Abstract

We study the effects of banking regulation on market liquidity, corporate funding, and welfare through the lens of a general equilibrium model. The model features heterogeneous banks that extend loans to firms and facilitate households’ bond purchases by “making markets”. Moreover, banks can trade loans in an interbank market. Standard frictions on this market give rise to pecuniary externalities. While regulation helps banks overcome these frictions, it also raises banks’ cost of funding bond inventory, inducing banks to widen bid–ask spreads on the bond market. By changing the relative return on assets, regulation also has costly distributional effects on households. We derive the welfare-maximising capital requirements in this environment, as well as the associated degree of market liquidity.

Keywords: Leverage ratio, market–making, funding liquidity

JEL Class.: E44, E60, G28
1 Introduction

Bank loans and corporate bonds represent two key sources of corporate funding. Firms’ ability to access both sources mitigates the impact of adverse shocks to banks or markets. The supply of bank- and of market-intermediated funding, however, is not independent. Banks, by issuing loans and “making markets”, represent a crucial link between the two. In the United States, banks continue to arrange the bulk of the trades in corporate bond markets. At the same time, firms increasingly rely on market-based funding, which has become, over the last decade, more than twice as large as bank-based funding.

Banks’ crucial role in market-based intermediation has prompted concerns about unintended consequences on market liquidity, of the post-crisis regulatory overhaul to strengthen the banking sector. Critics notably argue that leverage regulation, which forces banks to fund all assets, regardless of their underlying risk and purpose, with a minimum amount of – costly – equity, discourages banks from holding bonds for market-making purposes. Regulation would thus inadvertently induce banks to cut back on their market-making activities, and ultimately reduce market liquidity (e.g. Duffie (2017)).

To the extent that leverage regulation improves the functioning of the banking sector at the detriment of bond markets, its net effect on corporate funding – and the economy at large – is ambiguous. The macro-finance literature, which neglects the role of banks as market-makers, has not yet addressed this question.

In this paper, we study the effects of leverage regulation on market liquidity, corporate funding and welfare through the lens of a new macro-model. In this model, banks operate multiple business lines: they extend loans to firms, invest in corporate bonds and facilitate households’ bond purchases by “making markets” in corporate bonds. As in Boissay and Collard (2016), banks are assumed to be heterogeneous in terms of their loan servicing costs. By facilitating the migration of funds from the high-cost to the low-cost banks, an interbank market enhances lending efficiency. The usual agency problem in this market arises: asymmetric information and moral hazard prevent borrowers from credibly committing not to default. To enforce market discipline, lenders limit the amount of funds they lend in the interbank market, hampering an efficient allocation of funds to low-cost banks. Households invest in deposits, bank equity, and purchase corporate bonds from banks on the secondary market. The bond market is not perfectly liquid, in the sense that, to make markets, banks must hold bond inventories. If the return on bonds is low compared to the return on loans, then banks only hold bonds for market-making purposes, and charge a “bid-ask spread” for the

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1 Misrach (2017), for example, reports that the ten largest dealer-banks account for more than 60% of the trading volume in US secondary corporate bond markets.

2 As measured by US nonfinancial corporate business sector’s debt securities to loans ratio; based on national financial accounts.

3 The empirical evidence of such unintended consequences remains mixed. Kotidis and van Horen (2018) and Haselmann et al. (2018), for example, find a negative impact on market liquidity, whereas Adrian et al. (2017) and Trebbi and Xiao (2015) find little or no effect.
provision of liquidity.

We introduce banking regulation and describe the regulator’s trade–off in this environment. Calibrating the model to US data, we derive the welfare–maximising leverage requirements and the associated degree of market liquidity. Leverage ratio requirements have both benefits and costs. On the one hand, they help banks to overcome frictions in the interbank market, improve the allocation of funds across banks, and lower the overall cost of financial intermediation. On the other hand, by changing the relative return on assets, regulation also has costly distributional effects on households. Based on our calibration, most households prefer holding deposits over purchasing bonds, while preferring bond purchases over investing in bank equity. Hence, the regulatory–induced increase in the return on equity benefits only few households, whereas the decline in the deposit rate harms many. There are also opposing effects on the bond market. Higher capital requirements spur an increase in bond issuance and trading to compensate for the reduction in bank lending. At the same time, they raise banks’ cost of funding bond inventory, leading to higher bid–ask spreads. We characterise and quantify how a conscious regulatory approach strikes a balance between all these effects. This yields broad benchmarks for optimal capital requirements. In this context, a decline in market liquidity – as measured by the bid–ask spread – reflects an optimal regulatory response to excessive bank leverage rather than an “unintended consequence”.

Our paper is related to several recent studies on the effects of banking regulation using dynamic general equilibrium frameworks. A common feature with the work of, for example, Martinez-Miera and Suarez (2014), Christiano and Ikeda (2014, 2016), Begeneau (2015), Golec (2016), and Boissay and Collard (2016), is that we show how leverage regulation can improve welfare by making banks internalise pecuniary externalities. One advantage of this line of research is that it helps inform policy by providing benchmarks for optimal regulation. On this, we note that the optimal leverage ratio requirement following from our calibration, about 8.8%, is lower than, for example, the ratios of 12.4% and 14% suggested by Martinez-Miera and Suarez (2014) and Begeneau (2015), respectively. One difference is that, in our case, the regulator takes into account a potentially detrimental effect of the leverage ratio on banks’ market–making activity. This is important, given the growing role of markets as a source of corporate funding.

Our paper also speaks to the literature on the transmission of financial shocks when credit is intermediated via both banks and markets (Adrian and Shin (2010), Adrian et al. (2013), De Fiore and Uhlig (2011, 2015), Crouzet (2018)). In line with this literature, we find that markets cushion the macroeconomic effects of adverse shocks to banks. [TBC]

Finally, there is a growing empirical literature on the impact of regulation on bank lending, e.g. Fraisse et al. (2017), Benetton et al. (2017), Jimenez et al. (2017), and Glancy and Kurtzman (2018); as well as on the impact on market–making, e.g. Trebbi and Xiao (2015), Adrian et al. (2017), Bao et al. (2018), Kotidis and van Horen (2018), and Haselmann et al. (2018). These studies provide important contributions to our understanding of how regulation affects individual banking
activities. Our study complements this research by assessing the overall regulatory impact on welfare, taking general equilibrium effects into account.

The remainder of this paper is organised as follows. Section 2 presents the model, highlighting the interaction of firms, households and banks. We calibrate our model to US data, as discussed in Section 3. Based on this, we discuss the effects of banking regulation on bank lending, market liquidity and the macroeconomy in Section 4, before turning to the discussion of optimal regulation in Section 5. We then consider a dynamic version of the model in Section 6 [to be completed], and conclude in Section 7.

2 Model

This section presents our macroeconomic model. We first provide an overview of how the different agents interact. Next, we describe their objectives and constraints. We conclude this section by characterising the economy’s general equilibrium, which provides the basis for our analysis in the following sections.

We consider an economy populated by three types of agents: households, banks, and a representative firm. Households are risk–averse, whereas banks and the firm are risk–neutral. At the end of each period $t-1$, households consume $c_{t-1}$ and save $a_t$ for the next period. Then, at the beginning of period $t$, they choose whether to hold their savings as bank deposits, $d_t$, or to invest in corporate bonds, $b_h^t$, or bank equity, $e_t$. Banks, in turn, use their deposits and equity to fund corporate loans, $\ell_t$, and corporate bonds, $b_b^t$. In addition, they trade claims on loans in the interbank market. The firm, finally, decides how much capital to raise by issuing bonds or lending from banks in order to produce. There are two sources of aggregate shocks in the economy: the first one affects the firm’s productivity; the second one affects banks’ ability to raise funding in the interbank market. Figure 13 in the Appendix depicts the timing of shocks and of agents’ decisions.

Figure 1 summarises the flows of funds between agents in the economy, and highlights two important features of the banking sector. First, banks act as market–makers in the corporate bond market by channeling bonds from firms to households and, as such, must hold bond inventories, $\kappa b_h^t$. Second, banks not only lend to firms, but also lend funds to other banks (or borrow from them) in the interbank market.

