Liquidity Risk and the Dynamics of Arbitrage Capital

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- When non-financial firms/individuals trade, almost always some specialized agents like market makers, brokers, and hedge funds take the other side
- A view of financial markets: non-financial firms/individuals face various risks which financial firms are willing to partially absorb for compensation
  - $\rightarrow$ main issue: no immediate new inflows of financial capital (even if returns are high)
- We take this view in this paper and characterize the joint dynamics of asset prices and financial capital
- Main application: outcome generate "liquidity facts" in essentially frictionless economy

Liquidity facts

- assets' illiquidity: difficulty to sell
  - measures: price impact/ negative autocorrelation/ spread
- Liquidity varies over time and in correlated manner across assets (aggregate illiquidity)
- Liquidity can be priced risk factor: higher expected return for
  - ...stocks paying off when liquidity high?
  - ...stocks more liquid when liquidity high?
  - ...stocks more liquid when aggregate return high?
- Aggregate illiquidity depends on financial institutions' level of capital (Market makers, arbitrageurs, speculators, hedge funds, trading desks of investment banks...)

A dynamic model of risk-sharing

- Continuous time, infinite horizon, $t \in [0, \infty)$.

Hedgers.(e.g. non-financial firms, farmers, individuals)

- Endowment $u^\top dD_t$ at $t + dt \Rightarrow$ hedging demand at t, where

$$dD_t = \bar{D}dt + \sigma^\top dB_t,$$

  and B_t is N-dimensional Brownian motion. Payoff covariance matrix $\Sigma \equiv \sigma^\top \sigma$.

- Mean-variance preferences over change dv_t in wealth between t and $t + dt$

$$\frac{\mathbb{E}_t(dv_t)}{dt} - \frac{\alpha}{2}\frac{\mathbb{V}\mathrm{ar}_t(dv_t)}{dt}$$

  $\Rightarrow$ Demand for insurance is constant over time.

Arbitrageurs.(e.g. dealers, brokers, hedge funds, insurance companies,etc..)

- CRRA preferences over intertemporal consumption

$$\mathbb{E}_t\left(\int_t^{\infty}\frac{c_s^{1-\gamma}}{1-\gamma}e^{-\rho(s-t)}ds\right)$$

$\Rightarrow$ Supply for insurance is time-varying because of wealth effects.

Assets.

- (for now), N short-lived risk-sharing contracts at each time t.
  - Payoff dD_t at time $t + dt$
  - Price $\pi_t dt$ at time t.
  - Zero net supply.
  - (we'll introduce long-lived financial assets in a bit)

- exogenous riskless rate r

Equilibrium Prices and Positions

- Arbitrageur positions:

$$y_t = \frac{\alpha}{\alpha + A(w_t)} u.$$

  - Arbitrageurs hold fraction of portfolio u that hedgers want to sell.
  - Standard risk-sharing rule, but with effective risk aversion $A(w_t)$.

$$A(w_t) \equiv \underbrace{\frac{\gamma}{w_t}}_{\text{Static ARA}} - \underbrace{\frac{q'(w_t)}{q(w_t)}}_{\text{Intertemporal hedging}}$$

  - Asset prices:

$$\pi_t = \bar{D} - \frac{\alpha A(w_t)}{\alpha + A(w_t)} \Sigma u.$$

    - Portfolio u that hedgers want to sell is single pricing factor.

Effective Risk Aversion

$$A(w_t) \equiv \underbrace{\frac{\gamma}{w_t}}_{\text{Static ARA}} - \underbrace{\frac{q'(w_t)}{q(w_t)}}_{\text{Intertemporal hedging}}$$

is **effective risk aversion**

- Logarithmic preferences ($\gamma = 1$).
  - consumption proportional to wealth
  - Effective risk aversion is

