Aggregate Bank Capital and Credit Dynamics

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First draft: December, 2014
This version: March, 2016

Abstract

Central banks need models where the long term impact of macroprudential policies can be assessed. This paper proposes a model with this feature. In our model commercial banks finance their loans with deposits and equity, while facing equity issuance costs. Because of this financial friction, banks build equity buffers to absorb negative shocks. Aggregate bank capital determines the dynamics of credit. Notably, the equilibrium loan rate is a decreasing function of aggregate capitalization. The competitive equilibrium is constrained inefficient, because banks do not internalize the effect of their individual lending decisions on the future loss-absorbing capacity of the banking sector. In particular, we find that undercapitalized banks lend too much. Minimum capital ratios help tame excessive lending, which enhances stability of the banking system.

Keywords: macro-model with a banking sector, aggregate bank capital, pecuniary externality, capital requirements
JEL: E21, E32, F44, G21, G28

Acknowledgements: we thank Toni Ahnert, Hengjie Ai, George-Marios Angeletos, Manuel Arellano, Philippe Bacchetta, Bruno Biais, Markus Brunnermeier, Catherine Casamatta, Julien Daubanes, Peter DeMarzo, Jean-Paul Décamps, Sebastian Di Tella, John Geanakoplos, Hans Gersbach, Hendrik Hakenes, Lars Peter Hansen, Zhiguo He, Christian Hellwig, Florian Hoffmann, Eric Jondeau, Simas Kucinskas, Li (Erica) Xuenan, Michael Magill, Semyon Malamud, Loriano Mancini, David Martinez-Miera, Erwan Morellec, Alistair Milne, Steven Ongena, Henri Pagès, Bruno Parigi, Greg Phelan, Guillaume Plantin, Martine Quinzii, Raphael Repullo, Yuliy Sannikov, Enrique Sentana, Amit Seru, Jean Tirole and Stéphane Villeneuve, and seminar participants at the Banque de France, CEMFI, CICF 2015 conference, Chicago Booth, EPF Lausanne, ETH Zürich, IMF, MFM Winter 2016 Meeting, Princeton University, Stanford University, Toulouse School of Economics, the University of Bonn, the University of Nanterre, Yale University.

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1 Introduction

In the context of their new macro-prudential responsibilities, central banks have recently been endowed with powerful regulatory tools. These tools include the setting of capital requirements for all banks, determining capital add-ons for systemic institutions, and deciding when to activate counter-cyclical capital buffers. The problem is that very little is known about the long term impact of these regulations on growth and financial stability. The DSGE models that central banks currently have at their disposal have been designed for very different purposes, namely assessing the short term impact of monetary policy decisions on inflation and economic activity. Until recently, DSGE models did not even include banks in their representation of the economy. DSGE models are very complex and use very special assumptions, because they have been specifically calibrated to reproduce the short term reaction of prices and employment to movements in central banks' policy rates. It seems therefore clear that complementary models designed to assess the long term impact of macro-prudential policies on bank credit, GDP growth and financial stability are needed. This paper proposes an example of such a model.

Building on the recent literature on macro models with financial frictions, we develop a tractable dynamic model where aggregate bank capital determines the dynamics of lending. Though highly stylized, the model is able to generate predictions in line with empirical evidence. Moreover, we show that our model framework is flexible enough to accommodate the analysis of regulatory policies by looking at the long-run impact of capital regulation on lending and financial stability.

We consider an economy where firms borrow from banks that are financed by deposits, secured debt and equity. The aggregate supply of bank loans is confronted with the firms' demand for credit, which determines the equilibrium loan rate. Aggregate shocks impact the firms' default probability, which ultimately translates into profits or losses for banks. Banks can continuously adjust their volumes of lending to firms. They also decide when to distribute dividends and when to issue new equity. Equity issuance is subject to deadweight costs, which constitutes the main financial friction in our economy and creates room for the loss-absorbing role of bank capital.[1]

In a set-up without the financial friction (i.e., no issuance costs for bank equity) and i.i.d. aggregate shocks, the equilibrium volume of lending and the nominal loan rate would be constant. Furthermore, dividend payment and equity issuance policies would be trivial in this case: Banks would immediately distribute all profits as dividends and would issue new shares to offset losses and honor obligations to depositors. This implies that, in a frictionless world, there would be no need to build up capital buffers and all loans would be entirely financed by debt.

When the financial friction is taken into account, banks' dividend and equity issuance strategies become less trivial. We show that there is a unique competitive equilibrium, where all variables

[1] Empirical studies report sizable costs of seasoned equity offerings (see e.g. Lee, Lochhead, Ritter, and Zhao (1996), Hennessy and Whited (2007)). Here we follow the corporate finance literature (see e.g. Décamps et al. (2011) or Bolton et al. (2011)) and the banking literature (see e.g. De Nicolò et al. (2014)) by assuming that issuing new equity entails a deadweight cost proportional to the size of the issuance.

of interest are deterministic functions of the aggregate book value of bank equity, which follows a Markov process reflected at two boundaries. Banks issue new shares at the lower boundary, where aggregate book equity of the banking system is depleted. When aggregate bank equity reaches its upper boundary, any further earnings are paid out to shareholders as dividends. Between these boundaries, the changes in banks' equity are only due to their profits and losses. Banks retain earnings in order to increase their loss-absorbing equity buffer that allows them to guarantee the safety of issued debt claims, while avoiding raising new equity too frequently.[2] The target size of this loss-absorbing buffer is increasing with the magnitude of the financial friction.

We start by exploring the properties of the competitive equilibrium in the "laissez-faire" environment, in which banks face no regulation. Even though all agents are risk neutral, our model generates a positive spread for bank loans. This spread is decreasing in the level of aggregate bank capital. To get an intuition for this result, note that bank equity is more valuable when it is scarce. Therefore, the marginal (or market-to-book) value of equity is higher when total bank equity is lower. Moreover, as profits and losses are positively correlated across banks, each bank anticipates that aggregate bank equity will be lower (higher) in the states of the world where it makes losses (profits). Individual losses are thus amplified by a simultaneous increase in the market-to-book value, whereas individual profits are moderated by a simultaneous decrease in the market-to-book value. As a result, banks only lend to firms when the loan rate incorporates an appropriate premium.

To show that the competitive equilibrium is constrained inefficient, we compare it with the socially optimal allocation. Our analysis reveals two channels of welfare-reducing pecuniary externalities. On the one hand, we find that competitive banks do not internalize the impact of their individual lending decisions on i) the banking system's exposure to aggregate shocks and ii) the profit margin on lending. When the banking system is poorly capitalized, banks lend too much as compared to the socially optimal level, thereby, creating inefficiently high exposure to aggregate risk. Furthermore, as excessive lending put downward pressure on banks' profit margins, it undermines the banking system's ability to accumulate loss absorbing capital through retained earnings. On the other hand, overexposure to aggregate risk in the states with poor capitalization makes banks effectively more risk-averse when the banking system is well capitalized. As a result, well-capitalized banks lend too little as compared to the social optimum.

As an illustration of the potential of our framework to accommodate the analysis of macroprudential policy tools, we use our model to explore the effects of minimum capital requirements. A standard argument against high capital requirements is that they would reduce lending and growth. By contrast, the proponents of higher capital requirements put emphasis on their positive impact on financial stability. To consider the interplay between the aforementioned effects and get some

[2]The idea that bank equity is needed to guarantee the safety of banks' debt claims is explored in several recent papers. Stein (2012) shows its implication for the design of monetary policy. Hellwig (2015) develops a static general equilibrium model where bank equity is necessary to support the provision of safe and liquid investments to consumers. DeAngelo and Stulz (2014) and Gornall and Strebulaev (2015) argue that, due to the banks' ability to diversify risk, the actual size of this equity buffer may be very small.

insights into the *long run* consequences of capital regulation, we solve the competitive equilibrium under the regulatory constraint that requires banks to maintain capital ratios above a constant minimum level.

This analysis yields several results. First, when the minimum capital ratio is not too high, the regulatory constraint is binding in poorly-capitalized states and is slack in well-capitalized states. In other words, faced with moderate capital requirements, well-capitalized banks tend to build buffers on top of the required minimum level of capital. By contrast, above some critical level of minimum capital ratios, the regulatory constraint is always binding, so that banks operate without extra capital cushions.

Second, a higher capital ratio translates into a higher loan rate and thus reduces lending for any given level of bank capitalization. Importantly, this effect is present even when the regulatory constraint is slack, because banks anticipate that capital requirements might be binding in the future and require a higher lending premium for precautionary motives. Thus, provided it is not too high, a constant capital ratio mitigate inefficiencies (excessive lending) in poorly capitalized states, but exacerbates inefficiencies (insufficient lending) in well capitalized states. By contrast, for high minimum capital ratios, lending is inefficiently low in all states.

Finally, we look at the impact of a constant minimum capital ratio on financial stability, by analyzing the properties of the probability distribution of aggregate capital across states. Surprisingly, we find that imposing mild capital requirements induces the system to spend a lot of time in the states with low aggregate capitalization. The explanation for this “trap" induced by regulation is rooted in the interplay between the scale of endogenous volatility in the region where the regulatory constraint is slack and the region where the regulatory constraint is binding. In fact, even a very low minimum capital ratio induces a substantial reduction in lending and, thus, in the endogenous volatility in the constrained region (i.e., in the poorly capitalized states). By contrast, in the unconstrained region (i.e., in the well capitalized states), the endogenous volatility is almost as high as in the unregulated set-up. Thus, the system enters the poorly capitalized states almost as often as in the unregulated set-up, but gets trapped there because of the substantially reduced endogenous volatility. As the minimum capital ratio increases, the differences in the endogenous volatility across states becomes less pronounced. Furthermore, due to higher loan rates, banks benefit from higher expected profits, which fosters accumulation of earnings in the banking sector. As a result, the system enters undercapitalized states less frequently and the "trap" disappears. For very high levels of capital requirements, the regulated system becomes even more stable than in the social planner's allocation. However, this “extra” stability comes at the cost of severely reduced lending and output.

Related literature. Our paper pursues the effort of the growing body of the continuous-time macroeconomic models with financial frictions (see e.g. Brunnermeier and Sannikov (2014, 2015), Di Tella (2015), He and Krishnamurthy (2012, 2013), Phelan (2015)). Seeking for a better understanding of the transmission mechanisms and the consequences of financial instability, all these papers point out to the key role that balance-sheet constraints and net-worth of financial intermediaries may play in (de)stabilizing the economy in the presence of financing frictions and aggregate shocks. However, the aforementioned works do not explicitly distinguish financial intermediaries from the productive sector (even all of them use "financial intermediaries" as a metaphor for the most productive agents in the economy). We extend this literature by modeling the banking system explicitly, i.e., by separating the production technology from financial intermediation, which enables us to explore the impact of the bank lending channel on financial stability.

A common feature of the above-mentioned papers is the existence of fire-sale externalities in the spirit of Kiyotaki and Moore (1997) and Lorenzoni (2008) that arise when forced asset sales depress prices, triggering amplifying feedback effects. In our model, welfare-reducing pecuniary externalities in the credit market emerge in the absence of costly fire-sales, stemming from the difference between the social and private implied risk aversion with respect to the variations of aggregate bank capital. The main channel of inefficiencies imposed by individual banks' lending decisions is that each bank fails to internalize the impact of its lending choice on the banking system's exposure to aggregate risk, which increases endogenous volatility. An additional channel of inefficiencies is that individual banks do not internalize the impact of their lending decisions on the size of the expected profit margins, which is similar to the effect described in Malherbe (2015). However, whereas in Malherbe (2015), this “margin channel” affects social welfare via the size of the bankruptcy costs for all banks in the economy, in our framework it has the dynamic welfare implications. Namely, expected profit margins affect the ability of banks to accumulate loss-absorbing capital and thus their future capacity to lend.

From a technical perspective, our paper is related to the corporate liquidity management literature (see e.g. Jeanblanc and Shiryaev (1996), Milne and Robertson (1996), Décamps et al. (2011), Bolton et al. (2011, 2013), Hugonnier et Morellec (2015) among others) that places emphasis on the loss-absorbing role of corporate liquid reserves in the presence of financial frictions. In our model, the role of book equity is very similar to the role of liquidity buffers in those models. However, we differentiate from this literature by allowing for the feedback loop between the individual decisions and the dynamics of individual book equity via the general equilibrium mechanism that determines the loan rate and thus affects the expected earnings of banks. As a result, the individual bank's policies are driven by aggregate rather than individual book equity.

Our paper is also linked to the vast literature exploring the welfare effects of bank capital regulation. Most of the literature dealing with this issue is focused on the trade-off between the welfare gains from the mitigation of risk-taking incentives on the one hand and welfare losses caused by lower liquidity provision (e.g., Begenau (2015), Van den Heuvel (2008)), lower lending and output (e.g., Nguyên (2014), Martinez-Miera and Suarez (2014)) on the other hand.[3] In contrast to the

[3]The only exception is the work by De Nicolò et al. (2014) that conducts the analysis of bank risk choices under capital and liquidity regulation in a fully dynamic model, where capital plays the role of a shock absorber, and

above-mentioned studies, the focus of our model is entirely shifted from the incentive effect of bank capital towards its role of a loss absorbing buffer - the concept that is often put forward by bank regulators. Moreover, the main contribution of our paper is qualitative: we seek to identify the long run effects of capital regulation rather than to provide a quantitative guidance on the optimal level of a minimum capital ratio.

More broadly, this paper relates to the literature on credit cycles that has brought forward a number of alternative explanations for their occurrence. Fisher (1933) identified the famous debt deflation mechanism, that has been further formalized by Bernanke et al. (1996) and Kiyotaki and Moore (1997). It attributes the origin of credit cycles to the fluctuations of the prices of collateral. Several studies also emphasize the role of financial intermediaries, by pointing out the fact that credit expansion is often accompanied by a loosening of lending standards and "systemic" risk-taking, whereas materialization of risk accumulated on the balance sheets of financial intermediaries leads to the contraction of credit (see e.g. Aikman et al. (2014), Dell'Ariccia and Marquez (2006), Jimenez and Saurina (2006)). In our model, quasi-cyclical lending patterns emerge due to the reflection property of aggregate bank capital that follows from the optimality of "barrier" recapitalization and dividend strategies.

The rest of the paper is organized as follows. Section 2 presents the model. In Section 3 we characterize the competitive equilibrium. In Section 4, we discuss the long run behavior of the economy. Section 5 studies the sources of inefficiencies and compares the competitive equilibrium with the social planner's allocation. In Section 6 we introduce capital regulation, analyzing its implications for bank policies and the lending-stability trade-off. Section 7 concludes. All proofs and computational details are gathered in the Appendix. The empirical analysis supporting the key model predictions and additional materials can be found in the Online Appendix.

2 Model

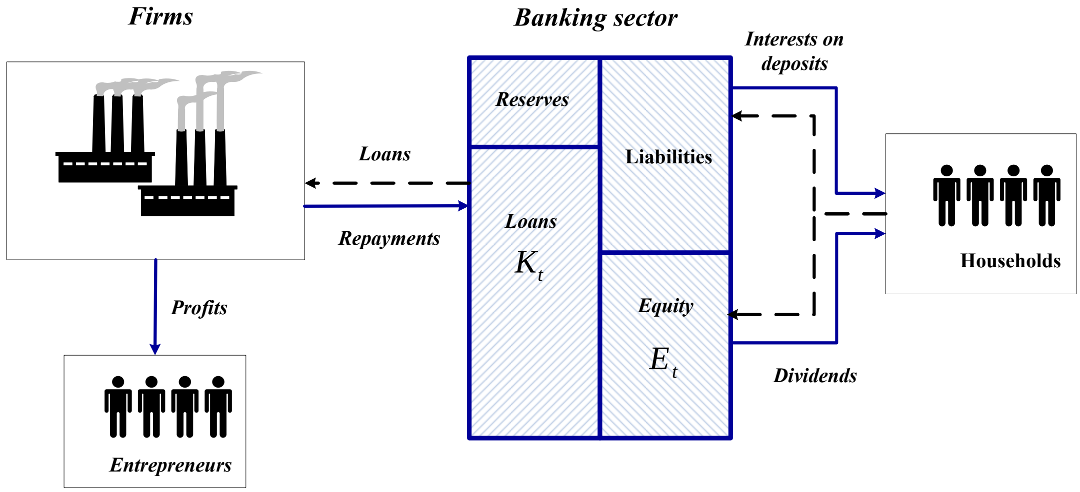
The economy is populated by households, who own and manage banks, and entrepreneurs, who own and manage firms (see Figure 1). All agents are risk-neutral and have the same discount rate ρ. An entity labelled "central bank" serves to equilibrate the interbank lending market, by offering (perfectly elastic) reserve and refinancing facilities. There is one physical good, taken as a numeraire, which can be consumed or invested.

2.1 Economy

Firms. We consider an economy where the volume of bank lending determines the volume of productive investment. Entrepreneurs can consume only positive amounts and cannot save. At

dividends, retained earnings and equity issuance are modeled under a financial friction captured by a constraint on collateralized debt.

Figure 1: The structure of the economy



any time t each firm is endowed with a project that requires an investment of 1 unit of good and produces xh units of good at time $t+h$. The productivity parameter x is distributed according to a continuous distribution with density function $f(x)$ defined on a bounded support $[0, \overline{R}]$. To finance the investment project, a firm can take up a bank loan, for which it has to repay $1 + R_t h$ (to be determined in equilibrium) at time $t + h$.[4]

For the sake of exposition, we assume that a firm can always repay the interest, while the productive capital (principal) is destroyed with probability $\tilde{p}_t(h)$.[5] Since the return on investment for a firm with productivity x is always equal to $x - R_t$, a firm asks for a bank loan and invests if and only if $x \geq R_t$. The total demand for bank loans at time t is, thus, given by

$$L(R_t) \equiv \int_{R_t}^{\overline{R}} f(x)dx. \tag{1}$$

Firms are subject to aggregate shocks that affect the probability of productive capital being destroyed. More specifically, firms' default probability $\tilde{p}_t(h)$ is higher under a negative shock and is lower under a positive shock, i.e.,

$$\tilde{p}_t(h) = \begin{cases} ph - \sigma_0\sqrt{h}, & \text{with probability } 1/2 \textit{ (positive shock)} \\ ph + \sigma_0\sqrt{h}, & \text{with probability } 1/2 \textit{ (negative shock)} \end{cases}, \tag{2}$$

[4]Firms in our model should be thought of as small and medium-sized enterprises (SMEs), which typically rely on bank financing. As is well known, the importance of bank financing varies across countries. For example, according to the TheCityUK research report (October 2013), in EU area, bank loans account for 81% of the long term debt in the real sector, whereas in the U.S. the same ratio amounts to 19%.

