Reconciling Hayek's and Keynes' views of recessions

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Abstract

Recessions often happen after periods of rapid accumulation of houses, consumer durables and business capital. This observation has led some economists, most notably Friedrich Hayek, to conclude that recessions mainly reflect periods of needed liquidation resulting from past over-investment. According to the main proponents of this view, government spending should not be used to mitigate such a liquidation process, as doing so would simply result in a needed adjustment being postponed. In contrast, ever since the work of Keynes, many economists have viewed recessions as periods of deficient demand that should be countered by activist fiscal policy. In this paper we reexamine the liquidation perspective of recessions in a setup where prices are flexible but where not all trades are coordinated by centralized markets. We show why and how liquidations can produce periods where the economy functions particularly inefficiently, with many socially desirable trades between individuals remaining unexploited when the economy inherits too many capital goods. In this sense, our model illustrates how liquidations can cause recessions characterized by deficient aggregate demand and accordingly suggests that Keynes' and Hayek's views of recessions may be much more closely linked than previously recognized. In our framework, interventions aimed at stimulating aggregate demand face the trade-off emphasized by Hayek whereby current stimulus mainly postpones the adjustment process and therefore prolongs the recessions. However, when examining this trade-off, we find that some stimulative policies may nevertheless remain desirable even if they postpone a recovery.

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1 Introduction

There remains considerable debate regarding the causes and consequences of recessions. Two views that are often presented as opposing, and which created controversy in the recent recession and its aftermath, are those associated with the ideas of Hayek and Keynes.[1] The Hayekian perspective is generally associated with viewing recessions as a necessary evil. According to this view, recessions mainly reflect periods of liquidation resulting from past over-accumulation of capital goods. A situation where the economy needs to liquidate such an excess can quite naturally give rise to a recession, but government spending aimed at stimulating activity, it is argued, is not warranted since it would mainly delay the needed adjustment process and thereby postpone the recovery. In contrast, the Keynesian view suggests that recessions reflect periods of deficient aggregate demand where the economy is not effectively exploiting the gains from trade between individuals. According to this view, policy interventions aimed at increasing investment and consumption are generally desirable, as they favor the resumption of mutually beneficial trade between individuals.[2]

In this paper we reexamine the liquidationist perspective of recessions in an environment with decentralized markets, flexible prices and search frictions. In particular, we examine how the economy adjusts when it inherits from the past an excessive amount of capital goods, which could be in the form of houses, durable goods or productive capital. Our goal is not to focus on why the economy may have over-accumulated in the past,[3] but to ask how it reacts to such an over-accumulation once it is realized. As suggested by Hayek, such a situation can readily lead to a recession as less economic activity is generally warranted when agents want to deplete past over-accumulation. However, because of the endogenous emergence of unemployment risk in our set-up, the size and duration of the recession implied

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[1] In response to the large recession in the US and abroad in 2008-2009, a high-profile debate around these two views was organized by Reuters. See `http://www.reuters.com/subjects/keynes-hayek`. See also Wapshott [2012] for a popular account of the Hayek-Keynes controversy.

[2] See Caballero and Hammour [2004] for an alternative view on the inefficiency of liquidations, based on the reduction of cumulative reallocation and inefficient restructuring in recessions.

[3] There are several reason why an economy may over-accumulate capital. For example, agents may have had overly optimistic expectations about future expected economic growth that did not materialize, as in Beaudry and Portier [2004], or it could have been the case that credit supply was unduly subsidized either through explicit policy, as argued in Mian and Sufi [2010] and Mian, Sufi, and Trebbi [2010], or as a by-product of monetary policy, as studied by Bordo and Landon-Lane [2013].

by the need for liquidation is not socially optimal. In effect, the reduced gains from trade induced by the need for liquidation creates a multiplier process that leads to an excessive reduction in activity. Although prices are free to adjust, the liquidation creates a period of deficient aggregate demand where economic activity is too low because people spend too cautiously due to increased unemployment risk. In this sense, we argue that liquidation and deficient aggregate demand should not be viewed as alternative theories of recessions but instead should be seen as complements, where past over-accumulation may be a key driver of periods of deficient aggregate demand. This perspective also makes salient the trade-offs faced by policy. In particular, a policy-maker in our environment faces an unpleasant trade-off between the prescriptions emphasized by Keynes and Hayek. On the one hand, a policy-maker would want to stimulate economic activity during a liquidation-induced recession because precautionary savings is excessively high. On the other hand, the policy-maker also needs to recognize that intervention will likely postpone recovery, since it slows down the needed depletion of excess capital. The model offers a simple framework where both of these forces are present and can be compared.

On a more general note, one of the contributions of this paper is to show why an economy can function quite efficiently in growth periods when it is far from its steady state, while simultaneously functioning particularly inefficiently when it is going through a liquidation phase near its steady state. When the economy is far from its steady-state level of capital, demand for capital is very strong and unemployment risk is therefore minimal. In contrast, when there is excessive capital, we show that reduced labor demand shows up at least in part as increased unemployment even if workers and firms bargain pair-wise efficiently on wages and hours-worked. The increased unemployment risk then causes households to increase precautionary savings, which in turn amplifies the initial fall in output and employment. The result is an over-reaction to the initial impetus induced by a need to liquidate capital.[4] As a presentation device, we show how this process can be represented on a diagram somewhat similar to a Keynesian cross, but where the micro-foundation and many comparative statics

[4] It is now common in the macroeconomic literature to summarize the functioning of model by indicating where and how they create distortions or wedges, as exemplified by Chari, Kehoe, and McGrattan [2007]. Accordingly, one way to view the working of our model economy is as generating an endogenous labor market wedge driven by unemployment risk, where the size of the wedge reacts to the extent to which inherited capital is above or below the steady state.

differ substantially from the sticky-price interpretation commonly used to discuss multipliers. Moreover, by clarifying why this process does not depend on sticky prices, our analysis suggest that monetary policy may be of limited help in addressing the difficulties associated with a period of liquidation.

One potential criticism of a pure liquidationist view of recessions is that, if markets functioned efficiently, such periods should not be socially painful. In particular, if economic agents interact in perfect markets and realize they have over-accumulated in the past, this should lead them to enjoy a type of holiday paid for by their past excessive work. Looking backwards in such a situation, agents may resent the whole episode, but looking forward after a period of over-accumulation, they should nonetheless feel content to enjoy the proceeds of the pass excessive work, even if it is associated with a recession. In contrast, in our environment we will show that liquidation periods are generally socially painful because of the multiplier process induced by precautionary savings and unemployment risk. In effect, we will show that everyone in our model economy can be worse off when they inherit too many capital goods from the past. This type of effect, whereby abundance creates scarcity, may appear quite counter-intuitive at first pass. To make as clear as possible the mechanism that can cause welfare to be reduced by such abundance, much of our analysis will focus on the case where the inherited capital takes the form of a good that directly contributes to utility, such as houses or durable goods. In this situation we will show why inheriting more houses or durables can make everyone worse off. However, as we shall show, this result is a local result that is most likely to be present around an economy's steady state. In contrast, if we were to destroy all capital goods in our model economy, this would always reduce welfare, as the direct effects on utility would out-weight the inefficiencies induced by unemployment risk. Accordingly, our model has the characteristic that behavior can be quite different when it inherits a large or small amount of capital from the past.

The structure of our model builds on the literature related to search models of decentralized trading. In particular, we share with Lucas [1990] and Shi [1998] a model in which households are composed of agents that act in different markets without full coordination. Moreover, as in Lagos and Wright [2005] and Rocheteau and Wright [2005], we exploit alternating decentralized and centralized markets to allow for a simple characterization of the equilibrium. However, unlike those papers, we do not have money in our setup. The paper also shares key features with the long tradition of macro models emphasizing strategic complementarities, aggregate demand externalities and multipliers, such as Diamond [1982] and Cooper and John [1988], but we do not emphasize multiple equilibrium. Instead we focus on situations where the equilibrium remains unique, which allows standard comparative statics exercises to be conducted without needing to worry about equilibrium-selection issues. The multiplier process derived in the paper therefore shares similarities with that found in the recent literature with strategic complementarities such as Angeletos and La'O [2013], in the sense that it amplifies demand shocks. However, the underlying mechanism in this paper is very different, operating through unemployment risk rather than through direct demand complementarities as in Angeletos and La'O [2013].

Unemployment risk and its effects on consumption decisions is at the core of our model. The empirical relevance of precautionary saving related to unemployment risk has been documented by many, starting with Carroll [1992]. For example, Carroll and Dunn [1997] have shown that expectations of unemployment are robustly and negatively correlated with every measure of consumer expenditure (non-durable goods, durable goods and home sales). Carroll, Sommer, and Slacalek [2012] confirm this finding and show why business cycle fluctuations may be driven to a large extent by changes in unemployment uncertainty. Alan, Crossley, and Low [2012] use U.K. micro data to show that increases in saving rates in recessions appear largely driven by uncertainty related to unemployment.[5] There are also recent theoretical papers that emphasized how unemployment risk and precautionary savings can amplify shocks and cause business cycle fluctuations. These papers are the closest to our work. In particular, our model structure is closely related to that presented in Guerrieri and Lorenzoni [2009]. However, their model emphasizes why the economy may exhibit excessive responses to productivity shocks, while our framework offers a mechanism that amplifies demand-type shocks. Our paper also shares many features with Heathcote and Perri [2012], who develop a model in which unemployment risk and wealth impact consumption decisions and precautionary savings. Wealth matters in their setup because of financial frictions that

[5] Using these empirical insights, Challe and Ragot [2013] have recently proposed a tractable quantitative model in which uninsurable unemployment risk is the source of wealth heterogeneity.

make credit more expensive for wealth-poor agents. They obtain a strong form of demand externality that gives rise to multiple equilibria and, accordingly, they emphasize self-fulfilling cycles as the important source of fluctuations. [6] Finally, the work by Ravn and Sterk [2012] emphasizes as we do how unemployment risk and precautionary savings can amplify demand shocks, but their mechanism differs substantially from ours since it relies on sticky nominal prices.

While the main mechanism in our model has many precursors in the literature, we believe that our setup illustrates most clearly *(i)* how unemployment risk gives rise to a multiplier process for demand shocks even in the absence of price stickiness or increasing returns, *(ii)* how this multiplier process can be ignited by periods of liquidation, and *(iii)* how fiscal policy can and cannot be used to counter the process.

The remaining sections of the paper are structured as follows. In Section 2, we present a static model where agents inherit from the past different levels of capital goods, and we describe how and why high values of inherited capital can lead to poor economic outcomes. The static setup allows for a clear exposition of the nature of the demand externality that arises in our setting with decentralized trade. We focus on the case where the inherited capital is in the form of a good which directly increases utility so as to make clear how more goods can reduce welfare. In Section 3, we discuss a set of extensions, including a discussion of the case where the inherited capital takes the form of a productive good. In Section 4, we extend the model to an infinite-period dynamic setting. We take particular care in contrasting the behavior of the economy when it is close to and far from its steady state. Finally, in Section 5, we discuss the trade-offs faced by a policy-maker when inheriting an excessive amount of capital from the past, while Section 6 concludes.

[6] The existence of aggregate demand externalities and self-fulfilling expectations is also present in the work of Farmer [2010] and in the work of Chamley [2014]. In a model with search in both labor and goods markets, Kaplan and Menzio [2013] also obtain multiple equilibria, as employed workers have more income to spend and less time to shop for low prices. As already underlined, and contrarily to those studies, our analysis is restricted to configurations in which the equilibrium is unique.

2 Static model

In this section, we present a very stripped-down static model in order to illustrate why an economy may function particularly inefficiently when it inherits a large stock of capital from the past. In particular, we will want to make clear why agents in an economy can be worse off when inherited capital goods are too high. For the mechanism to be as transparent as possible, we focus mainly on the case where the inherited capital produces services which directly enter agents' utility functions. Accordingly, this type of capital can be considered as representing houses or other durable consumer goods. In a later section, we will discuss how the analysis carries over to the case of productive capital.

In our model, trades are decentralized, and there are two imperfections which cause unemployment risk to emerge and generate precautionary savings behavior. First, there will be a matching friction in the spirit of Diamond-Mortensen-Pissarides, which will create the possibility that a household may not find employment when looking for a job. Second, there will be adverse selection in the insurance market that will limit the pooling of this risk. Since the adverse selection problem can be analyzed separately, we will begin the presentation by simply assuming that unemployment insurance is not available. Later we will introduce the adverse selection problem which rationalizes this missing market, and show that all main results are maintained. The key exogenous variable in the static model will be a stock of consumer durables that households inherit from the past. Our goal is to show why and when high values of this stock can cause the economy to function inefficiently and possibly even cause a decrease in welfare. We will also explore the role of governement spending in affecting economic activity in our setup.

2.1 Setup

Consider an environment populated by a mass L of households indexed by j. In this economy there are two sub-periods. In the first sub-period, households buy good 1, which we will call clothes, and try to find employment in the clothing sector. We refer to this good as clothes since in the dynamic version of the model it will represent a partially durable good. The good produced in the second sub-period, good 2, will be referred to as household services since it will have no durability. As there is no money in this economy, when the household buys clothes its bank account is debited, and when (and if) it receives employment income its bank account is credited. Then, in the second sub-period, households balance their books by repaying any outstanding debts or receiving a payment for any surplus. These payments are made in terms of good 2, which is also the numeraire in this economy.[7]

Preferences for the first sub-period are represented by

$$U(c_j) - \nu(\ell_j)$$

where c represents consumption of clothes and ℓ is the labor supplied by households in the production of clothes. The function $U(\cdot)$ is assumed to be increasing in c and strictly concave with $\lim_{c\to\infty} U' \leq 0$ and $U''' > 0$. The dis-utility of work function $\nu(\cdot)$ is assumed to be increasing and convex in ℓ, with $\nu(0) = 0$. The agents are initially endowed with X_j units of clothes, which they can either consume or trade. We assume symmetric endowments, so that $X_j = X\ \forall j$.[8] In the dynamic version of the model, X will represent the stock of durable goods and will be endogenous.

Trade in clothing will be subject to a coordination problem because of frictions in the labor market. At the beginning of the first sub-period, the household splits up responsibilities between two members. The first member, called the buyer, goes to the clothes market to make purchases. The second member searches for employment opportunities in the labor market. The market for clothes functions in a Walrasian fashion, with both buyers and firms that sell clothes taking prices as given. The market for labor in this first sub-period is subject to a matching friction, with sellers of labor searching for employers and employers searching for labor. The important information assumption is that buyers do not know, when choosing their consumption of clothes, whether the worker member of the household has secured a match. This assumption implies that buyers will worry about unemployment risk when making purchases of clothes.

There is a large set of potential clothes firms in the economy who can decide to search for workers in view of supplying clothes to the market. Each firm can hire one worker and

[7] We remain agnostic about the precise details of how good 2 is produced for the time being. One possible interpretation is discussed in the following sub-section.

[8] In what follows, we will drop the j index except where doing so may cause confusion.

has access to a decreasing-returns-to-scale production function $\theta F(\ell)$, where ℓ is the number of hours worked for the firm and $\theta > 0$ is a technology shift factor. Production also requires a fixed cost $\theta\Phi$ in terms of the output good, so that the net production of a firm hiring ℓ hours of labor is $\theta\left[F(\ell) - \Phi\right]$. For now, we will normalize θ to 1, and will reintroduce θ in its general form when we want to talk about the effects of technological change and balanced growth. We will also assume throughout that $\Omega(\ell) \equiv F'(\ell)\ell$ is increasing in ℓ.[9] Moreover, we will assume that Φ is sufficiently small such that there exists an $\ell^\star > 0$ satisfying $F\left(\ell^\star\right) - F'\left(\ell^\star\right)\ell^\star = \Phi$. These restrictions on the production technology are always satisfied if, for example, $F(\ell) = \ell^\alpha$, with $0 < \alpha < 1$.

Firms search for workers and, upon finding a worker, they jointly decide on the number of hours worked and on the wage to be paid. The fixed cost Φ is paid before firms can look for workers. Upon a match, the determination of the wage and hours-worked within a firm is done efficiently though a competitive bargaining process. In effect, upon a match, one can view a Walrasian auctioneer as calling out a wage w that equilibrates the demand for and supply of labor among the two parties in the match. Assuming such a process for wage and employment determination has the feature of limiting within-pair distortions that could muddle the understanding of the main mechanisms of the model. In Appendix B we show that the main results of the paper are robust to alternative bargaining protocols. Note that we have deliberately chosen a random-matching – rather than a directed-search – framework as we want to illustrate how inefficiencies in the labor market can interact with the liquidation process to create periods of deficient aggregate demand. Given the wage, the demand for labor from the firm is described by the marginal productivity condition

$$pF'(\ell) = w$$

where p is the relative price of clothes in terms of the non-durable good produced in the second sub-period.[10] The supply of labor is chosen optimally by the worker in a manner to be derived shortly.

[9] Because we assume free-entry for clothes firms, the quantity $\theta\Omega(\ell)$ will equal net output of clothes (after subtracting firms' fixed costs) by a single employed worker. The assumption that this quantity is increasing in ℓ is satisfied, for example, if F is a CES combination of labor and some other input in fixed supply, with an elasticity of substitution between these inputs of at least 1, which nests the case where F is Cobb-Douglas.

[10] As will become clear, p can be given an interpretation as an interest rate.

Letting N represent the number of firms who decide to search for workers, the number of matches is then given by the constant-returns-to-scale matching function $M(N,L)$, with $M(N,L) \leq \min\{N,L\}$. The equilibrium condition for the clothes market is given by

$$L \cdot (c - X) = M(N,L)F(\ell) - N\Phi$$

where the left-hand side is total purchases of new clothes and the right-hand side is the total available supply after subtracting search costs.

Firms will enter the market up to the point where expected profits are zero. The zero-profit condition can be written as[11]

$$\frac{M}{N}[pF(\ell) - w\ell] = \frac{M}{N}[pF(\ell) - pF'(\ell)\ell] = p\Phi$$

At the end of the first sub-period, household j's net asset position a_j, expressed in units of good 2, is given by $w\ell_j - p(c_j - X)$. We model the second sub-period so that it is costly to arrive in that sub-period with debt. For now, we can simply denote the value of entering the second sub-period with assets a_j by $V(a_j)$, where we assume that $V(\cdot)$ is increasing, with $V'(a_1) > V'(a_2)$ whenever $a_1 < 0 < a_2$; that is, we are assuming that the marginal value of a unit of assets is greater if one is in debt than if one is in a creditor position. In the following sub-section we specify preferences and a market structure for the second sub-period that rationalizes this $V(\cdot)$ function.

Taking the function $V(a)$ as given, we can specify the household's consumption decision as well as his labor-supply decision conditional on a match. The buyer's problem in household j is given by

$$\max_{c_j} U(c_j) + \mu V(w\ell_j - p(c_j - X)) + (1-\mu)V(-p(c_j - X))$$

where μ is the probability that a worker finds a job and is given by $\mu \equiv M(N,L)/L$. From this expression, we can see that the consumption decision is made in the presence of unemployment risk.

[11] We assume that searching firms pool their ex-post profits and losses so that they make exactly zero profits in equilibrium, regardless of whether they match.

The worker's problem in household j when matched, taking w as given, can be expressed as choosing a level of hours to supply in the first sub-period so as to solve

$$\max_{\ell_j} -\nu(\ell_j) + V(w\ell_j - p(c_j - X))$$

2.2 Deriving the value function $V(a)$

$V(a)$ represents the value function associated with entering the second sub-period with a net asset position a. In this subsection, we derive such a value function by specifying primitives in terms of preferences, technology and market organization. We choose to model this sub-period in such a way that if there were no friction in the first sub-period, there would be no trade between agents in the second sub-period. For this reason let us call "services" the good produced in the second period household, with preferences given by

$$\widetilde{U}(\widetilde{c}) - \widetilde{\nu}(\widetilde{\ell})$$

where $\widetilde{c}$ is consumption of these services, $\widetilde{U}(\cdot)$ is increasing and strictly concave in $\widetilde{c}$, $\widetilde{\ell}$ is the labor used to produced household services, and $\widetilde{\nu}(\cdot)$ is increasing and convex in $\widetilde{\ell}$.

To ensure that a unit of net assets is more valuable when in debt than when in surplus, let us assume that households in the second sub-period can produce services for their own consumption, using one unit of labor to produce $\widetilde{\theta}$ unit of services. However, if a household in the second sub-period has to produce market services – that is, services that can be sold to others in order to satisfy debt – then to produce $\widetilde{\theta}$ units of market services requires them to supply $1 + \tau$ units of labor, $\tau > 0$. To simplify notation, we can set $\widetilde{\theta} = 1$ for now and return to the more general formulation when talking about effects of technological change. The continuation value function $V(a)$ can accordingly be defined as

$$V(a) = \max_{\widetilde{c},\widetilde{\ell}} \widetilde{U}(\widetilde{c}) - \widetilde{\nu}(\widetilde{\ell})$$

subject to

$$\widetilde{c} = \widetilde{\ell} + a \text{ if } a \geq 0$$

and

$$\widetilde{c} = \widetilde{\ell} + a(1 + \tau) \text{ if } a < 0$$

It is easy to verify that $V(a)$ is increasing in assets and concave. If $\widetilde{\nu}(\widetilde{\ell})$ is strictly convex, then $V(a)$ will be strictly concave, regardless of the value of τ, with the key property that $V'(a_1) > V'(a_2)$ if $a_1 < 0 < a_2$; that is, the marginal value of an increase in assets is greater if one is in debt than if one is in surplus.[12] In the case where $\widetilde{\nu}(\ell)$ is linear, then $V(a)$ will be piecewise linear and will not be differentiable at zero. Nonetheless, it will maintain the key property that $V'(a_1) > V'(a_2)$ if $a_1 < 0 < a_2$. We will mainly work with this case, and in particular, will assume that $\widetilde{\nu}(\widetilde{\ell}) = v \cdot \widetilde{\ell}$, which implies that $V(a)$ is piecewise linear with a kink at zero.

