# Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound

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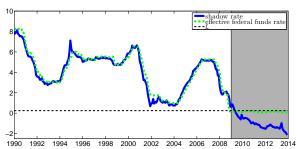
### Key question

What is the macroeconomic impact of monetary policy at the ZLB?

#### Conventional approach before ZLB

VAR with the fed funds rate

But since December 2008, the fed funds rate has been near zero



# Challenges of zero lower bound

#### Challenges

- Conventional monetary policy doesn't work. Fed has implemented unconventional policy tools
  - large-scale asset purchases
  - forward guidance
- What framework to study unconventional monetary policy?
- Gaussian ATSM allows negative interest rates

### Shadow rate term structure model: Black (1995)

- Non-negative short rate:  $r_t = max(\underline{r}, s_t)$
- Analytical solution does not exist in general



### Contributions

### This paper

an analytical approximation for SRTSM



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- an analytical approximation for SRTSM
- shadow rate has similar dynamic correlations with macro variables as the fed funds rate did previously
- our shadow rate updated monthly by Atlanta Fed www.frbatlanta.org/cqer/researchcq/shadow\_rate.cfm

- Shadow rate
- Macroeconomic Implications
- Conclusion

### Bond pricing

Risk-neutral factor dynamics:

$$X_{t+1} = \mu^{\mathbb{Q}} + \rho^{\mathbb{Q}} X_t + \Sigma \varepsilon_{t+1}^{\mathbb{Q}}, \quad \varepsilon_{t+1}^{\mathbb{Q}} \stackrel{\mathbb{Q}}{\sim} N(0, I).$$

Driging kornel

Pricing equation

$$P_t^n = \mathbb{E}_t^{\mathbb{Q}}[\exp(-r_t - r_{t+1} - ... - r_{t+n-1})]$$

Yield

$$y_t^n = -\frac{1}{n}\log(P_t^n)$$

Forward rate

$$f_{n,n+1,t} = (n+1)y_{n+1,t} - ny_{nt}$$

### SRTSM and GATSM

#### **SRTSM**

$$r_t = max(\underline{r}, s_t)$$
  
 $s_t = \delta_0 + \delta_1' X_t$ 

#### Forward rate

$$f_{n,n+1,t}^{SRTSM} = \underline{r} + \sigma_n^{\mathbb{Q}} g\left(\frac{a_n + b_n' X_t - \underline{r}}{\sigma_n^{\mathbb{Q}}}\right)$$

where 
$$g(z) = z\Phi(z) + \phi(z)$$

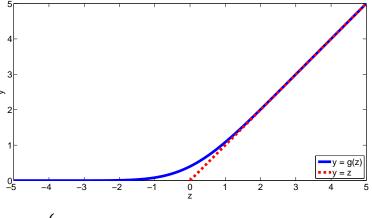
#### **GATSM**

$$r_t = \delta_0 + \delta_1' X_t$$

#### Forward rate

$$f_{n,n+1,t}^{GATSM} = a_n + b_n' X_t.$$

# Property of g(.)



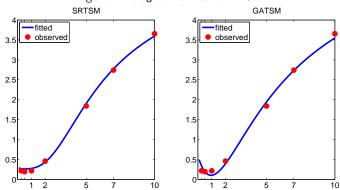
$$f_{n,n+1,t}^{SRTSM}$$
  $\left\{ egin{array}{l} pprox \underline{r}, ext{ at the ZLB} \\ pprox a_n + b_n' X_t = f_{n,n+1,t}^{GATSM}, ext{ when interest rates are high} \end{array} 
ight.$ 

