Public and Private Information in Monetary Policy Models*

Jeffery D. Amato
Bank for International Settlements
jeffery.amato@bis.org

Hyun Song Shin
London School of Economics
h.s.shin@lse.ac.uk

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Abstract

Monetary policy entails a dual role for the central bank. As well as being a vigilant observer of events, the central bank must also be able to shape expectations through its words and deeds. This paper examines the impact of central bank forecasts and other sources of public information in an economy where agents also have diverse private information. In an otherwise standard macro model, the disproportionate role of public information degrades the information value of economic outcomes, alters the welfare consequences of increased precision of public information and generates distinctive time series characteristics of some macro variables.

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1 Introduction

One of the often-cited virtues of a decentralized economy is the ability of the market mechanism to aggregate the private information of the individual economic agents. Each economic agent has a window on the world. This window is a partial vantage point for the underlying state of the economy. But when all the individual perspectives are brought together, one can gain a much fuller picture of the economy. If the pooling of information is effective, and economic agents have precise information concerning their respective sectors or geographical regions, the picture that emerges for the whole economy would be a very detailed one. When can policy makers rely on the effective pooling of information from individual decisions?

This question is a very pertinent one for the conduct of monetary policy. Central banks that attempt to regulate aggregate demand by adjusting interest rates rely on timely and accurate generation of information on any potential inflationary forces operating in the economy. The role of the central bank in this context is of a vigilant observer of events to detect any nascent signs of pricing pressure. Such signs can be met by prompt central bank action to head off any inflationary forces through the use of monetary policy instruments. More generally, these actions can be codified in a more systematic framework for the setting of nominal interest rates, for instance as part of an 'inflation-forecast targeting' regime.

However, by the nature of its task, the central bank cannot confine its role merely to be a vigilant, but detached observer. Its monetary policy role implies that it must also engage in the active shaping and influencing of events (see Blinder, Goodhart, Hildebrand, Lipton and Wyplosz (2001)). For economic agents, who are all interested parties in the future course of action of the central bank, the signals conveyed by the central bank in its deeds and words have a material impact on how economic decisions are arrived at. For this reason, Svensson (2002) and Svensson and Woodford (2002) have advocated the announcement of the future path of the short term policy interest rate as part of a central bank's overall policy of inflation-forecast targeting.

Monetary policy thus entails a dual role. As well as being a vigilant *observer* of outcomes, the central bank must also be able to *shape* the outcomes. There is, however, a tension in this dual role. To the extent that the central bank is effective in shaping the outcome, the informational value of this outcome for the purpose of inferring the underlying state of the economy may be impaired. Since the actions of economic agents reflect in part the central bank's own assessment of the underlying state, the mirror that is held up to the economy for signs of potential imbalances may simply reflect the central bank's own assessment of the same issue. The more authority that the central bank commands among the economic agents, the greater is the danger that the aggregate outcome is tinged with the central bank's own prior beliefs.

In a situation of common knowledge, potential problems associated with the feedback between central bank actions and the expectations of private agents can largely be avoided under standard policies.[1] By contrast, under differential information, the central bank's actions and the information it releases constitute a shared benchmark in the information processing decisions of economic agents. In particular, the central bank's disclosures — or, in general, any type of credible public information — become a powerful focal point for the coordination of expectations among such agents. There is the potential for a feedback process that degrades the information value of signals generated in the economy. Central bank disclosures push events in the direction of bringing about what was disclosed. Thus, any reaction function used by the central bank in setting policy has to rely on less informative signals of the underlying fundamentals. To compensate, the central bank would be forced to adopt a reaction function that is more sensitive to the signals emanating from the economy. When the signal to noise ratio is low, such a move would invite unwelcome side-effects.

Against this backdrop, this paper assesses the implications of public information in a small-scale monetary-policy model in which agents have imperfect

[1] See Bernanke and Woodford (1997) for a treatment of the issues as pertains to forward-looking interest rate rules.

common knowledge on the state of the economy. We employ a model that is standard in most respects, but one that recognizes the importance of decentralized information gathering and the resulting differential information in the economy. In particular, building on recent work by Woodford (2002), our focus is on the pricing behaviour of monopolistically competitive firms with access to both private and public information.

Our analysis proceeds in two steps. Beginning with a series of simplified examples, we show how differentially informed firms follow pricing rules that suppress their own information, but instead put disporportionately large weight on commonly shared information; that is, firms suppress their private information on the underlying demand and cost conditions far more than is justified than when the estimates of fundamentals are common knowledge. For reasonable values for the strength of strategic complementarity, the aggregate price suffers substantial information loss, and therefore ceases to be an informative signal of the underlying demand and cost conditions.

Following up on our partial equilibrium example, we then develop a general equilibrium model incorporating households and the central bank. A complete monetary-policy model allows us to consider the implications of specific monetary policies, in particular the properties of interest rate rules based on inflation forecasts as explored in the recent monetary policy literature. Our objectives here are threefold. First, under a particular parametrization of the policy rule, we wish to assess the impact of public information on the volatility of macroeconomic aggregates. Second, we investigate the dynamic responses of the variables to shocks in the underlying economic fundmentals. Third, we trace out the welfare implications of different parametrizations of the policy rule with respect to the relative precision of private and public signals of the fundamentals.

In the next section we provide a brief overview of related literature. Section 3 provides some theoretical background by means of simplified examples of pricing differential information. Section 4 develops a complete macroeconomic model that is standard except for the presence of differential information amongst some agents. The general equilibrium properties of this model are explored in Section

5. Section 6 concludes. The Appendix contains further technical results.

2 Related Literature

From a theoretical perspective, we have good grounds to conjecture that the 'climate of opinion' as embodied in the commonly shared information in an economy will play a disproportionate role in determining the outcome. A strand of the macroeconomics literature begun by Townsend (1983) and Phelps (1983), and recently developed and quantified by Woodford (2001), examines the impact of decentralized information processing by individual agents in an environment where their interests are intertwined. Indeed, Phelps's paper is explicitly couched in terms of the importance of higher order beliefs — that is, beliefs about the beliefs of others. For Woodford, the intertwining of interests arise from the strategic complementarities in the pricing decisions of firms. In setting prices, firms try to second-guess the pricing strategies of their potential competitors for market share. Even when there are no nominal rigidities, the outcome of navigating through the higher-order beliefs entailed by the second-guessing of others leads firms to set prices that are far less sensitive to firms' best estimates of the underlying fundamentals. The implication is that average prices suffer some impairment in serving as a barometer of the underlying cost and demand conditions.

These results are bolstered by recent theoretical studies into the impact of public and private information in a number of related contexts. They suggest that there is potential for the aggregate outcome to be overly sensitive to commonly shared information relative to reactions that are justified when all the available information is used in a socially efficient way. Morris and Shin (2002) note how increased precision of public information may impair social welfare in a game of second-guessing in the manner of Keynes's 'beauty contest' that has close formal similarities with the papers by Phelps and Woodford. Allen, Morris and Shin (2002) note that an asset's trading price may be a biased signal of its true value in a rational expectations equilibrium with uncertain supply, where the bias is toward the ex ante value of the asset.