2.3 Equilibrium in the first sub-period

Given the function $V(a)$, a symmetric equilibrium for the first sub-period is represented by five objects: two relative prices (the price of clothing p and the wage rate w), two quantities (consumption of clothes by each household c and the amount worked in each match ℓ), and a number N of active firms, such that

1. c solves the buyer's problem taking μ, p, w and ℓ as given.
2. The labor supply ℓ solves the worker's problem conditional on a match, taking p, w and c as given.
3. The demand for labor ℓ maximizes the firm's profits given a match, taking p and w as given.
4. The goods market clears; that is, $L \cdot (c - X) = M(N, L)F(\ell) - N\Phi$.
5. Firms' entry decisions ensure zero profits.

[12] To avoid backward-bending supply curves, we will also assume that $\widetilde{\nu}(\cdot)$ and $\widetilde{U}(\cdot)$ are such that $V'''(a) \geq 0$. This assumption is sufficient but not necessary for later results. Note that a sufficient condition for $V'''(a) \geq 0$ is that both $\widetilde{U}'''(\cdot) \geq 0$ and $\widetilde{\nu}'''(\cdot) < 0$.

The equilibrium in the first sub-period can therefore be represented by the following system of five equations:

$$U'(c) = p\Bigg\{\frac{M(N,L)}{L}V'\left(w\ell - p\left(c-X\right)\right) + \left[1-\frac{M(N,L)}{L}\right]V'\left(-p\left(c-X\right)\right)\Bigg\} \tag{1}$$

$$\nu'(\ell) = V'\left(w\ell - p\left(c-X\right)\right)w \tag{2}$$

$$pF'(\ell) = w \tag{3}$$

$$M(N,L)F(\ell) = L(c-X) + N\Phi \tag{4}$$

$$M(N,L)[pF(\ell) - w\ell] = Np\Phi \tag{5}$$

In the above system,[13] equations (1) and (2) represent the first-order conditions for the household's choice of consumption and supply of labor. Equations (3) and (5) represent a firm's labor demand condition and its entry decision. Finally, (4) is the goods market clearing condition.

At this level of generality it is difficult to derive many results. Nonetheless, we can combine (1), (2) and (3) to obtain the following important expression regarding a characteristic of the equilibrium,

$$\frac{\nu'(\ell)}{U'(c)}\left\{1+(1-\mu)\left[\frac{V'\left(-p\left(c-X\right)\right)}{V'\left(w\ell-p\left(c-X\right)\right)}-1\right]\right\} = F'(\ell) \tag{6}$$

From equation (6), we see that as long as $\mu < 1$, the marginal rate of substitution between leisure and consumption will not be equal to the marginal productivity of work; that is, the labor market will exhibit a wedge given by

$$(1-\mu)\left[\frac{V'\left(-p\left(c-X\right)\right)}{V'\left(w\ell-p\left(c-X\right)\right)}-1\right]$$

In fact, in this environment, the possibility of being unemployed leads to precautionary savings, which in turn causes the marginal rate of substitution between leisure and consumption to be low relative to the marginal productivity of labor. As we will see, changes in X will cause this wedge to vary, which will cause a feedback effect on economic activity. Obviously,

[13] To ensure that an employed worker's optimal choice of labor is strictly positive, we assume that $\lim_{c\to 0} U'(c) > \lim_{\ell\to 0}\frac{\nu'(\ell)}{F'(\ell)}$.

in this environment there would be a desire for agents to share the risk of being unemployed, which could reduce or even eliminate the wedge. As noted earlier, the reason that this type of insurance may be limited is the presence of adverse selection, an issue to which we will return.

Our main goal now is to explore the effects of changes in X on equilibrium outcomes. In particular, we are interested in clarifying why and when an increase in X can actually lead to a reduction in consumption and/or welfare. The reason we are interested in this comparative static is that we are interested in knowing why periods of liquidations – that is, periods where agents inherit excessive levels of durable goods from the past – may be socially painful.

To clarify the analysis, we will make two simplifying assumptions. First, we will assume that the matching function takes the form $M(N, L) = \min\{N, L\}$; that is, matches are determined by the short side of the market. This assumption creates a clear and useful dichotomy, with the economy characterized as being either in an unemployment regime if $L > N$ or in a full-employment regime if $N > L$. We will also assume that $V(a)$ is piece-wise linear, with $V'(a) = v \cdot a$ if $a \geq 0$ and $V'(a) = v \cdot a \cdot (1 + \tau)$ if $a < 0$, with $\tau > 0$ and $v > 0$. This form of the $V(\cdot)$ function corresponds to the case discussed in subsection 2.2 where the dis-utility of work in the second sub-period is linear. The important element here is τ. In effect, $1 + \tau$ represents the ratio of the marginal value of an extra unit of assets when one is in debt relative to its value when one is in surplus. A value of $\tau > 0$ can be justified in many ways, one of which is presented in subsection 2.2. Alternatively, $\tau > 0$ could reflect a financial friction related to the cost of borrowing versus savings.

Under these two functional-form assumptions, the equilibrium conditions can be reduced to the following:

$$U'(c) = \frac{\nu'(\ell)}{F'(\ell)}\left(1 + \tau - \frac{\min\{N, L\}}{L}\tau\right) \tag{7}$$

$$\frac{\min\{N, L\}}{L} = \frac{c - X}{F'(\ell)\ell} \tag{8}$$

$$\frac{\min\{N, L\}}{N}[F(\ell) - F'(\ell)\ell] = \Phi \tag{9}$$

$$w = \frac{\nu'(\ell)}{v} \tag{10}$$

$$p = \frac{\nu'(\ell)}{vF'(\ell)} \tag{11}$$

This system of equations now has the feature of being block-recursive. Equations (16), (8) and (9) can be solved for c, ℓ and N, with equations (10) and (11) then providing the wage and the price. From equations (16) and (8), one can immediately notice the complementarity that can arise between consumption and employment in the case where $N < L$ (the unemployment regime). From (16) we see that, if $N < L$, agents will tend to increase their consumption if they believe there are many firms looking for workers (N expected to be large). Then from equation (8) we see that more firms will be looking to hire workers if they believe that consumption will be high. So greater consumption favors greater employment, which in turn reinforces consumption. This feedback effect arises as the result of consumption and employment playing the role of strategic complements. Workers demand higher consumption when they believe that many firms are searching to hire, as they view a high N as reducing their probability of entering the second sub-period in debt. It is important to notice that this multiplier argument is implicitly taking ℓ, the number of hours worked by agents, as given. But, in the case where the economy is characterized by unemployment, this is precisely the right equilibrium conjecture. In particular, from (9) we can see that if the economy is in a state of unemployment, then ℓ is simply given by $\ell^\star$, the solution to the equation $F(\ell^\star) - F'(\ell^\star)\ell^\star = \Phi$, and is therefore locally independent of X or c. Hence, in the presence of unemployment, consumption and firm hiring will act as strategic complements. As is common in the case of strategic complements, multiple equilibria can arise. This possibility is stated in Proposition 1.

Proposition 1. *There exists a $\bar{\tau} > 0$[14] such that (a) if $\tau < \bar{\tau}$, then there exists a unique equilibrium for any value of X; and (b) if $\tau > \bar{\tau}$, then there exists a range of X for which there are multiple equilibria.*

The proofs of all propositions are presented in Appendix A.

While situations with multiple equilibria may be interesting, in this paper we will mainly focus on the case where the equilibrium is unique, as we believe this is more likely to be the empirically relevant case. Accordingly, Proposition 1 tells us that our setup will have a unique equilibrium if the marginal cost of debt is not too large. For the remainder of this section, we will assume that $\tau < \bar{\tau}$. Proposition 2 focuses on this case and provides a first step in the characterization of the equilibrium.

Proposition 2. *When $\tau < \bar{\tau}$, there exists an $X^\star$ such that if $X \leq X^\star$ then the equilibrium is characterized by full employment, while if $X > X^\star$ it is characterized by unemployment. Furthermore, there exists an $X^{\star\star} > X^\star$ such that if $X > X^{\star\star}$, then employment is zero and agents simply consume their endowment (i.e., $c = X$).*[15]

The content of Proposition 2 is very intuitive as it simply states that if agents have a low endowment of the consumption good, then there are substantial gains from trade, and that will favor full employment. In contrast, if the endowment is very high, this will reduce the demand for the good sufficiently as to create unemployment. Finally, if X is extremely high, all trade among agents will stop as people are content to simply consume their endowment.

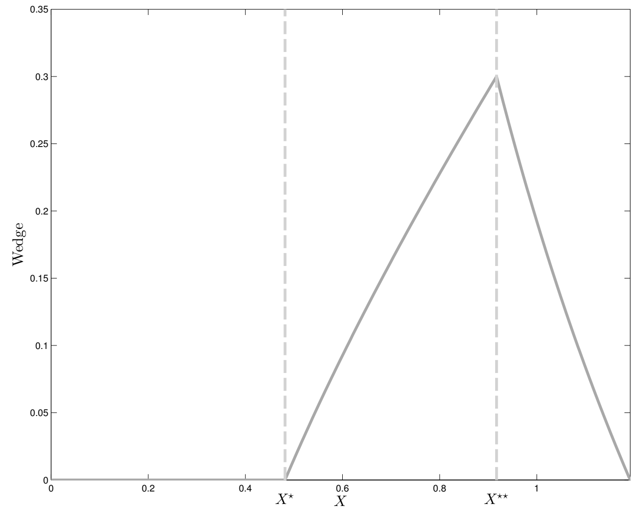
Proposition 2 can also be used to provide insight regarding the relationship between the labor wedge in this economy and the inherited endowment of X, where the labor wedge is defined as $\left[U'(c) - \frac{\nu'(\ell)}{F'(\ell)}\right] / \frac{\nu'(\ell)}{F'(\ell)}$. In Figure 1, we plot the labor wedge as a function of X. As can be seen, for $X < X^\star$,[16] the labor wedge is zero, while for $X \in [X^\star, X^{\star\star}]$, the labor wedge rises monotonically, reaching a peak at the point $X^{\star\star}$ where trade collapses. Then,

[14] $\bar{\tau} = -U''\left(U'^{-1}\left(\frac{\nu'(\ell^\star)}{F'(\ell^\star)}\right)\right)\frac{F'(\ell^\star)[F(\ell^\star)-\Phi]}{\nu'(\ell^\star)}$.

[15] $X^\star = U'^{-1}\left(\frac{\nu'(\ell^\star)}{F'(\ell^\star)}\right) - F'(\ell^\star)\ell^\star$ and $X^{\star\star} = U'^{-1}\left(\frac{\nu'(\ell^\star)}{F'(\ell^\star)}(1+\tau)\right)$.

[16] We assume here and throughout the remainder of this paper that $U'\left(F(\ell^\star) - \Phi\right) > \frac{\nu'(\ell^\star)}{F'(\ell^\star)}$, so that $X^\star > 0$.

Figure 1: Labor wedge as function of X.



Note: Labor wedge is defined as $\left[U'(c) - \frac{\nu'(\ell)}{F'(\ell)}\right] / \frac{\nu'(\ell)}{F'(\ell)}$. *Example is constructed assuming the functional forms* $U(c) = \log(c)$, $\nu(\ell) = \frac{\nu\ell^{1+\omega}}{1+\omega}$ *and* $F(\ell) = A\ell^{\alpha}$, *with parameters* $\omega = 1$, $\nu = 0.5$, $\alpha = 0.67$, $A = 1$, $\Phi = 0.35$ *and* $\tau = 0.3$.

for $X > X^{\star\star}$, we enter the no-employment zone and the wedge declines gradually until it reaches zero anew at a point where the no-employment outcome is socially optimal. This figure nicely illustrates that the degree of distortion in this economy varies with X, with low values of X being associated with a more efficient economy, while higher values of X generate a positive and growing wedge as long as trade remains present. From this observation, we can see how a higher inherited capital stock can increase inefficiency. Proposition 3 complements Proposition 2 by indicating how consumption is determined in each regime.

Proposition 3. *When the economy exhibits unemployment ($X^{\star\star} > X > X^{\star}$), the level of consumption is given as the unique solution to*

$$c = U'^{-1}\left(\frac{\nu'(\ell^{\star})}{F'(\ell^{\star})}\left[1 + \tau - \frac{c - X}{F'(\ell^{\star})\ell^{\star}}\tau\right]\right)$$

When the economy exhibits full employment ($X \leq X^{\star}$), consumption is the unique solution to

$$c = U'^{-1}\left(\frac{\nu'(\Omega^{-1}(c - X))}{F'(\Omega^{-1}(c - X))}\right)$$

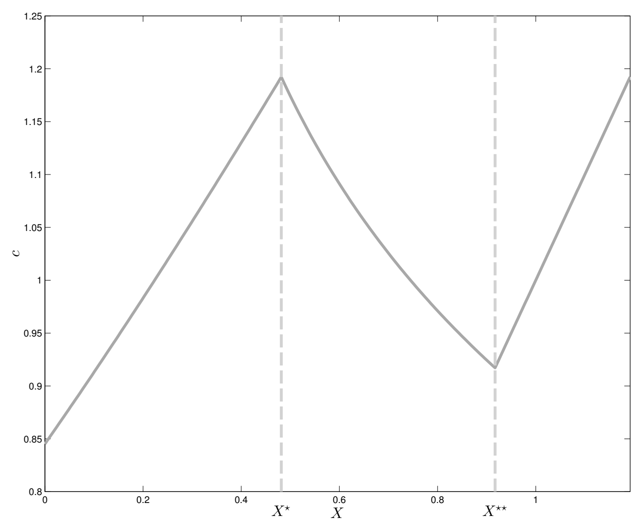
Finally, when $X \geq X^{\star\star}$, consumption is given by $c = X$.

Given the above propositions, we are now in a position to examine an issue of main interest, which is how an increase in X affects consumption. In particular, we want to ask whether an increase in X, which acts as an increase in the supply of goods, can lead to a decrease in the actual consumption of goods. Proposition 4 addresses this issue.

Proposition 4. *If $X^{\star\star} > X > X^{\star}$, then c is decreasing in X. If $X \leq X^{\star}$ or $X > X^{\star\star}$, then c is increasing in X .*

The content of Proposition 4 is illustrated in Figure 2. Proposition 4 indicates that,

Figure 2: Consumption as function of X.



Note: Example is constructed assuming the functional forms $U(c) = \log(c)$, $\nu(\ell) = \frac{\nu\ell^{1+\omega}}{1+\omega}$ and $F(\ell) = A\ell^{\alpha}$, with parameters $\omega = 1$, $\nu = 0.5$, $\alpha = 0.67$, $A = 1$, $\Phi = 0.35$ and $\tau = 0.3$.

starting at $X = 0$, consumption will continuously increase in X as long as X is compatible with full employment. Then, when X is greater than $X^{\star}$, the economy enters the unemployment regime and consumption starts to decrease as X is increased. Finally, beyond $X^{\star\star}$ trade collapses and consumption becomes equal to X and hence it increases with X. The reason that consumption decreases with a higher supply of X in the unemployment region is precisely because of the multiplier process described earlier. In this region, an increase in X leads to a fall in expenditures on new consumption, where we define expenditures as $e \equiv c - X$. The decrease in expenditures reduces the demand for goods as perceived by firms. Less firms then search for workers, which increases the risk of unemployment. The increase in unemployment risk leads households to cut their expenditures further, which further amplifies the initial effect of an increase in X on expenditures. It is because of this type of multiplier process that an increase in the supply of the good can lead to a decrease in its total consumption $(X+e)$. Note that such a negative effect does not happen when the economy is at full employment, as an increase in X does not cause an increase in precautionary savings, which is the key mechanism at play causing consumption to fall.

The link noted above between household j's expenditure, which we can denote by $e_j \equiv c_j - X_j$, and its expectation about the expenditures by other agents in the economy, which can denote by e, can be captured by rewriting the relations determining e_j implied by the elements of Proposition 3 as

$$e_j = Z(e) - X \tag{12}$$

with

$$Z(e) \equiv U'^{-1}\left(Q(e)\right) \tag{13}$$

and

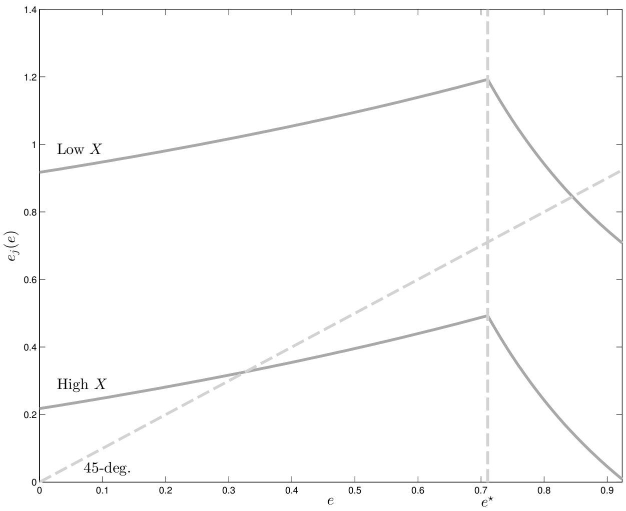
$$Q\left(e\right) \equiv \begin{cases} \frac{\nu'(\ell^\star)}{F'(\ell^\star)}\left(1+\tau-\tau\frac{e}{e^\star}\right) & \text{if } 0 < e < e^\star \\ \frac{\nu'\left(\Omega^{-1}(e)\right)}{F'(\Omega^{-1}(e))} & \text{if } e \geq e^\star \end{cases} \tag{14}$$

Here, $e^\star \equiv \Omega(\ell^\star)$ is the level of output (net of firms' search costs) that would be produced if all workers were employed, with hours per employed worker equal to $\ell^\star$. In equilibrium we have the additional requirement that $e_j = e$ for all j.

The equilibrium determination of e is illustrated in Figure 3, which somewhat resembles a Keynesian cross. In the figure, we plot the function $e_j = Z(e) - X$ for two values of X: a first value of X which places the economy in an unemployment regime, and a second value of X which places the economy in a full-employment regime. An equilibrium in this figure corresponds to the point where the function $e_j = Z(e) - X$ crosses the 45° line. Note that changes in X simply move the $e_j = Z(e) - X$ curve vertically.

There are several features to note about Figure 3. First, in the case where $X \in (X^\star, X^{\star\star})$, so that the equilibrium of the economy is in an unemployment regime with positive trade (i.e., $0 < e < e^\star$), the diagram is similar to a Keynesian cross. We can see graphically how an increase in X by one unit shifts down the $Z(e) - X$ curve and, since the slope of

Figure 3: Equilibrium determination



Note: Example is constructed assuming the functional forms $U(c) = \log(c)$, $\nu(\ell) = \frac{\nu\ell^{1+\omega}}{1+\omega}$ and $F(\ell) = A\ell^{\alpha}$, with parameters $\omega = 1$, $\nu = 0.5$, $\alpha = 0.67$, $A = 1$, $\Phi = 0.35$ and $\tau = 0.3$. Values of X used were $X = 0$ for the full-employment equilibrium and $X = 0.7$ for the unemployment equilibrium.

$Z(e) - X$ is positive and less than one, a multiplier process kicks in which causes e to fall by more than one. Because of this multiplier process, total consumption of clothes, which is equal to $e + X$, decreases, which is the essence of the first part of Proposition 4. Second, when $X < X^\star$, so that the economy is in a full-employment regime (i.e., the equilibrium is such that $e > e^\star$), the diagram is different from the Keynesian cross. The most notable difference is the negative slope of the function $Z(e) - X$ for values of $e > e^\star$. This reflects the fact that unemployment risk is not present in this regime. In fact, when X is sufficiently small so that the economy is in the full-employment regime, an increase in X by one unit leads to a decrease in e that is less than one, compared to a decrease of greater than one as exhibited in the unemployment regime. Here, expenditure by others actually plays the role of a strategic substitute with one's own expenditure – as opposed to playing the role of a strategic complement as is the case in the unemployment regime – through its effects on real wages and prices. Accordingly, in this region, an increase in X leads to an increase in total consumption of clothes. Another more subtle difference with the Keynesian cross is in how the intercept of $Z(e) - X$ is determined. The intercept is given by $U'^{-1}(\frac{\nu'(\ell^\star)}{F'(\ell^\star)}(1+\tau)) - X$.

The X term in the intercept can be interpreted as capturing a pure aggregate-demand effect, whereby higher values of X reduce aggregate demand. However, the remaining term, $U'^{-1}(\frac{\nu'(\ell^\star)}{F'(\ell^\star)}(1+\tau))$, reflects technology and preferences. In particular, we can generalize this term by re-introducing the technology parameter θ, in which case the intercept becomes $U'^{-1}(\frac{\nu'(\ell^\star)}{\theta F'(\ell^\star)}(1+\tau))$.[17] In this case, we see that an improvement in technology shifts up the intercept, and will lead to an increase in expenditures. This feature of the $Z(e) - X$ curve illustrates its equilibrium nature, which incorporates both demand and supply effects, as opposed to a Keynesian cross that only reflects demand effects.

2.4 Is there deficient demand in the unemployment regime?

In the case where X is large enough for the economy to be in the unemployment regime ($X^\star < X < X^{\star\star}$), we have already noted that the marginal rate of substitution between consumption and leisure is greater the marginal product of labor, with this distortion increasing the larger is X . In this sense, the economy is clearly working inefficiently in the unemployment regime. In this section, we want to examine whether this regime can also be appropriately characterized as suffering from deficient aggregate demand. In particular, suppose the structure of markets were not changed and $X^\star < X < X^{\star\star}$. Now suppose that all households deviated from their equilibrium strategies by increasing slightly their demand for consumption goods. If in this case the expected utility of the household would be increased, then it appears reasonable to characterize the situation as one of deficient demand. Using this definition, Proposition 5 indicates that the unemployment regime of our model is in fact characterized by deficient demand.

Proposition 5. *When the economy is in the unemployment regime ($X^\star < X < X^{\star\star}$), a coordinated increase by households in the purchase of the first sub-period consumption good increases the expected utility of all households.*

Proposition 5 can alternatively be interpreting as confirming that the consumption choices of individual households play the role of strategic complements in the unemployment regime.

[17] Recall that an increase in θ is associated with a proportional change in the search cost, so that $\ell^\star$ remains unchanged.

