The Optimal Supply of Central Bank Reserves under Uncertainty

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Motivation

- What is the optimal supply of reserves the central bank should provide?
- Central banks have multiple goals in supplying reserves, including
  - Targeting policy rate
  - Managing balance-sheet size
- Different jurisdictions have different implementation frameworks
- Banks' demand for reserves is highly nonlinear and uncertain

This Paper

- Tractable analytical framework for optimal supply of reserves
- Demand for reserves is nonlinear and uncertain
  - ◇ Regions: abundant (zero slope), ample (gentle slope), scarce (steep slope)
  - ◇ Uncertainty: horizontal shocks, vertical shocks, and slope shocks
- Central bank targets: interbank market rate & aggregate reserve level
- Central bank controls reserve supply
  - ◇ Extension: lending facility rate

Our Main Results

- Fully characterize optimal supply of reserves and associated equilibrium

1) Optimal supply under uncertainty exceeds that absent risk

2) *Abundant* reserves are optimal with sufficient degree of uncertainty
   - Even if central bank targets ample reserves

3) Optimal mean market rate can be higher or lower than absent uncertainty

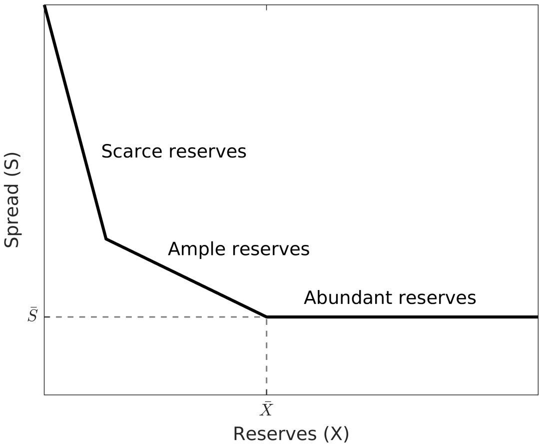
4) Lending facility reduces optimal reserve supply (if its rate is chosen optimally)

Reserve Demand Curve

- Price at which banks are willing to trade reserves as a function of total reserves
- Nonlinear function:
  - ◇ High reserve levels, demand curve is flat (satiated): *abundant reserves*
  - ◇ Intermediate reserve levels, demand curve is gently sloped: *ample reserves*
  - ◇ Low reserve levels, demand curve is steeply sloped: *scarce reserves*
- Reserve ampleness based on slope of reserve demand curve

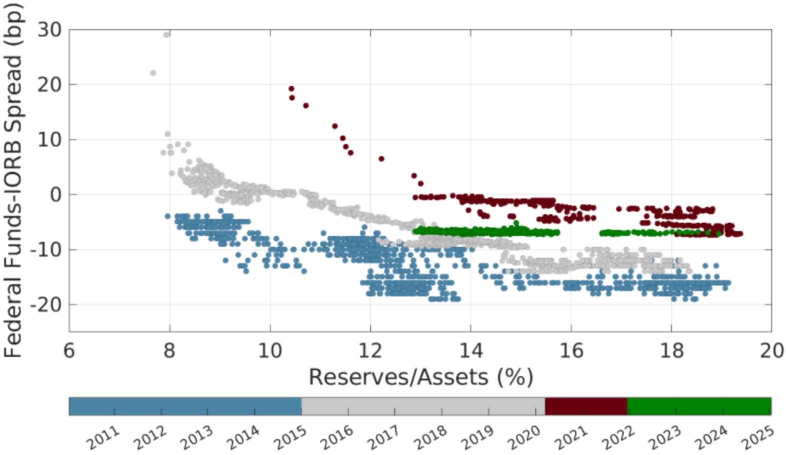
Reserve Demand Curve

- S: spread between market rate and interest on reserve balances; X: reserves



- $\bar{X}$: satiation point (banks' reserve targets + distribution of liquidity shocks)
- $\bar{S}$: lower asymptote (banks' balance-sheet costs + outside option)

Empirical Evidence: Nonlinear and Uncertain Demand



- Afonso, Giannone, La Spada, Williams (2022): time-varying estimates of slope
  - ◇ Evidence of horizontal & vertical shifts + time-varying estimation uncertainty

Simple Model with Two Regions

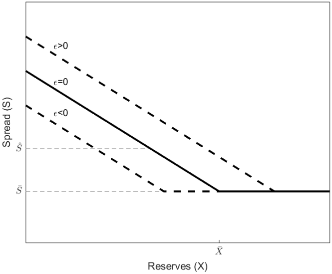
$$S(X) = \begin{cases} \bar{S} + \nu - (\alpha + \eta)(X - \bar{X} - \epsilon) & \text{if } \epsilon > X - \bar{X} \\ \bar{S} + \nu & \text{else} \end{cases}$$