A number of recent papers have revisited macroeconomic models with imperfect common knowledge by drawing on the recent modelling innovations for dealing with differential information. Hellwig (2002) analyses the impact of public announcements in a new Keynesian model with imperfect competition. He shows that public announcements allow quicker adjustment to fundamentals, but at the cost of greater noise. Bacchetta and van Wincoop (2002) explore the impact of public information in an asset pricing context. Pearlman and Sargent (2002) and Kasa (2000) are other recent papers that have pushed the boundaries of this literature.

There has also been growing interest in examining more deeply the underlying rationale for imperfect common knowledge among agents. Is it possible that agents observe only noisy signals of aggregate fundamentals? If so, why do agents lack common knowledge? The latter question is easier to address, since it is presumed to be self-evident that agents have access to (at least partially) private information in the conduct of their own activities. One answer to the first question is that data on macroeconomic aggregates are subject to persistent measurement errors. Publicly available statistics rarely provide a completely accurate measure of the true underlying aggregates of economic interest. Bomfim (2001) has analysed the general equilibrium implications of measurement error in a common knowledge rational expectations setting. A second answer is that agents have limited information processing capabilities, along the lines of Sims (2002). The story is as follows. Consider dividing agents' activities into two parts: an information processing stage and a decision-making stage. Given the vast quantity of information at their disposal, both private and public in nature, it is conjectured that agents can only imperfectly filter this data into a set of statistics upon which to base decisions. But conditional upon their information sets, agents act optimally. A related argument is that a good deal of public information that agents pay attention to is imperfectly filtered by public sources, for example, newspaper reports or commentators on television.

The existence and likely use of both public and private information suggests that models with disparately-informed agents should take both types of signals into account. The strong likelihood that measurement errors in some key macroeconomic data series or that processing errors by agents persist indefinitely into the future suggests that the true state is never revealed. Combining these two features in a monetary-policy model is a novel contribution of this paper.

One potential argument against the plausibility of the importance of higher-order beliefs in agents' behaviour is the degree of complexity involved in forming these beliefs (see, for instance, Svensson's (2001) comments on Woodford (2002)). If agents have only limited information processing capabilities, then how could they be expected to form expectations about others' expectations about others' expectations and so on? However, there is a clear distinction between the behaviours exhibited by agents and the informational constraints they face. Agents form and act upon higher-order beliefs because it is rational for them to do so. Invoking the well-known billiard player analogy, agents act *as if* they have knowledge of the workings of the economy, which in our setting requires that they implictly second-guess others. By contrast, it is not clear how they can act *as if* they have perfect common knowledge of the economy's state. Indeed, a differential-information rational expectations economy places less stringent requirements upon agents than full information rational expectations models that are typical in the literature. The elegance of these latter models can be misleading regarding the enormous demands placed upon agents in both their behaviour, which we also impose, and information processing abilities, which we relax.

3 Theoretical Background

Before developing our main arguments in a general equilibrium setting, we will introduce our conceptual building blocks by means of a series of simplified examples. Our focus is on the equilibrium consequences of the pricing rule for firms that takes the form:

$$p_i = E_i p + \xi E_i x \tag{1}$$

where p_i is the (log) price set by firm i, p is the (log) average price across firms, x denotes the output gap (in real terms) - our "fundamental variable" - and ξ is a constant between 0 and 1. A rigorous development of (1) is presented in section 4. The operator E_i denotes the conditional expectation with respect to firm i's information set. The pricing rule given by (1) arises in the classic treatment by Phelps (1983), and has been developed more recently by Woodford (2002) for an economy with imperfectly competitive firms.

In a discussion that has subsequently proved to be influential, Phelps (1983) compared this pricing rule to the 'beauty contest' game discussed in Keynes's General Theory (1936), in which the optimal action involves second-guessing the choices of other players. Townsend (1983) also emphasized the importance of higher order expectations - that of forecasting the forecasts of others. To see this, rewrite (1) in terms of the nominal output gap, defined as $q \equiv x + p$, yielding

$$p_i = (1 - \xi) E_i p + \xi E_i q \tag{2}$$

If we then take the average of (2) across firms, we get

$$p = (1 - \xi) \bar{E} p + \xi \bar{E} q \tag{3}$$

where $\bar{E}(\cdot)$ is the "average expectations operator", defined as $\bar{E}(\cdot) \equiv \int E_i(\cdot) di$. By repeated substitution, we have

$$p = \sum_{k=1}^{\infty} \xi (1 - \xi)^{k-1} \bar{E}^k q \tag{4}$$

where $\bar{E}^k$ is the k-fold iterated average expectations operator. With differential information, the k-fold iterated average expectations do not collapse to the single average expectation. Morris and Shin (2002) show how such the failure of the law of iterated expectations affect the welfare consequences of decision rules of this form, and note that increased precision of public information may be detrimental to welfare.

The size of the parameter ξ proves to be crucial in determining the impact of differential information, through higher-order expectations, on aggregate price dynamics. In a monopolistically competitive model, the parameter ξ reflects, among other things, the degree of competition between firms. The more intense is competition - that is, the larger is the elasticity of substitution between firms' goods - the smaller will be ξ, and hence the more important higher-order expectations in determining prices.

3.1 Differential Information in a Static Context

Let us explore the consequences of the pricing rule given by (1) in a single period context when the firms have differential information on the underlying fundamental variable - the nominal output gap q. For ease of illustration, let us suppose for the moment that the nominal output gap q can take on finitely many possible values. No firm observes q perfectly, but firm i observes an imperfect signal z_i of q, where z_i takes on finitely many possible values. Each firm observes the realization of its own signal, but not the signals of other firms. Let us further suppose that the firms can be partitioned into a finite number N of equally-sized subclasses, where firms in each subclass are identical, and commonly known to be so. We define a *state* ω to be an ordered tuple:

$$\omega \equiv (q, z_1, z_2, \cdots, z_N)$$

that specifies the outcomes of all random variables of relevance. We will denote by Ω the *state space* that consists of all possible states. The state space is finite given our assumptions.

There is a known *prior density* ϕ over the state space Ω that is implied by the joint density over q and the signals z_i. The prior is known to all firms, and represents the commonly shared assessment of the likelihood of various outcomes. However, once the firm observes its own signal z_i, it makes inferences on the economy based on the realization of its own signal z_i. Firm i's information partition over Ω is generated by the equivalence relation $\sim_i$ over Ω, where $\omega \sim_i \omega'$ if and only if the realization of z_i is the same at ω and ω'.