2.5 Effects of changes in X on welfare

We have shown that when X is high enough, then the economy will be in the unemployment regime, where a local increase in X causes consumption to fall. We now want to ask how expected welfare is affected in these cases, where expected welfare is defined as $U(c) + \mu[-\nu(\ell) + V(w\ell - p(c-X))] + (1-\mu)V(-p(c-X))$. In particular, we want to ask whether welfare can decrease when the economy is endowed with more goods. Proposition 6 answers this question in the affirmative. Proposition 6 actually goes a step further and indicates two sufficient conditions for there to exist a range of X in the unemployment regime where an increase in X leads to a fall in welfare.

Proposition 6. *An increase in X can lead to a fall in expected welfare. In particular, if either (i) τ is close enough to $\bar{\tau}$ or (ii) the average cost of work $\frac{\nu(\ell^\star)}{\ell^\star}$ is low enough relative to the marginal cost of work $\nu'(\ell^\star)$, then there is always a range of $X \in [X^\star, X^{\star\star}]$ such that an increase in X leads to a decrease in expected welfare.*

Proposition 6 provides a step toward answering whether more goods can make everyone worse off. In effect, the proposition indicates that the economy can function in a very perverse fashion when households have inherited many goods. We saw from Proposition 4 that an increase in X always leads to a decrease in consumption when we are in the unemployment regime. In comparison, Proposition 6 is weaker as it only indicates the possibility of a fall in welfare in the unemployment region when X rises. In response to a rise in X in the unemployment regime, there are three distinct channels through which expected welfare is affected. First, as discussed above, consumption falls, which tends to directly decrease welfare. Second, this fall in consumption is associated with a fall in the probability of being employed. It can be verified that the net benefit of being employed is strictly positive, so that this second effect also tends to decrease welfare. Finally, a rise in X means that a given quantity of consumption can be obtained with a lower level of expenditure, which increases assets for the employed and decreases debt for the unemployed, and therefore tends to increase welfare. Whether this final effect is outweighed by the first two depends on the factors discussed in Proposition 6.

As noted in Proposition 6, the effects of an increase in X on welfare depends, among other things, on the difference between the marginal utility cost of work and the average utility cost of work. This distinction is relevant because an important component of the net benefit of being employed is the utility value of wages earned, net of the value of foregone leisure.[18] In the current model, the average utility cost of work can be arbitrarily small relative to its marginal cost. When the average cost of work is low, the net benefit of being employed is large, and therefore a rise in the unemployment rate caused by a rise in X will have a larger negative effect on welfare (i.e., the second channel discussed above becomes more important). Hence, in our model, when employment is not perceived as very painful, and we are in the unemployment regime, then an increase in X leads to decreased welfare.

2.6 Allowing for offers of unemployment insurance

In our analysis thus far, we have assumed that agents do not have access to unemployment insurance. It may be thought that allowing for the private provision of unemployment insurance would necessarily eliminate the mechanisms we have highlighted. For this reason, in this subsection we want to briefly indicate how our analysis can be extended rather trivially to include an adverse selection problem that will justify the absence of unemployment insurance, without changing the main results. In particular, suppose there is a fraction ρ of households that behave as the households we have modeled to date, which we call participant households, and suppose the remaining $(1-\rho)$ fraction of households, which we can call the non-participant households, are simply not interested in work within the period. These latter households are happy to consume their endowment without wanting to search for work. Now suppose that some private agent wanted to offer unemployment insurance before the matching process, but could not differentiate between the two types of households. In this case, an insurer will not be able to offer contracts that will only be attractive to the participant households, because any unemployment insurance contract with a positive net payment to unemployed individuals will be desirable to non-participants. Therefore, as indicated in Proposition 7, as long as ρ is sufficiently low, this type of adverse

[18] The other component is the net welfare gain that stems from consumption expenditures being made in the positive-asset state rather than the more costly (in utility terms) negative-asset state.

selection problem implies that the only equilibrium outcome is one where no insurance is offered. Accordingly, in this setup, the mechanisms we have emphasized regarding how changes in X affect outcomes will directly apply.

Proposition 7. *In the presence of both participant households and non-participant households, if $\rho < \frac{1}{1+\tau}$, i.e., if the fraction of participant households is sufficiently low, then no unemployment-insurance contracts are traded in equilibrium.*

2.7 Introducing government spending

We now turn to examining how changes in government spending can affect economic activity. To do this, we extend the model by simply adding a government to the first sub-period. The government undertakes two activities in this sub-period: it buys goods, and it taxes employed individuals. We assume that the government runs a balanced budget so that its expenditure on goods is equal to the lump-sum tax per employed worker times the number of employed workers. It turns out that the effects of government spending in this setup depend crucially on what the government does with the goods. Accordingly, we will consider two types of government purchases: wasteful, and non-wasteful. Wasteful government purchases, denoted G_w, are not valued by households, [19] while non-wasteful purchases, denoted G_n, are assumed to directly affect agents' utility by entering as a substitute to private consumption. Note that G_w and G_n are per-capita government expenditures. If we return to the set of equilibrium conditions given by equations (16) to (11), the only condition that changes with the introduction of a government is equation (8), the goods-market equilibrium condition. The other conditions remain the same once the variable c is interpreted as total consumption including consumption of non-wasteful government purchases. The goods market equilibrium condition, equation (8), therefore has to be rewritten as[20]

$$\frac{\min\{N, L\}}{L} = \frac{c - X + G_w}{F'(\ell)\ell}$$

[19]We can as well assume that they are valued by households but that utility is linearly separable in G_w.

[20] We assume throughout this subsection that $\tau < \bar{\tau}$, and that total government expenditures are sufficiently low so that the lump-sum tax on employed workers is not so large as to cause households to prefer to be unemployed.

since $c - X + G_w$ now represents the total purchases of clothes in the sub-period. If we again allow e to represent these total purchases ($e = c - X + G_w$), then the determination of e takes a form almost identical to that described previously by equations (12)-(14). In fact, the determination of total expenditures e is now given by the solution to

$$e = Z(e) - X + G_w \tag{15}$$

where $Z(e)$ was defined in equation (13).

There are two key things to notice about equation (15). First, non-wasteful government expenditure G_n does not enter into this condition, and therefore does not affect the equilibrium level of economic activity e; that is, non-wasteful government expenditure crowds out private expenditure one-to-one. Second, in contrast, *wasteful* government expenditure will tend to stimulate activity in a manner parallel to a decrease in X. To understand why non-wasteful government purchases do not affect activity, it is helpful consider how people would behave simply if they conjectured the outcome. In this case, since they would conjecture that unemployment risk is not changing, they would want to consume at the same overall level as before the increase in G_n. But if they consume at the exact same overall level, it requires households to decrease their private purchases by exactly the same amount as the purchases made by the government. Hence, activity will not be increased and agents' initial conjecture is rationalized. This is why non-wasteful government purchases do not affect activity in our setup, even when the economy exhibits unemployment. Note that this logic does not hold in the case of wasteful government purchases. If government purchases are wasteful, and people conjecture that unemployment risk is unaffected, their overall consumption will be unchanged, and, with no increased utility from government purchases, private purchases would also be unchanged. But total purchases – including those made by the government – would necessarily be increased. If the economy were in the unemployment regime, this additional demand would be met by a rise in the employment rate μ, and hence households' conjecture that unemployment risk is unchanged would be false. Recognizing that unemployment risk in fact fell, households would reduce their precautionary savings and increase their private purchases, further increasing demand, and leading to a multiplier greater than one. If the economy had instead been in the full-employment regime, the additional demand would be met by a rise in hours per worker l, which is associated with a rise in the price p and a corresponding fall in private purchases, mitigating to some extent the rise in demand caused by the government and leading to a multiplier less than one. These results are summarized in Proposition 8.

Proposition 8. *An increase in non-wasteful government purchases has no effect on economic activity. An increase in wasteful government purchases leads to an increase in economic activity. If the economy is in the unemployment regime, wasteful government purchases are associated with a multiplier that is greater than one, while if the economy is in the full-employment regime, wasteful government purchases are associated with a multiplier that is less than one.*

From Proposition 8 we see that the multiplier associated with wasteful government purchases depends on the state of the economy and the type of purchases. In particular, the multiplier for wasteful government purchases is greater than one when the economy has a high level of X and is therefore in the unemployment regime. In contrast, when the economy has a low level of X and is therefore in the full-employment regime, the multiplier for wasteful government purchases is less than one. The interesting aspect of Proposition 8 is that it emphasizes why the effects of government purchases may vary drastically, from zero to more than one, depending on the circumstances.

While wasteful government purchases increase economic activity, this does not imply that they increase welfare. In fact, it can be easily verified that an increase in wasteful government purchases necessarily decreases welfare when the economy is in the full-employment regime, as it reduces private consumption and increases hours worked. On the other hand, when the economy is in the unemployment regime (due to a high value of X), the effect on welfare depends on a number of factors, in much the same way that the effect on welfare of a change in X depends on a number of factors. For example, the change in welfare depends on the ratio of the average dis-utility of labor relative to the marginal dis-utility of labor. As discussed earlier, when this ratio is low, the net benefit to being employed is high, and since one of the effects of an increase in wasteful government purchases is to increase the employment rate, the resulting increase in welfare through this channel is also high. As such, welfare is overall more likely to increase when the average dis-utility of work is low.

It turns out that sufficient conditions under which an increase in wasteful government purchases increases welfare are given by those contained in Proposition 6 regarding the welfare effects of a change X. This is stated in Proposition 9.

Proposition 9. *If the economy is in the unemployment regime and if X is in the range such that a fall in X would increase welfare, then an increase in wasteful government purchases will increase welfare.*

3 Further discussions and relaxing of assumptions

3.1 Relaxing functional-form assumptions

One of the important simplifying assumptions of our model is the use of a matching function of the "min" form. This specification has the nice feature of creating two distinct employment regimes: one where there is unemployment and one where there is full employment. However, this stark dichotomy, while useful, is not central to the main results of the model. In fact, as we now discuss, the important feature for our purposes is that there be one regime in which expenditures by individual agents play the role of strategic substitutes, and another in which they play the role of strategic complements. To see this, it is helpful to re-examine the equilibrium condition for the determination of expenditure for a general matching function. This is given by

$$U'(X + e_j) = vp(e)\left[1 + \tau - \frac{M(N(e), L)}{L}\tau\right] \tag{16}$$

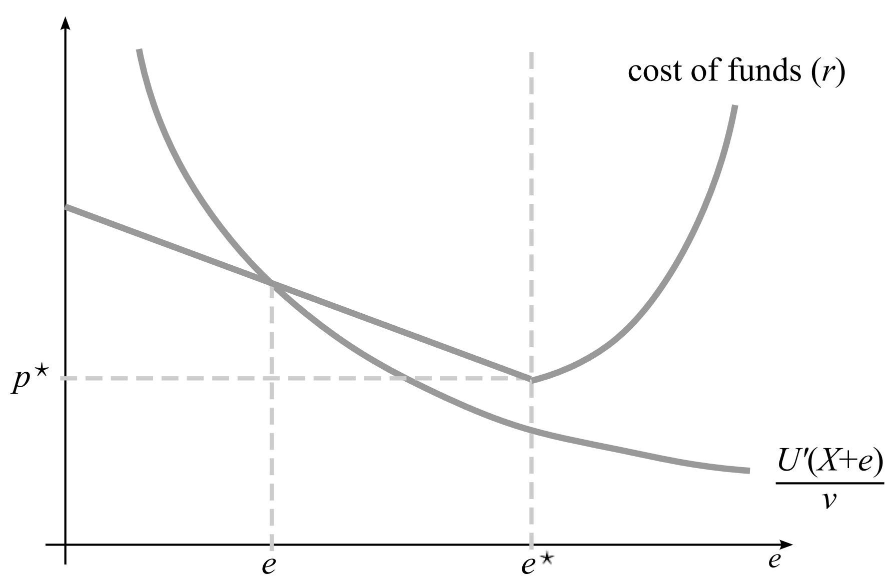
where $M(N, L)$ is a CRS matching function satisfying $M(N, L) \leq \min\{N, L\}$. In (16), we have made explicit the dependence of N and p on e, where this dependence comes from viewing the remaining four equilibrium conditions as determining N, p w and ℓ as functions of e.[21] Note that these other equilibrium conditions imply that $p(e)$ and $N(e)$ are always

[21] These remaining four equilibrium conditions can be written

$$\begin{aligned} \nu'(\ell) &= vw \\ pF'(\ell) &= w \\ M(N, L)F(\ell) &= L(c - X) + N\Phi \\ M(N, L)[pF(\ell) - w\ell] &= Np\Phi \end{aligned}$$

weakly increasing in e. In (16) we have once again made clear that this condition relates the determination of expenditure for agent j, e_j, to the average expenditure of all agents, e. From this equation, we can see that average expenditure can play either the role of strategic substitute or strategic complement to the expenditure decision of agent j. In particular, through its effect on the price p, e plays the role of a strategic substitute, while through its effect on firm entry N and, in turn, unemployment, it plays the role of strategic complement. The sign of the net effect of e on e_j therefore depends on whether the price effect or the unemployment effect dominates. In the case where $M(N, L) = \min\{N, L\}$, the equilibrium features the stark dichotomy whereby $\partial p(e)/\partial e = 0$ and $\partial M(N(e), L)/\partial e > 0$ for $e < e^\star$, while $\partial p(e)/\partial e > 0$ and $\partial M(N(e), L)/\partial e = 0$ for $e > e^\star$. In other words, for low values of e the expenditures of others plays the role of strategic complement to j's decision since the price effect is not operative, while for high values of e it plays the role of strategic substitute since the risk-of-unemployment channel is non-operative. This reversal in the role of e from acting as a complement to acting as a substitute is illustrated in Figure 4, where we first plot a cost-of-funds schedule for agents, defined by $r = p(e)\left[1 + \tau - \frac{\min\{N(e),L\}}{L}\tau\right]$, where r represents the total cost of funds to agent j when average expenditure is e. Our notion

Figure 4: Cost of Funds



of the total cost of funds reflects both the direct cost of borrowing, $p(e)$, and the extra cost associated with the presence of unemployment risk. We superimpose on this figure the demand for e as a function of the total cost of funds, which is implicitly given by the function $U'(X+e)/v = r$. This latter relationship, which can be interpreted as a type of aggregate demand curve, is always downward-sloping since U is concave. The important element to note in this figure is that the cost-of-funds schedule $r = p(e)\left[1+\tau - \frac{\min\{N(e),L\}}{L}\tau\right]$ is first decreasing and then increasing in e. Over the range $e < e^\star$, the cost of funds to an agent is declining in aggregate e, since N is increasing while p is staying constant. Therefore, in the range $e < e^\star$, a rise in e reduces unemployment and makes borrowing less costly to agents. This is the complementarity zone. In contrast, over the range $e \geq e^\star$, the effect of e on the cost of funds is positive since the unemployment channel is no longer operative, while the price channel is. This is the strategic substitute zone. In the figure, a change in X moves the demand curve $U'(X+e)/v = r$ without affecting the cost-of-funds curve. A change in X therefore has the equilibrium property $\partial e/\partial X < -1$ when $e < e^\star$ because the cost-of-funds curve is downward-sloping in this region, while $\partial e/\partial X > -1$ in the region $e \geq e^\star$ because the cost-of-funds curve is upward-sloping.

From the above discussion it should now be clear that our main results do not hinge on the "min" form of the matching function, but instead depend on the existence of two regions: one where the total cost of borrowing by agents at low levels of e is decreasing in e because the effect of e on unemployment risk dominates its effect on p, with a second region where the price effect dominates the effect running through the unemployment-risk channel. It can be easily verified that a sufficient condition for this feature is that the elasticity of $M(N,L)$ with respect to N tends towards one when N becomes sufficiently small, while simultaneously having this elasticity tending to zero when N is sufficiently large. This property is clearly captured by the "min" function, but is in fact also captured by a large class of matching functions, as the following proposition establishes.

Proposition 10. *For any non-trivial matching function $M(N,L)$[22] that is (i) non-decreasing and weakly concave in N and (ii) satisfies $0 \leq M(N,L) \leq \min\{N,L\}$, the elasticity of M with respect to N approaches one as $N \to 0$ and approaches zero as $N \to \infty$.*

[22] By "non-trivial matching function" we mean a function satisfying, for any $L > 0$, $M(N,L) > 0$ for some N.

While this proposition guarantees under quite general conditions that the cost-of-funds locus will be negatively sloped at low levels of e and positively sloped at high values of e, it is interesting to ask if this non-monotonicity property can be ensured by other means over a region where the matching function has a constant elasticity. In effect, this property can be ensured through assumptions on $\nu(\ell)$ and $F(\ell)$. In particular, if the elasticities of $\nu'(\ell)$ and $F'(\ell)$ with respect to ℓ tend toward zero when ℓ is sufficiently low – that is, if $\nu(\ell)$ and $F(\ell)$ become close to linear when ℓ is low – this will guarantee a downward-sloping cost-of-funds schedule even if the matching function has a constant elasticity. Furthermore, if the elasticity of either $\nu'(\ell)$ or $F'(\ell)$ with respect to ℓ tends toward infinity when ℓ is large, this will guarantee that the cost-of-funds schedule will be upward-sloping at high values of e. While it is an open empirical question whether any of these conditions are met in reality over an economically significant range, it appears at least plausible to us that for low values of activity *(i)* congestion effects in matching associated with increases in N are small, *(ii)* the returns to labor in production exhibit little decreasing returns, and *(iii)* the dis-utility of work is close to linear. All these conditions will favor a downward-sloping cost-of-funds curve at low levels of activity, which is what is needed for the main results of this paper to hold.

A second important functional-form assumption we have used to derive our results is that the dis-utility of work in the second sub-period be linear so as to obtain a piecewise linear $V(a)$ function. This restriction is again not necessary to obtain our main results. However, if we depart substantially from the linearity assumption for second-sub-period dis-utility of labor, income effects can greatly complicate our simple characterizations.

3.2 A version with productive capital

We have shown how a rise in the supply of the capital good X, by decreasing demand for employment and causing households to increase precautionary savings, can perversely lead to a decrease in consumption. While thus far we have considered the case where X enters directly into the utility function, in this section we show that Proposition 4 can be extended to the case where X is introduced as a productive capital good. To explore this in the simplest possible setting, suppose there are now two types of firms and that the capital stock X no longer enters directly into the agents' utility function. The first type of firm remains identical to those in the first version of the model, except that instead of producing a consumption good they produce an intermediate good, the amount of which is given by $\mathcal{M}$. There is also now a continuum of competitive firms who rent the productive capital good X from the households and combine it with goods purchased from the intermediate goods firms in order to produce the consumption good according to the production function $g(X, \mathcal{M})$. We assume that g is strictly increasing in both arguments and concave, and exhibits constant returns to scale. Given X, it can be verified that the equilibrium determination of $\mathcal{M}$ will then be given as the solution to

$$g_{\mathcal{M}}(X, \mathcal{M})U'(g(X, \mathcal{M})) = Q(\mathcal{M}) \tag{17}$$

where $Q(\cdot)$ is defined in equation (14).

Note the similarity between condition (17) and the corresponding equilibrium condition for the durable-goods version of the model, which can be written $U'(X + e) = Q(e)$. In fact, if $g(X, \mathcal{M}) = X + \mathcal{M}$, so that the elasticity of substitution between capital and the intermediate good $\mathcal{M}$ is infinite, then the two conditions become identical, and therefore X affects economic activity in the productive-capital version of the model in exactly the same way as it does in the durable-goods model. Thus, a rise in X leads to a fall in consumption when the economy is in the unemployment regime. In fact, as stated in Proposition 11, this latter result will hold for a more general g as long as g does not feature too little substitutability between X and $\mathcal{M}$.[23]

Proposition 11. *If the equilibrium is in the full-employment regime, then an increase in productive capital leads to an increase in consumption. If the equilibrium is in the unemployment regime, then an increase in productive capital leads to a decrease in consumption if and only if the elasticity of substitution between X and $\mathcal{M}$ is not too small.*

The reason for the requirement in Proposition 11 that the elasticity of substitution be sufficiently large relates to the degree to which an increase in X causes an initial impetus

[23] We assume throughout this section that an equilibrium exists and is unique. Conditions under which this is true are similar to the ones obtained for the durable-goods model, though the presence of non-linearities in g makes explicitly characterizing them less straightforward in this case.

that favors less employment. If the substitutability between X and $\mathcal{M}$ is small, so that complementarity is large, then even though the same level of consumption could be achieved at a lower level of employment, a social planner would nonetheless want to increase employment. Since the multiplier process in our model simply amplifies – and can never reverse – this initial impetus, strong complementarity would lead to a rise in employment and therefore a rise in consumption, rather than a fall. In contrast, if this complementarity is not too large, then an increase in X generates an initial impetus that favors less employment, which is in turn amplified by the multiplier process, so that a decrease in consumption becomes more likely.[24]

Let us emphasize that the manner in which we have just introduced productive capital into our setup is incomplete – and possibly unsatisfying – since we are maintaining a static environment with no investment decision. In particular, it is reasonable to think that the more interesting aspect of introducing productive capital into our setup would be its effect on investment demand. To this end, we now consider extending the model to a simple two-period version that features investment. The main result from this endeavor is to emphasize that the conditions under which a rise in X leads to a fall in consumption are weaker than those required for the same result in the absence of investment. In other words, our results from the previous section extend more easily to a situation where X is interpreted as physical capital if we simultaneously introduce an investment decision. The reason for this is that, in the presence of an investment decision, a rise in X is more likely to cause an initial impetus in favor of less activity.

