity preference parameter α_t, intermediation by foreign dealers f_t, TGA account a_t, Treasuries b_t, and the central bank's balance sheet $\underline{b}_t$ are subject to aggregate shocks. The vector of time-varying parameters, denoted by $\mathbf{x}_t \equiv \{\alpha_t, f_t, a_t, b_t, \underline{b}_t\}$, follows

$$d\mathbf{x}_t = \begin{cases} (\mathbf{x}^s - \mathbf{x}_t)dN_t & \text{if} \quad \mathbf{x}_t \neq \mathbf{x}^s, \\ (\mathbf{x}' - \mathbf{x}_t)dN_t & \text{if} \quad \mathbf{x}_t = \mathbf{x}^s, \end{cases} \tag{6}$$

where dN_t is a Poisson process with intensity $\lambda_t > 0$ and $\mathbf{x}'$ is a random variable independently and identically distributed. We specify the exact distribution of $\mathbf{x}'$ in the following sections when we study shocks to different parameters. As shown in equation (6), the economy features a steady state $\mathbf{x}^s$. Upon the arrival of a Poisson shock in state $\mathbf{x}_t = \mathbf{x}^s$, the state $\mathbf{x}_t$ takes on a new random value $\mathbf{x}'$. Upon the arrival of a Poisson shock in state $\mathbf{x}_t \neq \mathbf{x}^s$, $\mathbf{x}_t$ reverts to $\mathbf{x}^s$. The Poisson intensity λ_t is equal to λ if $\mathbf{x}_t = \mathbf{x}^s$ and equal to λ' otherwise. Hence, λ represents the likelihood of an aggregate shock while $1/\lambda'$ determines the expected duration of the shock.

2.2 Agents' Problems

Traditional Banks Traditional banks face a Merton's (1969) portfolio choice problem augmented by transaction costs and a balance sheet cost. Traditional banks maximize their lifetime

[9] This assumption aligns with the actual institutional framework in the US, where the vast majority of repo transactions are effectively intermediated by securities dealers. For further institutional details on repo markets, we refer to the work of Copeland, Martin, and Walker (2014).

[10] We adopt these terminologies to match the US institutional practice whereby money market funds (here abstracted from) lend in triparty repo, in which a third-party clearing bank handles the management of the collateral, while hedge funds tend to borrow in direct bilateral repo markets.

[11] Variations in the foreign dealer's balance sheet size primarily correspond to quarter-end window-dressing (Munyan, 2015) but can also be interpreted as any event that negatively affects aggregate dealer balance sheet capacity. For instance, the increasingly popular practice of FICC-sponsored cleared repo, which allows netting for regulatory purposes, could be interpreted in our setting as equivalent to an increase in the foreign dealer repo capacity.

expected profits:

$$\max_{\{w_u^k \geq 0, w_u^b \geq 0, w_u^m \geq 0, w_u^p, w_u^x \geq 0, w_u^d \geq 0\}_{u=t}^{\infty}} \mathbb{E}_t \left[\int_t^{\infty} e^{-\rho(u-t)} d\pi_u \right], \tag{7}$$

subject to the budget constraint:

$$\begin{aligned} d\pi_t = &\left(w_t^k r_t^k + w_t^b r_t^b + w_t^m r_t^m + w_t^p r_t^p + w_t^x (r_t^p - r_t^{pt}) - w_t^d r_t^d \right) dt \\ &- \frac{\chi}{2} \ell_t^2 dt - \nu |dw_t^b|, \end{aligned} \tag{8}$$

the balance sheet constraint:

$$w_t^k + w_t^b + w_t^m + w_t^p = w_t^d, \tag{9}$$

and the intraday liquidity (IL) constraint:

$$w_t^p \leq \kappa w_t^m. \tag{10}$$

Traditional bankers choose their portfolio weights for capital w_t^k, Treasury bonds w_t^b, reserves w_t^m, and deposits w_t^d given their respective interest rates r^k, r_t^b, r_t^m, and r_t^d.[12] Traditional banks also either lend or borrow in bilateral repo w_t^p given the interest rates in this market r_t^p. In addition, traditional banks select the size of their dealer balance sheet w_t^x and profit from the spread between the bilateral (post-intermediation) and triparty repo rates (pre-intermediation). In making these decisions, traditional banks incur a balance sheet cost that is quadratic in their balance sheet size ℓ_t, defined as

$$\ell_t \equiv w_t^d - \min\{0, w_t^p\} + w_t^x. \tag{11}$$

Note that through the minimum function, the portfolio weight for repos is added to deposits only when traditional banks are net borrowers ($w_t^p < 0$). Importantly, the dealer subsidiary balance sheet w_t^x is added to the consolidated traditional bank balance sheet. The latter follows from the practice whereby internal repos between a bank and a dealer subsidiary are not netted under existing regulatory practices.

Additionally, traditional banks consider the existence of transaction costs when trading Treasury bonds: $-\nu|w_t - w_{t\text{-}}|$, where ν is the transaction cost and t- is the time prior to the

[12] Although Treasury bonds are long-duration assets, we assume they have an instantaneous return because we are interested in the liquidity risk of shadow banks rather than their interest rate risk exposures. A straightforward interpretation of a Treasury holding in our model is the combination of holding a Treasury together with a futures contract selling that Treasury in the future, thereby hedging its duration risk. For this reason, the spread between the Treasury yield and the repo rate is to be interpreted as the cash-future basis (see Barth and Kahn (2023) for an in-depth account of this trade by hedge funds in the post-Basel III environment and Du, Hébert, and Li (2023) for the effect of dealer balance sheet costs on the yield curve).

transaction. Lastly, traditional banks are subject to the (IL) constraint (10), which limits repo lending to a fraction κ of reserve holdings.

Shadow Banks Shadow banks[13] face a similar problem to traditional banks but without balance sheet costs, the intraday liquidity constraint, and the ability to hold central bank reserves. Shadow banks maximize their expected profits:

$$\max_{\{\overline{w}_u^b \geq 0, \overline{w}_u^p\}_{u=t}^{\infty}} \left[\int_t^{\infty} e^{-\rho(u-t)} d\overline{\pi}_u \right], \tag{12}$$

subject to the budget constraint:

$$d\overline{\pi}_t = \left(\overline{w}_t^b r_t^b - \overline{w}_t^p r_t^p \right) dt - \nu |d\overline{w}_t^b|, \tag{13}$$

and the balance sheet constraint:

$$\overline{w}_t^b = \overline{w}_t^p. \tag{14}$$

Shadow banks choose their holdings of Treasury bonds $\overline{w}_t^b$ and repo financing $\overline{w}_t^p$ given the respective interest rates r_t^b and r_t^p. Like traditional banks, shadow banks incur a similar transaction cost when purchasing or selling Treasury bonds.

Households Households maximize their lifetime expected utility of consumption and liquidity benefits:

$$\max_{\{w_u^{h,d} \geq 0, w_u^{h,p} \geq 0\}_{u=t}^{\infty}} \mathbb{E}_t \left[\int_t^{\infty} e^{-\rho(u-t)} dc_u^h \right] + \mathbb{E}_t \left[\int_t^{\infty} e^{-\rho(u-t)} \beta \log(h(w_u^{h,p}, w_u^{h,d}, \alpha_u)) du \right], \tag{15}$$

subject to the budget constraint:

$$dc_t^h = (w_t^{h,p} r_t^{p,t} + w_t^{h,d} r_t^d - r_t^{\tau} + \pi_t^f) dt + d\pi_t + d\overline{\pi}_t, \tag{16}$$

and the balance sheet constraint:

$$w_t^{h,p} + w_t^{h,d} = \tau_t^h. \tag{17}$$

Households choose their portfolio holdings of deposits $w_t^{h,d}$ and repos $w_t^{h,p}$, given their liquidity preference α_t. They receive profits from traditional and shadow banks, π_t and $\overline{\pi}_t$, and foreign

[13] Although our primary interpretation for shadow banks is relative-value hedge funds, it can also incorporate any institution involved in Treasury arbitrage trades with repo leverage. This includes some securities dealers themselves, which renders the assumption that dealers are exclusively intermediating repos and do not hold Treasuries without loss of generality. A dealer holding Treasuries directly is to be interpreted as a combination of a repo dealer and a shadow bank.

dealers, $\pi_t^f \equiv (r_t^p - r_t^{pt}) f_t$, and pay lump-sum taxes, r_t^{τ}.[14]

2.3 Discussion of Assumptions

We discuss the three key frictions in our framework: the balance sheet cost, the intraday liquidity requirement, and households' demand for repos and deposits.

Balance Sheet Cost Through equation (8), traditional banks and their dealer subsidiaries are subject to a convex cost in the size of their balance sheet. This assumption captures in reduced form the implication of subjecting traditional banks to non-risk-weighted regulations that restrict their leverage, such as a supplementary leverage ratio (SLR) of 5% on large and systemic banks in the US. For micro-foundations, we refer to the modified-CAPM model of Frazzini and Pedersen (2014), in which a leverage constraint pushes investors to penalize low-beta investments, such as low-yield risk-free arbitrage, in favor of a high-beta asset tilt needed to achieve an optimal risk exposure. Alternatively, Andersen, Duffie, and Song (2019) show that a leverage ratio constraint creates a debt-overhang cost, which generates increased hurdle rates for shareholders for funding an arbitrage trade, referred to as funding value adjustments. Irrespective of the specific microfoundation, the convexity of the balance sheet cost is to be interpreted as the cost for banks to use an additional unit of balance sheet space to fund an arbitrage rather than some higher-yielding trade, and thereby captures banks' demand elasticity for on-balance-sheet arbitrage trades.

Intraday Liquidity Requirement Following inequality (10), the intraday liquidity constraint restricts traditional banks' ability to lend in repo to a multiple of their reserve balances. This constraint stems from new regulations such as resolution liquidity execution need (RLEN) which penalizes banks for using the Fed's intraday overdraft and thereby causes the quantity of reserves available to limit the capacity for intraday settlement that is required for repo transactions. We refer to d'Avernas, Han, and Vandeweyer (2023a); Correa, Du, and Liao (2020); Copeland, Duffie, and Yang (2021); and Pozsar (2019) for further details on this intraday liquidity constraint and how it differs from traditional bank reserve requirements, which were revoked in the US in February 2023.

Imperfect Substitution Between Repo and Deposit In practice, households and firms hold cash-like assets largely in the form of bank deposits and money market fund shares. Since the reform of money markets in 2016, most money market fund shares have been invested in either triparty repos with dealers or in short-duration Treasury securities and close substitutes

[14] To facilitate exposition, we assume that households do not invest directly in Treasuries. This condition would be guaranteed in equilibrium for a parameter β that is high enough. Our results are qualitatively robust to relaxing this assumption as long as households have a strong preference for money-like assets over Treasuries.

(Cipriani and La Spada, 2021). In our model, we abstract from money market funds and assume a perfect pass-through for money market fund share demand to repo.[15] The imperfect substitutability between repo and deposits captures the fact that deposits and money market fund shares have important diverging characteristics. For instance, deposits are more liquid, have lower interest, and are riskier when held above FDIC insurance quantity limits and safer below this limit. Empirically, Krishnamurthy and Li (2022) estimate this elasticity of substitution to be smaller than one on average, consistent with our imperfect substitute assumption.

2.4 Solving

Equilibrium Definition Given the assumptions on the aggregate shocks, the state space of the economy is given by the vector of time-varying parameters $\mathbf{x}_t$.[16]

Definition 1. *Given central bank policies $\{r^{rp}, r^{rrp}\}$ and the return on capital r^k, a **Markov equilibrium** $\mathcal{M}$ in $\mathbf{x}_t$ is a set of functions $g_t = g(\mathbf{x}_t)$ for (i) interest rates $\{r_t^b, r_t^m, r_t^p, r_t^{pt}, r_t^d\}$ and (ii) individual controls for traditional banks $\{w_t^k, w_t^b, w_t^m, w_t^p, w_t^x, w_t^d\}$, shadow banks $\{\overline{w}_t^b, \overline{w}_t^p\}$, and households $\{w_t^{h,p}, w_t^{h,d}\}$ such that:*

1. *Agents' optimal controls (ii) solve their respective problems given prices (i).*
2. *The balance sheet constraint of the central bank is satisfied.*
3. *The balance sheet constraint and budget constraint of the Treasury are satisfied.*
4. *Markets clear:*

$$
\begin{aligned}
&(a)\ \textit{capital:} && w_t^k = 0,\\
&(b)\ \textit{Treasury bonds:} && \overline{w}_t^b + w_t^b = b_t - \underline{b}_t,\\
&(c)\ \textit{reserves:} && w_t^m = m_t,\\
&(d)\ \textit{triparty repo:} && w_t^{h,p} = w_t^x + rrp_t + f_t,\\
&(e)\ \textit{bilateral repo:} && w_t^x + w_t^p + rp_t + f_t = \overline{w}_t^p,\\
&(f)\ \textit{deposits:} && w_t^d = w_t^{h,d}.
\end{aligned}
$$

5. *The law of motion for $\mathbf{x}_t$ is consistent with agents' expectations.*

[15] d'Avernas and Vandeweyer (forthcoming) investigate the role of the supply of Treasury bills in driving triparty repo rates when banks face balance sheet costs, which is abstracted from in this work by assuming that households do not hold Treasuries directly.

[16] Aggregate Treasury portfolio weights are not aggregate state variables because the *marginal* cost of transactions is constant and not a function of the size of the transaction. Thus, if it is profitable to fire-sell Treasuries when entering state $\{\mathbf{x}_t, w_{t\text{-}}^b, \overline{w}_{t\text{-}}^b\}$, it is also profitable to fire-sell Treasuries when entering state $\{\mathbf{x}_t, w_{t\text{-}}^{b\prime}, \overline{w}_{t\text{-}}^{b\prime}\}$, for $w_{t\text{-}}^{b\prime} \neq w_{t\text{-}}^b$ and $\overline{w}_{t\text{-}}^{b\prime} \neq \overline{w}_{t\text{-}}^b$. Therefore, the Markov functions are such that $g(\mathbf{x}_t, w_{t\text{-}}^{b\prime}, \overline{w}_{t\text{-}}^{b\prime}) = g(\mathbf{x}_t)$ and the equilibrium is entirely determined by $\mathbf{x}_t$. See Appendix Section A.

Equilibrium Restrictions Because the aim of this article is the study of Treasury market disruptions, we simplify the exposition[17] by focusing our analysis on a subset of relevant equilibria, and thereby implicitly restrict the set of parameters and policies considered. First, we focus on equilibria in which the traditional bank dealer subsidiary has a positive balance sheet size: $w_t^x > 0$; and reserves are in strict positive supply: $m_t > 0$. Furthermore, in the steady state $\mathbf{x}^s$, traditional banks hold some Treasuries: $w^b(\mathbf{x}^s) > 0$. Finally, the household balance sheet must be larger than the quantity of repo available from traditional banks: $\tau_t^h > \kappa m_t$.

Value Functions In Appendix A, we guess and verify that the value functions for traditional and shadow banks take the following form:

$$V(w^b;\mathbf{x}) = \xi(\mathbf{x}) + \theta(\mathbf{x})w^b, \qquad \overline{V}(\overline{w}^b;\mathbf{x}) = \overline{\xi}(\mathbf{x}) + \overline{\theta}(\mathbf{x})\overline{w}^b. \tag{18}$$

To solve for the marginal value of holding Treasuries, $V_w(w^b;\mathbf{x}) = \theta(\mathbf{x})$ and $\overline{V}_w(\overline{w}^b;\mathbf{x}) = \overline{\theta}(\mathbf{x})$, we apply the envelope theorem to the Hamilton-Jacobi-Bellman equations:

$$(\rho + \lambda')\theta(\mathbf{x}') = r^b(\mathbf{x}') - r^d(\mathbf{x}') - \chi\ell(\mathbf{x}') + \lambda'\nu\,\text{sign}(w^b(\mathbf{x}^\mathbf{s}) - w^b(\mathbf{x}')), \tag{19}$$

$$(\rho + \lambda')\overline{\theta}(\mathbf{x}') = r^b(\mathbf{x}') - r^p(\mathbf{x}') + \lambda'\nu\,\text{sign}(\overline{w}^b(\mathbf{x}^\mathbf{s}) - \overline{w}^b(\mathbf{x}')), \tag{20}$$

for $\mathbf{x}'$ such that $\overline{w}^b(\mathbf{x}') \neq \overline{w}^b(\mathbf{x}^s)$.[18] This condition implies that, following a shock large enough to trigger a fire sale, the marginal value of holding more Treasuries for shadow banks equals the Treasury-repo spread plus expected transaction costs when returning to the steady state. For traditional banks, the relevant spread is the Treasury-deposit spread, adjusted for the marginal balance sheet cost.

First-order Conditions The first-order conditions for deposits, reserves, bilateral repo, and triparty repo of traditional banks are given by

$$r^k - r^d(\mathbf{x}) = \chi\ell(\mathbf{x}), \tag{21}$$

$$r^k - r^m(\mathbf{x}) = \kappa\vartheta^m(\mathbf{x}), \tag{22}$$

$$r^k - r^p(\mathbf{x}) \begin{cases} = -\vartheta^m(\mathbf{x}) & \text{if } w^p(\mathbf{x}) > 0, \\ \in [-\vartheta^m(\mathbf{x}), \chi\ell(\mathbf{x})] & \text{if } w^p(\mathbf{x}) = 0, \\ = \chi\ell(\mathbf{x}) & \text{if } w^p(\mathbf{x}) < 0, \end{cases} \tag{23}$$

$$r^p(\mathbf{x}) - r^{pt}(\mathbf{x}) = \chi\ell(\mathbf{x}), \tag{24}$$

[17]These restrictions do not change our main results, but limit the number of different cases we need to consider in the proofs.

[18]See Appendix A for the remaining cases.

where $\vartheta^m(\mathbf{x})$ is the shadow price of the IL constraint (10). In equation (21), traditional banks equalize the marginal benefits of issuing deposits (spread to capital return) to their marginal cost (marginal increase in the balance sheet cost). In equation (22), the marginal cost of holding an additional unit of reserves must equal the marginal benefit of loosening the IL constraint. Similarly, in equation (23), if a traditional bank invests in repos, the bilateral repo rate needs to compensate for the tightening of the IL constraint. If a traditional bank funds itself in repo, the repo rate must be sufficiently low to compensate for the increase in the balance sheet cost, similar to deposits. Finally, in equation (24), traditional banks require a spread between bilateral and triparty repo to compensate for the balance sheet cost incurred by intermediating repo at the dealer subsidiary.

In addition, the households' first-order condition for their relative holdings of triparty repo and deposit is given by

$$r^{pt}(\mathbf{x}) - r^d(\mathbf{x}) = \beta \left(\frac{\alpha}{w^{h,d}(\mathbf{x})} - \frac{1-\alpha}{w^{h,p}(\mathbf{x})} \right). \tag{25}$$

Households equalize the marginal benefit of investing in triparty repo over deposits, as given by the spread between the rates on these two assets, to the marginal convenience cost of reallocating one unit of wealth from deposits to repo on the right-hand side of equation (25).

Lastly, the first-order condition for Treasury portfolio weights yields

$$\nu \operatorname{sign}(w^b(\mathbf{x}) - w^b(\mathbf{x}_-)) = \theta(\mathbf{x}) \qquad \text{if } w^b(\mathbf{x}) \neq w^b(\mathbf{x}_-), \tag{26}$$

$$\nu \operatorname{sign}(\overline{w}^b(\mathbf{x}) - \overline{w}^b(\mathbf{x}_-)) = \overline{\theta}(\mathbf{x}) \qquad \text{if } \overline{w}^b(\mathbf{x}) \neq \overline{w}^b(\mathbf{x}_-). \tag{27}$$

Both traditional and shadow banks trade off the marginal cost of a transaction, on the left-hand side, against the marginal benefit of purchasing or selling Treasury bonds, on the right-hand side. The value functions depend on current and future rates and the stochastic process for the state variables. For example, as we show below, the marginal value of holding Treasuries for shadow banks increases when the Treasury-repo spread $r^b(\mathbf{x}) - r^p(\mathbf{x})$ is larger or when the arrival intensity of a disruption λ is lower. The sign function reflects the fact that to sell bonds, the marginal value of an additional bond needs to be negative (and larger than the marginal transaction cost), and vice-versa for purchases.

3 Main Theoretical Results

In this section, we derive the main theoretical results of the paper. We focus on a specific shock to the liquidity preference parameter α_t and relegate the discussion of additional shocks to the next section when comparing the model predictions to the data. That is, we assume that α_t is the only parameter that varies after an aggregate Poisson shock: $\mathbf{x}^s = \{\alpha^s, f, a, b, \underline{b}\}$ and $\mathbf{x}' = \{\alpha', f, a, b, \underline{b}\}$, where α' is independently and identically distributed according to a uniform distribution on $(\alpha^s, 1)$. Following this "flight-to-deposit" shock, α increases, and households seek to reduce their repo holdings relative to deposits. We further assume that repo and reverse repo facilities are inactive and relegate the study of adjustments of the central bank balance sheet to the next section. Finally, in this section, we confine our analysis to the parameter space such that, if α' is sufficiently high, it becomes optimal for shadow banks to sell Treasuries.[19] We relegate all proofs of propositions to the Online Appendix.

Uniqueness and Inaction Region First, we demonstrate in Proposition 1 that the equilibrium is unique in all quantities and prices except for the rate on Treasuries r_t^b. This rate r_t^b can take a continuum of values within the boundaries defined below because of the presence of transaction costs, which creates an inaction region within which no Treasuries are traded. Within these bounds, a deviation in the Treasury rate is not sufficient to compensate for the transaction cost, so agents do not rebalance their portfolios.

Proposition 1. *If the equilibrium exists, it is unique in prices* $\{r^m, r^p, r^{pt}, r^d\}$ *and quantities* $\{w^k, w^b, w^m, w^p, w^x, w^d, \overline{w}^b, \overline{w}^p, w^{h,p}, w^{h,d}\}$ *as functions of* $\alpha \in [\alpha^s, 1)$.

Also, the interest rate on Treasuries is bounded by $\underline{r}^b(\alpha) \leq r^b(\alpha) \leq \overline{r}^b(\alpha)$, *where*

$$\underline{r}^b(\alpha') \equiv \max\{r^d(\alpha') + \chi\ell(\alpha') - \rho\nu, r^p(\alpha') - (\rho + 2\lambda')\nu\} \tag{28}$$

$$\overline{r}^b(\alpha') \equiv \min\{r^d(\alpha') + \chi\ell(\alpha') + (\rho + 2\lambda')\nu, r^p(\alpha') + \rho\nu\} \tag{29}$$

for $\alpha' \in (\alpha^s, 1)$.[20]

The boundaries of the inaction region in Proposition 1 capture the opportunity costs of buying and selling Treasuries for either traditional or shadow banks. The max and min operators are such that, in equilibrium, both traditional and shadow banks are not willing to sell (lower bound) or purchase (upper bound) Treasuries outside of the fire-sale region.[21] For example, if the Treasury rate goes below $r^p(\alpha) - (\rho + 2\lambda')\nu$, then shadow banks would prefer to sell their Treasuries, as it considers the benefits of sparing the repo funding rate $r^p(\alpha)$ and compares it to paying transaction costs a first time upon arrival of the shock $(\rho + \lambda')\nu$ (time-discounted in the shock state) and then once more when returning to the steady state with expectation $\lambda'\nu$.

Inelastic Benchmark To gain intuition about the model, we first consider a benchmark case in which the portfolios of both traditional and shadow banks' balance sheets are set constant. In this case, the banking sector does not provide any elasticity, and all adjustments occur entirely through changes in prices. Combining equations (21), (22), (24), and (25), the repo-to-reserves

[19] That is, the set of parameters $\theta \equiv \{\rho, \nu, \chi, \beta, \lambda, \lambda', \mathbf{x}^s\}$ is such that $\exists \mathbf{x}' : \overline{w}^b(\mathbf{x}') < \overline{w}^b(\mathbf{x}^s)$.
[20] See the proof in Online Appendix A.1 for the boundaries at $\alpha = \alpha^s$.
[21] When α' is sufficiently large to trigger sales by shadow banks, $\underline{r}^b(\alpha') = r^b(\alpha') = \overline{r}^b(\alpha')$.

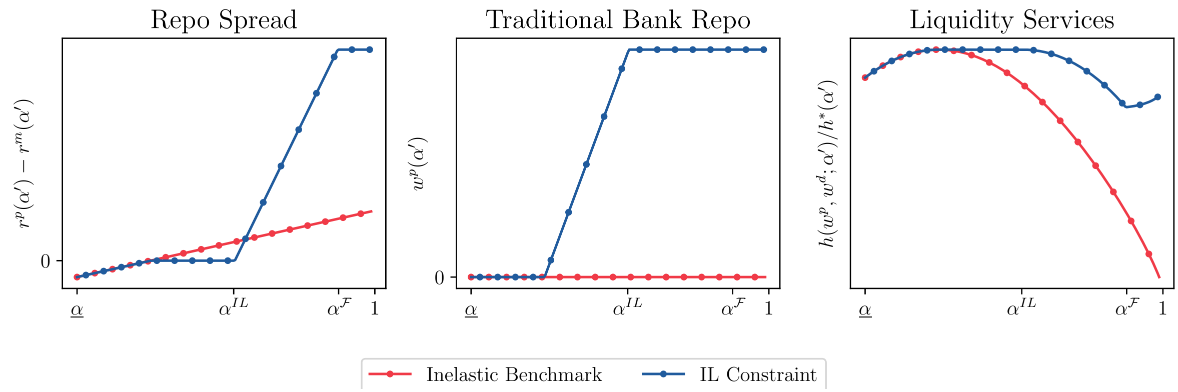

Figure 3: Inelastic Benchmark and Crisis State Dynamics. This figure shows equilibrium repo spreads $r^p(\alpha') - r^m(\alpha')$, traditional banks' repo lending $w^p(\alpha')$, and the ratio of liquidity services to its optimum $h(w^p, w^d; \alpha')/h^\star(\alpha')$ where $h^\star(\alpha') \equiv \max_w h(w, \tau^h - w; \alpha')$. We plot these variables under two different settings. In red, we provide an inelastic benchmark with no repo lending or borrowing by traditional banks or Treasury sales. In blue, we provide the solution of the full model, in which banks can adjust their balance sheets.

spread can be written in the inelastic benchmark as

$$r^p(\alpha') - r^m(\alpha') = \beta \left(\frac{\alpha'}{w^{h,d}} - \frac{1-\alpha'}{w^{h,p}} \right), \tag{30}$$

where the allocations $w^{h,d}$ and $w^{h,p}$ are held constant. That is, the repo-to-reserves spread reflects households' relative preferences for repos: When repos are less preferred than deposits, as captured by an increase in parameter α', repo rates have to increase to induce households to hold repos rather than deposits. This scenario is illustrated with red lines in the three panels of Figure 3. Since banks do not adjust their repo lending (middle panel), repo spreads have to increase as a function of the size of the shock α' (left panel) to compensate households for maintaining the fixed composition of repos and deposits following equation (30) (right panel). We next examine the model's reaction to liquidity shocks when balance sheets have some flexibility, but traditional banks are subject to both the balance sheet costs and the IL constraint.

