Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound

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Key question

What is the macroeconomic impact of monetary policy at the ZLB?

Conventional approach before ZLB

- VAR with the fed funds rate

But since December 2008, the fed funds rate has been near zero
Challenges of zero lower bound

Challenges

▶ Conventional monetary policy doesn’t work. Fed has implemented unconventional policy tools
  ▶ large-scale asset purchases
  ▶ forward guidance
▶ What framework to study unconventional monetary policy?
▶ Gaussian ATSM allows negative interest rates

Shadow rate term structure model: Black (1995)

▶ Non-negative short rate: $r_t = \max(r, s_t)$
▶ Analytical solution does not exist in general
Contributions

This paper

- an analytical approximation for SRTSM
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- shadow rate has similar dynamic correlations with macro variables as the fed funds rate did previously
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This paper

- an analytical approximation for SRTSM
- shadow rate has similar dynamic correlations with macro variables as the fed funds rate did previously
- our shadow rate updated monthly by Atlanta Fed

www.frbatlanta.org/cqer/researchcq/shadow_rate.cfm
Outline

1. Model
2. Shadow rate
3. Macroeconomic Implications
4. Conclusion
Bond pricing

Risk-neutral factor dynamics:

\[ X_{t+1} = \mu^Q + \rho^Q X_t + \sum \varepsilon^Q_{t+1}, \quad \varepsilon^Q_{t+1} \sim N(0, I). \]

Pricing kernel

Pricing equation

\[ P^n_t = \mathbb{E}^Q_t[\exp(-r_t - r_{t+1} - \ldots - r_{t+n-1})] \]

Yield

\[ y^n_t = -\frac{1}{n} \log(P^n_t) \]

Forward rate

\[ f_{n,n+1,t} = (n + 1)y_{n+1,t} - ny_{nt} \]
SRTSM and GATSM

SRTSM

\[
\begin{align*}
  r_t &= \max(r, s_t) \\
  s_t &= \delta_0 + \delta_1 X_t
\end{align*}
\]

Forward rate

\[
 f_{SRTSM}^{n,n+1,t} = r + \sigma_n^Q \frac{a_n + b_n X_t - r}{\sigma_n^Q} \left( z \Phi(z) + \phi(z) \right)
\]

where \( g(z) = z \Phi(z) + \phi(z) \)

\[ a_n, b_n \]

GATSM

\[
\begin{align*}
  r_t &= \delta_0 + \delta_1 X_t
\end{align*}
\]

Forward rate

\[
 f_{GATSM}^{n,n+1,t} = a_n + b_n X_t
\]
Property of $g(.)$

\[
\begin{align*}
    f_{SRTSM}^{n,n+1,t} &\approx r, \text{ at the ZLB} \\
    &\approx a_n + b'_n X_t = f_{GATSM}^{n,n+1,t}, \text{ when interest rates are high}
\end{align*}
\]
Model fit

GSW Data: monthly 1990-2013; maturities: 3m, 6m, 1y, 2y, 5y, 7y, 10y
Estimation: Kalman filters
Log likelihood values
- SRTSM: 850; GATSM: 750

Figure: Average forward curve in 2012

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# Approximation error

Average absolute approximation error between 1990M1 and 2013M1

<table>
<thead>
<tr>
<th></th>
<th>3M</th>
<th>6M</th>
<th>1Y</th>
<th>2Y</th>
<th>5Y</th>
<th>7Y</th>
<th>10Y</th>
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</thead>
<tbody>
<tr>
<td>forward rate error</td>
<td>0.01</td>
<td>0.02</td>
<td>0.04</td>
<td>0.13</td>
<td>0.69</td>
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<td>346</td>
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<td>384</td>
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<td>600</td>
<td>636</td>
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<td>yield error</td>
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<td>0.01</td>
<td>0.01</td>
<td>0.04</td>
<td>0.24</td>
<td>0.42</td>
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Summary for unconventional monetary policy?
Yield curve on May 21, 2013
## Hint of tapering (yield)

<table>
<thead>
<tr>
<th>Maturity</th>
<th>3M</th>
<th>1Y</th>
<th>3Y</th>
<th>5Y</th>
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<tbody>
<tr>
<td>May 21, 2013</td>
<td></td>
<td></td>
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<tr>
<td>May 22, 2013</td>
<td></td>
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</table>

May 22: Bernanke tells Congress Fed may decrease the size of monthly large-scale asset purchases
Hint of tapering (forward rate)

May 22: Bernanke tells Congress Fed may decrease the size of monthly large-scale asset purchases
Shift in shadow rate might summarize the effect
Monetary policy

$s_t^o = \begin{cases} 
\text{effective federal funds rate} & \text{before 2009} \\
\text{shadow rate} & \text{since 2009}
\end{cases}$

Can we use shadow rate as similar summary of Fed actions as fed funds rate provided historically?
Factor augmented vector autoregression