Model

Estimation: Kalman filters details
Log likelihood values specification

► SRTSM: 850; GATSM: 750

Figure: Average forward curve in 2012



# Approximation error

Average absolute approximation error between 1990M1 and 2013M1

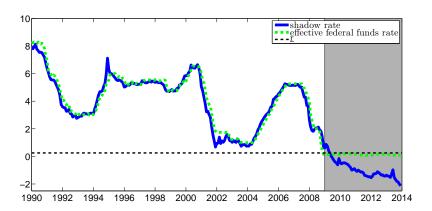
	3M	6M	1Y	2Y	5Y	7Y	10Y
forward rate error forward rate level yield error	0.01	0.02	0.04	0.13	0.69	1.14	2.29
forward rate level	346	357	384	435	551	600	636
yield error	0.00	0.01	0.01	0.04	0.24	0.42	0.78



Model



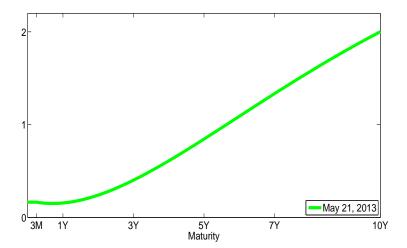
### Shadow rate



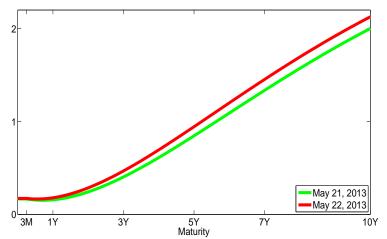
Summary for unconventional monetary policy?



## Yield curve on May 21, 2013



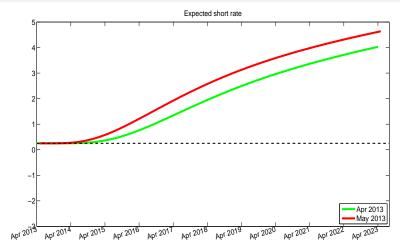
# Hint of tapering (yield)



May 22: Bernanke tells Congress Fed may decrease the size of monthly large-scale asset purchases

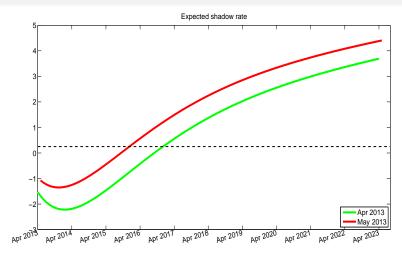


# Hint of tapering (forward rate)



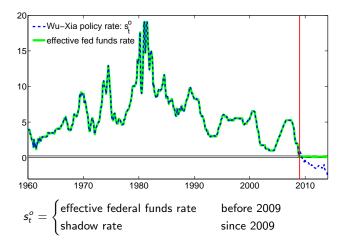
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# Shift in shadow rate might summarize the effect





# Monetary policy



Can we use shadow rate as similar summary of Fed actions as fed funds rate provided historically?

### Factor augmented vector autoregression

Replace the fed funds rate with  $s_t^o$  in Bernanke, Boivin, and Eliasz (2005)

$$Y_t^m = a_m + b_x x_t^m + b_s s_t^o + \eta_t^m, \quad \eta_t^m \sim N(0, \Omega)$$

- $Y_t^m$ : 97 economic variables from 1960 to 2013
- $\triangleright x_t^m$ : 3 underlying macro factors

#### Factor dynamics:

$$\begin{bmatrix} \mathbf{x}_t^m \\ \mathbf{s}_t^o \end{bmatrix} = \begin{bmatrix} \boldsymbol{\mu}^{\mathbf{x}} \\ \boldsymbol{\mu}^{\mathbf{s}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\rho}^{\mathbf{x}\mathbf{x}} & \boldsymbol{\rho}^{\mathbf{x}\mathbf{s}} \\ \boldsymbol{\rho}^{\mathbf{s}\mathbf{x}} & \boldsymbol{\rho}^{\mathbf{s}\mathbf{s}} \end{bmatrix} \begin{bmatrix} \boldsymbol{X}_{t-1}^m \\ \boldsymbol{S}_{t-1}^o \end{bmatrix} + \boldsymbol{\Sigma}^m \begin{bmatrix} \boldsymbol{\varepsilon}_t^m \\ \boldsymbol{\varepsilon}_t^{\mathrm{MP}} \end{bmatrix}, \quad \begin{bmatrix} \boldsymbol{\varepsilon}_t^m \\ \boldsymbol{\varepsilon}_t^{\mathrm{MP}} \end{bmatrix} \sim \textit{N}(\mathbf{0}, \textit{I})$$