Some matrix notation is useful here. Index the state space Ω by the set $\{1, 2, \cdots, |\Omega|\}$. We will use the convention of denoting a random variable $f : \Omega \to \mathbb{R}$ as a *column vector* of length $|\Omega|$, while denoting any probability density over Ω as a *row vector* of the same dimension. Thus, from here on, the prior density ϕ will be understood to be a row vector of length $|\Omega|$. We will denote by $b_i(k)$ the row vector that gives the posterior density for firm i at the state indexed by k. By gathering together the conditional densities across all states for a particular firm i, we can construct the matrix of posterior probabilities for that firm. Define the matrix B_i as the matrix whose kth row is given by firm i's posterior density at the state indexed by k. That is

$$B_i \equiv \begin{bmatrix} - & b_i(1) & - \\ - & b_i(2) & - \\ & \vdots & \\ - & b_i(|\Omega|) & - \end{bmatrix}$$

We note one important general property of this matrix. We know that the average of the rows of B_i weighted by the prior probability of each state must be equal to the prior density itself. This is just the consequence of the consistency between the prior density and the posterior densities. In our matrix notation, this means that

$$\phi = \phi B_i \tag{5}$$

for all firms i. In other words, ϕ is a fixed point of the mapping defined by B_i. More specifically, note that B_i is a stochastic matrix in the sense that it is a matrix of non-negative entries where each row sums to one. Hence, it is associated with a Markov chain defined on the state space Ω. Then equation (5) implies that the prior density ϕ is an *invariant distribution* over the states for this Markov chain. We will make much use of this property in what follows. This formalization of differential information environments in terms of Markov chains follows Shin and Williamson (1996) and Samet (1998).

For any random variable $f : \Omega \to \mathbb{R}$, denote by $E_i f$ the conditional expectation of f with respect to i's information. $E_i f$ is itself a random variable, and so we can denote it as a column vector whose kth component is the conditional expectation of firm i at the state indexed by k. In terms of our matrix notation, we can write:

$$E_i f = B_i f$$

As well as the conditional expectation of any particular firm, we will also be interested in the average expectation across all firms. Define $\bar{E}f$ as

$$\bar{E}f = \frac{1}{N}\sum_{i=1}^{N} E_i f$$

$\bar{E}f$ is the random variable whose value at state ω gives the average expectation of f at that state. The matrix that corresponds to the average expectations operator $\bar{E}$ is simply the average of the conditional belief matrices $\{B_i\}$, namely

$$B \equiv \frac{1}{N}\sum_{i=1}^{N} B_i$$

Then, for any random variable f, the average expectation random variable $\bar{E}f$ is given by the product Bf. Since Bf is itself a random variable, we can define

$$B^2 f \equiv BBf$$

as the average expectation of the average expectation of f. Iterating further, we can define $B^k f$ as the kth order iterated average expectation of f. Then, the equilibrium pricing rule (1) can be expressed in matrix form as

$$p_i = \xi B_i q + (1-\xi) B_i p$$

Taking the average across firms, we have

$$p = \xi Bq + (1-\xi) Bp \tag{6}$$

By successive substitution, and from the fact that $0 < \xi < 1$, we have

$$\begin{aligned} p &= \xi \sum_{i=0}^{\infty} \left((1-\xi) B\right)^k Bq \\ &= \xi \left(I - (1-\xi) B\right)^{-1} Bq \\ &= MBq \end{aligned} \tag{7}$$

where M is the matrix

$$M = \xi \left(I - (1-\xi) B\right)^{-1}$$

Thus, equilibrium average price p is given by (7). Let us note some preliminary observations on the comparison between (7) and the case where all firms observe the same signal, and hence where the law of iterated expectations holds. When all firms observe the same signal, the k-fold iterated average expectation collapses to the single average expectation, and we have the pricing rule:

$$p = Bq \tag{8}$$

The difference between (7) and (8) lies in the role played by matrix M. Note that M is a stochastic matrix (i.e. a matrix of non-negative entries whose rows sum to one) since each row of the matrix $((1-\xi)B)^k$ sums to $(1-\xi)^k$ so that the matrix $(I-(1-\xi)B)^{-1} = \sum_{i=0}^{\infty}((1-\xi)B)^k$ has rows which sum to $1 + (1-\xi) + (1-\xi)^2 + \cdots = 1/\xi$. Thus, the matrix $M = \xi(I-(1-\xi)B)^{-1}$ is a stochastic matrix.

The matrix M serves the role of "adding noise" (in the sense of Blackwell) to the average expectation of the fundamentals q. The effect of the noise is to smooth out the variability of prices across states. Thus, in going from (8) to (7) the average price becomes a less reliable signal of the output gap. Since the noise matrix M is a convex combination of the higher order beliefs $\{B^k\}$, we must first understand what determines these higher order beliefs. In general, higher order expectations contain much less information than lower order expectations in the following precise sense. For any random variable f, denote by $\max f$ the highest realization of f, and define $\min f$ analogously as the smallest realization of f. Then for any stochastic matrices C and D and any random variable f,

$$\begin{aligned} \max CDf &\leq \max Df \\ \min CDf &\geq \min Df \end{aligned}$$

CD is a "smoother" or "noisier" version of D in the sense of Blackwell. So, the higher is the order of the iterated expectation, the more rounded are the peaks and troughs of the iterated expectation across states.

The importance of the parameter ξ is now apparent. The smaller is this parameter, the greater is the weighting received by the higher order beliefs in the noise matrix M, so that the prices are much less informative about the underlying fundamentals.

In particular, the limiting case for higher order beliefs B^k as k becomes large has a very special property. From (5), we know that

$$\phi = \phi B \tag{9}$$

so that the prior density ϕ is an invariant distribution for the Markov chain defined by the average belief matrix B. By post-mulitiplying both sides by B, we have

$$\phi = \phi B = \phi B^2$$

so that ϕ is an invariant density for B^2 also. By extension, we can see that ϕ is an invariant density for B^k for any kth order average belief operator. We also know from the elementary theory of Markov chains that under certain regularity conditions (which we will discuss below), the sequence $\left\{B^k\right\}_{k=1}^{\infty}$ converges to a matrix B^∞ whose rows are identical, and given by the unique stationary distribution over Ω. Since we know that the prior density ϕ is an invariant distribution, we can conclude that under the regularity conditions, all the rows of B^∞ are given by ϕ. That is

$$B^\infty = \begin{bmatrix} - & \phi & - \\ - & \phi & - \\ & \vdots & \\ - & \phi & - \end{bmatrix} \tag{10}$$

In other words, the limiting case of higher order beliefs B^k as k becomes large is so noisy that all information is lost, and the average beliefs converge to the prior density ϕ at every state. In particular, for any random variable f, successively higher order beliefs are so noisy that all all peaks and troughs into a constant function, where the constant is given by the prior expectation $\bar{f}$ (i.e. the expectation of f with respect to the prior density ϕ). In other words,

$$B^k f \to \begin{bmatrix} \bar{f} \\ \bar{f} \\ \vdots \\ \bar{f} \end{bmatrix} \quad \text{as } k \to \infty \tag{11}$$

To introduce the regularity conditions that ensure this, and to delve further into the underlying structure of our results, let us denote the (j,k)th entry of B by

$$b(j,k)$$

This is the probability of one-step transition from state j to state k in this Markov chain. The condition that guarantees (10) is the following.