- $\alpha > 0$: slope in the ample region

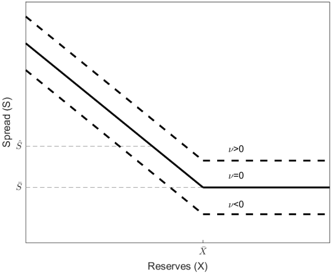
1) Horizontal uncertainty: ϵ shifts the kink in the demand curve ($\bar{X}$)
2) Vertical uncertainty: ν shifts the demand curve up and down ($\bar{S}$)
3) Slope uncertainty: η changes the slope of the demand curve (α)

- ϵ, ν, and η: independent mean-zero random variables with known variances
- ϵ: pdf $g(\cdot)$ and cdf $G(\cdot)$

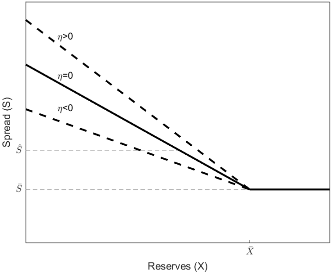
A Graphical Representation of Uncertainty



Horizontal ϵ shocks



Vertical ν shocks



Slope η shocks

Model Implications under Uncertainty

- Define cutoff level of the shock when reserves become ample as

$$\bar{\epsilon}(X) = X - \bar{X}$$

- $\mathscr{G}(\cdot)$ is the super-cumulative distribution function

$$\mathscr{G}(\bar{\epsilon}) = \int_{-\infty}^{\bar{\epsilon}} G(\epsilon) d\epsilon \geq \max\left[0, \bar{\epsilon}\right]$$

- The mean spread is greater than absent uncertainty for every reserve level

$$\mathbb{E}S = \bar{S} + \alpha\left(\mathscr{G}(\bar{\epsilon}) - \bar{\epsilon}\right) \geq \max\left[\bar{S} - \alpha(X - \bar{X}), \bar{S}\right]$$

Central Bank Optimization

$$\mathscr{L} = \min_X \frac{1}{2}\mathbb{E}\left[(S - \hat{S})^2 + \lambda(X - \hat{X})^2\right]$$

$\diamond$ $S = S(X)$ piecewise-linear demand for reserves described above

- Targets: spread & reserves

  $\diamond$ $\hat{S}$: efficiency goals based on prices (Friedman rule: $\hat{S} = \bar{S}$. Frictions $\Rightarrow \hat{S} > \bar{S}$)

  $\diamond$ $\hat{X}$: goals based on quantity of public liquidity (e.g., financial stability)

- Minimize weighted sum of expected squared deviations ($\lambda \geq 0$)

Assumptions

1) X is chosen before the resolution of uncertainty

2) No ability to learn

3) Ample reserves are optimal absent uncertainty: $\hat{S} > \bar{S}$ and $\hat{X} < \bar{X}$

Optimal Reserves Absent Uncertainty

- Optimal deterministic supply of reserves:

$$X^* = \bar{X} - \frac{\lambda}{\alpha^2 + \lambda}\left(\bar{X} - \hat{X}\right) - \frac{\alpha}{\alpha^2 + \lambda}(\hat{S} - \bar{S}).$$

  - ◇ X^* increases less than one-for-one with the target level of reserves $\hat{X}$
  - ◇ X^* decreases with the target spread $\hat{S}$

- Resulting optimal spread S^*:

$$S^* = \hat{S} - \frac{\lambda}{\alpha^2 + \lambda}(\hat{S} - \bar{S}) + \frac{\alpha\lambda}{\alpha^2 + \lambda}(\bar{X} - \hat{X})$$

Optimal Supply of Reserves under Uncertainty

- FOC determines implicit function for X^{**}

$$X^{**} = X^{*} + \frac{\alpha^2 + \sigma_\eta^2}{\alpha^2 + \sigma_\eta^2 + \lambda}\mathscr{G}(\bar{\epsilon}) + \frac{\alpha}{\alpha^2 + \lambda}(\hat{S} - \bar{S})G(\bar{\epsilon}) + \frac{\sigma_\eta^2}{(\alpha^2 + \lambda)(\alpha^2 + \sigma_\eta^2 + \lambda)}\Big(\alpha(1 - G(\bar{\epsilon}))(\hat{S} - \bar{S}) + \lambda(\bar{X} - \hat{X})\Big)$$