Shock-state Dynamics When banks' portfolio allocations are flexible, the dynamics of price and quantity adjustments are more complex because they depend on whether the shock is large enough for the IL constraint to bind or Treasury sales to be optimal. The blue lines in Figure 3 depict these dynamics upon entering the shock state. As long as the IL constraint is slack ($\alpha' < \alpha^{IL}$), the repo-to-reserves spread $r^p(\alpha') - r^m(\alpha')$ remains bounded above at zero (left panel) since banks would otherwise arbitrage any positive spread by swapping reserves for repo. Banks' ability to lend in repo markets (middle panel) results in households receiving their preferred mix of reduced repo and increased deposits (right panel) until the IL constraint is reached. At this threshold, banks cannot lend in repo further, and equilibrium requires households to absorb a suboptimal mix of repos and deposits (right panel), which leads to an increase in repo spreads. Because the IL constraint is binding, the repo-to-reserves spread increases at a faster rate compared to the inelastic benchmark, reflecting a positive value for reserves' shadow cost from the IL constraint $\vartheta^m(\alpha')$ in equation (22). For some shock with a magnitude beyond $\alpha^{\mathcal{F}}$, it becomes optimal for shadow banks to pay the transaction cost and sell Treasuries to traditional banks rather than to fund these positions in repo. These Treasury sales provide an outside option for shadow banks and cap how high the repo rate can spike (left panel).

Equilibrium Treasury Holdings We next characterize how the steady state holdings of Treasuries and endogenous risk are affected by changes in the balance sheet cost through Proposition 2.

Proposition 2. *Higher balance sheet costs χ correspond to higher steady-state shadow banks' Treasury holdings $\overline{w}^b(\alpha^s)$ and probability of fire sales. That is,*

$$\frac{\partial \overline{w}^b(\alpha^s)}{\partial \chi} > 0 \quad \text{and} \quad \frac{\partial \mathbb{P}[\overline{w}^b(\alpha') < \overline{w}^b(\alpha^s)]}{\partial \chi} > 0.$$

Proposition 2 highlights an unintended consequence of leverage regulations: Increasing banks' balance sheet costs pushes equilibrium Treasury holdings to the unregulated shadow banking sector and generates liquidity risk.[22] Shadow banks have a comparative advantage in holding Treasuries financed by repo, in that they are not subject to the balance sheet cost. Proposition 2 shows that this comparative advantage is increasing in banks' balance sheet costs, leading to an increase in equilibrium shadow banks' bond holdings. However, this comparative advantage is not absolute. Because shadow banks finance Treasuries with repos from households, they are exposed to liquidity risk. Upon the arrival of the Poisson shock (α jumps to α'), shadow banks have to pay a higher repo rate for the duration of the shock or sell Treasuries and incur the transaction cost. The balance between regulatory arbitrage and liquidity risk determines Treasury holdings allocation in equilibrium. As the probability of fire sales $\mathbb{P}[\overline{w}^b(\alpha') < \overline{w}^b(\alpha^s)]$ is increasing in shadow banks' bond holdings, it is also increasing in the balance sheet cost.

Volatility Paradox in Treasury Markets We next characterize how equilibrium portfolio allocations and endogenous risk are affected by a change in the likelihood of a flight-to-deposit shock.

Proposition 3. *A lower shock arrival intensity λ corresponds to larger shadow banks' Treasury portfolio holdings $\overline{w}^b_t$ in the steady state, a higher probability of fire sales, and higher expected*

[22] Such a mechanism corresponds to empirical findings by Allahrakha, Cetina, and Munyan (2018) and Barth and Kahn (2023).

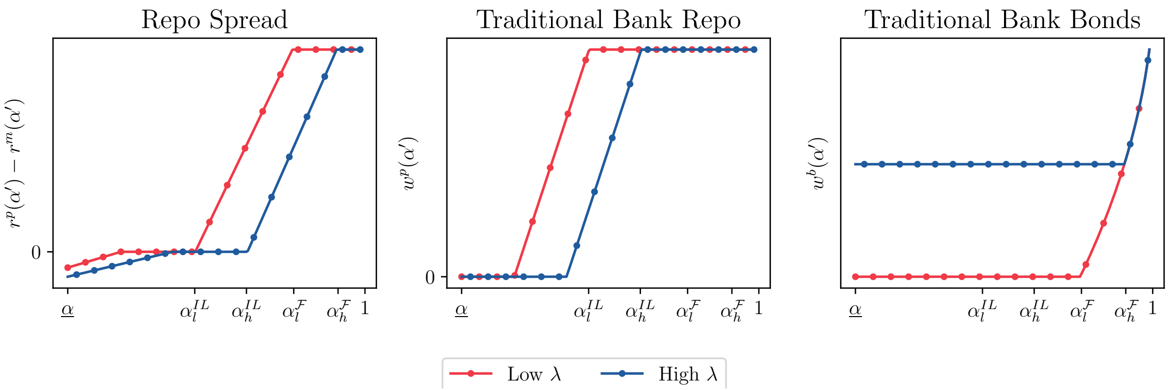

Figure 4: Shock Frequency and Intensity. This figure shows equilibrium repo spreads $r^p(\alpha') - r^m(\alpha')$, traditional banks' repo lending $w^p(\alpha')$, and traditional banks' bond holdings $w^b(\alpha')$ for low (red) and high (blue) Poisson arrival intensity λ (probability of a flight-to-deposit shock).

repo rates. That is,

$$\frac{\partial \overline{w}^b(\alpha^s)}{\partial \lambda} < 0, \quad \frac{\partial \mathbb{P}[\overline{w}^b(\alpha') < \overline{w}^b(\alpha^s)]}{\partial \lambda} < 0, \quad \textit{and} \quad \frac{\partial \mathbb{E}[r^p(\alpha')]}{\partial \lambda} < 0. \tag{31}$$

Proposition 3 underscores the implication of shadow banks trading off the spread between Treasury and repo rates with potential losses upon the arrival of a flight-to-deposit shock. A decrease in the likelihood of a shock (lower λ) amplifies the incentives for shadow banks to acquire additional repo-financed Treasuries, which in turn leads to a higher probability of fire sales conditional on a shock and higher expected repo spikes, as shown in Figure 4. This result is similar to the "volatility paradox" in Brunnermeier and Sannikov (2014), with the additional trade-off between risk frequency and risk intensity. As the frequency of shocks decreases (lower λ), their intensity increases through the endogenous take up in shadow bank risk-taking. This insight underscores the drawbacks of depending on discretionary interventions in the repo market. When investors anticipate interventions with a high likelihood, the few times a negative shock happens without intervention, it does so with an outsized magnitude. Hence, a standing repo facility that provides systematic intervention may, therefore, be necessary to avoid this paradox of volatility.

Crisis Duration and Dynamics Next, we study how expectations about the duration of the shock affect equilibrium prices and portfolio allocations in repo and Treasury markets.

Proposition 4. *A lower shock duration (a higher λ') corresponds to larger shadow banks' Treasury portfolio holdings in the steady state, higher expected repo rates, and lower expected Treasury-repo spreads. That is,*

$$\frac{\partial \overline{w}^b(\alpha^s)}{\partial \lambda'} > 0, \quad \frac{\partial \mathbb{E}[r^p(\alpha')]}{\partial \lambda'} > 0, \quad \textit{and} \quad \frac{\partial \mathbb{E}[\underline{r}^b(\alpha') - r^p(\alpha')]}{\partial \lambda'} < 0. \tag{32}$$

Proposition 4 demonstrates that long-lived shocks result in lower repo rate spikes and higher Treasury yield surges. To understand this result, we recall the envelope theorem for shadow banks' holdings of Treasuries in the shock states in equation (20). This equation implies that in the shock state, the marginal value of holding Treasuries equals the Treasury-repo spread plus the expected transaction costs when returning to the steady state. In the fire-sale region, from the optimality condition (27), it must be that $\overline{\theta}(\alpha') = -\nu$ in equilibrium if shadow banks sell Treasuries, so that we can rewrite equation (20) as

$$\nu = \frac{r^p(\alpha') - r^b(\alpha')}{(\rho + 2\lambda')}. \tag{33}$$

Shadow banks equalize the transaction cost of Treasuries ν to their expected losses from the position in the crisis state. The latter is simply the loss from having to pay a repo rate $r^p(\alpha')$ above the received Treasury rate $r^b(\alpha')$ discounted at the time discount rate ρ and the likelihood of exiting the shock state $(2\lambda')$. In equilibrium, this equation pins down how large the shock needs to be to enter the fire-sale region. As can be observed in Figure 5, a higher likelihood of exiting the shock (high λ')—i.e., lower expected duration of the shock—results in an increase in the (negative) spread that shadow banks are willing to tolerate before selling their Treasuries to traditional banks. Therefore, when a repo supply shock is expected to be short-lived (high λ'), shadow banks are willing to pay a higher repo rate for a short period of time to avoid paying costly transaction fees, which reduces the likelihood of a fire sale. In contrast, if a shock is expected to last for a long period (low λ'), shadow banks prefer to sell Treasuries rather than pay high repo rates for a potentially long time.[23] Overall, when λ' is higher, repo funding is less fragile and shadow banks can take on more leverage and tolerate higher funding rates during these short-lived disruptions.[24] This asymmetry helps explain why repo rates experienced a dramatic spike in September 2019 while Treasury markets remained relatively stable, as opposed to the events of 2020. In 2020, Treasury yields rose sharply but repo rates did not surge significantly, which our model would explain to the extent that the latter event was anticipated to be longer-lasting than the liquidity shortage event of 2019.

Central Bank Balance Sheet and Disruption Likelihood We next consider how the overall size of the central bank balance sheet affects market dynamics.

[23] In practice, although costs on Treasuries may appear relatively small, they ought to be compared with annualized interest rates on repos. For instance, a 2.5 bps trading cost corresponds to $(1 + 0.00025)^{360} - 1 \approx 950$ bps annualized loss in return for a single trip trade. In addition, those transactions are more significant for less liquid "off-the-run" issues typically held by hedge funds arbitraging Treasury basis (Barth and Kahn, 2023) and significantly increase during adverse events. To allow for tractable analytical results, our model does not feature fire-sale spirals, as in Brunnermeier and Pedersen (2009).

[24] We also derive in the Online Appendix a sufficient condition for the probability of a fire sale to be decreasing in λ'.

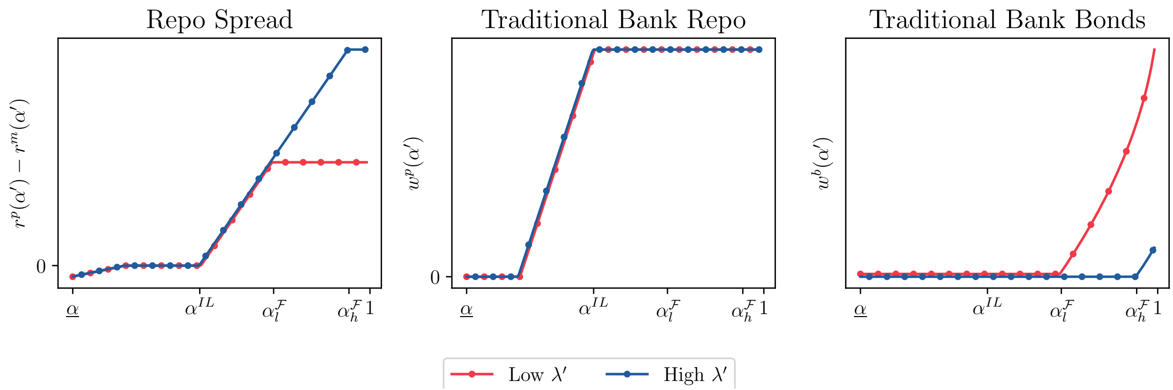

Figure 5: Shock Duration and Shock State Dynamics. This figure shows equilibrium repo spreads $r^p(\alpha') - r^m(\alpha')$, traditional banks' repo lending $w^p(\alpha')$, and traditional banks' bond holdings $w^b(\alpha')$ for low (red) and high (blue) Poisson arrival intensity λ' (probability of exiting the shock state and returning to steady state).

Lemma 1. *In the absence of repo and reverse repo facilities, $r_t^p > r_t^m$ if and only if*

$$\underbrace{b - \underline{b} - w_t^b}_{\textit{shadow banks' repo demand}} > \underbrace{(1-\alpha_t)\tau^h + \kappa m}_{\textit{largest possible repo supply}}. \tag{34}$$

Proposition 5. *A smaller central bank balance sheet $\underline{b}$ results in a larger probability of a fire sale and larger repo spikes. That is,*

$$\frac{\partial \mathbb{P}[\overline{w}^b(\alpha') < \overline{w}^b(\alpha^s)]}{\partial \underline{b}} < 0 \quad \textit{and} \quad \frac{\partial \mathbb{E}[r^p(\alpha')]}{\partial \underline{b}} < 0. \tag{35}$$

Lemma 1 first provides the necessary and sufficient conditions to observe the bilateral repo rate spiking above the interest on reserves when shadow banks' equilibrium repo demand is above the maximum possible combined repo supply from households and traditional banks. As described above, a large demand for repo from shadow banks, $b - \underline{b} - w_t^b$, can be met by traditional banks up to the IL constraint ($w_t^p = \kappa m$) and complement household supply when the liquidity aggregator is at its optimum $\arg\max_w h(w, \tau^h - w; \alpha_t) = (1-\alpha_t)\tau^h$. When the condition in Lemma 1 holds, the IL constraint is binding, and repo rates must increase to incentivize households to provide more repo than their optimal portfolio allocation. Importantly, both sides of the inequality (34) depend on the central bank's balance sheet. On the asset side, an increase in the central bank's Treasury holdings $\underline{b}$ relaxes the balance sheet cost and reduces shadow banks' repo demand, as appears on the left-hand side of condition (34). On the liability side, an increase in reserves m allows banks to lend more in the repo market and support a larger flight-to-deposit shock, as appears on the right-hand side of condition (34). A contraction in the size of the central bank's balance sheet, therefore, unambiguously increases both the size of repo spikes and the probability of Treasury liquidations, as shown in Proposition 5.

	Repo-IOR Spread	Interm. Spread	RRP vol.	TGA vol.
Quarter-End	+	+	+	+
Tax Deadline	+	0	0	+
Treasury Issuance	+	+	0	+

Table 1: Qualitative Summary of Empirical Evidence The table provides a qualitative summary of the empirically observed relationship between the following three salient shocks: Quarter-Ends, Tax Deadlines, and Treasury Issuance. The GC Repo-IOR Spread is the spread between the inter-dealer rate—the GCF rate for Treasuries—and the IOR. The intermediation spread is measured as the spread between GCF and the dealer-to-money fund rate—the tri-party general collateral rate (TGCR). RRP vol. represents reverse repo volumes at the Fed and TGA vol. represents TGA volumes at the Fed. See the Online Appendix for the table of coefficient estimates.

4 Empirical Validation and Policy Implications

In this section, we use our model to study the reaction of repo and Treasury markets to three shocks known to cause disruptions in repo markets: (i) a reduction in foreign repo intermediation during quarter ends, (ii) the payment of tax liabilities from households to the Treasury around tax deadlines, and (iii) the issuance of new Treasury bonds. This exercise serves three purposes. First, we validate the empirical relevance of our model. Second, we examine the mechanisms at play for each shock. Third, we investigate how altering some policy parameters in the model, such as introducing repo facilities with various designs or changing regulations, affects Treasury market dynamics.

4.1 Empirical Evidence on Repo Funding Shocks

An extensive literature has empirically investigated the instability of repo markets in the post-GFC institutional environment. As a prelude to our analysis, we provide a summary of this literature and characterize the relationship between various shocks found to affect repo and Treasury markets. Table 1 summarizes the qualitative reaction of repo markets to the most salient shocks in the general collateral financing (GCF) repo[25] to IOR (corresponding to $r_t^p - r_t^m$ in the model); repo intermediation spreads ($r_t^p - r_t^{pt}$ in the model); RRP volumes (rrp_t in the model); and TGA balances (a_t in the model). This qualitative table is informed by previous studies and is consistent with the results of a linear regression replication exercise, which we report in Appendix B.[26]

[25] Bilateral repo rates between hedge funds and dealers are unfortunately not directly observed, but can be proxied by an index of inter-dealer repo rates that represents the rate at which dealers are willing to lend, referred to as GCF. We further follow He, Nagel, and Song (2022) and Correa, Du, and Liao (2020) and measure the intermediation spread as the difference between GCF and TGCR—an index of repo rates based on dealer-to-money-fund transactions, corresponding to the rate at which dealers are willing to borrow. Hence, this difference is considered a good indicator for repo intermediation margins.

[26] We relegate these regression tables to the Online Appendix and focus on a qualitative summary due to the highly nonlinear nature of the dynamic system we study which, combined with the limited

As documented by Munyan (2015); Du, Tepper, and Verdelhan (2018); Klingler and Syrstad (2021); Correa, Du, and Liao (2020); Paddrik, Young, Kahn, McCormick, and Nguyen (2023); and Bassi, Behn, Grill, and Waibel (2023), quarter-ends represent an important source of disruptions in repo markets because foreign dealer intermediation contracts on those days following window-dressing practices. Contrarily to the US, most foreign jurisdictions calculate the Basel III-mandated leverage ratios based on a snapshot of banks' balance sheets at quarter-ends rather than an average over all days of the quarter, as is the practice in the US. Foreign dealer banks tend to contract their balance sheets and cut back on low-yielding activities such as repo intermediation on those days to account for increased funding value adjustments (Andersen, Duffie, and Song, 2019). Accordingly, the first row of Table 1 shows that quarter-ends are associated with an increase in repo rates relative to the interest on reserves, as well as an increase in intermediation spreads, an increase in the Fed's reverse repo facility usage (RRP), and an increase in the TGA. Although Klingler and Syrstad (2021) find a consistent upward response for the US benchmark SOFR, they show that the pattern is reversed for the UK and the euro area, in which the central bank does not offer a reverse repo facility. Our model provides a natural explanation for this fact, which we discuss below. In addition, Correa, Du, and Liao (2020) makes use of confidential regulatory data to document that, at quarter ends, globally systemic banks engage in both matched-book intermediation—simultaneously increasing repo borrowing and repo lending—and reserves-draining intermediation—replacing reserves by repo lending. Those patterns also align with the observation of Pozsar (2019) that large banks act as "lender-of-next-to-last-resort" in repo markets by "fracking" reserves.

Several papers (Copeland, Duffie, and Yang, 2021; Correa, Du, and Liao, 2020; d'Avernas, Han, and Vandeweyer, 2023a) also point to tax deadlines as an important driver of market disruptions. In particular, as noted by Anbil, Anderson, and Senyuz (2021), the largest spike in repo rates on record took place on September 16 and 17, 2019, at the same time as a corporate tax deadline. On those dates, corporations withdraw cash from their money market accounts and credit the TGA to extinguish their tax obligations. These flows generate both a net decrease in repo supply and a decrease in reserves available to banks. The third row of Table 1 correspondingly shows larger repo rates near tax deadlines (Column 1) that are not accompanied by a significant increase in intermediation spread (Column 2) nor in volumes at the reverse repo facility (Column 3). As anticipated, the TGA balance increases on tax deadline days (Column 4).

Lastly, Treasury issuance has also been shown to be associated with repo market stresses (Correa, Du, and Liao, 2020; Klingler and Syrstad, 2021; Paddrik, Young, Kahn, McCormick, and Nguyen, 2023). When the Treasury issues additional debt securities, dealers and asset managers tend to purchase and finance those purchases in the repo market, thereby increasing the demand for repo. As can be seen in the fifth row of Table 1, the issuance of Treasuries is

number of observations for large shocks, does not allow for meaningful quantitative estimates. For the same reasons, we also abstain from attempting to calibrate our model. Rather, we focus on the qualitative heterogeneity in the reactions of spreads and volumes to various shocks to discipline our model.

indeed associated with increased repo rates (Column 1), increased repo intermediation spreads (Column 2), and larger balances in the TGA (Column 3). Consistent with our replication exercise, Correa, Du, and Liao (2020) find the variations of TGA balances driven by Treasury issuance to be positively associated with increased intermediation spreads whereas it is not the case for changes in TGA balances originating from other sources, such as tax payments.

In the next subsections, we compare model predictions with these empirical observations and discuss the mechanisms behind the heterogeneous patterns across shocks.

4.2 Theoretical Preliminaries

To study those shocks in the current institutional setting, we allow for endogenous adjustments to the central bank's balance sheet through reverse and standing repo facilities. To do so, we first provide some helpful preliminary theoretical results.

Lemma 2. *In an economy without facilities, liquidity services are maximized if and only if traditional banks are marginal lenders of repo; that is,* $h(w^p(\mathbf{x}), w^d(\mathbf{x}); \alpha) = \max_w h(w, \tau^h(\mathbf{x}) - w; \alpha)$ *if and only if* $r^p(\mathbf{x}) = r^m(\mathbf{x})$.

Lemma 2 shows that whenever traditional banks are net borrowers of repos or the IL constraint is binding, households deviate from their optimal allocation of liquid assets. In that case, repo rates diverge from the interest on reserves to compensate households for having to provide more or less repo funding than their optimal portfolio composition. This result is akin to the discussion of repo spikes in Section 3 but generalized beyond the flight-to-deposit shock α. Lemma 2 highlights that persistent deviations of repo rates from interest on reserves are a signal of suboptimal liquidity provision in the economy. In addition, we note that Lemma 1, which provides a threshold for the disconnection of repo rates from interest on reserves, does not depend on the type of shock studied and also applies to the general setting. In Lemma 3, we further characterize the effect of opening repo and reverse repo facilities. To allow for a detailed discussion of the design of facilities, we consider two types of standing repo facilities: a *broad-access* facility open to all agents (including shadow banks) and a *narrow-access* facility only open to traditional banks and their dealers.

Lemma 3. *A reverse repo facility with rate* r^{rrp} *acts as a floor in the triparty repo market:* $r^{pt}(\mathbf{x}) \geq r^{rrp}$. *A repo facility with rate* r^{rp} *that is open only to traditional banks acts as a ceiling in the triparty repo market:* $r^{pt}(\mathbf{x}) \leq r^{rp}$. *A broad-access repo facility open to both traditional and shadow banks acts as a ceiling in the bilateral repo market:* $r^p(\mathbf{x}) \leq r^{rp}$.

Lemma 3 presents specific results for various central bank facility designs. First, a reverse repo facility offers a fixed-rate outside option r^{rrp} for households when investing in repos. Since the dealers' funding rate (triparty repo rate r_t^{pt}) is consistently lower than the dealers' lending rate (bilateral rate: r_t^p) to compensate for balance sheet costs, this outside option functions as a floor for the triparty repo rate. Second, a narrow-access repo facility accessible only to traditional banks serves as a ceiling on the dealers' funding rate (triparty rate r_t^{pt}) as dealers would never borrow from households at a higher rate. Similarly, a broad-access repo facility that is available to shadow banks serves as a ceiling directly on the bilateral repo rate as shadow banks borrowing in the bilateral repo market would never borrow at a higher rate than the facility rate. We further discuss the implication of those Lemmas in the context of specific shocks below.

Below, we study shocks to (i) intermediation capacity f_t, (ii) tax payments a_t, or (iii) Treasury issuances b_t. As in the analysis of liquidity shocks α_t, for each shock, we solve for the equilibrium assuming that either f_t, a_t, or b_t is the only parameter that varies after an aggregate Poisson shock. We then display in Figure 7, 8, and 9 the equilibrium variables after such shocks away from the steady state. We relegate details regarding the solution method to Appendix C.

4.3 Intermediation Shocks

We begin by studying the effect of variations in foreign dealer intermediation volumes f_t. This exercise aims to capture the contraction of foreign dealer intermediaries at quarter ends (corresponding to the first row of Table 1) but also represent any outright increase in repo intermediation costs, which could arise from changes in regulation or negative shocks to bank equity. Figure 7 displays the mapping between key variables of the model and the level of foreign dealer intermediation shocks under three scenarios.

Baseline The red lines in Figure 8 correspond to a baseline scenario without repo facilities. As the foreign dealer sector contracts—i.e., moving leftward on the graphs—the triparty repo spread to IOR, $r_t^{pt} - r_t^m$, declines, which allows for an increase in intermediation spread, $r_t^p - r_t^{pt}$, as compensation for a larger marginal balance sheet cost for banks $\chi \ell_t$, while maintaining the bilateral repo rate equal to IOR, $r_t^p = r_t^m$. This no-arbitrage condition is driven by equation (24) when the IL constraint is not binding ($\vartheta_t^m = 0$). In this baseline case, all other variables remain constant, including households' liquidity benefits, because the shock does not prompt any portfolio adjustment from non-dealer agents.

Reverse Repo Facility The blue lines in Figure 8 illustrate the scenario with a reverse repo facility as introduced by the Fed in 2014. Following Lemma 3, the reverse repo facility establishes a lower bound on the triparty repo to IOR spread, $r_t^{pt} - r_t^m$. Upon reaching this floor, households start to lend repo directly to the Fed through the reverse repo facility at rate r^{rrp} (see RRP Quantity panel). When the triparty repo rate reaches the RRP floor, traditional banks begin lending in repo to shadow banks, thereby preventing bilateral repo rates from rising above IOR (see Traditional Bank Repo panel). This adjustment is made possible by the shift of households' portfolio into reverse repos with the Fed, consequently diminishing the quantity of reserves on traditional banks' balance sheets and generating room for repo lending. As noted by Diamond, Jiang, and Ma (2023), by occupying space on banks' balance sheets, reserves crowd out potential lending opportunities. In our model, the decrease in reserves allows traditional banks to lend in repos and prevents repo spikes. The model thereby underscores that the "reserves-draining" repo lending from banks observed at quarter ends by Correa, Du, and Liao (2020) is a side-product of the concurrent surge in reverse repo facility volumes, as reserves cannot leave traditional banks' balance sheets otherwise. However, traditional banks can lend in repo only until the IL constraint becomes binding—the treshold f^{IL} (see Traditional Bank Repo panel). Beyond that point, the reserve quantity on banks' balance sheets restricts their repo lending capacity, resulting in the bilateral repo rate r_t^p rising above IOR as compensation to households for having to lend more in repo (see Bilateral Repo Spread panel). In this situation, it is the bilateral repo rate that rises above the IOR because traditional banks can no longer arbitrage the repo to IOR spread $r_t^p - r_t^m$, while the triparty rate r_t^{pt} is bounded below by the reverse repo facility rate r^{rrp}. In comparison to the baseline scenario without a reverse repo facility, this finding is consistent with the fact that upward repo spikes only started appearing after the introduction of the facility in the US (row 1 of Table 1) and that other jurisdictions without such a facility are observing downward, rather than upward, shocks to repo spreads at quarter ends (Corradin, Eisenschmidt, Hoerova, Linzert, Schepens, and Sigaux, 2020; Klingler and Syrstad, 2021).