Condition 1 *For any two states j and k, there is a positive probability of making a transition from j to k in finite time.*

Condition 1 ensures that the matrix B corresponds to a Markov chain that is *irreducible*, *persistent* and *aperiodic*. It is irreducible since all states are accessible from all other states. For finite chains, this also means that all states are visited infinitely often, and hence persistent. Finally, the aperiodicity is trivial, since all diagonal entries of B are non-zero irrespective of condition 1. We can then prove lemma 2. Samet (1998) proves an analogous result for the iteration of individual beliefs.

Lemma 2 *Suppose B satisfies condition 1. Then, the prior density ϕ is the unique stationary distribution, and $B^k \to B^\infty$, where B^∞ is the matrix whose rows are all identical and given by ϕ.*

Condition 1 has an interpretation in terms of the degree of information shared between the firms. It corresponds to the condition that

$$\bigcap_i \mathcal{I}_i = \emptyset \tag{12}$$

In other words, the intersection of the information sets across all firms is empty. There is no signal that figures in the information set of all the firms. For instance, if the firms' costs are highly correlated, but not exactly identical, then (12) holds so that condition 1 is satisfied. Another way to phrase this is to say that there is no non-trivial event that is common knowledge among the firms. The only event that is common knowledge is the trivial event Ω, which is the whole space itself.

When the intersection $\bigcap_i \mathcal{I}_i$ is non-empty, then this means that there are signals that are observed by every firm. Hence, the outcomes of signals in $\bigcap_i \mathcal{I}_i$ becomes common knowledge among all firms. One such example would be the publicly announced inflation forecast of the central bank. Information contained in $\bigcap_i \mathcal{I}_i$ is thus *public*. The equilibrium pricing decision of firms can be analysed for this more general case in which firms have access to public information, as well as their private information.

In this more general case, the limiting results for the higher order average belief matrices B^k correspond to the beliefs conditional on *public signals*. In order to introduce these ideas, let us recall the notion of an information partition for a firm. Let firm i's information partition be defined by the equivalence relation $\sim_i$ where $\omega \sim_i \omega'$ if firm i cannot distinguish between states ω and ω'. Denote firm i's information partition by $\mathcal{P}_i$, and consider set of all information partitions $\{\mathcal{P}_i\}$ across firms. The *meet* of $\{\mathcal{P}_i\}$ is defined as the finest partition that is at least as coarse as all of the parit
itions in $\{\mathcal{P}_i\}$. The meet of $\{\mathcal{P}_i\}$ is thus the greatest lower bound of all the individual partitions in the lattice over partitions ordered by the relation "is finer than". The meet of $\{\mathcal{P}_i\}$ is denoted by

$$\bigwedge_i \mathcal{P}_i$$

The meet is the information partition that is generated by the public signals - i.e. those signals that are in the information set of every firm, and hence in the intersection

$$\bigcap_i \mathcal{I}_i$$

The meet has the following property whose proof is given in Shin and Williamson (1996).

Lemma 3 *If two states ω and ω' belong to the same element of the meet $\bigwedge_i \mathcal{P}_i$, then there is positive probability of making a transition from ω to ω' in finite time in the Markov chain associated with B.*

Lemma 3 gives a generalization of condition 1. The idea is that the Markov chain defined by the average belief matrix B can be expressed in block diagonal form:

$$B = \begin{bmatrix} A_1 & & & \\ & A_2 & & \\ & & \ddots & \\ & & & A_J \end{bmatrix}$$

and where each sub-matrix A_j defines an irreducible Markov chain that corresponds to an element of the meet $\bigwedge_i \mathcal{P}_i$. Then, the higher-order belief limit is given by

$$B^{\infty} = \begin{bmatrix} A_1^{\infty} & & & \\ & A_2^{\infty} & & \\ & & \ddots & \\ & & & A_J^{\infty} \end{bmatrix}$$

Furthermore, we have

$$\phi = \phi B^{\infty} = \phi \begin{bmatrix} A_1^{\infty} & & & \\ & A_2^{\infty} & & \\ & & \ddots & \\ & & & A_J^{\infty} \end{bmatrix}$$

and so for any random variable f, the higher order expectation of f at each state has the following limiting property, in which the limit of the higher order expectation is the conditional expectation based on the public signals only.

$$B^k f \to \begin{bmatrix} E\left(f | \cap_i \mathcal{I}_i\right)(\omega_1) \\ E\left(f | \cap_i \mathcal{I}_i\right)(\omega_2) \\ \vdots \\ E\left(f | \cap_i \mathcal{I}_i\right)(\omega_N) \end{bmatrix} \quad \text{as } k \to \infty$$

where $E\left(f | \cap_i \mathcal{I}_i\right)(\omega_s)$ is the conditional expectation of f at state ω_s based on public information only.

This result has important implications for the pricing equilibrium for firms. For small values of the parameter ξ, the dominant influence in determining the average price level p is given by the set of *public signals*. In particular, if the central bank's forecast is publicly announced, and is a sufficient statistic for any public signals available to the firms, then the equilibrium average price p will be an extremely noisy signal of the underlying cost conditions in the economy. Rather than reflecting the underlying average marginal costs of the firms, p will simply reflect the public information only. In this sense, price will be an uninformative signal of the underlying state of fundamentals.

3.2 Extension to Dynamic Context

So far we have examined a static example of price setting by firms. However, many economic decisions (such as consumption and investment) are inherently dynamic, and so we have to face the task of showing whether the results shown so far translate into a more general framework that can accommodate dynamic decisions over time. This entails generalizing the argument above, but it turns out that the key results of our example hold in analogous form in a more general economy with time. We turn first to the definition of the state space Ω.

Time is discrete, and indexed by the non-negative integers. Let there be a countable set of economic variables

$$\{f_1, f_2, f_3, \cdots\}$$

that are of relevance to the economy. This list includes all economic variables that affect the fundamentals of the economy such as productivity, preferences and exogenous shocks, together with all signals observed by any economic agent of these variables. We assume that each economic variable f_k can take on a countable number of realizations, drawn from the set S_k. The *outcome space* is the product space

$$S \equiv \prod_k S_k.$$

The *outcome* of the economy at time t - given by a specified outcome for each of the economic variables f_s - is thus an element of S. Since each S_k is countable, so is the outcome space S.

The *state space* Ω is defined to be set of all sequences drawn from the set S. Thus, a typical state ω is given by the sequence

$$\omega = (s_0, s_1, s_2, \cdots)$$

where each s_t is an element of the outcome space S. Thus, a state ω specifies the outcome of all economic variables at every date, and so is a maximally specific description of the world over the past, present and future.