$$A(w_t) = \frac{1}{w_t}.$$

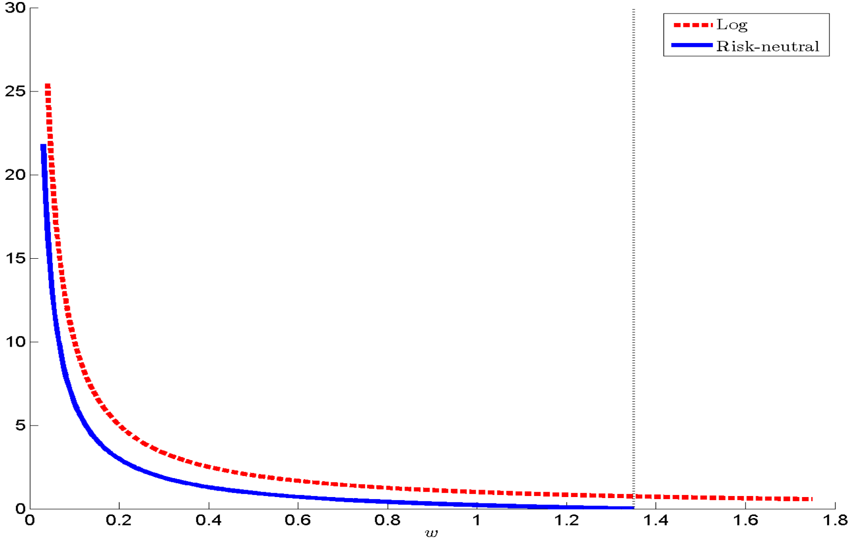
  - Static ARA. No intertemporal hedging.

- Risk-neutral preferences ($\gamma \to 0$) and riskless rate $r \to 0$.
  - Consumption is zero for $w_t \in (0, \bar{w})$ and at infinite rate for $w_t \in (\bar{w}, \infty)$.
  - Effective risk aversion is

$$A(w_t) = \frac{\alpha}{1+z}\left(\sqrt{z}\cot\left(\frac{\alpha w_t}{\sqrt{z}}\right) - 1\right) \quad \text{for} \quad w_t \in (0, \bar{w}),$$

  where $z \equiv \frac{\alpha^2 u^\top \Sigma u}{2\rho}$ and $\cot\left(\frac{\alpha \bar{w}}{\sqrt{z}}\right) \equiv \frac{1}{\sqrt{z}}$.
  - Static ARA=0. Only intertemporal hedging.

Effective Risk Aversion



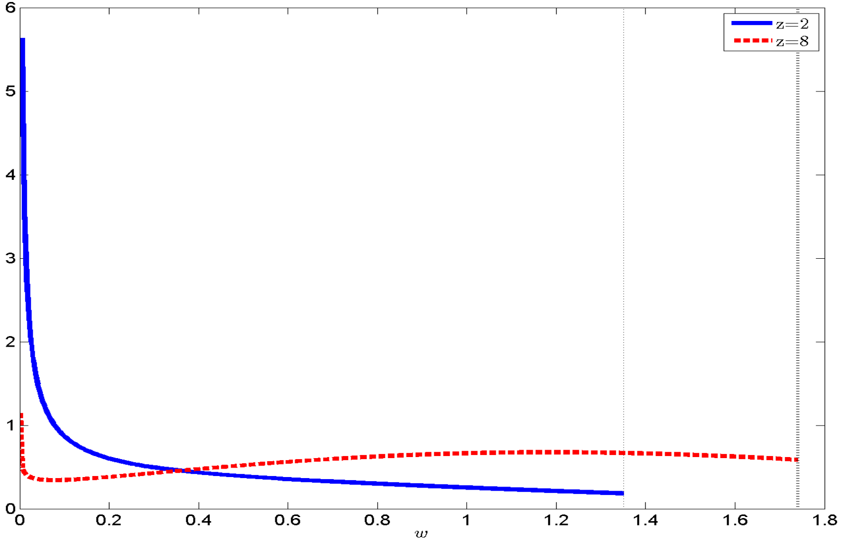
Stationary Distribution

- Self-correcting dynamics: Arbitrageur Sharpe ratio (SR)
- Depends only on $z \equiv \frac{\alpha^2 u^\top \Sigma u}{2(\rho - r)}$.