[5]The assumption that R is always repaid is, of course, completely inconsequential in the continuous-time limit that we consider throughout the main analysis.

where ph denotes the unconditional probability of default and σ_0 the exposure to aggregate shocks. Taking the limit for period length $h \to 0$ yields the following dynamics of net aggregate output:

$$F(L(R_t))dt - [pdt - \sigma_0 dZ_t]L(R_t), \tag{3}$$

where $\{Z_t, t \geq 0\}$ is a standard Brownian motion and $F(L(R_t))$ denotes the aggregate instantaneous production function:

$$F(L(R_t)) \equiv \int_{R_t}^{\overline{R}} xf(x)dx. \tag{4}$$

Differentiating (4) with respect to R, using (1), immediately yields that at any point in time, the loan demand adjusts so that the marginal product of capital equals marginal costs (i.e., the loan rate):

$$F'(L(R_t)) \equiv R_t.$$

Households. Households are risk-neutral and have a discount rate ρ. As, for instance, in Brunnermeier and Sannikov (2014), they can consume both positive and negative amounts.[6] Households allocate their wealth between bank equity and bank deposits.[7] We also assume that households value the payment services provided by liquid deposits and, thus, enjoy additional utility flow of $\lambda(D_t)dt$, which is a concave non-decreasing function of the supply of deposits D_t.[8] Lifetime utility of households is, therefore, given by:

$$\mathbb{E}\left[\int_0^{+\infty} e^{-\rho t}\Big(dC_t + \lambda(D_t)\Big)dt\right], \tag{5}$$

where dC_t denote the consumption flow at time t.

Deposits pay interest rate r_t^d. Provided that $\rho > r_t^d$ (this will be shown in Section 3), maximization of the lifetime utility of households with respect to the volume of deposits yields the equilibrium condition

$$\lambda'(D_t) = \rho - r_t^d, \tag{6}$$

which implicitly determines the aggregate volume of deposits in the economy. This condition implies that households invest in deposits up to the point where the marginal utility from transaction services equals the difference between households' discount rate and the interest rate on deposits (i.e., liquidity premium).[9]

[6] Negative consumption can be interpreted as disutility from labor to produce additional goods.

[7] The assumption that households cannot invest directly in firms' projects can be justified by technological and informational frictions that are not modeled here explicitly (see e.g. Freixas and Rochet(2008), Chapter 2).

[8] See e.g., Stein (2012), Begenau (2015) and Phelan (2015). As will be shown further, this additional utility from holding deposits introduces the wedge between the cost of debt and equity financing, which makes debt financing attractive even in the absence of tax benefits.

[9] Intuitively, as households value the transaction services provided by deposits, they are willing to accept an interest

Banks. Each bank has access to central bank reserves and refinancing facilities (or, equivalently, to interbank lending) at the exogenously given, constant central bank (or, equivalently, money market) rate $r < \rho$. At time t, a given bank with $e_t \geq 0$ units of equity chooses the amount of deposits, $d_t \geq 0$, and the volume of lending to firms, $k_t \geq 0$. Its net position of reserves in the central bank (or/and lending on the interbank lending market) is determined by the following accounting identity:

$$m_t = e_t + d_t - k_t. \tag{7}$$

A bank also chooses the amount of dividends to be distributed to existing shareholders $d\delta_t \geq 0$, and the amount of new equity to be raised, $di_t \geq 0$. Issuing equity entails a proportional (dead-weight) cost γ, which constitutes the main financial friction in our economy. Note that in the presence of this financial friction and aggregate shocks, bank equity will serve the purpose to buffer losses on loans.

Banks supply loans in a perfectly competitive market and, thus, take the loan rate R_t as given. When a bank grants a loan to firms, it bears the risk that the principal will be destroyed, implying that the instantaneous cash flow generated by 1 unit of loans is given by $(1 + R_t - p)dt - \sigma_0 dZ_t$. Thus, book equity of a given bank evolves according to:

$$e_{t+dt} = k_t[(1 + R_t - p)dt - \sigma_0 dZ_t] - d\delta_t + di_t + (1 + r)m_t dt - (1 + r_t^d)d_t dt,$$

which, given accounting identity (7), can be rewritten as follows:

$$de_t = re_t dt + k_t[(R_t - p - r)dt - \sigma_0 dZ_t] - d\delta_t + di_t + d_t(r - r_t^d)dt. \tag{8}$$

Similarly, aggregate bank equity E_t, evolves according to:

$$dE_t = rE_t dt + K_t[(R_t - p - r)dt - \sigma_0 dZ_t] - d\Delta_t + dI_t + D_t(r - r_t^d)dt, \tag{9}$$

where K_t, $d\Delta_t$, dI_t, and D_t denote, respectively, the aggregate volumes of lending, dividends, equity issuance, and deposits at time t.

Each bank is run in the interest of its shareholders, implying that lending, deposit, dividend, and recapitalization policies are chosen so as to maximize shareholder value

$$v(e, E) = \max_{k_t, d_t, d\delta_t, di_t} \mathbb{E}\left[\int_0^\tau e^{-\rho t}\Big(d\delta_t - (1 + \gamma)di_t\Big)|e_0 = e, E_0 = E\right], \tag{10}$$

where individual and aggregate equity follow (8) and (9) respectively, and $\tau := \inf\{t : e_t < 0\}$ denotes the first time when the book value of bank equity becomes negative, which triggers the bank's default. If it is optimal to inject new equity instead of defaulting, $\tau \equiv \infty$. We assume

rate below the discount rate.

throughout that this is, in fact, the case and provide the appropriate condition in the proof of Proposition 2.

2.2 One-period example

Before turning to the analysis of the competitive equilibrium of the dynamic model, it is instructive to briefly consider a one-period problem to illustrate the basic frictions in our framework. In $t = 0$, each bank has an initial endowment of equity e_0, deposit d, issues new equity i, distributes dividends δ, and grants loans k to firms. Returns are realized, claims are repaid, and consumption takes place at $t = 1$. For a period length normalized to one, firms' default probability is $\tilde{p} \equiv p \mp \sigma_0$. The accounting identity (7) then becomes

$$m = e_0 - \delta + i + d - k, \tag{11}$$

and bank profits at date $t = 1$ can be written as

$$\pi_B = (1 + r)e + k(R - \tilde{p} - r) + d(r - r^d), \tag{12}$$

where $e \equiv e_0 - \delta + i$ denotes an individual bank's equity after recapitalization and dividend distribution. Note that for the deposit market to clear, the deposit rate must coincide with the central bank rate, $r^d \equiv r$, implying that the last term in (12) vanishes. Conditional on the realization of the aggregate shock, a given bank's equity in $t = 1$ is then given by

$$e^+ \equiv (1 + r)e + k\big[R - r - (p - \sigma_0)\big],$$
$$e^- \equiv (1 + r)e + k\big[R - r - (p + \sigma_0)\big].$$

To make the non-negativity constraint for equity meaningful in the one-period problem, we assume that interbank lending is fully collateralized.[10] Thus, bank capital must be sufficiently high to cover the worst possible loss:

$$e^- \geq 0. \tag{13}$$

Imposing this collateral constraint generates a role for bank capital as a loss absorbing buffer in this static set-up. However, this is merely a short-cut formulation for the endogenous benefits of capital buffers that arise in the dynamic setting.

A competitive equilibrium is characterized by a loan rate R which, by market clearing, determines the aggregate volume of lending $K \equiv L(R)$. Each bank takes the loan rate R as given and chooses

[10]This condition easily extends to the case when depositors accept some probability of default (Value-at-Risk constraint similar to Adrian and Shin (2010)). An equivalent interpretation is that banks finance themselves by repos, and the lender applies a hair cut equal to the maximum possible value of the asset (the loan portfolio) that is used as collateral.

dividend policy $\delta \geq 0$, recapitalization policy $i \geq 0$ and the volume of lending $k \geq 0$ so as to maximize shareholder value,

$$v = \max_{\delta,i,k} \left\{ \delta - (1+\gamma)i + \frac{\left(\frac{1}{2}\right)e^{+} + \left(\frac{1}{2}+\theta\right)e^{-}}{1+\rho} \right\},$$

where θ denotes the Lagrange multiplier associated with the constraint (13).[11] Note that this problem is separable, i.e.,

$$v = e_0 u + \max_{\delta \geq 0} \delta\big[1-u\big] + \max_{i \geq 0} \quad i\big[u-(1+\gamma)\big] + \max_{k \geq 0} \quad k\Big[\frac{(R-p-r)(1+\theta)-\theta\sigma_0}{1+\rho}\Big], \tag{14}$$

where

$$u \equiv \frac{(1+r)(1+\theta)}{1+\rho}. \tag{15}$$

Optimizing with respect to the bank's policies yields the following conditions:

$$1-u \leq 0 \quad (= \text{ if } \delta > 0), \tag{16}$$

$$u-(1+\gamma) \leq 0 \quad (= \text{ if } i > 0), \tag{17}$$

$$-\frac{R-r-p}{R-r-(p+\sigma_0)} \geq \theta \quad (= \text{ if } k > 0). \tag{18}$$

Condition (16) shows that $\theta > 0$, which implies that constraint (13) is always binding at both the individual and aggregate level. This determines the loan rate as a function of aggregate bank capitalization, i.e. $R \equiv R(E)$. More specifically, rewriting the binding constraint (13) for aggregate equity yields

$$(1+r)E + L(R(E))\big[R(E)-r-(p+\sigma_0)\big] = 0. \tag{19}$$

Two implications immediately follow from this expression: i) the equilibrium loan rate $R \equiv R(E)$ is a decreasing function of aggregate capital and ii) the aggregate volume of lending is strictly positive. Hence, equation (18) must hold with equality and pins down the shadow cost function $\theta \equiv \theta(E)$. Substituting (15) through (18) into (14), we find that the shareholder value function of each bank is proportional to its book equity and explicitly given by

$$v = e_0 u(E) = e_0 \left(\frac{1+r}{1+\rho}\right)\left(\frac{-\sigma_0}{R(E)-r-(p+\sigma_0)}\right), \tag{20}$$

where $u \equiv u(E)$ is the market-to-book value of equity. Note that $u(E)$ is a decreasing function of E, implying that bank capital becomes more valuable when it is getting scarce.

Conditions (16) and (17) show that the optimal dividend and recapitalization policies also depend on aggregate capital via the market-to-book value of bank equity. Let E_{max} denote the unique level

[11] In this one-period model, shareholders receive terminal dividends at the end of the period.

of aggregate equity such that (16) holds with equality, and E_{min} the one such that (17) holds with equality. Then, if $E < E_{min}$, shareholders will recapitalize banks by raising in aggregate $E_{min} - E$. Similarly, if $E > E_{max}$, aggregate dividends $E - E_{max}$ are distributed to shareholders. As a result, the market-to-book ratio of the banking sector always belongs to the range $[1, 1+\gamma]$. The following proposition summarizes our results for this static set-up:

Proposition 1 *The one-period model has a unique competitive equilibrium, where*

a) The loan rate $R \equiv R(E)$ is a decreasing function of aggregate capital and it is implicitly given by the binding non-negativity constraint for aggregate capital (19)*.*

b) All banks have the same market-to-book ratio of equity given by (20)*, which is a decreasing function of aggregate capital.*

c) Banks pay dividends when $E \geq E_{max} \equiv u^{-1}(1)$ and recapitalize when $E \leq E_{min} \equiv u^{-1}(1+\gamma)$.

Despite the simplistic approach, modelling the need for capital buffers in such a reduced form allows us to illustrate several important features that prevail in the continuous-time equilibrium. First, only the level of aggregate bank capital E matters for banks' policies, whereas the individual banks' sizes do not play any role. Second, banks' recapitalization and dividend policies are of the "barrier type" and are driven by the market-to-book value of equity which, in turn, is strictly decreasing in aggregate bank capital and remains in the interval $[1, 1+\gamma]$. Finally, the loan rate is decreasing in aggregate bank capital.

3 Competitive Equilibrium

3.1 Solving for the equilibrium

In this section we solve for a Markovian competitive equilibrium, where all aggregate variables are deterministic functions of the single state variable, namely, aggregate bank equity, E_t. In a competitive equilibrium, i) loan and deposit markets must clear; ii) the dynamic of individual and aggregate bank capital is given by (8) and (9), respectively; iii)banks maximize (10).

Consider first the optimal decision problem of an individual bank that chooses lending $k_t \geq 0$, deposit $d_t \geq 0$, dividend $d\delta_t \geq 0$ and recapitalization $di_t \geq 0$, so as to maximize shareholder value. Making use of the homotheticity property of the equity market value defined in (10), we define the market-to-book value of bank capital,

$$u(E) \equiv \frac{v(e, E)}{e},$$

which can also be interpreted as the marginal value of an individual bank's book equity, e. Note that this allows to work with aggregate bank capital, E, as the single state variable and, thus, to focus on the capitalization of the banking sector as a whole, instead of the capitalization of each individual bank. By standard dynamic programming arguments, the market-to-book value of a given bank in the economy satisfies the following Bellman equation:

$$
\begin{aligned}
(\rho - r)u(E) = &\max_{d\delta \geq 0}\Big\{\frac{d\delta}{e}[1-u(E)]\Big\} - \max_{di\geq 0}\Big\{\frac{di}{e}[1+\gamma-u(E)]\Big\} + \max_{d\geq 0}\Big\{\frac{d}{e}(r-r_t^d)u(E)\Big\} \\
&+ \max_{k\geq 0}\Big\{\frac{k}{e}\Big[(R(E)-p-r)u(E)+\sigma_0^2 K(E)u'(E)\Big]\Big\} \qquad (21) \\
&+ \Big(rE + K(E)[R(E)-p-r]\Big)u'(E) + \frac{\sigma_0^2 K^2(E)}{2}u''(E).
\end{aligned}
$$

Maximization with respect to the volume of deposits shows that the deposit market clears only if the deposit rate coincides with the central bank rate, i.e. $r_t^d \equiv r$. This implies that an individual bank is indifferent with respect to the volume of deposits. Using (6), it is easy to see that, on the aggregate level, the volume of deposits is constant and can be pinned down from equation $\lambda'(D) = \rho - r$. Moreover, with $r_t^d \equiv r$, the last terms in the equations (8) and (9) describing the dynamics of individual and aggregate equity vanish.

Since banks are competitive, each bank takes the loan rate $R(E)$ as given. For the loan market to clear, the *aggregate* supply of loans has to equal firms' demand for loans in (1) under the prevailing loan rate, i.e. $K(E) = L(R(E))$.[12] Maximizing (21) with respect to the loan supply of each *individual* bank shows that for lending to be interior, the risk-adjusted loan spread must equal banks' risk-premium:

$$R(E) - p - r = -\frac{u'(E)}{u(E)}\sigma_0^2 L(R(E)). \qquad (22)$$

The risk-premium required by any given bank is driven by its implied risk aversion with respect to variation in aggregate capital, $-u'(E)/u(E)$, multiplied by the endogenous volatility of aggregate capital, $\sigma_0^2 L(R(E))$. Since an additional unit of loss absorbing capital in the banking sector is more valuable when it is scarce, the market-to-book ratio $u(E)$ is (weakly) decreasing in aggregate capital (this is shown formally in the proof of Proposition 2). Intuitively, risk-neutral banks become risk-averse with respect to variations in aggregate capital and therefore require a positive risk premium due to the following amplification effect: Upon the realization of a negative aggregate shock, the capitalization of the banking system deteriorates, which drives up the market-to-book value $u(E)$ and, thus, aggravates the impact of the negative shock on each individual bank's shareholder value (recall that $v(e, E) \equiv eu(E)$). Conversely, upon the realization of a positive aggregate shock, the banking system will be better capitalized, which reduces the market to book ratio $u(E)$ and, thus, reduces the impact of the positive shock on shareholder value of any individual bank.

Next, optimization with respect to $d\delta$ and di shows that the optimal dividend and recapitaliza-

[12] This specification is analogous to an economy with constant returns to scale, in which the equilibrium price of any output is only determined by technology (constant marginal cost), whereas the volume of activity is determined by the demand side.

tion policies are of the "barrier" type. That is, each individual bank distributes dividends ($d\delta > 0$) when the level of aggregate bank capital exceeds a critical level E_{max}, that satisfies

$$u(E_{max}) = 1, \tag{23}$$

i.e., the marginal value of equity capital equals the shareholders' marginal value of consumption. Aggregate dividend payments therefore consist of all profits in excess of E_{max}, which induces the aggregate capital process to be reflected at that point.

Moreover, the absence of arbitrage opportunities on the bank equity market implies that, at E_{max}, the marginal change in the value of total bank equity claim must equal the marginal value of consumtion, i.e., $[Eu(E)]' = 1$. Together with (23), this condition implies that $u'(E_{max}) = 0$. As a result, at the dividend payout boundary, the required risk-premium in (22) vanishes, i.e.,

$$R(E_{max}) = p + r. \tag{24}$$

That is, as banks make zero expected profits on the marginal loan, all capital in excess of E_{max} is distributed to shareholders rather than invested in the corporate loan market.

Similarly, new equity is issued ($di > 0$) only when aggregate capital reaches a critical threshold E_{min} at which the marginal value of equity equals the total marginal cost of equity issuance, i.e.,

$$u(E_{min}) = 1 + \gamma. \tag{25}$$

At that point, all banks issue new equity to offset further losses on their loan portfolios and, thus, to prevent aggregate equity from falling below E_{min}. Since the market to book ratio $u(E)$ is (weakly) decreasing in aggregate capital, banks recapitalize only if all capital is completely exhausted and, thus, the non-negativity constraint binds on the individual and on the aggregate level.[13] Taken together, aggregate bank equity fluctuates between the reflecting barriers,

$$E_{min} = 0,$$

where all banks recapitalize, and E_{max}, where all banks pay dividends. In the interior region $(0, E_{max})$, banks retain all profits to build up capital buffers and use these buffers to offset losses.