- monthly VAR(13)
- $\triangleright$   $\Sigma^m$ : Cholesky decomposition



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#### Hypothesis I

$$H_0: \rho^{xs}(t < \mathsf{Great} \; \mathsf{Recession}) = \rho^{xs}(t > \mathsf{Great} \; \mathsf{Recession})$$

▶ 
$$p = 0.29 \text{ for } s_t^o$$



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- p = 0.0007 for EFFR

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#### Hypothesis II

$$H_0: \rho^{sx}(t < \text{Great Recession}) = \rho^{sx}(t > \text{Great Recession})$$

- p=1 for  $s_t^o$
- p = 1 for EFFR

Can we use shadow rate as similar summary of Fed actions as fed funds rate provided historically?

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Implication: researchers can use shadow rate to update earlier studies that had been based on the historical fed funds rate. Robustness

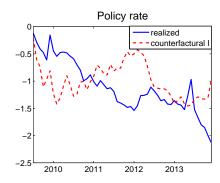
# Historical decomposition

#### What if there had been no monetary policy shocks?

- ightharpoonup realized:  $\varepsilon_t^{\mathrm{MP}} = \hat{\varepsilon}_t^{\mathrm{MP}}$
- counterfactual:  $\varepsilon_t^{MP} = 0$  for ZLB

#### Unconventional monetary policy

reduced the shadow rate by 0.4% between 2011 and 2013.





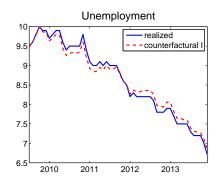
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#### Unconventional monetary policy

▶ reduced unemployment by 0.13% in Dec 2013. ▶ More





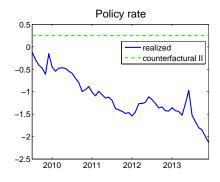
### Counterfactual II

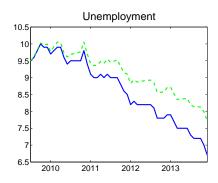
#### What if the shadow rate had been kept at r?

• counterfactual:  $\varepsilon_t^{\text{MP}}$  is such that  $s_t^o = \underline{r}$  at ZLB

#### Unconventional monetary policy

reduced unemployment by 1% in December 2013 More

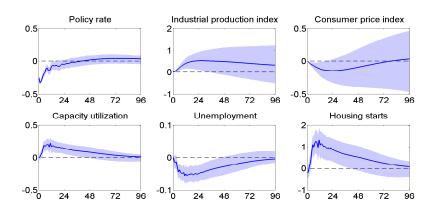






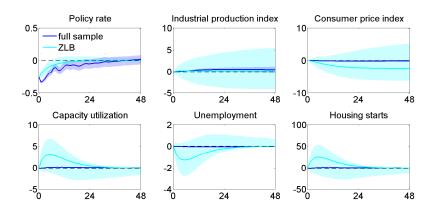
### Impulse resposne: full sample

### A -25bps monetary policy shock



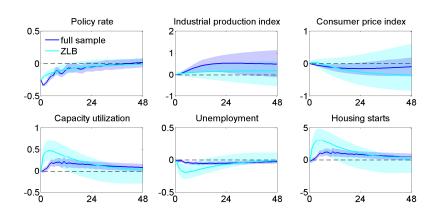
# Full sample FAVAR(13) vs. ZLB FAVAR(1)