We discuss the details and implications of these features in Section 2.3, after having outlined the firm’s and households’ optimisation problems in Sections 2.1 and 2.2 respectively.
2.1 Firm

There is a representative, competitive firm, established at the beginning of period $t$ and closed at the end of period $t$. The firm produces $z_t k_t^\alpha$ of homogeneous goods with $k_t$ units of capital and capital elasticity $\alpha$. Productivity, $z_t$, is assumed to be stochastic and follows the process

$$\log(z_t) = \rho \log(z_{t-1}) + \epsilon^z_t,$$

where the productivity shock, $\epsilon^z_t$, is log–normally distributed, and drawn at the beginning of period $t$. The firm acquires its capital at the beginning of period $t$, before the aggregate shocks are realised. Absent any endowment, the firm finances its capital by issuing $b_t$ corporate bonds, at rate $r^b_t$, and by borrowing $\ell_t$ from the banks, at rate $r^\ell_t$. It maximises its discounted expected profit,

$$\max_{k_t, b_t, \ell_t} \mathbb{E}_{t-1} \left( \Psi_{t-1, t} \pi_t \right),$$

where $\pi_t \equiv z_t k_t^\alpha + (1 - \delta) k_t - r^\ell_t \ell_t - r^b_t b_t$, given the constraint $k_t \leq \ell_t + b_t$ and the depreciation rate $\delta$. The term $\Psi_{t-1, t} \equiv \beta \frac{w'(c_t)}{w'(c_{t-1})}$ is the household’s discount factor (see Section 2.2 below). $\mathbb{E}_{t-1}()$ denotes the expectation operator taken over aggregate shocks to productivity and bank funding, conditional on the information available at the end of period $t - 1$ (see Section 2.3 below). The first order conditions yield:

$$k_t = \ell_t + b_t,$$

$$r^b_t = r^\ell_t,$$

$$r^\ell_t = \alpha z_t k_t^{\alpha-1} + 1 - \delta.$$

The interest rates on bonds and loans, $r^b_t$ and $r^\ell_t$, are assumed to be state contingent, and the profit, $\pi_t$, is assumed to be rebated lump–sum to the household.

2.2 Households

There is a continuum of atomistic households with total mass one. Households take their decisions in two stages. First, at the end of period $t - 1$, they consume $c_{t-1}$ and save $a_t$. Second, at the
beginning of period $t$, they decide whether to purchase corporate bonds, $b^h$, to keep their savings as deposits with banks, $d$, or to invest in bank equity, $e$:

**Assumption 1 (Households’ transaction costs)** At the beginning of period $t$, households learn about the idiosyncratic financial transaction costs they will have to pay at the end of period $t$.

This portfolio allocation takes place before the realisation of the aggregate shocks, but after households draw idiosyncratic financial transaction costs (See Assumption 1). The latter relate to the effort of screening, the cost of gathering financial intelligence, financial advisor fees, or to any cost associated with contract enforcement (e.g. obtaining legal advice).

Specifically, we assume that investing in corporate bonds, bank deposits, and bank equity is costly, and that the costs are idiosyncratic. We refer to household $(q^b_h, q^d, q^e_e) \in [0, 1]^3$ as the household with transaction costs $1 - q^b_h$ for corporate bonds, $1 - q^d$ for bank deposits, and $1 - q^e$ for bank equity. This household’s net unit return on asset $j$ is $q^j r^j$, with $j \in \{b^h, d, e\}$, where $r^j$ denotes the asset’s gross return. Formally, $q^b_h$, $q^d$, and $q^e$ are drawn randomly and independently from the joint cumulative distribution $\mu_{(b^h,d,e)}(q^b_h, q^d, q^e)$. This setup provides a flexible way of introducing realistic and endogenous spreads across the returns of multiple financial assets in a tractable manner.

As illustrated in Figure 1, households purchase the corporate bonds from banks on a secondary market. Banks charge a commission fee (or a “bid–ask” spread) for this intermediation service, which we denote by $\omega_t$ (we provide more detail on this commission fee in Section 2.3). This fee is paid at the end of period $t$. As a result, households’ return on bonds (gross of transaction costs), $r^b_h$, is a fraction of the corporate bond yield:

$$r^b_h \equiv (1 - \omega_t) r^b_t.$$ (6)

Since returns are linear, each household invests its savings in only one asset. We refer to this asset as the household’s “preferred habitat” – loosely borrowing the term from the literature that studies the term structure of interest rates (e.g. Vayanos and Vila (2009)). In response to changes in relative returns, households may leave their initially preferred habitat, and invest in another asset. We denote the decision to invest into asset $j$ by $1^j_{t+1}$, with $1^j_{t+1} = 1$ if there is an investment and $1^j_{t+1} = 0$ otherwise, for all $j \in \{b^h, d, e\}$, with $\sum_{j \in \{b^h, d, e\}} 1^j_{t+1} = 1$. Households maximize their discounted expected utility:

$$\max_{\{a_{t+1}, c_t\}_{t=0,...,\infty}} E_q \left[ \mathbb{E}_d \left( \sum_{i=0}^{\infty} \beta^i \max_{\{1^j_{t+1}\}} \{1 \in \{b^h, d, e\}\} u(c_{t+i}) \right) \right],$$ (7)

We note that financial transaction costs are relevant in practice. Hung et al. (2008), for example, report that 73% of US households consult a financial advisor before purchasing shares or mutual funds. In the US mutual fund industry, the so-called “expense ratio 12b-1” is typically between 0.25% and 1% (the maximum allowed) of a fund’s assets under management. Foerster et al. (2017) report that for Canadian households the sum of management fees and portfolio advice amounts to 2.5% of the households’ assets under management per year.
subject to the constraints:

\[ c_t + a_{t+1} = r_t a_t + \pi_t \] (8)

\[ r_t \equiv Q_t^b r_t^b \frac{d_t}{a_t} + Q_t^d r_t^d \frac{b_t}{a_t} + Q_t^e r_t^e \frac{c_t}{a_t}, \] (9)

\[ j_t \equiv a_t \int_0^1 \int_0^1 \int_0^1 \frac{1}{2} \mu_{bh} \left( q_t^{bh} \right) d\mu_{dh} \left( q_t^d \right) d\mu_{eh} \left( q_t^e \right) \forall j \in \{b^h, d, e\} \] (10)

\[ Q_t^j \equiv \frac{a_t}{j_t} \int_0^1 \int_0^1 \int_0^1 \frac{1}{2} q_t^j \mu_{bh} \left( q_t^{bh} \right) d\mu_{dh} \left( q_t^d \right) d\mu_{eh} \left( q_t^e \right) \forall j \in \{b^h, d, e\}, \] (11)

where \( u(\cdot) \) satisfies the usual regularity conditions. The term \( E_q \) is the expectation operator taken over \((q_t^{bh}, q_t^d, q_t^e)\) and \( r_t \) denotes the overall gross return on savings. The term \( Q_t^j \), defined in equation (11), corresponds to (one minus) the household’s average expense ratio for security \( j \). The higher it is, the lower the average transaction cost on the security. In terms of timing, we recall that the return on deposits, \( r_{td}^d \), is determined before the aggregate shocks occur, whereas the other returns, \( r_{tb}^b \) and \( r_{te}^e \), are state–contingent (see Figure 13).

We solve the maximization problem backwards, starting with the second stage.

**Second Stage: Asset Portfolio Choice** Given its transaction costs, household \((q_t^{bh}, q_t^d, q_t^e)\) invests \( a_t \) into asset \( j \) if and only if its expected net return on \( j \) is higher than its expected net return on any other asset \( j^- \neq j \):

\[ 1^j_t = 1 \Leftrightarrow \left\{ q^j > q^{j^-} \frac{E_{t-1} (\Psi_{t-1, t} r_{t}^{j^-})}{E_{t-1} (\Psi_{t-1, t} r_{t}^{j})} \right\} \forall j^- \in \{b^h, d, e\} \backslash \{j\} \] (12)

for all assets \( j \in \{b^h, d, e\} \). Thus, asset \( j \) is household \((q_t^{bh}, q_t^d, q_t^e)\)’s preferred habitat.

**First Stage: Consumption and Savings** At the end of period \( t-1 \), each household chooses the level of consumption, \( c_{t-1} \), and savings, \( a_t \), anticipating the second stage portfolio choices. This yields the Euler equation

\[ E_{t-1} (\Psi_{t-1, t} r_t) = 1 \] (13)

as optimality condition.

### 2.3 Banks

The banking sector is represented by a continuum of one–period lived banks with total mass one. Each bank operates from the beginning to the end of period \( t \), i.e. for one period.

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\(^5\) Notice that our model with short–lived banks is isomorphic to one, where banks live infinitely. The reason is that, in our model, banks have a frictionless access to the equity market and no reputation risk (indeed, they draw their types afresh at the beginning of every period; see Assumption 3). Given that each household is risk averse and can freely reallocate equity and deposits across banks, it is optimal for it to diversify its equity holdings and deposits across the continuum of banks every period. The upshot is that, even if they lived infinitely, banks would start out...
two types of activities. On the one hand, they lend to firms, and buy the bonds that firms issue on
the primary market. On the other hand, they act as dealers, or “market makers”, on the secondary
bond market, by channelling corporate bonds to households.