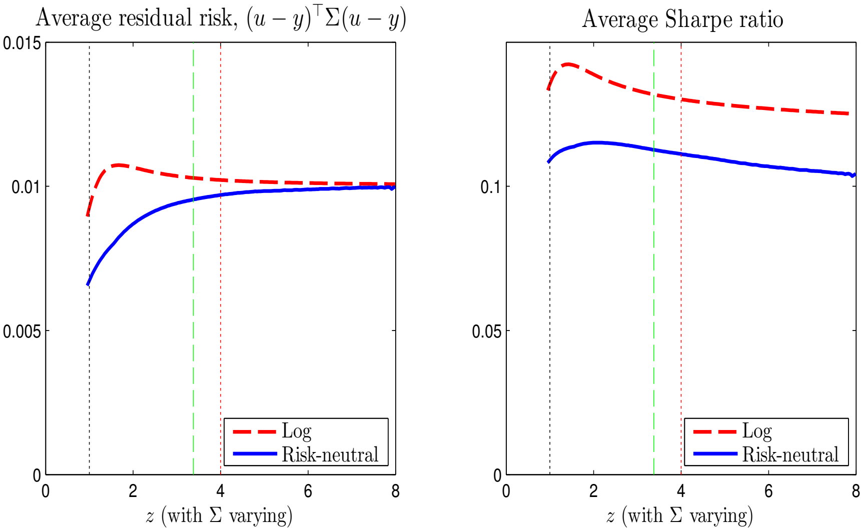
$0 < z < 1$	$1 < z < \bar{z}$	$\bar{z} < z$
w_t converges to 0	decreasing pdf	bimodal pdf

- $\bar{z} = \frac{27}{8}$ in logarithmic case, $\bar{z} = 4$ in risk-neutral case decreases in wealth.
- z pushes distribution to the left in a Monotone Likelihood Ratio-sense
- bimodal distribution, sign of sytemic risk?

Shape of Stationary Density



Unconditional objects



Long-Lived Assets

- N risky assets.
  - Price S_t at time t.
  - Payoff $dD_{t'}$ for times $t' > t$. (Infinite stream of short-lived assets' payoffs.)
  - Zero net supply.
- Comparison with short-lived assets:
  - Same allocation of risk and market prices of risk.
  - But an asset $\neq$ a claim on unit risk: return depends on (endogenous) price-dynamics $\rightarrow$ Liquidity risk.
    - Zero with short-lived assets.
  - Time-varying volatilities and correlations.
    - Constant with short-lived assets.
  - key: how dD_t shocks are transformed to wealth shocks and how this affects expected returns.

Endogenous wealth shocks and expected returns

long-lived assets do not affect risk sharing

↓

risk holding proportional to u

↓

dD_t changes wealth proportionally to u

↓

more compensation for assets covarying with u

↓

price negatively, expected return positively proportional to Σu

Equilibrium

- Asset prices proportional to Σu ($g(0) = 0, \frac{\alpha}{r} > g(w_t) > 0, g'(w_t) > 0$):

$$S(w_t) = \underbrace{\frac{\bar{D}}{r}}_{\text{risk-neutral price}} - \underbrace{\left(\frac{\alpha}{r} - g(w_t)\right)\Sigma u.}_{\text{premium}}$$

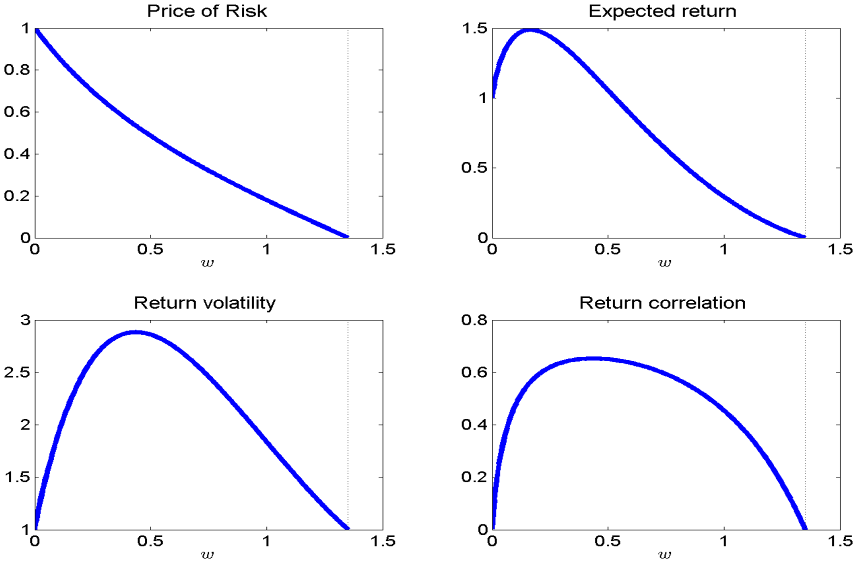
- expected return also proportional to Σu:

$$\frac{\mathbb{E}_t(dR_t)}{dt} = \underbrace{\frac{\alpha A(w_t)}{\alpha + A(w_t)}\left[\frac{\alpha g'(w_t) u^\top \Sigma u}{\alpha + A(w_t)} + 1\right]}_{\text{scalar, hump-shaped in } w_t}\Sigma u.$$