Following this reasoning, Proposition 6 demonstrates that an intermediation shock requires the strict combination of three frictions to generate a repo spike: a binding IL, a positive balance sheet cost, and a reverse repo facility.

Proposition 6. *Given*

$$b - (1 + \kappa)\underline{b} + \kappa a - w_t^b \leq (1 - \alpha)\tau^h, \tag{36}$$

the bilateral repo rate is above the interest on reserves, $r_t^p > r_t^m$, if and only if (i) the IL constraint is binding $\vartheta_t^m > 0$, (ii) balance sheet costs are positive $\chi > 0$, and (iii) the RRP facility rate is binding $rrp_t > 0$.

Condition (36) guarantees that the baseline demand for repos by shadow banks (when $r_t^{pt} = r_t^d$ and liquidity services to households are at their optimum) is not already above the capacity of the system unrestricted by the IL constraint. The role of the reverse repo facility as a necessary condition for upward bilateral repo spikes is particularly noteworthy. In its absence, the intermediation shock is absorbed by a continuous decrease in triparty rates. By establishing a lower bound on triparty rates, the central bank introduces a market distortion by subsidizing triparty repo markets through the direct provision of repo assets. This subsidy is a necessary condition for causing excessive household portfolio allocation to repo when the IL constraint becomes binding, as seen in the Liquidity Services panel. Overall, the model matches and rationalizes the empirical pattern documented in the first row of Table 1. The only exception is to TGA balances, which is due to our model orthogonalizing the effect of a change in the TGA.[27]

Standing Repo Facility Last, the yellow lines in Figure 8 illustrate a scenario with both a reverse repo and a broad-access standing repo facility. Following Lemma 3, the design of a standing repo facility is key to its efficacy in preventing repo spikes. With broad access, the bilateral repo rate does not surge above the repo facility rate (see Bilateral Repo Spread panel). This result requires the central bank to effectively act as *dealer of last resort* by simultaneously borrowing from households in triparty repo markets and lending to shadow banks in the bilateral repo, thereby economizing on dealers' balance sheets. Notably, in that case, both the ceiling and the floor on repo market rates are active simultaneously (see RRP Quantity, RP Quantity, Bilateral Repo Spread, and Triparty Repo Spread panels). In contrast, a repo facility that is inaccessible to shadow banks is not effective in preventing repo spikes following an intermediation shock. This result follows from the nature of the intermediation shock as an encumbrance of dealers' balance sheets. In that case, the narrow-access repo facility cannot alleviate the shock as repos borrowed from the central bank still need to be intermediated by dealers and require the same amount of balance sheet space. This finding echoes arguments by Duffie, Geithner, Parkinson, and Stein (2022) in favor of broadening access to the Fed's standing repo facility.[28]

4.4 Tax Payment Shocks

We now study the effect of a tax payment shock that simultaneously reduces households' tax liabilities τ_t^h and increases TGA a_t (corresponding to tax deadlines in the second row of Table 1). The dynamics of tax deadline adjustments are illustrated in Figure 8.

Baseline We start with a baseline case without facilities (red lines in Figure 8). In contrast to a repo intermediation shock, a tax deadline shock is a net repo supply contraction that does not lead to an increase in the repo intermediation spread ($r_t^p - r_t^{pt}$). Indeed, balance sheet costs are decreasing in TGA balances. The reduction in repo supply from households exerts simultaneous upward pressure on both bilateral and triparty repo rates up until reaching the threshold $a^{\mathcal{F}}$ at which shadow banks prefer to fire-sell their Treasuries. As for the flight-to-deposit shock, this outside option limits how high the repo rate can spike.

[27]In practice, quarter ends also correspond to high Treasury issuance periods and result in surges in TGA balances with the additional effect discussed in section 4.4 below.

[28]"Without direct access, central bank liquidity support for the US Treasury market, which relies heavily on nonbank financial intermediaries for market liquidity, has been excessively dependent on the willingness and ability of the primary dealers to intermediate between the Federal Reserve and those other nonbank financial intermediaries. In principle, banks and the primary dealers can obtain liquidity from the Fed and on-lend it to others. But in a crisis, the willingness of those firms to intermediate has tended to be constrained by a combination of regulations and internal limits on the firms' risk-taking, both of which have tended to tighten under stress." Duffie, Geithner, Parkinson, and Stein (2022, pp.7)

Concomitantly to the reduction of repo supply, the payment of taxes by households into the TGA reduces the supply of reserves to banks. Similar to the intermediation shock, the sterilization of the reserves (here, in the TGA) initially allows banks to lend more to shadow banks through "reserves-draining" intermediation (Correa, Du, and Liao, 2020). This drawdown of reserves however becomes problematic once reaching the threshold a^{IL} at which the IL constraint becomes binding. From that point onward, tighter intraday regulations restrict traditional banks' capacity to lend in repo and exacerbate the repo supply shock, causing an upward spike in bilateral repo rates (see Bilateral Repo Spread panel). This mechanism corresponds to the events of September 2019, which took place during a tax deadline (Anbil, Anderson, and Senyuz, 2021). In addition, note that in this scenario, due to the net reduction in repo supply from households and the subsequent increase in triparty repo rates, the reverse repo facility does not come into play.

Standing Repo Facility In contrast to the intermediation shock, we find that a narrow-access repo facility that is not available directly to shadow banks is effective in preventing sharp repo rate spikes (yellow lines in Figure 8). This discrepancy arises because, during a tax deadline shock, when traditional banks borrow from the central bank to lend to shadow banks in repo, they simply replace the loss in repo funding stemming from households having paid their taxes. Therefore, they are able to lend to shadow banks without expanding their balance sheets. With a broad-access repo facility, shadow banks can directly borrow from the central bank instead, and thereby economize traditional banks' balance sheets, which renders it a more efficient tool (blue lines in Figure 8). Because the broad-access facility is directly accessible to shadow banks, it becomes active earlier than a narrow-access facility, which sets a ceiling on the triparty repo rate rather than on bilateral rates.

4.5 Treasury Issuance Shocks

We next investigate the impact of fiscal expansion shocks on repo and Treasury markets, with the proceeds of Treasury issuance entirely absorbed by the TGA account, as illustrated in Figure 9. For this shock, we further assume that shadow banks purchasing from the Treasury in primary markets are not subject to transaction costs to account for the special relationship between primary dealers and the Treasury in the US.

Baseline In the baseline case without facilities (red lines), an increase in outstanding Treasuries increases repo demand since, in equilibrium, additional Treasuries are purchased by shadow banks and financed with repos (see Shadow Bank Treasuries panel). Similarly to the tax deadline shocks, the increase in Treasury issuance increases the TGA, resulting in a reduction of the reserves available to traditional banks (see Reserves panel). As for the previous shocks, this drawdown in reserves is initially helpful as it allows banks to increase repo lending without impacting their balance sheet costs. However, once reaching the IL threshold b^{IL}, the increasing demand for repo financing from shadow banks exerts upward pressure on both bilateral and triparty repo rates. Indeed, the market clearing conditions now require households to supply more repo and deviate from their optimal portfolio allocation. Lastly, the reverse repo facility never comes into play following a fiscal shock (see RRP Quantity panel). Taken together, these predictions correspond to the third row of Table 1 and the empirical observations of Klingler and Syrstad (2021) and Correa, Du, and Liao (2020).

An important difference between fiscal shocks and both intermediation and tax deadlines shocks, however, is that a fiscal expansion shock (absorbed entirely in the TGA) does not increase the balance sheet cost of banks because the increase in the repo dealer subsidiary's balance sheet is entirely compensated by a reduction in the traditional bank's balance sheet. This result is a side-product of the adjustment to the TGA account that is assumed in this subsection. In contrast, we provide in Figure 10, a graphical representation of the case in which the newly issued Treasuries result in an increase in tax liabilities from households rather than an increase in the TGA account. This scenario corresponds to longer-term adjustments when the Treasury uses the proceeds of the Treasury issuances to pay for public spending and the salary of public servants and other public spending. In this second case, the fiscal shock also results in an increase in the balance sheet cost as the requirement for repo intermediation is increased.

Standing Repo Facility As with the previous shocks, the presence of a broad-access repo facility effectively caps repo rate spikes and prevents fire sales of Treasuries (blue lines in Figure 9). For a Treasury issuance shock beyond b^{BRP}, shadow banks fund their new Treasury purchases with repo loans from the central bank (see RP Quantity panel). Consequently, households do not need to make further adjustments to their repo supply, thereby stabilizing both the bilateral and triparty repo rates. With a narrow-access repo facility only available to traditional banks (yellow lines in Figure 9), dealers can borrow at the repo facility rate r^{nrp}, imposing an upper bound on the triparty repo spread as dealers would refuse to borrow at a higher rate from households (see Triparty Repo Spread panel). However, two main drawbacks render the narrow-access repo facility less effective compared to the broad-access facility. First, for the same facility rate, the narrow-access facility only comes into play for a larger shock as it acts as a ceiling on the triparty rate and not directly on the bilateral rate, similar to the tax deadline shock. Second, because the narrow-access facility requires the intermediation of dealers to reach shadow banks, it is accompanied by an increase in balance sheet costs. Thus, the bilateral repo rate is not bounded by the facility rate because the increase in balance sheet costs requires a corresponding surge in the intermediation spread. The facility is nonetheless useful in slowing down the rate of increase in the bilateral repo rate and delaying the point at which the system enters the fire-sale region (see Bilateral Repo Spread and Liquidity Services panels).

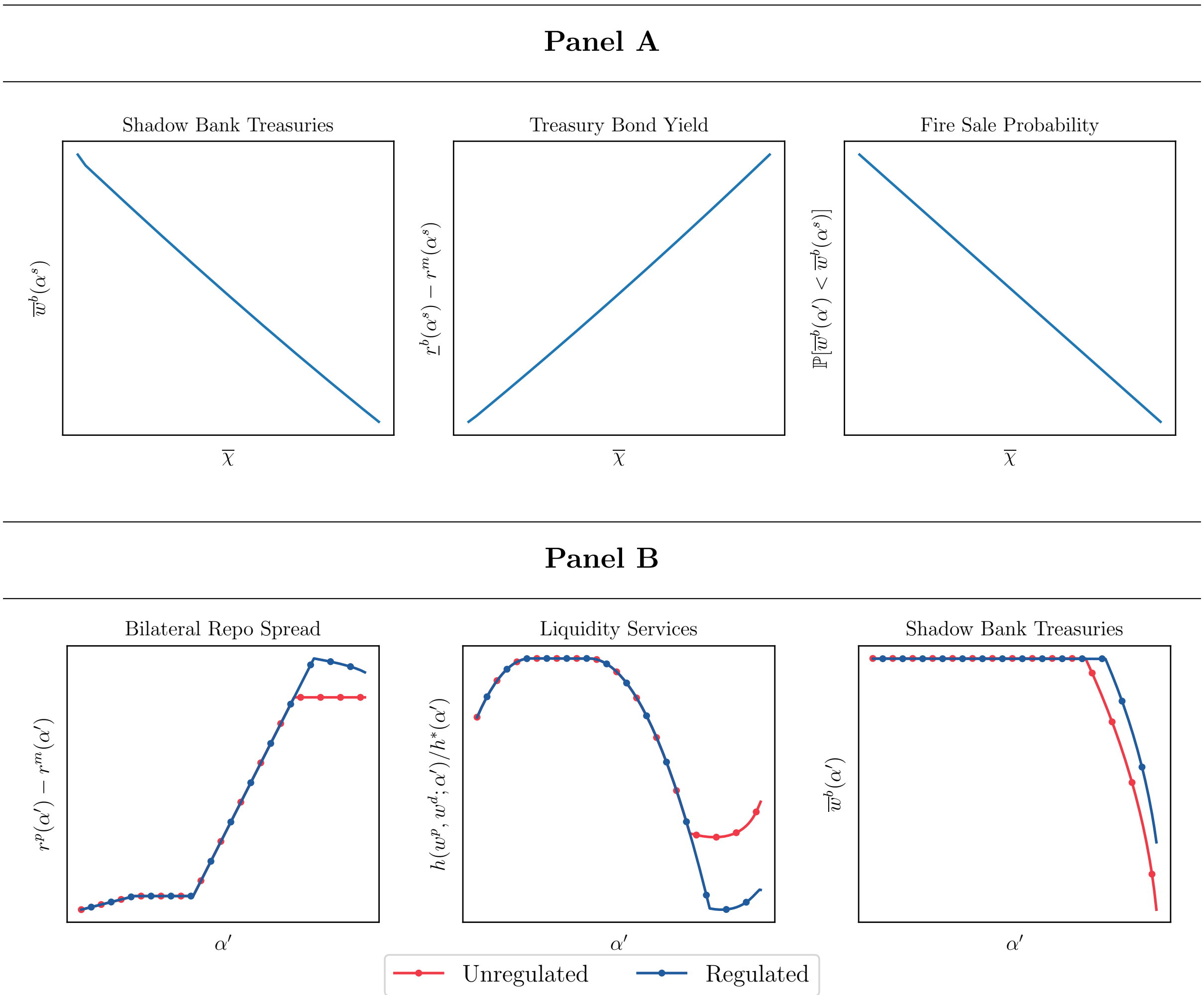

Figure 6: Increased Shadow Banks' Regulatory Balance Sheet Costs. Panel A displays steady-state shadow banks' Treasury holdings $\overline{w}^b(\alpha^s)$, Treasury yields $\underline{r}^b(\alpha^s) - r^m(\alpha^s)$, and fire-sale probability $\mathbb{P}[\overline{w}^b(\alpha') < \overline{w}^b(\alpha^s)]$ as a function of shadow banks' regulatory balance sheet cost parameter $\overline{\chi}$. Panel B displays the bilateral repo spread $r^p(\alpha') - r^m(\alpha')$, liquidity services to its optimum $h(w^p, w^d; \alpha')/h^\star(\alpha')$ where $h^\star(\alpha') \equiv \max_w h(w, \tau^h - w; \alpha')$, and shadow banks' treasury holdings $\overline{w}^b(\alpha')$ for the unregulated $(\overline{\chi} = 0)$ and regulated $(\overline{\chi} > 0)$ equilibria.

4.6 Counterfactual Exercise

In this section, we make use of the model to derive the counterfactual reaction of the repo and Treasury markets to shocks under alternative regulatory frameworks.

Excluding Treasuries and Reserves from Balance Sheet Cost We firs study the effect of removing both reserves and Treasuries from banks' balance sheet costs. In our model, removing both reserves and Treasuries from the balance sheet cost amounts to setting the balance sheet cost parameter χ to zero. In that case, shadow banks do not have any comparative advantage for holding Treasuries, which means that traditional banks become the marginal holders of Treasuries and both repo and Treasury markets become fully stable. Such a policy of removing both Treasuries and Reserves from SLR calculations was implemented by the Fed in the US between April 2020 and March 2021. Consistently with our model, a series of empirical works have investigated the impact of the measure and found a positive impact on repo and Treasury market stability (Favara, Infante, and Rezende, 2022; Koont and Walz, 2021; Kroen, 2022). Although this policy appears unambiguously positive in our model, it is not equipped to tackle the full normative question, as the regulation is modeled in a reduced form to focus on the positive questions relating Treasury market disruptions to the central bank's balance sheet.

Proposition 7. *When there is no balance sheet cost, $\chi = 0$, shadow banks never fire sell their Treasuries: $\overline{w}^b(\mathbf{x}') \geq \overline{w}^b(\mathbf{x}^s)$ for all $\mathbf{x}'$.*

Regulating Shadow Banks Next, we consider counterfactual regulatory changes according to which hedge funds involved in Treasury basis trades are more heavily regulated. Two additional proposals for regulation are currently being discussed by regulators: requiring hedge funds involved in Treasury bases trades to be classified as dealers (and thereby be regulated as such) or requiring that hedge funds borrow in centrally cleared repo. Because centrally cleared repos have stricter margin requirements,[29] both regulatory proposals amount to limiting hedge funds' ability to leverage, albeit to different magnitudes. Such regulatory changes correspond to adding a quadratic balance sheet cost with modulating parameter $\overline{\chi}$ to shadow banks. Figure 6 Panel A displays comparative statics results for various values of $\overline{\chi}$ and Figure 6 Panel B displays liquidity shock equilibria with ($\overline{\chi} > 0$) and without ($\overline{\chi} = 0$) shadow bank regulation. As can be seen, an increase in shadow banks' regulation results in shadow banks reducing their leveraged holdings of Treasury bonds, as well as increased steady-state Treasury yields, and reduced expectations of Treasury fire sales in the crisis state. In other words, by regulating shadow banks, the government reduces the likelihood of observing fire sales during liquidity events at the cost of paying higher yields on its debt in the steady state. Our model thereby suggests that the policy discussions on this topic should consider both sides of this trade-off.

5 Conclusion

This article proposes a dynamic model of the Treasury market that captures disruptions observed over recent years. It emphasizes the role of the central bank's balance sheet, portfolio allocations, and regulatory frictions in Treasury market stability. In particular, our framework identifies

[29] Barth, Kahn, and Mann (2023) find that most uncleared repos are applied zero margins, which renders leverage virtually unconstrained. Note that under current regulations, cleared repos can be netted for regulatory metrics purposes. This would amount to reducing the balance sheet cost from dealer intermediation. However, the fact that most hedge funds choose not to rely on this option if not mandated to is suggestive that they find the zero margins to be more attractive than borrowing at a lower repo cost. We consider this as evidence that such a requirement would result in a net increase in the balance sheet cost of the shadow banks, even accounting for reduced intermediation spreads.

the frictions to explain these disruptions, stresses the key role of both sides of the central bank balance sheet, highlights the dual nature of reserve drawdowns, and investigates the effectiveness of repo facilities. To allow for tractable exposition, our framework nonetheless abstracts from potential interactions with other markets. Exploring those interactions is left to future research. For instance, our model could serve as a starting point to study the financing of international investors of dollar assets through FX derivatives, also subject to intermittent disruptions as pointed out by Correa, Du, and Liao (2020). Overall, this work provides theoretical foundations for research on the sources of capital market instability stemming from money market disruptions with implications for the optimal design of regulation and monetary policy implementation frameworks.

6 Relegated Figures

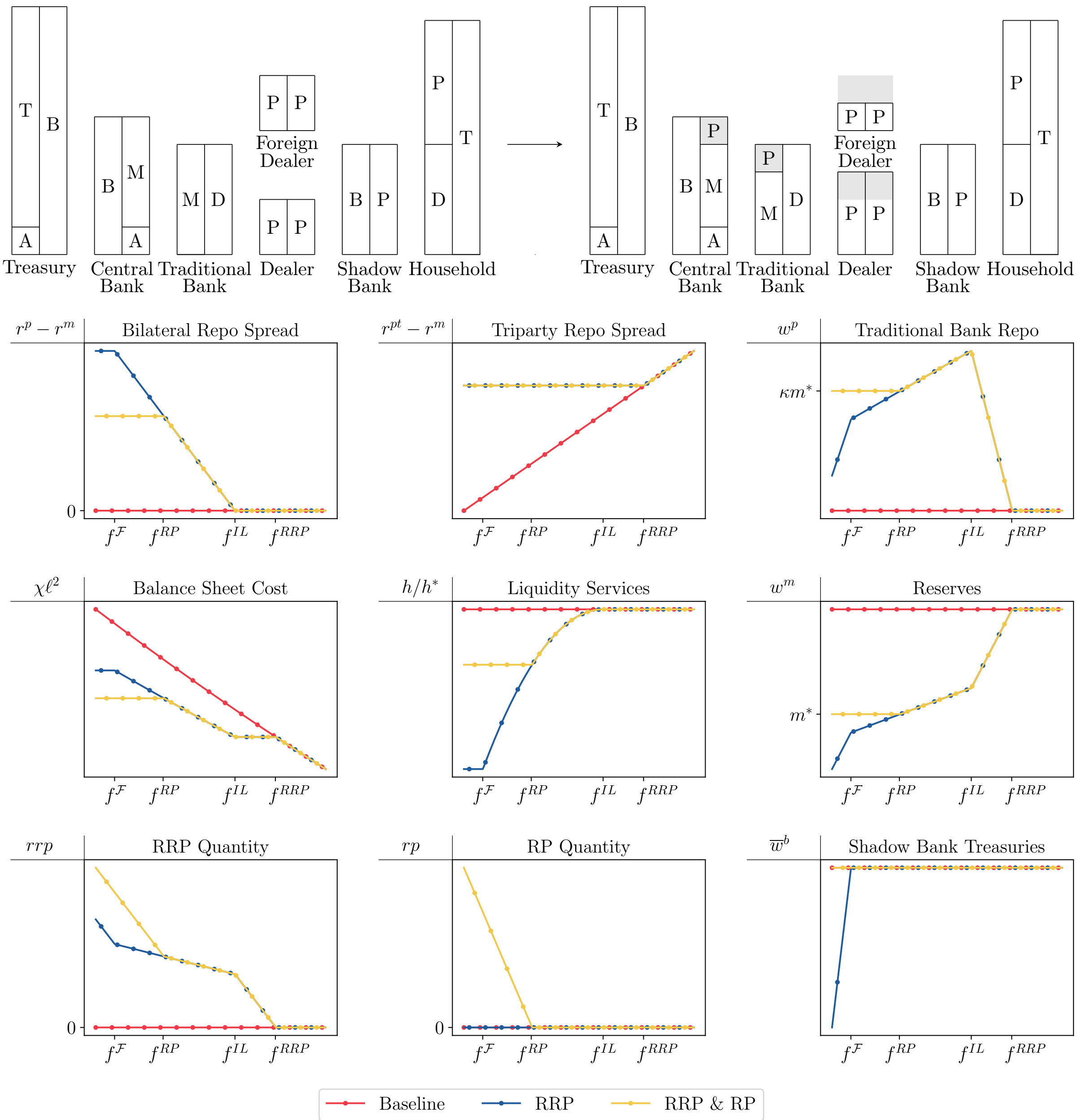

Figure 7: Intermediation Shock. The top panels illustrate the impact of an intermediation shock, such as foreign dealer window-dressing at quarter-ends, on the balance sheets of the economy. T denotes the present value of future taxes, A the TGA, B Treasury bonds, M reserves, P repo, and D deposits. A negative shock to intermediation corresponds to a reduction in f on the x-axis of the figures. On the x-axis of the figures, f^{RRP} refers to the threshold at which the reverse facility rate is binding, f^{IL} refers to the threshold at which the IL constraint is binding, and f^{RP} refers to the threshold at which the repo facility rate is binding. We show the dynamics for three scenarios: without facilities (red), with a reverse repo facility (blue), and with a reverse repo facility and a broad-access repo facility (yellow).

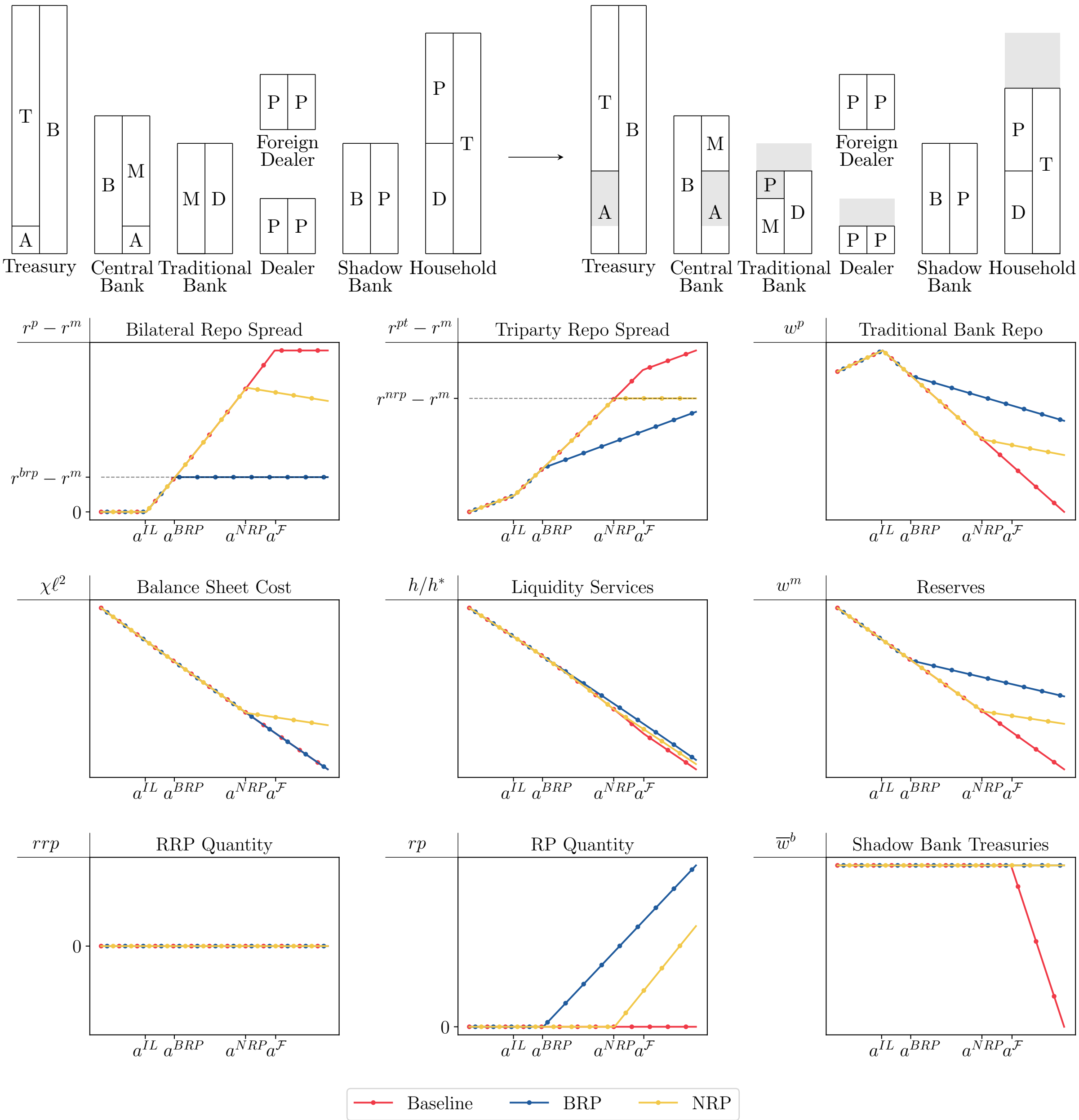

Figure 8: Tax Deadline Shock. The top panels illustrate the impact of a tax deadline shock. T denotes the present value of future taxes, A the TGA, B Treasury bonds, M reserves, P repo, and D deposits. A positive shock to the Treasury General Account (TGA) corresponds to an increase in a on the x-axis of the figures. On the x-axis of the figures, a^{IL} refers to the threshold at which the IL constraint is binding, a^{BRP} refers to the threshold at which the broad-access repo facility rate is binding, a^{NRP} refers to the threshold at which the narrow-access repo facility rate is binding, and $a^{\mathcal{F}}$ refers to the threshold at which shadow banks start selling Treasuries in the baseline scenario. We show the dynamics for three scenarios: without a repo facility (red), with a broad-access repo facility (blue), and with a narrow-access repo facility (yellow).