2.3.1 Market–Making and Lending Activities

Market–making requires banks to warehouse assets.

**Assumption 2** *(Bond inventory)* A bank must hold $\kappa$ bonds as inventory for every unit of bond
it sells to households.

Parameter $\kappa$ is one determinant of banks’ ability to make the bond market and, as such, is
meant to capture the degree of “intrinsic” market liquidity in the secondary bond market. Since
we focus on the balance sheet implications of banks’ market–making, we abstract from modelling
explicitly how agents trade in this market. Assumption 2 captures the idea that, in relatively
liquid markets, frequent trading allows banks to easily match customer demand and supply. This
implies that market–makers require inventory to provide immediacy services. In less liquid markets,
customer orders arrive infrequently and market–makers must maintain a sizeable inventory in order
to facilitate trading, which is captured by $\kappa$ (see, e.g. O’Hara and Oldfield (1986) or CGFS (2014)).

We also assume that banks incur a cost of managing their bond portfolio, which amounts to a
fraction $1 - Q^b$ of the bond return, and is paid at the end of the period. A bank’s net unit return
on corporate bonds is thus $Q^b r_t^b$. Later, when we calibrate the model (see Section 3), we will see
that $Q^b$ must be set sufficiently low so that banks’ net return on holding bonds does not always
dominate their net return on loans.

Banks are born identical and take their decisions in two stages at the beginning of period $t$. In
the first stage, given equity $e_t$ from its shareholders (i.e. the households), the representative bank
raises $d_t$ deposits, issues $\ell_t$ loans, and purchases $b_t^b$ corporate bonds on the primary market. It may
also simultaneously sell $s_t$ bonds to households on the secondary market, so that $e_t + d_t = \ell_t + b_t^b - s_t$.
The bank thus ultimately holds $b_t^b - s_t$ bonds on its balance sheet, of which $\kappa s_t$ are for market–making
purposes and $b_t^b - (1 + \kappa) s_t$ are for proprietary trading purposes, with $b_t^b \geq (1 + \kappa) s_t$. The markets
for deposits, equity, bonds and loans close at this end of this first stage.

In the second stage, each bank draws a random idiosyncratic loan servicing cost, (one minus) $q^\ell$,
from a cumulative distribution $\mu_\ell(q^\ell)$:

**Assumption 3** *(Banks’ loan servicing costs)* At the beginning of period $t$, each bank learns
every period with identical balance sheets, like new-born banks. In other terms, bank equity is not a state variable. In
this respect, our approach is similar to Begenau and Landvoigt (2017) or Coimbra and Rey (2017), but differs from
other models with infinitely–lived banks that assume the absence of equity markets, like Gertler and Kiyotaki (2011).
In this latter model, banks accumulate equity by retaining earnings. We do not make this assumption here, because
this would bar us from studying the effects of bank capital regulation.
about the idiosyncratic cost $1 - q^\ell$ it will have to pay at the end of period $t$, if it services corporate loans.

For a bank that draws $q^\ell$ (henceforth “bank $q^\ell$”), loan servicing costs amount to $(1 - q^\ell) r^\ell_t$ per unit of loan. This yields a net return on loans of $q^\ell r^\ell_t$, with $q^\ell \in [0, 1]$. Given its type $q^\ell$, each bank decides whether to service it loans or to sell them (or a fraction thereof) to another bank in the interbank market. Through this market, high–$q^\ell$ banks can obtain corporate loans from low–$q^\ell$ banks, thus raising the overall sector’s lending efficiency. Specifically, we assume that banks acquire loans by issuing claims that promise a unit return $r^i_t$ at the end of the period. We refer to these transactions as “interbank loans”, and to the (high–$q^\ell$) banks that issue such claims – i.e. borrow funds – as “borrowers”. By analogy, we call the (low–$q^\ell$) banks that purchase those claims “lenders”. From the perspective of lenders, the interbank market helps to insure against unfavourable cost draws. From the perspective of borrowers, it helps to take advantage of favourable cost draws.

In a frictionless world, only the banks with the lowest cost, $q^\ell = 1$, would borrow and service corporate loans. Those banks would purchase loans from the rest of the banking sector, and – given their small size – would be infinitely leveraged. To motivate a limit on the bank’s ability to issue claims, we follow Boissay et al. (2016), and introduce the following frictions:

**Assumption 4 (Agency problem)** The idiosyncratic loan servicing costs are private information. A bank can extract up to $\zeta_t$ of cash per unit of loan, and this cash cannot be seized by its creditors.

The first part of the assumption introduces asymmetric information: a bank’s $q^\ell$ is private information and cannot be revealed by the bank in a credible manner. The second part entails an agency problem: a bank has the possibility to extract up to $\zeta_t$ of cash from the firm, per unit of loan (as in the model of e.g. Holmstrom and Tirole (1997)). As in Bolton et al. (2017), the bank has the option to revoke its loans, force the firm into early liquidation, and obtain a private benefit $\zeta_t$ per unit of loan. Since the cash extracted cannot be seized by its creditors (i.e. neither the lending banks nor depositors), early liquidation gives the bank the possibility to default on its debt and abscond.

For simplicity, we assume that the cash flow from the $b^t - s_t$ bonds held on the balance sheet can be seized by lending banks, and therefore can be pledged as collateral in the interbank market. Similarly, the cash flow from market–making activities is also seizable. To motivate capital regulation, we however assume that the bank can default on its deposits. Lending banks protect themselves against a default of their counter–party by keeping the amount of interbank funding, $\phi_t$, in check. Since lenders do not observe a borrower’s $q^\ell$, they need to ensure that even the banks with the lowest $q^\ell$ (i.e. $q^\ell = 0$) do not borrow only to abscond later. Such a bank earns $\zeta_t (\ell_t + \phi_t)$ if it borrows $\phi_t$, extracts the non–seizable cash, and defaults. In this case, it forgoes the commission
fees from market–making, $\omega_t r_t^b s_t$, and the cash flow from the corporate bonds, $Q^b r_t^b (b_t^b - s_t)$. If instead the bank lends $\ell_t$ on the interbank market, it earns $r_t^{i} \ell_t + Q^b r_t^b (b_t^b - s_t) + \omega_t r_t^b s_t - r_t^d d_t$. Accordingly, the incentive compatibility constraint that ensures banks choose not to defaults is:

$$\zeta_t (\ell_t + \phi_t) \leq r_t^{i} \ell_t + Q^b r_t^b b_t^b + (\omega_t - Q^b) r_t^b s_t - r_t^d d_t. \quad (14)$$

Variable $\zeta_t$ determines the bank’s ability to raise interbank (“wholesale”) funding. The term on the right–hand side of the incentive constraint (14) increases with interbank rates, $r_t^{i}$, suggesting that higher rates support banks’ funding liquidity. Yet, as price–takers, banks fail to internalise this effect. As we discuss later, this creates scope for regulatory intervention.

Empirical research has highlighted the time–varying nature of funding liquidity conditions, which is marked by sharp contractions during episodes of stress (e.g. CGFS (2014)). Against this background, we assume that the private benefit follows the stochastic process

$$\log(\zeta_t) = \rho \log(\zeta_{t-1}) + \epsilon_t^\zeta,$$

where $\epsilon_t^\zeta$ is a log–normally distributed aggregate funding liquidity shock, drawn at the beginning of period $t$ (see Figure 13).

### 2.3.2 Banking Regulation

We assume that a regulator seeks to maximize steady state welfare, by imposing the following regulatory constraint.