To keep this extension as simple as possible, let us consider a two-period version of our model with productive capital (where there remains two sub-periods in each period). In this case, it can be verified that the continuation value for household j for the second period is of the form $R(X_2) \cdot X_{2,j}$, where $X_{2,j}$ is capital brought by household j into the second period and X_2 is capital brought into that period by all other households. In order to rule out the possibility of multiple equilibria that could arise in the presence of strategic complementarity in investment, we assume we are in the case where $R'(X_2) < 0$. The description of the model

[24] Note that a rise in X also increases output for any given level of employment. To ensure that consumption falls in equilibrium, we require that the substitutability between X and $\mathcal{M}$ be large enough so that the drop in employment more than offsets this effect.

is then completed by specifying the capital accumulation equation,

$$X_2 = (1-\delta)X_1 + i \tag{18}$$

where i denotes investment in the first period and X_1 is the initial capital stock, as well as the new first-period resource constraint,

$$c + i = g(X_1, \mathcal{M}) \tag{19}$$

Given this setup, we need to replace the equilibrium condition from the static model (equation (17)) with the constraints (18) and (19) plus the following two first-order conditions,

$$g_{\mathcal{M}}(X_1, \mathcal{M})U'(c) = Q(\mathcal{M}) \tag{20}$$

$$U'(c) = R(X_2) \tag{21}$$

Equation (20) is the household's optimality condition for its choice of consumption, and is similar to its static counterpart (17), while equation (21) is the intertemporal optimality condition equating the marginal value of consumption with the marginal value of investment.

Of immediate interest is whether, in an unemployment-regime equilibrium, a rise in X_1 will produce an equilibrium fall in consumption and/or employment in the first period. As Proposition 12 indicates, the conditions under which our previous results extend are weaker than those required in Proposition 11 for the static case, in the sense that lower substitution between X and $\mathcal{M}$ is possible.

Proposition 12. *In the two-period model with productive capital,*[25] *an increase in capital leads to a decrease in both consumption and investment if and only if the elasticity of substitution between X and $\mathcal{M}$ is not too small. Furthermore, for a given level of equilibrium employment, this minimum elasticity of substitution is lower than that required in Proposition 11 in the absence of investment decisions.*

The intuition for why consumption and investment fall when the elasticity of substitution is high is similar to in the static case. The addition of the investment decision has the effect

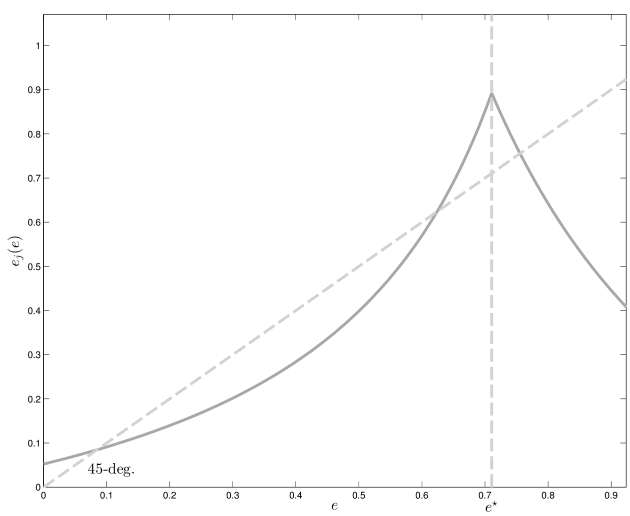
[25] We are again assuming that the equilibrium exists, is unique, and is in the unemployment regime.

of making it more likely that an increase in X leads to a fall in consumption because the increase in X decreases investment demand, which in turn increases unemployment and precautionary savings.

3.3 Multiple equilibria

Before discussing the welfare effects of changes in X, let us briefly discuss how multiple equilibria can arise in this model when $\tau > \bar{\tau}$. It can be verified that, when $\tau > \bar{\tau}$, the equilibrium determination of expenditures can still be expressed as the solution to the pair of equations $e_j = Z(e) - X$ and $e_j = e$. The problem that arises is that this system may no longer have a unique solution. Instead, depending on the value of X, it may have multiple solutions, an example of which is illustrated in Figure 5. In the figure, we see that, for this value of X, there are three such solutions.

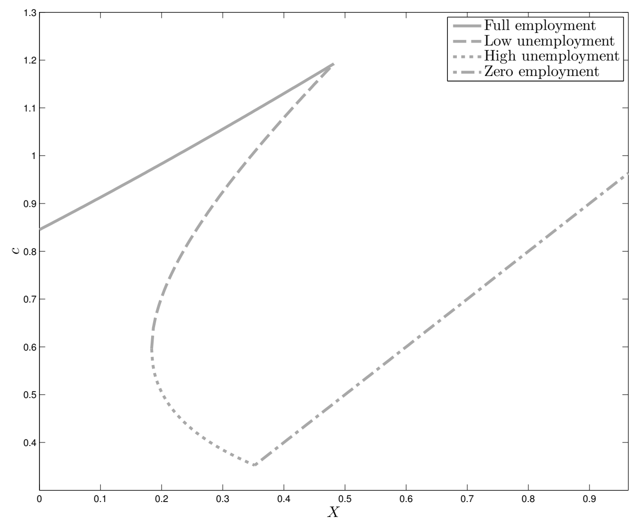
Figure 5: Equilibrium determination (multiple equilibria)



Note: Example is constructed assuming the functional forms $U(c) = \log(c)$, $\nu(\ell) = \frac{\nu\ell^{1+\omega}}{1+\omega}$ *and* $F(\ell) = A\ell^{\alpha}$, *with parameters* $\omega = 1$, $\nu = 0.5$, $\alpha = 0.67$, $A = 1$, $\Phi = 0.35$, $\tau = 1.2$ *and* $X = 0.3$.

Figure 6 shows how the set of possible equilibrium values of consumption depends on X when $\tau > \bar{\tau}$. As can be seen, when X is in the right range, there is more than one such equilibrium, with at least one in the unemployment regime and one in the full-employment

Figure 6: Consumption as function of X (multiple equilibria)



Note: Example is constructed assuming the functional forms $U(c) = \log(c)$, $\nu(\ell) = \frac{\nu\ell^{1+\omega}}{1+\omega}$ *and* $F(\ell) = A\ell^{\alpha}$, *with parameters* $\omega = 1$, $\nu_1 = 0.5$, $\alpha = 0.67$, $A = 1$, $\Phi = 0.35$ *and* $\tau = 1.2$.

regime. When this is the case, the selection of the equilibrium will depend on people's sentiment. If people are pessimistic, they cut back on consumption, which leads firms to cut back on employment, which can rationalize the initial pessimism. In contrast, if households are optimistic, they tend to buy more, which justifies many firms wanting to hire, which reduces unemployment and supports the optimistic beliefs. This type of environment featuring multiple equilibria driven by demand externalities is at the core of many papers. On this front, this paper has little to add. The only novel aspect of the current paper in terms of multiple equilibria is to emphasize how the possibility of multiple equilibria may depend on the economy's holding of capital goods.

3.4 The role of beliefs

There is another aspect in which the current model differs from a Keyesnian-cross setup, and that is with respect to the role of beliefs. The current setup should be thought of as part of the family of coordination games, and accordingly can potentially be analyzed with the tools and concepts used in the global games literature. Because of our assumption of homogeneity across households, we have not been very specific about agents' beliefs up to now. Nonetheless, it is worth emphasizing that the type of multiplier process present in the unemployment regime is the equilibrium outcome of a simultaneous-move game, rather than the outcome of events occurring sequentially over time. As such, prior beliefs of the players in the game are potentially a key driving force in the multiplier process. To clarify the potential role of these beliefs in our setup, it is helpful to briefly consider the case where agents have different holdings of X. For example, suppose that each agent j has an X_j drawn from a distribution with mean χ. The first-order condition for household j, assuming he thinks he is in the unemployment regime, can then be stated as

$$U'(e_j + X_j) = \frac{\nu'(\ell^\star)}{F'(\ell^\star)} (1 + \tau - E_j[\mu]\tau)$$

What is unknown to the household in this setup is the match probability μ, and therefore the expenditure decision, e_j, depends on household j's expectation of μ, which we write as $E_j[\mu]$. But μ in turn depends on firms' entry decisions, which depends on firms' expectation of aggregate consumption. This latter expectation can be expressed as $E_f[\int e_i di]$, where the operator $E_f[\cdot]$ represents expectations by firms, and $\int e_i di$ is the aggregate level of expenditures. So the first-order condition for household j would be given by

$$U'(e_j + X_j) = \frac{\nu'(\ell^\star)}{F'(\ell^\star)} \left(1 + \tau - \frac{E_j[E_f[\int e_i di]/L]}{\Omega(l^\star)}\tau\right)$$

We can now see that agent j's consumption decision will depend on his expectation of firms' expectation of the aggregate level of expenditure. This type of setup therefore involves forecasting the forecasts of others. If we assume that $U(\cdot)$ is quadratic and all relevant random variables jointly normally distributed, then this problem can be solved analytically, and will lead agent j to have a decision rule for consumption which depends on both X_j and χ, the prior about the average level of X_i across all other agents. Hence, both actual X_j's and beliefs regarding the average value of X_j in the economy will be main forces that drive expenditure and employment. For example, if agents believe that other agents have a high holding of X, this will depress consumption for all agents regardless of the actual holdings of X. Furthermore this effect can potentially be large because of the amplification mechanisms running through precautionary savings.

4 Dynamics

In this section we want to explore a dynamic extension of our static durable-goods model where current consumption contributes to the accumulation of X. In particular, we want to consider the case where the accumulation of X obeys the accumulation equation

$$X_{t+1} = (1-\delta)X_t + \gamma e_t \qquad 0 < \delta \leq 1 \quad , \quad 0 < \gamma \leq 1 - \delta \tag{22}$$

where the parameter γ represents the fraction of current consumption expenditures, $e_t = c_t - X_t$, which take the form of durable goods. Since we do not want to allow heterogeneity between individuals to expand over time, we will allow individuals to borrow and lend only within a period but not across periods; in other words, households are allowed to spend more than their income in the first sub-period of a period, but must repay any resulting debt in the second sub-period.[26] The problem facing a household in the first sub-period of a period is therefore to choose how much clothing to buy and, conditional on a match, how much labor to supply. We model the second sub-period as in sub-section 2.2, where households use labor to produce household services either for their own consumption or, at a level of productivity that is lower by a factor $1+\tau$, for the consumption of others. In each second sub-period, then, the household chooses how much to consume of household services and how much to produce of household services to both satisfy his needs and to pay back any accumulated debt. In order to keep the model very tractable, we will continue to assume that dis-utility of work in the second sub-period is linear (i.e., equal to $v \cdot \widetilde{\ell}$). Under this assumption, all households will choose the same level of consumption of household services in each second sub-period, while the production of household services will vary across households depending on whether they entered the sub-period in debt or in surplus. Since there are no interesting equilibrium interactions in second sub-periods, we can maintain most of our focus on equilibrium outcomes in the sequence of first sub-periods.

Relative to the static case, the only difference in equilibrium relationships (aside from the addition of the accumulation equation (22)) is that the first-order condition associated

[26] This lack of borrowing across periods can be rationalized if one assumes that the transaction cost of intermediating loans across periods is greater than $1+\tau$.

with the households' choice of consumption of clothes is now given by the Euler equation

$$U'\left(X_t+e_t\right)-Q\left(e_t\right)=\beta\left[(1-\delta-\gamma)U'\left(X_{t+1}+e_{t+1}\right)-(1-\delta)Q\left(e_{t+1}\right)\right] \tag{23}$$

where Q is as defined in equation (14). In this dynamic setting, an equilibrium will be represented as a sequence of the previous equilibrium conditions (8) to (11) plus the accumulation equation (22) and the Euler equation (23).

There are many complications that arise in the dynamic version of this model, which makes characterizing equilibrium behavior difficult. In particular, there can be multiple equilibrium paths and multiple steady-state solutions. Luckily, the problem can be simplified if we focus on cases where δ is small; that is, on cases where the durability of goods is long. In addition to simplifying the analysis, focusing on the low-δ case appears reasonable to us, as many consumer durables are long-lived, especially if we include housing in that category. In the case where δ is sufficiently small, as stated in Proposition 13, the economy will have only one steady state and that steady state will have the property of exhibiting unemployment.

Proposition 13. *If δ is sufficiently small, then the model has a unique steady state and this steady state is characterized by unemployment.*

Proposition 13 is very useful, as it will allow us to analyze the equilibrium behavior around the steady state without worrying about equilibrium selection. Accordingly, for the remainder of this section, we will assume that δ is sufficiently small so that Proposition 13 applies. However, before examining local properties in some generality, we believe that it is helpful to first illustrate global equilibrium behavior for a simple case that builds directly on our static analysis. The reason that we want to illustrate global behavior for at least one example is to emphasize that local behavior in our setup is likely to differ substantially and meaningfully from global behavior. Moreover, the example will allow us to gain some intuition on how the latter local results should best be interpreted.

Before discussing the transitional dynamics of the model, we first briefly discuss the conditions under which the model would exhibit a balanced growth path. In particular, suppose production in the first sub-periods is given by $\theta_t F(\ell_t)$ where θ_t is a technology index that is assumed to grow at a rate g_θ. Then it is easy to verify that our economy will admit an equilibrium growth path where both e and X grow at rate g_θ if the following three conditions are satisfied *(i)* the fixed cost of creating jobs grows at rate g_θ, *(ii)* the productivity of labor in the second sub-periods grows at rate g_θ, and *(iii)* the utility of consumption is represented by the log function. These conditions are not surprising, as they parallel those needed for a balanced growth path in many common macro models. The important aspect to note about this balanced-growth property is that the notion of high or low levels of capital should be interpreted as relative to the balanced growth path. In other words, the key endogenous state variable in the system should be viewed as the ratio of X_t to the growth component of θ_t. Accordingly, this justifies why, in the initial empirical motivation section of the paper, we deflated the measures of capital by a technology index.

4.1 Global dynamics for a simple case

The difficulty in analyzing the global dynamics for our model is related to the issue of multiple equilibria we discussed in the static setting. If the static setting exhibits multiple equilibria then the dynamic setting will likely exhibit multiple equilibrium paths. To see this, it is useful to recognize that our problem of describing equilibrium paths can be reduced to finding the household's decision rule for consumption. Since the only state variable in the system is X_t, the household's decision rule for consumption will likely be representable by a relationship (which may be stochastic) of the form $c(X_t)$. Given $c(X_t)$, the equilibrium dynamics of the system are given by

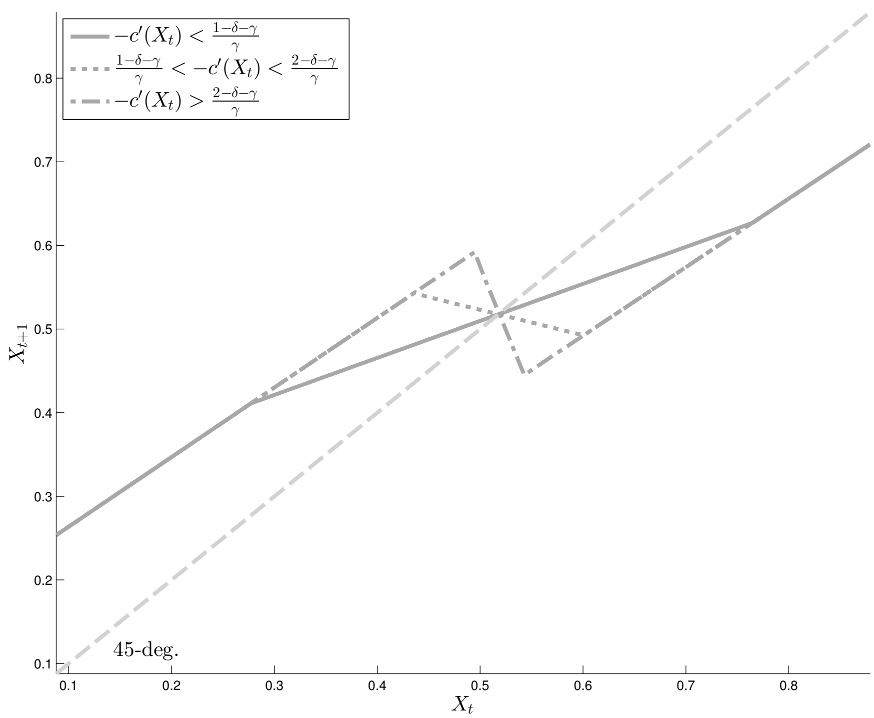
$$X_{t+1} = (1 - \delta - \gamma)X_t + \gamma c(X_t) \tag{24}$$

If the relationship $c(X_t)$ is a function, then equilibrium dynamics are deterministic. However, if we consider the case with $\beta = 0$ – so that households are not forward-looking and thus the dynamic equilibrium is simply a sequence of static equilibria – we already know that the household's decision rule $c(X_t)$ may not be a function. For example, if $\tau > \bar{\tau}$, then the household's decision rule may be a correspondence of the form given in Figure 6. Therefore, even for the rather simple case where $\beta = 0$ and $\tau > \bar{\tau}$ we know that the equilibrium dynamics need not be unique, in which case some equilibrium-selection device will be needed to solve the model. In contrast, for the case where $\beta = 0$ and $\tau < \bar{\tau}$, then we know from

Proposition 3 that $c(X_t)$ is a function. Hence, in the case where $\beta = 0$ and $\tau < \bar{\tau}$, we can describe the global dynamics of the system rather easily, and this is what we will do in this section. In particular, when $\beta = 0$ and $\tau < \bar{\tau}$, the stock of durables evolves according to equation (24), with $c(X_t)$ given by the value of c obtained using Proposition 3 with X_t in place of X.

Figure 7 plots the equilibrium transition function for X for three cases; that is, it plots $(1 - \delta - \gamma)X_t + \gamma c(X_t)$ for different possible $c(X_t)$ functions. The figure is drawn so that the steady state is in the unemployment region, which is consistent with a low value of δ as implied by Proposition 13. As can be seen from the figure, when X_t is not too great (i.e., less

Figure 7: X_{t+1} as a function of X_t



Note: Figure shows transition functions $X_{t+1}(X_t)$ for three different decision rules $c(X_t)$ that all yield the same steady state. Decision rules are identical when the economy features either full employment or zero employment, and differ when the economy features partial unemployment. Legend entries refer to value of $c'(X_t)$ in partial-unemployment regime.

than $X^{\star}$) the economy is in the full-employment regime and $X_{t+1} > X_t$. So if the economy starts with a low value of X_t it will generally go through a phase of full employment. During this phase, we know from Proposition 3 that consumption is also increasing. Eventually, X_t will exceed $X^\star$ and the economy enters the unemployment regime, at which point the dynamics depend on the derivative of the equilibrium decision rule, i.e., $c'(X_t)$, where in this regime $c(X_t)$ solves

$$U'(c) = \frac{\nu'(\ell^\star)}{F'(\ell^\star)}\left(1 + \tau - \tau\frac{c - X_t}{e^\star}\right)$$

If $-c'(X_t) < \frac{1-\delta-\gamma}{\gamma}$ when $X^\star < X_t < X^{\star\star}$, then the transition function maintains a positive slope near the steady state and the economy will converge monotonically to its steady state. However, note that even if X converges monotonically to its steady state in such a case, this will not be the case for consumption. Again, from Proposition 3 we know that consumption is decreasing in X in the unemployment region. Hence, starting from $X = 0$, in this case consumption would initially increase, reaching a maximum just as the economy enters the unemployment regime, then decline towards its eventual steady-state level which is lower than the peak obtained during the transition. If instead $-c'(X_t) > \frac{1-\delta-\gamma}{\gamma}$, then the transition function for X will exhibit a negative slope in the unemployment regime. In this case, X will no longer converge monotonically to the steady state. In fact, if the slope of this function (which depends on the elasticity of c with respect to X at the steady state) is negative but greater than -1, the system will converge with oscillations. However, if this slope is smaller than -1, which can arise for very large negative values of $c'(X_t)$, then the system will not converge and instead can exhibit rich dynamics, including cycles and chaos. In general, however, even in the case where $c'(X_t)$ is very negative, the system will not necessarily be explosive, since once it moves sufficiently far away from the steady state, forces kick in that work to push it back. Such rich dynamics, with the possibility of limit cycles, are certainly intriguing, but we will not dwell on them since it appears unlikely to us that this type of configuration is relevant.

There are two main messages to take away from exploring the global dynamics in this special case with $\beta = 0$. First, the behavior of the state variable X can be well-behaved, exhibiting monotonic convergence throughout. Second, the behavior of consumption (and therefore possibly welfare), can nonetheless exhibit interesting non-monotonic dynamics, with steady-state consumption actually being below the highest level it achieved during the transition. It is worth noting that if the steady state were to be in the full-employment regime (due, for example, to a higher δ), then from $X = 0$ both X_t and c_t would always converge monotonically to the steady state when $\beta = 0$ and $\tau < \bar{\tau}$

The most interesting aspect about the global dynamics in this case is that it allows us to illustrate the following possibility: If the economy is near its steady state, then a small reduction in X_t will increase consumption and can potentially increase welfare, while a large decrease in X_t will certainly decrease welfare. In this sense, the model exhibits behavior around the unemployment steady state that can differ substantially from behavior far away from the steady state, with the behavior far away from the steady state being more akin to that generally associated with classical economics, while behavior in the unemployment regime being more similar to that suggested by a Keynesian perspective.[27]

4.2 Local dynamics in the general case

In this subsection, we explore the local dynamics of the general model when $\beta > 0$, still assuming that δ is sufficiently small so that the steady state is unique and in the unemployment regime. From our analysis of the case with $\beta = 0$, we know that local dynamics can exhibit convergence or divergence depending on how responsive consumption is to X around the steady state. The one question we could not address when $\beta = 0$ is whether dynamics could exhibit local indeterminacy. In other words, can forward-looking behavior give rise to an additional potential local source of multiple equilibria in our setup? Proposition 14 indicates that this is not possible; that is, the roots of the system around the unique steady state can not both be smaller than one.[28]

Proposition 14. *The local dynamics around the steady state can either exhibit monotonic convergence in c and X, convergence with oscillations, or divergence. Locally indeterminacy is not possible.*

Proposition 14 is useful as it tells us that the decision rule for consumption around the

[27] It is worth noting that this type of synthesis, which emphasizes differences between being near to the steady state versus far from the steady state, is substantially different from the new neo-classical synthesis, which emphasizes differences in the long run and the short run because of sticky prices.