[13]In the Online Appendix, we show that, in the discrete-time dynamic version of the model, banks are recapitalized at a strictly positive level of E. The reason is that, in the discrete-time dynamic set-up, the non-negativity constraint for capital is binding for low levels of equity both at the individual and at the aggregate level. Accelerating recapitalizations allows shareholders to reduce the "shadow costs" of the binding non-negativity constraint. However, when the length of the time period goes to zero, the region on which the non-negativity constraint is binding shrinks (and collapses to a single point, 0, in the continuous-time limit). It is important to stress, however, that the property $E_{min} = 0$ is *not* a general feature of the continuous-time set-up. In the Online Appendix we show that the recapitalization barrier may depend on the state of the business cycle, similar in spirit to the "market timing" result of Bolton et al. (2013).

For $E \in (0, E_{max})$, the market-to-book value $u(E)$ satisfies

$$(\rho - r)u(E) = \Big[rE + L(R(E))\Big(R(E) - p - r\Big)\Big]u'(E) + \frac{\sigma_0^2 L(R(E))^2}{2}u''(E). \tag{26}$$

The following proposition characterizes the competitive equilibrium.

Proposition 2 *There exists a unique Markovian equilibrium, in which aggregate bank capital evolves according to:*

$$dE_t = rE_t dt + L(R(E_t))\big[(R(E_t) - p - r)dt - \sigma_0 dZ_t\big]. \tag{27}$$

The equilibrium loan rate $R(E)$ and market-to-book value $u(E)$ satisfy (22) *through* (26)*. Banks distribute dividends when E reaches the threshold E_{max} and recapitalize when E reaches* 0*.*

To illustrate the properties of the obtained equilibrium, we use expression (22) to eliminate the market-to-book value (and its derivatives) in the second order differential equation (26). This yields a relatively simple first order differential equation in $R(E)$,

$$R'(E) \equiv H(E, R(E)) := -\left(\frac{1}{\sigma_0^2}\right)\frac{2(\rho - r)\sigma_0^2 + \big[R(E) - p - r\big]^2 + 2rE\big[R(E) - p - r\big]L(R(E))^{-1}}{L(R(E)) - L'(R(E))\big[R(E) - p - r\big]}. \tag{28}$$

From (28) it is immediate, that the loan rate $R(E)$ is strictly decreasing in E. That is, as the banking system becomes better capitalized, banks' implied risk-aversion $-u'(E)/u(E)$ decreases which, despite an increase in the endogenous volatility of aggregate capital $\sigma_0^2 L(R(E))$, results in a lower risk-premium in expression (22).[14]

Moreover, solving equation (22) subject to boundary condition (23), yields the quasi-explicit expression for the market-to-book value:

$$u(E) = \exp\Big(\int_E^{E_{max}} \frac{R(s) - p - r}{\sigma_0^2 L(R(s))}ds\Big), \tag{29}$$

where $R(E)$ satisfies (28) with the boundary condition $R(E_{max}) = p + r$.

The free boundary E_{max} is then pinned down by boundary condition (25), which transforms to:

$$\int_0^{E_{max}} \frac{R(s) - p - r}{\sigma_0^2 L(R(s))}ds = \ln(1 + \gamma). \tag{30}$$

The fact that $R(E)$ is a decreasing function of E that satisfies the boundary condition $R(E_{max}) = p + r$ implies that the loan rate increases with E_{max} for any level of E. It is then easy to see that the left-hand side of Expression (30) is increasing in E_{max}. This immediately leads to the following result:

[14] Since aggregate output in our economy is completely determined by the volume of bank lending, this gives rise to pro-cyclical lending patterns in line with empirical evidence reported, for instance, by Becker and Ivashina (2014).

Corollary 1 *The target level of aggregate bank capital, E_{max}, is increasing with the magnitude of financial friction γ.*

Intuitively, stronger financial frictions make bank shareholders effectively more risk averse, which induces them to postpone dividend payments to build larger capital buffers. Simultaneously, a higher effective risk aversion implies higher loan rates. Thus, our model suggests that lending and, thereby, output should exhibit more dispersion in the economies with stronger financial frictions.

3.2 Testable implications

Overall, the analysis conducted above delivers two key testable predictions: the equilibrium loan rate and the banks' market-to-book ratio of equity should be decreasing functions of aggregate bank capital. To examine whether these predictions are rejected by the data or not, in Appendix III we conduct statistical tests on a data set covering a large panel of publicly traded banks in 43 advanced and emerging market economies for the period 1992-2012. We find that our predictions fit the data extremely well. Table 4 in Appendix III shows the results of simple regressions of loan rates and market-to-book ratios of banks equity in four sub-panels: U.S. banks, Japanese banks, Advanced countries' (excluding U.S. and Japan) banks and Emerging countries' banks. In all cases, the coefficients of Total Bank Equity are negative with p-values indistinguishable from zero, which indicates that two key predictions of our model are consistent with the data.

3.3 Parameter choice

For the rest of the paper, we adopt the particular specification of the demand for loans:

$$L(R) = \Big(\frac{\overline{R} - R}{\overline{R} - p - r}\Big)^{\beta}, \tag{31}$$

where $\beta \geq 0$ is the elasticity parameter. In the above specification the volume of credit is normalized by its frictionless level.

Table 1: Parameter values

Parameter	Symbol	Feasible values
Discount rate	ρ	[0.02, 0.05]
Money market rate	r	$[0, \rho)$
Unconditional default probability of firms	p	[0.02, 0.04]
Maximum feasible loan rate	$\overline{R}$	[0.2, 0.3]
Elasticity parameter	β	[0, 1.2]
Risk exposure	σ_0	[0.05, 0.1]
Financial friction	γ	[0.01, 0.5]

Table 1 summarizes the set of parameter values used in our further numerical analysis. The admissible range for the discount rate ρ generates the discount factors in the range [0.98, 0.95], which is consistent with empirical evidence. The range of the maximum feasible loan rate is consistent with the maximum rates of returns on bank assets observed in our data sample. The range for the unconditional probabilities of default in the real sector reflects the average default probabilities in the industrial sector estimated by Condrad et al. (2013). Elasticity parameter β is restricted to ensure that the absolute value of the semi-elasticity of the loan demand (i.e., $|L'(R)/L(R)| = \frac{\beta}{\bar{R}-R}$) implied by our model does not exceed 10 percentage points. The range of values of the exposure to exogenous shocks, σ_0, is set so as to generate non-negligible variations in lending. Finally, the range of admissible values for the financial friction is chosen based on the market-to-book values observed in our data sample. For example, for U.S. sub-sample, the average market-to-book value is about 1.40 which, according to our model, implies that $\gamma > 0.4$. It should be mentioned, however, that the values of the financial friction implied by the data are much higher than the typical values of the flotation costs of equity issuance used in the recent quantitative models of banking (e.g. 0.06 in De Nicolò et al. (2014)) or documented by the corporate finance literature (e.g., $0.05 - 0.1$ in Hennessy and Whited (2007)), which suggests that γ might admit a broader interpretation.

4 The long-run behavior of the economy

In this section we show that the endogenous risk is the main driver of the long run behavior of the economy, so that the "deterministic" steady state approach employed by the traditional marcoeconomic models to analyse the long run macrodynamics would lead to substantial misestimations.

4.1 Loan rate dynamics

To illustrate the long run behavior of our economy we focus on the dynamics of the loan rate R_t.[15] For the special case in which the central bank rate r is set to zero, the dynamics of the loan rate can be obtained in closed form. Let $R_t = R(E_t)$ be some twice continuously differentiable function of aggregate capital. Applying Itô's lemma to $R(E_t)$, while using the equilibrium dynamics of $E_t \in (0, E_{max})$ given in (27), yields:

$$dR_t = \underbrace{L(R(E_t))\Big((R(E_t)-p)R'(E_t) + \frac{\sigma_0^2 L(R(E_t))}{2} R''(E_t)\Big)}_{\mu(R_t)} dt - \underbrace{\sigma_0 L(R(E_t))R'(E_t)}_{\sigma(R_t)} dZ_t. \tag{32}$$

Recall that the loan rate satisfies the first-order differential equation (28), which allows us to eliminate its derivatives from expression (32). After some algebra, one can thus obtain the drift and

[15] One could equivalently look at the dynamics of the state variable E_t. However, in the simple set-up with no regulation, working with R_t instead of E_t enables us to analytically characterize the system's behavior, because the drift and volatility of R_t are closed-form expressions.

the volatility of R_t in closed form, which is summarized in the following proposition.

Corollary 2 *When $r = 0$, the loan rate $R_t = R(E_t)$ has explicit dynamics*

$$dR_t = \mu(R_t)dt + \sigma(R_t)dZ_t, \qquad p \leq R_t \leq R_{max}, \tag{33}$$

with reflections at both ends of the interval $[p, R_{max}]$. The volatility function is given by

$$\sigma(R) = \frac{2\rho\sigma_0^2 + (R-p)^2}{\sigma_0\Big(1 - (R-p)\frac{L'(R)}{L(R)}\Big)}. \tag{34}$$

The drift function is

$$\mu(R) = \frac{\sigma(R)}{2}\Big[\frac{\sigma(p) - \sigma(R)}{R-p} - \frac{R-p}{\sigma_0} + \sigma'(R)\Big]. \tag{35}$$

The closed-form specifications of the loan rate dynamics allows the explicit analysis of the long-run behavior of the economy. We show below that relying on the results of the *impulse response* analysis that is usually employed to study the long-run dynamics of macro-variables in the traditional marcoeconomic models does not provide a correct picture of the asymptotic behavior of the economy in our framework, as the latter is mainly driven by the endogenous volatility neglected by the impulse response analysis.

4.2 Comparing the "deterministic" steady state and the ergodic distribution

Recall that the usual approach to studying the macrodynamics in a DSGE model requires linearizing this model around the deterministic steady-state and perturbing the system by a single unanticipated shock. The equivalent approach in our framework would be to consider a particular trajectory of realization of aggregate shocks such that $dZ_t = 0$ for $t > 0$. Then, the dynamics of the system can be described by the ordinary differential equation (linearization is not needed here):

$$dR_t = \mu(R_t)dt,$$

where the initial shock determines $R_0 > p$.

The "deterministic" steady state satisfies the value(s) of R at which the drift of the proccess vanishes, i.e., $\mu(R) = 0$.

Consider the particular case where $r = 0$ and the demand for loans is linear (i.e., $\beta = 1$). In this simple case, the volatility function of R becomes

$$\sigma(R) = \frac{2\rho\sigma_0^2 + (R-p)^2}{\sigma_0}\Big(\frac{\overline{R} - R}{\overline{R} - p}\Big) > 0, \tag{36}$$

and the drift function can be written as

$$\mu(R) = -\frac{\sigma(R)}{\sigma_0}\Big(\frac{R-p}{\overline{R}-p}\Big). \tag{37}$$

It is easy to see that the frictionless loan rate $R = p$ is a unique equilibrium of the deterministic system. As $\mu'(p) = 0$ and $\mu(R) < 0$ for all R, it is locally and globally stable.

While the outcomes of the impulse response analysis applied to our model might suggest that the economy should remain most of the time at the "deterministic" steady state $R = p$, it is actually never the case in our set-up because of the significant impact of endogenous risk. To demonstrate this, we solve for the ergodic density function of R that shows how frequently each state is visited in the long run.

Let $g(t, R)$ denote the probability density function of R_t. Given the loan rate dynamics defined in Proposition 2, it must satisfy the forward Kolmogorov equation:

$$\frac{\partial g(t,R)}{\partial t} = -\frac{\partial}{\partial R}\Big\{\mu(R)g(t,R) - \frac{1}{2}\frac{\partial}{\partial R}\Big[\sigma^2(R)g(t,R)\Big]\Big\}. \tag{38}$$

Since the process R_t is stationary, we have $\frac{\partial g(t,R)}{\partial t} = 0$ and thus $g(t,R) \equiv g(R)$. Integrating Equation (38) over R yields:

$$\mu(R)g(R) = \frac{1}{2}\frac{\partial}{\partial R}\Big[\sigma^2(R)g(R)\Big],$$

where the constant of integration is set to zero because of reflection properties of the process. Solving the above equation by using the change of variable $\hat{g}(R) \equiv \sigma^2(R)g(R)$ ultimately yields:

$$g(R) = \frac{C_0}{\sigma^2(R)} exp\Big(\int_p^{R_{max}} \frac{2\mu(s)}{\sigma^2(s)} ds\Big), \tag{39}$$

where the constant C_0 is chosen so as to normalize the solution to 1 over the region $[p, R_{max}]$, i.e. $\int_p^{R_{max}} g(R)dR = 1$.[16]

By differentiating the logarithm of the ergodic density defined in (39), we obtain:

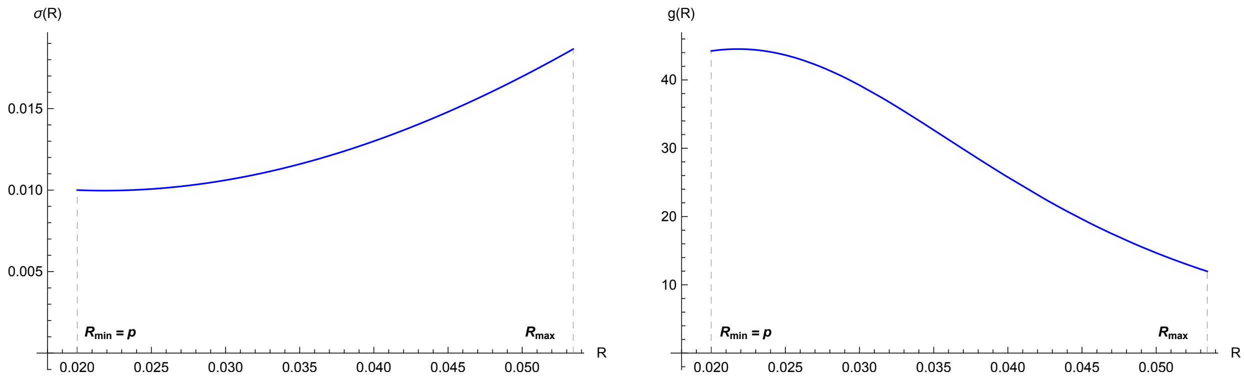
$$\frac{g'(R)}{g(R)} = \frac{2\mu(R)}{\sigma^2(R)} - \frac{2\sigma'(R)}{\sigma(R)}. \tag{40}$$

Using expressions (36) and (37), it is easy to see that $\sigma(p) = 2\rho\sigma_0$, $\sigma'(p) = -2\rho\sigma_0/(\overline{R} - p) < 0$ and $\mu(p) = 0$. Hence, $g'(p) > 0$, which means that the state $R = p$ that corresponds to the "deterministic" steady state is definitely *not* the one at which the economy spends most of the time in the stochastic set up. The reason is that the endogenous risk plays an important role in the system's dynamics. This feature is illustrated in Figure 2 that reports the typical patterns of the

[16] To ensure that the distribution of R is non-degenerate, it is sufficient to check that $\sigma(R) > 0$ for any $R \in [p, R_{max}]$. From the expression of $\sigma(R)$, it is easy to see that this condition holds for any loan demand specifications such that $L'(R) < 0$ and $L(R) > 0$.

endogenous volatility $\sigma(R)$ (the left-hand side panel) and the ergodic density $g(R)$ (the right-hand side panel) for the specification of the loan demand stated in (31). It shows that the extrema of the ergodic density mirror those of the volatility function, i.e., the economy spends most of the time in the states with the lowest loan rate volatility.[17]

Figure 2: Volatility and ergodic density of R



Notes: this figure reports the typical patterns of the loan rate volatility (left panel) and the ergodic density of the loan rate (right panel). Parameter values: $\rho = 0.05$, $r = 0$, $\sigma_0 = 0.1$, $p = 0.02$, $\beta = 1$, $\overline{R} = 0.3$, $\gamma = 0.5$.

5 Welfare analysis

In this section we show that the competitive equilibrium is not constrained efficient due to the pecuniary externalities that each bank inflicts on its competitors. More precisely, competitive banks do not take into account the effect of their individual lending decisions on the dynamics of aggregate bank equity. This leads to excessive lending when banks are poorly capitalized, implying overexposure of the banking system to aggregate shocks and the erosion of its ability to accumulate earnings. The failure to reduce exposure in bad times in turn aggravates banks' implied risk aversion with respect to variations in aggregate capital and leads to inefficiently low lending in good times (i.e. when the banking system is well capitalized). Thus, our model shows that welfare-reducing pecuniary externalities can emerge in credit markets even if there are no costly fire sales.[18]

5.1 Welfare in the CE

Social welfare in our framework can be computed as the expected present value of the entrepreneurs' and households' consumption flows, namely, firms' profits (which are immediately con-

[17]Note that the magnitude of financial friction γ does not affect the shape of the ergodic density function. The reason is that the financial friction affects only the support of the loan rate distribution (namely, R_{max}), and does not have any impact on the drift and the volatility of the loan rate. Thus $g(.)$ is just truncated and rescaled on $[p, R_{max}]$ when γ varies.

[18]Examples of macro models with costly fire sales are Brunnermeier and Sannikov (2015), Phelan (2015) and Stein (2012).

sumed by entrepreneurs), banks' dividend payments net of capital injections and instantaneous surplus of depositors:[19]

$$\mathbb{E}\left[\int_0^{+\infty} e^{-\rho t}\Big\{\pi_F(K_t)dt + d\Delta_t - (1+\gamma)\, dI_t + (rD + \lambda(D))dt\Big\}|E_0 = E\right], \tag{41}$$

where firms' profits (i.e., instantaneous output net of firms' costs of credit) are given by:

$$\pi_F(K) \equiv F(K) - KF'(K).$$

Since the volumet of deposits is constant (recall that it satisfies $\lambda'(D) = \rho - p$), the expected value of the discounted surplus of depositors is just $(rD + \lambda(D))/\rho$. Up to this constant, social welfare equals

$$W(E) = \mathbb{E}\left[\int_0^{+\infty} e^{-\rho t}\Big\{\pi_F(K)dt + d\Delta_t - (1+\gamma)\, dI_t\Big\}|E_0 = E\right].$$

With a slight abuse of terminology, we refer to $W(E)$ as the *welfare function*. As long as banks neither distribute dividends nor recapitalize, firms' profits are the only state-dependent instantaneous utility flow in our economy. Therefore, in the region $E \in (0, E_{max})$, the welfare function, $W(E)$, must satisfy the following differential equation:[20]

$$\rho W(E) = \pi_F(K) + \Big(rE + K[F'(K) - p - r]\Big)W'(E) + \frac{\sigma_0^2}{2}K^2 W''(E), \tag{42}$$

subject to the boundary conditions:

$$W'(0) = 1 + \gamma, \tag{43}$$

$$W'(E_{max}) = 1. \tag{44}$$

These boundary conditions reflect the fact that when bank recapitalizations occur, the (social) marginal value of bank capital has to equal households' total costs of injecting additional equity. Likewise, when dividend distributions occur, the (social) marginal value of bank capital has to equal households' marginal utility from consumption.