**Assumption 5 (Banking regulation)** The regulator imposes a minimum leverage ratio,

$$\frac{e_t}{d_t + e_t} \geq \tau. \quad (16)$$

We refer to parameter $\tau$ as the minimum leverage ratio requirement. Constraint (16) aims at relaxing banks’ funding liquidity constraint. Since lending banks enforce enough discipline on borrowing banks to avoid defaults in our model, regulation is not meant to address solvency risk. In the absence of default risk, the regulator also stays away from restricting interbank funding, as this would only impair the reallocation of loans from low–$q^\ell$ to high–$q^\ell$ banks after the idiosyncratic loan

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6 Implicitly, we view those cash flows as assets that can be pledged as collateral. A borrower can obtain $Q^b r_t^b (b_t^b - s_t)/r_t^\ell$ loans by pledging the cash flow from its corporate bonds, and $\omega_t r_t^b s_t/r_t^\ell$ loans by pledging the cash flow from its market–making activities. Of course, high–$q^\ell$ banks could alternatively (and equivalently) sell bonds outright, on an (interbank) bond market, to acquire loans from low–$q^\ell$ banks. Let $p_t$ be the unit price of such a bond, in terms of loans. By a simple no–arbitrage argument, it is easy to see that for a low–$q^\ell$ bank to enter a trade, the return of a bond, $Q^b r_t^b/p_t$, must be above – or equal to – the opportunity cost of selling $p_t$ loans, $r_t^\ell p_t$. Similarly, high–$q^\ell$ banks participate only if the opportunity cost of purchasing a loan, $Q^b r_t^\ell/p_t$, is below – or equal to – the cost of interbank loans, $r_t^\ell$. Given this, $p_t = Q^b r_t^b/r_t^\ell$.

7 In the calibrated version of the model (see Section 3), we obtain $r_t^\ell \ell_t + Q^b r_t^b (b_t^b - s_t) + \omega_t r_t^b s_t > r_t^d d_t$ in the steady state, which means that no bank defaults ex post in the neighbourhood of the steady state.
servicing costs are revealed. Accordingly, Assumption 5 applies to banks’ *ex ante* balance sheets, i.e. at the beginning of period $t$.

As we discuss in more detail Section 5.4 other – more targeted – regulatory requirements could also be envisaged in the context of our model. That said, the constraint in (16) provides a reasonable proxy of leverage regulation in practice.

### 2.3.3 Optimisation Problem

All elements are now in place to consider a representative bank’s optimisation problem. The latter consists of maximising the discounted expected end–of–period $t$ dividend payout, by choosing optimally $d_t$, $b_t^b$, $s_t$, $1_t$, and $\phi_t$:

$$\max_{d_t, b_t^b, s_t} E_{t-1} (\Psi_{t-1,t} r_t^e) e_t \equiv E_{t-1} \left( \Psi_{t-1,t} \int_{1_t, \phi_t} (1 - 1_t) r_t^l \ell_t + 1_t \left( q^l r_t^l (\ell_t + \phi_t) - r_t^d \phi_t \right) \right. \left. + Q^b b_t^b + (\omega_t - Q^b) r_t^b s_t - r_t^d d_t \right) \text{d} \mu_{q^l}(q^l)$$

(17)

subject to the constraints on interbank borrowing (14), the regulatory requirement (16), bond inventory,

$$b_t^b \geq (1 + \kappa) s_t$$

(18)

and the balance sheet identity $e_t + d_t = \ell_t + b_t^b - s_t$. The first term in (17) is the return if the bank is a lender in the interbank market ($1_t = 0$). The second term is the return if the bank is a borrower ($1_t = 1$). The last term is return on bonds holdings and market–making, net of the interest expenses on deposits, $Q^b b_t^b + (\omega_t - Q^b) r_t^b s_t - r_t^d d_t$. Constraint (18) formalises Assumption 2. A bank’s bond holdings, $b_t^b - s_t$, must exceed a multiple $\kappa$ of the quantity of bonds the bank sells on the secondary market, $s_t$.

The structure of the objective function in (17) reflects the bank’s two–stage decision problem. First, the bank chooses $(d_t, b_t^b, s_t)$ knowing neither its loan servicing cost, $q^l$, nor the aggregate shocks, $e_t^z$ and $e_t^\zeta$. Second, after learning $q^l$ and observing the shocks, the bank chooses $(1_t, \phi_t)$. Accordingly, we solve this problem by backward induction, starting with the choice of $1_t$ and $\phi_t$.

**Second Stage: Choice of $1_t$ and $\phi_t$.** The linearity of the objective function in (17) implies that bank $q^l$ borrows funds on the interbank market whenever the net marginal return of the leveraged funds (i.e. the first derivative with respect to $\phi_t$) is positive:

$$1_t = 1 \iff q^l \geq \tilde{q}_t^l \equiv \frac{r_t^l}{r_t^d}.$$  

(19)

---

8 This amounts to assuming that banks are supervised before the realisation of the idiosyncratic shocks and the opening of the interbank market; see Figure 13.
A bank with \( q^f > q_t^l \) always exhausts its borrowing capacity, and its incentive compatibility constraint (14) binds. The threshold \( q^l \) is an important variable in the model. It reflects the efficiency of the interbank market in re-allocating loans from low-\( q^f \) banks to high-\( q^f \) banks. The higher \( q^l \), the more efficient the pool of borrowers, the lower the borrowers’ incentive to extract cash and abscond, and the higher the borrowing capacity. Given (14), (19), and the accounting identity \( e + d = \ell + b^b - s \), the latter amounts to:

\[
\phi_t = \frac{\ell_t}{\zeta_t} \left( r^i_t - \zeta_t + Q^b b^b \frac{b^b}{d_t + e_t} + (\omega_t - Q^b) r^i_t \frac{s_t}{d_t + e_t} - r^d_t - r^i_t \frac{1 - e_t}{d_t + e_t} \right). \tag{20}
\]

For a given amount of loan issuance, \( \ell_t \), the borrowing capacity decreases with the private benefit from early liquidation, \( \zeta_t \), and increases with the interbank rate, \( r^i_t \) and the leverage ratio, \( e_t/(d_t + e_t) \). Thus, equity helps to relax banks’ borrowing constraint by ensuring that banks have enough “skin in the game”.

**First Stage: Choice of \( d_t, b^b_t, \) and \( s_t \).** The representative bank chooses \( d_t, b^b_t, \) and \( s_t \) to maximize its expected profit based on (17), taking the second stage solutions (19), and (20) into account:

\[
\max_{d_t, b^b_t, s_t} \mathbb{E}_{t-1} \left( \Psi_{t-1,t} \left( (1+\Delta_t) r^i_t e_t - (1+\Delta_t) (r^i_t - Q^b b^b_t) s_t + (1+\Delta_t) (r^i_t + (\omega_t - Q^b) r^i_t) s_t + (1+\Delta_t) (r^i_t - r^d_t) d_t \right) \right)
\]

subject to the inventory constraint (18) and the regulatory constraint (16), where

\[
\Delta_t = \frac{1 - \mu (q^l_t)}{\zeta_t} (Q^l r^i_t - r^e_t), \tag{21}
\]

is the excess return on leveraged funds, and

\[
Q^l_t \equiv \int_{q^l_t}^1 q^f d\mu_{q^f}(q^f) \frac{1 - \mu (q^l_t)}{\mu (q^l_t)} \tag{22}
\]

is (one minus) banks’ average non-interest expense ratio. The higher this ratio, the higher is the banking sector’s lending efficiency. Let \( \Gamma_t \) and \( \Lambda_t \) be the Lagrange multipliers associated with constraints (18) and (16), respectively. The first order conditions are given by:

\[
\tau \Lambda_t = \mathbb{E}_{t-1} \left( \Psi_{t-1,t} (1+\Delta_t) (r^i_t - r^d_t) \right), \tag{23}
\]

\[
\Gamma_t + \mathbb{E}_{t-1} \left( \Psi_{t-1,t} (1+\Delta_t) Q^b r^b_t \right) = \mathbb{E}_{t-1} \left( \Psi_{t-1,t} (1+\Delta_t) r^i_t \right), \tag{24}
\]

\footnote{For US banks, this ratio ranges from 0.5% to 3.5%, depending on the type of expenses included (e.g. employees’ compensation, occupancy expenses, other non-interest expenses). Obviously, not all non-interest expenses can be imputed to loan servicing activities. To fix ideas, we will target an expense ratio of 2.3% in the calibration (see Section 3).}
and

\[ \kappa \Gamma_t = \omega_t \Psi_{t-1, t} (1 + \Delta_t) r_t^b. \]  \hspace{1cm} (25)

Relation (25) determines \( \Gamma_t \) as the shadow value of holding bonds for market-making purposes. The latter is proportional to the discounted expected value of the commission fee, \( \omega_t \Psi_{t-1, t} (1 + \Delta_t) r_t^b \), and takes into account that the cash flow from market-making will help the bank to borrow funds on the interbank market, if it draws a high \( q^\ell \) (term \( \Delta_t \)). In relation (24), the bank compares the expected returns on loans (right side), with that on bonds (left side), taking into account that both bonds and loans have some collateral value (term \( \Delta_t \)), and that the marginal unit of bonds relaxes the bank’s inventory constraint (term \( \Gamma_t \)). Relation (23) determines the shadow value of equity, \( \Lambda_t \), and reflect that, by relaxing the regulatory constraint, the marginal unit of equity will allow the bank to borrow additional funds on the interbank market if it draws a high \( q^\ell \). As banks distribute their profits as dividends, the return on equity is given by:

\[ r_t^e \equiv (1 + \Delta_t) r_t^d + (1 + \Delta_t) (r_t^d - Q_t^b) \frac{d_t}{e_t} - (1 + \Delta_t) \left( r_t^d - Q_t^b \right) \frac{b_t^b}{e_t} + (1 + \Delta_t) \left( r_t^d + \left( \omega_t - Q_t^b \right) r_t^b \right) \frac{s_t}{e_t}. \]  \hspace{1cm} (26)