[28] In this section we only consider local dynamics around a unique unemployment-regime steady state. Nonetheless, it is straightforward to show that if the unique steady state is in the full-employment regime, then the local dynamics necessarily exhibit monotonic convergence.

steady state is a function.[29] Accordingly, we can now examine the sign of the derivative of this function. The question we want to examine is whether the decision rule for consumption around the steady state has the property that a larger X leads to a lower level of consumption, as was the case in our static model when in the unemployment regime. In other words, we want to know whether the results regarding the effect of X on consumption we derived for the static model extend to the steady state of the dynamic setting with $\beta > 0$. Proposition 15 indicates that if τ is not too large, then local dynamics will exhibit this property. Note that the condition on τ is a sufficient condition only.

Proposition 15. *If τ is sufficiently small, then in a neighborhood of the unique steady state, consumption is decreasing in X, with the dynamics for X converging monotonically to the steady state.*

From Proposition 15 we now know that, as long as τ is not too big, our model has the property that when the economy has over-accumulated relative to the steady state (i.e., if X slightly exceeds its steady-state value), then consumption will be lower than in the steady state throughout the transition period toward the steady state, which we can refer to as a period of liquidation. In this sense, the economy is overreacting to its inherited excess of capital goods during this liquidation period, since it is reducing it expenditures to such an extent that that people are consuming less even though there are more goods available to them in the economy. While such a response is not socially optimal, it remains unclear whether it is so excessive as to make people worse off in comparison to the steady state, since they are also working less during the liquidation phase. It turns out that, as in the static case, the welfare effect of such a liquidation period depends, among other things, on whether the average dis-utility of work is small enough relative to the marginal dis-utility. For example, if the average dis-utility of work is sufficiently low relative to its marginal value, then it can be verified that a liquidation period induced by inheriting an excess of X relative to the steady state will make average utility in all periods of the transition lower than the steady state level of utility. This result depends in addition on the unemployment rate not being too large in the steady state.

[29] This is a slight abuse of language since Proposition 14 does not rule out the existence of other equilibrium paths away from the steady state.

While we do not have a simple characterization of the global dynamics when $\beta > 0$, Propositions 14 and 15 suggest to us that the intuition we gained from the case where $\beta = 0$ likely extends to the more general problem as long as τ is not too large and δ is small. In particular, we take our analysis as suggesting that, starting from $X = 0$, the economy will generally go though a phase of full employment, with both X and c increasing over time. The economy then enters into the unemployment range once X is large enough. Then, as long as τ is not too great, X will continue to monotonically increase, converging toward its steady state. In contrast to X, upon entering the unemployment regime, consumption starts to decrease as unemployment risk leads to precautionary savings which depresses activity. Eventually, the economy will reach a steady state where consumption, employment, and possibly period welfare are below the peak levels reached during the transition.

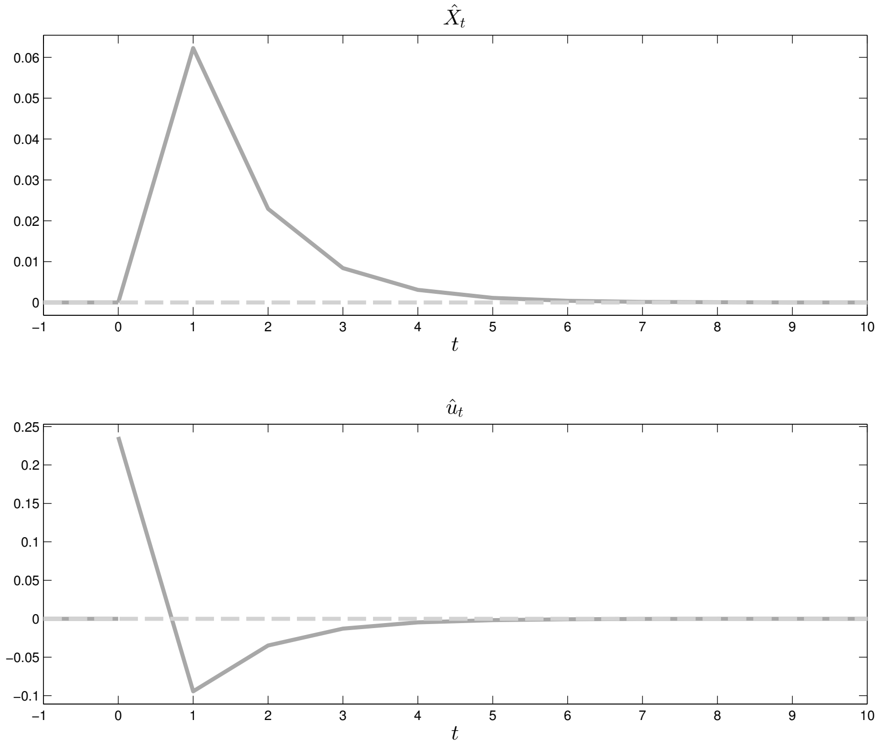
In the above discussion of liquidation, we have taken the level of inherited capital as given and have only examined how the economy responds over time to a situation where X is initially above its steady state. In particular, we have shown that such a liquidation phase can be associated with excessively low consumption, low welfare and high unemployment, all relative to their steady state values. While the focus of the paper is precisely to understand behavior during such a liquidation phase, it nonetheless remains interesting to ask how welfare would behave if we were to view the whole cycle, both the over-accumulation phase and the liquidation phase together. To briefly examine this issue, we build on the news-noise literature and consider a case where agents in an economy start at a steady state and then receive information about productivity.[30] Agents have to make their consumption decision based on the news, and we assume that they subsequently learn that the news is false. This leads to an initial high level of consumption during the period where agents are optimistic, followed by a period of low consumption during the liquidation phase after realizing that they had mistakenly over-accumulated. Details of this extension are presented in Appendix C.

In Figure 8 we report for illustration purposes two impulse responses associated with a simple calibration of such a noise-driven-boom-followed-by-liquidation model. We plot the dynamics for the stock of durables and the average period utility of households relative to the

[30] See Beaudry and Portier [2013] for a survey of this literature.

steady state. From the figure, we see that during the first period, when agents are acting on

Figure 8: Response of economy to a noise shock



Note: Impulse is associated with a 10% overly-optimistic belief by shoppers in the first sub-period of $t = 0$. $\hat{X}_t$ is the stock of durables and $\hat{u}_t$ is average period utility across all households, both expressed in deviations from steady state. Example is constructed assuming the functional forms $U(c) = \log(c)$, $\nu(l) = \frac{\nu_1 l^{1+\omega}}{1+\omega}$ and $F(l) = Al^{\alpha}$, with parameters $\beta = 0.9$, $\delta = 0.1$, $\gamma = 0.1$, $\omega = 1.2$, $\nu_1 = \nu_2 = 0.35$, $\alpha = 0.67$, $A = 1.2$, $\Phi = 0.5$ and $\tau = 0.3$.

optimistic beliefs about productivity, their period welfare increases even if they are working hard to ramp up their stocks of durable goods. After one period, they realize their error since productivity has not actually improved, and consequently cut back on their expenditures to start a liquidation process. The welfare of households from the second period on is lower than in steady state because of the excessively cautious behavior of households, which stops the economy from taking advantage of the excessively high inherited capital stock.

It is interesting to contrast this path with that which would happen if unemployment risk were perfectly insured or if matching frictions were absent. In such a case, the news would still lead to a boom, and the realization of the error would lead to a recession. However, the dynamics of period welfare would be very different. Instead of the boom being associated with high period welfare and the recession being associated with low period welfare, as in our model with unemployment risk, the opposite would happen. The boom would be associated with low period welfare, as agents would be working harder than normal, while in the recession welfare would be above the steady-state value since agents would take a vacation and benefit from past excess work. While evaluating welfare is certainly difficult, the path for period welfare in our model with unemployment risk appears to us as more in line with common perceptions about boom-bust cycles than that implied by a situation with no market frictions.

5 Policy trade-offs

In this last section, we turn to one of our motivating questions and ask whether or not stimulative policies should be used when an economy is going through a liquidation phase characterized by high unemployment. In particular, we consider the case where the economy has inherited from the past a level of X above its steady-state value and, in the absence of intervention, would experience a period of liquidation, with consumption below its steady-state level throughout the transition. Obviously, the first-best policies in this environment would be to remove the sources of frictions or to perfectly insure agents against unemployment risk. However, for a number of reasons, such-first best policies may not be possible. We therefore want to consider the value of a more limited type of policy: one that seeks only to temporarily boost expenditures. In particular, we are interested in asking whether welfare would be increased by stimulating expenditures for one period, knowing that this would imply a higher X tomorrow and therefore lower consumption in all subsequent periods until the liquidation is complete. This policy question is aimed at capturing the tension between the Keynesian and Hayekian prescriptions in recession. In answering this question, we will be examining the effects of such a policy without being very explicit about the precise policy tools used to engineer the stimulus, as we think it could come from several sources. However, it can be verified that the stimulus we consider can be engineered by a one period subsidy to consumption financed by a tax on the employed.

Examining how a temporary stimulus to expenditures affects welfare during a liquidation turns out to be quite involved. For this reason, we break down the question into two parts. First, we ask whether a temporary stimulus would increase welfare if the economy were initially in a steady state characterized by unemployment. Second, we ask whether the effect on welfare of such a stimulus would be greater if the economy were initially in a state of liquidation (i.e., with X_0 above its steady state) than in the case where it is initially at a steady state.

When looking at how a temporary boost in expenditures would affect welfare, one may expect it to depend on many factors, including the extent of risk-aversion and the dis-utility of work. However, since the level of expenditures represents a private optima, the present discounted welfare effect of a temporary boost in expenditures turns out to depend on a quite limited set of factors. In particular, if the economy is initially at a steady state in the unemployment regime, then to a first-order approximation the direction of the cumulative welfare effect depends simply on whether the stimulus induces an increase or decrease in the presented discounted value of the output stream. This is stated in Proposition 16.

Proposition 16. *Suppose the economy is in steady state in the unemployment regime. Then, to a first-order approximation, a (feasible) change in the path of expenditures from this steady state equilibrium will increase the present discounted value of expected welfare if and only if it increases the presented discounted sum of the resulting expenditure path,* $\sum_{i=0}^{\infty} \beta^i e_{t+i}$.

The logic behind Proposition 16 derives mainly from the envelope theorem. Since the consumption stream is optimally chosen from the individual's perspective, most of the effects of a change in the consumption path are only of second order and can therefore be neglected when the change is small. Moreover, in the unemployment region, prices, wages and hours worked are invariant to changes in expenditures. Hence the only effects needed to be taken into account for welfare purposes are the induced changes in the match probabilities times the marginal value of changing these match probabilities. When the economy is initially in a steady state, the marginal value of changing the match probabilities is the same at each point in time. Further, since the match probabilities are proportional to expenditures, this explains why welfare increases if and only if the perturbed path of expenditures has a positive presented discounted value. With this result in hand, it becomes rather simple to calculate whether, starting from steady state, a one-period increase in expenditures followed by a return to equilibrium decisions rules results in an increase in welfare. In particular, recall that the law of motion for X is given by

$$X_{t+1} = (1-\delta)X_t + \gamma e(X_t) \qquad 0 < \gamma < 1$$

where the function $e(X_t)$ is the equilibrium policy function for e_t. Now, beginning from steady state, suppose at $t = 0$ we stimulate expenditures by ϵ for one period such that the stock at $t = 1$ is now given by

$$\widetilde{X}_1 = (1-\delta)X_0 + \gamma(e + \epsilon)$$

As as result of this one-period perturbation, the path for expenditures for all subsequent periods will be changed even if there is no further policy intervention. The new sequence for X, which we denote $\widetilde{X}_t$, will be given by $\widetilde{X}_{t+1} = (1-\delta)\widetilde{X}_t + \gamma e(\widetilde{X}_t)$ for all $t \geq 1$. From Proposition 16, this perturbation increases present discounted welfare if and only if

$$\epsilon > -\sum_{t=1}^{\infty} \beta^t \left[e(\widetilde{X}_t) - e \right] \tag{25}$$

For ϵ small, we can use the linear approximation of the function $e(\cdot)$ around the steady state to make this calculation. Note that $e'(X) = -(1-\delta-\lambda_1)/\gamma$, where λ_1 is the smallest eigenvalue of the dynamic system in modulus.[31] Thus, in this case, one may show that condition (25) becomes

$$\frac{1-\beta(1-\delta)}{1-\beta\lambda_1} > 0$$

If the system is locally stable, then $\lambda_1 < 1$, and therefore this condition will always hold. Hence, if we are considering a situation where the economy is in an unemployment-regime steady state, and this steady state is locally stable, then a one-period policy of stimulating household expenditures will increase welfare. This arises even though most of the effect of the policy is to front-load utility by creating an initial boom followed by a liquidation bust.[32] While we knew that the initial steady state was sub-optimal, and that a policy that increases expenditures in all periods would likely be desirable, it is interesting to learn that a policy

[31] See the proof of Proposition 15.

[32] Note that this result does not depend on the welfare factors considered earlier in the static model, such as the magnitudes of τ and of the difference between the marginal and average disutility of work.

that favors expenditure today over expenditure tomorrow – when in the economy is in the unemployment regime – tends to increase welfare.

The question we now want to examine is whether the gains in welfare of a temporary stimulus are greater when the economy is initially in a liquidation phase than in steady state. We believe this is a relevant question since a case for stimulus during a liquidation can best be made if the gains are greater than when the economy is in steady state. Otherwise, there is no particular reason to favor stimuli more when unemployment is above normal than when it is at a normal level. Somewhat surprisingly to us, as long as U''' is not too big,[33] the answer to this question is negative, as stated in Proposition 17.

Proposition 17. *Assuming the economy's steady state is in the unemployment regime and U''' is not too big, then, to a second-order approximation around the steady state, a temporary stimulus increases the presented discounted value of welfare less when implemented during a liquidation phase then when implemented at the steady state.*

Although a period of liquidation is associated with a higher-than-normal level of unemployment, and the degree of distortion as captured by the labor wedge is higher in such periods when compared to the steady state, Proposition 17 indicates that the gains to a temporary stimulus are not greater during a liquidation period than in normal (steady-state) times. At first pass, one may be puzzled by this result, as one might have expected the gains to be highest when the marginal utility of consumption is highest. However, when the economy is in a liquidation phase, while the benefits from current stimulus are high, so are the costs associated with delaying the recovery. In fact, because consumption levels are at a private optimum, these two forces essentially cancel each other out. Moreover, when in the unemployment regime, the direct gain from employing one more individual – that is, the value of the additional production, net of the associated dis-utility of work – is the same regardless of whether unemployment is high or low. Hence, the only remaining difference between the value of stimulus in high- versus low-unemployment states relates to the net utility gain from employed workers entering the second sub-period in surplus rather than debt. In a lower-unemployment regime, households take less precaution, so that unemployed

[33] Note that this condition on U''' is sufficient but not necessary for this result.

workers end up with more debt, which is costly. It is this force which makes postponing an adjustment particularly costly when in a liquidation phase.[34]

With respect to the policy debate between the followers of Hayek and Keynes, we take our results are clarifying the scope of the arguments. On the one hand, we have found that a policy that stimulates current consumption at the cost of lower consumption in the future can often be welfare-improving when the economy features unemployment. However, at the same time, we have found that the rationale for such a policy does not increase simply because the level of unemployment is higher. Hence, if one believes that stimulus is not warranted in normal times (because of some currently un-modeled costs) and that normal times are characterized by excessive unemployment, then stimulus should not be recommended during liquidation periods. While this insight will likely not extinguish the debate on the issue, we believe it can help focus the dialogue.

6 Conclusion

There are three types of elements that motivated us to write this paper. First, there is the observation that most deep recessions arise after periods of fast accumulation of capital goods, either in the form of houses, consumer durables, or productive capital. This, in our view, gives plausibility to the hypothesis that recessions may often reflect periods of liquidation where the economy is trying to deplete excesses from past over-accumulation.[35,36] Second, during these apparent liquidation-driven recessions, the process of adjustment seems to be socially painful and excessive, in the sense that the level of unemployment does not seem to be consistent with the idea that the economy is simply "taking a vacation" after excessive past work. Instead, the economy seems to be exhibiting some coordination failure that makes the exploitation of gains from trade between individuals more difficult than in normal times. These two observations capture the tension we believe is often associated with

[34] There is an additional force at play here, which relates to the fact that the magnitude of the amplification mechanism will in general be different when the economy is away from the steady state. However, as long as U''' is not too big, this effect can safely be ignored.

[35] Note that this is a fundamentalist view of recessions, in that the main cause of a recession is viewed as an objective fundamental (in this case, the level of capital relative to technology) rather than a sunspot-driven change in beliefs.

[36] An alternative interpretation of this observation is that financial imbalances associated with the increase in capital goods are the main source of the subsequent recessions.

the Hayekian and Keynesian views of recessions. Finally, even when monetary authorities try to counter such recessions by easing policy, this does not seem to be fully effective. This leads us to believe that there are likely mechanisms at play beyond those related to nominal rigidities.[37] Hence, our objective in writing this paper was to offer a framework that is consistent with these three observations, and accordingly to provide an environment where the policy trade-offs inherent to the Hayekian and Keynesian views could be discussed.

A central contribution of the paper is to provide a simple macro model that explains, using real as opposed to nominal frictions, why an economy may become particularly inefficient when it inherits an excessive amount of capital goods from the past. The narrative behind the mechanism is quite straightforward. When the economy inherits a high level of capital, this decreases the desire for trade between agents in the economy, leading to less demand. When there are fixed costs associated with employment, this will generally lead to an increase in unemployment. If the risk of unemployment cannot be entirely insured away, households will react to the increased unemployment by increasing saving and thereby further depressing demand. This multiplier process will cause an excess reaction to the inherited goods and can be large enough to make society worse off even if – in a sense – it is richer since it has inherited a large stock of goods. Within this framework, we have shown that policies aimed at stimulating activity will face an unpleasant trade-off, as the main effect of stimulus will simply be to postpone the adjustment process. Nonetheless, we find that such stimulative policies may remain desirable even if they postpone recovery, but these gains do not increase simply because the rate of unemployment is higher. As noted, the mechanisms presented in the paper have many antecedents in the literature, but we believe that our framework offers a particularly tractable and clear way of capturing these ideas and of reconciling diverse views about the functioning of the macro-economy.

[37] We chose to analyze in this paper in an environment without any nominal rigidities so as to clarify the potential role of real rigidities in understanding behavior in recessions. However, in doing so, we are not claiming that the economy does not also exhibit nominal rigidities or that monetary policy is ineffective. We are simply suggesting that explanations based mainly on nominal rigidities may be missing important forces at play that cannot be easily overcome by monetary policy.

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Appendix

A Proofs of Propositions

Proof of Proposition 1

We first establish that there always exists an equilibrium of this model. Substituting equation (8) into equation (16) and letting $e \equiv c - X$ yields

$$U'(X+e) = \frac{\nu'(\ell)}{F'(\ell)}\left(1+\tau-\tau\frac{e}{\Omega(\ell)}\right) \tag{A.1}$$

where $\Omega(\ell) \equiv F'(\ell)\ell$ is output net of search costs per employed worker, which is assumed to be strictly increasing. When $N < L$ (i.e., the full-employment constraint is not binding), equation (9) implies that $\ell = \ell^\star$, and equation (8) implies that $e < e^\star$, where $e^\star \equiv \Omega(\ell^\star)$. On the other hand, when $N > L$ (i.e., the full-employment constraint binds), equation (8) implies that $\ell = \Omega^{-1}(e)$. Further, since $\min\{N, L\} < N$ and $F(\ell) - F'(\ell)\ell$ is assumed to be strictly increasing in ℓ, equation (9) implies that $\ell > \ell^\star$, and thus, by strict increasingness of Ω, we also have $e > e^\star$. Substituting these results into equation (A.1) yields that $e > 0$ is an equilibrium of this model if it satisfies

$$U'(X+e) = Q(e) \tag{A.2}$$

where the function $Q(e)$, defined in equation (14), is the expected marginal utility cost of consumption when aggregate expenditures are $e = c - X$. Note that Q is continuous, strictly decreasing on $[0, e^\star]$, and strictly increasing on $[e^\star, \infty)$.

Lemma A.1. *If $U'(X) \le Q(0)$, then there is an equilibrium with $e = 0$.*

Proof. To see this, suppose aggregate conditions are that $e = 0$. Then the marginal utility of consumption when the household simply consumes its endowment is no greater than its expected marginal cost, and thus households respond to aggregate conditions by making no purchases, which in turn validates $e = 0$. □

Lemma A.2. *If $U'(X) > Q(0)$, then there is an equilibrium with $e > 0$.*

Proof. We have that $\min Q(e) = \nu'(\ell^\star)/F'(\ell^\star) > 0$. Since we have assumed $\lim_{c\to\infty} U'(c) \leq 0$, it necessarily follows that for any X, there exists an e sufficiently large that $U'(X+e) < \min Q(e)$, and therefore, by the intermediate value theorem, there must exist a solution $e > 0$ to equation (A.2). □

Lemmas A.1 and A.2 together imply that an equilibrium necessarily exists. We turn now to showing under what conditions this equilibrium is unique for all values of X. As in equation (12), we may represent household j's optimal expenditure when aggregate expenditure is e as $e_j(e) = U'^{-1}(Q(e)) - X$, so that equilibrium is a fixed point $e_j(e) = e$. The function $e_j(e)$ is continuous everywhere, and differentiable everywhere except at $e = e^\star$, with

$$e_j'(e) = \frac{Q'(e)}{U''\left(U'^{-1}(Q(e))\right)}$$

Note that $e_j'(e)$ is independent of X, strictly increasing on $[0, e^\star]$ and strictly decreasing on $[e^\star, \infty)$.

Lemma A.3. *If*

$$\lim_{e\uparrow e^\star} e_j'(e) < 1 \tag{A.3}$$

then $e_j'(e) < 1$ *for all* e.