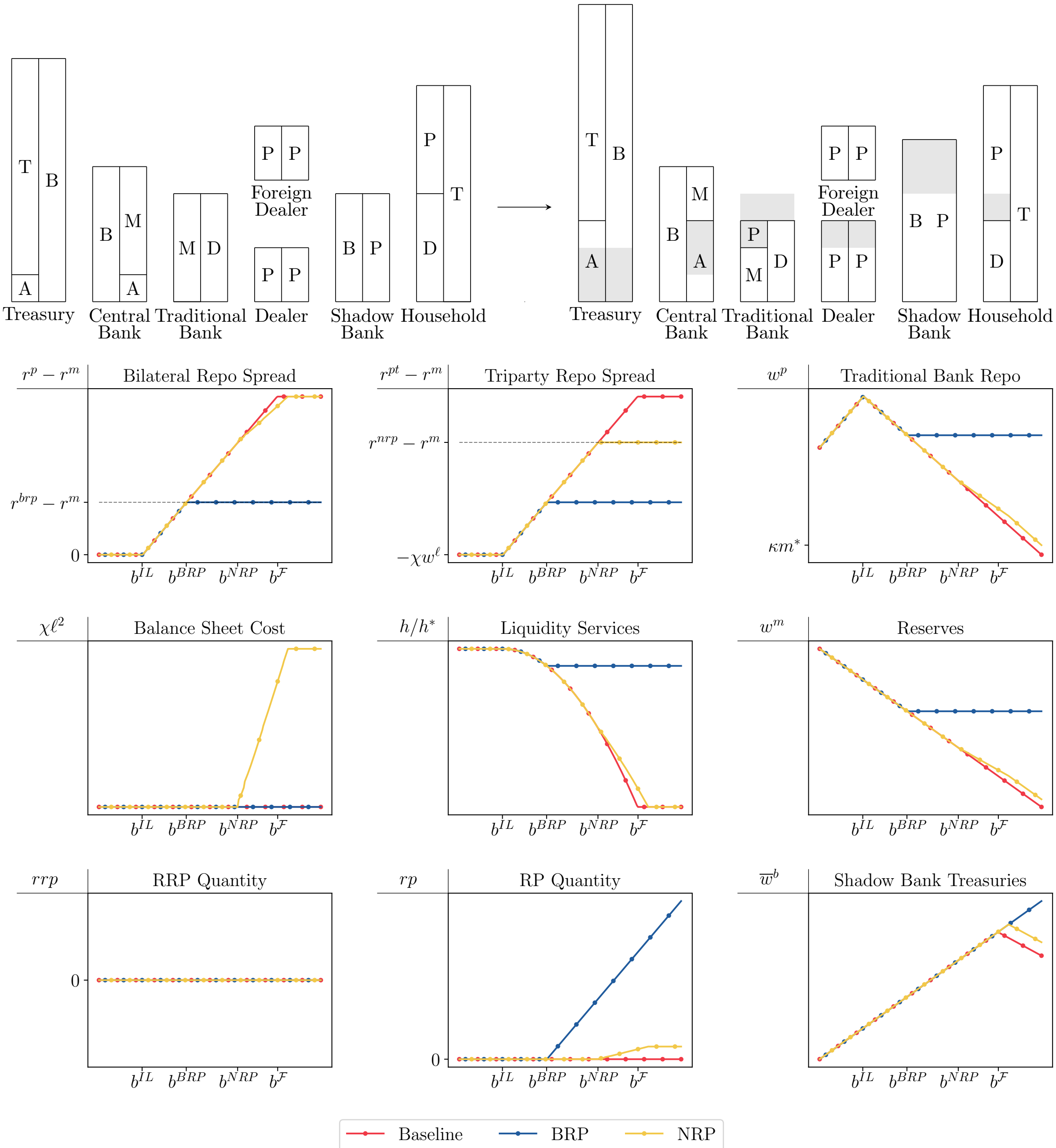

Figure 9: Fiscal Shock. The top panels illustrate the impact of a fiscal shock (issuance of Treasury bonds). T denotes the present value of future tax, B denotes Treasury bonds, N denotes net worth, M denotes reserves, K denotes capital, P denotes repo, and D denotes deposits. An increase in Treasuries corresponds to an increase in b on the x-axis of the figures. On the x-axis of the figures, b^{IL} refers to the threshold at which the IL constraint is binding, b^{BRP} refers to the threshold at which the broad-access repo facility rate is binding, b^{NRP} refers to the threshold at which the narrow-access repo facility rate is binding, and $b^{\mathcal{F}}$ refers to the threshold at which traditional banks begin purchasing Treasury bonds. We show the dynamics for three scenarios: without a repo facility (red), with a broad-access repo facility (blue), and with a narrow-access repo facility (yellow).

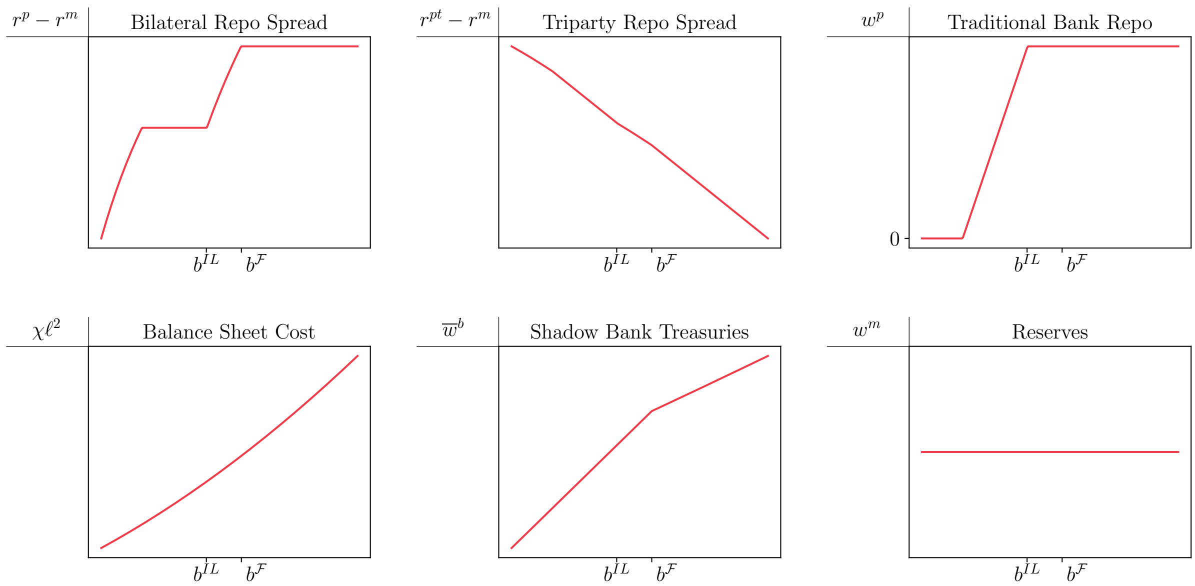

Figure 10: Fiscal Shock - Adjusting Tax Liabilities. A positive fiscal shock (Treasury issuance) corresponds to an increase in b on the x-axis of the figures. On the x-axis of the figures, b^{IL} refers to the threshold at which the IL constraint is binding and $b^{\mathcal{F}}$ refers to the threshold at which traditional banks begin purchasing Treasuries. Only the baseline (no central bank facilities) is displayed.

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Appendices

A Solving the Model

Notation and State Space Given our assumption on the law of motion of $\mathbf{x}_t$, the functional form of the transaction cost, and aggregate wealth dynamics, equilibrium prices are only a function of $\mathbf{x}_t$. In the following proofs, we rewrite agents' problems in recursive form and drop the time subscript for ease of notation. We denote by $f(\cdot)$ the distribution of $\mathbf{x}'$ given the arrival of a Poisson shock from the steady state $\mathbf{x}^s$.

First, we guess and verify that the value functions have the following form:

$$V(w^b;\mathbf{x}) = \xi(\mathbf{x}) + \theta(\mathbf{x})w^b, \tag{37}$$
$$\overline{V}(\overline{w}^b;\mathbf{x}) = \overline{\xi}(\mathbf{x}) + \overline{\theta}(\mathbf{x})\overline{w}^b, \tag{38}$$
$$V^h(\mathbf{x}) = \xi^h(\mathbf{x}). \tag{39}$$

Shadow Banks We can write the HJB for shadow banks as

$$\overline{V}(\overline{w}^b_-;\mathbf{x}) = \max_{\overline{w}^b}\Big\{\overline{w}^b_-(r^b(\mathbf{x}) - r^p(\mathbf{x}))dt + (1-\rho dt)(1-\lambda dt)\mathbb{E}_t\big[\overline{V}(\overline{w}^b;\mathbf{x}+d\mathbf{x})|dN=0\big] + (1-\rho dt)\lambda(\mathbf{x})dt\mathbb{E}\big[\overline{V}(\overline{w}^b;\mathbf{x}+d\mathbf{x})|dN=1\big]\Big\}. \tag{40}$$

Using Ito's lemma, the law of motion for $\mathbf{x}$, and the law of motion for $\overline{\pi}$, we can rewrite the HJB in equation (40) as

$$(\rho+\lambda(\mathbf{x}))\overline{V}(\overline{w}^b(\mathbf{x});\mathbf{x}) = \overline{w}^b(\mathbf{x})(r^b(\mathbf{x}) - r^p(\mathbf{x})) + \lambda(\mathbf{x})\int\Big(\overline{V}(\overline{w}^b(\mathbf{x}');\mathbf{x}') - \nu|\overline{w}^b(\mathbf{x}') - \overline{w}^b(\mathbf{x})|\Big)f(\mathbf{x}')d\mathbf{x}'. \tag{41}$$

Substituting with the guess for $\overline{V}$ obtains

$$(\rho+\lambda(\mathbf{x}))\overline{V}(\overline{w}^b(\mathbf{x});\mathbf{x}) = \overline{w}^b(\mathbf{x})(r^b(\mathbf{x}) - r^p(\mathbf{x})) + \lambda(\mathbf{x})\int\Big(\overline{\xi}(\mathbf{x}') + \overline{\theta}(\mathbf{x}')\overline{w}^b(\mathbf{x}') - \nu|\overline{w}^b(\mathbf{x}') - \overline{w}^b(\mathbf{x})|\Big)f(\mathbf{x}')d\mathbf{x}'. \tag{42}$$

As banks can adjust their holdings of Treasuries instantaneously by paying the transaction cost, the value function given $\overline{w}^b_-$ must be equal to the value that would obtain by changing the debt level to the optimum, which we denote $\overline{w}^{b\star}(\overline{w}^b_-;\mathbf{x})$:

$$\overline{V}(\overline{w}^b_-;\mathbf{x}) = \max_{\overline{w}^b}\Big\{\overline{V}(\overline{w}^b;\mathbf{x}) - \nu|\overline{w}^b - \overline{w}^b_-|\Big\} \tag{43}$$
$$= \overline{V}(\overline{w}^{b\star}(\overline{w}^b_-;\mathbf{x});\mathbf{x}) - \nu|\overline{w}^{b\star}(\overline{w}^b_-;\mathbf{x}) - \overline{w}^b_-|. \tag{44}$$

For ease of notation, we sometimes use the short notation $\overline{w}^{b\star} \equiv \overline{w}^{b\star}(\overline{w}^b_-;\mathbf{x})$. Thus, $\overline{w}^{b\star}$ is determined by

$$\overline{V}_w(\overline{w}^{b\star};\mathbf{x}) = \nu\,\mathrm{sign}(\overline{w}^{b\star} - \overline{w}^b_-) \quad \text{if } \overline{w}^{b\star} \neq -\overline{w}^b_-, \tag{45}$$

where we use the notation $f_x = \partial f/\partial x$ for partial derivatives. Substituting for the guess for $\overline{V}$, we get

$$\overline{\theta}(\mathbf{x}) = \nu\,\mathrm{sign}(\overline{w}^{b\star} - \overline{w}^{b}_{-}) \quad \text{if } \overline{w}^{b\star} \neq \overline{w}^{b}_{-}. \tag{46}$$

Then, we can write the optimal weight on Treasuries as follows:

$$\overline{w}^{b\star}(\overline{w}^{b}_{-};\mathbf{x}) = \begin{cases} 0 & \text{if } \ \overline{\theta}(\mathbf{x}) < -\nu \\ [0, \overline{w}^{b}_{-}] & \text{if } \ \overline{\theta}(\mathbf{x}) = -\nu, \\ \overline{w}^{b}_{-} & \text{if } \ -\nu < \overline{\theta}(\mathbf{x}) < \nu, \\ [\overline{w}^{b}_{-}, \infty] & \text{if } \ \overline{\theta}(\mathbf{x}) = \nu. \end{cases} \tag{47}$$

We omit the case for $\overline{\theta}(\mathbf{x}) > \nu$, as this would lead to an infinite holding of Treasuries, which is infeasible in equilibrium.

Using the envelope theorem with respect to w^b, we get

$$\begin{aligned}(\rho + \lambda(\mathbf{x}))\overline{\theta}(\mathbf{x}) =& r^b(\mathbf{x}) - r^p(\mathbf{x}) \\ &+ \lambda(\mathbf{x})\frac{\partial}{\partial \overline{w}^b(\mathbf{x})}\int \left(\overline{\theta}(\mathbf{x}')\overline{w}^b(\mathbf{x}') - \nu|\overline{w}^b(\mathbf{x}') - \overline{w}^b(\mathbf{x})|\right) f(\mathbf{x}')d\mathbf{x}'.\end{aligned} \tag{48}$$

For $\mathbf{x} = \mathbf{x}^s$, this becomes

$$\begin{aligned}(\rho + \lambda)\overline{\theta}(\mathbf{x}^s) =& r^b(\mathbf{x}^s) - r^p(\mathbf{x}^s) \\ &+ \lambda\nu \int_{\overline{w}^b(\mathbf{x}')\neq \overline{w}^b(\mathbf{x}^s)} \mathrm{sign}(\overline{w}^b(\mathbf{x}') - \overline{w}^b(\mathbf{x}^s)) f(\mathbf{x}')d\mathbf{x}' \\ &+ \lambda \int_{\overline{w}^b(\mathbf{x}') = \overline{w}^b(\mathbf{x}^s)} \overline{\theta}(\mathbf{x}') f(\mathbf{x}')d\mathbf{x}'.\end{aligned} \tag{49}$$

For $\mathbf{x} \neq \mathbf{x}^s$, since after a Poisson shock we return to the steady state $\mathbf{x}^s$, we get

$$\begin{aligned}(\rho + \lambda')\overline{\theta}(\mathbf{x}) =& r^b(\mathbf{x}) - r^p(\mathbf{x}) \\ &+ \lambda'\nu\,\mathrm{sign}(\overline{w}^b(\mathbf{x}^s) - \overline{w}^b(\mathbf{x}))\mathbb{1}\{\overline{w}^b(\mathbf{x}^s) \neq \overline{w}^b(\mathbf{x})\} \\ &+ \lambda'\overline{\theta}(\mathbf{x}^s)\mathbb{1}\{\overline{w}^b(\mathbf{x}^s) = \overline{w}^b(\mathbf{x})\}.\end{aligned} \tag{50}$$

Traditional Banks Similarly, we can write the HJB for traditional banks as

$$\begin{aligned}V(w^b_{-}(\mathbf{x});\mathbf{x}) = \max_{w^k,w^b,w^m,w^d,w^p} &\Big\{\mu(\mathbf{x})dt \\ &+ (1-\rho dt)(1-\lambda(\mathbf{x})dt)\mathbb{E}_t\big[V(w^b;\mathbf{x}+d\mathbf{x})|dN = 0\big] \\ &+ (1-\rho dt)\lambda(\mathbf{x})dt\mathbb{E}\big[V(w^b;\mathbf{x}+d\mathbf{x})|dN = 1\big]\Big\}\end{aligned} \tag{51}$$

where

$$\begin{aligned}\mu(\mathbf{x}) \equiv& w^k(\mathbf{x})r^k + w^b(\mathbf{x})r^b(\mathbf{x}) + w^m(\mathbf{x})r^m(\mathbf{x}) + w^p(\mathbf{x})r^p(\mathbf{x}) - w^d(\mathbf{x})r^d(\mathbf{x}) \\ &+ w^x(r^p(\mathbf{x}) - r^{pt}(\mathbf{x})) - \frac{\chi}{2}\ell(\mathbf{x})^2\end{aligned} \tag{52}$$

and such that $w^k + w^b + w^m + w^p = w^d$ and

$$w^p \leq \kappa w^m, \tag{53}$$

where

$$\ell \equiv w^d + w^x - \min\{0, w^p\}. \tag{54}$$

As before, the value function given w^b_- must be equal to the value that would obtain by changing the Treasury bond holdings to the optimum, which we denote $w^{b\star}(w^b_-; \mathbf{x})$. For ease of notation, we sometimes use the short notation $w^{b\star} \equiv w^{b\star}(w^b_-; \mathbf{x})$. That is,

$$V(w^b_-; \mathbf{x}) = \max_{w^b \geq 0} \left\{ V(w^b; \mathbf{x}) - \nu |w^b - w^b_-| \right\} \tag{55}$$

$$= V(w^{b\star}; \mathbf{x}) - \nu |w^{b\star} - w^b_-|. \tag{56}$$

Thus, substituting for the guess for V, we get

$$\theta(\mathbf{x}) = \nu \operatorname{sign}(w^{b\star} - w^b_-) \quad \text{if } w^{b\star} \neq w^b_-. \tag{57}$$

Thus,

$$w^{b\star}(w^b_-; \mathbf{x}) = \begin{cases} 0 & \text{if} \quad \theta(\mathbf{x}) < -\nu \\ [0, w^b_-] & \text{if} \quad \theta(\mathbf{x}) = -\nu, \\ w^b_- & \text{if} \quad -\nu < \theta(\mathbf{x}) < \nu, \\ [w^b_-, \infty] & \text{if} \quad \theta(\mathbf{x}) = \nu. \end{cases} \tag{58}$$

Using the same steps as for the shadow banks and substitute with the guess for V obtains

$$\begin{aligned} (\rho + \lambda(\mathbf{x}))V(w^b(\mathbf{x}); \mathbf{x}) = \mu(\mathbf{x}) \\ + \lambda(\mathbf{x}) \int \left(\xi(\mathbf{x}') + \theta(\mathbf{x}') w^b(\mathbf{x}') - \nu |w^b(\mathbf{x}') - w^b(\mathbf{x})| \right) f(\mathbf{x}') d\mathbf{x}' \\ + \vartheta^m(\mathbf{x})(\kappa w^m(\mathbf{x}) - w^p(\mathbf{x})), \end{aligned} \tag{59}$$

where $\vartheta^m(\mathbf{x})$ is the Lagrange multiplier on the constraint $\kappa w^m(\mathbf{x}) \geq w^p(\mathbf{x})$. Thus, given that in equilibrium $w^d > 0$, the first-order condition for c, w^k, w^m, w^x, and w^p are given by

$$r^k - r^d(\mathbf{x}) = \chi \ell(\mathbf{x}), \tag{60}$$

$$r^m(\mathbf{x}) - r^d(\mathbf{x}) = \chi \ell(\mathbf{x}) - \kappa \vartheta^m(\mathbf{x}), \tag{61}$$

$$r^p(\mathbf{x}) - r^{pt}(\mathbf{x}) = \chi \ell(\mathbf{x}), \tag{62}$$

$$r^p(\mathbf{x}) - r^d(\mathbf{x}) = \begin{cases} \chi \ell(\mathbf{x}) + \vartheta^m(\mathbf{x}) & \text{if } w^p(\mathbf{x}) > 0, \\ 0 & \text{if } w^p(\mathbf{x}) < 0. \end{cases} \tag{63}$$

Thus,

$$w^p(\mathbf{x}) = \begin{cases} \in (-\infty, 0] & \text{if } r^p(\mathbf{x}) - r^d(\mathbf{x}) = 0 \\ 0 & \text{if } 0 < r^p(\mathbf{x}) - r^d(\mathbf{x}) < \chi\ell(\mathbf{x}) \\ \in [0, \kappa w^m(\mathbf{x})] & \text{if } r^p(\mathbf{x}) - r^d(\mathbf{x}) = \chi\ell(\mathbf{x}) \\ = \kappa w^m(\mathbf{x}) & \text{if } r^p(\mathbf{x}) - r^d(\mathbf{x}) > \chi\ell(\mathbf{x}). \end{cases} \tag{64}$$

As before, the envelope theorem yields

$$(\rho + \lambda(\mathbf{x}))\theta(\mathbf{x}) = r^b(\mathbf{x}) - r^d(\mathbf{x}) - \chi\ell(\mathbf{x}) \tag{65}$$

$$+ \lambda(\mathbf{x}) \frac{\partial}{\partial w^b(\mathbf{x})} \int \Big(\theta(\mathbf{x}')w^b(\mathbf{x}') - \nu|w^b(\mathbf{x}') - w^b(\mathbf{x})|\Big) f(\mathbf{x}')d\mathbf{x}'. \tag{66}$$

For $\mathbf{x} = \mathbf{x}^s$, this becomes

$$\begin{aligned} (\rho + \lambda)\theta(\mathbf{x}^s) =& r^b(\mathbf{x}^s) - r^d(\mathbf{x}^s) - \chi\ell(\mathbf{x}^s) \\ &+ \lambda\nu \int_{w^b(\mathbf{x}') \neq w^b(\mathbf{x}^s)} \operatorname{sign}(w^b(\mathbf{x}') - w^b(\mathbf{x}^s)) f(\mathbf{x}')d\mathbf{x}' \\ &+ \lambda \int_{w^b(\mathbf{x}') = w^b(\mathbf{x}^s)} \theta(\mathbf{x}') f(\mathbf{x}')d\mathbf{x}'. \end{aligned} \tag{67}$$

For $\mathbf{x} \neq \mathbf{x}^s$, since after a Poisson shock we return to the steady state $\mathbf{x}^s$, we get

$$\begin{aligned} (\rho + \lambda')\theta(\mathbf{x}) =& r^b(\mathbf{x}) - r^d(\mathbf{x}) - \chi\ell(\mathbf{x}) \\ &+ \lambda'\nu \operatorname{sign}(w^b(\mathbf{x}^s) - w^b(\mathbf{x}))\mathbb{1}\{w^b(\mathbf{x}^s) \neq w^b(\mathbf{x})\} \\ &+ \lambda'\theta(\mathbf{x}^s)\mathbb{1}\{w^b(\mathbf{x}^s) = w^b(\mathbf{x})\}. \end{aligned} \tag{68}$$

Households Similarly, we can write the HJB for households as

$$\begin{aligned} V^h(\mathbf{x}) = \max_{w^{h,d}, w^{h,p}} \Big\{ & c^h(\mathbf{x})dt + \beta \log(h(w^{h,p}, w^{h,d}; \alpha))dt \\ &+ (1 - \rho dt)(1 - \lambda(\mathbf{x})dt)\mathbb{E}_t\big[V^h(\mathbf{x} + d\mathbf{x})|dN = 0\big] \\ &+ (1 - \rho dt)\lambda(\mathbf{x})dt\mathbb{E}\big[V^h(\mathbf{x} + d\mathbf{x})|dN = 1\big] \Big\} \end{aligned} \tag{69}$$

where

$$h(w^{h,p}, w^{h,d}; \alpha) = (w^{h,d})^{\alpha}(w^{h,p})^{1-\alpha} \tag{70}$$

and such that $w^{h,p} + w^{h,d} = \tau^h$ and

$$c^h(\mathbf{x}) = w^{h,d} r^d(\mathbf{x}) + w^{h,p} r^{pt}(\mathbf{x}) + \pi(\mathbf{x}) + \overline{\pi}(\mathbf{x}) - r^{\tau}(\mathbf{x}). \tag{71}$$

We can rewrite the HJB as follows:

$$(\rho + \lambda(\mathbf{x}))V^h(\mathbf{x}) = \max_{w^{h,p}, w^{h,d}} \Big\{ \pi^h(\mathbf{x}) + \beta \log(h(w_u^{h,p}, w_u^{h,d}; \alpha)) + \lambda(\mathbf{x}) \int \xi^h(\mathbf{x}')f(\mathbf{x}')d\mathbf{x}' \Big\}. \tag{72}$$

The first-order condition for households is given by

$$r^{pt}(\mathbf{x}) - r^{d}(\mathbf{x}) = \beta \left(\frac{\alpha}{w^{h,d}(\mathbf{x})} - \frac{1-\alpha}{w^{h,p}(\mathbf{x})} \right). \tag{73}$$

B Relegated Table

Table 2: Impact of Common Shock to Repo Markets. This table presents results from daily regressions of the first-difference repo to interest on reserves spreads ($\Delta GCF - IOR$), the repo intermediation spreads between inter-dealer repo and dealer-to-money-fund repo ($\Delta GCF - TGCR$), as well as volumes at the reverse repo facility (ΔRRP), and balance in the Treasury General Account (ΔTGA) on dummy variables indicating if the day is the last day of a quarter (Quarter-End), the first day of a quarter (Quarter-End + 1), a tax deadline day (Tax Deadline), or a day following a tax deadline day (Tax Deadline+1), as well as a continuous variable indicating the daily change in Treasury outstanding (Δ Treasury). Data for GCF rates are from DTCC, TGCR from the Federal Reserve Bank of New York, the TGA balance from the US Treasury Department, the RRP volumes from the Federal Reserve Bank of New York, the IOR from the Federal Reserve Bank of St Louis FRED. Tax deadlines for companies and individuals are retrieved from the Treasury Department website. Newey-West Standard Errors with a maximum lag of seven days are reported with significant levels: $^{*}p < 0.05$, $^{**}p < 0.01$, $^{***}p < 0.001$.

	Dependent Variables			
	$\Delta GCF - IOR$	$\Delta GCF - TGCR$	ΔRRP	ΔTGA
Quarter-End	9.625***	0.069***	102.4***	29.53***
	(1.967)	(0.017)	(12.117)	(4.732)
Quarter-End +1	−5.940	−0.046	−118.7***	−40.93***
	(4.111)	(0.028)	(18.006)	(5.245)
Tax Deadline	2.739***	0.009	−0.304	47.10***
	(0.446)	(0.006)	(3.002)	(6.614)
Tax Deadline +1	4.547	−0.010	11.02***	14.57***
	(6.230)	(0.012)	(2.544)	(3.066)
Δ Treasury	1.646***	0.009***	0.256	4.176***
	(0.300)	(0.001)	(0.804)	(0.7)
Constant	−0.275	0.00	−0.511	−4.056***
	(0.210)	(0.001)	(0.499)	(0.363)
Observations	2,010	1,971	1,277	2,010

C Solution Method

We use the following algorithm to numerically solve for steady state ($\mathbf{x}^s$) and shock states ($\mathbf{x}'$) equilibrium variables:[30]

1. Select guess values for $\mathbb{P}(\mathcal{M}(\mathbf{x}') \in \mathcal{F})$ and $\int_{\mathcal{M}(\mathbf{x}')\notin\mathcal{F}} (\theta(\mathbf{x}') - \overline{\theta}(\mathbf{x}'))f(\mathbf{x}')d\mathbf{x}'$.