Before solving for the constrained welfare optimum in the next subsection, we first evaluate the welfare function at the competitive equilibrium in order to get a first grasp of the prevailing distortions. To this end, let $\mathcal{L}$ denote the right-hand side of (42). Taking the first derivative of $\mathcal{L}$

[19] Note that total depositors surplus generally writes as $dD_t + rDdt + \lambda(D)dt$. However, as in our economy the level of debt remains constant, $dD_t \equiv 0$.

[20] For the sake of space, we omit the argument of $K(E)$. When deriving the equation (42), we have used the fact that $R(E) \equiv F'(K(E))$.

with respect to K yields:

$$\frac{\partial \mathcal{L}}{\partial K} = \pi_F'(K) + [F'(K) - p - r + KF''(K)]W'(E) + \sigma_0^2 K W''(E). \tag{45}$$

Recall that, at the competitive equilibrium, the loan rate satisfies

$$R(E) \equiv F'(K(E)) = p + r - \frac{u'(E)}{u(E)}\sigma_0^2 K(E). \tag{46}$$

Substituting $F'(K)$ in (45) by the above expression and rearranging terms yields:

$$\frac{\partial \mathcal{L}}{\partial K} = W'(E)\Big[\frac{W''(E)}{W'(E)} - \frac{u'(E)}{u(E)}\Big]\sigma_0^2 K + F''(K)K\Big[W'(E) - 1\Big]. \tag{47}$$

The first term in (47) captures an inefficiency stemming from the fact that competitive banks do not take into account how their individual lending decisions affect the exposure of the banking system to aggregate shocks (we call it *"exposure channel"*). The term in square brackets captures the difference between the social planner's, $-W''(E)/W'(E)$, and individual banks', $-u'(E)/u(E)$, implied risk aversion with respect to variations in aggregate capital.

The second term in (47) reflects a distributive inefficiency in the competitive equilibrium: An increase in aggregate lending drives down the loan rate ($F''(K) = \partial R/\partial K < 0$). This increases firms' profits but bites into the profits of banks and, thus, impairs banks' ability to accumulate loss absorbing capital in the form of retained earnings (we call it *"margin channel"*).[21]

The socially optimal level of lending in the economy is attained when the sum of both terms in expression (47) is zero. Competitive banks lend too little (i.e., welfare could be improved by increasing lending) when this sum is positive and too much (i.e., welfare could be improved by reducing lending) when it is negative. As expression (47) cannot be signed globally without making further assumptions, we proceed by solving for the social planner's allocation and comparing its outcomes with the outcomes of the competitive equilibrium.

5.2 Second Best Allocation

The second best (social planner's) allocation is characterized by the welfare maximizing lending, dividend and recapitalization policies, subject to the same friction γ as in the competitive equilibrium. The single state variable in the social planner's problem is aggregate bank equity E and the market for bank credit must clear, i.e. $K = L[R(E)]$.

Provided that the welfare function $W(E)$ is concave (this has to be verified ex-post), it is immediate that optimal dividend and recapitalization policies are of the "barrier type" and thus aggregate equity fluctuates within a bounded support $[E^{sb}_{min}, E^{sb}_{max}]$, where E^{sb}_{min} and E^{sb}_{max} denote

[21] The negative effect of competition of banks' profit margins is also acknowledged in Martinez-Miera and Repullo (2010) albeit in an entirely different application.

the recapitalization and the dividend distribution barriers, respectively. For $E \in [E^{sb}_{min}, E^{sb}_{max}]$, the social welfare function satisfies the following HJB equation:

$$\rho W(E) = \max_{K \geq 0} \quad F(K) - KF'(K) + \Big(rE + K[F'(K) - p - r]\Big) W'(E) + \frac{\sigma_0^2}{2} K^2 W''(E). \tag{48}$$

Taking the first-order condition of the right-hand side of (48) with respect to K, and rearranging terms shows that the second-best loan rate, $R^{sb}(E)$, satisfies:

$$R^{sb}(E) - p - r = -\frac{W''(E)}{W'(E)} \sigma_0^2 K^{sb}(E) + \Big(\frac{1}{W'(E)} - 1\Big) K^{sb} F''(K^{sb}), \tag{49}$$

where $K^{sb} = L(R^{sb})$. Given that $W'(E^{sb}_{max}) = 1$ and $W''(E^{sb}_{max}) = 0$, it is easy to see that $R^{sb}(E^{sb}_{max}) = p + r$, like in the competitive equilibrium.

Note that the above expression shares some similarities with the expression (22) in the competitive equilibrium, with the notable distinction that the loan rate depends on the *social* implied risk aversion with respect to variation in aggregate equity, $-W''(E)/W'(E)$, whereas in the competitive equilibrium, it is driven by the *private* implied risk aversion $-u'(E)/u(E)$. Moreover, there is an additional term which is always positive or zero, as the (social) marginal value of earnings retained in banks is weakly higher than the marginal value of instantaneously consumed firms' profits, i.e., $W'(E) \geq 1$.

The following proposition summarizes the characterization of the second best allocation.

Proposition 3 *The Second Best allocation is characterized by the aggregate lending function $K^{sb}(E) = L(R^{sb}(E))$, where $R^{sb}(E)$ is implicitly given by (49). The welfare function $W_{sb}(E)$ satisfies the ODE*

$$\rho W_{sb}(E) = \quad F(K^{sb}) - K^{sb} F'(K^{sb}) + \Big(rE + K^{sb}[F'(K^{sb}) - p - r]\Big) W'_{sb}(E) + \frac{\sigma_0^2}{2} [K^{sb}]^2 W''_{sb}(E),$$

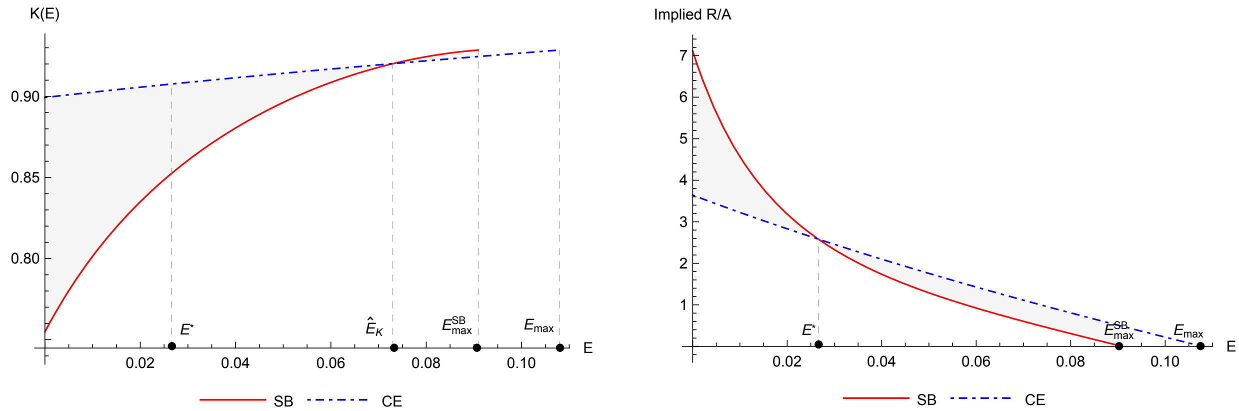
subject to the boundary conditions $W'_{sb}(0) = W'_{sb}(E^{sb}_{max}) - \gamma = 1$. Banks distribute dividends when E_t reaches $E^{sb}_{max} = [W''_{sb}]^{-1}(0)$ and recapitalize when E_t hits zero.

To illustrate credit market inefficiencies emerging in our model set-up, we resort to the numerical example using a simple linear specification of the demand for loans that is obtained from the one introduced in (31) by setting $\beta = 1$.[22]

The left panel in Figure 3 shows that in the competitive equilibrium (CE), a poorly-capitalized banking system lends more than under the second best (SB) allocation, which reflects the pecuniary externality inflicted by each bank on its competitors. The social planner internalizes this externality and reduces the exposure to macro shocks ($\sigma_0 K(E)$) in the states of the world where the implied risk aversion with respect to fluctuations in E is most severe (i.e., close to the recapitalization barrier

[22]Using a simple linear specification of the loan demand allows us to compute the Second Best allocation without using value-iteration algorithms.

Figure 3: Credit volume and implied risk aversion: CE vs SB



Notes: this figure reports the credit volume (left panel) and implied risk-aversion (right panel) in the competitive equilibrium (dash-dotted lines) and the second best allocation (solid lines). Parameter values: $\rho = 0.05$, $r = 0.02$, $\sigma_0 = 0.05$, $p = 0.02$, $\gamma = 0.2$, $\overline{R} = 0.3$ and $\beta = 1$.

$E_{min} = 0$). As illustrated in the right panel of Figure 3, this "exposure" externality is reflected by a higher implied risk-aversion of the social planner compared to individual banks (i.e., the first term in (47) is strictly negative) when the system is poorly capitalized, i.e., when $E < E^*$ where E^* is such that $-W''_{sb}(E^*)/W'_{sb}(E^*) = -u'(E^*)/u(E^*)$.

Moreover, lower lending in poorly capitalized states of the second best allocation increases banks' profit margins, which makes it easier to rebuild equity buffers after negative shocks.[23] Importantly, this insight remains true even though restricting lending reduces firm profits. The reason is that, in the second best allocation, the social value of banks' profits (which are used to build loss-absorbing equity buffers) is always larger than the social value of firms' profits (which are consumed immediately). Due to this "margin channel", second-best lending remains lower than competitive lending even when the social planner's implied risk-aversion becomes lower than that of banks (namely, this happens on the interval $[E^*, \hat{E}_K]$, where $\hat{E}_K$ is such that $K^{ce}(\hat{E}_K) = K^{sb}(\hat{E}_K)$). This margin channel is captured by the second term in (47), which is strictly negative as long as $W'(E) > 1$.[24]

Due to the lower risk-exposure and higher expected profits of banks in poorly-capitalized states, the target aggregate equity buffer in the second best allocation is lower than in the competitive equilibrium, i.e. $E^{sb}_{max} < E^{ce}_{max}$. Intuitively the social planner curbs the banking system's exposure

[23]It is important to stress, however, that this "margin channel" is of less importance as compared to the "exposure channel". In the Online Appendix we compute the Second Best allocation in the case of the inelastic demand for loans. In the second best, $R^{sb}(E) \equiv \overline{R}$, such that firms make zero profits and the "margin channel" of inefficiencies is shut down. However, when the social implied risk aversion in the states with poor bank capitalization is sufficiently high, it is still socially optimal to restrict lending of the undercapitalized banking system.

[24]Recall that, in the competitive equilibrium, $W'(0) = 1 + \gamma$ and thus inequality $W'(E) > 1$ holds for the lower levels of aggregate capitalization. However, when $E \to E_{max}$, retaining earnings in banks is no longer valuable from the social perspective and $W'(E) < 1$.

to macro shocks when they are most harmful (i.e. when implied risk-aversion is most severe), while individual banks fail to efficiently control aggregate risk. As a result, the critical capital buffer at which the social planner no longer requires a lending premium is lower than the one at which this is the case for individual banks (recall that $R^{ce}(E^{ce}_{max}) = R^{sb}(E^{sb}_{max}) = p + r$). Hence, if the target aggregate equity buffer E^{ce}_{max} is sufficiently high in the competitive set-up, banks are effectively more risk-averse than the social planner and, thus, lending is inefficiently low in the vicinity of E^{ce}_{max}.[25]

6 Impact of capital regulation

The evidence of welfare distortions in the competitive equilibrium calls for macroprudential regulation. Our objective in this section is to investigate the impact of capital regulation on bank policies and the long run behavior of the economy.

6.1 Regulated equilibrium

We consider a requirement for each bank to finance at least fraction Λ of its risky loans by equity, i.e.,

$$e_t \geq \Lambda k_t. \tag{50}$$

Facing a binding capital requirement, banks can either issue new equity or de-lever by reducing lending and debt. We show below that, compared to the unregulated case, banks on the one hand recapitalize earlier (i.e., $E^{\Lambda}_{min} > 0$) and on the other hand reduce lending when the constraint is binding but, for precautionary reasons, also when it is slack. As in the unregulated case, individual banks choose lending, recapitalization and dividend policies subject to the regulatory constraint to maximize the market value of equity:[26]

$$v_\Lambda(e, E) \equiv e u_\Lambda(E) = \max_{k_t \leq \frac{e}{\Lambda}, d\delta_t, di_t} \mathbb{E}\left[\int_0^{+\infty} e^{-\rho t}\left(d\delta_t - (1+\gamma)di_t\right)|e_0 = e, E_0 = E\right]. \tag{51}$$

Recall that in the unregulated equilibrium bank lending is always strictly positive and banks recapitalize only when equity is completely depleted. Capital regulation must therefore bind for sufficiently low levels of capital. In this case, lending is determined by the binding constraint (50). If it is slack, the interior level of lending must satisfy an individual rationality condition equivalent to (22) in the unregulated case. In both cases, the relation between equilibrium lending and loan rate is pinned down by the market clearing condition.

[25] Note that the regulatory restrictions on lending changes the marginal value of bank equity. Namely, in the social planner's allocation, $u(E)$ spikes above $1 + \gamma$ for the low levels of aggregate bank capital. This implies that, if bank shareholders were free to choose the recapitalization barrier, they would recapitalize at a strictly positive level of aggregate capital.

[26] Note that homotheticity is not affected by the considered form of regulation.

Proposition 4 *For all $\Lambda \in (0,1]$, there exists a unique regulated equilibrium, where banks' market to book value of equity satisfies the HJB equation*

$$\begin{aligned}(\rho - r)u_\Lambda(E) =& \Big[rE + L(R(E))(R(E) - p - r)\Big]u'_\Lambda(E) + \frac{\sigma_0^2 L(R(E))^2}{2}u''_\Lambda(E) \\ &+ \frac{1}{\Lambda}\big[(R(E) - p - r)u_\Lambda(E) + \sigma_0^2 L(R(E))u'_\Lambda(E)\big],\end{aligned} \tag{52}$$

for $E \in [E^\Lambda_{min}, E^\Lambda_{max}]$, subject to the boundary conditions $u_\Lambda(E^\Lambda_{max}) = u_\Lambda(E^\Lambda_{min}) - \gamma = 1$ and $u'_\Lambda(E^\Lambda_{max}) = u'_\Lambda(E^\Lambda_{min}) = 0$. Furthermore, there exists a unique threshold E^Λ_c such that

a) for $E \in [E^\Lambda_{min}, E^\Lambda_c]$ the regulatory constraint binds, i.e., $K(E) = E/\Lambda$. The equilibrium loan rate is explicitly given by the market clearing condition:

$$R(E) = L^{-1}(E/\Lambda), \tag{53}$$

where L^{-1} is the inverse function of the loan demand. The evolution of aggregate bank capital is given by:

$$\frac{dE_t}{E_t} = rEdt + \frac{1}{\Lambda}\Big((R(E_t) - p - r)dt - \sigma_0 dZ_t\Big), \qquad E \in (E^\Lambda_{min}, E^\Lambda_c).$$

b) for $E_t \in (E^\Lambda_c, E^\Lambda_{max}]$ the regulatory constraint is slack. The equilibrium loan rate satisfies the first-order differential equation (28) subject to the boundary condition $R(E^\Lambda_c) = L^{-1}[E^\Lambda_c/\Lambda]$.

Finally, there exists a unique threshold $\Lambda^* \in (0,1)$ such that $E^\Lambda_c = E^\Lambda_{max}$, i.e., the constraint binds for all $E \in [E^\Lambda_{min}, E^\Lambda_{max}]$, when $\Lambda > \Lambda^*$.

Note that, when the constraint is not binding, the equilibrium loan rate is determined by the same condition as in the unregulated equilibrium (see condition (22)), so that the term in square brackets in the second line of HJB (52) vanishes. If the constraint is binding, however, this term is strictly positive and can be interpreted as the shadow costs associated with the constraint.

We turn to the results of our numerical analysis to illustrate the impact of minimum capital regulation on bank policies, systemic stability and the distribution of welfare across states.[27]

6.2 Dividend and recapitalization policies

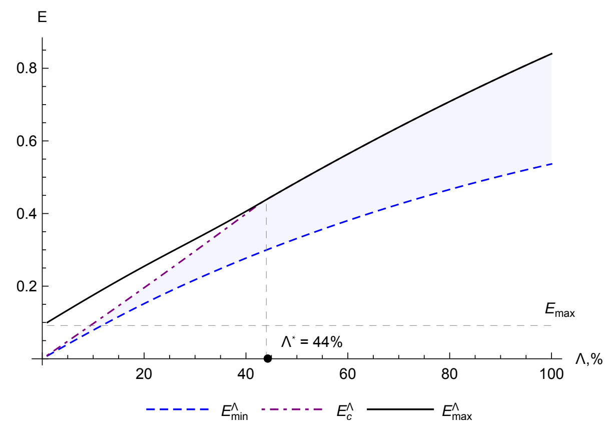
Figure 4 illustrates the impact of the minimum capital ratio on the optimal recapitalization and dividend policies of banks. In contrast to the unregulated case, it is no longer optimal to postpone recapitalizations until equity is fully depleted ($E^\Lambda_{min} > 0$). This is an immediate consequence of the fact that banks are allowed to lend at most fraction $1/\Lambda$ of their equity, so that a bank with zero

[27] In Appendix we provide a detailed description of the computational procedure implemented to solve for the regulated equilibrium.

equity and, thus, zero loans, would be permanently out of business. Therefore, as illustrated in Figure 4, the recapitalization boundary E^{Λ}_{min} is strictly increasing in the minimum capital ratio Λ.