- Arbitrageurs' holding in assets is also proportional to u and grows from 0 to u

Closed-Form Solutions

- Compute $g'(w_t)$ in logarithmic and risk-neutral cases when $r \to 0$.
- Volatilities are hump-shaped in arbitrageur wealth.
  - Price volatility = Volatility of arb. wealth $\times$ Price sensitivity to wealth.
  - $w_t \approx 0 \Rightarrow$ Volatility of arb. wealth ≈ 0.
  - w_t large $\Rightarrow$ Price sensitivity to wealth ≈ 0.
- Expected returns are also hump-shaped
  - Price of risk still decreasing but volatility is hump-shaped
- Non-fundamental covariance because of wealth shocks: humped shape in wealth
  - effect all returns the most when their non-fundamental volatility is highest


Price of Risk
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
w
Expected return
1.5
1
0.5
0
0
0.5
1
1.5
w
Return volatility
3
2.5
2
1.5
1
0
0.5
1
1.5
w
Return correlation
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
w

Illiquidity of an asset: Kyle's lambda

- Define illiquidity λ_{nt} of asset n as price change per unit of quantity traded, following shock to u_n, hedgers' willingness to hold asset n
  - Kyle's lambda: $\lambda_{nt} \equiv \frac{\frac{\partial S_{nt}}{\partial u_n}}{\frac{\partial X_{nt}}{\partial u_n}}$
  - Can also interpret λ as concerning price difference between asset pair with different u's.
- Illiquidity λ_{nt} of asset n is equal to

$$\left(1 + \frac{A(w_t)}{\alpha} + g'(w_t)u^\top \Sigma u\right)\left(\frac{\alpha}{r} - g(w_t)\right)\Sigma_{nn}.$$

  - Depends on n through variance Σ_{nn} of asset's payoff (consistent with Stoll (1978), Chen,Lesmond and Wei (2007) etc) .
  - Decreasing in arbitrageur wealth.

Illiquidity factor

- consider one of A-P illiquidity betas (covariance of asset's return and aggregate illiquidity,$\Lambda_t = -\frac{\sum \lambda_{nt}}{N}$):

$$\beta_n(w) \equiv \mathbb{C}\mathrm{ov}(d\Lambda_t, dR_{nt})$$

- as Λ_t monotonic in wealth and wealth shocks affect prices proportional to Σu, we have $\beta_n(w) = \beta^C(w)\Sigma u$ (while illiquidity proportional to Σ_{nn})

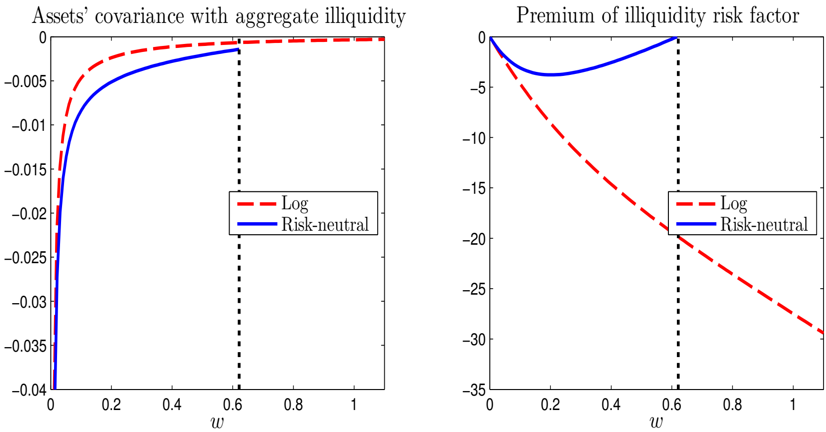
- as expected returns are also proportional to Σu we can write

$$\frac{\mathbb{E}_t(dR_n t)}{dt} = \Pi(w)\beta_n(w)$$

with $\Pi(w)$ premium for illiquidity risk

- $\Rightarrow$ Asset n's expected return is explained by:
  - Covariance between asset's return and aggregate liquidity.
  - Not by covariance between asset's liquidity and aggregate liquidity or return.