### 2.4 General Equilibrium

We conclude this section by characterising the competitive general equilibrium of the economy:

**Definition 1 (Competitive General Equilibrium)**

A competitive general equilibrium is:

- a sequence of prices, \( P_t \equiv \{ r_{t+i}^i, r_{t+i}^d, r_{t+i}^b, r_{t+i}^e, r_{t+i}^t, \omega_{t+i} \}_{i=0}^\infty \),
- a sequence of quantities, \( Q_t \equiv \{ c_{t+i}, k_{t+i}, d_{t+i}, e_{t+i}, b_{t+i}^h, b_{t+i}^b, \ell_{t+i}, s_{t+i} \}_{i=0}^\infty \),

such that (i) for a given sequence of prices, \( P_t \), the sequence of quantities, \( Q_t \), solves the optimization problems of the agents, and (ii) for a sequence of quantities, \( Q_t \), the sequence of prices, \( P_t \), clears the markets, for \( i = 0, ..., +\infty \).

**Primary Bond Market**  The primary market for corporate bonds clears when

\[ b_t = b_t^b. \]  \hspace{1cm} (27)

**Secondary Bond Market**  The secondary bond market clears when

\[ b_t^b = s_t. \]  \hspace{1cm} (28)
Interbank Market  The interbank market clears when

\[(1 - \mu_\ell (\pi^\ell_t)) \phi_t = \mu_\ell (\pi^\ell_t) \ell_t. \tag{29}\]

Goods Market  The goods market clears when

\[y_t = c_t + i_t + \chi_t, \tag{30}\]

where investment, \(i_t\), is given by \(i_t = k_{t+1} - (1 - \delta)k_t\) and the deadweight loss, \(\chi_t\), is defined as:

\[\chi_t \equiv (1 - \Pi^c_t) r_t^c \ell_t + (1 - \Pi^{bh}_t) r_t^{bh} (b_t^{bh} - s_t) + (1 - \Pi^d_t) r_t^d d_t + (1 - \Pi^{bh}_t) r_t^{bh} b_t^{bh} + (1 - \Pi^e_t) r_t^e e_t. \tag{31}\]

\(\chi_t^i\) represents the deadweight loss associated with the banking sector. It is the sum of the banks’ loan servicing costs, \(\chi_t^l\), and bond portfolio management costs, \(\chi_t^{bh}\). The term \(\chi_t^a\) denotes households’ total cost of savings. It comprises the deadweight losses \(\chi_t^d\), \(\chi_t^{bh}\) and \(\chi_t^e\), which are incurred by depositors, bond holders, and equity investors, respectively.

3 Calibration

For the calibration of the real sector, we use standard parameter values. The capital elasticity in the production function, \(\alpha\), is set to 0.35, and the annual rate at which capital depreciates is set to \(\delta = 0.06\). The steady state level of TFP is normalised to one, \(z = 1\). These parameter choices are listed in Table 2. The households’ preferences are represented by the log utility function,

\[u(c_t) = \ln(c_t). \tag{32}\]

The remaining parameters pertain to the financing of the economy. We assume that the transaction costs are distributed independently across households, according to the following distributions:

\[\mu_j(q) = q^{1_j} \tag{33}\]

for \(j \in \{b^h, d, e, \ell\}\). The form of the distributions is admittedly restrictive. But this parsimony limits the number of parameters and targets – an advantage given the lack of data on the second order moments.

\[\text{In Boissay et al. (2016), we show that there may be multiple solutions to this market clearing condition, i.e. there can be equilibria for “normal times” and “crisis times”. For the purpose of this paper, we consider only the former. We thus focus on states in the neighborhood of the deterministic steady state, where the crisis probability is null.}\]
Table 1: Financial Targets

<table>
<thead>
<tr>
<th>Target</th>
<th>Values</th>
<th>Data sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r^b$</td>
<td>1.0428</td>
<td>Federal Reserve Bank of Saint Louis FRED database; <em>Moody’s seasoned Baa corporate bond yield</em>; BAA</td>
</tr>
<tr>
<td>$r^i$</td>
<td>1.0194</td>
<td>Federal Reserve Bank of Saint Louis FRED database; <em>Federal funds effective rate</em>; <em>RIFSPFF.N.A</em></td>
</tr>
<tr>
<td>$b/\ell$</td>
<td>1.3019</td>
<td>US Financial Accounts; Firms; <em>Bond-to-loan ratio</em>; <em>FL104122005.A/FL104123005.A</em></td>
</tr>
<tr>
<td>$e/(d+e)$</td>
<td>0.0814</td>
<td>US Financial Accounts; Depository institutions; <em>Leverage ratio</em>; <em>(FL704194005.A-FL704190005.A)/FL704194005.A</em></td>
</tr>
<tr>
<td>$(b^b-s_t)/(d+e)$</td>
<td>0.0386</td>
<td>US Financial Accounts; Depository institutions; <em>Liquidity ratio</em>; <em>(FL703063005.A/FL704194005.A)</em></td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.0100</td>
<td>Adrian et al. (2017)</td>
</tr>
<tr>
<td>$\chi^i/(d+e)$</td>
<td>0.0230</td>
<td>FDIC Tables CB07 and CB09; banks’ total non-interest expenses to total assets</td>
</tr>
<tr>
<td>$\chi^a/a$</td>
<td>0.0250</td>
<td>Foerster et al. (2017); Households; <em>Asset-management-expenses-to-total-asset ratio</em></td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>0</td>
<td>The shadow cost of the leverage ratio rule is zero</td>
</tr>
</tbody>
</table>

We calibrate the model on annual US data for the years from 1988 to 2003. The sample thus pre-dates the run-up to the systemic banking crisis of 2007–09 and the associated build-up of bank leverage which, with the benefit of hindsight, proved unsustainable (Adrian and Shin (2010)). US banks were subject to a variety of regulatory constraints during that period. Our focus is on capital “guidelines” and subsequent revisions issued in the early 80s by the US regulatory authorities. Those guidelines, among other requirements, established minimum leverage ratio requirements based on the ratio of a narrowly defined measure of capital and the bank’s total assets. This ratio tallies closely with the stylised leverage ratio requirement, $\tau$, in our model. Minimum requirements differed for different types of banks, but were broadly in a range of 5.5% to 7% (see Wall (1989)). This compares with an average leverage ratio of 8.14% (see Table 1) for the period of observation. On average, leverage ratio requirements were thus not strictly binding for banks. Nevertheless they are likely to have constrained the banks’ risk-taking and leverage since banks generally maintain a capital buffer above the minimum requirements to avoid supervisory intervention.\footnote{Indeed, minimum requirements do not reflect supervisors’ – explicit or implicit – expectation that banks exceed those requirements, in order for them to be considered well-capitalised.}

Against this background, we assume that the regulatory constraint\footnote{Equation (16)} binds in the steady state, but at the same time also assume that the shadow cost of the constraint is negligible, i.e. $\Lambda_t = 0$ in the steady state. In other words, we calibrate the model so that, in the steady state, the privately optimal capitalisation level coincides with the regulatory minimum, $\tau = 8.14\%$. This provides a useful steady state benchmark.
Overall, we need to calibrate eight additional financial parameters (see Table 2). These parameters (and the leverage ratio) are jointly calibrated so that, in steady state, the model matches the financial ratios and interest rates listed in Table 1.

We target the main interest rates of the model, the structure of firms’ liabilities, households’ expense ratio, and banks’ expense and leverage ratios. Households’ expense ratio is based on Canadian data taken from Foerster et al. (2017), since, to our knowledge, this ratio is not available for the US. Banks’ total non–interest expenses–to–total–assets ratio is calculated using FDIC data over the 1988–2003 sample period. On this basis, we obtain $\zeta = 5.45\%$ for the private benefit from early liquidation; $\kappa = 3.18\%$ for the parameter of the inventory constraint; and $Q^b_h = 66.33\%$ for the parameter of banks’ bond management cost. The other financial parameters govern households’ transaction costs, which, in turn, determine their preferred habitat. We find that most households prefer deposits to holding corporate bonds, while preferring bonds to bank equity. Table 2 summarizes the outcome of the calibration.