Proof. Note first that $e_j'(e) < 0$ for $e > e^\star$, so that this condition is obviously satisfied in that case. For $e < e^\star$, note that

$$e_j''(e) = \frac{Q''(e) - U'''\left(X + e_j(e)\right)\left[e_j'(e)\right]^2}{U''\left(X + e_j(e)\right)}$$

Since $Q''(e) = 0$ on this range and $U''' > 0$, we have $e_j''(e) > 0$, and thus $e_j'(e) < \lim_{e\uparrow e^\star} e_j'(e)$, which completes the proof. □

Lemma A.4. *Inequality* (A.3) *holds if and only if*

$$\tau < \bar{\tau} \equiv -U''\left(U'^{-1}\left(\frac{\nu'(\ell^\star)}{f'(\ell^\star)}\right)\right)\frac{f'(\ell^\star)\left[f(\ell^\star) - \Phi\right]}{\nu'(\ell^\star)}$$

Proof. We have that

$$\lim_{e\uparrow e^\star} e_j'(e) = \frac{\nu'(\ell^\star)\tau}{-U''\left(U'^{-1}\left(\frac{\nu'(\ell^\star)}{f'(\ell^\star)}\right)\right) f'(\ell^\star)\left[f(\ell^\star) - \Phi\right]}$$

which is clearly less than one if and only if $\tau < \bar{\tau}$. □

Lemma A.5. *If $\tau < \bar{\tau}$, then there always exists a unique equilibrium regardless of the value of X. If $\tau > \bar{\tau}$, then there exists values of $X \in \mathbb{R}$ such that there are multiple equilibria.*

Proof. We have already established that there always exists an equilibrium. Note that equilibrium occurs at the point where the $e_j = e_j(e)$ locus intersects with the locus characterizing the equilibrium condition, i.e., $e_j = e$. To see the first part of the lemma, suppose $\tau < \bar{\tau}$ so that inequality (A.3) holds. Then since the slope of the equilibrium locus is one, and the slope of the $e_j = e_j(e)$ locus is strictly less than one by Lemma A.3, there can be at most one intersection, and therefore the equilibrium is unique.

To see the second part of the lemma, suppose that $\tau > \bar{\tau}$ and thus (A.3) does not hold. Then by strict convexity of $e_j(e)$ on $(0, e^\star)$, there exists a value $\underline{e} < e^\star$ such that $e_j'(e) > 1$ on $(\underline{e}, e^\star)$. Define $\widetilde{X}(e) \equiv U'^{-1}(Q(e)) - e$, and note that e is an equilibrium when $X = \widetilde{X}(e)$. We show that there are at least two equilibria when $X = \widetilde{X}(e)$ with $e \in (\underline{e}, e^\star)$. To see this, choose $e_0 \in (\underline{e}, e^\star)$, and note that, for $X = \widetilde{X}(e_0)$, $e_j(e_0) = e_0$ and $e_j'(e) > 1$ on $(e_0, e^\star)$. Thus, it must also be the case that $e_j(e^\star) > e^\star$. But since $e_j(e)$ is continuous everywhere and strictly decreasing on $e > e^\star$, this implies that there exists some value $e > e^\star$ such that $e_j(e) = e$, which would represent an equilibrium. Since $e_0 < e^\star$ is also an equilibrium, there are at least two equilibria. □

This completes the proof of Proposition 1.

Proof of Proposition 2

Lemma A.6. *If $\tau < \bar{\tau}$ and X is such that $e > 0$, then $de/dX < 0$.*

Proof. Totally differentiating equilibrium condition (A.2) with respect to X yields

$$\frac{de}{dX} = \frac{U''(X+e)}{Q'(e) - U''(X+e)} \tag{A.4}$$

From Lemma A.4, we see that $Q'(e) > U''\left(U'^{-1}(Q(e))\right)$. In equilibrium, $U'^{-1}(Q(e)) = X + e$, so that this inequality becomes $Q'(e) > U''(X+e)$, and thus the desired conclusion follows by inspection. □

Given Lemma A.6 and the fact that the economy exhibits unemployment when $e < e^\star$ and full employment when $e \geq e^\star$, it is clear that the economy will exhibit unemployment if and only if X is smaller than the level such that $e = e^\star$ is the equilibrium; that is, if $X \leq X^\star$, where

$$X^\star \equiv U'^{-1}\left(\frac{\nu'(\ell^\star)}{F'(\ell^\star)}\right) - F'(\ell^\star)\ell^\star$$

This completes the proof of the first part of the proposition.

Next, from Lemma A.1, we see that there is a zero-employment equilibrium if and only if $U'(X) \leq \frac{\nu'(\ell^\star)}{F'(\ell^\star)}(1+\tau)$, which holds when $X \geq X^{\star\star}$, where

$$X^{\star\star} \equiv U'^{-1}\left(\frac{\nu'(\ell^\star)}{F'(\ell^\star)}(1+\tau)\right)$$

This completes the proof of Proposition 2.

Proof of Proposition 3

If $X < X^{\star\star}$, we know from Proposition 2 that $e > 0$, and therefore e solves equation (A.2). Substituting $e = c - X$ for e yields the desired result in this case. From Proposition 2, we also know that if $X \geq X^{\star\star}$ then $e = 0$, in which case $c = X$, which completes the proof.

Proof of Proposition 4

If $X > X^{\star\star}$, so that the economy features zero employment and therefore $c = X$, then clearly c is increasing in X. Thus, suppose $X < X^{\star\star}$, so that $e > 0$. Totally differentiating the expression $c = X + e$ with respect to X and using equation (A.4), we obtain

$$\frac{dc}{dX} = \frac{Q'(e)}{Q'(e) - U''(X+e)} \tag{A.5}$$

Since the denominator of this expression is positive (see the proof of Lemma A.6), the sign of dc/dX is given by the sign of $Q'(e)$, which is negative if $e < e^\star$ (i.e., if $X^\star < X < X^{\star\star}$) and positive if $e > e^\star$ (i.e., if $X < X^\star$). This completes the proof.

Proof of Proposition 5

Letting $\mathcal{U}(e)$ denote welfare conditional on the coordinated level of e, we may obtain that

$$\mathcal{U}(e) = U(X+e) + \mu(e)\left[\mathcal{L}^\star - \frac{\nu'(\ell^\star)}{F'(\ell^\star)}e\right] - [1-\mu(e)](1+\tau)\frac{\nu'(\ell^\star)}{F'(\ell^\star)}e$$

where $\mu(e) = e/[F'(\ell^\star)\ell^\star]$ denotes employment conditional on e. Using the envelope theorem, it is straightforward to see that the only welfare effects of a marginal change in e from its decentralized equilibrium value are those that occur through the resulting change in employment. Thus,

$$\mathcal{U}'(e) = \left[\mathcal{L}^\star + \tau\frac{\nu'(\ell^\star)}{F'(\ell^\star)}e\right]\mu'(e) > 0$$

where $\mathcal{L}^\star \equiv \nu'(\ell^\star)\ell^\star - \nu(\ell^\star) \geq 0$, and therefore a coordinated rise in e would increase expected utility of all households.

Proof of Proposition 6

Denote welfare as a function of X by

$$\mathcal{U}(X) \equiv U(X+e) + \mu\left[-\nu(\ell) + V(w\ell - pe)\right] + (1-\mu)V(-pe)$$

If $X < X^\star$, so that the economy is in the full-employment regime, or if $X > X^{\star\star}$, so that the economy is in the zero-employment regime, we may show that $\mathcal{U}'(X) > 0$ always holds. Thus, we focus on the case where $X \in (X^\star, X^{\star\star})$. When this is true, some algebra yields

$$\mathcal{U}(X) = U(X+e) + \left\{\ell^\star\left[\nu'(\ell^\star) - \frac{\nu(\ell^\star)}{\ell^\star}\right] + \frac{\nu'(\ell^\star)}{F'(\ell^\star)}\tau e\right\}\mu - \frac{\nu'(\ell^\star)}{F'(\ell^\star)}(1+\tau)e$$

Using the envelope theorem, we may differentiate this expression with respect to X to obtain

$$\mathcal{U}'(X) = U'(X+e) + \left[\mathcal{L}^\star + \frac{\nu'(\ell^\star)}{F'(\ell^\star)}\tau e\right]\frac{d\mu}{dX} \tag{A.6}$$

where $\mathcal{L}^\star \equiv \nu'(\ell^\star)\ell^\star - \nu(\ell^\star) \geq 0$.

Lemma A.7. *$\mathcal{U}''(X) > 0$ on $(X^\star, X^{\star\star})$.*

Proof. Substituting the equilibrium condition (A.2) into (A.6) and using the fact that

$$\frac{d\mu}{dX} = \frac{1}{F'(\ell^\star)\ell^\star}\frac{de}{dX}$$

after some algebra, we obtain

$$\mathcal{U}'(X) = \frac{\nu'(\ell^\star)}{F'(\ell^\star)}\left[1 + \tau + \tau\mu\left(\frac{de}{dX} - 1\right)\right] + \frac{\mathcal{L}^\star}{F'(\ell^\star)\ell^\star}\frac{de}{dX} \tag{A.7}$$

From (A.4), we may also obtain that

$$\frac{de}{dX} = \left(\frac{\nu'(\ell^\star)\tau}{-U''(X+e)\left[F'(\ell^\star)\right]^2 \ell^\star} - 1 \right)^{-1}$$

$$\frac{d^2e}{dX^2} = \frac{U'''(X+e)}{U''(X+e)} \frac{de}{dX} \left[\frac{dc}{dX} \right]^2 > 0$$

and therefore

$$\mathcal{U}''(X) = \frac{\nu'(\ell^\star)}{F'(\ell^\star)} \tau \frac{d\mu}{dX} \left(\frac{de}{dX} - 1 \right) + \left[\frac{\nu'(\ell^\star)}{F'(\ell^\star)} \tau\mu + \frac{\mathcal{L}^\star}{F'(\ell^\star) l^\star} \right] \frac{d^2e}{dX^2}$$

Since $de/dX < 0$, $d\mu/dx < 0$, and thus the first term is positive, as is the second term, and the proof is complete. □

Lemma A.8. *If*

$$\tau > \underline{\tau} \equiv \frac{\nu(\ell^\star)}{\nu'(\ell^\star)\ell^\star} \left(\frac{\bar{\tau}}{1+\bar{\tau}} \right)$$

then there exists a range of X *such that* $\mathcal{U}'(X) < 0$.

Proof. Since $\mathcal{U}$ is convex by Lemma A.7, $\mathcal{U}'(X) < 0$ for some values of X if and only if $\lim_{X \downarrow X^\star} \mathcal{U}'(X) < 0$. Taking limits of equation (A.7), and using the facts that

$$\lim_{X \downarrow X^\star} \frac{de}{dX} = -\frac{\bar{\tau}}{\bar{\tau} - \tau}$$

and $\lim_{X \downarrow X^\star} \mu = 1$, we obtain that

$$\lim_{X \downarrow X^\star} \mathcal{U}'(X) = \frac{\nu'(\ell^\star)}{F'(\ell^\star)} \left(1 - \frac{\tau\bar{\tau}}{\bar{\tau} - \tau} \right) - \frac{\mathcal{L}^\star}{F'(\ell^\star)\ell^\star} \left(\frac{\bar{\tau}}{\bar{\tau} - \tau} \right)$$

Substituting in from the definition of $\mathcal{L}^\star$, straightforward algebra yields that this expression is less than one if and only if $\tau > \underline{\tau}$. □

Note that, by convexity of $\nu(\ell)$ and the fact that $\nu(0) = 0$, we have $\nu(\ell^\star) \leq \nu'(\ell^\star)\ell^\star$, and thus $\underline{\tau} < \bar{\tau}$, so that there always exists values of τ such that $\underline{\tau} < \tau < \bar{\tau}$. From the definition of $\underline{\tau}$, we also see that, holding τ and $\nu'(\ell^\star)$ constant, if $\nu(\ell^\star)/\ell^\star$ is small, this inequality is more likely to be satisfied.

Proof of Proposition 7

We suppose there is a competitive insurance industry offering a menu of unemployment insurance contracts. A typical contract is denoted (h, q), where h is the premium, paid in all states, and q is the coverage, which the purchaser of the contract receives if and only if he is unemployed. Both h and q are expressed in units of good 1. Since insurance is only potentially useful when $0 < \mu < 1$, we henceforth assume that this is true. Note also that zero profit of insurers requires that $h = (1 - \mu\hat{\rho})q$, where $\hat{\rho}$ is the fraction of purchasers of the contract that are participant households. This implies that non-participant households will not purchase any such zero-profit contract featuring $q < 0$.

Lemma A.9. *In any separating equilibrium, no contracts are purchased by participant households.*[38]

Proof. Suppose there is a separating equilibrium, and let (h_p, q_p) denote the contract purchased by participant households, and (h_n, q_n) that purchased by non-participant households. From the insurer's zero-profit condition, we must have $h_p = (1 - \mu)q_p$ and $h_n = q_n$. Since non-participant households will always deviate to any contract with $h_p < q_p$, this implies that we must have $q_p < 0$ in such an equilibrium.

Next, for any zero-profit separating contract, the assets of employed participant households are given by $A_e = w\ell - p[(1 - \mu)q_p + e]$ and of unemployed participant households by $A_u = p(\mu q_p - e)$. Note that, since $q_p < 0$ and from the resource constraint $wl > pc$, we must have $A_u < 0 < A_e$. Also, the derivative of the household's objective function with respect to q_p along the locus of zero-profit contracts is given by

$$\frac{\partial \mathcal{U}}{\partial q_p} = p\mu(1 - \mu)\left[V'(A_u) - V'(A_e)\right] > 0$$

wherever such a derivative exists. Since $A_u < 0 < A_e$, this derivative must exist at the candidate equilibrium, and therefore in a neighborhood of that equilibrium the objective function is strictly increasing on $q_p < 0$. Thus, given any candidate zero-profit equilibrium contract with $q_p < 0$, there exists an alternative contract (h'_p, q'_p) with $q'_p > q_p$ which satisfies

[38] Technically, agents are always indifferent between not purchasing a contract and purchasing the contract $(0, 0)$. For ease of terminology, we will assume that the contract $(0, 0)$ does not exist.

that $h'_p - (1-\mu)q'_p$ is strictly greater than but sufficiently close to zero so that participant households would choose it over (h_p, q_p), while non-participant households would not choose it, and therefore insurers could make a positive profit selling it. Thus, (h_p, q_p) cannot be an equilibrium contract. Since this holds for all $q_p < 0$, it follows that no separating equilibrium exists in which contracts are purchased by participant households. $\square$

Next, consider a pooling equilibrium, so that $\hat{\rho} = \rho$. As argued above, we must have $q \geq 0$ in any such equilibrium. Assets of an employed worker when choosing a zero-profit pooling contract $(h, q) = ((1-\mu\rho)q, q)$ are given by $A_e = w\ell - p[(1-\mu\rho)q + e]$, while $A_u = p(\mu\rho q - e)$ are those of an unemployed worker. Let $\mathcal{U}(q)$ denote the value of the household's objective function when choosing such a zero-profit pooling contract.

Lemma A.10. *If $\mathcal{U}(q)$ is strictly decreasing in q whenever $A_e > A_u$, then a pooling equilibrium does not exist.*

Proof. Note first that if $A_e \leq A_u$, then being unemployed is always strictly preferred to being employed by participant households, so that this cannot represent an equilibrium. Furthermore, as argued above, we must have $q \geq 0$ in any pooling equilibrium. Thus, suppose $A_e > A_u$ and $q > 0$. We show that such a q cannot represent an equilibrium. To see this, let (h', q') denote an alternative contract with $0 < q' < q$ and $h' = (1-\mu\rho)q'$. Since $\mathcal{U}$ is strictly decreasing in q, this contract is strictly preferred by participant households. Furthermore, since non-participant households would get net payment $\mu\rho(q' - q) < 0$ from deviating to this new contract, only participant households would deviate to it, and therefore the expected profit to an insurer offering it would be $(1-\rho)\mu q' > 0$. Thus, this deviation is mutually beneficial for participants and insurers, and so q cannot be an equilibrium. $\square$

Lemma A.11. *If $\rho < 1/(1+\tau)$, then there is no equilibrium in which an insurance contract is purchased by participant households.*

Proof. Note that $\mathcal{U}(q)$ is continuous, with

$$\mathcal{U}'(q) = p\mu \left[(1-\mu)\rho V'(A_u) - (1-\mu\rho)V'(A_e)\right]$$

wherever this derivative exists (i.e., whenever $A_e A_u \neq 0$). If $A_e A_u > 0$, then $V'(A_e) = V'(A_u)$, and therefore $\mathcal{U}'(q) = -p\mu(1-\rho)V'(A_e) < 0$. Suppose on the other hand that

$A_e A_u < 0$. If in addition $A_e > A_u$, we must have $A_u < 0 < A_e$, and therefore $\mathcal{U}'(q) = -pv\mu\{1-\rho[1+\tau(1-\mu)]\}$. Since $\rho < 1/(1+\tau)$, it follows that $\mathcal{U}'(q) < 0$. Thus, $\mathcal{U}(q)$ is strictly decreasing whenever $A_e > A_u$, and therefore by Lemma A.10, no pooling equilibrium exists. Since, by Lemma A.9, there does not exist a separating equilibrium either, no equilibrium exists. □

Proof of Proposition 8

We may re-write the equilibrium condition (15) as

$$U'(X + e - G_w) = Q(e) \tag{A.8}$$

where $Q(e)$ is as defined in equation (14).

That non-wasteful government purchases have no effect on economic activity can be seen directly from the fact that G_n does not appear in equation (A.8). Totally differentiating equation (A.8) with respect to G_w, we obtain

$$\frac{de}{dG_w} = \frac{-U''(X + e - G_w)}{Q'(e) - U''(X + e - G_w)}$$

Under the assumption that $\tau < \bar{\tau}$, the denominator of this expression is positive, and thus $de/dG_w > 0$. Further, if the economy is in the unemployment regime, then $Q'(e) < 0$ and therefore $de/dG_w > 1$, while if the economy is in the unemployment regime, then $Q'(e) > 0$ and therefore $de/dG_w < 1$, which completes the proof.

Proof of Proposition 9

First, note that a balanced budget requires that employed workers be taxed $(G_n + G_w)/\mu$. Letting $e_p = e - G_n - G_w$ denote private expenditures, we may therefore obtain welfare as a function of X, G_n and G_w as

$$\mathcal{U}(X, G_n, G_w) = U(X + e_p + G_n) + \frac{\nu'(\ell^\star)}{F'(\ell^\star)}\left[\left(\frac{F'(\ell^\star)}{\nu'(\ell^\star)}\mathcal{L}^\star + \tau e_p\right)\mu \quad -G_n - G_w - (1+\tau)e_p\right] \tag{A.9}$$

where as before $\mathcal{L}^\star \equiv \nu'(\ell^\star)\ell^\star - \nu(\ell^\star)$. Taking derivatives with respect to G_w and applying the envelope theorem, we may obtain that

$$\mathcal{U}_3(X, G_n, G_w) = \frac{\nu'(\ell^\star)}{F'(\ell^\star)}\left[\left(\frac{F'(\ell^\star)}{\nu'(\ell^\star)}\mathcal{L}^\star + \tau e_p\right)\frac{d\mu}{dG_w} - 1\right] \tag{A.10}$$

Meanwhile, differentiating (A.9) with respect to X, applying the envelope theorem and using the equilibrium condition $U'(X + e_p + G_n) = Q(e_p + G_n + G_w)$, we may obtain that

$$\frac{F'(\ell^\star)}{\nu'(\ell^\star)}\mathcal{L}^\star + \tau e_p = \frac{F'(\ell^\star)}{\nu'(\ell^\star)}\left[\mathcal{U}_1(X, G_n, G_w) - \frac{\nu'(\ell^\star)}{F'(\ell^\star)}(1 + \tau - \tau\mu)\right]\left(\frac{d\mu}{dX}\right)^{-1} \tag{A.11}$$

We may also obtain from the equilibrium condition that $d\mu/dG_w = -d\mu/dX$. Substituting this and (A.11) into (A.10), we may obtain

$$\mathcal{U}_3(X, G_n, G_w) = \frac{\nu'(\ell^\star)}{F'(\ell^\star)}(1 - \mu)\tau - \mathcal{U}_1(X, G_n, G_w)$$

Since the first term on the left-hand side is positive, if X is in the range such that $\mathcal{U}_1(X, G_n, G_w) < 0$, then we necessarily have $\mathcal{U}_3(X, G_n, G_w) > 0$, which completes the proof.

Proof of Proposition 10

Let $m(N) \equiv M(N, L)$ and note that the restrictions on M imply, among other things, that (a) $m(0) = 0$, (b) $m'(0) \in (0, 1]$, and (c) $\lim_{N\to\infty} m'(N) = 0$. We have that

$$\lim_{N\to 0}\frac{m'(N)\,N}{m(N)} = \lim_{N\to 0}\frac{m'(N)}{[m(N) - m(0)]/N}$$

where we have used property (a). The limit of the numerator is clearly just $m'(0)$, while the limit of the denominator is, by definition, also equal to $m'(0)$ and thus, since by property (b) $m'(0)$ is non-zero and bounded, we have that

$$\lim_{N\to 0}\frac{m'(N)\,N}{m(N)} = \frac{m'(0)}{m'(0)} = 1$$

Next, suppose $\lim_{N\to\infty} m'(N)N/m(N) > 0$. Since $0 < \lim_{N\to\infty} m(N) < \infty$, this implies that $\lim_{N\to\infty} N/g(N) > 0$ where $g(N) \equiv 1/m'(N)$. This in turn implies that $g(N) = O(N)$ as $N \to \infty$, or, equivalently, that there exists an $N_0 > 0$ such that, for $N \geq N_0$,

$$\frac{g'(N)}{g(N)} \leq \frac{1}{N}$$

where the right-hand side of this inequality is simply the growth rate of N. We may therefore obtain, for $N \geq N_0$,

$$\begin{aligned} g(N) &= g(N_0)\exp\left\{\int_{N_0}^{N}\frac{g'(s)}{g(s)}ds\right\} \\ &\leq g(N_0)\exp\left\{\int_{N_0}^{N}\frac{1}{s}ds\right\} \\ &= \frac{g(N_0)N}{N_0} \end{aligned}$$

and thus $m'(N) \geq m'(N_0)N_0/N$. But

$$\begin{aligned} m(N) &= m(N_0) + \int_{N_0}^{N} m'(s)ds \\ &\geq m(N_0) + m'(N_0)N_0 \int_{N_0}^{N} \frac{1}{s} ds \\ &= m(N_0) + m'(N_0)N_0 \left[\log(N) - \log(N_0)\right] \end{aligned}$$

The expression on the last line above is clearly unbounded as $N \to \infty$, which would imply the same for $m(N)$, a clear contradiction of the requirement that $M(N, L) \leq L$. Thus, we cannot have $\lim_{N\to\infty} m'(N)N/m(N) > 0$, i.e., we must have $\lim_{N\to\infty} m'(N)N/m(N) = 0$.