When the regulator imposes a higher minimum capital ratio, Λ, the dividend boundary grows at a higher rate than the recapitalization boundary, such that the maximum loss absorbing capacity of the banking system, $E^{\Lambda}_{max} - E^{\Lambda}_{min}$, expands as well. To understand this effect, note first that, as long as the constraint does not bind globally ($\Lambda \leq \Lambda^*$), banks distribute dividends when the marginal loan is no longer profitable, i.e. the equilibrium loan rate at the dividend boundary E^{Λ}_{max} equals the unconditional default rate ($R^{\Lambda}_{min} = p + r$). As regulation reduces the supply of bank loans, banks' profit margins increase at any given level of aggregate capital, which in turn drives up E^{Λ}_{max}. This effect is even more pronounced for sufficiently high minimum capital ratios ($\Lambda > \Lambda^*$), where the constraint binds globally and $R^{\Lambda}_{min} > p + r$.

Figure 4: Minimum capital requirements and bank policies



Notes: this figure illustrates the impact of minimum capital ratios on the optimal dividend and recapitalization policies. The dashed line indicates the values of the maximum level of aggregate capital in the absence of regulation (E_{max}). The shaded area marks the states of E in which the regulatory constraint is binding. For $\Lambda > \Lambda^*$ ($\Lambda^* = 44\%$ in this numerical example), the regulatory constraint is binding for any $E \in [E^{\Lambda}_{min}, E^{\Lambda}_{max}]$. Parameter values: $\rho = 0.05$, $r = 0$, $\sigma_0 = 0.05$, $p = 0.02$, $\gamma = 0.2$, $\overline{R} = 0.3$, $\beta = 1$.

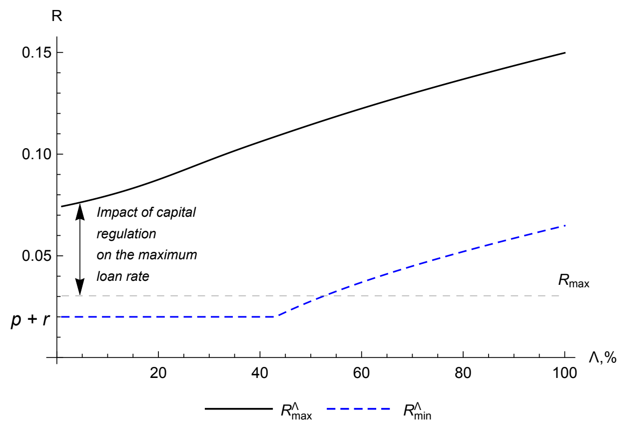
6.3 Loan rates

Figure 5 illustrates the impact of a minimum capital ratio on the minimum and maximum levels of the equilibrium loan rate. In particular, it shows that, as long as the constrained and unconstrained region coexist (in this numerical example this occurs for any $\Lambda < \Lambda^*$) the minimum loan rate R^{Λ}_{min} remains at its frictionless level $p + r$. For capital ratios that make the regulatory constraint binding for any level of aggregate capital, the minimum loan rate is determined by the binding regulatory constraint and thus stays strictly above its frictionless level.

It is also worthwhile to note that for $\Lambda \to 0$, the equilibrium loan rate at the recapitalization boundary R^{Λ}_{max} (indicated by the solid line) does not converge to its unregulated counterpart R_{max}

(the dashed line). The reason is that the loan rate has to ensure market clearing and, thus, a capital constraint that binds over a nonempty (albeit small) region drives up the market clearing loan rate by discrete amount. As a result, even a tiny minimum capital ratio leads to significant increase in R^{Λ}_{max}.

Figure 5: Minimum capital requirements and loan rates



Notes: this figure illustrates the impact of minimum capital ratios on the equilibrium loan rates. The dashed line indicates the values of the maximum loan rate in the absence of regulation (R_{max}). For $\Lambda > \Lambda^*$ ($\Lambda^* = 44\%$ in this numerical example), the regulatory constraint is binding for any level of aggregate capital and thus $R^{\Lambda}_{min} > p + r$. Parameter values: $\rho = 0.05$, $r = 0$, $\sigma_0 = 0.05$, $p = 0.02$, $\gamma = 0.2$, $\overline{R} = 0.3$, $\beta = 1$.

6.4 Lending

The direct impact of the minimum capital ratio on lending is illustrated in Figure 6.

Consider first a relatively mild minimum capital ratio of, say 3%, which in fact leads to a significant reduction in exposure when the constrained is binding (i.e., when the system is poorly capitalized). Interestingly, the effect of the minimum capital requirements propagates over the unconstrained region. This result is driven by the banks' precautionary motive: anticipating a potentially binding constraint in the future, banks also reduce lending when the constraint is slack, albeit less severely. Thus, while mitigating inefficiencies (excessive lending) in poorly capitalized states, the constant minimum capital ratio exacerbates inefficiencies (insufficient lending) in good states.

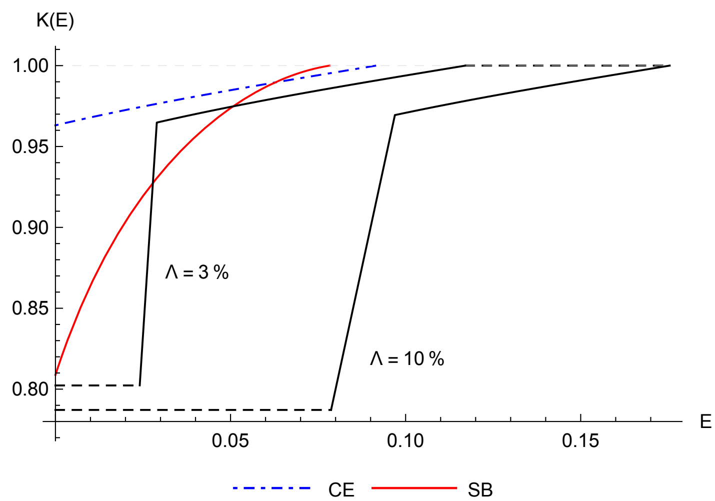
Above some critical level of Λ, further increases in capital requirements will lead to inefficiently low lending for all feasible levels of aggregate capitalization. Indeed, upon raising the minimum capital ratio to 10%, the region in which the constraint binds expands (cf. the shaded area in Figure 4), and lending in the constrained region declines further, falling far below the second best level.[28] Moreover, with a stricter minimum capital ratio also the precautionary motive and, thus,

[28]Note that banks react to the stricter regulation by increasing E^{Λ}_{min}, yet, not sufficiently to maintain the same level of lending. Therefore, R^{Λ}_{max}, which is represented by the solid line in Figure 5, is strictly increasing in Λ.

the reduction of lending in the unconstrained region becomes more pronounced.

Overall, our results show that minimum capital requirements reduce the exposure of the banking system to aggregate shocks and, thus, reduce the endogenous volatility of aggregate capital. However, this comes at a cost of severe reductions in lending.

Figure 6: Minimum capital ratio and aggregate lending



Notes: this figure illustrates the impact of minimum capital requirements on aggregate lending (dashed black lines). The slope of the lending curves gets sharper for the range of E in which the regulatory constraint is binding. The solid and dash-dotted lines correspond to aggregate lending in the Second Best allocation (SB) and the Competitive Equilibrium (CE), respectively. Parameter values: $\rho = 0.05$, $r = 0$, $\sigma_0 = 0.05$, $p = 0.02$, $\gamma = 0.2$, $\overline{R} = 0.3$, $\beta = 1$.

6.5 Long-run dynamics of aggregate capital

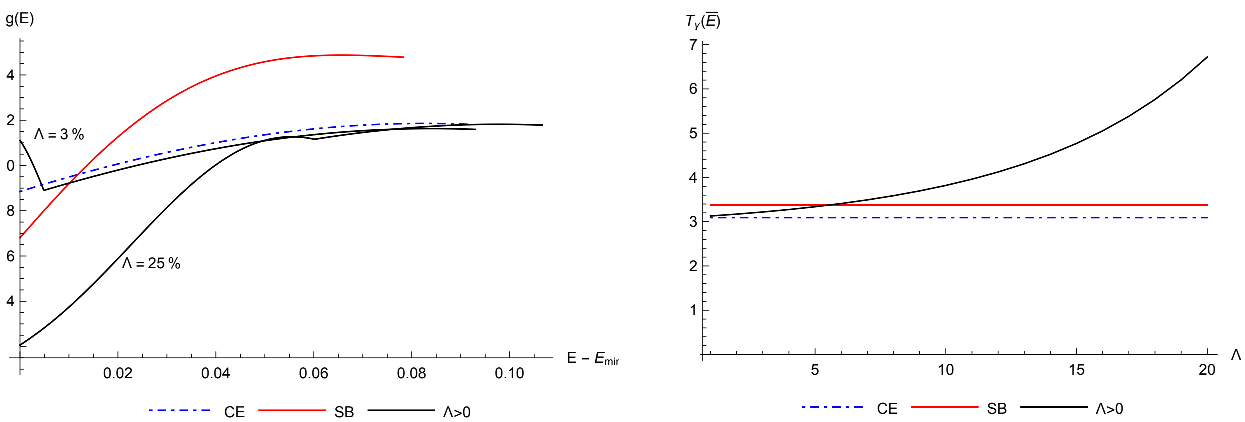
We now study the long-run impact of the minimum capital ratio on the long-run dynamics of aggregate capital. To this end, we numerically compute the ergodic density (the left panel of Figure 7) and the cumulative density (the right panel of Figure 7) functions of E. For the sake of comparability, both are plotted as the functions of the actual capital buffer $E - E^{\Lambda}_{min}$.

The left panel of Figure 7 illustrates the patterns of the ergodic density function $g_{\Lambda}(.)$ for two different levels of capital ratios. An interesting phenomenon revealed by this graph is the density concentration in the states with poor capitalization that emerges for mild minimum capital requirements. The reason is that, for low capital ratios, banks substantially un-lever only when the constraint is binding, which occurs in the states where the banking system is poorly capitalized. For most states, however, the constraint is not binding and exposure remains high, almost as high as in the unregulated set-up. The system, therefore, enters the poorly capitalized region almost as often as in the unregulated equilibrium. Yet, as banks have to severely un-lever when the system enters the poorly capitalized region, it can get trapped for some time in the states where lending (and, thus, the endogenous volatility of aggregate capital) is substantially lower. Thus, in contrast to the models in the similar style (see e.g. Brunnermeier and Sannikov (2014) and Phelan (2015)), in our model the bimodal density pattern can emerge only due to the impact of regulation, whereas, in the unregulated framework, the ergodic density is unimodal for the whole range of acceptable parameter values.

For sufficiently high minimum capital ratios exposure to aggregate shocks (lending) is also substantially reduced when the system is well capitalized. Furthermore, cutting on lending boosts banks' expected profits, and, thus, improves the banking system's capacity to accumulate capital. As a result of these two effects, the spike in the density pattern disappears and the density concentration occurs only in highly-capitalized states. Thus, enhancing capital requirements makes the banking system more stable.

This feature is also illustrated in the right-hand side panel of Figure 7, which reports the banking system's stability, measured by the average time to recapitalization ($\overline{T}_\gamma(\overline{E})$) (see Appendix for the computational details). As stability is strictly increasing in the minimum capital ratio Λ, for sufficiently high capital ratios the banking system becomes even more stable than in the second best allocation. This, however, comes at the cost of substantial reductions in lending and, thus, output.

Figure 7: The ergodic density of E and average time to recapitalization



Notes: this figure illustrates the impact of a constant minimum capital ratio on the ergodic density of E (left panel) and average time to recapitalization. Parameter values: $\rho = 0.05$, $r = 0$, $\sigma_0 = 0.05$, $p = 0.02$, $\gamma = 0.2$, $\overline{R} = 0.3$, $\beta = 1$.

7 Conclusion

This paper develops a general equilibrium model of commercial banking, in which banks satisfy households' needs for safe deposits and channel funds to the productive sector. Bank capital plays the role of a loss-absorbing buffer that insulates banks from the need to undertake costly recapitalizations too often. In our model, the aggregate level of bank capitalization drives the cost and the volume of lending. Specifically, we establish a negative relation between the equilibrium loan rate and the level of aggregate bank capital that is supported by data. The competitive equilibrium is not constrained efficient because of a pecuniary externality which leads to excessive lending in the states with poor bank capitalization and insufficient lending in well-capitalized states. Minimum capital requirements curb lending, which enhances stability of the banking system.

Our model is a first step toward the analysis of bank regulation policies in a fully specified dynamic model. Several extensions appear important. First, in future research, we plan to explore the welfare effects of "countercyclical" capital regulation in the extended version of our model that explicitly accounts for business cycles. Second, introducing market activities such as investment in securities in addition to the current focus on commercial banking activities (deposit taking and lending) might be important to understand the dynamics of endogenous risk. Finally, in this paper we have focused on the scenario in which private bank recapitalizations prevent systemic crises from happening. A potential direction of further investigations would be to explore the alternative scenarios in which shareholders do not have incentives to inject new capital, and public authorities are forced to intervene.

Appendix

I. Proofs

Proof of Proposition 1. Omitted.

Proof of Proposition 2. By the standard dynamic programming arguments, shareholder value $v(e, E)$ must satisfy the Bellman equation:[29]

$$\begin{aligned}\rho v = \max_{k\geq 0, d\delta\geq 0, di\geq 0} & \Big\{ d\delta(1-v_e) - di(1+\gamma - v_e) \\ & + \Big[re + k(R(E)-p-r)\Big]v_e + kK(E)\sigma_0^2 v_{eE} + \frac{k^2\sigma_0^2}{2}v_{ee} \\ & + \Big[rE + K(E)(R(E)-p-r)\Big]v_E + \frac{\sigma_0^2 K^2(E)}{2}v_{EE}\Big\}.\end{aligned} \tag{I.1}$$

Using the fact that $v(e, E) = eu(E)$, one can rewrite the Bellman equation (I.2) as follows:

$$\begin{aligned}(\rho - r)u(E) = \max_{d\delta\geq 0} & \Big\{\frac{d\delta}{e}[1-u(E)]\Big\} - \max_{di\geq 0}\Big\{\frac{di}{e}[1+\gamma-u(E)]\Big\} \\ & + \max_{k\geq 0}\Big\{\frac{k}{e}\Big[(R(E)-p-r)u(E) + \sigma_0^2 K(E)u'(E)\Big]\Big\} \\ & + \Big(rE + K(E)[R(E)-p-r]\Big)u'(E) + \frac{\sigma_0^2 K^2(E)}{2}u''(E).\end{aligned} \tag{I.2}$$

A solution to the maximization problem in k only exists when

$$\frac{u'(E)}{u(E)} \leq -\frac{R(E)-p-r}{\sigma_0^2 K(E)}, \tag{I.3}$$

with equality when $k > 0$.

Assuming that $R(E) \geq p + r$ (which is verified ex-post), it follows from the above expression that $u(E)$ is a decreasing function of E. Then, the optimal payout policy maximizing the right-hand side of (I.2) is characterized by a critical barrier E_{max} satisfying

$$u(E_{max}) = 1, \tag{I.4}$$

and the optimal recapitalization policy is characterized by a barrier E_{min} such that

$$u(E_{min}) = 1 + \gamma. \tag{I.5}$$

In other words, dividends are only distributed when E_t reaches E_{max}, whereas recapitalization occurs only when E_t reaches E_{min}. Given (I.3), (I.4), (I.5) and $k > 0$, it it easy to see that, in the region $E \in (E_{min}, E_{max})$, market-to-book value $u(E)$ satisfies:

$$(\rho - r)u(E) = \Big[rE + K(E)(R(E)-p-r)\Big]u'(E) + \frac{\sigma_0^2 K^2(E)}{2}u''(E). \tag{I.6}$$

[29] For the sake of space, we omit the arguments of function $v(e, E)$.

Note that, at equilibrium, $K(E) = L(R(E))$. Taking the first derivative of (I.3), we can compute $u''(E)$. Inserting $u''(E)$ and $u'(E)$ into (I.6) and rearranging terms yields:

$$R'(E) = -\frac{1}{\sigma_0^2} \frac{2(\rho - r)\sigma_0^2 + (R(E) - p - r)^2 + 2(R(E) - p - r)rE/L(R(E))}{L(R(E)) - (R(E) - p - r)L'(R(E))}. \tag{I.7}$$

Since $L'(R(E)) < 0$, it is clear that $R'(E) < 0$ if $R(E) > p + r$. To verify that $R(E) > p + r$ for any $E \in [E_{min}, E_{max})$, it is sufficient to show that $R_{min} \equiv R(E_{max}) \geq p + r$.

To obtain R_{min}, let

$$V(E) \equiv Eu(E)$$

denote the market value of the entire banking system. At equilibrium, dividends are distributed when the marginal value of bank capital equals the marginal value of dividends, which implies

$$V'(E_{max}) = u(E_{max}) + E_{max}u'(E_{max}) = 1.$$

Similarly, recapitalizations take place when the marginal value of bank capital equals the marginal costs of recapitalizing the banks, which implies

$$V'(E_{min}) = u(E_{min}) + E_{min}u'(E_{min}) = 1 + \gamma.$$

Given (I.4) and (I.5), it must hold that[30]

$$u'(E_{max}) = 0,$$

and

$$E_{min} = 0.$$

Inserting $u'(E_{max}) = 0$ into the binding condition (I.3) immediately shows that $R_{min} = p + r$, so that $R(E) > p + r$ for any $E \in [E_{min}, E_{max})$.

Hence, for any given E_{max}, the loan rate $R(E)$ can be computed as a solution to the differential equation (I.7) with the boundary condition $R(E_{max}; E_{max}) = p + r$.

To obtain E_{max}, we use the fact that individual banks' optimization with respect to the recapitalization policy implies $u(0) = 1 + \gamma$. Integrating equation (I.3) in between $E_{min} = 0$ and E_{max}, while taking into account the condition $u(E_{max}) = 1$, yields an equation that implicitly determines E_{max}:

$$\underbrace{u(E_{max})exp\Big(\int_0^{E_{max}} \frac{R(E; E_{max}) - p - r}{\sigma_0^2 L(R(E; E_{max}))} dE\Big)}_{u(0)} = 1 + \gamma. \tag{I.8}$$

Note that, since $R(E; E_{max})$ is increasing with E_{max}, the right-hand side of Equation (I.8) increases with E_{max}. When the left-hand side of equation (I.8) is larger than $1+\gamma$ when $E_{max} \to \infty$, it has the (unique) solution.[31] ***Q.E.D.***

Proof of Proposition 3. For any given barriers E^{sb}_{min} and E^{sb}_{max}, solving for the second best

[30] As we show in Appendix B, these properties can be alternatively established by looking at the limit of the discrete-time equilibrium characterization for $h \to 0$.