Table 2: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intertemporal Elast. of Subst.</td>
<td>$\sigma_c$</td>
</tr>
<tr>
<td>Capital elasticity</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>Capital depreciation rate</td>
<td>$\delta$</td>
</tr>
<tr>
<td>Exogenous TFP</td>
<td>$z$</td>
</tr>
<tr>
<td>Regulatory leverage ratio</td>
<td>$\tau$</td>
</tr>
<tr>
<td>Private benefit</td>
<td>$\zeta$</td>
</tr>
<tr>
<td>Bond inventory</td>
<td>$\kappa$</td>
</tr>
<tr>
<td>Distribution – $\mu(q^d)$</td>
<td>$\lambda_t$</td>
</tr>
<tr>
<td>Distribution – $\mu_d(q^d)$</td>
<td>$\lambda_d$</td>
</tr>
<tr>
<td>Distribution – $\mu_e(q^e)$</td>
<td>$\lambda_e$</td>
</tr>
<tr>
<td>Distribution – $\mu_b(q^b)$</td>
<td>$\lambda_b$</td>
</tr>
<tr>
<td>Bond management cost</td>
<td>$Q^b_b$</td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
</tr>
</tbody>
</table>

4 Effects of Banking Regulation

This section studies the effects of banking regulation on banks, the corporate bond market, and households at the steady state of the economy. Our focus is on the effects of a marginal increase in $\tau$ above 8.14%.

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12 All rates are deflated with the CPI (source: FRED, series CPALT01USA659N).
13 That is, $\lambda_d > \lambda_b > \lambda_e$. With the form of the cumulative distributions in (33), those parameters are also indicative of the degree of substitution between various assets. Thus, our calibration indicates that bonds are a closer substitute to deposits than bank equity, i.e. $\lambda_d - \lambda_b < \lambda_d - \lambda_e$.
14 We derive the optimal regulation later, in Section 5.
4.1 Bank-Based Intermediation

Banks pull several levers to adjust to an increase in capital requirements. Their first margin of adjustment is the funding mix. To meet the new requirements, banks raise their demand for equity and cut back on their demand for deposits. This results in an increase in the expected return on equity, while lowering the return on deposits. As banks de-leverage, their *ex ante* balance sheets shrinks. Reduced leverage boosts banks’ demand for interbank loans *ex post*, which drives up the interbank lending rate, $r^I_t$ (Figure 14 in the Appendix illustrates the adjustment in the return on each asset).

Bank de-leveraging affects loan issuance in several ways (Figure 2). Initially, a more stringent leverage ratio requirement induces banks to cut-back their supply of loans, pushing the loan rate, $r^L_t$, higher. Yet, banks’ efficiency gains dominate the adjustment, leading to an overall decline in $r^L_t$. Specifically, the regulatory tightening mitigates the agency problem on the interbank market, and improves banks’ funding conditions. As interbank rates are being bid up, the cutoff for servicing loans profitably, $\bar{q}^L_t$, increases, and funding liquidity further improves. Anticipating that they will offload loans in the interbank market more easily in case they face high loan servicing costs, banks raise they loan supply, and the return on loans, $r^L_t$, declines in equilibrium.

**Figure 2: Regulation and bank-based intermediation**

(a) Corporate loans  
(b) Lending efficiency  
(c) Interbank loans

Note: ★ Optimum at $\tau^* = 8.76\%$ ● Calibrated economy with $\tau = 8.14\%$.

4.2 Market-Based Intermediation

The above adjustments in bank-based intermediation spill over onto the bond market (Figure 3). The decline in the return on loans implies a reduction in the return on bonds, $r^B_t$, as firms arbitrage the two options of applying for a bank loan or issuing bonds (see the no-arbitrage condition (4)). As a result, firms issue more bonds, thus meeting increased demand from households (see below). Overall, financial intermediation shifts in favour of bond finance, at the expense of bank lending (the ratio of $b_t/\ell_t$ increases).

Figure 4 contrasts the partial and the general equilibrium effects of the regulatory tightening.
on measures of market liquidity. Starting from point $A$, we recall that an increase in the leverage requirement raises the interbank lending rate, $r_i^t$. Thus, banks’ opportunity cost of supplying market-making services – i.e. lending on the interbank market as opposed to holding bonds with net unit return $Q^h r^h_t$ – increases, raising the bid-ask spread. This prompts a decline in volume, $s_t$. Pausing at this partial equilibrium, point $B$, we would conclude that the tightening harms market liquidity, consistent with a number of empirical studies (e.g. Kotidis and van Horen (2018), Haselmann et al. (2018)). In general equilibrium, though, following the change in the relative returns on assets, households raise their demand for bonds (see Section 4.3) and, accordingly, for market-making services. This shift in demand (from $s^d_t$ to $s^d_i$) raises volumes to point $C$. As rising bond demand lowers the return on bonds, it induces yet another upward adjustment in the bid-ask spread, resulting in a new equilibrium at point $D$. Overall, the effect on trade volumes is positive, though.
Typical measures of market liquidity thus depict a varied impact of leverage regulation on bond markets. On the one hand, banks raise the bid-ask spread, reflecting an increase in their opportunity cost of warehousing bonds. On the other hand, the market deepens amid a decline in the cost of issuing bonds in the primary market. We also note that the wedge between firms’ cost of bonds, $r^b_t$, and the households’ net return on bonds, $Q^{bh}_t r^b_t (1 - \omega_t)$, widens on the back of not only a higher bid–ask spread, but also due to an increase in the average transaction cost on bonds (i.e. a lower $Q^{bh}_t$) as discussed below.

### 4.3 Households’ Portfolio Re–balancing

The changes in the relative return on assets and the associated adjustments in the balance sheets of banks and the firm, discussed above, are the mirror image of households’ portfolio re–balancing (Figure 5). Foremost are declining deposit returns, which encourage some depositors to leave their “preferred habitat” and search for alternative investment opportunities. As supported by our calibration, bonds represent the closest substitute to deposits for most households once transaction costs are taken into account. As such, the decline in the return on deposit initiates increasing household demand for bonds, adding to the reduction in the return on bonds. Rising (expected) return on equity, by comparison, induces only a modest increase in household equity holdings, reflecting high transaction costs for most households.
5 Optimal Regulation

The aim of this section is to describe the regulator’s trade–off, and to devise the optimal leverage ratio requirement. We begin by characterizing the market inefficiencies that warrant regulatory intervention in the first place.
Figure 7: Output, Consumption, Savings

(a) Output

(b) Consumption

(c) Savings

Note: ★ Optimum at $\tau^* = 8.76\%$ ● Calibrated economy with $\tau = 8.14\%$.

5.1 Why Regulate Banks?

The introduction of a minimum leverage ratio requirement, $\tau$ in $(16)$, is motivated by the existence of a pecuniary externality in the interbank market. As is clear from $(20)$, a bank’s borrowing limit increases with the interbank rate, $\tau^i_t$, and depends on the bank’s balance sheet structure. An increase in its leverage ratio, $e_t/(d_t + e_t)$, for instance, allows the bank to borrow more. As demand goes up, the equilibrium interbank rate increases, and the least efficient borrowers switch to become lenders. The average quality of the remaining borrowers improves, further raising their borrowing limit. Each individual bank fully understands the first round effects of de-leveraging. Yet, as a price-taker, it fails to internalise the second round effects, which operate via the interbank rate. As a consequence, banks tend to be excessively leveraged, compared with the regulator’s objective.

To better understand how these pecuniary externalities arise in the model, it is useful to consider a “counter-factual” economy, in which the cash flows from corporate bonds and market-making cannot be seized and the bank cannot default on its deposits. In such an economy, a bank’s balance sheet structure has no effect on its incentives to extract cash, and incentive-compatible borrowing limit in $(20)$ becomes:

$$\phi_t = \frac{\ell_t}{\zeta_t} (\tau^i_t - \zeta_t).$$

Since this limit is independent of leverage, the bank values equity less than in the benchmark economy. The privately optimal leverage ratio is thus much lower than in our benchmark calibration, i.e. less than 1%. Is there a pecuniary externality in the counter-factual economy? If there is none, we expect that any minimum leverage ratio requirement reduces welfare. In Panel (a) of Figure 8, we report the level of welfare in the counter-factual economy for various values of $\tau$. The welfare reaches its maximum at the private equilibrium outcome (the dot and the star coincide). This means that there is no need for regulation in this economy, and no pecuniary externality. For comparison, we report in Panel (b) the level of welfare for our benchmark economy. In this case, the equilibrium of the optimally regulated economy is distinctively higher than in the calibrated
economy. This result only highlights that the pecuniary externalities present in the benchmark model arise exclusively from the dependence of $\phi_t$ on banks’ balance sheet structure (see (20)).