Proof of Proposition 11

The following result will be useful.

Lemma A.12. *Let $\mathcal{E}^g_{X\mathcal{M}}$ denote the elasticity of substitution between X and $\mathcal{M}$ embodied in g. Then*

$$\mathcal{E}^g_{X\mathcal{M}} = \frac{g_X(X,\mathcal{M})g_{\mathcal{M}}(X,\mathcal{M})}{g_{X\mathcal{M}}(X,\mathcal{M})g(X,\mathcal{M})} \tag{A.12}$$

Proof. Letting H^k denote homogeneity of degree k, note first that, since g is H^1, for $a, b \in \{X, \mathcal{M}\}$, g_a is H^0 and g_{ab} is H^{-1}.

Next, by definition, we have

$$\mathcal{E}^g_{X\mathcal{M}} \equiv \left[\frac{d\log\left(g_X(X,\mathcal{M})/g_{\mathcal{M}}(X,\mathcal{M})\right)}{d\log\left(\mathcal{M}/X\right)}\right]^{-1}$$

Letting $\widetilde{\mathcal{M}} \equiv \mathcal{M}/X$ and using H^0 of g_X and $g_{\mathcal{M}}$, we may obtain

$$\begin{aligned} \mathcal{E}^g_{X\mathcal{M}} &= \frac{g_X(1,\widetilde{\mathcal{M}})}{g_I(1,\widetilde{\mathcal{M}})\widetilde{\mathcal{M}}}\left[\frac{d}{d\widetilde{\mathcal{M}}}\left(\frac{g_X(1,\widetilde{\mathcal{M}})}{g_{\mathcal{M}}(1,\widetilde{\mathcal{M}})}\right)\right]^{-1} \\ &= \frac{g_X(1,\widetilde{\mathcal{M}})g_{\mathcal{M}}(1,\widetilde{\mathcal{M}})}{\widetilde{\mathcal{M}}\left[g_{X\mathcal{M}}(1,\widetilde{\mathcal{M}})g_I(1,\widetilde{\mathcal{M}}) - g_X(1,\widetilde{\mathcal{M}})g_{\mathcal{M}\mathcal{M}}(1,\widetilde{\mathcal{M}})\right]} \\ &= \frac{g_X(X,\mathcal{M})g_{\mathcal{M}}(X,\mathcal{M})}{\mathcal{M}\left[g_{X\mathcal{M}}(X,\mathcal{M})g_{\mathcal{M}}(X,\mathcal{M}) - g_X(X,\mathcal{M})g_{\mathcal{M}\mathcal{M}}(X,\mathcal{M})\right]} \end{aligned}$$

where the last line follows from H^0 of g_a and H^{-1} of g_{ab}. Adding and subtracting $g_{X\mathcal{M}}(X,\mathcal{M})g_X(X,\mathcal{M})X$ in the denominator and grouping terms yields

$$\mathcal{E}^g_{X\mathcal{M}} = \frac{g_X(X,\mathcal{M})g_{\mathcal{M}}(X,\mathcal{M})}{g_{X\mathcal{M}}(X,\mathcal{M})\left[g_X(X,\mathcal{M})X + g_{\mathcal{M}}(X,\mathcal{M})\mathcal{M}\right] - g_X(X,\mathcal{M})\left[g_{X\mathcal{M}}(X,\mathcal{M})X + g_{\mathcal{M}\mathcal{M}}(X,\mathcal{M})\mathcal{M}\right]}$$

The first bracketed term in the denominator equals $g(X, \mathcal{M})$ by H^1 of g, while the second bracketed term equals 0 by H^0 of $g_{\mathcal{M}}$, and thus equation (A.12) follows. $\square$

Next, let $W(X, \mathcal{M}) \equiv U(g(X, \mathcal{M}))$. Then the equilibrium condition (17) can be written

$$W_{\mathcal{M}}(X, \mathcal{M}) = Q(\mathcal{M}) \tag{A.13}$$

Note that

$$W_{\mathcal{M}\mathcal{M}}(X, \mathcal{M}) = [g_{\mathcal{M}}(X, \mathcal{M})]^2 U''(g(X, \mathcal{M})) + g_{\mathcal{M}\mathcal{M}} U'(g(X, \mathcal{M})) < 0$$

so that the left-hand side of equation (A.13) is strictly decreasing in $\mathcal{M}$. To ensure the existence of an equilibrium with $\mathcal{M} > 0$, we assume that $W_{\mathcal{M}}(X, 0) > Q(0)$. We further assume that $g_{\mathcal{M}\mathcal{M}\mathcal{M}}(X, \mathcal{M}) \geq 0$, which ensures that $Q_{\mathcal{M}\mathcal{M}\mathcal{M}} > 0$, and therefore, similar to in the durable-goods model, there are at most three equilibria: at most two in the unemployment regime, and at most one in the full-employment regime. Additional conditions under which we can ensure that there exists a unique equilibrium are similar in flavor to in the durable-goods case, though less easily characterized explicitly. We henceforth simply assume conditions are such that the equilibrium is unique, and note that this implies that

$$W_{\mathcal{M}\mathcal{M}}(X, \mathcal{M}) < Q'(\mathcal{M}) \tag{A.14}$$

at the equilibrium value of $\mathcal{M}$. Define also

$$\mathcal{E}^Q_{\mathcal{M}} \equiv \frac{Q'(\mathcal{M})\mathcal{M}}{Q(\mathcal{M})}$$

as the elasticity of Q with respect to $\mathcal{M}$.

Lemma A.13. *$dc/dX < 0$ if and only if*

$$-\mathcal{E}^Q_{\mathcal{M}} \mathcal{E}^g_{X\mathcal{M}} > 1 \tag{A.15}$$

Proof. Totally differentiating the equilibrium condition (A.13) with respect to X yields that

$$\frac{d\mathcal{M}}{dX} = \frac{W_{X\mathcal{M}}(X, \mathcal{M})}{Q'(\mathcal{M}) - W_{\mathcal{M}\mathcal{M}}(X, \mathcal{M})} \tag{A.16}$$

Doing the same with the equilibrium condition $c = g(X, \mathcal{M})$ yields

$$\begin{aligned}
\frac{dc}{dX} &= g_X(X,\mathcal{M}) + g_{\mathcal{M}}(X,\mathcal{M})\frac{d\mathcal{M}}{dX} \\
&= \frac{g_X(X,\mathcal{M})\left[Q'(\mathcal{M}) - W_{\mathcal{M}\mathcal{M}}(X,\mathcal{M})\right] + g_{\mathcal{M}}(X,\mathcal{M})W_{X\mathcal{M}}(X,\mathcal{M})}{Q'(\mathcal{M}) - W_{\mathcal{M}\mathcal{M}}(X,\mathcal{M})}
\end{aligned}$$

where the second line has used (A.16). By (A.14), the denominator is positive, so that this expression is of the same sign as the numerator. Substituting in for $W_{\mathcal{M}\mathcal{M}}$ and $W_{X\mathcal{M}}$ and using the equilibrium condition (A.13), we may obtain that $dc/dX < 0$ if and only if

$$\left[\frac{g_{X\mathcal{M}}(X,\mathcal{M})}{g_X(X,\mathcal{M})} - \frac{g_{\mathcal{M}\mathcal{M}}(X,\mathcal{M})}{g_{\mathcal{M}}(X,\mathcal{M})}\right]\mathcal{M} < -\mathcal{E}^Q_{\mathcal{M}} \tag{A.17}$$

The term in square brackets, meanwhile, can be written as

$$\frac{g_{X\mathcal{M}}(X,\mathcal{M})\left[g_X(X,\mathcal{M})X + g_{\mathcal{M}}(X,\mathcal{M})\mathcal{M}\right] - g_X(X,\mathcal{M})\left[g_{X\mathcal{M}}(X,\mathcal{M})X + g_{\mathcal{M}\mathcal{M}}(X,\mathcal{M})\mathcal{M}\right]}{g_X(X,\mathcal{M})g_{\mathcal{M}}(X,\mathcal{M})\mathcal{M}}$$

By H^0 of $g_{\mathcal{M}}$, the second term in the numerator equals zero, and thus by H^1 of g, we have that

$$\frac{g_{X\mathcal{M}}(X,\mathcal{M})}{g_X(X,\mathcal{M})} - \frac{g_{\mathcal{M}\mathcal{M}}(X,\mathcal{M})}{g_{\mathcal{M}}(X,\mathcal{M})} = \frac{g_{X\mathcal{M}}(X,\mathcal{M})g(X,\mathcal{M})}{g_X(X,\mathcal{M})g_{\mathcal{M}}(X,\mathcal{M})\mathcal{M}}$$

Substituting this into (A.17) and using (A.12) yields (A.15). □

If the economy is in the full-employment regime, $\mathcal{E}^Q_{\mathcal{M}} > 0$ and therefore, since $\mathcal{E}^g_{X\mathcal{M}} > 0$, condition (A.15) cannot hold. Thus, from Lemma A.13, if the economy is in the full-employment regime, $dc/dX > 0$. If instead the economy is in the unemployment regime, then $\mathcal{E}^Q_{\mathcal{M}} < 0$, and therefore condition (A.15) can hold as long as $\mathcal{E}^g_{X\mathcal{M}}$ is sufficiently large, which completes the proof of the proposition.

Proof of Proposition 12

Let $y = g(X_1, \mathcal{M})$ denote output of the final good in the first period. Furthermore, let $B(X_2) \equiv U'^{-1}(R(X_2)) + X_2$ denote the total resources (output plus undepreciated first-period capital) that would be required for the choice X_2 to satisfy the constraints (18) and (19) as well as the intertemporal optimality condition (21), and note that

$$B'(X_2) = \frac{R'(X_2)}{U''(c)} + 1 > 1 \tag{A.18}$$

where the inequality follows from the assumption made that $R'(X_2) < 0$. Since total resources actually available are $(1-\delta)X_1+g(X_1,\mathcal{M})$, we have $X_2 = B^{-1}((1-\delta)X_1+g(X_1,\mathcal{M}))$, and therefore from condition (20) equilibrium can be characterized by a solution to

$$G(X_1,\mathcal{M}) = Q(\mathcal{M}) \tag{A.19}$$

for $\mathcal{M}$, where $G(X,\mathcal{M}) \equiv g_{\mathcal{M}}(X,\mathcal{M})R(B^{-1}((1-\delta)X+g(X,\mathcal{M})))$. Note that

$$G_{\mathcal{M}}(X_1,\mathcal{M}) = g_{\mathcal{M}\mathcal{M}}(X_1,\mathcal{M})R(X_2) + \frac{R'(X_2)\left[g_{\mathcal{M}}(X_1,\mathcal{M})\right]^2}{B'(X_2)} < 0$$

Similar to in the static case, we assume that $G(X,0) > Q(0)$ so that there is an equilibrium with $\mathcal{M} > 0$, and further, conditions are such that this equilibrium is unique, which implies that

$$G_{\mathcal{M}}(X_1,\mathcal{M}) < Q'(\mathcal{M}) \tag{A.20}$$

at the equilibrium value of $\mathcal{M}$.

Lemma A.14. *If $dX_2/dX_1 < 0$ then $dc/dX_1 < 0$ and $di/dX_1 < 0$.*

Proof. Since in equilibrium $c + X_2 = B(X_2)$, we have that

$$\frac{dc}{dX_1} = \left[B'(X_2) - 1\right]\frac{dX_2}{dX_1}$$

Since $B'(X_2) > 1$, if $dX_2/dX_1 < 0$ then $dc/dX_1 < 0$. Further, if X_2 falls when X_1 rises, from the capital accumulation equation (18) we see that i must also fall. □

Lemma A.15. *$dX_2/dX_1 < 0$ if and only if*

$$\left\{-\mathcal{E}^Q_{\mathcal{M}} + \frac{(1-\delta)g_{X\mathcal{M}}(X,\mathcal{M})}{g_X(X,\mathcal{M})\left[g_X(X,\mathcal{M})+1-\delta\right]}\right\}\mathcal{E}^g_{X\mathcal{M}} > 1 \tag{A.21}$$

Proof. Totally differentiating the equilibrium condition (A.19) with respect to X_1 yields that

$$\frac{d\mathcal{M}}{dX_1} = \frac{G_X(X_1,\mathcal{M})}{Q'(\mathcal{M}) - G_{\mathcal{M}}(X_1,\mathcal{M})} \tag{A.22}$$

Doing the same with $y = g(X,\mathcal{M})$ yields

$$\frac{dy}{dX_1} = g_X(X_1,\mathcal{M}) + g_{\mathcal{M}}(X_1,\mathcal{M})\frac{d\mathcal{M}}{dX_1} \tag{A.23}$$

while differentiating $X_2 = B^{-1}((1-\delta)X_1 + g(X_1, \mathcal{M}))$ yields

$$\begin{aligned}\frac{dX_2}{dX_1} &= \frac{1}{B'(X_2)}\left(1-\delta+\frac{dy}{dX_1}\right)\\ &= \frac{[1-\delta+g_X(X_1,\mathcal{M})]\,[Q'(\mathcal{M})-G_{\mathcal{M}}(X_1,\mathcal{M})]+g_{\mathcal{M}}(X_1,\mathcal{M})G_X(X_1,\mathcal{M})}{B'(X_2)\,[Q'(\mathcal{M})-G_{\mathcal{M}}(X_1,\mathcal{M})]}\end{aligned}$$

where the second line has used equations (A.22) and (A.23). Since the denominator of this expression is positive by (A.18) and (A.20), the sign of dX_2/dX_1 is given by the sign of the numerator. Substituting in for $G_{\mathcal{M}}$ and G_X and using (A.19), some algebra yields that this expression is negative if and only if condition (A.21) holds. □

Lemmas A.14 and A.15 together indicate that $dc/dX_1 < 0$ and $di/dX_1 < 0$ both hold if and only if condition (A.21) holds. Further, for a given equilibrium level of $\mathcal{M}$, it is clear that the minimum level of $\mathcal{E}^g_{X\mathcal{M}}$ needed to satisfy (A.21) is (weakly) greater than that needed to satisfy (A.15) in the static case.

Proof of Proposition 13

It can be verified that the steady-state level of purchases e solves

$$U'\left(\frac{\delta+\gamma}{\delta}e\right) = \zeta Q(e) \tag{A.24}$$

where

$$\zeta \equiv \frac{1-\beta(1-\delta)}{1-\beta(1-\delta)+\beta\gamma} \in (0,1)$$

Lemma A.16. *For δ sufficiently small, a steady state exists and is unique.*

Proof. Similar to in the static case, we may express individual j's optimal choice of steady-state expenditure e_j given aggregate steady-state expenditure e as

$$e_j(e) = \frac{\delta}{\delta+\gamma}U'^{-1}\left(\zeta Q(e)\right)$$

As before, we can verify that $e_j'(e) < 0$ for $e > e^\star$, while $e_j'(e) > 0$ and $e_j''(e) > 0$ for $e < e^\star$. Thus, an equilibrium necessarily exists and is unique if $e_j'(e) < 1$ for $e < e^\star$, which is equivalent to the condition that $\lim_{e\uparrow e^\star} e_j'(e) < 1$. This is in turn equivalent to the condition $\tau < \widetilde{\tau}$, where

$$\widetilde{\tau} \equiv -\frac{\delta+\gamma}{\delta\zeta}U''\left(U'^{-1}\left(\zeta\frac{\nu'(\ell^\star)}{F'(\ell^\star)}\right)\right)\frac{F'(\ell^\star)\,[F(\ell^\star)-\Phi]}{\nu'(\ell^\star)} \tag{A.25}$$

As $\delta \to 0$, $\widetilde{\tau}$ approaches infinity, and thus it will hold for any τ, which completes the proof. $\square$

Note for future reference that if $e_j'(e) < 1$ then

$$(\delta + \gamma)U''(X + e) < \delta\zeta Q'(e) \tag{A.26}$$

Lemma A.17. *For δ sufficiently small, there exists a steady state in the unemployment regime.*

Proof. Since $U'(0) > Q(0)$ by assumption, we also have $U'(0) > \zeta Q(0)$. Thus, if

$$U'\left(\frac{\delta+\gamma}{\delta}e^\star\right) < \zeta Q(e^\star)$$

then by the intermediate value theorem, equation A.24 holds for at least one value of $e < e^\star$. Note that

$$\lim_{\delta\to 0}\frac{\delta+\gamma}{\delta}e^\star = \infty$$

and $\lim_{\delta\to 0}\zeta t = (1-\beta)/(1-\beta+\beta\gamma) > 0$. Thus, since $\lim_{c\to\infty} U'(c) \leq 0$ by assumption, it follows that

$$\lim_{\delta\to 0} U'\left(\frac{\delta+\gamma}{\delta}e^\star\right) \leq 0 < \lim_{\delta\to 0}\zeta Z(e^\star)$$

and thus the desired property holds for δ close enough to zero. $\square$

Lemmas A.16 and A.17 together prove the proposition.

Proof of Proposition 14

Linearizing the system in e_t and X_t around the steady state and letting variables with hats denote deviations from steady state and variables without subscripts denote steady-state quantities, we have

$$\hat{X}_{t+1} = (1-\delta)\hat{X}_t + \gamma\hat{e}_t$$

$$\hat{e}_{t+1} = -\frac{[1-\beta(1-\delta)(1-\delta-\gamma)]U''X+e)}{\beta\left[(1-\delta)Q'(e)-(1-\delta-\gamma)U''(X+e)\right]}\hat{X}_t + \frac{Q'(e)-\left[1-\beta\gamma(1-\delta-\gamma)\right]U''(X+e)}{\beta\left[(1-\delta)Q'(e)-(1-\delta-\gamma)U''(X+e)\right]}\hat{e}_t$$

or

$$\hat{x}_{t+1} \equiv \begin{pmatrix} \hat{X}_{t+1} \\ \hat{e}_{t+1} \end{pmatrix} = \begin{pmatrix} 1-\delta & \gamma \\ a_{eX} & a_{ee} \end{pmatrix} \begin{pmatrix} \hat{X}_t \\ \hat{e}_t \end{pmatrix} \equiv A\hat{x}_t$$

where a_{eX} and a_{ee} are the coefficients on $\hat{X}_t$ and $\hat{e}_t$ in the expression for $\hat{e}_{t+1}$. The eigenvalues of A are then given by

$$\lambda_1 \equiv \frac{1-\delta+a_{ee}-\sqrt{(1-\delta+a_{ee})^2-4\beta^{-1}}}{2}$$

$$\lambda_2 \equiv \frac{1-\delta+a_{ee}+\sqrt{(1-\delta+a_{ee})^2-4\beta^{-1}}}{2}$$

We may obtain that

$$\lambda_1\lambda_2 = \beta^{-1} > 1 \tag{A.27}$$

so that $|\lambda_i| > 1$ for at least one $i \in \{1, 2\}$. Thus, this system cannot exhibit local indeterminacy (see, e.g., Blanchard and Kahn (1980)), which completes the proof.

Proof of Proposition 15

Note for future reference that (A.27) implies that if the eigenvalues are real then they are of the same sign, with $\lambda_2 > \lambda_1$.

Lemma A.18. *The system is saddle-path stable if and only if*

$$|1-\delta+a_{ee}| > \frac{1+\beta}{\beta} \tag{A.28}$$

in which case the eigenvalues are real and of the same sign as $1-\delta+a_{ee}$.

Proof. To see the "if" part, suppose (A.28) holds, and note that this implies

$$(1-\delta+a_{ee})^2 > \left(\frac{1+\beta}{\beta}\right)^2 > 4\beta^{-1}$$

and therefore the eigenvalues are real. If $1-\delta+a_{ee} > (1+\beta)/\beta$, then this implies that $\lambda_2 > \lambda_1 > 0$, and therefore the system is stable as long as $\lambda_1 < 1$, which is equivalent to the condition

$$(1-\delta+a_{ee}) - 2 < \sqrt{(1-\delta+a_{ee})^2-4\beta^{-1}} \tag{A.29}$$

Since $1 - \delta + a_{ee} > (1+\beta)/\beta > 2$, both sides of this inequality are positive, and therefore, squaring both sides and rearranging, it is equivalent to

$$1 - \delta + a_{ee} > \frac{1+\beta}{\beta} \tag{A.30}$$

which holds by hypothesis. A similar argument can be used to establish the claim for the case that $-(1 - \delta + a_{ee}) > (1+\beta)/\beta$.

To see the "only if" part, suppose the system is stable. If the eigenvalues had non-zero complex part, then $|\lambda_1| = |\lambda_2| > 1$, in which case the system would be unstable. Thus, the eigenvalues must be real, i.e., $(1 - \delta + a_{ee})^2 > 4\beta^{-1}$, which in turn implies that

$$|1 - \delta + a_{ee}| > 2\sqrt{\beta^{-1}}$$

If $1 - \delta + a_{ee} > 2\sqrt{\beta^{-1}}$, then, reasoning as before, $\lambda_2 > \lambda_1 > 0$, and therefore if the system is stable then (A.29) must hold. Since $(1 - \delta + a_{ee}) > 2\sqrt{\beta^{-1}} > 2$, then again both sides of (A.29) are positive, and thus that inequality is equivalent to (A.30), which in turn implies (A.28). Similar arguments establish (A.28) for the case where $-(1 - \delta + a_{ee}) > 2\sqrt{\beta^{-1}}$. □

Lemma A.19. *The system is saddle-path stable with positive eigenvalues if and only if*

$$(1 - \delta - \gamma)U''(X + e) < (1 - \delta)Q'(e) \tag{A.31}$$

Proof. Note that the system is stable with positive eigenvalues if and only if (A.30) holds. We have that

$$1 - \delta + a_{ee} - \frac{1+\beta}{\beta} = \frac{[1 - \beta(1 - \delta - \gamma)][\delta\zeta Q'(e) - (\delta + \gamma)U''(X + e)]}{\beta[(1 - \delta)Q'(e) - (1 - \delta - \gamma)U''(X + e)]}$$

Since the numerator is positive by (A.26), inequality (A.30) holds if and only if (A.31) holds. □

Lemma A.20. *If*

$$\tau < \widetilde{\tau}^\star \equiv -\frac{1 - \delta - \gamma}{1 - \delta} U''\left(U'^{-1}\left(\zeta\frac{\nu'(\ell^\star)}{F'(\ell^\star)}\right)\right)\frac{F'(\ell^\star)[F(\ell^\star) - \Phi]}{\nu'(\ell^\star)}$$

then the system is saddle-path stable with positive eigenvalues.