Let Ω be endowed with a prior probability measure ϕ. Each economic variable f_s then defines a stochastic process in the usual way in terms of the sequence

$$(f_{s,0}, f_{s,1}, f_{s,2}, \cdots)$$

where $f_{s,t}$ is the random variable that maps each state ω to the outcome of the economic variable f_s at time t. The information set of agent i at date t is a set of random variables whose outcomes are observed by firm i at date t. We denote by

$$\mathcal{I}_{i,t}$$

the information set of firm i at date t. The information set $\mathcal{I}_{i,t}$ defines the information partition of agent i at date t over the state space Ω. This information partition is denoted by

$$\mathcal{P}_{i,t}$$

The *meet* of the individual partitions at t is the finest partition of Ω that is at least as coarse as each of the partitions in $\{\mathcal{P}_{i,t}\}$. The meet at t is denote by $\mathcal{P}_t$. It is the partition generated by the intersection of all information sets at date t, as in our earlier discussion. The meet $\mathcal{P}_t$ represents the set of events that are common knowledge at date t.

The analysis of pricing decisions by firms can then be generalized to this new setting. By construction, the state space Ω is countable. Much of the notation and apparatus developed in the previous section can then be used in our new setting by using matrix notation for random variables and probability measures, provided that we are mindful of those rules for matrix manipulation that are not valid for infinite matrices. Kemeny, Snell and Knapp (1966) is a textbook reference for how infinite matrices can be used in the context of countable state spaces.

As before, any probability measure over Ω is denoted as a *row vector*, while a random variable f is denoted as a column vector. For each date t, the *average belief matrix* B_t is defined in the natural way. The sth row of B_t is the probability measure over Ω that represents the mean across firms of their conditional beliefs over Ω at date t. Then, the average price at date t satisfies

$$p_t = \xi B q_t + (1 - \xi) B_t p_t \tag{13}$$

where p_t is the average price at t, and q_t is the date t version of the random variable q in the static case. By successive substitution, and from the fact that $0 < \xi < 1$, we can solve for p_t.

$$p_t = \xi \sum_{i=0}^{\infty} ((1 - \xi) B_t)^k B q_t \tag{14}$$

Here, we encounter the first difference between our more general dynamic framework and the simple static framework developed earlier. For finite Ω, we can express the infinite sum $\sum_{i=0}^{\infty} ((1 - \xi) B_t)^k$ as being the inverse matrix $(I - (1 - \xi) B)^{-1}$. However, for infinite matrices, the notion of an inverse is not well defined, and so we cannot simplify (14) any further (see Kemeny, Snell and Knapp (1966, chapter 1)).

There is also a more substantial change to our results in this more general framework. The results concerning the prior information limit and the public information limit examined in the previous section is no longer valid. Let us first consider the prior information limit. For our finite state space example, Condition 1 was sufficient for the limiting result that, for any random variable f,

$$B^k f \to \begin{bmatrix} \phi f \\ \phi f \\ \phi f \\ \vdots \end{bmatrix} \quad \text{as } k \to \infty \tag{15}$$

where ϕf is the expectation of f with respect to the prior density ϕ. For our more general framework, we must strengthen Condition 1 by stipulating that the Markov chain associated with B_t is also *recurrent* in the sense of every state being visited infinitely often by the Markov chain. With this additional strengthening, we can then appeal to the standard results for Markov chains on the convergence to stationary distributions (see Karlin and Taylor (1975, p.35)) to conclude that

$$B_t^k \to \begin{bmatrix} - & \phi & - \\ - & \phi & - \\ - & \phi & - \\ & \vdots & \end{bmatrix} \quad \text{as } k \to \infty$$

Similar modifications must be made to the public information limit. Here, the additional restriction that needs to be imposed is the requirement that each irreducible *sub-chain* associated with elements of the meet $\bigwedge_i \mathcal{P}_{i,t}$ is recurrent. Then, we have the result that for any random variable f,

$$B_t^k f \to \begin{bmatrix} E\left(f|\cap_i \mathcal{I}_{i,t}\right)(\omega_1) \\ E\left(f|\cap_i \mathcal{I}_{i,t}\right)(\omega_2) \\ \vdots \\ E\left(f|\cap_i \mathcal{I}_{i,t}\right)(\omega_N) \end{bmatrix} \quad \text{as } k \to \infty$$

where $E\left(f|\cap_i \mathcal{I}_{i,t}\right)(\omega_s)$ is the conditional expectation of f at state ω_s based on date t public information only.

4 General Equilibrium Monetary Policy Model

We now consider the general equilibrium implications of the presence of both public and private information in monetary-policy models. Our analysis is based on a model with standard behavioural assumptions on households and firms. All agents are rational, in the sense that they know the structure of the economy and make optimal decisions based on their information sets. The only departure we make from the benchmark full information rational expectations setting is the absence of common knowledge of the state of the economy among some agents. Specifically, as in the partial equilibrium example studied in the previous section, we assume that firms receive private and public noisy signals of current shocks. By contrast, households and the central bank are assumed to observe these shocks perfectly. This helps keep the focus on the pricing decisions, where the presence of strategic complementarities allows differential information to have important dynamic effects.

In this section we describe the behaviour and information sets of households, firms and the central bank, respectively. In the next section, we begin by characterising equilibrium, and we then provide some simulation results illustrating the impact of public information on macroeconomic volatility and the effects of different monetary policies on economic dynamics.

4.1 Households

Households maximize their discounted expected utility of consumption subject to their budget constraint. One issue that must be addressed at the outset is the potential implications of having households possess private information. As mentioned above, we assume that households have full knowledge of the state. This allows households to mitigate idiosyncractic risk in incomes through insurance markets without greatly complicating our analysis. Households make identical consumption choices and we avoid having to keep track of the distribution of wealth. However, our assumption of perfect income insurance is only reasonable if we assume that households have perfect common knowledge without introducing complications regarding costly state verification. In addition, we would need to consider how rational expectations equilibria are established in asset markets under differential information. Incorporating asset market issues would take us too far astray, and divert attention from the main focus of our paper. Thus, both for the purpose of ensuring identical consumption decisions, and also for the purpose of avoiding asset market complications with differential information, we model households as having maximally-specific information sets with regard to all economic variables that have been realized to date.

To be more specific, we will assume that at any date t, households' information sets are identical, and include the realizations of all current and past economic variables $\{f_1, f_2, \cdots\}$. Thus, at date t, all households have the information set

$$\mathcal{I}_t^* \equiv \cup_s \{f_{s,0}, f_{s,1}, \cdots, f_{s,t}\}$$

Households' conditional expectations operator at date t is given by

$$E_t(\cdot) \equiv E(\cdot|\mathcal{I}_t^*)$$

At date t, households know at least as much as any other agent in the economy, including Nature, who has chosen the latest realizations of the economic variables.