#### ZLB with effective federal funds rate



# Full sample FAVAR(13) vs. ZLB FAVAR(1)

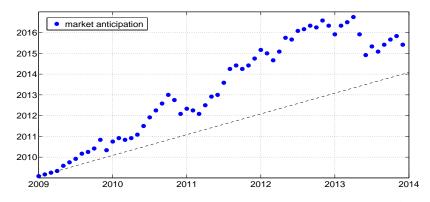
#### ZLB with shadow rate



# Forward guidance

#### ZLB duration

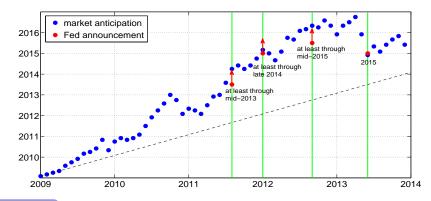
$$\tau_t = \inf\{\tau_t \ge 0 | s_{t+\tau} \ge \underline{r}\}.$$



### Forward guidance

#### ZLB duration

$$\tau_t = \inf\{\tau_t \ge 0 | s_{t+\tau} \ge \underline{r}\}.$$



### Conclusion

#### Method

Develop an approximation for bond prices in the SRTSM

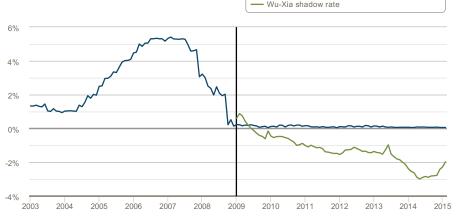
#### **Economics**

- The shadow rate exhibits similar dynamic correlations with economic variables after the Great Recession as the fed funds rate did earlier in data.
- Unconventional monetary policy lowered the unemployment rate by 0.13% in December 2013.

#### Wu-Xia Shadow Federal Funds Rate

through February 2015

- Effective federal funds rate, end-of-month - Wu-Xia shadow rate

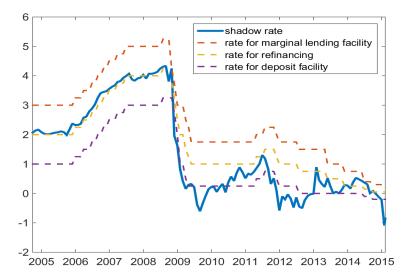


Sources: Board of Governors of the Federal Reserve System and Wu and Xia (2014)

Source: www.frbatlanta.org/cqer/researchcq/shadow\_rate.cfm



### ECB shadow rate



### Pricing kernel

Factor dynamics:

$$X_{t+1} = \mu + \rho X_t + \Sigma \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, I).$$

Pricing kernel

$$m_{t+1} = r_t + \frac{1}{2}\lambda_t'\lambda_t + \lambda_t'\varepsilon_{t+1}$$
$$\lambda_t = \lambda_0 + \lambda_1 X_t$$

where  $\mu^{\mathbb{Q}} = \mu - \Sigma \lambda_0$ , and  $\rho^{\mathbb{Q}} = \rho - \Sigma \lambda_1$ 

Pricing equation

$$P_t^n = \mathbb{E}_t[\exp(-m_{t+1})P_{t+1}^{n-1}]$$



### Bond recursions

$$a_{n} = \delta_{0} + \delta'_{1} \left( \sum_{j=0}^{n-1} \left( \rho^{\mathbb{Q}} \right)^{j} \right) \mu^{\mathbb{Q}} - \frac{1}{2} \delta'_{1} \left( \sum_{j=0}^{n-1} \left( \rho^{\mathbb{Q}} \right)^{j} \right) \Sigma \Sigma' \left( \sum_{j=0}^{n-1} \left( \rho^{\mathbb{Q}} \right)^{j} \right)' \delta_{1},$$

$$b'_{n} = \delta'_{1} \left( \rho^{\mathbb{Q}} \right)^{n}.$$



### Model specification

r = 0.25, interest rate on reserves

three factors

Normalization: restrict Q parameters

Repeated eigenvalues

$$ho^{\mathbb{Q}} = egin{bmatrix} 
ho_1^{\mathbb{Q}} & 0 & 0 \ 0 & 
ho_2^{\mathbb{Q}} & 1 \ 0 & 0 & 
ho_2^{\mathbb{Q}} \end{bmatrix}.$$