2. Using guesses from Step 1, solve for steady state allocations $\{w^b(\mathbf{x}^s), w^m(\mathbf{x}^s), w^p(\mathbf{x}^s), w^x(\mathbf{x}^s), w^d(\mathbf{x}^s)\}$, $\{\overline{w}^b(\mathbf{x}^s), \overline{w}^p(\mathbf{x}^s)\}$, $\{w^{h,p}(\mathbf{x}^s), w^{h,d}(\mathbf{x}^s)\}$ and rates $\{r^b(\mathbf{x}^s), r^m(\mathbf{x}^s), r^p(\mathbf{x}^s), r^{pt}(\mathbf{x}^s), r^d(\mathbf{x}^s)\}$.

3. Calculate shock state allocations and rates for all shocks $\mathbf{x}'$ conditional on the steady-state allocations and rates computed in the previous step.

4. Compute $\mathbb{P}(\mathcal{M}(\mathbf{x}') \in \mathcal{F})$ and $\int_{\mathcal{M}(\mathbf{x}')\notin\mathcal{F}} (\theta(\mathbf{x}') - \overline{\theta}(\mathbf{x}'))f(\mathbf{x}')d\mathbf{x}'$ from the shock state equilibria in the previous step. Update the guesses with these values.

5. Repeat steps 2-4 until convergence.

We evaluate Step 3 on a grid of $\mathbf{x}'$ spanning the sample space. Since $\{\mathcal{A}, \mathcal{S}, \mathcal{U}, \mathcal{C} \setminus \mathcal{F}, \mathcal{F}\}$ forms a partition of the state space of $\mathbf{x}'$, for each $\mathbf{x}'$ in Step 3 we can sequentially compute the equilibrium for each partition subset and halt when the equilibrium is feasible as one and only one partition subset has a feasible equilibrium for each $\mathbf{x}'$. We use numerical integration in Step 4.

Importantly, we assume that steady-state allocations in cases for which central bank intervention choices result in $\mathcal{F} = \emptyset$ are given by the baseline steady-state allocations.

[30] The sets used below are defined in the Online Appendix.

Online Appendix

State Space Partitioning We define four disjoint sets of equilibria that correspond to different dynamics in the pricing of the bilateral repo. Our analysis below characterizes how shocks to state variables $\mathbf{x}$ are shifting equilibrium across those sets.

Definition 2. *Let $\mathcal{A}$ be the set of* **arbitraged** *repo market equilibria, defined as* $\{\mathcal{M}(\mathbf{x}) \in \mathcal{A} \mid w^p(\mathbf{x}) < 0\}$.

Definition 3. *Let $\mathcal{S}$ be the set of* **segmented** *repo market states, defined as* $\{\mathcal{M}(\mathbf{x}) \in \mathcal{S} \mid w^p(\mathbf{x}) = 0$ *and* $r^p(\mathbf{x}) < r^m(\mathbf{x})\}$.

Definition 4. *Let $\mathcal{U}$ be the set of* **unconstrained** *repo market states, defined as* $\{\mathcal{M}(\mathbf{x}) \in \mathcal{U} \mid r^p(\mathbf{x}) = r^m(\mathbf{x})\}$.

Definition 5. *Let $\mathcal{C}$ be the set of* **constrained** *repo market states, defined as* $\{\mathcal{M}(\mathbf{x}) \in \mathcal{C} \mid r^p(\mathbf{x}) > r^m(\mathbf{x})\ \}$.

Definition 6. *Let $\mathcal{F}$ be the set of* **fire-sale** *states, a subset of constrained repo market states $\mathcal{C}$, defined as* $\{\mathcal{M}(\mathbf{x}) \in \mathcal{F} \mid w^b(\mathbf{x}) \neq w^b(\mathbf{x}^s)\}$.

We define as *arbitraged* the set of equilibria in which traditional banks are borrowing in bilateral repos; as *segmented* the ones in which traditional banks are not marginal in bilateral repos due to the balance sheet cost; as *unconstrained* equilibria in which the IL constraint is not binding for traditional banks and bilateral repo rates equal to the interest on reserves; as *constrained* the ones in which traditional banks are constrained by the IL regulation, and where bank repo rates are above the interest on reserves; and as *fire sale*, the ones in which traditional banks adjust their Treasury bonds holdings when entering the shock state.

A Proofs of Section 3

In Section 3, we focus on the specific shock to the liquidity preference parameter α_t. Hence, we write variables as functions of the state variable α_t instead of $\mathbf{x}_t$. We begin with a set of intermediate results before providing the proofs of our propositions.

Below, we take stock of the assumptions made in the main text for the case in which the only time-varying parameter is α.

Assumption 1. *There is always a state in which shadow banks fire-sell treasuries to traditional banks:* $\exists \alpha' \in (\alpha^s, 1), \overline{w}^b(\alpha') < \overline{w}^b(\alpha^s)$.

Assumption 2. *In the steady state, traditional banks hold some Treasuries:* $w^b(\alpha^s) > 0$.

Assumption 3. *The traditional bank dealer subsidiary has a positive balance sheet size:* $w^x(\alpha) > 0$.

Assumption 4. *The household balance sheet is larger than the quantity of repo available from traditional banks:* $\tau^h > \kappa m$.

Lemma A1. *If there exists α' such that* $0 < w^b(\alpha^s) < w^b(\alpha')$, *then* $\theta(\alpha^s) = -\overline{\theta}(\alpha^s) = -\nu$, $\theta(\alpha') = -\overline{\theta}(\alpha') = \nu$, $\overline{w}^b(\alpha^s) > \overline{w}^b(\alpha')$, *and there does not exist a shock state α' such that* $w^b(\alpha') < w^b(\alpha^s)$ *or* $\overline{w}^b(\alpha^s) < \overline{w}^b(\alpha')$.

Proof. From the envelope theorem for w^b in equation (58), we have that for the traditional bank to be incentivized to increase its holding of Treasuries and pay the adjustment cost when moving from α^s to α' and vice versa, then it must be that $\theta(\alpha') = \nu$ and $\theta(\alpha^s) \leq -\nu$. Thus, there cannot exist another state such that $w^b(\alpha') < w^b(\alpha^s)$.

The market-clearing condition for the Treasury market is given by

$$w^b(\alpha) + \overline{w}^b(\alpha) + \underline{b} = b \quad \forall\alpha. \tag{74}$$

Thus, the reverse must be true for $\overline{w}^b(\alpha)$ and $\overline{\theta}(\alpha)$. In addition, given that $w^b(\alpha^s) > 0$ in Assumption 2, we have $\theta(\alpha^s) = -\nu$. This further implies that

$$r^p(\alpha') - r^d(\alpha') = \chi\ell(\alpha)' + (\rho + \lambda')(\nu - \overline{\theta}(\alpha')) + 2\lambda'\nu \tag{75}$$

from the envelope theorem for traditional and shadow banks. Given $\overline{\theta}(\alpha') < -\nu$, $r^p(\alpha') - r^d(\alpha') > \chi\ell(\alpha)'$. By the traditional bank first-order conditions, we have $w^p(\alpha') > 0$. Combining the triparty and bilateral repo market clearing conditions, we have $\overline{w}^p(\alpha') = w^{h,p}(\alpha') + w^p(\alpha') > 0$. Hence, $\theta(\alpha') = -\nu$. ☐

Lemma A1 pins down $\overline{\theta}(\alpha^s) = -\theta(\alpha^s) = \nu$ in the steady state and $\overline{\theta}(\alpha') = -\theta(\alpha') = -\nu$ in the fire sale region under Assumption 1. Below, we formalize what happens in each state.

Steady State: $\alpha = \alpha^s$. Given Lemma A1, the envelope theorem for traditional and shadow banks leads to the following expressions for rates relative to $r^d(\alpha^s)$:

$$r^b(\alpha^s) - r^d(\alpha^s) = \chi\ell(\alpha^s) - (\rho + \lambda)\nu - \lambda\nu\mathbb{P}(\mathcal{M}(\alpha) \in \mathcal{F}) - \lambda \int_{\mathcal{M}(\alpha')\notin\mathcal{F}} \theta(\alpha')f(\alpha')d\alpha', \tag{76}$$

$$r^b(\alpha^s) - r^p(\alpha^s) = (\rho + \lambda)\nu + \lambda\nu\mathbb{P}(\mathcal{M}(\alpha) \in \mathcal{F}) - \lambda \int_{\mathcal{M}(\alpha')\notin\mathcal{F}} \overline{\theta}(\alpha')f(\alpha')d\alpha'. \tag{77}$$

Thus,

$$r^p(\alpha^s) - r^d(\alpha^s) = \chi\ell(\alpha^s) - 2\nu(\rho + \lambda) - 2\lambda\nu\mathbb{P}(\mathcal{M}(\alpha) \in \mathcal{F}) + \lambda \int_{\mathcal{M}(\alpha')\notin\mathcal{F}} (\overline{\theta}(\alpha') - \theta(\alpha'))f(\alpha')d\alpha'. \tag{78}$$

Thus, $0 \leq r^p(\alpha^s) - r^d(\alpha^s) < \chi\ell(\alpha^s)$ as $\overline{\theta}(\alpha') - \theta(\alpha') \leq 2\nu$. This further implies that $w^p(\alpha^s) \leq 0$ from the first-order condition for w^p. Hence, $\vartheta^m(\alpha^s) = 0$ and the IL constraint is not binding in the steady state. From the first-order condition for $w^x(\alpha^s)$, we get

$$r^{pt}(\alpha^s) - r^d(\alpha^s) = -2\nu(\rho + \lambda) - 2\lambda\nu\mathbb{P}(\mathcal{M}(\alpha) \in \mathcal{F}) + \lambda \int_{\mathcal{M}(\alpha')\notin\mathcal{F}} (\overline{\theta}(\alpha') - \theta(\alpha'))f(\alpha')d\alpha'. \tag{79}$$

For ease of notation, define $\Theta^s \equiv r^{pt}(\alpha^s) - r^d(\alpha^s)$. We have two cases.

- Case $w^p(\alpha^s) < 0$, thus $\mathcal{M}(\alpha^s) \in \mathcal{A}$. Then $\Theta^s = r^{pt}(\alpha^s) - r^d(\alpha^s) = -\chi\ell(\alpha^s)$ by traditional bank first-order condition. We also have that $\ell(\alpha^s) = w^d(\alpha^s) + w^x(\alpha^s) - w^p(\alpha^s) = \tau^h - f - w^p(\alpha^s)$ by the market-clearing conditions for triparty repo and deposits. Hence, $w^p(\alpha^s)$ is pinned down by $\chi(\tau^h - f - w^p(\alpha^s)) = -\Theta^s$.

- Case $w^p(\alpha^s) = 0$, thus $\mathcal{M}(\alpha^s) \in \mathcal{S}$.

In both cases, we can pin down the value of traditional banks' repo holdings in the steady state with Θ^s. Households' holdings of repo in the steady state $w^{h,p}(\alpha^s)$ is also determined by Θ^s. For ease of notation, we define $\mathcal{W}$ as

$$\mathcal{W}(s,\alpha) \equiv \begin{cases} \frac{s\tau^h-\beta+\mathcal{G}(s,\alpha)}{2s} & \text{if } s \neq 0, \\ \tau^h(1-\alpha) & \text{if } s = 0. \end{cases} \tag{80}$$

where

$$\mathcal{G}(s,\alpha) \equiv \sqrt{\beta^2 + s^2(\tau^h)^2 + 2\beta s\tau^h(1-2\alpha)}. \tag{81}$$

Then $\mathcal{W}(r^{pt}-r^d,\alpha)$ is the solution[31] of equation (73) for $w^{h,p}$ in terms of a spread $r^{pt}-r^d$ and a liquidity preference parameter α. We have $w^{h,p}(\alpha^s) = \mathcal{W}(\Theta^s,\alpha^s)$.

Then, combined with the shadow bank balance sheet constraint, repo markets clearing conditions, and the Treasury bond market-clearing condition, we get

$$\overline{w}^b(\alpha^s) = \mathcal{W}(\Theta^s,\alpha^s) + w^p(\alpha^s) \tag{82}$$

and

$$w^b(\alpha^s) = b - \underline{b} - \mathcal{W}(\Theta^s,\alpha^s) - w^p(\alpha^s). \tag{83}$$

Shock States: $\alpha' \in (\alpha^s, 1)$**.** Given Lemma A1, the envelope theorem for traditional and shadow banks leads to the following expressions for rates relative to $r^b(\alpha')$:

$$(\rho+\lambda')\theta(\alpha') = r^b(\alpha') - r^d(\alpha') - \chi\ell(\alpha') - \lambda'\nu, \tag{84}$$

$$(\rho+\lambda')\overline{\theta}(\alpha') = r^b(\alpha') - r^p(\alpha') + \lambda'\nu, \tag{85}$$

and

$$(\rho+\lambda')(\theta(\alpha') - \overline{\theta}(\alpha')) = r^p(\alpha') - r^d(\alpha') - \chi\ell(\alpha') - 2\lambda'\nu. \tag{86}$$

From the first-order condition for $w^x(\alpha')$, we get

$$r^{pt}(\alpha') - r^d(\alpha') = (\rho+\lambda')(\theta(\alpha') - \overline{\theta}(\alpha')) + 2\lambda'\nu. \tag{87}$$

For ease of notation, define $\Theta' \equiv r^{pt}(\alpha') - r^d(\alpha')$. Since $\theta(\alpha') - \overline{\theta}(\alpha') \leq 2\nu$, we have $\Theta' \leq 2\rho\nu + 4\lambda'\nu$, and $\Theta' = 2\rho\nu + 4\lambda'\nu$ when there is a fire sale. There is no discontinuity in the range of possible values for Θ'.

States with the IL Constraint Not Binding: $\alpha'|\vartheta^m(\alpha') = 0$**.** Consider some shock $\alpha' \in (\alpha^s, 1)$ such that the IL constraint does not bind. From the traditional bank first-order conditions, we get $\Theta' = r^{pt}(\alpha') - r^d(\alpha') \leq 0$. Hence, it is impossible that $\theta(\alpha') = -\overline{\theta}(\alpha') = \nu$. By Lemma A1, there is no fire sale: $w^b(\alpha') = w^b(\alpha^s)$ and $\overline{w}^b(\alpha') = \overline{w}^b(\alpha^s)$. We get

$$\overline{w}^p(\alpha') = \mathcal{W}(\Theta^s,\alpha^s) + w^p(\alpha^s), \tag{88}$$

$$w^{h,p}(\alpha') = \mathcal{W}(\Theta',\alpha'). \tag{89}$$

[31] Only this root is relevant, since the other root implies that either $w^{h,p}$ or $w^{h,d}$ is negative.

Combining the bilateral and triparty repo market-clearing conditions then yields

$$w^p(\alpha') = \mathcal{W}(\Theta^s, \alpha^s) - \mathcal{W}(\Theta', \alpha') + w^p(\alpha^s). \tag{90}$$

We have three cases.

- Case $w^p(\alpha') < 0$, thus $\mathcal{M}(\alpha') \in \mathcal{A}$. Then $\Theta' = r^{pt}(\alpha') - r^d(\alpha') = -\chi\ell(\alpha') < 0$ by traditional bank first-order condition. We also have that $\ell(\alpha') = w^d(\alpha') + w^x(\alpha') - w^p(\alpha') = \tau^h - f - w^p(\alpha')$ by the market-clearing conditions for triparty repo and deposits. Hence, $\chi(\tau^h - f - w^p(\alpha')) = -\Theta'$. Together with equations (73) and (90), we can solve for the three unknowns $w^p(\alpha')$, $\mathcal{W}(\Theta', \alpha')$, and Θ'. Combining the three equations, we get

$$\chi\left[\tau^h - f - \mathcal{W}(\Theta^s, \alpha^s) + \mathcal{W}(\Theta', \alpha') - w^p(\alpha^s)\right] \\ = -\beta\left(\frac{\alpha'}{\tau^h - \mathcal{W}(\Theta', \alpha')} - \frac{1-\alpha'}{\mathcal{W}(\Theta', \alpha')}\right). \tag{91}$$

  The left-hand side increases in $\mathcal{W}(\Theta', \alpha')$ and the rigt-hand side decreases in $\mathcal{W}(\Theta', \alpha')$. Also, the left-hand side is bounded when $\mathcal{W}(\Theta', \alpha') \in (0, \tau^h)$, while the RHS approaches $+\infty$ when $\mathcal{W}(\Theta', \alpha') \to 0$ and approaches $-\infty$ when $\mathcal{W}(\Theta', \alpha') \to \tau^h$. Hence, there exists a unique solution of $\mathcal{W}(\Theta', \alpha')$ between 0 and τ^h. By the household first-order condition we get Θ', and by equation (90), we get $w^p(\alpha')$.

- Case $\Theta' < 0$ and $w^p(\alpha') \geq 0$. Then $r^{pt}(\alpha') < r^d(\alpha')$ and $w^p(\alpha') = 0$ by traditional bank first-order condition. Thus, $\mathcal{M}(\alpha') \in \mathcal{S}$. We can also pin down $\mathcal{W}(\Theta', \alpha')$ by equation (90).

- Case $\Theta' = 0$, thus $\mathcal{M}(\alpha') \in \mathcal{U}$. By equation (80), $\mathcal{W}(\Theta', \alpha') = \tau^h(1-\alpha')$. We can also pin down $w^p(\alpha')$ by equation (90).

In every case, we can pin down $w^{h,p}(\alpha') = \mathcal{W}(\Theta', \alpha')$ and $w^p(\alpha')$ using the equilibrium outcome in the steady state. Then, by the household first-order conditions, we get Θ'.

States with the IL Constraint Binding: $\alpha'|\vartheta^m(\alpha') > 0$. Consider some shock $\alpha' \in (\alpha^s, 1)$ such that the IL constraint binds. Then we have $w^p(\alpha') = \kappa w^m(\alpha')$ and $\vartheta^m > 0$. Therefore, from the first-order conditions of the traditional banks, it must be that $r^p(\alpha') - r^d(\alpha') > \chi\ell(\alpha')$ and $\Theta' = r^{pt}(\alpha') - r^d(\alpha') > 0$.

We first consider the case when no fire sale occurs: $\overline{w}^b(\alpha') = \overline{w}^b(\alpha^s)$. We get

$$\overline{w}^p(\alpha')\overline{n} = \mathcal{W}(\Theta^s, \alpha^s) + w^p(\alpha^s), \tag{92}$$
$$w^{h,p}(\alpha') = \mathcal{W}(\Theta', \alpha'). \tag{93}$$

which, combined with the bilateral and triparty repo market clearing conditions, yields

$$w^p(\alpha') = \mathcal{W}(\Theta^s, \alpha^s) - \mathcal{W}(\Theta', \alpha') + w^p(\alpha^s). \tag{94}$$

Since $w^p(\alpha') = \kappa w^m(\alpha') = \kappa m$ from the reserve market clearing condition, we can solve for $\mathcal{W}(\Theta', \alpha')$ using the equilibrium outcome in the steady state. Then, by the household first-order conditions, we get Θ'.

If the solution gives $\Theta' > 2\nu\rho + 4\lambda'\nu$, then this contradicts with $\theta - \overline{\theta} \leq 2\nu$. A fire sale must occur. In this case, we have $\Theta' = 2\nu\rho + 4\lambda'\nu$ and we can further pin down $\mathcal{W}(\Theta', \alpha')$ by equation

(80). Together with $w^p(\alpha') = \kappa m$, we pin down shadow banks repo holdings by the bilateral and triparty repo market-clearing conditions

$$\overline{w}^p(\alpha') = \mathcal{W}(\Theta', \alpha') + \kappa m. \tag{95}$$

In all, the threshold of a fire sale is that $\Theta' = 2\nu(\rho + \lambda') + 2\lambda'\nu$, and we can pin down $\mathcal{W}(\Theta', \alpha')$ and $\overline{w}^p(\alpha')$ using the equilibrium outcome in the steady state whether or not there is a fire sale.

Lemma A2. *For all α' such that $w^p(\alpha') \geq 0$, $\ell(\alpha') = \tau^h - f$.*

Proof. From the triparty repo market-clearing, we get $w^x(\alpha') = w^{h,p}(\alpha') - f$. From the deposit market-clearing condition and the household balance sheet constraint, we obtain $w^d(\alpha') = \tau^h - w^{h,p}(\alpha')$. By definition, $\ell(\alpha') = w^d(\alpha') + w^x(\alpha') - [w^p(\alpha')]^-$. Hence, when $w^p(\alpha') \geq 0$, we get $\ell(\alpha') = w^x(\alpha') + w^d(\alpha') = \tau^h - f$. ☐

When $w^p(\alpha') < 0$, we have $\ell(\alpha') = w^x(\alpha') + w^d(\alpha') - w^p(\alpha') > \tau^h - f$. Define $\underline{\ell} = \tau^h - f$. Also, note that we only have $w^p(\alpha') < 0$ on $\mathcal{A}$. Given Lemma A2, we can now write $\underline{\ell}$ instead of $\ell(\alpha)$ when $\mathcal{M}(\alpha) \notin \mathcal{A}$.

Lemma A3. *$\mathcal{A}$, $\mathcal{S}$, $\mathcal{U}$, and $\mathcal{C}$ are intervals. Furthermore, $\mathcal{A}$, $\mathcal{S}$, $\mathcal{U}$, and $\mathcal{C}$ form a partition of $(\alpha^s, 1)$ and $\mathcal{F}$ is an interval subset of $\mathcal{C}$. Finally, we can write*

$$\forall \mathcal{M}(\alpha') \in \mathcal{A}, \mathcal{M}(\alpha'') \in \mathcal{S} \cup \mathcal{U} \cup \mathcal{C} : \alpha' < \alpha''; \tag{96}$$
$$\forall \mathcal{M}(\alpha') \in \mathcal{S}, \mathcal{M}(\alpha'') \in \mathcal{U} \cup \mathcal{C} : \alpha' < \alpha''; \tag{97}$$
$$\forall \mathcal{M}(\alpha') \in \mathcal{U}, \mathcal{M}(\alpha'') \in \mathcal{C} : \alpha' < \alpha''; \tag{98}$$
$$\forall \mathcal{M}(\alpha') \in \mathcal{C} \setminus \mathcal{F}, \mathcal{M}(\alpha'') \in \mathcal{F} : \alpha' < \alpha''. \tag{99}$$

Proof. We first take a look at the rate spread between the triparty repo and deposits. Define

$$g(x, y) \equiv \frac{x}{\tau^h - y} - \frac{1 - x}{y}, \text{ where } x \in (0, 1) \text{ and } y \in (0, \tau^h). \tag{100}$$

Then $g(x, y)$ increases in both x and y. By the household first-order condition, $r^{pt}(\alpha) - r^d(\alpha) = \beta \cdot g(\alpha, w^{h,p}(\alpha))$. Given that $\beta > 0$, if $\alpha' \geq \alpha''$ and $w^{h,p}(\alpha') \geq w^{h,p}(\alpha'')$, then it must be that $r^{pt}(\alpha') - r^d(\alpha') \geq r^{pt}(\alpha'') - r^d(\alpha'')$. If $\alpha' \geq \alpha''$ and $w^{h,p}(\alpha') > w^{h,p}(\alpha'')$, then it must be that $r^{pt}(\alpha') - r^d(\alpha') > r^{pt}(\alpha'') - r^d(\alpha'')$

Next, We prove every statement sequentially.

- Statement 96. Consider arbitrary $\mathcal{M}(\alpha') \in \mathcal{A}$ and $\mathcal{M}(\alpha'') \in \mathcal{S}$. From the first-order conditions of traditional banks, $\Theta' = r^{pt}(\alpha') - r^d(\alpha') = -\chi\ell(\alpha')$ and $\Theta'' = r^{pt}(\alpha'') - r^d(\alpha'') \geq -\chi\underline{\ell}$. We also have that $\ell(\alpha') > \underline{\ell}$. Hence, $\Theta' < \Theta''$. By definition, we have that $w^p(\alpha') < 0$ and $w^p(\alpha'') = 0$. Also, since there are no fire sales in $\mathcal{A}$ and $\mathcal{S}$, $\overline{w}^p(\alpha') = \overline{w}^p(\alpha'') = \overline{w}^p(\alpha^s)$. Thus, combining the bilateral and triparty repo market-clearing conditions, we must have $w^{h,p}(\alpha'') < w^{h,p}(\alpha')$. Assume by way of contradiction that $\alpha'' \leq \alpha'$. This implies that $r^{pt}(\alpha'') - r^d(\alpha'') \leq r^{pt}(\alpha') - r^d(\alpha')$, a contradiction with $\Theta' < \Theta''$ we derived earlier. Therefore $\alpha' < \alpha''$ for any $\mathcal{M}(\alpha') \in \mathcal{A}$ and $\mathcal{M}(\alpha'') \in \mathcal{S}$.

- Statement 97. Consider arbitrary $\mathcal{M}(\alpha') \in \mathcal{S}$ and $\mathcal{M}(\alpha'') \in \mathcal{U}$. Then, $\Theta' = r^{pt}(\alpha') - r^d(\alpha') < 0$ and $\Theta'' = r^{pt}(\alpha'') - r^d(\alpha'') = 0$. We have that $w^p(\alpha') = 0$ and by the traditional bank first-order condition $w^p(\alpha'') \geq 0$. Since there are no fire sales in $\mathcal{S}$ and

$\mathcal{U}$, $\overline{w}^p(\alpha') = \overline{w}^p(\alpha'') = \overline{w}^p(\alpha^s)$. Thus, combining the bilateral and triparty repo market-clearing conditions, we must have $w^{h,p}(\alpha'') \leq w^{h,p}(\alpha')$. Assume by way of contradiction that $\alpha'' \leq \alpha'$. This implies that $r^{pt}(\alpha'') - r^d(\alpha'') \leq r^{pt}(\alpha') - r^d(\alpha')$, a contradiction with $\Theta' < 0 = \Theta''$. Therefore $\alpha' < \alpha''$ for any $\mathcal{M}(\alpha') \in \mathcal{S}$ and $\mathcal{M}(\alpha'') \in \mathcal{U}$.