Each household z supplies labour services of one type, $H_t(z,i)$, for firm i, and seeks to maximise

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[u(C_t(z)) - v(H_t(z,i)) \right] \right\} \tag{16}$$

subject to the budget constraint

$$E_t[\delta_{t,t+1}\Xi_{t+1}] \leq \Xi_t + W_t(i)H_t(z,i) + \Phi_t - P_t C_t(z) \tag{17}$$

Within each period, the household derives utility, $u(\cdot)$, from consuming the Dixit-Stiglitz aggregate, $C_t(z)$, defined as

$$C_t(z) \equiv \left[\int_0^1 C_t(z,i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}} \tag{18}$$

where $C_t(z,i)$ is household z's consumption of product i and $\epsilon > 1$ is the elasticity of substitution between differentiated products. As ϵ increases, goods become ever closer substitutes (i.e. firms have *less* market power), and hence the degree of strategic complementarity increases. Supplying $H_t(z,i)$ hours reduces welfare, as indicated by the function $v(\cdot)$. We assume that labour markets are competitive and a equal number of households supply labour of type i.

Households can insure against idiosyncratic risk in incomes (as mentioned above) and therefore consume the identical amount given by C_t. In the budget constraint, P_t denotes the price index corresponding to the aggregate C_t defined as

$$P_t \equiv \left[\int_0^1 P_t(i)^{1-\epsilon} di \right]^{\frac{1}{1-\epsilon}} \tag{19}$$

where $P_t(i)$ is the price of product i; Ξ_t denotes the nominal value of the household's holdings of financial assets at the beginning of period t; $W_t(i)$ is the nominal hourly wage for supplying labour of type i; Φ_t is the household's share of firms' profits, which we assume are distributed lump-sum to households, and $\delta_{t,s}$ is a stochastic discount factor, pricing in period t assets whose payoffs are realised in period s. We assume there exists a riskless one-period nominal bond, the gross return on which is given by $R_t \equiv (E_t \delta_{t,t+1})^{-1}$. Finally, notice that we have not assumed that housholds can insure against idiosyncratic variation in labour supply, although, in equilibrium, households who supply labour to firm i will work the same amount, $H_t(i)$.

Given the overall level of consumption, households allocate their expenditures across goods according to

$$C_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\epsilon} C_t \tag{20}$$

The first-order condition for determining the optimal level of consumption, given the allocation of consumption across goods expressed in (20), is $\Lambda_t = u_c(C_t)$, where Λ_t is the marginal utility of real income, and the standard Euler equation is given by

$$\Lambda_t / P_t = \beta R_t E_t [\Lambda_{t+1} / P_{t+1}] \tag{21}$$

A log-linear approximation of (21) around $\Lambda_t = \bar{\Lambda}$, $R_t = \bar{R}$ and $P_{t+1}/P_{t+1} = 1$ results in

$$\lambda_t = E_t \lambda_{t+1} + r_t - E_t \pi_{t+1} \tag{22}$$

where $\pi_{t+1} \equiv \log(P_{t+1}/P_t)$ is the inflation rate and lower case represents percent deviation of a variable from its steady state.

Market clearing requires that $C_t = Y_t - G_t$, where Y_t is the aggregate demand for output and G_t is an exogenous component of demand (e.g. exogenous government expenditures). Since $\Lambda_t = u_c(Y_t - G_t)$, λ_t can be expressed as

$$\lambda_t = -\sigma\left(y_t - g_t\right) \tag{23}$$

where $\sigma \equiv u_{cc}(\bar{C})\bar{C}/u_c(\bar{C})$ is the inverse of the intertemporal elasticity of substitution. Substituting out for λ_t in (22) yields a "forward-looking IS equation":

$$y_t - g_t = E_t\left(y_{t+1} - g_{t+1}\right) - \sigma^{-1}\left[r_t - E_t\pi_{t+1}\right] \tag{24}$$

The (log of) the demand shock, g_t, is assumed to follow a Markov process given by

$$g_t = \rho_g g_{t-1} + \varepsilon_t^g, \qquad \varepsilon_t^g \overset{iid}{\sim} N(0, \sigma_{\varepsilon,g}^2) \tag{25}$$

Finally, the first-order condition for optimal labour supply is found by equating the marginal rate of substitution of consumption for leisure with the real wage

$$\frac{W_t(i)}{P_t} = \frac{v_h(H_t(i))}{\Lambda_t} \tag{26}$$

4.2 Firms

Consider first the optimal pricing decisions of firms, taking as given each firm's information set. Each firm i faces a Cobb-Douglas production technology with constant returns to scale

$$Y_t(i) = K_t(i)^{\zeta}(A_t H_t(i))^{1-\zeta} \tag{27}$$

where $K_t(i)$ is the capital input of firm i, A_t denotes a labour-augmenting technology shock and $0 < \zeta < 1$. For simplicity, we assume that the level of the capital stock is fixed and equal across firms (i.e. $K_t(i) = \bar{K}$). This assumption means that the demand for each good has the same form as (20), namely

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\epsilon} Y_t \tag{28}$$

Analogous to g_t, the technology shock, a_t, is assumed to follow the Markov process

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a, \qquad \varepsilon_t^a \overset{iid}{\sim} N(0, \sigma_{\varepsilon,a}^2) \tag{29}$$

The pricing decision by the firm is a static optimisation problem, where the first-order condition is given by

$$E_t^i \left[\frac{\partial \Pi_t(i)}{\partial P_t(i)} \right] = E_t^i \left[(1-\epsilon) \frac{Y_t(i)}{P_t} + \epsilon \frac{Y_t(i)}{P_t(i)} \frac{MC_t(i)}{P_t} \right] = 0 \tag{30}$$

where $\Pi_t(i)$ is firm i's real profit function and $MC_t(i)$ is its nominal marginal cost of producing an extra unit of output. Firms' conditional expectations operator at date t is given by

$$E_t^i(\cdot) \equiv E\left(\cdot | \mathcal{I}_t^i\right)$$

where $\mathcal{I}_t^i$ is the information set of firm i (see below).

Rearranging (30) yields

$$E_t^i \left[\frac{P_t(i)}{P_t} - \frac{\epsilon}{\epsilon - 1} \frac{MC_t(i)}{P_t} \right] = 0 \tag{31}$$

Thus, the firm chooses its price such that its expected relative price is a constant mark-up over expected real marginal cost. In a situation of complete common knowledge, equation (31) reduces to the familiar condition that firms set their price equal to a fixed mark-up over marginal cost.

A log-linear approximation of (31) around $P_t(i)/P_t = 1$ and $S_t(i) \equiv MC_t(i)/P_t = (\epsilon - 1)/\epsilon$ gives

$$E_t^i \left[\hat{p}_t(i) - s_t(i) \right] = 0 \tag{32}$$

where $\hat{p}_t(i) \equiv \log(P_t(i)/P_t)$.