### Kalman filters

State equation

$$X_{t+1} = \mu + \rho X_t + \Sigma \varepsilon_{t+1}, \varepsilon_{t+1} \sim N(0, I)$$

observation equation for SRTSM  $\Rightarrow$  extended Kalman filter

$$f_{n,n+1,t}^{o} = \underbrace{\underline{r} + \sigma_{n}^{\mathbb{Q}} g\left(\frac{a_{n} + b_{n}' X_{t} - \underline{r}}{\sigma_{n}^{\mathbb{Q}}}\right)}_{f_{n,n+1,t}^{SRTSM}} + \eta_{nt}, \eta_{nt} \sim N(0,\omega)$$

observation equation for GATSM ⇒ Kalman filter

$$f_{n,n+1,t}^{o} = \underbrace{a_n + b_n' X_t}_{f_{n,n+1}^{GATSM}} + \eta_{nt}, \eta_{nt} \sim N(0,\omega)$$



### Approximation error for ZLB

Average absolute approximation error between 2009M1 and 2013M1

	3M	6M	1Y	2Y	5Y	7Y	10Y
forward rate error forward rate level yield error	0.00	0.01	0.06	0.43	2.50	3.51	5.41
forward rate level	23	26	46	111	326	418	481
yield error	0.00	0.00	0.01	0.10	0.91	1.50	2.37

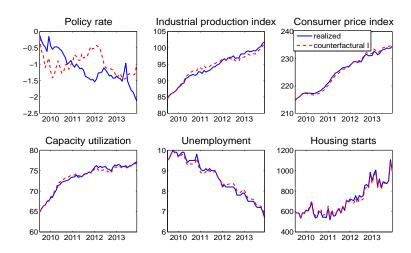


### Robustness

		$\mid$ <i>p</i> -value for $ ho_1^{xs} =  ho_3^{xs}$	$\mid$ <i>p</i> -value for $ ho_1^{ extsf{sx}}= ho_3^{ extsf{sx}}$
	Baseline	0.29	1.00
A1	estimate <u>r</u>	0.18	1.00
A2	2-factor SRTSM	0.13	0.97
A3	Fama-Bliss	0.38	1.00
A4	5-factor FAVAR	0.70	1.00
A5	6-lag FAVAR	0.09	0.98
	7-lag FAVAR	0.19	0.97
	12-lag FAVAR	0.22	1.00

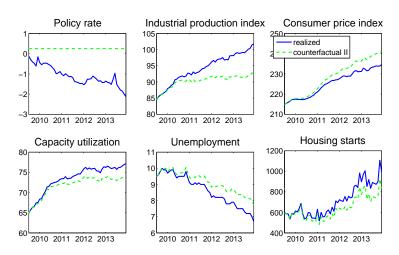


### Historical decomposition





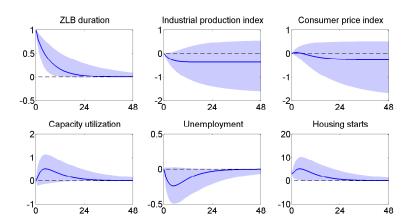
### Counterfactual II



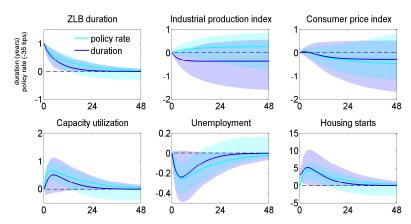


### Impulse responses: forward guidance

A monetary policy shock to increase the ZLB by 1 year



### Forward guidance vs. shadow rate



Unemployment rate decreases by 0.25% with

- a one year increase in the expected ZLB duration
- ▶ 35 basis-point decrease in the policy rate ▶ Back □