Illiquidity Factor: Covariance and Premium



$w_t \approx 0$: Illiquidity is large and highly sensitive to $w_t \Rightarrow$ Covariance is large and premium is small.

- Intuition
  - as arbitrageurs' take one side of each trade, their f.o.c determines prices: their portfolio is the pricing factor
  - Assets covarying most with hedgers' portfolio:
    - Have high expected returns.
    - And drop the most when arbitrageur wealth decreases.
  - empirical measures of illiquidity factor are proxying this pricing factor

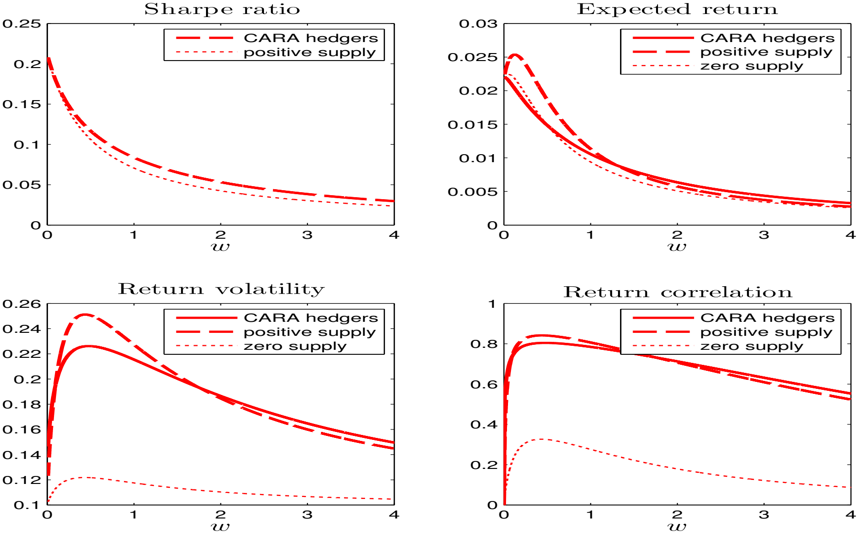
Infinitely lived CARA hedgers and positive supply

- add a positive supply vector of assets, and...
- CARA preferences over intertemporal consumption:

$$\mathbb{E}_t \left(\int_t^\infty - \exp(-\alpha \hat{c}_s) e^{-\hat{\rho}(s-t)} ds \right).$$

  - Preserves no wealth effects for hedgers.
  - Adds intertemporal hedging demand.
    - Hedge against changes in arbitrageur wealth: effective risk-aversion for hedgers depends on w_t also
  - Solutions become numerical (ODE), but main results remain the same.

Infinitely lived CARA hedgers and positive supply



Related Literature

Arbitrageur Capital, Asset Prices and Liquidity.

- Constraints on equity capital: Shleifer-Vishny (1997), He-Krishnamurthy (2012,2013a).
- Wealth effects: Kyle-Xiong (2001), Xiong (2001), Basak-Pavlova (2013).
- Margin/VaR constraints: Gromb-Vayanos (2002), Brunnermeier-Pedersen (2009), Garleanu-Pedersen (2011), Chabakauri (2013), Danielsson-Shin-Zigrand (2013).
- Holding costs: Tuckman-Vila (1992), Kondor (2009).
- Macro: Brunnermeier-Sannikov (2013), He-Krishnamurthy (2013b).
- Survey by Gromb-Vayanos (2010).

Dynamic Risk-Sharing with CRRA Agents.

- Production economy: Dumas (1989).
- Endowment economy: Wang (1996), Chan-Kogan (2002), Bhamra-Uppal (2009), Garleanu-Panageas (2013), Longstaff-Wang (2013).

Conclusion

- Continuous-time, multi-asset model of liquidity provision with wealth effects.
- Implications for: Expected asset returns, volatilities, correlations, arbitrageur positions, short-run and long-run dynamics.
- Pricing of illiquidity factors:
  - as arbitrageurs' take one side of each trade, their f.o.c determines prices: their portfolio is the pricing factor
  - Assets covarying most with hedgers' portfolio:
    - Have high expected returns.
    - And drop the most when arbitrageur wealth decreases.
  - empirical measures of illiquidity factor are proxying this pricing factor
- Extensions in online appendix (positive supply assets, long-horizon hedgers, stochastic u_t)