Figure 8: Pecuniary Externality

(a) Welfare in the counter-factual Economy

(b) Welfare in the benchmark Economy

Note: ★ Optimum at $\tau^* = 0.8\%$ (Panel (a)) or $\tau^* = 8.76\%$ (Panel (b)) • Calibrated economy with $\tau = 0.8\%$ (Panel (a)) or $\tau = 8.14\%$ (Panel (b)).

5.2 Optimal Leverage Ratio Requirement

As discussed in Section 5.1, regulation is beneficial because it relaxes banks’ wholesale funding constraints and, by doing so, improves the re-allocation of funds from the least efficient, low–$q^\ell$ banks to the most efficient, high–$q^\ell$ banks. This, in turn, reduces the deadweight loss of inefficiently allocated bank lending, as measured by $\chi^\ell_t$ (see relation (31)). Yet regulation is also costly. By constraining banks’ balance sheets, it alters households’ portfolio decisions and raises the associated financial transaction costs, as measured by $\chi^a_t$ (see relation (31) and the discussion in Section 4.3). Figure 9 illustrates this trade-off. Households’ financial transaction costs monotonically increase with $\tau$, whereas banks’ financial intermediation costs monotonically decrease with $\tau$. The regulator maximises welfare, by striking a balance between the reduction in $\chi^\ell_t$ and the increase in $\chi^a_t$.

The optimal leverage requirement, $\tau^*$, at about 8.76%, minimises $\chi_t$. At this point, the economy reaches its constraint–efficient optimum, as depicted in Figure 8. We calculate that the net welfare gain corresponds to a 0.074% increase in permanent consumption, relative to the unregulated economy.

5.3 Optimal Regulatory Response to Changes in Market Frictions

The aim of this section is to discuss how the optimal regulatory leverage ratio, $\tau^*$, varies, as the frictions driving funding liquidity (captured by parameter $\zeta$) or market liquidity (captured by
Figure 9: The Regulator’s Trade–off

(a) Banks’ Deadweight Loss  
(b) Households’ Deadweight Loss  
(c) Overall Deadweight Loss

Note: ⋆ Optimum at \( \tau^* = 8.76\% \) ● Calibrated economy with \( \tau = 8.14\% \).

parameter \( \kappa \) become more severe.\(^{15}\) Panel (a) of Figure 10 (dashed green line) shows that when bank’s moral hazard problem becomes worse (\( \zeta \) goes up) and pecuniary externalities are stronger, it is optimal for the regulator to tighten leverage regulation. This result is consistent with our earlier discussion in Section 5.1.

Interestingly, Panel (b) shows that the regulator also adjusts its optimal requirement based on the degree of liquidity of the bond market. This result highlights that the role of banks as market–makers is an important component of the regulator’s trade–off. To comply with regulation, banks have essentially two margins of adjustment. One consists of issuing equity, keeping the size of the balance sheet unchanged. Given households’ high financial transaction costs related to equity investment,\(^{16}\) such an adjustment is relatively costly. The other margin consists in shrinking in size. This can be achieved if households re–balance their portfolio away from deposits and bank equity, toward corporate bonds. In effect, the bond market acts as a buffer that helps to mitigate the overall cost of regulation. However, when banks must keep more inventory to make the corporate bond market, the bid–ask spread goes up and households’ appetite for bonds subsides, undermining the role of the bond market as a buffer. Lower market liquidity thus makes bank de–leveraging more costly. For the regulator, it is optimal to react by loosening leverage ratio requirements, implying that \( \tau^* \) diminishes as \( \kappa \) goes up.

5.4 “Narrow” Leverage Ratio Requirements

Since requiring banks to fund bonds with a minimum amount of equity raises the bid–ask spread, and a higher bid–ask spread raises the cost of regulation, one natural question is whether the regulator can further raise welfare by exempting bonds from capital requirements. To answer this

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\(^{15}\)When \( \zeta = 0 \), banks reap no gain from extracting cash and, therefore, there is no moral hazard in the interbank market. When \( \kappa = 0 \), banks do not need to hold an inventory to make markets. When \( \zeta = \kappa = 0 \), the model boils down to a standard, frictionless, Real Business Cycle model.

\(^{16}\)This is reflected in the distribution of households being skewed toward depositors, i.e. in parameter \( \lambda_d \) being very low relative to parameters \( \lambda_d \) and \( \lambda_b \); see Table 2.
question, we compare the effects of the “comprehensive” leverage regulation in Assumption 5 with those of a “narrow” leverage regulation, as in Assumption 6.

**Assumption 6 (Banking regulation)** The regulator imposes a minimum “narrow” leverage ratio,  
\[
\frac{e_t}{\ell_t} \geq \theta. \quad (16')
\]

Under Assumption 6, the regulator does not require banks to fund their bonds with any equity. It only requires banks to have “skin in the game” to back their loans. Given that only the cash flow from loans can be diverted – i.e. only loans are “risky” (see Assumption 4) – a carve–out for bonds is appealing.

We thus assume that the regulator exempts bonds and initially sets \(\theta\) at 8.76%, the value of \(\tau^*\). Since a bank’s narrowly defined leverage ratio is, all things equal, higher than its comprehensively measured one, the exemption loosens the regulatory stance. Following the loosening, banks leverage up, without internalising the negative effect on their borrowing capacity. Panel (a) of Figure 11 shows that it is then optimal for the regulator to tighten the narrow leverage requirement, from \(\theta = 8.76\%\) to \(\theta^* = 9.12\%\).

Furthermore, Panel (a) of Figure 12 illustrates that, given our calibration, the optimal narrow leverage requirement in effect leads banks to raise their leverage ratio up to the level that would prevail under the optimal leverage requirement, i.e \(e_t/(d_t + e_t) = 8.76\%\). Put differently, banks finance their bond inventories with as much equity under the narrow leverage regulation, as they do under the more comprehensive regulation. This can be seen by comparing the green (optimal leverage ratio) and the blue (optimal narrow leverage ratio) stars in Figure 12a. The upshot is that, for a given regulatory stance, exempting bonds does not change the funding mix of bond inventories.
Figure 11: Narrow versus comprehensive leverage regulation: welfare and bid-ask spread

Note: ----- Optimal leverage ratio requirement  ----- Optimal narrow leverage ratio requirement  *  (*) Optimum at $\tau^* = 8.76\%$ ($\theta^* = 9.12\%$).

Figure 12: Narrow versus comprehensive leverage regulation: balance sheet structures

Note: ----- Leverage ratio requirement  ----- Narrow leverage ratio requirement  *  (*) Optimum at $\tau^* = 8.76\%$ ($\theta^* = 9.12\%$)  ●(●) Calibrated economy.
Notwithstanding this equivalence, exempting bonds from the leverage ratio has distributional effects on households. Since banks are not required to fund inventories with equity, the bid–ask spread is lower (compare the green and the blue stars in Panel (b) of Figure 11). The lower cost of market–making services has material implications for households and firms. Panel (b) of Figure 12 shows that households re–balance their portfolio toward bonds. This is intuitive, given that their return on bonds, net of market–making costs, increases (see Panel (a) of Figure 15), relative to that on bank equity and deposits. The higher demand for bonds is matched by a higher supply by firms, whose loan–to–credit ratio diminishes (Panel (c) of Figure 12). Overall, the change from the optimal leverage ratio $\tau^*$ to the optimal narrow leverage ratio $\theta^*$ affects households, banks, and firms’ balance sheet structures. Yet these compositional effects neither affect the size of the economy (Panel (b) of Figure 15 and Panel (c) of 15) nor welfare (Panel (a) of Figure 11).

Finally, Figure 10 suggests that exempting bonds from leverage ratio requirements does not spare the regulator from adjusting $\theta^*$ to changes in the frictions that affect funding liquidity (Panel (a)) or market liquidity (Panel (b)).

6 Model Dynamics

TBC

7 Conclusion

This paper presents a first attempt to study the general equilibrium effects of leverage regulation on market liquidity, corporate funding and welfare. To do so, we develop a model that takes account of banks’ dual role as lenders to firms and market–makers in bond markets. In this setup, raising capital requirements helps banks overcome frictions in the interbank market, which lowers the overall cost of financial intermediation by improving the allocation of funds across banks. At the same time, regulation supports the expansion of the bond market, while inducing banks to raise bid–ask spreads. As such, it has a varied impact on different measures of market liquidity. In addition, by altering the relative return on assets, regulation redistributes income across households.