- Statement 98. Next, consider arbitrary $\mathcal{M}(\alpha') \in \mathcal{U}$ and $\mathcal{M}(\alpha'') \in \mathcal{C} \setminus \mathcal{F}$. Then, $\Theta' = r^{pt}(\alpha') - r^d(\alpha') = 0$ and $\Theta'' = r^{pt}(\alpha'') - r^d(\alpha'') > 0$. We have that $w^p(\alpha') \in [0, \kappa m]$ and $w^p(\alpha'') = \kappa m$ by the traditional bank first-order condition for w^p. Since there are no fire sales in $\mathcal{U}$ and $\mathcal{C} \setminus \mathcal{F}$, $\overline{w}^p(\alpha') = \overline{w}^p(\alpha'') = \overline{w}^p(\alpha^s)$. Thus, combining the bilateral and triparty repo market-clearing conditions, we must have $w^{h,p}(\alpha'') \leq w^{h,p}(\alpha')$. Assume by way of contradiction that $\alpha'' \leq \alpha'$. This implies that $r^{pt}(\alpha'') - r^d(\alpha'') \leq r^{pt}(\alpha') - r^d(\alpha')$, a contradiction with $\Theta' = 0 < \Theta''$. Therefore $\alpha' < \alpha''$ for any $\mathcal{M}(\alpha') \in \mathcal{U}$ and $\mathcal{M}(\alpha'') \in \mathcal{C} \setminus \mathcal{F}$.

- Statement 99. Finally, consider arbitrary $\mathcal{M}(\alpha') \in \mathcal{C} \setminus \mathcal{F}$ and $\mathcal{M}(\alpha'') \in \mathcal{F}$. We have that $w^p(\alpha') = w^p(\alpha'') = \kappa m$. Given that there is a firesale in $\mathcal{F}$, $\overline{w}^p(\alpha') = \overline{w}^p(\alpha^s) > \overline{w}^p(\alpha'')$. Thus, combining the bilateral and triparty repo market-clearing conditions, we must have $w^{h,p}(\alpha'') < w^{h,p}(\alpha')$. Assume by way of contradiction that $\alpha'' \leq \alpha'$. This implies that $r^{pt}(\alpha'') - r^d(\alpha'') < r^{pt}(\alpha') - r^d(\alpha')$. We have shown that $w^p(\alpha^s) \leq 0$. Hence, both α' and α'' are in shock states. Then, it must be that $\Theta' = r^{pt}(\alpha') - r^d(\alpha') \leq 2\rho\nu + 4\lambda'\nu$ and $\Theta'' = r^{pt}(\alpha'') - r^d(\alpha'') = 2\rho\nu + 4\lambda'\nu$, a contradiction with $\Theta'' < \Theta'$. Therefore $\alpha' < \alpha''$ for any $\mathcal{M}(\alpha') \in \mathcal{C} \setminus \mathcal{F}$ and $\mathcal{M}(\alpha'') \in \mathcal{F}$.

□

Given Lemma A3, we can show that $w^p(\alpha^s) < 0$. Assume by way of contradiction $w^p(\alpha^s) = 0$. Then, $\Theta' \to \Theta^s$ when $\alpha' \to \alpha^s$ from the household first-order condition

$$\Theta' = \beta \left(\frac{\alpha'}{\tau - w^{h,p}(\alpha')} - \frac{1 - \alpha'}{w^{h,p}(\alpha')} \right) \tag{101}$$

where $w^{h,p}(\alpha') = \mathcal{W}^s = \overline{w}^p(\alpha^s)$ when $\alpha' \to \alpha^s$. However, we know that the steady state spread is

$$\begin{aligned}
\Theta^s &= -2\nu(\rho + \lambda) - 2\lambda\nu\mathbb{P}(\mathcal{M}(\alpha) \in \mathcal{F}) + \lambda \int_{\mathcal{M}(\alpha') \notin \mathcal{F}} (\overline{\theta}(\alpha') - \theta(\alpha')) f(\alpha') d\alpha' \\
&= -2\nu\rho - 4\lambda\nu\mathbb{P}(\mathcal{M}(\alpha) \in \mathcal{F}) + \lambda \int_{\mathcal{M}(\alpha') \notin \mathcal{F}} (\overline{\theta}(\alpha') - \theta(\alpha') - 2\nu) f(\alpha') d\alpha' \\
&\leq -2\nu\rho - 4\lambda\nu\mathbb{P}(\mathcal{M}(\alpha) \in \mathcal{F})
\end{aligned} \tag{102}$$

given $\overline{\theta} - \theta \leq 2\nu$. Also, we have that

$$\Theta' = (\rho + \lambda')(\theta(\alpha') - \overline{\theta}(\alpha')) + 2\lambda'\nu \geq -2\nu\rho \tag{103}$$

So long as $\mathbb{P}(\mathcal{M}(\alpha) \in \mathcal{F}) > 0$, $\Theta' \to \Theta^s$ when $\alpha' \to \alpha^s$ does not hold, a contradiction. Hence, $w^p(\alpha^s) < 0$ and we can define the thresholds of each partition as follows:

$$\mathcal{A} = [\alpha^s, \alpha^{\mathcal{A}}), \;\; \mathcal{S} = [\alpha^{\mathcal{A}}, \alpha^{\mathcal{U}}), \; \mathcal{U} = [\alpha^{\mathcal{U}}, \alpha^{\mathcal{C}}], \; \mathcal{C} = (\alpha^{\mathcal{C}}, 1), \; \mathcal{F} = (\alpha^{\mathcal{F}}, 1) \tag{104}$$

and $\alpha^s < \alpha^{\mathcal{A}} \leq \alpha^{\mathcal{U}} \leq \alpha^{\mathcal{C}} \leq \alpha^{\mathcal{F}}$. We also have that $\Theta^s < -\chi\underline{\ell}$.

Define $\Theta(\alpha') \equiv r^{pt}(\alpha') - r^d(\alpha')$, $\mathcal{W}^s \equiv \mathcal{W}(\Theta^s, \alpha^s) = w^{h,p}(\alpha^s)$, $\mathcal{W}^{\mathcal{U}} \equiv \mathcal{W}(\Theta(\alpha^{\mathcal{U}}), \alpha^{\mathcal{U}}) = w^{h,p}(\alpha^{\mathcal{U}})$, and $\mathcal{W}^{\mathcal{F}} \equiv \mathcal{W}(\Theta^{\mathcal{F}}, \alpha^{\mathcal{F}}) = w^{h,p}(\alpha^{\mathcal{F}})$ where $\Theta^{\mathcal{F}} \equiv r^{pt}(\alpha^{\mathcal{F}}) - r^d(\alpha^{\mathcal{F}})$ and $\mathcal{W}(\Theta, \alpha)$ is defined in equation (80).

Lemma A4. *The thresholds solve for*

$$\mathcal{W}^{\mathcal{U}} = \mathcal{W}\left(-\chi\underline{\ell}, \alpha^{\mathcal{A}}\right), \tag{105}$$

$$\alpha^{\mathcal{U}} = \frac{\tau^h - \mathcal{W}^{\mathcal{U}}}{\tau^h}, \tag{106}$$

$$\alpha^{\mathcal{C}} = \frac{\tau^h - \mathcal{W}^{\mathcal{F}}}{\tau^h}. \tag{107}$$

Furthermore,

$$\mathcal{W}^{\mathcal{U}} = \mathcal{W}^s + \underline{\ell} + \Theta^s/\chi, \tag{108}$$

$$\mathcal{W}^{\mathcal{F}} = \mathcal{W}^{\mathcal{U}} - \kappa m, \tag{109}$$

$$\Theta^{\mathcal{F}} = 2\nu(\rho + \lambda') + 2\lambda'\nu. \tag{110}$$

Proof. We have that $w^p(\alpha^{\mathcal{A}}) = w^p(\alpha^{\mathcal{U}}) = 0$ and $w^p(\alpha^{\mathcal{C}}) = w^p(\alpha^{\mathcal{F}}) = \kappa m$. By the market-clearing conditions for repo markets, we further obtain that

$$\mathcal{W}(\Theta(\alpha^{\mathcal{A}}), \alpha^{\mathcal{A}}) = \mathcal{W}^{\mathcal{U}} = \mathcal{W}(\Theta(\alpha^{\mathcal{C}}), \alpha^{\mathcal{C}}) + \kappa m = \mathcal{W}^{\mathcal{F}} + \kappa m \tag{111}$$

given that $\overline{w}^p(\alpha)$ remains constant when $\alpha \leq \alpha^{\mathcal{F}}$. This yields equation (109).

By definition of $\mathcal{A}$ and $\mathcal{S}$, $\alpha^{\mathcal{A}}$ is such that $\Theta(\alpha^{\mathcal{A}}) = -\chi\underline{\ell}$ and $\mathcal{W}(\Theta(\alpha^{\mathcal{A}}), \alpha^{\mathcal{A}}) = \mathcal{W}^{\mathcal{U}}$. These two conditions together yield equation (105).

By definition of $\mathcal{S}$ and $\mathcal{U}$, $\alpha^{\mathcal{U}}$ is such that $\Theta(\alpha^{\mathcal{U}}) = 0$ and $\mathcal{W}(\Theta(\alpha^{\mathcal{U}}), \alpha^{\mathcal{U}}) = \mathcal{W}^{\mathcal{U}}$. These two conditions together yield equation (106).

By definition of $\mathcal{U}$ and $\mathcal{C}$, $\alpha^{\mathcal{C}}$ is such that $\Theta(\alpha^{\mathcal{C}}) = 0$ and $\mathcal{W}^{\mathcal{U}} - \mathcal{W}(\Theta(\alpha^{\mathcal{C}}), \alpha^{\mathcal{C}}) = \kappa m$. These two conditions together yield equation (107).

We also have that $\Theta^s = -\chi\ell(\alpha^s) = -\chi\left(\underline{\ell} - w^p(\alpha^s)\right)$. By equation (90), we have that

$$0 = w^p(\alpha^{\mathcal{U}}) = \mathcal{W}^s - \mathcal{W}^{\mathcal{U}} + w^p(\alpha^s) = \mathcal{W}^s - \mathcal{W}^{\mathcal{U}} + \underline{\ell} + \Theta^s/\chi \tag{112}$$

This yields equation (108). □

A.1 Proof of Proposition 1

Given Lemma A3, we get that for $\alpha' \in [\alpha^s, \alpha^{\mathcal{F}}]$, $\overline{w}^b(\alpha') = \overline{w}^b(\alpha^s)$ and $w^b(\alpha') = w^b(\alpha^s)$, and for $\alpha'' \in (\alpha^{\mathcal{F}}, 1]$, $\overline{w}^b(\alpha'') < \overline{w}^b(\alpha^s)$ and $w^b(\alpha'') > w^b(\alpha^s)$. Note that $f(\alpha) = 1/(1-\alpha^s)$. Thus, given $\Theta' = (\rho + \lambda')(\theta(\alpha') - \overline{\theta}(\alpha')) + 2\lambda'\nu$, we can rewrite equation (79) as

$$\Theta^s = -2\nu(\rho + \lambda) - 2\lambda\nu\frac{1 - \alpha^{\mathcal{F}}}{1 - \alpha^s} + \frac{2\lambda\lambda'\nu(\alpha^{\mathcal{F}} - \alpha^s)}{(1 - \alpha^s)(\rho + \lambda')} - \frac{\lambda}{(1 - \alpha^s)(\rho + \lambda')}\int_{\alpha^s}^{\alpha^{\mathcal{F}}} \Theta' d\alpha'. \tag{113}$$

Thus, the equilibrium is characterized by a system of equations

$$\mathcal{W}^{\mathcal{F}} = \mathcal{W}^s + \underline{\ell} + \Theta^s/\chi - \kappa m, \tag{114}$$

$$\Theta^s = \beta\left(\frac{\alpha^s}{\tau^h - \mathcal{W}^s} - \frac{1-\alpha^s}{\mathcal{W}^s}\right), \tag{115}$$

$$\Theta^{\mathcal{F}} = \beta\left(\frac{\alpha^{\mathcal{F}}}{\tau^h - \mathcal{W}^{\mathcal{F}}} - \frac{1-\alpha^{\mathcal{F}}}{\mathcal{W}^{\mathcal{F}}}\right) = 2\lambda'\nu + 2\nu(\rho+\lambda'), \tag{116}$$

$$\Theta^s = -2\nu(\rho+\lambda) - 2\lambda\nu\frac{1-\alpha^{\mathcal{F}}}{1-\alpha^s} + \frac{2\lambda\lambda'\nu(\alpha^{\mathcal{F}}-\alpha^s)}{(1-\alpha^s)(\rho+\lambda')} - \frac{\lambda}{(1-\alpha^s)(\rho+\lambda')}\int_{\alpha^s}^{\alpha^{\mathcal{F}}}\Theta' d\alpha'. \tag{117}$$

where Θ' for $\alpha' \in (\alpha^s, 1)$ is given by

$$\Theta' = \begin{cases} -\chi\ell(\alpha') & \text{if } \mathcal{M}(\alpha') \in \mathcal{A}, \\ 0 & \text{if } \mathcal{M}(\alpha') \in \mathcal{U}, \\ 2\nu(\rho+\lambda') + 2\lambda'\nu & \text{if } \mathcal{M}(\alpha') \in \mathcal{F}, \end{cases} \tag{118}$$

and

$$\mathcal{W}^s + w^p(\alpha^s) = \mathcal{W}(\Theta', \alpha') \tag{119}$$

if $\mathcal{M}(\alpha') \in \mathcal{S}$ and

$$\kappa m = \mathcal{W}^s - \mathcal{W}(\Theta', \alpha') + w^p(\alpha^s) \tag{120}$$

if $\mathcal{M}(\alpha') \in \mathcal{C}\backslash\mathcal{F}$.

The first equation is given by definition of $\mathcal{W}^{\mathcal{F}}$ in (109). The second equation is the household first-order condition in the steady state. The third equation is the household first-order condition at $\alpha' = \alpha^{\mathcal{F}}$. The fourth equation is derived above. Finally, the values for Θ' come from the above discussion of states with the IL constraint not binding and states with the IL constraint binding. We have already solved for $w^p(\alpha^s)$ given Θ^s and Θ' when $\mathcal{M}(\alpha') \in \mathcal{A}$ in the discussion around equation (91). Hence, there are only four unknowns $(\Theta^s, \mathcal{W}^s, \mathcal{W}^{\mathcal{F}}, \alpha^{\mathcal{F}})$.

Note that if we find the solution for $\mathcal{W}^s$, then from equation (115), we get Θ^s. Subsequently, equation (114) yields $\mathcal{W}^{\mathcal{F}}$, and equation (116) yields $\alpha^{\mathcal{F}}$. Thus, to prove that this nonlinear system of equations has a unique solution, we just need to show that equation (117) gives a unique solution for $\mathcal{W}^s$.

Rearranging (117), we get

$$\begin{aligned} &\Theta^s + \frac{\lambda}{(1-\alpha^s)(\rho+\lambda')}\int_{\alpha^s}^{\alpha^{\mathcal{U}}}\Theta' d\alpha' + 2\nu(\rho+\lambda) + \frac{2\lambda\nu}{1-\alpha^s} + \frac{2\lambda\lambda'\nu\alpha^s}{(1-\alpha^s)(\rho+\lambda')} \\ =&\frac{\lambda(\alpha^{\mathcal{F}}+\alpha^{\mathcal{C}})\Theta^{\mathcal{F}}}{2(1-\alpha^s)(\rho+\lambda')} \end{aligned} \tag{121}$$

Next, we show that the term $\int_{\alpha^s}^{\alpha^{\mathcal{U}}}\Theta' d\alpha'$ is increasing with respect to $\mathcal{W}^s$.

$$\int_{\alpha^s}^{\alpha^{\mathcal{U}}}\Theta' d\alpha' = \int_{\alpha^s}^{\alpha^{\mathcal{A}}}(-\chi\ell(\alpha'))d\alpha' + \int_{\alpha^{\mathcal{A}}}^{\alpha^{\mathcal{U}}}\Theta' d\alpha' = \chi\int_{\alpha^s}^{\alpha^{\mathcal{A}}} w^p(\alpha')d\alpha' - \chi\underline{\ell}\left(\frac{\alpha^{\mathcal{U}}+\alpha^{\mathcal{A}}}{2} - \alpha^s\right) \tag{122}$$

By household first-order conditions and market-clearing conditions, we have

$$-\chi\left(\underline{\ell}-w^{p}(\alpha')\right)=\Theta'=\beta\left(\frac{\alpha'}{\tau^{h}-w^{h,p}(\alpha')}-\frac{1-\alpha'}{w^{h,p}(\alpha')}\right) \tag{123}$$

$$w^{h,p}(\alpha')+w^{p}(\alpha')=\mathcal{W}^{\mathcal{U}} \tag{124}$$

From equation (123), for any given level of α', $w^{h,p}(\alpha')$ increases with $w^{p}(\alpha')$. Hence, by equation (124), when $\mathcal{W}^{\mathcal{U}}$ increases, both $w^{h,p}(\alpha')$ and $w^{p}(\alpha')$ must increase. We can then denote $w^{p}(\alpha')$ as an increasing function $g_w(\mathcal{W}^{\mathcal{U}},\alpha')$ of $\mathcal{W}^{\mathcal{U}}$. Also, by equation (105), $\alpha^{\mathcal{A}}$ is a decreasing function of $\mathcal{W}^{\mathcal{U}}$. We denote it as $g_\alpha(\mathcal{W}^{\mathcal{U}})$. Then for $x_1>x_2$, $g_\alpha(x_1)<g_\alpha(x_2)$ and $g_w(x_1,\alpha')>g_w(x_2,\alpha')$. Given that $g_w(x,\alpha')\le 0$ when $\alpha'\in[\alpha^s,\alpha^{\mathcal{A}}]$, we get that

$$\int_{\alpha^s}^{g_\alpha(x_1)} g_w(x_1,\alpha')d\alpha'-\int_{\alpha^s}^{g_\alpha(x_2)} g_w(x_2,\alpha')d\alpha' \tag{125}$$

$$=\int_{\alpha^s}^{g_\alpha(x_1)}(g_w(x_1,\alpha')-g_w(x_2,\alpha'))d\alpha'-\int_{g_\alpha(x_1)}^{g_\alpha(x_2)} g_w(x_2,\alpha')d\alpha'>0 \tag{126}$$

Hence, $\int_{\alpha^s}^{\alpha^{\mathcal{A}}} w^{p}(\alpha')d\alpha'$ increases with $\mathcal{W}^{\mathcal{U}}$.

Given that Θ^s increases with $\mathcal{W}^s$, $\mathcal{W}^{\mathcal{U}}$ increases with $\mathcal{W}^s$ by equation (108). Then $\int_{\alpha^s}^{\alpha^{\mathcal{A}}} w^{p}(\alpha')d\alpha'$ increases with $\mathcal{W}^s$. By equation (106) $\alpha^{\mathcal{U}}$ decreases with $\mathcal{W}^s$ and by equation (105) $\alpha^{\mathcal{A}}$ decreases with $\mathcal{W}^s$. These together yield that $\int_{\alpha^s}^{\alpha^{\mathcal{U}}}\Theta' d\alpha'$ increases with $\mathcal{W}^s$.

Together with Θ^s increasing with $\mathcal{W}^s$, we get that the left-hand side of (121) increases with $\mathcal{W}^s$. On the other hand, given that Θ^s increases with $\mathcal{W}^s$, we further get that $\mathcal{W}^{\mathcal{F}}$ increases with $\mathcal{W}^s$ by equation (114). Hence, by equations (107) and (116), $\alpha^{\mathcal{C}}$ and $\alpha^{\mathcal{F}}$ decrease with $\mathcal{W}^s$. Thus, the right-hand side of (121) decreases with $\mathcal{W}^s$. So equation (117) gives a unique solution $\mathcal{W}^s$.

We now consider the bounds on $\overline{\theta}$. Note that θ and $\overline{\theta}$ are between $-\nu$ and ν and that

$$\theta(\alpha')-\overline{\theta}(\alpha')=\frac{\Theta'-2\lambda'\nu}{\rho+\lambda'}. \tag{127}$$

We can write the bounds of $\overline{\theta}$ as follows:

$$\overline{\theta}(\alpha')\in\left[-\nu-\frac{\Theta'-2\lambda'\nu}{\rho+\lambda'},\nu\right]\quad \text{if } \alpha'\le\alpha^{\Theta}, \tag{128}$$

and

$$\overline{\theta}(\alpha')\in\left[-\nu,\nu-\frac{\Theta'-2\lambda'\nu}{\rho+\lambda'}\right]\quad \text{if } \alpha'>\alpha^{\Theta}, \tag{129}$$

where α^{Θ} is defined as the solution to $\Theta(\alpha^{\Theta})=2\lambda'\nu$. Since $0<\Theta(\alpha^{\Theta})<\Theta^{\mathcal{F}}$, we have $\alpha^{\Theta}\in[\alpha^{\mathcal{C}},\alpha^{\mathcal{F}})$. Given that Θ' is increasing in α' when $\mathcal{M}(\alpha')\in\mathcal{C}\backslash\mathcal{F}$ from equation (120), it is well-defined and given by

$$2\lambda'\nu=\beta\left(\frac{\alpha^{\Theta}}{\tau^{h}-\mathcal{W}^{\mathcal{F}}}-\frac{1-\alpha^{\Theta}}{\mathcal{W}^{\mathcal{F}}}\right). \tag{130}$$

By the shadow bank first-order conditions, we have

$$r^b(\alpha') = r^p(\alpha') + (\rho + \lambda')\overline{\theta}(\alpha') - \lambda'\nu. \tag{131}$$

Using the definition of θ and $\overline{\theta}$ and first-order conditions, we get that the interest rate on Treasuries is bounded by $\underline{r}^b(\alpha) \leq r^b(\alpha) \leq \overline{r}^b(\alpha)$, where

$$\underline{r}^b(\alpha) \equiv \max\{r^d(\alpha) + \chi\ell(\alpha) - \rho\nu, r^p(\alpha) - (\rho + 2\lambda')\nu\}, \tag{132}$$

and

$$\overline{r}^b(\alpha) \equiv \min\{r^d(\alpha) + \chi\ell(\alpha) + (\rho + 2\lambda')\nu, r^p(\alpha) + \rho\nu\}. \tag{133}$$

for $\alpha \in (\alpha^s, 1)$. When $\mathcal{M}(\alpha) \in \mathcal{F}$, the interest rate on Treasuries is pinned down by $r^b(\alpha) = r^k + (\rho + 2\lambda')\nu$.

The normal state envelope theorem (from s-banks) gives

$$r^b(\alpha^s) - r^p(\alpha^s) = (\rho + \lambda)\nu + \lambda\nu\mathbb{P}(\mathcal{M}(\alpha') \in \mathcal{F}) - \frac{\lambda}{1-\alpha^s}\int_{\alpha^s}^{\alpha^{\mathcal{F}}} \overline{\theta}(\alpha')d\alpha' \tag{134}$$

We can then integrate over $\min\{\overline{\theta}(\alpha')\}$ and $\max\{\overline{\theta}(\alpha')\}$ to get the bounds for the spreads in the normal state.

A.2 Proof of Proposition 2

we first introduce a number of intermediate values as follows:

$$\begin{aligned}
\mathcal{P}(\alpha') &\equiv \frac{\alpha'}{(\tau^h - w^{h,p}(\alpha'))^2} + \frac{1-\alpha'}{(w^{h,p}(\alpha'))^2} \quad \forall \alpha' \in [\alpha^s, 1) \\
D &\equiv \frac{1}{\tau^h - \mathcal{W}^{\mathcal{F}}} + \frac{1}{\mathcal{W}^{\mathcal{F}}} \\
E &\equiv \frac{2\lambda\nu}{1-\alpha^s} \\
G &\equiv (\rho + \lambda')(1-\alpha^s) \\
H &\equiv \frac{\chi\underline{\ell}}{2}\frac{(\mathcal{P}(\alpha^{\mathcal{A}}) + \mathcal{P}(\alpha^{\mathcal{U}}))(\tau^h - \mathcal{W}^{\mathcal{U}})\mathcal{W}^{\mathcal{U}}}{\tau^h} \\
I &\equiv -\frac{\lambda\beta}{2G}\left(\frac{(\alpha^{\mathcal{F}})^2}{(\tau^h - \mathcal{W}^{\mathcal{F}})^2} - \frac{(\alpha^{\mathcal{F}} - 1)^2}{(\mathcal{W}^{\mathcal{F}})^2}\right) \\
J &\equiv -\frac{\lambda\beta}{G}\left(\frac{\alpha^{\mathcal{F}}}{\tau^h - \mathcal{W}^{\mathcal{F}}} - \frac{1-\alpha^{\mathcal{F}}}{\mathcal{W}^{\mathcal{F}}}\right) \\
K &\equiv \frac{2\lambda\lambda'\nu}{G}
\end{aligned}$$

and we denote $A \equiv \mathcal{P}(\alpha^s)$, $C \equiv \mathcal{P}(\alpha^{\mathcal{F}})$, and

$$B \equiv \int_{\alpha^s}^{\alpha^{\mathcal{A}}} \frac{\beta\mathcal{P}(\alpha')\chi}{\beta\mathcal{P}(\alpha') + \chi} d\alpha'.$$

Since $\alpha' \in (0,1)$ for all $\alpha' \in [\alpha^s, 1)$, $\mathcal{P}(\alpha') > 0$. Together with $\chi, \beta > 0$, we have $B > 0$. Since

$w^{h,p}(\alpha) \in (0, \tau^h)$ for any α, $D > 0$. Together with $\underline{\ell}, \tau^h > 0$, $H > 0$. In addition, $\alpha^s \in (0,1)$ and $\rho, \lambda, \lambda', \nu > 0$ imply that $E, G, K > 0$. Also, we can simplify J as $J = -\lambda\Theta^{\mathcal{F}}/G$. Since $\Theta^{\mathcal{F}} = 2\nu(\rho+\lambda') + 2\lambda'\nu > 0$ and $\lambda, G > 0$, $J < 0$. In addition, we have that

$$E + K + J = \frac{1}{G}\left(2\lambda\nu(\rho+\lambda') + 2\lambda\lambda'\nu - \lambda(2\nu(\rho+\lambda') + 2\lambda'\nu)\right) = 0$$

We can use algebraic factorization and rewrite I as

$$I = -\frac{\lambda\Theta^{\mathcal{F}}}{2G}\left(\frac{\alpha^{\mathcal{F}}}{\tau^h - \mathcal{W}^{\mathcal{F}}} + \frac{1-\alpha^{\mathcal{F}}}{\mathcal{W}^{\mathcal{F}}}\right). \tag{135}$$

Since $\lambda, G, \Theta^{\mathcal{F}} > 0$, $\mathcal{W}^{\mathcal{F}} \in (0, \tau^h)$ and $\alpha^{\mathcal{F}} \in (0,1)$, we have $I < 0$.