[31] Throughout the paper we focus on this case. In the alternative case all banks default simultaneously when E_t falls to zero.

allocation requires solving for the functions $R^{sb}(E)$ and $W(E)$ that simultaneously satisfy equations (49) and (42), subject to the boundary conditions $W'(E^{sb}_{min}) = W'(E^{sb}_{max}) - \gamma = 1$. From the concavity of the welfare function it immediately follows that $E^{sb}_{min} = 0$, whereas the optimal target level of equity E^{sb}_{max} must satisfy the super-contact condition $W''(E^{sb}_{max}) = 0$. ***Q.E.D.***

Proof of Proposition 4. Consider the shareholders' maximization problem stated in (51). By the standard dynamic programming arguments and the fact that $v_\Lambda(e,E) = eu_\Lambda(E)$, where $u_\Lambda(E)$ satisfies the following Bellman equation:

$$\begin{aligned}(\rho - r)u_\Lambda(E) = &\max_{d\delta\geq 0, di\geq 0} \left\{\frac{d\delta}{e}[1 - u_\Lambda(E)] - \frac{di}{e}[1+\gamma - u_\Lambda(E)]\right\} + \\ &+ \max_{0<k\leq e/\Lambda} \left\{\frac{k}{e}[(R(E) - p - r)u_\Lambda(E) + \sigma_0^2 K(E)u'_\Lambda(E)]\right\} + \\ &+ \Big(rE + K(E)[R(E) - p - r]\Big)u'_\Lambda(E) + \frac{\sigma_0^2 K^2(E)}{2}u''_\Lambda(E).\end{aligned} \tag{I.9}$$

A solution to (I.9) exists only if $K(E) \leq E/\Lambda$, and

$$B(E) := \frac{R(E) - p - r}{\sigma_0^2 K(E)} \geq -\frac{u'_\Lambda(E)}{u_\Lambda(E)} \equiv y(E), \tag{I.10}$$

with equality when $K(E) < E/\Lambda$.

The optimal dividend and recapitalization policies are characterized by barriers E^Λ_{max} and E^Λ_{min} such that $u_\Lambda(E^\Lambda_{max}) = 1$ and $u_\Lambda(E^\Lambda_{min}) = 1 + \gamma$. Moreover, by the same reason that in the unregulated equilibrium, it must hold that

$$E^\Lambda_{max} u'_\Lambda(E^\Lambda_{max}) = 0,$$

and

$$E^\Lambda_{min} u'_\Lambda(E^\Lambda_{min}) = 0.$$

From the first equation we immediately have $u'_\Lambda(E^\Lambda_{max}) = 0$. The implication of the second equation is less trivial. Recall that in the unregulated equilibrium, we had $E_{min} = 0$. However, under capital regulation, this is no longer possible, as $K(E) = E/\Lambda$ on the constrained region and thus equation (I.9) could not hold when $E \to 0$. Therefore, it must be that $u'_\Lambda(E^\Lambda_{min}) = 0$.

With the conjecture that there exists some critical threshold E^Λ_c such that the regulatory constraint is binding for $E \in [E^\Lambda_{min}, E^\Lambda_c]$ and is slack for $E \in [E^\Lambda_c, E^\Lambda_{max}]$, equation (I.9) can be rewritten as the first-order differential equation:[32]

$$\rho - r = -\mu(E)y(E) + \frac{\sigma_0^2 K^2(E)}{2}[y^2(E) - y'(E)] + \frac{[R(E) - p - r - \sigma_0^2 K(E)y(E)]}{\Lambda} \tag{I.11}$$

where $\mu(E)$ denotes the drift of aggregate capital (banks' expected profits):

$$\mu(E) = rE + K(E)[R(E) - p - r],$$

[32]Note that the last term in the right-hand side of equation (I.11) vanishes when $E \in [E^\Lambda_{min}, E^\Lambda_c]$, because condition (I.10) holds with equality.

the volume of lending $K(E)$ satisfies

$$K(E) = \begin{cases} E/\Lambda, & E \in [E^{\Lambda}_{min}, E^{\Lambda}_{c}] \\ L(R(E)), & E \in (E^{\Lambda}_{c}, E^{\Lambda}_{max}], \end{cases} \tag{I.12}$$

and the loan rate $R(E)$ is given by

$$R(E) = \begin{cases} L^{-1}(E/\Lambda), & E \in [E^{\Lambda}_{min}, E^{\Lambda}_{c}] \\ R'(E) = H(R(E)), \quad R(E^{\Lambda}_{c}) = L^{-1}(E^{\Lambda}_{c}/\Lambda), & E \in (E^{\Lambda}_{c}, E^{\Lambda}_{max}], \end{cases} \tag{I.13}$$

where $H(R(E))$ is defined in (28) and L^{-1} is the inverse function of the demand for loans.

Note that the value-matching condition for the loan rate function at E^{Λ}_{c} is implied by the fact the function $u(E)$ and its first derivative are continuous functions of their argument and the constraint (I.10) holds with equality at E^{Λ}_{c}.

The critical threshold E^{Λ}_{c} must satisfy equation

$$y(E^{\Lambda}_{c}) = B(E^{\Lambda}_{c}).$$

If $y(E) < B(E)$ for any $E \in [E^{\Lambda}_{min}, E^{\Lambda}_{max}]$, then the regulatory constraint is always binding and $y(E)$ satisfies equation (I.11) with $E^{\Lambda}_{c} = E^{\Lambda}_{max}$. The condition $u'_{\Lambda}(E^{\Lambda}_{min}) = 0$ yields the boundary condition $y(E_{min}) = 0$. Similarly, the condition $u'_{\Lambda}(E^{\Lambda}_{max}) = 0$ translates into the boundary condition $y(E_{max}) = 0$. ***Q.E.D.***

II. Solving for the regulated equilibrium

a) Numerical procedure

This numerical algorithm solving for the competitive equilibrium with minimum capital regulation can be implemented with the *Mathematica* software:[33]

- Pick a candidate value $\hat{E}^{\Lambda}_{min}$.
- Assume that the regulatory constraint always binds. Solve ODE (I.11) for $y(E)$ under the boundary condition $y(\hat{E}_{min}) = 0$.
- Compute a candidate value $\hat{E}^{\Lambda}_{max}$ such that satisfies equation $y(\hat{E}^{\Lambda}_{max}) = 0$.
- Check whether $y(\hat{E}^{\Lambda}_{max}) \leq B(\hat{E}^{\Lambda}_{max})$.
- Conditional on the results of the previous step, one of the two scenarios is possible:

  a) if $y(\hat{E}^{\Lambda}_{max}) \leq B(\hat{E}^{\Lambda}_{max})$, then the regulatory constraint is always binding for a given Λ, i.e., there is a single "constrained" region. In this case market-to-book value $u_{\Lambda}(\hat{E}^{\Lambda}_{min})$ can be computed according to

$$u_{\Lambda}(\hat{E}^{\Lambda}_{min}) = u_{\Lambda}(\hat{E}^{\Lambda}_{max}) exp\Big(\int_{\hat{E}^{\Lambda}_{min}}^{\hat{E}^{\Lambda}_{max}} y(E)dE\Big) = exp\Big(\int_{\hat{E}^{\Lambda}_{min}}^{\hat{E}^{\Lambda}_{max}} y(E)dE\Big).$$

[33] In all computations Λ is taken as a parameter.

b) observing $y(\hat{E}^{\Lambda}_{max}) > B(\hat{E}^{\Lambda}_{max})$ means that, for given $\hat{E}^{\Lambda}_{min}$, the constrained and unconstrained regions coexist. To find the critical level of aggregate equity $\hat{E}^{\Lambda}_{c}$ above which the regulatory constraint is slack, one needs to solve the following equation:

$$y(\hat{E}^{\Lambda}_{c}) = B(\hat{E}^{\Lambda}_{c}).$$

- using $\hat{E}^{\Lambda}_{c}$ and (I.13), define the function $R(E)$ for $E > \hat{E}^{\Lambda}_{c}$;
- compute a new candidate for the dividend boundary, $\check{E}^{\Lambda}_{max}$, such that $B(\check{E}^{\Lambda}_{max}) = 0$ (note that this is equivalent to solving equation $R(\check{E}^{\Lambda}_{max}) = p$);
- compute the market-to-book value $u_{\Lambda}(\hat{E}^{\Lambda}_{min})$ according to:

$$u_{\Lambda}(\hat{E}^{\Lambda}_{min}) = exp\Big(\int_{\hat{E}^{\Lambda}_{min}}^{\hat{E}^{\Lambda}_{c}} y(E)dE\Big) exp\Big(\int_{\hat{E}^{\Lambda}_{c}}^{\check{E}^{\Lambda}_{max}} B(E)dE\Big).$$

- If $u_{\Lambda}(\hat{E}^{\Lambda}_{min}) = 1 + \gamma$, then $E^{\Lambda}_{min} = \hat{E}^{\Lambda}_{min}$, $E^{\Lambda}_{max} = \hat{E}^{\Lambda}_{max}$ (or $\check{E}^{\Lambda}_{max}$ if 2 regions) and $E^{\Lambda}_{c} = \hat{E}^{\Lambda}_{c}$ (if 2 regions). Otherwise, pick a different $\hat{E}^{\Lambda}_{min}$, repeat the procedure from the beginning.

b) Ergodic density

Since all variables of interest are the deterministic functions of E, we directly use the ergodic density of aggregate bank capitalization in order to compute their average values. Given the dynamics of E defined in the Proposition 4 , the ergodic density function of E in the regulated competitive equilibrium can be computed according to:

$$g_{\Lambda}(E) = \frac{C_{\Lambda}}{\sigma_0^2 K^2(E)} exp\Big(\int_{E^{\Lambda}_{min}}^{E^{\Lambda}_{max}} \frac{2[rE + K(E)(R(E) - p - r)]}{\sigma_0^2 K^2(E)} dE\Big),$$

where $K(E)$ and $R(E)$ are defined in (I.12) and (I.13) respectively, and the constant C_{Λ} is such that $\int_{E^{\Lambda}_{min}}^{E^{\Lambda}_{max}} g_{\Lambda}(E)dE = 1$.

c) Expected time to reach the recapitalization barrier

Let $\overline{T}_{\gamma}(E)$ denote the expected time it takes to reach the recapitalization boundary E^{Λ}_{min} starting from any $E \geq E^{\Lambda}_{min}$. Since $t + \overline{T}_{\gamma}(E_t)$ is a martingale, function $\overline{T}_{\gamma}(E)$ satisfies the following differential equation:

$$\frac{[K(E)\sigma_0]^2}{2}\overline{T}''_{\gamma}(E) + \Big[rE + K(E)(R(E) - p - r)\Big]\overline{T}'_{\gamma}(E) + 1 = 0, \qquad (E^{\Lambda}_{min}, E^{\Lambda}_{max}),$$

where functions $K(E)$ and $R(E)$ are defined in (I.12) and (I.13) respectively.

The above equation is subject to the following two boundary conditions: $\overline{T}_{\gamma}(E^{\Lambda}_{min}) = 0$ (i.e., it takes no time to reach E^{Λ}_{min} from E^{Λ}_{min}), and $\overline{T}'_{\gamma}(E^{\Lambda}_{max}) = 0$, which emerges due to the reflection property of aggregate equity. To measure the impact of minimum capital requirements on financial stability in Section 6, for each level of Λ, we compute $\overline{T}_{\gamma}(\overline{E})$ - the expected time to reach the recapitalization boundary E^{Λ}_{min} starting from the long-run average level of aggregate equity, $\overline{E}$.

III. Empirical analysis

This Appendix presents a simple assessment of the consistency of the key predictions of our model with the data. As stated in Section 3.3, our model delivers two key predictions: bank loan rates and the ratio of market-to-book equity are (weakly) decreasing functions of aggregate bank equity. We assess these predictions by estimating panel regressions between measures of bank gross returns on earning assets, the ratio of market-to-book equity, and aggregate bank equity. We use a large bank-level panel dataset. Its use, and the attendant heterogeneity of the data for banks belonging to a specific country group, place a strong consistency requirement on the predictions of our model, which is constructed under the simplifying assumption of homogeneous banks.

Our results indicate that the key predictions of our model are not rejected by standard statistical tests, are consistent with a large variety of country circumstances, and are robust to data heterogeneity owing to our use of a firm-level panel dataset.

a) Data and statistics

The data consists of consolidated accounts and market data for panels of publicly traded banks in 43 advanced and emerging market economies for the period 1992-2012[34] taken from the Wordscope database retrieved from Datastream.[35] The sample is split into four sub-samples: U.S. banks; Japanese banks; banks in advanced economies (excluding the U.S. and Japan); banks in emerging market economies. Table 2 summarizes the definitions of the variables considered.

Table 2: Definition of variables

Identifier	*Variable*	*Measurement*
ret	bank gross return on assets	total interest income/earning assets
mtb	market-to-book equity ratio	market equity/book equity
logta	bank size	Log(assets)
loanasset	% of loans to assets	total loans/total assets
bequity	bank book equity	bank book equity
npl	non-performing loans	non-performing loans in % of total assets
TBE	total bank equity	sum of bequity

Note that bank gross return on assets include revenues accruing from investments other than loans; however, in the analysis below we will condition our estimates on asset composition using the % of loans to assets as a bank control. Furthermore, total bank equity is the sum of the equity of banks belonging to a particular country: this amounts to assuming that the relevant banking market is the country. All other variables are exact empirical counterparts of the variables defined in the model.

[34] We do not include the 80s in our time period because of a low number of observations.

[35] The Standard Industrial Classification (SIC) System is used to identify the types of financial institutions included in the sample, which are: National Commercial Banks (6021), State Commercial Banks (6022), Commercial Banks Not Elsewhere Classified (6029), Savings Institution Federally Chartered (6035), Savings Institutions Not Federally Chartered (6036). In essence, the sample includes all publicly quoted depository institutions in the database. This panel dataset is unbalanced due to mergers and acquisitions, but all banks active in each period are included in the sample to avoid survivorship biases.

Table 3 reports sample statistics (Panel A) and some (unconditional) correlations (Panel B). Note that the correlations between bank returns, the market-to-book equity ratio, and total bank equity are negative and significant. However, we wish to gauge conditional correlations, to which we now turn.

b) Panel regressions

We test whether there exists a negative conditional correlation between bank returns, market-to-book equity and total bank equity by estimating panel regressions with $Y_{it} \in (ret, mtb)$ as the dependent variable of the form:

$$Y_{it} = \alpha + \beta E_{t-1} + \gamma_1 bequity_{it-1} + \gamma_2 Logta_{it-1} + \gamma_3 loanasset_{it-1} + \gamma_4 npl_{t-1} + \gamma_5 Timedummy_{it} + \epsilon_{it}. \tag{D1}$$

All variables are lagged to mitigate potential endogenity problems. Model (D1) is used for the US and Japan samples, while we add to Model (D1) country specific effects in the estimation for the advanced economies and emerging market samples. Our focus in on the coefficient β. Bank specific effects are controlled for by the set of four variables (*bequity*, *logta*, *loanasset*, *npl*), where *npl* proxies risk in the bank's loan portfolio. The variable *Timedummy* denotes time dummies for the US and Japan sample, and country-time dummies for the advanced economies and emerging markets samples: these dummy variables control for all time-varying country specific effects.

Table 4 reports the results. The coefficient β is negative and (strongly) statistically significant in all regressions. The quantitative impact of changes in total bank equity on both bank returns and the market-to-book ratio is substantial as well. For example, a one standard deviation increase in total book equity (TBE) entails about 1.5 percentage point reduction in the loan rate. Thus, we conclude that the two key predictions of our model are consistent with the data.

Online Appendix

A. Discrete-time dynamic model

A.1. Competitive equilibrium in the dynamic discrete-time model

In this Appendix we develop a discrete-time dynamic model and show that, when the length of time periods tends to zero, the outcomes of this model converges to the outcomes of the continuous-time model.

Let h denote the length of each time period. We work with the particular specification of the firms' default probability which allows the convergence to the diffusion process in the continuous

Table 3: Sample statistics and unconditional correlations

Panel A: Sample Statistics

	US			Japan			Advanced (ex. US and Japan)			Emerging		
Variable	*Obs.*	*Mean*	*Std.Dev.*	*Obs.*	*Mean*	*Std.Dev.*	*Obs.*	*Mean*	*Std.Dev.*	*Obs.*	*Mean*	*Std.Dev.*
ret	10213	6.49	1.57	2116	3.18	1.66	4779	7.34	4.09	3015	9.99	5.01
mtb	9542	1.42	0.71	2151	1.19	0.64	4788	1.4	0.85	2914	1.61	0.99
logta	10991	13.54	1.65	2342	17.12	1.22	5148	16.26	2.39	3473	15.65	1.94
loanasset	10812	65.96	13.42	2091	68.25	9.98	4572	70.38	16.53	3074	66.43	15.74
bequity (US$ billion)	10923	0.98	9.19	2318	2.83	7.61	5133	5.23	14.16	3419	2.87	11.49
npl	10299	1.59	2.68	1770	4.11	2.89	2710	3.37	5.36	1937	5.92	8.78
TBE (US$ billion)	16742	486.69	352.6	3061	297.38	108.18	7185	59.77	76.73	5696	35.65	108.21

Panel B: Correlations

	ret	*mtb*	*logta*	*ret*	*mtb*	*logta*	*ret*	*mtb*	*logta*	*ret*	*mtb*	*logta*
mtb	0.2368*	1		0.5481*	1		−0.0179	1		0.0581*	1	
logta	−0.2101*	0.2302*	1	0.1220*	0.2694*	1	−0.2597*	0.1992*	1	−0.3720*	0.0909*	1
TBE	−0.7703*	−0.3557*	0.2300*	−0.1764*	−0.3131*	0.1456*	−0.3283*	−0.0938*	0.3128*	−0.1974*	−0.0535*	0.3772*

Notes: * indicates significance at 5% level.