Proof. Note that condition (A.31) always holds around a full-employment steady state. If the steady state is in the unemployment regime, then it can be verified that condition (A.31) holds if and only if

$$e_j'(e) < \frac{\delta}{\delta+\gamma}\zeta\frac{1-\delta-\gamma}{1-\delta} \in (0,1)$$

where $e_j(e)$ is as defined in Lemma A.16. As before, this condition holds for all e if it holds for $\lim_{e\uparrow e^\star} e_j'(e)$, which it can be verified is equivalent to the condition $\tau < \widetilde{\tau}^\star$. Note also that $\widetilde{\tau}^\star < \widetilde{\tau}$, where $\widetilde{\tau}$ was defined in equation (A.25), so that this condition is strictly stronger than the one required to ensure the existence of a unique steady state. □

Lemmas A.19 and A.20 together establish that, for τ sufficiently small (e.g., $\tau < \widetilde{\tau}^\star$), the system converges monotonicaly to the steady state. It remains to show that consumption is decreasing in the stock of durables. Assuming τ is sufficiently small so that the system is saddle-path stable with positive eigenvalues, it is straightforward to obtain the solution

$$\hat{X}_t = \lambda_1^t \hat{X}_0$$
$$\hat{e}_t = \psi \hat{X}_t$$
$$\hat{c}_t = (1+\psi)\,\hat{X}_t$$

where $\psi \equiv -(1-\delta-\lambda_1)/\gamma$. Thus, consumption is decreasing in the stock of durables if and only if $\psi < -1$.

Proposition 18. *If* (A.31) *holds and the steady state is in the unemployment regime, then* $\psi < -1$.

Proof. We may write

$$1-\delta-\gamma-\lambda_1 = \frac{\sqrt{[a_{ee}+2\gamma-(1-\delta)]^2 + 4\beta^{-1}[\beta(1-\delta-\gamma)(a_{ee}+\gamma)-1]} - [a_{ee}+2\gamma-(1-\delta)]}{2}$$

Now, $a_{ee}+2\gamma-(1-\delta) > a_{ee}-(1-\delta) > 0$, so that $1-\delta-\gamma-\lambda_1$ is positive if and only if $\beta(1-\delta-\gamma)(a_{ee}+\gamma) > 1$. We have

$$\beta(1-\delta-\gamma)(a_{ee}+\gamma) = \frac{[1+\beta\gamma(1-\delta)]Q'(e) - U''(X+e)}{\left(\frac{1-\delta}{1-\delta-\gamma}\right)Q'(e) - U''(X+e)}$$

Note by earlier assumptions that this expression is strictly positive, and that

$$\frac{1-\delta}{1-\delta-\gamma} - [1+\beta\gamma(1-\delta)] = \gamma\frac{1-\beta(1-\delta)(1-\delta-\gamma)}{1-\delta-\gamma} > 0$$

Thus, if $Q'(e) < 0$ (i.e., the steady state is in the unemployment regime) then $\beta(1-\delta-\gamma)(a_{ee}+\gamma) > 1$, in which case $1-\delta-\gamma-\lambda_1 > 0$ and therefore $\psi < -1$. $\square$

Proof of Proposition 16

Without loss of generality, assume the alternative path begins at $t = 0$, and let $\widetilde{e}_t(\Delta) \equiv e + \Delta \cdot \epsilon_t$ denote the alternative feasible path of expenditures, where ϵ_t is the change in the path of expenditures, and Δ is a perturbation parameter, which is equal to zero in the steady-state equilibrium and equal to one for the alternative path. Let $\widetilde{X}_t(\Delta)$ denote the associated path for the stock of durables, and note that $\widetilde{X}_0(\Delta) = X$, i.e., this alternative path does not affect the initial stock of durables. Welfare can then be written as a function of Δ as

$$\mathcal{U}(\Delta) = \sum_{t=0}^{\infty} \beta^t \left\{ U(\widetilde{X}_t(\Delta) + \widetilde{e}_t(\Delta)) + \frac{\widetilde{e}_t(\Delta)}{F'(\ell^\star)\ell^\star} [-\nu(\ell^\star) + V(w^\star \ell^\star - p^\star \widetilde{e}_t(\Delta))] \right.$$
$$\left. + \left(1 - \frac{\widetilde{e}_t(\Delta)}{F'(\ell^\star) l^\star}\right) V(-p^\star \widetilde{e}_t(\Delta)) \right\}$$

From the envelope theorem, beginning from the steady state path (i.e., $\Delta = 0$), for a marginal change in Δ the net effect on welfare through the resulting changes in U and V in each period is zero. Thus, we need only consider effects that occur through changes in the employment rate term, $\widetilde{e}_t(\Delta)/[F'(\ell^\star)\ell^\star]$. A first-order approximation to $\mathcal{U}(1)$ around $\mathcal{U}(0)$ is therefore given by

$$\mathcal{U}(1) \approx \mathcal{U}(0) + \frac{1}{F'(\ell^\star)\ell^\star} \left[\mathcal{L}^\star + \tau \frac{\nu'(\ell^\star)}{F'(\ell^\star)} e \right] \sum_{t=0}^{\infty} \beta^t \widetilde{e}'_t(0)$$

Substituting in $\widetilde{e}'_t(0) = \epsilon_t$, the desired result obtains.

Proof of Proposition 17

Let $\widetilde{e}_t(\epsilon)$ and $\widetilde{X}_t(\epsilon)$ denote alternative paths for expenditure and the stock of durables, with $\widetilde{e}_t(\epsilon) \equiv e(\widetilde{X}_t(\epsilon)) + \epsilon_t$ and $\widetilde{X}_{t+1}(\epsilon) = (1-\delta)\widetilde{X}_t(\epsilon) + \gamma \widetilde{e}_t(\epsilon)$. Here, $e(\cdot)$ is the equilibrium policy function for expenditures, while $\epsilon_0 = \epsilon$ and $\epsilon_t = 0$ for $t \geq 1$. Letting $\mathcal{U}(X_0, \epsilon)$ denote the corresponding welfare as a function of X_0 and ϵ, we may write a second-order approximation to this function around $(X_0, \epsilon) = (X, 0)$ as

$$\mathcal{U}(X_0, \epsilon) \approx \mathcal{U}(X, 0) + \mathcal{U}_X \hat{X}_0 + \mathcal{U}_\epsilon \epsilon + \frac{1}{2}\left[\mathcal{U}_{XX}\hat{X}_0^2 + \mathcal{U}_{\epsilon\epsilon}\epsilon^2\right] + \mathcal{U}_{X\epsilon}\hat{X}_0\epsilon$$

where variables with hats indicate deviations from steady state and partial derivatives of $\mathcal{U}$ are evaluated at the point $(X_0, \epsilon) = (X, 0)$. Clearly, to a second-order approximation, the welfare effect of a temporary stimulus is smaller when the economy is in a liquidation phase if and only if $\mathcal{U}_{X\epsilon} < 0$.

Next, using the envelope condition as in the proof of Proposition 16, it is straightforward to obtain that

$$\mathcal{U}_\epsilon(X_0, 0) = \frac{1}{F'(\ell^\star)l^\star}\sum_{t=0}^{\infty}\beta^t\left[\mathcal{L}^\star + \tau\frac{\nu'(\ell^\star)}{F'(\ell^\star)}e(X_t)\right]\widetilde{e}_t'(0)$$

where $X_t = \widetilde{X}_t(0)$ is the stock of durables that would occur in the absence of stimulus. One may also obtain that

$$\widetilde{e}_t'(0) = \begin{cases} 1 & : t = 0 \\ \gamma e'(X_t)\left\{\prod_{i=1}^{t-1}\left[1 - \delta + \gamma e'(X_{t-i})\right]\right\} & : t \geq 1 \end{cases}$$

so that

$$\mathcal{U}_\epsilon(X_0, 0) = \frac{1}{F'(\ell^\star)\ell^\star}\Bigg\{\left[\mathcal{L}^\star + \tau\frac{\nu'(\ell^\star)}{F'(\ell^\star)}e(X_0)\right] + \gamma\sum_{t=1}^{\infty}\beta^t\left[\mathcal{L}^\star + \tau\frac{\nu'(\ell^\star)}{F'(\ell^\star)}e(X_t(X_0))\right]\cdot$$
$$e'(X_t(X_0))\left(\prod_{i=1}^{t-1}\left[1 - \delta + \gamma e'(X_{t-i}(X_0))\right]\right)\Bigg\}$$

where $X_t(X_0)$ indicates the equilibrium value of X_t given X_0. Taking the derivative of this expression with respect to X_0 and evaluating at $X_0 = X$ yields

$$\mathcal{U}_{X\epsilon}(X, 0) = \frac{1}{F'(\ell^\star)\ell^\star}\tau\frac{\nu'(\ell^\star)}{F'(\ell^\star)}\cdot\frac{1 - \beta\lambda_1(1-\delta)}{1 - \beta\lambda_1^2}\psi + \Xi e''(X)$$

where $\psi \equiv e'(X) < 0$, which was computed above, and Ξ is some strictly positive number. Since $\lambda_1 < 1$, the first term on the right-hand side of this expression is clearly negative. Thus, there is a strictly positive number ξ such that if $e''(X) < \xi$ we will have $\mathcal{U}_{X\epsilon}(X, 0) < 0$, which is the desired result.

Letting $\chi(X_t)$ denote the equilibrium value of X_{t+1} given X_t, we may re-express the equilibrium equations governing the dynamics of the system (i.e., equations (22) and (23)) as

$$\chi(X_t) = (1-\delta)X_t + \gamma e(X_t)$$

and

$$U'(X_t + e(X_t)) - Q(e(X_t)) = \beta\left[(1-\delta-\gamma)U'(\chi(X_t) + e(\chi(X_t))) - (1-\delta)Q(e(\chi(X_t)))\right]$$

Taking derivatives of both sides of these equations twice with respect to X_t, evaluating at $X_t = X$ and solving for $e''(X)$, we may obtain that $e''(X) = bU'''(X+e)$, where b is some number that does not depend on $U'''(X+e)$. Thus, if U''' is sufficiently close to zero, $e''(X) < \xi$ and the desired result holds.

B Introducing Nash bargaining

Here we consider the static model of section 2 and replace the "competitive" determination of w and ℓ within a match by Nash bargaining.

The gain from a match for a firm is $pF(\ell) - w\ell$ while outside option is zero. The gain for the household is $-\nu(\ell) + V(w\ell - p(c-X)$ while the outside option is $V(-p(c-X))$. Using the piecewise linear specification for V, the Nash-Bargaining criterion $\mathcal{W}$ is:

$$\mathcal{W} = \Big(pF(\ell) - w\ell\Big)^{\psi}\Big(-\nu(\ell) + vw\ell + v\tau p(c-X)\Big)^{\psi}$$

Maximizing $\mathcal{W}$ w.r.t. ℓ and w gives the following F.O.C.:

$$\begin{aligned}
\frac{\psi\mathcal{W}}{pF(\ell)-w\ell}\Big(pF'(\ell) - w\Big) &= \frac{(1-\psi)\mathcal{W}}{-\nu(\ell)+vw\ell+v\tau p(c-X)}\Big(vw - \nu'(\ell)\Big) \\
\frac{\psi\mathcal{W}}{pF(\ell)-w\ell} &= \frac{(1-\psi)\mathcal{W}}{-\nu(\ell)+vw\ell+v\tau p(c-X)}v
\end{aligned}$$

Rearranging gives the two equations

$$\begin{aligned}
vpF'(\ell) &= \nu'(\ell) \\
vw\ell &= (1-\psi)vpF(\ell) + \psi\nu(\ell) - \psi v\tau p(c-X)
\end{aligned}$$

Assuming that the matching function is "min", the equilibrium is given by the five following equations:

$$u'(c) = \frac{\nu'(\ell)}{F'(\ell)}\left(1+\tau-\frac{\min\{N,L\}}{L}\tau\right) \tag{B.1}$$

$$w\ell = (1-\psi)pF(\ell)+\frac{\psi}{v}\nu(\ell)-\psi\tau p(c-X) \tag{B.2}$$

$$vpF'(\ell) = \nu'(\ell) \tag{B.3}$$

$$\min\{N,L\}F(\ell) = L(c-X)+N\Phi \tag{B.4}$$

$$\min\{N,L\}\big(pF(\ell)-w\ell\big) = pN\Phi \tag{B.5}$$

Equations (B.2) and (B.3) determine p and w once N, c and ℓ are determined by the three other equations. After some manipulations, those three equations (B.1), (B.4) and (B.5) can be written:

$$u'(c) = \frac{\nu'(\ell)}{F'(\ell)}\left(1+\tau-\frac{\min\{N,L\}}{L}\tau\right) \tag{B.6}$$

$$\frac{\min\{N,L\}}{L} = \frac{(c-X)}{(1-\psi)F(\ell)+\psi F'(\ell)\frac{\nu(\ell)}{\nu'(\ell)}-\psi\tau(c-X)} \tag{B.7}$$

$$\psi\frac{\min\{N,L\}}{N} = \Phi\left(F(\ell)-F'(\ell)\frac{\nu(\ell)}{\nu'(\ell)}+\tau(c-X)\right)^{-1} \tag{B.8}$$

In the unemployment regime, those equations write

$$u'(c) = \frac{\nu'(\ell)}{F'(\ell)}\left(1+\tau-\frac{N}{L}\tau\right) \tag{B.9}$$

$$\frac{N}{L} = \frac{(c-X)}{(1-\psi)F(\ell)+\psi F'(\ell)\frac{\nu(\ell)}{\nu'(\ell)}-\psi\tau(c-X)} \tag{B.10}$$

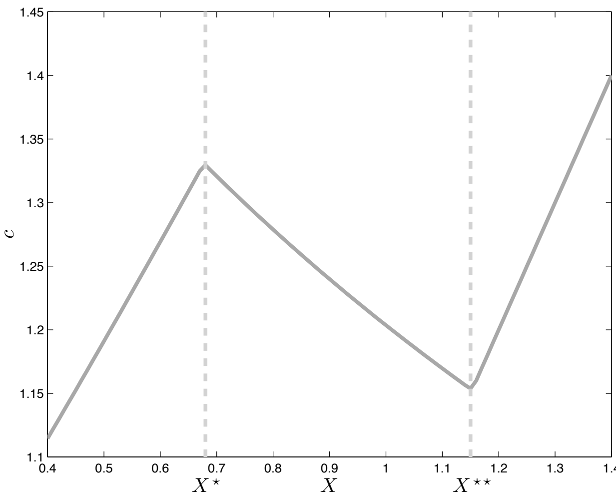
$$\frac{\Phi}{\psi} = F(\ell)-F'(\ell)\frac{\nu(\ell)}{\nu'(\ell)}+\tau(c-X) \tag{B.11}$$

Main difference with the model of the main text is that (B.11) does not determine ℓ independently of (B.9) and (B.10). But it is still the case that, assuming ℓ is fixed, (B.9) implies that if N is high, c will be high and (B.10) implies that if c is high, N will be high. As far as (B.11) implies that ℓ does not vary too much, subsequent results of section 2 hold. This can be illustrated with a numerical example that reproduces Figures 2, 3 and 5.

Consider the functional forms $\nu(\ell) = \frac{\nu_1\ell^{1+\omega}}{1+\omega}$, $F(\ell) = \theta_1 A\ell^{\alpha}$, $u(c) = \ln c$, $V(a)$ is ν_2/θ_2 if $a \geq 0$ and $(1+\tau)\nu_2/\theta_2$ if $a < 0$. Common parameters values are $\psi = .5$, $\omega = 1.2$, $\nu_1 = .5$,

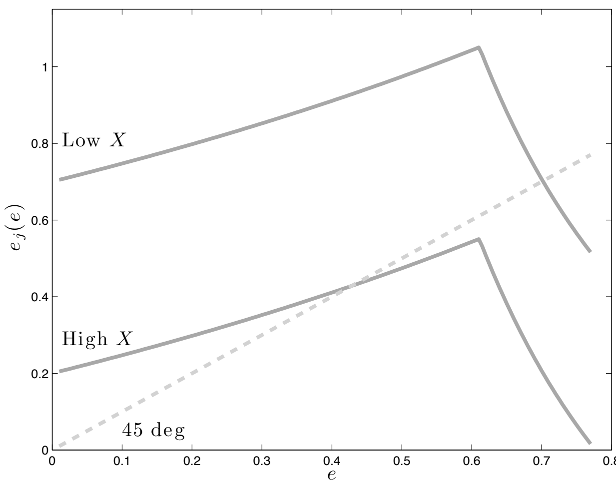
$\alpha = .67$, $A = 1$, $\Phi = .35$, $L = 1$. Solving for the equilibrium in such a case produce Figures 9, 10 and 11, which are qualitatively similar to Figures 2, 3 and 5.

Figure 9: The Model with Nash bargaining, Consumption as function of X.



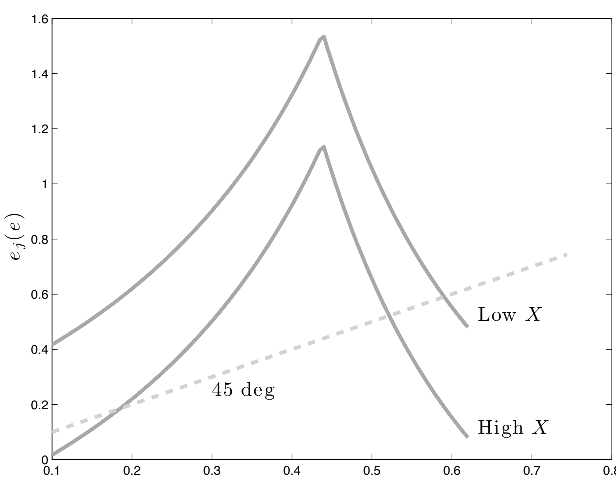
Note: Example is constructed assuming the functional forms $\nu(\ell) = \frac{\nu\ell^{1+\omega}}{1+\omega}$, $F(\ell) = A\ell^\alpha$, $u(c) = \ln c$, $V(a)$ *is* av *if* $a \geq 0$ *and* $(1+\tau)av$ *if* $a < 0$. *Parameters values are* $\psi = .5$, $\omega = 1.2$, $\nu = v = .5$, $\alpha = .67$, $A = 1$, $\Phi = .35$, $L = 1$ *and* $\tau = .05$.

Figure 10: The Model with Nash bargaining, Equilibrium determination



Note: Example is constructed assuming the functional forms $\nu(\ell) = \frac{\nu \ell^{1+\omega}}{1+\omega}$, $F(\ell) = A\ell^{\alpha}$, $u(c) = \ln c$, *$V(a)$ is va if $a \geq 0$ and $(1+\tau)va$ if $a < 0$. Parameters values are $\psi = .5$, $\omega = 1.2$, $\nu = v = .5$, $\alpha = .67$, $A = 1$, $\Phi = .35$, $L = 1$ and $\tau = .05$. Values of X used were $X = .3$ for the full-employment equilibrium and $X = 0.9$ for the unemployment equilibrium.*

Figure 11: The Model with Nash bargaining, Equilibrium determination (multiple equilibria)



Note: Example is constructed assuming the functional forms $\nu(\ell) = \frac{\nu \ell^{1+\omega}}{1+\omega}$, $F(\ell) = A\ell^{\alpha}$, $u(c) = \ln c$, *$V(a)$ is va if $a \geq 0$ and $(1+\tau)va$ if $a < 0$. Parameters values are $\psi = .5$, $\omega = 1.2$, $\nu = v = .5$, $\alpha = .67$, $A = 1$, $\Phi = .35$, $L = 1$, $\tau = .4$, $X = .6$ or $X = 1$.*

C Noise shock extension

For the extension discussed at the end of Section 4.2, we re-introduce the first-sub-period (θ) and second-sub-period ($\widetilde{\theta}$) productivity factors to the model, and assume that $\widetilde{\theta}_t = \theta_t$. We assume that the economy is always in the unemployment regime, and that all agents come into the first sub-period of period t with the same belief about the value of θ_t, but that after the household splits to go to market, the true value is revealed to the workers and firms, while the shoppers retain their initial belief.

To abstract from issues relating to uncertainty about the true value of θ_t, we assume that all agents are subjectively certain – though possibly incorrect – about the entire stream of productivity values θ_t, only updating such a belief if they receive some information that contradicts it. One may verify that, in the unemployment regime, shoppers' prior beliefs are never contradicted until re-uniting with the workers after making their purchases. We denote agents' belief about θ_t at the beginning of date s by $\bar{\theta}_{t|s}$

In the example constructed, we assume that productivity is constant at $\theta_t = 1$ for all $t \in \mathbb{Z}$, but that at the beginning of $t = 0$, agents receive information such that $\bar{\theta}_{t|0} = \theta > 1$ for all $t \geq 0$, i.e., that productivity has risen permanently. After the households split, workers and firms learn that in fact productivity has not changed, nor will it in the future. Shoppers do not receive this information until after making their purchases, so that for one shopping period they are overly optimistic. In all subsequent periods $s \geq 1$, however, we have $\bar{\theta}_{t|s} = \theta_t = 1$.