Next, we show the proof of Proposition 2. We have that $\alpha^s \le \alpha^{\mathcal{A}} \le \alpha^{\mathcal{U}} \le \alpha^{\mathcal{C}} \le \alpha^{\mathcal{F}}$. This further implies that $\Theta(\alpha') = 0$ on $[\alpha^{\mathcal{U}}, \alpha^{\mathcal{C}}]$, $\Theta(\alpha') = \beta\left(\frac{\alpha'}{\tau^h - \mathcal{W}^{\mathcal{U}}} - \frac{1-\alpha'}{\mathcal{W}^{\mathcal{U}}}\right)$ for $\alpha' \in [\alpha^{\mathcal{A}}, \alpha^{\mathcal{U}})$, and $\Theta(\alpha') = \beta\left(\frac{\alpha'}{\tau^h - \mathcal{W}^{\mathcal{F}}} - \frac{1-\alpha'}{\mathcal{W}^{\mathcal{F}}}\right)$ for $\alpha' \in (\alpha^{\mathcal{C}}, \alpha^{\mathcal{F}}]$. Hence, equation (117) can be written as

$$\begin{aligned}\Theta^s =& -2\nu(\rho+\lambda) - 2\lambda\nu\frac{1-\alpha^{\mathcal{F}}}{1-\alpha^s} + \frac{2\lambda\lambda'\nu(\alpha^{\mathcal{F}} - \alpha^s)}{(1-\alpha^s)(\rho+\lambda')} \\ & - \frac{\lambda}{(1-\alpha^s)(\rho+\lambda')}\left(\int_{\alpha^s}^{\alpha^{\mathcal{A}}} \Theta' d\alpha' + \frac{(\alpha^{\mathcal{U}} - \alpha^{\mathcal{A}})\Theta(\alpha^{\mathcal{A}})}{2} + \frac{(\alpha^{\mathcal{F}} - \alpha^{\mathcal{C}})\Theta^{\mathcal{F}}}{2}\right)\end{aligned} \tag{136}$$

We then take a look at $\Theta(\alpha')$ when $\alpha' \in [\alpha^s, \alpha^{\mathcal{A}})$. If $\alpha' \in [\alpha^s, \alpha^{\mathcal{A}})$, we have $w^p(\alpha') < 0$ and

$$-\chi\left(\underline{\ell} - w^p(\alpha')\right) = \beta\left(\frac{\alpha'}{\tau^h - w^{h,p}(\alpha')} - \frac{1-\alpha'}{w^{h,p}(\alpha')}\right) = \Theta(\alpha'), \tag{137}$$

$$w^{h,p}(\alpha') + w^p(\alpha') = \overline{w}^p(\alpha') = \overline{w}^p(\alpha^s) = \mathcal{W}^s + w^p(\alpha^s) = \mathcal{W}^{\mathcal{U}}. \tag{138}$$

Taking derivatives with respect to χ, we get

$$\beta\mathcal{P}(\alpha')\frac{\partial w^{h,p}(\alpha')}{\partial\chi} = -\left(\underline{\ell} - w^p(\alpha')\right) + \chi\frac{\partial w^p(\alpha')}{\partial\chi} \tag{139}$$

$$\frac{\partial w^{h,p}(\alpha')}{\partial\chi} + \frac{\partial w^p(\alpha')}{\partial\chi} = \frac{\partial\mathcal{W}^s}{\partial\chi} + \frac{\partial w^p(\alpha^s)}{\partial\chi} = \frac{\partial\mathcal{W}^{\mathcal{U}}}{\partial\chi} \tag{140}$$

Combining, we obtain

$$\frac{\partial w^p(\alpha')}{\partial\chi} = \frac{\beta\mathcal{P}(\alpha')\frac{\partial\mathcal{W}^{\mathcal{U}}}{\partial\chi} + \underline{\ell} - w^p(\alpha')}{\beta\mathcal{P}(\alpha') + \chi} \tag{141}$$

This implies that

$$\frac{\partial\Theta(\alpha')}{\partial\chi} = \beta\mathcal{P}(\alpha')\frac{\chi\frac{\partial\mathcal{W}^{\mathcal{U}}}{\partial\chi} - (\underline{\ell} - w^p(\alpha'))}{\beta\mathcal{P}(\alpha') + \chi} \tag{142}$$

Plugging in $\alpha' = \alpha^s$, we get

$$\frac{\partial\Theta^s}{\partial\chi} = A\beta\frac{\chi\frac{\partial\mathcal{W}^{\mathcal{U}}}{\partial\chi} - (\underline{\ell} - w^p(\alpha^s))}{A\beta + \chi} \tag{143}$$

Taking derivatives with respect to χ for equation (105) and (106), we have

$$-\underline{\ell} = \beta\left(\mathcal{P}(\alpha^{\mathcal{A}})\frac{\partial \mathcal{W}^{\mathcal{U}}}{\partial \chi} + \left(\frac{1}{\tau^h - \mathcal{W}^{\mathcal{U}}} + \frac{1}{\mathcal{W}^{\mathcal{U}}}\right)\frac{\partial \alpha^{\mathcal{A}}}{\partial \chi}\right), \tag{144}$$

$$0 = \beta\left(\mathcal{P}(\alpha^{\mathcal{U}})\frac{\partial \mathcal{W}^{\mathcal{U}}}{\partial \chi} + \left(\frac{1}{\tau^h - \mathcal{W}^{\mathcal{U}}} + \frac{1}{\mathcal{W}^{\mathcal{U}}}\right)\frac{\partial \alpha^{\mathcal{U}}}{\partial \chi}\right). \tag{145}$$

Hence,

$$\frac{\partial \alpha^{\mathcal{A}}}{\partial \chi} = -\frac{\underline{\ell}(\tau^h - \mathcal{W}^{\mathcal{U}})\mathcal{W}^{\mathcal{U}}}{\beta\tau^h} - \frac{\mathcal{P}(\alpha^{\mathcal{A}})(\tau^h - \mathcal{W}^{\mathcal{U}})\mathcal{W}^{\mathcal{U}}}{\tau^h}\frac{\partial \mathcal{W}^{\mathcal{U}}}{\partial \chi}, \tag{146}$$

$$\frac{\partial \alpha^{\mathcal{U}}}{\partial \chi} = -\frac{\mathcal{P}(\alpha^{\mathcal{U}})(\tau^h - \mathcal{W}^{\mathcal{U}})\mathcal{W}^{\mathcal{U}}}{\tau^h}\frac{\partial \mathcal{W}^{\mathcal{U}}}{\partial \chi}. \tag{147}$$

We now go back to equation (136). Note that $\partial \mathcal{W}^{\mathcal{F}}/\partial \chi = \partial \mathcal{W}^{\mathcal{U}}/\partial \chi$. Taking derivatives of both sides

$$\begin{aligned}\frac{\partial \Theta^s}{\partial \chi} =& (E+K+J)\frac{\partial \alpha^{\mathcal{F}}}{\partial \chi} + I\frac{\partial \mathcal{W}^{\mathcal{U}}}{\partial \chi} \\ & - \frac{\lambda}{(1-\alpha^s)(\rho+\lambda')}\left(\frac{\partial}{\partial \chi}\int_{\alpha^s}^{\alpha^{\mathcal{A}}} \Theta' d\alpha' - \frac{(\alpha^{\mathcal{U}} - \alpha^{\mathcal{A}})\underline{\ell}}{2} - \frac{\chi\underline{\ell}}{2}\left(\frac{\partial \alpha^{\mathcal{U}}}{\partial \chi} - \frac{\partial \alpha^{\mathcal{A}}}{\partial \chi}\right)\right) \\ =& I\frac{\partial \mathcal{W}^{\mathcal{U}}}{\partial \chi} + \frac{\lambda}{(1-\alpha^s)(\rho+\lambda')}\left(\frac{(\alpha^{\mathcal{U}} - \alpha^{\mathcal{A}})\underline{\ell}}{2} + \frac{\chi\underline{\ell}}{2}\left(\frac{\partial \alpha^{\mathcal{U}}}{\partial \chi} - \frac{\partial \alpha^{\mathcal{A}}}{\partial \chi}\right)\right) \\ & - \frac{\lambda}{(1-\alpha^s)(\rho+\lambda')}\left(-\chi\underline{\ell}\frac{\partial \alpha^{\mathcal{A}}}{\partial \chi} + \int_{\alpha^s}^{\alpha^{\mathcal{A}}} \frac{\partial \Theta(\alpha')}{\partial \chi} d\alpha'\right).\end{aligned} \tag{148}$$

Plugging in equations (142), (143), $\partial \alpha^{\mathcal{A}}/\partial \chi$ and $\partial \alpha^{\mathcal{U}}/\partial \chi$, we get

$$\begin{aligned}&\left(\frac{A\beta\chi}{A\beta+\chi} - I + \frac{\lambda H}{G} + \frac{\lambda B}{G}\right)\frac{\partial \mathcal{W}^{\mathcal{U}}}{\partial \chi} \\ =& \frac{\lambda\underline{\ell}}{2G}\left(\alpha^{\mathcal{U}} - \alpha^{\mathcal{A}} - \frac{\chi\underline{\ell}(\tau^h - \mathcal{W}^{\mathcal{U}})\mathcal{W}^{\mathcal{U}}}{\beta\tau^h}\right) + \frac{\lambda}{G}\int_{\alpha^s}^{\alpha^{\mathcal{A}}} \frac{\beta\mathcal{P}(\alpha')\left(\underline{\ell} - w^p(\alpha')\right)}{\beta\mathcal{P}(\alpha') + \chi} d\alpha' + \frac{A\beta\left(\underline{\ell} - w^p(\alpha^s)\right)}{A\beta+\chi}.\end{aligned} \tag{149}$$

Note that

$$\alpha^{\mathcal{U}} - \alpha^{\mathcal{A}} - \frac{\chi\underline{\ell}(\tau^h - \mathcal{W}^{\mathcal{U}})\mathcal{W}^{\mathcal{U}}}{\beta\tau^h} = 0. \tag{150}$$

Hence, equation (149) can be simplified as

$$\begin{aligned}&\left(\frac{A\beta\chi}{A\beta+\chi} - I + \frac{\lambda H}{G} + \frac{\lambda B}{G}\right)\frac{\partial \mathcal{W}^{\mathcal{U}}}{\partial \chi} \\ =& \frac{\lambda}{G}\int_{\alpha^s}^{\alpha^{\mathcal{A}}} \frac{\beta\mathcal{P}(\alpha')\left(\underline{\ell} - w^p(\alpha')\right)}{\beta\mathcal{P}(\alpha') + \chi} d\alpha' + \frac{A\beta\left(\underline{\ell} - w^p(\alpha^s)\right)}{A\beta+\chi}\end{aligned} \tag{151}$$

Since $A, B, \beta, G, H, \underline{\ell}, \chi, \lambda > 0$, $I < 0$, $\mathcal{W}^{\mathcal{U}} \in (0, \tau^h)$, $\forall \alpha' \in (0,1), \mathcal{P}(\alpha') > 0$, and $\forall \alpha' \in$

$[\alpha^s, \alpha^{\mathcal{A}}), w^p(\alpha') < 0$, we have

$$\frac{A\beta\chi}{A\beta+\chi} - I + \frac{\lambda H}{G} + \frac{\lambda B}{G} > 0, \tag{152}$$

$$\frac{\lambda}{G}\int_{\alpha^s}^{\alpha^{\mathcal{A}}} \frac{\beta\mathcal{P}(\alpha')\left(\underline{\ell} - w^p(\alpha')\right)}{\beta\mathcal{P}(\alpha')+\chi} d\alpha' + \frac{A\beta\left(\underline{\ell} - w^p(\alpha^s)\right)}{A\beta+\chi} > 0. \tag{153}$$

Hence, $\partial\mathcal{W}^{\mathcal{U}}/\partial\chi > 0$. We also have that $\overline{w}^b(\alpha^s) = \overline{w}^p(\alpha^s) = \mathcal{W}^{\mathcal{U}}$. Hence, $\partial\overline{w}^b(\alpha^s)/\partial\chi = \partial\mathcal{W}^{\mathcal{U}}/\partial\chi > 0$.

From equation (109), we know that $\mathcal{W}^{\mathcal{F}} = \mathcal{W}^{\mathcal{U}} - \kappa m$. Hence, $\partial\mathcal{W}^{\mathcal{F}}/\partial\chi = \partial\mathcal{W}^{\mathcal{U}}/\partial\chi > 0$. Taking derivatives with respect to χ for equation (116), we get that

$$0 = \beta\left(C\frac{\partial\mathcal{W}^{\mathcal{F}}}{\partial\chi} + D\frac{\partial\alpha^{\mathcal{F}}}{\partial\chi}\right). \tag{154}$$

Given that $\beta, C, D > 0$, we have $\partial\alpha^{\mathcal{F}}/\partial\chi = -(D/C)(\partial\mathcal{W}^{\mathcal{F}}/\partial\chi) < 0$. This further implies that

$$\frac{\partial\mathbb{P}[\mathcal{M}(\alpha') \in \mathcal{F}]}{\partial\chi} = \frac{\partial(1-\alpha^{\mathcal{F}})}{\partial\chi} = -\frac{\partial\alpha^{\mathcal{F}}}{\partial\chi} > 0. \tag{155}$$

A.3 Proof of Proposition 3

Lemma A5.

$$\frac{\partial\alpha^{\mathcal{F}}}{\partial\lambda} > 0 \text{ and } \frac{\partial\overline{w}^b(\alpha^s)}{\partial\lambda} < 0$$

Proof. Taking derivatives of equations (115)-(117), we get

$$\begin{aligned}
\frac{\partial\Theta^s}{\partial\lambda} &= A\beta\frac{\partial\mathcal{W}^s}{\partial\lambda} = A\beta\left(\frac{\partial\mathcal{W}^{\mathcal{U}}}{\partial\lambda} - \frac{1}{\chi}\frac{\partial\Theta^s}{\partial\lambda}\right) \\
0 &= \beta\left[C\frac{\partial\mathcal{W}^{\mathcal{F}}}{\partial\lambda} + D\frac{\partial\alpha^{\mathcal{F}}}{\partial\lambda}\right] \\
\frac{\partial\Theta^s}{\partial\lambda} &= F + (E+K+J)\frac{\partial\alpha^{\mathcal{F}}}{\partial\lambda} + I\frac{\partial\mathcal{W}^{\mathcal{F}}}{\partial\lambda} - \frac{\lambda}{G}(B+H)\frac{\partial\mathcal{W}^{\mathcal{U}}}{\partial\lambda}
\end{aligned} \tag{156}$$

where F is another intermediate value given by

$$F \equiv -2\nu\left(1 + \frac{1-\alpha^{\mathcal{F}}}{1-\alpha^s}\right) + \frac{2\lambda'\nu(\alpha^{\mathcal{F}} - \alpha^s)}{(1-\alpha^s)(\rho+\lambda')} - \frac{1}{(1-\alpha^s)(\rho+\lambda')}\int_{\alpha^s}^{\alpha^{\mathcal{F}}} \Theta' d\alpha'$$

This is a system with only three unknowns, $\partial\mathcal{W}^{\mathcal{U}}/\partial\lambda$, $\partial\Theta^s/\partial\lambda$, and $\partial\alpha^{\mathcal{F}}/\partial\lambda$, because $\partial\mathcal{W}^{\mathcal{F}}/\partial\lambda = \partial\mathcal{W}^{\mathcal{U}}/\partial\lambda$, which derives from $\mathcal{W}^{\mathcal{F}} = \mathcal{W}^{\mathcal{U}} - \kappa m$. We can solve $\partial\alpha^{\mathcal{F}}/\partial\lambda$ as

$$\frac{\partial\alpha^{\mathcal{F}}}{\partial\lambda} = \frac{F}{\left(I - \frac{\lambda}{G}(B+H) - \frac{A\beta\chi}{A\beta+\chi}\right)(D/C) - (E+K+J)}. \tag{157}$$

To recap, we have

$$I < 0 \tag{158}$$
$$A, B, C, D, G, H > 0 \tag{159}$$
$$E + K + J = 0 \tag{160}$$

Note that $\lambda F = \Theta^s + 2\rho\nu < 0$. Hence, given $\lambda > 0$, we have $F < 0$. Together with $\beta, \chi > 0$, we have $\partial\alpha^{\mathcal{F}}/\partial\lambda > 0$.

Since $0 = C\cdot\partial\mathcal{W}^{\mathcal{U}}/\partial\lambda + D\cdot\partial\alpha^{\mathcal{F}}/\partial\lambda$ and $C, D, \partial\alpha^{\mathcal{F}}/\partial\lambda > 0$, then $\partial\mathcal{W}^{\mathcal{U}}/\partial\lambda = \partial w^{h,p}(\alpha^{\mathcal{U}})/\partial\lambda < 0$. By definition, $w^p(\alpha^{\mathcal{U}}) = 0$. Combining the triparty and bilateral repo market clearing conditions, we arrive at $w^{h,p}(\alpha^{\mathcal{U}}) = \overline{w}^p(\alpha^{\mathcal{U}}) = \overline{w}^p(\alpha^s)$. Noting the shadow bank balance sheet constraint, we have $\overline{w}^b(\alpha^s) = \overline{w}^p(\alpha^s)$. Hence, $\partial\overline{w}^b(\alpha^s)/\partial\lambda = \partial\mathcal{W}^{\mathcal{U}}/\partial\lambda < 0$. □

Lemma A6.

$$\frac{\partial}{\partial\lambda}\mathbb{E}[r^p(\alpha')] < 0$$

Proof. From the traditional bank first-order conditions, we get that

$$r^p(\alpha') = r^p(\alpha') - r^{pt}(\alpha') + r^{pt}(\alpha') - r^d(\alpha') + r^d(\alpha') - r^k + r^k = \Theta' + r^k \tag{161}$$

This can be further written as

$$r^p(\alpha') = \begin{cases} -\chi(\underline{\ell} - w^p(\alpha')) + r^k & \text{if } \alpha' \in (\alpha^s, \alpha^{\mathcal{A}}) \\ \beta\left(\frac{\alpha'}{\tau^h - \mathcal{W}^s} - \frac{1-\alpha'}{\mathcal{W}^s}\right) + r^k & \text{if } \alpha' \in [\alpha^{\mathcal{A}}, \alpha^{\mathcal{U}}) \\ r^k & \text{if } \alpha' \in [\alpha^{\mathcal{U}}, \alpha^{\mathcal{C}}] \\ \beta\left(\frac{\alpha'}{\tau^h - \mathcal{W}^{\mathcal{F}}} - \frac{1-\alpha'}{\mathcal{W}^{\mathcal{F}}}\right) + r^k & \text{if } \alpha' \in (\alpha^{\mathcal{C}}, \alpha^{\mathcal{F}}) \\ 4\lambda'\nu + 2\rho\nu + r^k & \text{if } \alpha' \in [\alpha^{\mathcal{F}}, 1) \end{cases}$$

It can be seen that $r^p(\alpha')$ is a non-decreasing function of α'. We have $\partial\mathcal{W}^{\mathcal{U}}/\partial\lambda = \partial\mathcal{W}^{\mathcal{F}}/\partial\lambda < 0$ from Lemma A5. This further implies

$$\frac{\partial}{\partial\lambda}\left[-\chi(\underline{\ell} - w^p(\alpha')) + r^k\right] = \frac{\beta\mathcal{P}(\alpha')\chi}{\beta\mathcal{P}(\alpha') + \chi}\frac{\partial\mathcal{W}^{\mathcal{U}}}{\partial\lambda} < 0, \tag{162}$$
$$\frac{\partial}{\partial\lambda}\left[\beta\left(\frac{\alpha'}{\tau^h - \mathcal{W}^{\mathcal{U}}} - \frac{1-\alpha'}{\mathcal{W}^{\mathcal{U}}}\right) + r^k\right] = \beta\left(\frac{\alpha'}{(\tau^h - \mathcal{W}^{\mathcal{U}})^2} + \frac{1-\alpha'}{(\mathcal{W}^{\mathcal{U}})^2}\right)\frac{\partial\mathcal{W}^{\mathcal{U}}}{\partial\lambda} < 0, \tag{163}$$
$$\frac{\partial}{\partial\lambda}\left[\beta\left(\frac{\alpha'}{\tau^h - \mathcal{W}^{\mathcal{F}}} - \frac{1-\alpha'}{\mathcal{W}^{\mathcal{F}}}\right) + r^k\right] = \beta\left(\frac{\alpha'}{(\tau^h - \mathcal{W}^{\mathcal{F}})^2} + \frac{1-\alpha'}{(\mathcal{W}^{\mathcal{F}})^2}\right)\frac{\partial\mathcal{W}^{\mathcal{F}}}{\partial\lambda} < 0, \tag{164}$$
$$\frac{\partial\alpha^{\mathcal{A}}}{\partial\lambda} = -\frac{\mathcal{P}(\alpha^{\mathcal{A}})(\tau^h - \mathcal{W}^{\mathcal{U}})\mathcal{W}^{\mathcal{U}}}{\tau^h}\frac{\partial\mathcal{W}^{\mathcal{U}}}{\partial\lambda} > 0, \tag{165}$$
$$\frac{\partial\alpha^{\mathcal{U}}}{\partial\lambda} = \frac{\partial}{\partial\lambda}\left[\frac{\tau^h - \mathcal{W}^{\mathcal{U}}}{\tau^h}\right] = -\frac{1}{\tau^h}\frac{\partial\mathcal{W}^{\mathcal{U}}}{\partial\lambda} > 0, \tag{166}$$
$$\frac{\partial\alpha^{\mathcal{C}}}{\partial\lambda} = \frac{\partial}{\partial\lambda}\left[\frac{\tau^h - \mathcal{W}^{\mathcal{F}}}{\tau^h}\right] = -\frac{1}{\tau^h}\frac{\partial\mathcal{W}^{\mathcal{F}}}{\partial\lambda} > 0. \tag{167}$$

Together with $\partial\alpha^{\mathcal{F}}/\partial\lambda > 0$ and $r^p(\alpha^{\mathcal{A}}) = -\chi\underline{\ell} + r^k$ and $4\lambda'\nu + 2\rho\nu + r^k$ irrelevant with λ, we have that $\partial r^p(\alpha')/\partial\lambda < 0$ when $\alpha \in (\alpha^s, \alpha^{\mathcal{U}})$ or $\alpha \in (\alpha^{\mathcal{C}}, \alpha^{\mathcal{F}})$, and $\partial r^p(\alpha')/\partial\lambda = 0$ when $\alpha \in [\alpha^{\mathcal{U}}, \alpha^{\mathcal{C}}]$ or $\alpha \in [\alpha^{\mathcal{F}}, 1)$.

This further implies that

$$\frac{\partial \mathbb{E}[r^p(\alpha')]}{\partial \lambda} < 0. \tag{168}$$

□

Proposition 3 then directly follows from Lemmas A5 and A6.

A.4 Proof of Proposition 4

Lemma A7.

$$\frac{\partial \overline{w}^b(\alpha^s)}{\partial \lambda'} > 0 \text{ and } \frac{\partial \mathbb{E}[r^p(\alpha')]}{\partial \lambda'} > 0$$

Proof. Taking derivatives of equations (115)-(117), we get

$$\begin{aligned}
\frac{\partial \Theta^s}{\partial \lambda'} &= A\beta \frac{\partial \mathcal{W}^s}{\partial \lambda'} = A\beta \left(\frac{\partial \mathcal{W}^{\mathcal{U}}}{\partial \lambda'} - \frac{1}{\chi} \frac{\partial \Theta^s}{\partial \lambda'} \right) \\
\frac{4\nu}{\beta} &= C \frac{\partial \mathcal{W}^{\mathcal{U}}}{\partial \lambda'} + D \frac{\partial \alpha^{\mathcal{F}}}{\partial \lambda'} \\
\frac{\partial \Theta^s}{\partial \lambda'} &= L + (E + K + J) \frac{\partial \alpha^{\mathcal{F}}}{\partial \lambda'} + \left(I - \frac{\lambda}{G}(B + H) \right) \frac{\partial \mathcal{W}^{\mathcal{U}}}{\partial \lambda'}
\end{aligned}$$

where the new intermediate L is defined as

$$L \equiv \frac{2\lambda\nu\rho(\alpha^{\mathcal{F}} - \alpha^s)}{(1 - \alpha^s)(\rho + \lambda')^2} + \frac{\lambda}{(\rho + \lambda')^2(1 - \alpha^s)} \int_{\alpha^s}^{\alpha^{\mathcal{F}}} \Theta' d\alpha'$$

Given $E + K + J = 0$, we can solve for $\partial \mathcal{W}^{\mathcal{U}} / \partial \lambda'$ as

$$\frac{\partial \mathcal{W}^{\mathcal{U}}}{\partial \lambda'} = \frac{L}{\frac{A\beta\chi}{A\beta + \chi} + \frac{\lambda}{G}(B + H) - I}. \tag{169}$$

Note that $\Theta' \geq 0$ when $\alpha' \in [\alpha^{\mathcal{U}}, \alpha^{\mathcal{F}}]$. This implies

$$\begin{aligned}
L &= \frac{\lambda}{(\rho + \lambda')^2(1 - \alpha^s)} \left(2\nu\rho(\alpha^{\mathcal{F}} - \alpha^s) + \int_{\alpha^s}^{\alpha^{\mathcal{F}}} \Theta' d\alpha' \right) \\
&\geq \frac{\lambda}{(\rho + \lambda')^2(1 - \alpha^s)} \left(2\nu\rho(\alpha^{\mathcal{F}} - \alpha^s) + \int_{\alpha^s}^{\alpha^{\mathcal{U}}} \Theta' d\alpha' \right) \\
&\geq \frac{\lambda}{(\rho + \lambda')^2(1 - \alpha^s)} \left(2\nu\rho(\alpha^{\mathcal{F}} - \alpha^s) - 2\nu\rho(\alpha^{\mathcal{U}} - \alpha^s) \right)
\end{aligned} \tag{170}$$

The last inequality stems from the fact that $\Theta' = (\rho + \lambda')(\theta(\alpha') - \overline{\theta}(\alpha')) + 2\lambda'\nu$ where $\theta - \overline{\theta} \geq -2\nu$. Hence, $L > 0$ given that $\alpha^{\mathcal{F}} > \alpha^{\mathcal{U}}$. Together with $A, B, G, H, \beta, \lambda, \chi > 0$ and $I < 0$, $\partial \mathcal{W}^{\mathcal{U}} / \partial \lambda' > 0$. By definition, $w^p(\alpha^{\mathcal{U}}) = 0$. Combining the triparty and bilateral repo market clearing conditions, we arrive at $\mathcal{W}^{\mathcal{U}} = w^{h,p}(\alpha^{\mathcal{U}}) = \overline{w}^p(\alpha^{\mathcal{U}}) = \overline{w}^p(\alpha^s)$. Noting the shadow bank balance sheet constraint, we have $\overline{w}^b(\alpha^s) = \overline{w}^p(\alpha^s)$. Hence, $\partial \overline{w}^b(\alpha^s)/\partial \lambda' = \partial \mathcal{W}^{\mathcal{U}} / \partial \lambda' > 0$.