Replace the fed funds rate with $s^o_t$ in Bernanke, Boivin, and Eliasz (2005)

$$Y^m_t = a_m + b_x x^m_t + b_s s^o_t + \eta^m_t, \quad \eta^m_t \sim N(0, \Omega)$$

- $Y^m_t$: 97 economic variables from 1960 to 2013
- $x^m_t$: 3 underlying macro factors

Factor dynamics:

$$\begin{bmatrix} x^m_t \\ s^o_t \end{bmatrix} = \begin{bmatrix} \mu^x \\ \mu^s \end{bmatrix} + \begin{bmatrix} \rho^{xx} & \rho^{xs} \\ \rho^{sx} & \rho^{ss} \end{bmatrix} \begin{bmatrix} X^m_{t-1} \\ S^o_{t-1} \end{bmatrix} + \Sigma^m \begin{bmatrix} \varepsilon^m_t \\ \varepsilon_{MP}^t \end{bmatrix}, \quad \begin{bmatrix} \varepsilon^m_t \\ \varepsilon_{MP}^t \end{bmatrix} \sim N(0, I)$$

- monthly VAR(13)
- $\Sigma^m$: Cholesky decomposition
Measures of monetary policy

Can we use shadow rate as similar summary of Fed actions as fed funds rate provided historically?
Measures of monetary policy

Can we use shadow rate as similar summary of Fed actions as fed funds rate provided historically?

Hypothesis I

\[ H_0 : \rho^{xs}(t < \text{Great Recession}) = \rho^{xs}(t > \text{Great Recession}) \]

▶ \( p = 0.29 \) for \( s_t^\circ \)
Measures of monetary policy

Can we use shadow rate as similar summary of Fed actions as fed funds rate provided historically?

Hypothesis I

$H_0 : \rho^{xs}(t < \text{Great Recession}) = \rho^{xs}(t > \text{Great Recession})$

- $p = 0.29$ for $s_t$
- $p = 0.0007$ for EFFR

Implication: researchers can use shadow rate to update earlier studies that had been based on the historical fed funds rate.
Measures of monetary policy

Can we use shadow rate as similar summary of Fed actions as fed funds rate provided historically?

Hypothesis I

\[ H_0 : \rho^{xs}(t < \text{Great Recession}) = \rho^{xs}(t > \text{Great Recession}) \]

- \( p = 0.29 \) for \( s_t^o \)
- \( p = 0.0007 \) for EFFR

Hypothesis II

\[ H_0 : \rho^{sx}(t < \text{Great Recession}) = \rho^{sx}(t > \text{Great Recession}) \]

- \( p = 1 \) for \( s_t^o \)
- \( p = 1 \) for EFFR
Measures of monetary policy

Can we use shadow rate as similar summary of Fed actions as fed funds rate provided historically?

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Implication: researchers can use shadow rate to update earlier studies that had been based on the historical fed funds rate.
Historical decomposition

What if there had been no monetary policy shocks?

- **realized**: $\varepsilon_{t}^{MP} = \hat{\varepsilon}_{t}^{MP}$
- **counterfactual**: $\varepsilon_{t}^{MP} = 0$ for ZLB

Unconventional monetary policy

- reduced the shadow rate by 0.4% between 2011 and 2013.
Historical decomposition

What if there had been no monetary policy shocks?

- **realized:** $\varepsilon^\text{MP}_t = \hat{\varepsilon}^\text{MP}_t$

- **counterfactual:** $\varepsilon^\text{MP}_t = 0$ for ZLB

Unconventional monetary policy

- reduced unemployment by 0.13% in Dec 2013.
Counterfactual II

What if the shadow rate had been kept at $r$?

- **counterfactual**: $\varepsilon_t^{MP}$ is such that $s_t^o = r$ at ZLB

Unconventional monetary policy

- reduced unemployment by 1% in December 2013
Impulse response: full sample

A -25bps monetary policy shock

Graphs showing the impact of a -25bps monetary policy shock on various economic indicators such as policy rate, industrial production index, consumer price index, capacity utilization, unemployment, and housing starts.
Full sample FAVAR(13) vs. ZLB FAVAR(1)

ZLB with effective federal funds rate

Graphs showing policy rate, industrial production index, consumer price index, capacity utilization, unemployment, and housing starts under full sample and ZLB FAVAR(1) models.
Full sample FAVAR(13) vs. ZLB FAVAR(1)

ZLB with shadow rate
**Forward guidance**

**ZLB duration**

\[ \tau_t = \inf \{ \tau_t \geq 0 | s_{t+\tau} \geq r \} . \]
Forward guidance

ZLB duration

\[ \tau_t = \inf \{ \tau_t \geq 0 \mid s_{t+\tau} \geq r \} . \]
Conclusion

Method

- Develop an approximation for bond prices in the SRTSM

Economics

- The shadow rate exhibits similar dynamic correlations with economic variables after the Great Recession as the fed funds rate did earlier in data.