Since real marginal cost is equal to the ratio of the real wage to the marginal product of labour, and in equilibrium the real wage must also equal the marginal rate of substitution, as given in (26), a log-linear approximation of real marginal cost can be expressed as

$$s_t(i) = \omega y_t(i) - (\nu + 1) a_t - \lambda_t \tag{33}$$

where $\nu \equiv v_{hh}(\bar{H})\bar{H}/v_h(\bar{H})$ is the inverse of the Frisch elasticity of labour supply and $\omega \equiv \left(\frac{\nu+\zeta}{1-\zeta}\right)$. Substituting (23) into (33) and rearranging gives

$$s_t(i) = (\omega + \sigma)\left(y_t - y_t^n\right) - \omega\epsilon\hat{p}_t(i)$$

where y_t^n is the "natural rate of output", defined as

$$y_t^n \equiv \frac{1}{(\omega + \sigma)}\left[(\nu + 1)a_t + \sigma g_t\right] \tag{34}$$

We can now substitute the expression for marginal cost, given by (33), in the first-order condition for pricing, (32), to yield

$$p_t(i) = E_t^i p_t + \xi E_t^i\left(y_t - y_t^n\right) \tag{35}$$

where $\xi \equiv (\omega+\sigma)/(1+\omega\epsilon)$. This equation is analogous to (1). Following the same steps as in Section 2, by first averaging (35) across firms, rewriting the resulting expression in terms of nominal output, defined as $q_t \equiv y_t + p_t$, and solving by repeated substitution yields

$$p_t = \sum_{k=1}^{\infty} \xi\left(1 - \xi\right)^{k-1} \bar{E}_t^k\left(q_t - y_t^n\right) \tag{36}$$

Next consider the information sets of firms. There are two sources of aggregate disturbances in the model: the demand shock, g_t, and the productivity shock, a_t. To simplify matters, we assume that each firm observes one private and one public signal of each of these shocks. Specifically, firm i's information set is given by

$$\mathcal{I}_t^i \equiv \left\{g_t(i), a_t(i), g_t^P, a_t^P\right\}$$

where $g_t(i)$ and $a_t(i)$ are private signals of g_t and a_t, respectively, and, similarly, g_t^P and a_t^P are public signals of g_t and a_t. Each of the signals is assumed to have an *iid* Gaussian distribution, with conditional mean equal to the fundamental shock; namely,

$$g_t(i) = g_t + e_t^g(i), \qquad e_t^g(i) \overset{iid}{\sim} N(0, \sigma_{e,g}^2) \tag{37}$$

$$a_t(i) = a_t + e_t^a(i), \qquad e_t^a(i) \overset{iid}{\sim} N(0, \sigma_{e,a}^2) \tag{38}$$

$$g_t^P = g_t + \eta_t^g, \qquad \eta_t^g \overset{iid}{\sim} N(0, \sigma_{\eta,g}^2) \tag{39}$$

$$a_t^P = a_t + \eta_t^a, \qquad \eta_t^a \overset{iid}{\sim} N(0, \sigma_{\eta,a}^2) \tag{40}$$

The innovations in (25), (29) and (37)-(40) are assumed to be independent of each other at all leads and lags.

Other plausible assumptions on firms' information sets could also be incorporated into our framework. For example, one alternative approach would be to have firms obtain signals of endogenous variables directly, instead of the underlying fundamental shocks. For instance, firm i might observe a private signal of the price level such as $p_t^S(i) = p_t + e_t^p(i)$. A more obvious alternative is to allow firms to observe all of the variables involved in their own production activities, such as their own output, hours hired and wages paid. In the current set-up, if firms can observe their own output and hours employed when making pricing decisions, then they can infer without error the value of the technology shock, A_t (or equivalently, a_t), from the production function (27). However, by modifying the model with the further realistic assumption that firms are subject to idiosyncratic technology shocks and that they can only infer their own level of productivity from their production activities, we are back to the present case where firms do not perfectly observe aggregate productivity shocks. Thus, if we were to introduce firm-specific technology shocks, we could also include $y_t(i)$, $h_t(i)$ and $w_t(i)$ in $\mathcal{I}_t^i$ and still obtain qualitatively similar results as in the present set-up.

4.3 Monetary Policy

Most central banks conduct monetary policy by setting a target for a short-term nominal interest rate.[2] Much of the recent monetary policy literature, both

[2]This interest rate is typically an overnight rate in an interbank market, for example, the federal funds rate in the United States.

theoretical and empirical, has assumed that the short-term rate is set according to a rule. An example is the Taylor rule (1993), where the interest rate is a function of the deviation of current inflation from target and current output (or the output gap). Other rules have been investigated by various authors in a wide range of models, including, notably, forecast-based rules (see, for example, Batini and Haldane (1999)). In forecast-based rules, the central bank's forecast of inflation, and possibly output, replace observations on actual current values of the variables. The appeal of these types of rules is that they seem to correspond more closely to descriptions of central bank behaviour in reality. Whether or not a central bank has an explicit numerical target for inflation, forecasts seem to play a pivotal role in policy makers' decisions.

In this paper we consider a forecast-based rule for monetary policy. Specifically, following recent practice, we assume that the central bank sets the one-period riskless nominal interest rate, r_t, according to

$$r_t = \alpha_\pi E_t \pi_{t+1} + \alpha_y y_t \tag{41}$$

One important additional assumption we make is that the central bank has the same information set as households.[3] This means that policy makers observe, among other things, the current price level and output without error. The reason for assuming that the central bank observes the state perfectly is, once again, to keep our focus on the impact of differential information on firms' pricing behaviour and its macroeconomic consequences. Restricting monetary policy makers to have imperfect knowledge, while an interesting case to consider in its own right and certainly more realistic, would only serve to cloud the present analysis.

[3] In particular, the conditional expectation in (41) is computed using the same probability measure used by households. Recall that households' information sets are maximally-specific with regard to all random variables realized to date.

5 General Equilibrium

The complete model is given by the behavioural equations: (24), (36) and (41); the processes for the fundamental shocks: (25) and (29); and the processes for the signals: (37)-(40). We begin this section by setting up the model in state-space form, solving for the stochastic process followed by the state and then solving for the equilibrium of the price level, output and the interest rate. This is followed by an investigation of some of the equilibrium properties of the model, inclduing a comparison of the differential information model to a oft used version of a sticky price model.

5.1 Characterising Equilibrium

The first step in solving the model is to describe the state space and determine the stochastic process followed by the state. In the present model, the state, denoted by X_t, is given by

$$X_t \equiv \begin{bmatrix} \theta_t \\ \psi_t \end{bmatrix} \tag{42}$$

where θ_t is a vector of exogenous variables and ψ_t is defined as

$$\psi_t \equiv \sum_{k=1}^{\infty} \xi \left(1-\xi\right)^{k-1} \bar{E}_t^k \left(\theta_t\right) \tag{43}$$

In (43), the average expectations operator, $\bar{E}_t^k(\bullet)$, refers to average expectations across firms. From equations (24) and (43), it can be seen that the exogenous variables are

$$\theta_t \equiv \begin{bmatrix} a_t, & g_t, & \eta_t^a, & \eta_t^g \end{bmatrix}'$$

The vector θ_t follows a Markov process given by

$$\theta_t = B\theta_{t-1} + bu_t \tag{44}$$

where

$$u_t \equiv \begin{bmatrix} \varepsilon_t^a, & \varepsilon_t^g, & \eta_t^a, & \eta_t^g \end{bmatrix}', \qquad u_t \overset{iid}{\sim} N(0, \Omega_u)$$

$$\Omega_u \equiv diag\left(\left[\begin{array}{cccc} \sigma_{\varepsilon,a}^2, & \sigma_{\varepsilon,g}^2, & \sigma_{\eta,a}^2, & \sigma_{\eta,g}^2 \end{array}\right]\right)$$

$$B \equiv \left[\begin{array}{cc} B_1 & 0_4 \\ 0_4 & 0_4 \end{array}\right], B_1 \equiv \left[\begin{array}{cc} \rho_a & 0 \\ 0 & \rho_g \end{array}\right], b \equiv I_4$$

I_n and 0_n denote the $n \times n$ identity and null matrices, respectively.