From the proof of Lemma A6, we have that

$$\mathbb{E}[r^p(\alpha')] = \int_{\alpha^s}^{1} \Theta' f(\alpha') d\alpha' + r^k \tag{171}$$

Taking derivatives with respect to λ', we get

$$\begin{aligned}\frac{\partial \mathbb{E}[r^p(\alpha')]}{\partial \lambda'} =& \frac{1}{1-\alpha^s}\left(4\nu(1-\alpha^{\mathcal{F}}) + \left(B + H - \frac{GI}{\lambda}\right)\frac{\partial \mathcal{W}^{\mathcal{U}}}{\partial \lambda'}\right) \\ &- \left(\frac{\Theta^{\mathcal{F}}}{1-\alpha^s} + \frac{\rho+\lambda'}{\lambda}J\right)\frac{\partial \alpha^{\mathcal{F}}}{\partial \lambda'}.\end{aligned} \tag{172}$$

Given that $J = -\lambda\Theta^{\mathcal{F}}/G$, we get

$$\frac{\partial \mathbb{E}[r^p(\alpha')]}{\partial \lambda'} = \frac{1}{1-\alpha^s}\left(4\nu(1-\alpha^{\mathcal{F}}) + \left(B + H - \frac{GI}{\lambda}\right)\frac{\partial \mathcal{W}^{\mathcal{U}}}{\partial \lambda'}\right) \tag{173}$$

Given that $B, G, H, \nu, \lambda, \partial\mathcal{W}^{\mathcal{U}}/\partial\lambda' > 0$, $\alpha^s, \alpha^{\mathcal{F}} \in (0,1)$, and $I < 0$, we have $\partial\mathbb{E}[r^p(\alpha')]/\partial\lambda' > 0$. □

Lemma A8.

$$\frac{\partial \mathbb{E}[\underline{r}^b(\alpha') - r^p(\alpha')]}{\partial \lambda'} < 0. \tag{174}$$

Proof. From equation (132), we get that

$$\underline{r}^b(\alpha') - r^p(\alpha') = \max\{r^d(\alpha') - r^p(\alpha') + \chi\ell(\alpha') - \rho\nu, -(\rho+2\lambda')\nu\}, \tag{175}$$

By the traditional bank first-order conditions, we have that

$$\underline{r}^b(\alpha') - r^p(\alpha') = \max\{-\Theta' - \rho\nu, -(\rho+2\lambda')\nu\}. \tag{176}$$

This can be further written as

$$\underline{r}^b(\alpha') - r^p(\alpha') = \begin{cases} \chi(\underline{\ell} - w^p(\alpha')) - \rho\nu & \text{if } \alpha' \in (\alpha^s, \alpha^{\mathcal{A}}) \\ -\beta\left(\frac{\alpha'}{\tau^h - \mathcal{W}^{\mathcal{U}}} - \frac{1-\alpha'}{\mathcal{W}^{\mathcal{U}}}\right) - \rho\nu & \text{if } \alpha' \in [\alpha^{\mathcal{A}}, \alpha^{\mathcal{U}}) \\ -\rho\nu & \text{if } \alpha' \in [\alpha^{\mathcal{U}}, \alpha^{\mathcal{C}}] \\ -\beta\left(\frac{\alpha'}{\tau^h - \mathcal{W}^{\mathcal{F}}} - \frac{1-\alpha'}{\mathcal{W}^{\mathcal{F}}}\right) - \rho\nu & \text{if } \alpha' \in (\alpha^{\mathcal{C}}, \alpha^{\Theta}) \\ -(\rho+2\lambda')\nu & \text{if } \alpha' \in [\alpha^{\Theta}, 1). \end{cases} \tag{177}$$

It can be seen that $\underline{r}^b(\alpha') - r^p(\alpha')$ is a non-increasing function of α'. We also have $\partial\mathcal{W}^{\mathcal{U}}/\partial\lambda' =$

$\partial \mathcal{W}^{\mathcal{F}}/\partial \lambda' > 0$ from Lemma A7. This further implies

$$\frac{\partial}{\partial \lambda'}\left[\chi(\underline{\ell} - w^p(\alpha')) - \rho\nu\right] = -\frac{\beta\mathcal{P}(\alpha')\chi}{\beta\mathcal{P}(\alpha') + \chi}\frac{\partial \mathcal{W}^{\mathcal{U}}}{\partial \lambda'} < 0, \tag{178}$$

$$\frac{\partial}{\partial \lambda'}\left[-\beta\left(\frac{\alpha'}{\tau^h - \mathcal{W}^{\mathcal{U}}} - \frac{1-\alpha'}{\mathcal{W}^{\mathcal{U}}}\right) - \rho\nu\right] = -\beta\left(\frac{\alpha'}{(\tau^h - \mathcal{W}^{\mathcal{U}})^2} + \frac{1-\alpha'}{(\mathcal{W}^{\mathcal{U}})^2}\right)\frac{\partial \mathcal{W}^{\mathcal{U}}}{\partial \lambda'} < 0, \tag{179}$$

$$\frac{\partial}{\partial \lambda'}\left[-\beta\left(\frac{\alpha'}{\tau^h - \mathcal{W}^{\mathcal{F}}} - \frac{1-\alpha'}{\mathcal{W}^{\mathcal{F}}}\right) - \rho\nu\right] = -\beta\left(\frac{\alpha'}{(\tau^h - \mathcal{W}^{\mathcal{F}})^2} + \frac{1-\alpha'}{(\mathcal{W}^{\mathcal{F}})^2}\right)\frac{\partial \mathcal{W}^{\mathcal{F}}}{\partial \lambda'} < 0, \tag{180}$$

$$\frac{\partial \alpha^{\mathcal{A}}}{\partial \lambda'} = -\frac{\mathcal{P}(\alpha^{\mathcal{A}})(\tau^h - \mathcal{W}^{\mathcal{U}})\mathcal{W}^{\mathcal{U}}}{\tau^h}\frac{\partial \mathcal{W}^{\mathcal{U}}}{\partial \lambda'} < 0, \tag{181}$$

$$\frac{\partial \alpha^{\mathcal{U}}}{\partial \lambda'} = \frac{\partial}{\partial \lambda'}\left[\frac{\tau^h - \mathcal{W}^{\mathcal{U}}}{\tau^h}\right] = -\frac{1}{\tau^h}\frac{\partial \mathcal{W}^{\mathcal{U}}}{\partial \lambda'} < 0, \tag{182}$$

$$\frac{\partial \alpha^{\mathcal{C}}}{\partial \lambda'} = \frac{\partial}{\partial \lambda'}\left[\frac{\tau^h - \mathcal{W}^{\mathcal{F}}}{\tau^h}\right] = -\frac{1}{\tau^h}\frac{\partial \mathcal{W}^{\mathcal{F}}}{\partial \lambda'} < 0. \tag{183}$$

Together with $-(\rho + 2\lambda')\nu$ decreasing with λ', we have that $\partial(\underline{r}^b(\alpha') - r^p(\alpha'))/\partial \lambda' < 0$ when $\alpha \in (\alpha^s, \alpha^{\mathcal{U}})$ or $\alpha \in (\alpha^{\mathcal{C}}, 1)$, and $\partial(\underline{r}^b(\alpha') - r^p(\alpha'))/\partial \lambda' = 0$ when $\alpha \in [\alpha^{\mathcal{U}}, \alpha^{\mathcal{C}}]$.

This further implies that

$$\frac{\partial \mathbb{E}[\underline{r}^b(\alpha') - r^p(\alpha')]}{\partial \lambda'} < 0. \tag{184}$$

□

Proposition 4 then directly follows from Lemmas A7 and A8.

We also provide a lemma regarding how the probability of a fire sale changes with λ' here.

Lemma A9. *Under the restriction that*

$$\frac{\partial \Theta(\alpha^{\mathcal{F}}, \mathcal{W}^{\mathcal{F}})}{\partial \mathcal{W}^{\mathcal{F}}} \Bigg/ \frac{\partial \Theta^s}{\partial \mathcal{W}^{\mathcal{U}}} < \frac{\rho + \lambda'}{\lambda}, \tag{185}$$

we have

$$\frac{\partial \mathbb{P}(\mathcal{M}(\alpha') \in \mathcal{F})}{\partial \lambda'} < 0. \tag{186}$$

Proof. We can also solve for $\partial \alpha^{\mathcal{F}}/\partial \lambda'$ as

$$\frac{\partial \alpha^{\mathcal{F}}}{\partial \lambda'} = \frac{1}{D}\left(\frac{4\nu}{\beta} - C\frac{L}{\frac{A\beta\chi}{A\beta+\chi} + \frac{\lambda}{G}(B+H) - I}\right)$$

Given that $A, B, C, D, G, H, L, \beta, \chi > 0$ and $I < 0$, we have

$$\frac{\partial \alpha^{\mathcal{F}}}{\partial \lambda'} > \frac{1}{\beta D}\left(4\nu - \frac{C}{\frac{A\chi}{A\beta+\chi}}L\right) \tag{187}$$

Let's next put an upper bound on L:

$$\begin{aligned} L &= \frac{\lambda}{(1-\alpha^s)(\rho+\lambda')^2}\left(2\nu\rho(\alpha^{\mathcal{F}}-\alpha^s)+\int_{\alpha^s}^{\alpha^{\mathcal{F}}}\Theta' d\alpha'\right) \\ &\leq \frac{\lambda}{(1-\alpha^s)(\rho+\lambda')^2}\left(2\nu\rho(\alpha^{\mathcal{F}}-\alpha^s)+\int_{\alpha^{\mathcal{C}}}^{\alpha^{\mathcal{F}}}\Theta' d\alpha'\right) \\ &= \frac{\lambda\nu}{(1-\alpha^s)(\rho+\lambda')^2}\left(2\rho(\alpha^{\mathcal{F}}-\alpha^s)+(\rho+2\lambda')(\alpha^{\mathcal{F}}-\alpha^{\mathcal{C}})\right) \\ &< \frac{\lambda\nu}{(1-\alpha^s)(\rho+\lambda')^2}(3\rho+2\lambda')(\alpha^{\mathcal{F}}-\alpha^s) < \frac{3\lambda\nu(\alpha^{\mathcal{F}}-\alpha^s)}{(1-\alpha^s)(\rho+\lambda')} < \frac{3\lambda\nu}{\rho+\lambda'}. \end{aligned} \tag{188}$$

Also, the additional restriction (185) simply implies that

$$\frac{C}{\frac{A\chi}{A\beta+\chi}} < \frac{\rho+\lambda'}{\lambda} \tag{189}$$

Hence, we have

$$\frac{\partial\alpha^{\mathcal{F}}}{\partial\lambda'} > \frac{1}{\beta D}\left(4\nu - \frac{\rho+\lambda'}{\lambda}\frac{3\lambda\nu}{\rho+\lambda'}\right) > 0 \tag{190}$$

which further implies that

$$\frac{\partial\mathbb{P}(\mathcal{M}(\alpha')\in\mathcal{F})}{\partial\lambda'} = \frac{\partial}{\partial\lambda'}(1-\alpha^{\mathcal{F}}) = -\frac{\partial\alpha^{\mathcal{F}}}{\partial\lambda'} < 0. \tag{191}$$

□

A.5 Proof of Lemma 1

Lemma A10. *In the absence of repo and reverse repo facilities, the IL constraint binds ($\vartheta_t^m > 0$) if and only if $b - \underline{b} - w_t^b > (1-\alpha_t)\tau^h + \kappa m$.*

Proof. Assume that $b - \underline{b} - w_t^b > (1-\alpha_t)\tau^h + \kappa m$. Combining the market-clearing conditions, we get $w_t^{h,p} = b - \underline{b} - w_t^b - w_t^p$. Thus, $w_t^{h,p} > (1-\alpha_t)\tau^h + \kappa m_t - w_t^p \geq (1-\alpha_t)\tau^h$. Then, by the household first-order condition, $r_t^{pt} > r_t^d$ and by equations (21), (22), (23), and (24), $\vartheta_t^m > 0$.

If $\vartheta_t^m > 0$, then, $w_t^p = \kappa w_t^m = \kappa m > 0$ and by equations (21), (23), and (24), $r_t^{pt} > r_t^d$. Thus, by the household first-order condition, $w_t^{h,p} > (1-\alpha_t)\tau^h$. Combining the market-clearing conditions, we get $b - \underline{b} - w_t^b - \kappa m = w_t^{h,p} > (1-\alpha_t)\tau^h$. □

The proof of Lemma 1 then follows.

If $\vartheta_t^m > 0$, then $w_t^p = \kappa w_t^m = \kappa m > 0$. Thus, by equations (22) and (23), $r_t^p > r_t^m$. If $r_t^p > r_t^m$, by equation (22) and (23), $\vartheta_t^m > 0$.

Thus, given Lemma A10, $r_t^p > r_t^m$ if and only if $b - \underline{b} - w_t^b > (1-\alpha_t)\tau^h + \kappa m$.

A.6 Proof of Proposition 5

Lemma A11.

$$\frac{\partial \mathbb{P}(\mathcal{M}(\alpha') \in \mathcal{F})}{\partial m} < 0 \ \textit{and}\ \frac{\partial \overline{w}^b(\alpha^s)}{\partial m} > 0. \tag{192}$$

Proof. Taking derivatives of equations (115)-(117), we get

$$\begin{aligned}
\frac{\partial \Theta^s}{\partial m} &= A\beta \frac{\partial \mathcal{W}^s}{\partial m} = A\beta \left(\frac{\partial \mathcal{W}^{\mathcal{U}}}{\partial m} - \frac{1}{\chi} \frac{\partial \Theta^s}{\partial m} \right) \\
0 &= C \frac{\partial \mathcal{W}^{\mathcal{U}}}{\partial m} - C\kappa + D \frac{\partial \alpha^{\mathcal{F}}}{\partial m} \\
\frac{\partial \Theta^s}{\partial m} &= (E + K + J) \frac{\partial \alpha^{\mathcal{F}}}{\partial m} + \left(I - \frac{\lambda}{G}(B + H) \right) \frac{\partial \mathcal{W}^{\mathcal{U}}}{\partial m} - I\kappa
\end{aligned}$$

We combine these expressions to solve for $\partial \alpha^{\mathcal{F}} / \partial m$:

$$\frac{\partial \alpha^{\mathcal{F}}}{\partial m} = \frac{\kappa \left(\frac{A\beta\chi}{A\beta + \chi} + \frac{\lambda}{G}(B + H) \right)}{(E + K + J) + \frac{D}{C} \left(\frac{A\beta\chi}{A\beta + \chi} + \frac{\lambda}{G}(B + H) - I \right)}.$$

Since $A, B, C, D, G, H, \lambda, \beta, \chi, \kappa > 0, E + K + J = 0$ and $I < 0$, then $\partial \alpha^{\mathcal{F}} / \partial m > 0$ and $\partial \mathbb{P}(\mathcal{M}(\alpha') \in \mathcal{F}) / \partial m < 0$.

We can also solve for $\partial \mathcal{W}^{\mathcal{U}} / \partial m$ as

$$\begin{aligned}
\frac{\partial \mathcal{W}^{\mathcal{U}}}{\partial m} &= \kappa - \frac{D}{C} \frac{\kappa \left(\frac{A\beta\chi}{A\beta + \chi} + \frac{\lambda}{G}(B + H) \right)}{(E + K + J) + \frac{D}{C} \left(\frac{A\beta\chi}{A\beta + \chi} + \frac{\lambda}{G}(B + H) - I \right)} \\
&= \frac{-I\kappa}{\frac{A\beta\chi}{A\beta + \chi} + \frac{\lambda}{G}(B + H) - I} > 0.
\end{aligned} \tag{193}$$

Given $w^p(\alpha^{\mathcal{U}}) = 0$, combining the triparty and bilateral repo market clearing conditions, we arrive at $w^{h,p}(\alpha^{\mathcal{U}}) = \overline{w}^p(\alpha^{\mathcal{U}}) = \overline{w}^p(\alpha^s)$. Then, from the shadow bank balance sheet constraint we have $\overline{w}^b(\alpha^s) = \overline{w}^p(\alpha^s)$. Hence, $\partial \overline{w}^b(\alpha^s) / \partial m = \partial \mathcal{W}^{\mathcal{U}} / \partial m > 0$. ☐

Lemma A12.

$$\frac{\partial}{\partial m} \mathbb{E}[r^p(\alpha')] < 0$$

Proof. From Lemma A6, we have

$$\mathbb{E}[r^p(\alpha'))] = \frac{1}{1 - \alpha^s} \left(\int_{\alpha^s}^{1} \Theta(\alpha') d\alpha' \right) + r^k. \tag{194}$$

Taking the partial derivative with respect to m and noting that $\partial \mathcal{W}^{\mathcal{U}} / \partial m = \partial \mathcal{W}^{\mathcal{F}} / \partial m - \kappa$, we get

$$\frac{\partial}{\partial m}\mathbb{E}[r^p(\alpha')] = \frac{\rho+\lambda'}{\lambda}\left(\left(\frac{\lambda}{G}(B+H)-I\right)\frac{\partial \mathcal{W}^{\mathcal{U}}}{\partial m}+\kappa I\right) - \left(\frac{\Theta^{\mathcal{F}}}{1-\alpha^s}+\frac{\rho+\lambda'}{\lambda}J\right)\frac{\partial \alpha^{\mathcal{F}}}{\partial m}. \tag{195}$$

Given that $J = -\lambda\Theta^{\mathcal{F}}/G$ and together with equation (193), we get

$$\begin{aligned}\frac{\partial}{\partial m}\mathbb{E}[r^p(\alpha')] &= \frac{\rho+\lambda'}{\lambda}\kappa\left(-\frac{\left(\frac{\lambda}{G}(B+H)-I\right)I}{\frac{A\beta\chi}{A\beta+\chi}+\frac{\lambda}{G}(B+H)-I}+I\right)\\ &= \frac{\rho+\lambda'}{\lambda}\frac{\frac{A\beta\chi}{A\beta+\chi}I\kappa}{\frac{A\beta\chi}{A\beta+\chi}+\frac{\lambda}{G}(B+H)-I}.\end{aligned} \tag{196}$$

Since $\kappa, \rho, \lambda', \lambda, A, B, G, H, \beta, \chi > 0$ and $I < 0$, then

$$\frac{\partial}{\partial m}\mathbb{E}[r^p(\alpha')] < 0. \tag{197}$$

□

When there are no facilities, $\underline{b} = m + a$. Hence, $\partial\underline{b}/\partial m = 1$. This further implies that $\partial\mathbb{P}(\mathcal{M}(\alpha') \in \mathcal{F})/\partial\underline{b} = \partial\mathbb{P}(\mathcal{M}(\alpha') \in \mathcal{F})/\partial m < 0$ from Lemma A11, and $\partial E[r^p(\alpha')]/\partial\underline{b} = \partial E[r^p(\alpha')]/\partial m < 0$ from Lemma A12. This concludes the proof for Proposition 5.

B Proofs of Section 4

B.1 Proof of Lemma 2

Liquidity services $h(w_t^{h,p}, w_t^{h,d}, \alpha_t)$, given the budget constraint (17), are maximized when $w_t^{h,p} = (1-\alpha_t)\tau^h$ and $w_t^{h,d} = \alpha_t\tau^h$. Given households' first-order condition for triparty repo in equation (25), this occurs if and only if $r_t^{pt} = r_t^d$. Combining equations (21) and (24) obtains $r_t^p - r^k = r_t^{pt} - r_t^d$. Thus, $r_t^{pt} = r_t^d$ if and only if $r_t^p = r^k$.

If $\vartheta_t^m > 0$, then $w_t^p = \kappa w_t^m > 0$ since $m_t > 0$ and $r_t^p > r^k$ given the first-order condition for bilateral repo of traditional banks in equation (23). Thus, if $r_t^p = r^k$, then $\vartheta_t^m = 0$ and $\mathcal{M}(\mathbf{x}_t) \in \mathcal{U}$.

If $\mathcal{M}(\mathbf{x}_t) \in \mathcal{U}$, then $\vartheta_t^m = 0$. Assume the contrary: $\vartheta_t^m > 0$. Then $w_t^p = \kappa w_t^m > 0$ since $m_t > 0$ and $r^k = r_t^m + \kappa\vartheta_t^m = r_t^p - \vartheta_t^m$. Thus, $r_t^m < r_t^p$, a contradiction. Thus, $r^k = r_t^m = r_t^p$.

Therefore, liquidity services are at the optimum if and only if $\mathcal{M}(\mathbf{x}_t) \in \mathcal{U}$.

B.2 Proof of Lemma 3

Assume by way of contradiction that $r^{rrp} > r^{pt}$. Then households have a better investment opportunity than the market rate for triparty repo.

Assume by way of contradiction that $r^{rp} < r^{pt}$ with a repo facility open only to traditional banks. Then traditional bank dealers have a cheaper funding option available than the triparty repo rate.

Assume by way of contradiction that $r^{rp} < r^p$ with a broad-access repo facility. Then shadow banks have a better funding rate available than the market rate of bilateral repo.

B.3 Proof of Proposition 6

From the traditional bank first-order conditions, we have that

$$r_t^p = \begin{cases} r_t^m + \kappa\vartheta_t^m - \chi\ell_t & \text{if } w_t^p < 0, \\ \in [r_t^m + \kappa\vartheta_t^m - \chi\ell_t, r_t^m + (1+\kappa)\vartheta_t^m] & \text{if } w_t^p = 0, \\ r_t^m + (1+\kappa)\vartheta_t^m & \text{if } w_t^p > 0. \end{cases} \tag{198}$$

If $\vartheta_t^m = 0$, then $r_t^p \leq r_t^m$. Hence, if $r_t^p > r_t^m$, then $\vartheta_t^m > 0$.

Assume $\chi = 0$. From the traditional bank first-order conditions, we have $r_t^p = r_t^{pt} = r_t^d + \vartheta_t^m > r_t^d = r^k = r_t^m + \kappa\vartheta_t^m$. Thus, $r_t^{pt} > r_t^m > r^{rrp}$ and $rrp_t = 0$. For markets to clear, we need $\mathcal{W}(r_t^{pt} - r_t^d, \alpha) = b - \underline{b} - w_t^b - \kappa(\underline{b} - a) - (1+\kappa)rp_t$. Since $rp_t \geq 0$, $\mathcal{W}(r_t^{pt} - r_t^d, \alpha) \leq b - \underline{b} - w_t^b - \kappa(\underline{b} - a)$. Given that $r_t^{pt} > r_t^d$, $\mathcal{W}(r_t^{pt} - r_t^d, \alpha) > \mathcal{W}(0, \alpha) = (1-\alpha)\tau^h$, and we have a contradiction.

Assume $rrp_t = 0$. Then, $m_t + a \geq \underline{b}$. Furthermore, $r_t^d = r^k - \chi\ell_t = r_t^p - \vartheta_t^m - \chi\ell_t = r_t^{pt} - \vartheta_t^m$ and $r_t^{pt} > r_t^d$. For markets to clear, we need $\mathcal{W}(r_t^{pt} - r_t^d, \alpha) = b - \underline{b} - w_t^b - \kappa(\underline{b} - a) - (1+\kappa)rp_t$. Since $rp_t \geq 0$, $\mathcal{W}(r_t^{pt} - r_t^d, \alpha)n^h \leq b - \underline{b} - w_t^b - \kappa(\underline{b} - a)$. Given that $r_t^{pt} > r_t^d$, $\mathcal{W}(r_t^{pt} - r_t^d, \alpha) > \mathcal{W}(0, \alpha) = (1-\alpha)\tau^h$, we have a contradiction.

Thus, under Condition (36), if $r_t^p > r_t^m$, then $\vartheta_t^m > 0$, $\chi > 0$, and $rrp_t > 0$.

Finally, if $\vartheta_t^m > 0$ (and $\chi > 0$, $rrp_t > 0$)—i.e., the IL constraint binds—the traditional bank has positive repo supply $w_t^p > 0$ and $r_t^p > r_t^m$. This concludes the proof.

B.4 Proof of Proposition 7

Assume by way of contradiction that $\exists \mathbf{x}'$ such that $\overline{w}^b(\mathbf{x}') < \overline{w}^b(\mathbf{x}^s)$. Then from the optimal weight for $\overline{w}^b$, we have that for the shadow bank to be incentivized to decrease its holding of Treasuries and pay the adjustment cost when moving from $\mathbf{x}^s$ to $\mathbf{x}'$ and vice versa, then it must be that $\overline{\theta}(\mathbf{x}') \leq -\nu$ and $\overline{\theta}(\mathbf{x}^s) = \nu$. Thus, there cannot exist another state $\mathbf{x}''$ such that $\overline{w}^b(\mathbf{x}^s) < \overline{w}^b(\mathbf{x}'')$. Given the market-clearing condition for the Treasury market

$$\overline{w}_t^b + w_t^b = b_t - \underline{b}_t \tag{199}$$

the reverse must be true for $w^b(\mathbf{x})$ and $\theta(\mathbf{x})$[32]. In addition, given the assumption that $w^b(\mathbf{x}^s) > 0$, we have $\theta(\mathbf{x}^s) = -\nu$ and $\theta(\mathbf{x}'') \geq -\nu$ for all shock states $\mathbf{x}''$.

Then, the envelope theorem for shadow banks and traditional banks at the steady state implies

[32]For shock on α, f, and a, b and $\underline{b}$ do not change. For issuance shock, b increases, but this still implies that $w^b(\mathbf{x}^s) < w^b(\mathbf{x}')$.

that

$$
\begin{aligned}
r^p(\mathbf{x}^s) - r^d(\mathbf{x}^s) =& \chi\ell(\mathbf{x}^s) - 2\nu(\rho+\lambda) \\
& - \lambda\nu \int_{\overline{w}^b(\mathbf{x}')<\overline{w}^b(\mathbf{x}^s)} f(\mathbf{x}')d\mathbf{x}' + \lambda \int_{\overline{w}^b(\mathbf{x}')=\overline{w}^b(\mathbf{x}^s)} \overline{\theta}(\mathbf{x}')f(\mathbf{x}')d\mathbf{x}' \\
& - \lambda\nu \int_{w^b(\mathbf{x}')>w^b(\mathbf{x}^s)} f(\mathbf{x}')d\mathbf{x}' - \lambda \int_{w^b(\mathbf{x}')=w^b(\mathbf{x}^s)} \theta(\mathbf{x}')f(\mathbf{x}')d\mathbf{x}'
\end{aligned}
$$

Since $\overline{\theta} \leq \nu$ and $\theta \geq -\nu$, $r^p(\mathbf{x}^s) - r^d(\mathbf{x}^s) \leq \chi\ell(\mathbf{x}^s) - 2\nu\rho$. When $\chi = 0$, this further implies that $r^p(\mathbf{x}^s) - r^d(\mathbf{x}^s) \leq -2\nu\rho < 0$. However, by traditional bank first-order condition, we always have that $r_t^p - r_t^d \geq 0$, a contradiction with $r^p(\mathbf{x}^s) - r^d(\mathbf{x}^s) < 0$. Hence, if $\chi = 0$, shadow banks never sell their treasuries. This concludes the proof.