Table 4: Panel regressions

	US		Japan		Advanced (ex. US and Japan)		Emerging	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Explanatory variables	*ret*	*mtb*	*ret*	*mtb*	*ret*	*mtb*	*ret*	*mtb*
TBE	−0.00416***	−0.000139***	−0.0124***	−0.00466***	−0.0123***	−0.00374***	−0.00280***	−0.00215***
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
bequity	0.00232**	−0.00945***	0.0254***	−0.00316*	0.0117***	−0.00113*	0.0167***	−0.00160*
	[0.01]	[0.00]	[0.00]	[0.07]	[0.00]	[0.08]	[0.00]	[0.10]
logta	−0.0454***	0.157***	0.0220	0.204***	−0.366***	−0.0164**	−0.370***	−0.00726
	[0.00]	[0.00]	[0.45]	[0.00]	[0.00]	[0.01]	[0.00]	[0.41]
loanasset	0.0228***	−0.000590	0.00561***	0.00388	−0.0282***	−0.00195***	−0.0357***	−0.000738
	[0.00]	[0.23]	[0.00]	[0.29]	[0.00]	[0.00]	[0.00]	[0.43]
npl	0.0274***	−0.0414***	0.0386***	0.0369***	0.0181*	−0.0110***	−0.0103	−0.00892***
	[0.00]	[0.00]	[0.00]	[0.00]	[0.09]	[0.00]	[0.16]	[0.00]
Constant	8.117***	−0.527***	6.897***	−1.002	16.88***	2.633***	24.22***	1.551***
	[0.00]	[0.00]	[0.00]	[0.11]	[0.00]	[0.00]	[0.00]	[0.00]
Number of banks	728	728	128	128	248	248	183	183
Bank/years	9,736	8,899	1,534	1,598	2,600	2,600	1,772	1,772
R-squared	0.794	0.429	0.803	0.385	0.700	0.506	0.737	0.392

Notes: robust p-values in brackets (*** -$p < 0.01$, **- $p < 0.05$, *- $p < 0.1$).

time limit:[36]

$$\tilde{p}_t(h) = \begin{cases} p - \frac{\sigma_0}{\sqrt{h}}, & \text{with probability } 1/2 \textit{ (positive shock)}, \\ p + \frac{\sigma_0}{\sqrt{h}}, & \text{with probability } 1/2 \textit{ (negative shock)}. \end{cases}$$

To simplify the presentation of further results, let

$$\tilde{A}_{t+h}(E_t) = \begin{cases} (R^h(E_t) - p - r)h + \sigma_0\sqrt{h} \equiv A^+(E_t, h), \\ (R^h(E_t) - p - r)h - \sigma_0\sqrt{h} \equiv A^-(E_t, h), \end{cases}$$

denote the marginal loan return realized in period $(t, t+h]$.

Then, the dynamics of individual equity follows:

$$\tilde{e}_{t+h} = (1 + rh)(e_t + i_t + \delta_t) + k_t \tilde{A}_{t+h}(E_t). \tag{A1}$$

Similarly, aggregate equity evolves according to

$$\tilde{E}_{t+h} = (1 + rh)(E_t + I_t + \Delta_t) + L(R^h(E_t))\tilde{A}_{t+h}(E_t), \tag{A2}$$

where we have used the equilibrium condition $K_t = L(R^h(E_t))$.

The non-default constraint imposed by the interbank borrowing market implies

$$e^-_{t+h} := (1 + rh)(e_t + i_t + \delta_t) + k_t A^-(E_t, h) \geq 0. \tag{A3}$$

Let $\theta^h(E_t)$ denote the Lagrangian multiplier associated with the non-default constraint (A3). Given the dynamics of individual (equation (A1)) and aggregate (equation (A2)) equity, the maximization problem of an individual bank can be stated as follows:

$$v(e_t, E_t) \equiv e_t u^h(E_t) = \max_{\delta_t \geq 0, i_t \geq 0, k_t \geq 0} \quad \delta_t - (1+\gamma)i_t + \mathbb{E}\left[\frac{\tilde{e}_{t+h}u^h(\tilde{E}_{t+h}) + \theta^h(E_t)e^-_{t+h}}{1 + \rho h}\right], \quad t = 0, ..., +\infty. \tag{A4}$$

Then, the maximization problem (A4) can be rewritten in the following way:

$$\begin{aligned} e_t u^h_t(E_t) = e_t \frac{(1+rh)(\mathbb{E}[u(\tilde{E}_{t+h})] + \theta^h(E_t))}{1 + \rho h} &+ \max_{\delta_t \geq 0} \quad \delta_t \Big[1 - \frac{(1+rh)(\mathbb{E}[u^h(\tilde{E}_{t+h})] + \theta^h(E_t))}{1+\rho h}\Big] \\ &+ \max_{i_t \geq 0} \quad i_t \Big[-(1+\gamma) + \frac{(1+rh)(\mathbb{E}[u^h(\tilde{E}_{t+h})] + \theta^h(E_t))}{1+\rho h}\Big] \\ &+ \max_{k_t > 0} \quad k_t \Big[\frac{\mathbb{E}[\tilde{A}_{t+h}u^h(\tilde{E}_{t+h})] + \theta^h(E_t)A^-_{t+h}(E_t)}{1+\rho h}\Big]. \end{aligned} \tag{A5}$$

[36] Another interesting example could be constructed for the specification of the default probability generating a jump-process in the continuous-time limit:

$$\tilde{p}_t(h) = \begin{cases} p, & \text{with probability } 1 - \phi h \textit{ (positive shock)}, \\ p - \frac{l}{h}, & \text{with probability } \phi h \textit{ (negative shock)}, \end{cases}$$

where ϕ is the intensity of large losses and l is the size of a (proportional) large loss.

Since the value function is linear in all policy variables, optimizing with respect to the bank's policies yields:

$$1 - \frac{(1+rh)(\mathbb{E}[u^h(\tilde{E}_{t+h})] + \theta^h(E_t))}{1+\rho h} \le 0 \qquad (= \text{ if } \delta_t > 0), \tag{A6}$$

$$-(1+\gamma) + \frac{(1+rh)(\mathbb{E}[u^h(\tilde{E}_{t+h})] + \theta^h(E_t))}{1+\rho h} \le 0 \quad (= \text{ if } i_t > 0), \tag{A7}$$

$$\mathbb{E}[\tilde{A}_{t+h}(E_t)u^h(\tilde{E}_{t+h})] + \theta^h(E_t)A^-(E_t,h) = 0, \tag{A8}$$

and thus

$$u(E_t) = \frac{(1+rh)(\mathbb{E}[u^h(\tilde{E}_{t+h})] + \theta^h(E_t))}{1+\rho h}. \tag{A9}$$

Under (A9), condition (A6) transforms to

$$u(E_t) \ge 1, \quad (= \text{ if } \delta_t > 0), \tag{A10}$$

and condition (A7) can be rewritten as

$$u(E_t) \le 1+\gamma, \quad (= \text{ if } i_t > 0). \tag{A11}$$

Thus, provided that $u^h(.)$ is a decreasing function of its argument (this has to be verified ex-post), the optimal dividend and recapitalization policies are of the barrier type. Let E^h_{min} and E^h_{max} denote, respectively, the levels of aggregate capital at which (A10) and (A11) holds with equality. Banks distribute any excess profits as dividends so as to maintain aggregate equity at or below E^h_{max}. Similarly, recapitalizations are undertaken so as to offset losses and to maintain aggregate equity at or above E^h_{min}.

To complete the characterization of the equilibrium, one has to solve for the functions $\theta^h(E)$, $u^h(E)$ and $R^h(E)$. Rewriting (A8) and (A9), we obtain the system of equations:

$$\frac{1}{2}\Big[A^+(E,h)u^h(E^+) + A^-(E,h)u^h(E^-)\Big] = -\theta^h(E)A^-(E,h), \tag{A12}$$

$$\frac{1}{2}\Big[u^h(E^+) + u^h(E^-)\Big] = \frac{(1+\rho h)}{(1+rh)}u^h(E) - \theta^h(E), \tag{A13}$$

Moreover, the fact that the non-default constraint (A3) is binding at the individual level if and only if it is binding at the aggregate level implies an additional equation:

$$\theta(E)\Big\{(1+rh)E + L(R^h(E))A^-(E,h)\Big\} = 0 \tag{A14}$$

We proceed by conjecturing and verifying the existence of two non-empty regions: $[E^h_{min}, \hat{E}^h)$, where the non-default constraint is binding and $\theta^h(E) > 0$, and $[\hat{E}^h, E^h_{max}]$ where the non-default constraint is slack and $\theta^h(E) = 0$. The critical barrier $\hat{E}^h$ is implicitly given by $\theta^h(\hat{E}^h) = 0$.

First, consider the constrained region $[E^h_{min}, \hat{E}^h)$. The equilibrium interest rate, $R^h(E)$, is implicitly given by the binding non-default constraint:

$$L(R^h(E))[\sigma_0\sqrt{h} - (R^h(E) - p - r)h] = (1+rh)E.$$

The above equation implies that $E^- \equiv 0$ in the region $[E^h_{min}, \hat{E}^h)$. Furthermore, we have $E^+ = 2\sigma_0\sqrt{h}L(R^h(E))$ and $u(E^-) = u(E^h_{min}) = 1+\gamma$. Solving the system of equations (A13) and (A12) with respect to $\theta^h(E)$ and $u^h(E)$ yields:

$$\theta^h(E) = \frac{1}{2}\Big(\frac{1+\beta^h[R^h(E)]}{1-\beta^h[R^h(E)]}u(2\sigma_0\sqrt{h}L(R^h(E))) - 1 - \gamma\Big), \tag{A15}$$

$$u^h(E) = \Big(\frac{1+rh}{1+\rho h}\Big)\frac{u(2\sigma_0\sqrt{h}L(R^h(E)))}{1-\beta^h[R^h(E)]}, \tag{A16}$$

where

$$\beta^h[R^h(E)] = \frac{R^h(E) - p - r}{\sigma_0}\sqrt{h}.$$

In the unconstrained region, $[\hat{E}^h, E^h_{max}]$, the equilibrium loan rate $R^h(E)$ and the market-to-book ratio $u^h(E)$ satisfy the system of equations:

$$\Big(1+\beta^h[R^h(E)]\Big)u^h(E^+) = \Big(1-\beta^h[R^h(E)]\Big)u^h(E^-), \tag{A17}$$

$$2(1+\rho h)u^h(E) = (1+rh)\Big[u^h(E^+) + u^h(E^-]\Big). \tag{A18}$$

Rewriting Equation (A17) after taking expectation yields:

$$R^h(E) = p + r + \underbrace{\frac{\sigma_0}{\sqrt{h}}\Big[\frac{u^h(E^-) - u^h(E^+)}{u^h(E^+) + u^h(E^-)}\Big]}_{\text{lending premium, } > 0} > p + r.$$

It remains to check that $E^h_{max} > \hat{E}^h > E^h_{min} > 0$.

First, let us show that $E^h_{min} > 0$. Suppose by way of contradiction that $E^h_{min} = 0$. Then, from the binding leverage constraint at the aggregate level it follows that $R^h(0) = p + r + \frac{\sigma_0}{\sqrt{h}}$ and thus $\beta^h[R^h(0)] = 1$. The optimal recapitalization policy implies that at E^h_{min} it must hold that $u^h(E^h_{min}) = 1+\gamma$. However, evaluating (A16) at $E^h_{min} = 0$ yields $u(0) \equiv \infty$, which is incompatible with the previous statement. Therefore, $E^h_{min} > 0$, as claimed.

Next, we show that $\hat{E}^h > E^h_{min}$. Combining (A13) and $u^h(E^h_{min}) = 1+\gamma$, it is immediate to see that $\theta^h(E^h_{min}) > 0$. Then, by continuity, it must be that $\theta^h(E) > 0$ in the vicinity of E^h_{min}.

Finally, let us show that $E^h_{max} > \hat{E}^h$. Assume by way of contradiction that $E^h_{max} = \hat{E}^h$. Then it must hold that $\theta^h(E^h_{max}) \geq 0$. The optimal dividend policy implies that $u(E^+) = 1$ for $E = E^h_{max}$. Note that $\theta^h(E^h_{max}) \geq 0$ if and only if

$$\xi(h) := 1 + \beta^h[R^h(E^h_{max})] - (1+\gamma)(1 - \beta^h[R(E^h_{max})]) > 0.$$

Yet, when $h \to 0$, we have $\beta^h[R(E^h_{max})] \to 0$ and thus $\xi(h) < 0$, a contradiction. Hence, $E^h_{max} > \hat{E}^h$.

A.2. Convergence to the continuous-time results

We now establish convergence of the properties of the discrete-time version of the competitive equilibrium studied above to the properties of its continuous-time counterpart studied in Section 3. Namely, we are going to show that, for $h \to 0$,

- $\hat{E}^h \to E^h_{min}$ and $E^h_{min} \to 0$;
- $R(E^h_{max}) \to p + r$;
- Equations (A17) and (A18) converge to Equations (22) and (26).

a) First, let us show that $\hat{E}^h \to E^h_{min}$ when $h \to 0$. Recall that $\hat{E}^h$ is implicitly defined by Equation $\theta(\hat{E}^h) = 0$, which holds if and only if

$$(1 + \beta^h[R^h(\hat{E}^h)])u(2\sigma_0\sqrt{h}L(R^h(\hat{E}^h))) = (1+\gamma)(1 - \beta^h[R^h(\hat{E}^h)]).$$

When $h \to 0$, we have $\lim_{h\to 0}\beta^h[R^h(\hat{E}^h)] \to 0$, so that the above equality transforms to:

$$u(2\sigma_0\sqrt{h}L(R^h(\hat{E}^h))) = 1 + \gamma. \tag{A19}$$

At the same time, E^h_{min} satisfies $u^h(E^h_{min}) = 1 + \gamma$, which is equivalent to

$$\Big(\frac{1 + rh}{1 + \rho h}\Big)\frac{u(2\sigma_0\sqrt{h}L(R^h(E^h_{min})))}{1 - \beta^h[R^h(E^h_{min})]} = 1 + \gamma.$$

When $h \to 0$, the above equation transforms to

$$u(2\sigma_0\sqrt{h}L(R^h(E^h_{min}))) = 1 + \gamma. \tag{A20}$$

Given that $u^h(.)$ is a monotonically-decreasing function of its argument, equations (A19) and (A20) simultaneously hold only when $\hat{E}^h = E^h_{min}$.

Second, the property $E^h_{min} \to 0$ for $h \to 0$ immediately follows from the aggregate market leverage constraint that holds with equality at $E = E^h_{min}$.

b) Next, we demonstrate that $R(E^h_{max}) \to p + r$ when $h \to 0$. To this end, consider the system of Equations (A17) and (A18) evaluated at E^h_{max}:

$$\Big(1 + \beta^h[R^h(E^h_{max})]\Big)u^h(E^+) = \Big(1 - \beta^h[R^h(E^h_{max})]\Big)u^h(E^-), \tag{A21}$$

$$2(1 + \rho h)u^h(E^h_{max}) = (1 + rh)\Big[u^h(E^+) + u^h(E^-)\Big]. \tag{A22}$$

The optimal dividend policy implies that $E^+ \equiv E^h_{max}$ and $u^h(E^h_{max}) = 1$. Then, solving the above system one obtains:

$$u^h(E^-) = \frac{1 - \beta^h[R^h(E^h_{max})]}{1 + \beta^h[R^h(E^h_{max})]},$$

and

$$\beta^h[R^h(E^h_{max})] = \frac{(\rho - r)h}{1 + \rho h}.$$

Using the definition of $\beta^h(.)$, one can show that:

$$R^h(E^h_{max}) = p + r + \sigma_0\frac{(\rho - r)\sqrt{h}}{1 + \rho h}.$$

It is easy to see from the above equation that $R^h(E^h_{max}) \to p + r$ when $h \to 0$.

c) Finally, let us show that Equations (A17) and (A18) converge to Equations (22) and (26) when $h \to 0$. To see this, consider first a first-order Taylor expansion of Equation (A17). Neglecting the terms of order higher than $\sqrt{h}$, we get:

$$\Big(1+\beta^h[R^h(E)]\Big)\Big[u^h(E)+[u^h(E)]'L(R^h(E))\sigma_0\sqrt{h}\Big] = \Big(1-\beta^h[R^h(E)]\Big)\Big[u(E)-[u^h(E)]'L(R^h(E))\sigma_0\sqrt{h}\Big], \tag{A23}$$

which, after simplification, yields

$$u^h(E)(R^h(E) - p - r)h + \sigma_0^2[u^h(E)]'L(R^h(E))h = 0, \tag{A24}$$

or,

$$\frac{[u^h(E)]'}{u^h(E)} = -\frac{R^h(E) - p - r}{\sigma_0^2 L(R^h(E))},$$

which corresponds to (22).

Similarly, applying a second-order Taylor expansion to the right-hand side of Equation (A18) and neglecting terms of order higher than h yields:

$$(\rho - r)u^h(E) = [u^h(E)]'\Big(rE + (R^h(E) - p - r)L(R^h(E))\Big) + \frac{\sigma_0^2 L(R^h(E))^2}{2}[u^h(E)]''(E), \tag{A25}$$

which corresponds to (26).

Appendix B. Inelastic demand for loans

Consider the special case where $r = 0$ and the firms' demand for loans is constant and equal to 1 as long as the loan rate does not exceed some maximum rate $\overline{R}$:[37]

$$L(R) = \begin{cases} 1 & \text{for} \quad R \le \overline{R}, \\ 0 & \text{for} \quad R > \overline{R}. \end{cases} \tag{B1}$$

In this Appendix we show that, when the social implied risk aversion in the states with poor bank capitalization is sufficiently high, it is still socially optimal to restrict lending of the undercapitalized banking system, even though the "margin channel" of inefficiencies is shut down.

The single state variable in the social planner's problem is aggregate bank equity E and the market for bank credit must clear, i.e., $K_t = L(R_t)$. Over $[0, E^{sb}_{max}]$, the social welfare function satisfies the following ODE:[38]

[37] This loan demand specification can be obtained from the demand specification (31) by taking the limit case $\beta \equiv 0$.