- Unconventional monetary policy lowered the unemployment rate by 0.13% in December 2013.
Wu-Xia Shadow Federal Funds Rate through February 2015

Effective federal funds rate, end-of-month
Wu-Xia shadow rate

Sources: Board of Governors of the Federal Reserve System and Wu and Xia (2014)

Source: www.frbatlanta.org/cqer/researchcq/shadow_rate.cfm
ECB shadow rate
Pricing kernel

Factor dynamics:

\[ X_{t+1} = \mu + \rho X_t + \sum \epsilon_{t+1}, \quad \epsilon_{t+1} \sim N(0, I). \]

Pricing kernel

\[ m_{t+1} = r_t + \frac{1}{2} \lambda_t' \lambda_t + \lambda_t' \epsilon_{t+1}, \]
\[ \lambda_t = \lambda_0 + \lambda_1 X_t \]

where \( \mu^Q = \mu - \Sigma \lambda_0 \), and \( \rho^Q = \rho - \Sigma \lambda_1 \)

Pricing equation

\[ P^n_t = \mathbb{E}_t[\exp(-m_{t+1})P^{n-1}_{t+1}] \]
Bond recursions

\[ a_n = \delta_0 + \delta'_1 \left( \sum_{j=0}^{n-1} (\rho^Q)^j \right) \mu^Q - \frac{1}{2} \delta'_1 \left( \sum_{j=0}^{n-1} (\rho^Q)^j \right) \Sigma \Sigma' \left( \sum_{j=0}^{n-1} (\rho^Q)^j \right) \delta_1, \]

\[ b'_n = \delta'_1 (\rho^Q)^n. \]
Model specification

\[ r = 0.25, \text{ interest rate on reserves} \]

three factors

Normalization: restrict \( Q \) parameters

Repeated eigenvalues

\[
\rho^Q = \begin{bmatrix}
\rho_1^Q & 0 & 0 \\
0 & \rho_2^Q & 1 \\
0 & 0 & \rho_2^Q
\end{bmatrix}.
\]
Kalman filters

State equation

\[ X_{t+1} = \mu + \rho X_t + \Sigma \varepsilon_{t+1}, \varepsilon_{t+1} \sim N(0, I) \]

observation equation for SRTSM \( \Rightarrow \) extended Kalman filter

\[ f_{n,n+1,t}^S = r + \sigma_n^Q g \left( \frac{a_n + b'_n X_t - r}{\sigma_n^Q} \right) + \eta_{nt}, \eta_{nt} \sim N(0, \omega) \]

\[ f_{n,n+1,t}^{SRTSM} \]

observation equation for GATSM \( \Rightarrow \) Kalman filter

\[ f_{n,n+1,t}^G = a_n + b'_n X_t + \eta_{nt}, \eta_{nt} \sim N(0, \omega) \]

\[ f_{n,n+1,t}^{GATSM} \]
## Approximation error for ZLB

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<th>10Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>forward rate error</td>
<td>0.00</td>
<td>0.01</td>
<td>0.06</td>
<td>0.43</td>
<td>2.50</td>
<td>3.51</td>
<td>5.41</td>
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<tr>
<td>forward rate level</td>
<td>23</td>
<td>26</td>
<td>46</td>
<td>111</td>
<td>326</td>
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<td>yield error</td>
<td>0.00</td>
<td>0.00</td>
<td>0.01</td>
<td>0.10</td>
<td>0.91</td>
<td>1.50</td>
<td>2.37</td>
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# Robustness

<table>
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<tr>
<th></th>
<th>$p$-value for $\rho_{1s}^{xs} = \rho_{3s}^{xs}$</th>
<th>$p$-value for $\rho_{1s}^{sx} = \rho_{3s}^{sx}$</th>
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<tbody>
<tr>
<td>Baseline</td>
<td>0.29</td>
<td>1.00</td>
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<tr>
<td>A1</td>
<td>estimate $_r$</td>
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<tr>
<td>A2</td>
<td>2-factor SRTSM</td>
<td>0.97</td>
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<tr>
<td>A3</td>
<td>Fama-Bliss</td>
<td>1.00</td>
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<tr>
<td>A4</td>
<td>5-factor FAVAR</td>
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<tr>
<td>A5</td>
<td>6-lag FAVAR</td>
<td>0.98</td>
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<tr>
<td></td>
<td>7-lag FAVAR</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>12-lag FAVAR</td>
<td>1.00</td>
</tr>
</tbody>
</table>
Historical decomposition

- Policy rate
- Industrial production index
- Consumer price index
- Capacity utilization
- Unemployment
- Housing starts

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Counterfactual II

- Policy rate
- Industrial production index
- Consumer price index
- Capacity utilization
- Unemployment
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Impulse responses: forward guidance

A monetary policy shock to increase the ZLB by 1 year

- ZLB duration
- Industrial production index
- Consumer price index
- Capacity utilization
- Unemployment
- Housing starts
Forward guidance vs. shadow rate

Unemployment rate decreases by 0.25% with

- a one year increase in the expected ZLB duration
- 35 basis-point decrease in the policy rate