1) Uncertainty about horizontal shifts *increases* optimal reserves

2) Uncertainty about the slope of demand *increases* optimal reserves

3) Uncertainty about vertical shifts *has no effect* on optimal reserves

◇ Abundant reserves are optimal if horizontal uncertainty is sufficiently large

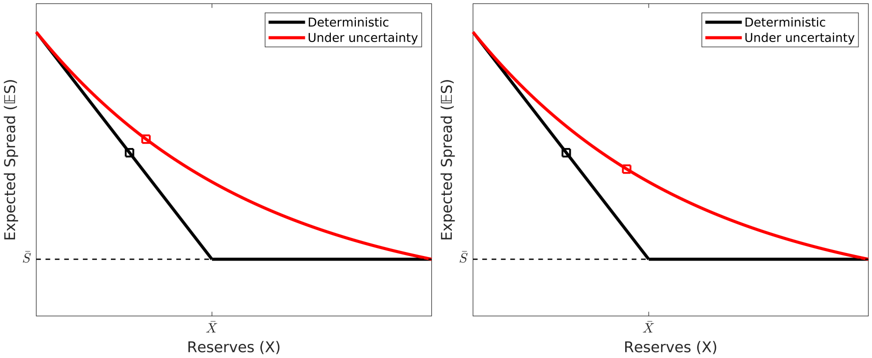
The Optimal Mean Spread

- Uncertainty may increase or decrease the optimal mean spread

$$\mathbb{E}S^{**} = \bar{S} + \left(\frac{\alpha}{\alpha^2 + \sigma_\eta^2 + \lambda}\right)\left(\lambda(\bar{X} - \hat{X}) + \lambda\mathscr{G}(\bar{\epsilon}) - (1 - G(\bar{\epsilon}))\,(\hat{S} - \bar{S})\right) \gtrless S^*$$

- Two effects in opposite directions
  - ◇ Positive (direct) effect of higher uncertainty for a given value of X
  - ◇ Negative (indirect) effect of a higher optimal level of X.
- Optimality cannot be judeged by the average level of the spread

The Optimal Mean Spread



Extension with Third Region of Scarce Reserves

- Scarce: $X < \bar{X}_2$ (steep slope)
- Ample: $\bar{X}_2 \leq X < \bar{X}_1$ (gentle slope)
- Abundant: $X \geq \bar{X}_1$ (zero slope)

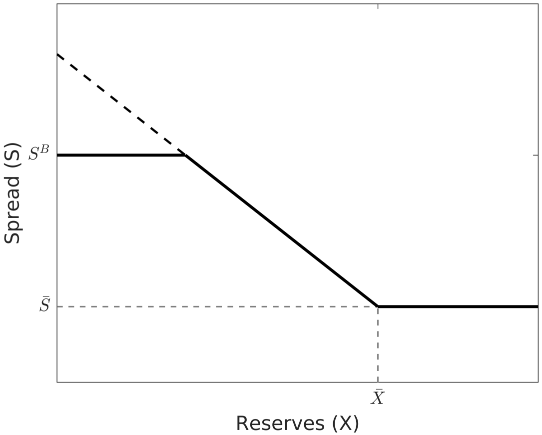
- Optimal supply increases further:

$$X^{***} = X^{**} + \frac{\beta(1 - G(\bar{\epsilon}_2))}{\alpha^2 + \sigma_\eta^2 + \lambda}\left(\bar{S} + \alpha(\bar{X}_1 - \bar{X}_2) - \hat{S}\right) > X^{**}$$

- Scarcity region relaxes condition for optimality of abundant supply

Reserve Demand Curve with Lending Facility

- Central bank offers frictionless lending facility at fixed spread S^B
- Demand above S^B is fully met by borrowing from the facility
- Assumption: only horizontal uncertainty



Central Bank Optimization with Lending Facility

$$\mathscr{L} = \min_{X, S^B} \frac{1}{2} \left\{ \mathbb{E} \left[(S - \hat{S})^2 + \lambda (X - \hat{X})^2 + \lambda^B B \right] \right\},$$

- Assumption: central bank may not like large facility utilization
  - Penalty: $\lambda^B B$, where $B \geq 0$ borrowing from facility and $\lambda^B \geq 0$ weight
- $B = B(S^B, \epsilon)$ depends on facility spread and demand shock
- Optimal supply with *exogenous* lending spread: $X^{**B} \gtrless X^{**}$
  - Lending facility $\Rightarrow$ cap on rates $\Rightarrow$ lower optimal reserves
  - Cost of facility usage $\Rightarrow$ reduce borrowing $\Rightarrow$ higher optimal reserves

Optimal Reserve Supply with Optimal Lending Facility

- Optimal lending spread is above the target spread: $S^B = \hat{S} + \frac{\lambda^B}{\alpha} \geq \hat{S}$
- Optimal supply of reserves is lower than absent the facility: $X^{**B} < X^{**}$
- Comparative statics with respect to λ^B (cost of facility usage):
  - ◇ Optimal lending spread increases: $\frac{dS^B}{d\lambda^B} > 0$
  - ◇ Optimal supply of reserves increases: $\frac{dX^{**B}}{d\lambda^B} \geq 0$

Conclusion

- We study the optimal supply of central bank reserves under uncertainty
- Analytically tractable framework that can be adjusted to different jurisdictions
- Uncertainty about demand for reserves implies larger optimal supply
- Abundant reserves can be optimal even if cental bank prefers ample reserves
- Lending facility lowers optimal supply of reserves (if rate chosen optimally)