[38] Note that the social welfare function coincides with the value function of a monopolistic bank. This is in contrast to the case with an elastic demand for loans, where firms are making profits and, thus, the socially optimal policies differ from the optimal policies of a monopolistic bank. Namely, in the case with an elastic demand for loans, the monopolistic loan rate is always strictly higher than the second best (and also the competitive) loan rate. Furthermore, with an elastic demand, the optimal dividend barrier of a monopolistic bank is strictly lower than the one corresponding to the second best allocation.

$$\rho W(E) = \max_{R \leq \overline{R}, K \leq 1} \quad \underbrace{K(\overline{R}-R)}_{\text{firms' profits}} + \underbrace{K(R-p)}_{\text{expected banks' profits}} W'(E) + \frac{\sigma_0^2 K^2}{2} W''(E), \tag{B2}$$

subject to the boundary conditions

$$W'(0) = 1 + \gamma, \tag{B3}$$

$$W'(E_{max}^{sb}) = 1. \tag{B4}$$

The optimal dividend barrier E_{max}^{sb} should satisfy the super-contact condition

$$W''(E_{max}^{sb}) = 0. \tag{B5}$$

As a consequence, it holds that $W'(E) > 1$ for $E \in [0, E_{max}^{sb})$, implying that social welfare is maximized at the highest possible ("reservation") loan rate $R^{sb}(E) \equiv \overline{R}$.[39] Therefore, with inelastic demand, firm profits are equal to zero and the social welfare function coincides with the value function of a monopolistic bank, satisfying ODE

$$\rho W(E) = \max_{K \leq 1} \quad K\left(\overline{R}-p\right) W'(E) + \frac{\sigma_0^2 K^2}{2} W''(E). \tag{B6}$$

Maximizing the right-hand side of (B6) with respect to aggregate lending yields:

$$K^{sb}(E) = \min\left\{1, -\frac{(\overline{R}-p)}{\sigma_0^2}\frac{W'(E)}{W''(E)}\right\}. \tag{B7}$$

Substituting K^{sb} back into HJB (B6) implies that for interior loan volumes ($K^{sb} < 1$), the social welfare function has an explicit solution given by

$$W_1(E) = c_2\left(\frac{E}{1-\eta} - c_1\right)^{1-\eta}, \quad \text{with } \eta \equiv \frac{(\overline{R}-p)^2}{(\overline{R}-p)^2 + 2\rho\sigma_0^2}, \tag{B8}$$

where $\{c_1, c_2\}$ are constants to be determined.[40]

Substituting the general solution of the welfare function (B8) into Equation (B7) shows that the optimal volume of lending, if interior, linearly increases with aggregate capital E:

$$K^{sb}(E) = \min\left\{1, \frac{(\overline{R}-p)}{\eta\sigma_0^2}\left(E - (1-\eta)c_1\right)\right\}. \tag{B9}$$

Next, let $\hat{E}_K$ denote the critical level of aggregate capital for which

$$K^{sb}(\hat{E}_K) = 1, \tag{B10}$$

such that the welfare maximizing loan volume satisfies $K^{sb}(E) < 1$ for $E < \hat{E}_K$ and $K^{sb}(E) = 1$ for $E \geq \hat{E}_K$. That is, for low levels of aggregate bank capital, the social planner reduces aggregate

[39] To see this, note that the partial derivative of (B2) with respect to R is equal to $K[W'(E) - 1] > 0$.

[40] We indicate the solution by W_1 in order to differentiate it from its counterpart W_2 in the region where the corner solution $R = 1$ applies.

lending below the level that is attained in a competitive equilibrium.

From (B9), we have[41]

$$\hat{E}_K = \max\left\{0, \frac{\eta\sigma_0^2}{\overline{R}-p} + c_1(1-\eta)\right\}.$$

Combining (B7) and (B3), it is easy to see that $\hat{E}_K > 0$ when the following condition is satisfied:

$$W''(0) < -\frac{(\overline{R}-p)}{\sigma_0^2}(1+\gamma).$$

Thus, when the implied social risk aversion in the undercapitalized states is high enough, the socially optimal level of lending will be lower than the competitive level of lending.

Appendix C. Time-varying financing conditions

In the core of the paper we have considered the setting in which the refinancing cost γ was constant over time. In this section we extend the basic model to time-varying financing conditions. Assume that the cost of issuing new equity depends on a macroeconomic state that can be Good or Bad, with the respective costs of recapitalization γ_G and γ_B, such that $\gamma_B > \gamma_G$. Let ψ_B denote the intensity of transition from the Good to the Bad state and ψ_G denote the intensity of transition from the Bad to the Good state.

Note that the homogeneity property of the individual decision problem is still preserved, so we can again work directly with the market-to-book value of banks. To ensure that the maximization problem of bank shareholders has a non-degenerate solution, the market-to-book value must satisfy the system of simultaneous equations:

$$\rho u_G(E) = \frac{[K_G(E)\sigma_0]^2}{2}u_G''(E) + K_G(E)\left[R_G(E)-p\right]u_G'(E) - \psi_B\left[u_G(E)-u_B(E)\right], \quad E \in (E_{min}^G, E_{max}^G) \tag{B1}$$

$$\rho u_B(E) = \frac{[K_B(E)\sigma_0]^2}{2}u_B''(E) + K_B(E)\left[R_B(E)-p\right]u_B'(E) - \psi_G\left[u_B(E)-u_G(E)\right], \quad E \in (E_{min}^B, E_{max}^B) \tag{B2}$$

along the system of the FOCs for the individual choices of lending in each state:

$$u_G(E)[R_G(E)-p] = -\sigma_0^2 K_G(E)u_G'(E), \tag{B3}$$

$$u_B(E)[R_B(E)-p] = -\sigma_0^2 K_B(E)u_B'(E). \tag{B4}$$

Thus, the market-to-book value, the loan rate and aggregate lending functions will have different expressions conditional on financing conditions. The differential equations characterizing them take into account the possibility of transitions between the states. Note that equations (B1)-(B2) are similar to equation (22) obtained in the setting with the time-invariant financing cost. However, compared to (22), each equation carries the additional term $-\psi_{\overline{j}}\left[u_j(E) - u_{\overline{j}}(E)\right]$ reflecting the

[41] Note that if γ and σ_0 are sufficiently low, $\hat{E}_K$ might not be interior, in which case $K^{sb}(E) = 1$ for all $E \in [E_{min}^{sb}, E_{max}^{sb}]$.

possibility that the cost of raising new equity can suddenly change from γ_j to $\gamma_{\bar{j}}$ (here $\bar{j}$ denotes the state complementary to the state j).

For given recapitalization (E^G_{min}, E^B_{min}) and dividend (E^G_{max}, E^B_{max}) boundaries, the boundary conditions can be established by solving for the optimal recapitalization and payout policies, which yields:

$$u_G(E^G_{max}) = u_B(E^B_{max}) = 1, \tag{B5}$$

$$u_G(E^G_{min}) = 1 + \gamma_G, \quad u_B(E^B_{min}) = 1 + \gamma_B. \tag{B6}$$

To define the boundaries E^G_{min}, E^G_{max}, E^B_{min}, E^B_{max}, we follow the same logic that was used in the setting with the time-invariant cost and consider the marginal value of the entire banking system at the boundaries. The absence of arbitrage opportunities implies that the following condition must hold at $E^j_{max}, j \in \{G, B\}$:

$$V'_j(E^j_{max}) = u_j(E^j_{max}) + E^j_{max} u'_j(E^j_{max}) = 1.$$

Combining the above condition with (B5) and taking into account the fact that $E^j_{max} > 0$ yields us a couple of equations needed to compute the values of $E^j_{max}, j \in \{G, B\}$:

$$u'_j(E^j_{max}) = 0, \qquad j \in \{G, B\}. \tag{B7}$$

Similarly, at refinancing boundaries $E^j_{min}, j \in \{G, B\}$, it must hold that

$$V'_j(E^j_{min}) = u_j(E^j_{min}) + E^j_{min} u'_j(E^j_{min}) = 1 + \gamma_j,$$

which implies

$$E^j_{min} u'_j(E^j_{min}) = 0, \qquad j \in \{G, B\}. \tag{B8}$$

Note that, compared with the setting involving the time-invariant refinancing cost, two polar cases are possible now: a) $u'_j(E^j_{min}) > 0$ with $E^j_{min} = 0$; or b) $u'_j(E^j_{min}) = 0$ with $E^j_{min} > 0$. As we discuss below, which of these cases ultimately materializes depends on the magnitude of the refinancing cost in the Bad state, γ_B.

Finally, before we turn to the numerical illustration of the equilibrium properties, note that the loan rate in state $j \in \{G, B\}$ satisfies the following differential equation:[42]

$$R'_j(E) = -\frac{1}{\sigma_0^2} \frac{2\rho\sigma_0^2 + (R_j(E) - p)^2 + \psi_{\bar{j}}\Big(1 - \frac{u_{\bar{j}}(E)}{u_j(E)}\Big)\sigma_0^2}{L(R_j(E)) - (R_j(E) - p)L'(R_j(E))}.$$

Compared with equation (28) in Section 3, the right-hand side of the above equation contains an additional term in the numerator, $\psi_{\bar{j}}\Big(1 - \frac{u_{\bar{j}}(E)}{u_j(E)}\Big)\sigma_0^2$. This suggests that in the current set-up the loan rate will carry an extra premium/discount.

Numerical example. To illustrate the numerical solution and properties of the competitive equilibrium in the setting with time-varying refinancing costs, we resort to the simple linear specification

[42]This equation can be obtained by combining equations (B1) and (B3) (or, equivalently, (B2) and (B4)).

of the demand for loans:

$$K_j(E) = \overline{R} - R_j(E), \qquad j \in \{G, B\}.$$

We first work with the systems of equations (B1)- (B2) and (B3)- (B4). Replacing $K_j(E)$ in (B3)- (B4), one can express $R_j(E)$ as a function of $u_j(E)$ and $u_j'(E)$. In our simple linear case, this yields:

$$R_j(E) = \frac{pu_j(E) - \overline{R}\sigma_0^2 u_j'(E)}{u_j(E) - \sigma_0^2 u_j'(E)}, \qquad j \in \{G, B\}. \tag{B9}$$

Substituting the above expression(s) in the system (B1)- (B2) leaves us with a system of two simultaneous second-order differential equations that can be solved numerically by using the four boundary conditions stated in (B5)-(B6). To obtain a solution to this system, we conjecture that $E^G_{max} < E^B_{max}$ (this is verified ex-post) and use the fact that $u_G(E) \equiv 1$ when $E \in [E^G_{max}, E^B_{max}]$.[43] To solve numerically for the equilibrium, we proceed in three steps:

i) first, we take the boundaries $\{E^G_{max}; E^B_{max}\}$ as given and solve for the optimal E^G_{min} and E^B_{min} that satisfy conditions (B8);

ii) second, we search for the couple $\{E^G_{max}; E^B_{max}\}$ that satisfies conditions (B7);

iii) finally, we uncover the equilibrium loan rates by substituting the functions $u_G(E)$ and $u_B(E)$ into equations (B9).

We now turn to the discussion of two possible cases that may arise depending on the magnitude of the refinancing cost γ_B. For this discussion, it is helpful to introduce two *benchmarks*: in the first benchmark, the economy never leaves the Good state, i.e., $\psi_B \equiv 0$; in the second benchmark, the economy always remains in the Bad state, i.e., $\psi_G \equiv 0$. The first benchmark gives us the lower bound for E^G_{max} that we will further label E^*_{max}, whereas the second benchmark gives the upper bound for E^B_{max} that we will label E^{**}_{max}. We also denote $u^{\psi_B=0}(E)$ ($R^{\psi_B=0}(E)$) the market-to-book value (loan rate) in the set-up with $\psi_B = 0$ and $u^{\psi_G=0}(E)(R^{\psi_G=0}(E))$ the market-to-book value (loan rate) in the set-up with $\psi_G = 0$.

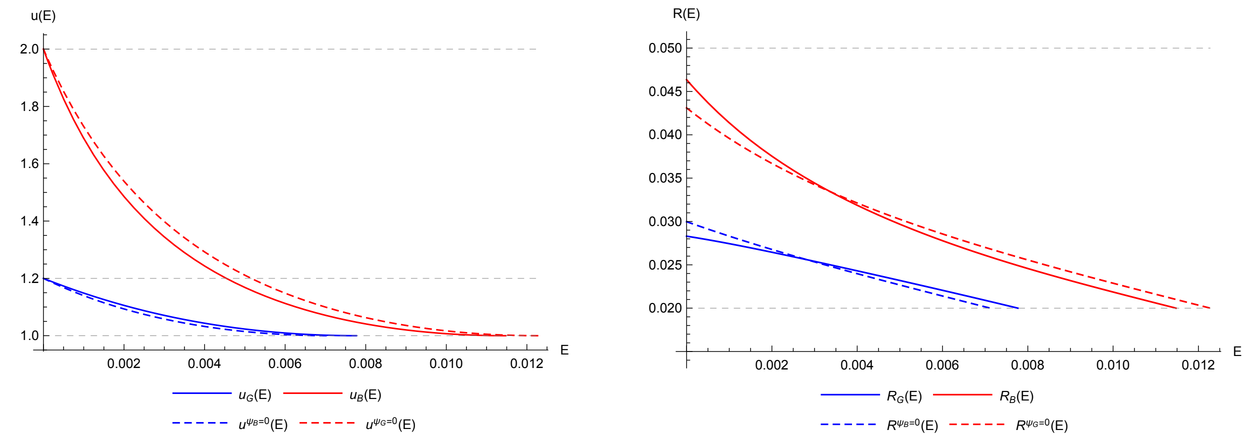
Case 1: $\boldsymbol{\gamma_B}$ low. First we consider the case in which the refinancing cost in the Bad state is relatively low. The solid lines in Figure 8 depict the typical patterns of the market-to-book ratios (left panel) and the corresponding loan rates (right panel) computed in each state. These outcomes are contrasted with the patterns emerging in the two benchmarks (dashed lines).

Several features of the equilibrium are worth mentioning at this stage. First, one can note that $E^G_{min} = E^B_{min} = 0$, so that banks do not change their recapitalization policies as compared to those implemented in the benchmark settings. By contrast, $E^G_{max} > E^*_{max}$ and $E^B_{max} < E^{**}_{max}$, which means that banks will delay the distribution of dividends as compared to the setting in which the economy permanently remains in the Good state and will accelerate dividend payments as compared to the setting in which the economy is permanently locked in the Bad state. Second, in the Good state, the loan rate carries a "discount" (as compared to its benchmark value obtained for $\psi_B \equiv 0$) when E is low and an extra "premium" when E is high enough. By contrast, the loan rate in the Bad state carries an extra "premium"(as compared to its benchmark value obtained for $\psi_G \equiv 0$) for

[43] To find a numerical solution, one can start with a guess for $u_B(E)$ and proceed iteratively until the difference between the solutions on the consequent iterations vanishes. It turns out to be convenient to take as the initial guess for $u_B(E)$ the solution to the ODE (B2) with $\psi_G \equiv 0$.

lower level of E and a "discount" when E is high. Finally, the fact that $R_B(E) > R_G(E)$ implies $L(R_B(E)) < L(R_G(E))$, i.e., lending is procyclical.

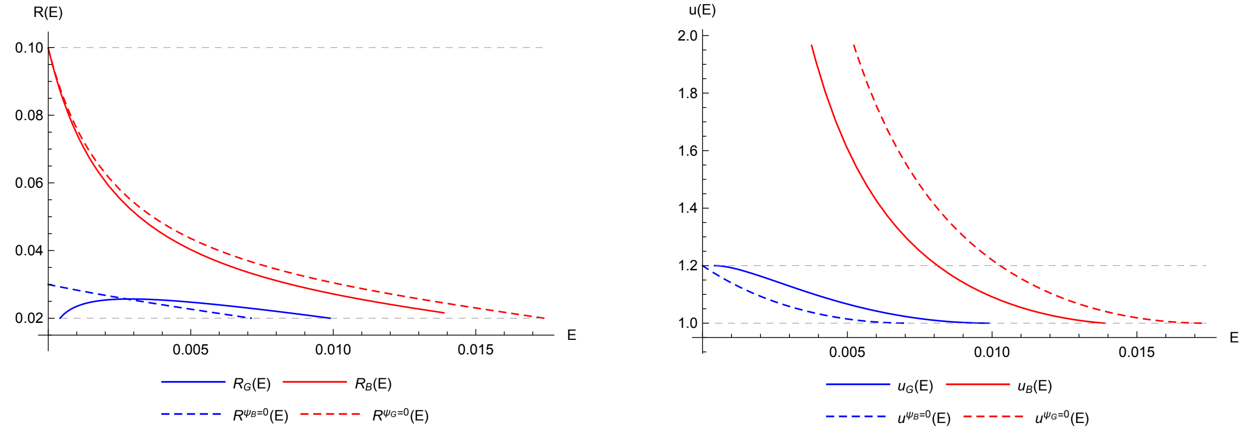
Figure 8: Competitive equilibrium with time-varying financing costs: γ_B low



Parameter values: $\rho = 0.05$, $\sigma_0 = 0.05$, $p = 0.02$, $\gamma_G = 0.2$, $\gamma_B = 1$, $\psi_G = 0.1$, $\psi_B = 0.05$; $L(R) = \overline{R} - R$ with $\overline{R} = 0.1$. The benchmark payout barriers are: $E^*_{max} \approx 0.007$ and $E^{**}_{max} \approx 0.012$. The payout barriers in the Good and Bad regimes are: $E^G_{max} \approx 0.008$ and $E^B_{max} \approx 0.011$.

Case 2: $\boldsymbol{\gamma_B}$ high. We now turn to the setting involving very high refinancing costs in the Bad state. The properties of the competitive equilibrium emerging in this case are illustrated in Figure 9. The key distinction from the previous case is that, for fear of incurring substantial recapitalization costs if the economy slides in the Bad state, in the Good state banks will raise new equity capital at a *strictly positive* level of aggregate capitalization, i.e., $E^G_{min} > 0$. In a dynamic setting, this phenomenon of "market timing" was first identified by Bolton et al. (2013) within a partial-equilibrium liquidity-management model with the stochastically changing fixed cost of equity issuance. The main general-equilibrium implication of this feature in our setting is that the loan rate (and, therefore, the volume of lending) converges to its First-Best level when $E \to E^G_{min}$ and financing conditions are good (i.e., $j = G$).

Figure 9: Competitive equilibrium with time-varying financing costs: γ_B high



Parameter values: $\rho = 0.05$, $\sigma_0 = 0.05$, $p = 0.02$, $\gamma_G = 0.2$, $\gamma_B = +\infty$, $\psi_G = 0.1$, $\psi_B = 0.05$; $L(R) = \overline{R} - R$ with $\overline{R} = 0.1$. The benchmark payout barriers are: $E^*_{max} \approx 0.007$ and $E^{**}_{max} \approx 0.017$. The payout barriers in the Good and Bad regimes are: $E^G_{max} \approx 0.01$ and $E^B_{max} \approx 0.014$. The recapitalization barriers are: $E^G_{min} \approx 0.0005$ and $E^B_{min} = 0$. Note that functions $u_G(E)$ and $u^{\psi_G=0}(E)$ tend to +

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