Calibrating our model to US data, we explain and quantify how a conscious regulatory approach strikes a balance between all these effects. We derive broad benchmarks for optimal capital requirements and discuss how regulation should react to changes in market liquidity and funding liquidity.

Our analysis underscores the importance of evaluating regulatory reforms holistically rather than considering individual markets in isolation or focussing solely on partial equilibrium effects. Calls for exempting specific financial instruments from leverage regulation are a case in point. While exempting banks’ market–making inventory would indeed reduce bid–ask spreads in our model, we find that this would neither raise output nor welfare.
The benefits of a parsimonious and tractable model come at a cost. In this paper, we have set aside several important features of financial intermediation. For one, we summarise banks’ market–making in an admittedly crude way, neglecting details of the market microstructure. We have also limited ourselves to the discussion of a single regulatory tool, notwithstanding the multiple ratios that form today’s regulatory framework. Enriching the model along these or other dimensions provides avenues for future research.
A Appendix

A.1 Timeline

Figure 13: Shocks and Decisions in Period $t$

- Agents’ decisions / market clearing
- Idiosyncratic shocks
- Aggregate shocks
A.2 Additional Figures

Figure 14: Return on assets

(a) Deposits  (b) Bond holdings  (c) Equity

(d) Bond issuance  (e) Interbank loans

Note: ⋆ Optimum at $\tau^* = 8.76\%$ • Calibrated economy with $\tau = 8.14\%$.

Figure 15: Net bond return and size of the economy

(a) Net bond return  (b) Output  (c) Savings

Note: • • • Leverage ratio requirement — Narrow leverage ratio requirement ⋆ (⋆) Optimum at $\tau^* = 8.76\%$ ($\theta^* = 9.12\%$) • (⋆) Calibrated economy.
A.3 Recap of the Model

This section recaps the equations of the model. The term \( L \) is a dummy equal to one if the regulator imposes the leverage ratio rule \( (16) \), and to zero if the regulator imposes the leverage ratio rule \( (16) \).

**Firms**

1. \( y_t = z_t k_t^a \)
2. \( k_t = \xi_t + b_t \)
3. \( r_t^b = r_t^b \)
4. \( r_t^b = \alpha z_t k_t^{a-1} + 1 - \delta \)

**Representative Household**

5. \( E_t (\Psi_{t+1} \tau_{t+1}) = 1 \)

6. \( Q_t^{d} = \int_{0}^{1} \mu_{b} \left( \frac{q^{d} E_{t-1} (\Psi_{t-1,t} r_{t}^{d})}{E_{t-1} \left(\Psi_{t-1,t} r_{t}^{d} \right)} \right) \mu_{d} \left( \frac{q^{d} E_{t-1} (\Psi_{t-1,t} r_{t}^{d})}{E_{t-1} \left(\Psi_{t-1,t} r_{t}^{d} \right)} \right) d\mu_{d} (q^{d}) \)

7. \( Q_t^{h} = \int_{0}^{1} \mu_{b} \left( \frac{q^{h} E_{t-1} (\Psi_{t-1,t} r_{t}^{h})}{E_{t-1} \left(\Psi_{t-1,t} r_{t}^{h} \right)} \right) \mu_{e} \left( \frac{q^{d} E_{t-1} (\Psi_{t-1,t} r_{t}^{d})}{E_{t-1} \left(\Psi_{t-1,t} r_{t}^{d} \right)} \right) d\mu_{d} (q^{h}) \)

\[
+ \frac{\alpha \delta}{b_t} \int_{0}^{1} \frac{\mu_{e} \left( \frac{q^{h} E_{t-1} (\Psi_{t-1,t} r_{t}^{h})}{E_{t-1} \left(\Psi_{t-1,t} r_{t}^{h} \right)} \right) \mu_{d} \left( \frac{q^{d} E_{t-1} (\Psi_{t-1,t} r_{t}^{d})}{E_{t-1} \left(\Psi_{t-1,t} r_{t}^{d} \right)} \right) d\mu_{d} (q^{h}) \)
\]

8. \( e_t = a_t - b_t^c - d_t \)

9. \( r_t^{h} = (1 - \omega_t) r_t^b \)

**Representative Bank**

10. \( \phi_t = \frac{(r_t^{c} - \zeta_t) \xi_t + Q_t^{h} b_t^c b_t^h + (\omega_t - Q_t^{h}) r_t^{c} s_t - r_t^{h} d_t}{\zeta_t} \)

11. \( E_{t-1} (\Psi_{t-1,t} (1 + \Delta_t) r_t^{c}) = E_{t-1} (\Psi_{t-1,t} (1 + \Delta_t)) r_t^{d} + \tau \Lambda_t \)

12. \( \Gamma_t + (1 - L) \tau \Lambda_t + E_{t-1} (\Psi_{t-1,t} (1 + \Delta_t) Q_t^{h} r_t^{h}) = E_{t-1} (\Psi_{t-1,t} (1 + \Delta_t) r_t^{h}) \)

13. \( \omega_t = \kappa \frac{E_{t-1} (\Psi_{t-1,t} (1 + \Delta_t) r_t^{c})}{E_{t-1} (\Psi_{t-1,t} (1 + \Delta_t) r_t^{d})} \)

14. \( \Lambda_t \left( e_t - \tau (\xi_t + L (b_t^h - s_t)) \right) = 0 \)

15. \( \Gamma_t (b_t^h - (1 + \kappa) s_t) = 0 \)

16. \( e_t + d_t = \ell_t + b_t^h - s_t \)

**Market Clearing**

17. \( y_t = c_t + k_t + (1 - \delta) k_t + \chi_t \)

18. \( (1 - \mu (\Psi_t^{c})) \phi_t = \mu (\Psi_t^{c}) \ell_t \)

19. \( b_t = b_t^h \)

20. \( b_t^h = s_t \)
21. \( r_t \equiv Q_t^r \frac{dt}{a_t} + Q_t^b \frac{dt}{a_t} + Q_t^c \frac{et}{a_t} \)

22. \( \Psi_{t-1,t} \equiv \beta u'(c_{t+1}) \omega'(c_t) \)

23. \( x_t \equiv (1 - Q_t^f) r_t^f e_t + (1 - Q_t^b) r_t^b (b_t^b - s_t) + (1 - Q_t^d) r_t^d d_t + (1 - Q_t^h) r_t^h b_t^h + (1 - Q_t^e) r_t^e e_t \)

24. \( \overline{q}_t^q \equiv \frac{q_t^q}{r_t^q} \)

25. \( \overline{Q}_t^q \equiv \int_0^1 q_t^q \frac{d\mu_{q_t}(q^q)}{1 - \mu_{q_t}(\overline{q}_t^q)} \)

26. \( \overline{Q}_t^q \equiv a_t \int_0^1 q_t^q \mu_{b_t} \left( q_t^q \frac{E_t - 1 (\Psi_t - 1, t^q_{r_t^q})}{E_t - 1 (\Psi_t - 1, t^q_{r_t^q})} \right) d\mu_{b_t}(q^b) \)

27. \( \overline{Q}_t^b \equiv a_t b_t \int_0^1 q_t^b \mu_{b_t} \left( q_t^b \frac{E_t - 1 (\Psi_t - 1, t^b_{r_t^b})}{E_t - 1 (\Psi_t - 1, t^b_{r_t^b})} \right) d\mu_{b_t}(q^b) \)

28. \( \overline{Q}_t^c \equiv a_t c_t \int_0^1 q_t^c \mu_{c_t} \left( q_t^c \frac{E_t - 1 (\Psi_t - 1, t^c_{r_t^c})}{E_t - 1 (\Psi_t - 1, t^c_{r_t^c})} \right) d\mu_{c_t}(q^c) \)

29. \( \Delta_t \equiv \frac{(1 - \mu_{\overline{q}_t^q}) (Q_t^q e_t - r_t^q)}{\overline{q}_t^q} \)

30. \( r_t^c \equiv (1 + \Delta_t) r_t^c + (1 + \Delta_t) \left( r_t^c - Q_t^b r_t^b \right) \frac{dt}{a_t} + (1 + \Delta_t) \left( r_t^c - Q_t^b r_t^b \right) b_t^b \frac{dt}{a_t} + (1 + \Delta_t) \left( r_t^c + (\omega_t - Q_t^b) r_t^b \right) \frac{dt}{a_t} \)
References