Each firm observes the vector of variables

$$y_t^{sig}(i) \equiv \left[\begin{array}{cccc} a_t(i), & g_t(i), & a_t^P, & g_t^P \end{array}\right]'$$

In terms of X_t, $y_t^{sig}(i)$ can be expressed as

$$y_t^{sig}(i) = ZX_t + zv_t(i) \tag{45}$$

where

$$v_t(i) \equiv \left[\begin{array}{cc} e_t^a(i), & e_t^g(i) \end{array}\right]', \qquad v_t(i) \overset{iid}{\sim} N(0, \Omega_v)$$

$$\Omega_v \equiv diag\left(\left[\begin{array}{cc} \sigma_{e,a}^2, & \sigma_{e,g}^2 \end{array}\right]\right)$$

$$Z \equiv \left[\begin{array}{cc} Z_1 & Z_2 \end{array}\right], Z_1 \equiv \left[\begin{array}{cc} I_2 & 0_2 \\ I_2 & I_2 \end{array}\right], Z_2 \equiv 0_4, z \equiv \left[\begin{array}{c} I_2 \\ 0_2 \end{array}\right]$$

Lemma 4 *Given equations (44) and (45), the state, X_t, defined in (42) follows the Markov process given by*

$$X_t = MX_{t-1} + mu_t$$

where

$$M \equiv \left[\begin{array}{cc} B & 0_n \\ G & H \end{array}\right], m \equiv \left[\begin{array}{c} b \\ h \end{array}\right],$$

and the matrices G, H and h are given in equations (58), (59) and (60), respectively.

Proof. See Appendix A.1. ■

We are now in a position to describe the equilibrium processes of p_t, y_t and r_t.

Theorem 5 *Consider the model given by equations (24), (36) and (41), and the assumptions of lemma 4. Under certain conditions on α_π and α_y, a Markov Perfect Equilibrium is given by*

$$
\begin{aligned}
p_t &= \lambda' X_t \\
y_t &= \left[(I - \mu_1 M')^{-1}\delta - \lambda\right]' X_t \\
r_t &= \alpha' X_t
\end{aligned}
$$

where μ_1, δ, λ and α are given in equations (65), (68), (71,76) and (77), respectively.

Proof. See Appendix A.2. ■

5.2 Model Properties

Here we examine several features of the model presented above. Before proceeding, we must choose values for the parameters. These are presented in Table 1. Putting aside the variances of the shocks, the numbers chosen for the other parameters fall within the range of values typically used in the literature. On the other hand, the variances have been chosen somewhat arbitrarily, since there is not much evidence we can draw upon for these parameters. However, it is not our objective to show that the simple version of the model investigated here provides a good *quantitative* description of actual economies. We simply wish to illustrate some properties of the model. Thus, as a baseline case, we set all the variances equal to each other — for the innovations of the fundamental shocks, as well as the shocks to the signals. The absolute values of the variances are not very important. By contrast, their relative size does matter, as is born out in the simulations we present.

5.2.1 Changing Weights on Higher-Order Beliefs

Recall that one of the key parameters of the model is ξ, which determines the relative weight attached to higher-order expectations in the pricing relation (36).

Among other things, ξ depends inversely upon the elasticity of substitution, ϵ. Thus, an increase in ϵ, which increases the coordination motive among firms and produces a smaller steady-state markup, gives a more prominent role to higher-order beliefs by lowering ξ.[4] One feature of the macro model we wish to highlight is the implication of changing ξ on the sample paths of the output gap and the price level. We do this by altering the value of ϵ, since it enters the model only through ξ.

The results of one such experiment are shown in Figure 1. Each panel of the figure plots one sample realisation (time series) of the price level against the output gap using the same randomly drawn sample of shocks. The cases in the panels are distinguished by their treatment of ϵ and the relative precision of the public signals. A markup of 10 percent (left-hand side panels) corresponds to our baseline parameterisation, whereas the markup has been cut to 5 percent to produce the right-hand side panels. In addition, the top panels report cases with high-precision public signals, whereas the lower panels are based on low-precision public signals. The plots suggest that, conditional on the output gap, an increase in competition (lower markup) or a decline in the precision of the public signal spreads out prices. This is most evident in the lower right panel, where prices depend relatively more on higher-order expectations (due to lower ξ), which in turn are adversely affected by noisier information (less precise public signals).

These scatter plots intimate the potential degradation of the information value of price as a signal of the output gap. For economies that have relatively noisy public signals and a high degree of competition, prices convey poor quality information about the underlying output gap.

5.2.2 Volatility and the Quality of Public Information

We next demonstrate that more precise public information does not necessarily lead to lower volatilty among endogenous variables. This result is evident in

[4] As already noted by Woodford (2002), such changes are more critical in the current setting than in standard sticky price models (see below), where an increase in competition lowers the elasticity of inflation to the output gap, but no more.

Figures 2 and 3. These figures plot values of the variances of the endogenous variables as a function of the precision of the public signals, where the precision is defined as $1/\sigma^2_{\eta,\cdot}$. Figure 2 documents the case when firms' private signals are of high precision, whereas in Figure 3 firms' signals are of low precision.[5] The result mentioned above is shown in Figure 2. When firms observe fairly precise private signals of aggregate shocks, the mere presence of the public signal, interpreted as a signal with precision greater than zero, actually makes inflation *more* volatile.[6] Moreover, increases in the precision of the public signal produce a higher variance of inflation over a certain range. These plots illustrate one key effect of public information. From the results in section 3, recall that more precise public signals get a higher weight in both individuals' and average k-fold expectations. A higher weight on a common (i.e. public) signal necessarily means that individuals' expectations are distributed more closely together around the public signal. However, this can lead to greater volatility in the aggregate if the public signal is not very precise relative to private information. This is exactly what is shown in Figure 2 for inflation. By contrast, when firms have sufficiently imprecise private information (Figure 3), increases in the precision of public always lead to lower volatility.

Since higher-order beliefs play a direct role only in firms' pricing decisions, it is perhaps not surprising that these effects largely pertain to inflation outcomes. These results affirm and extend the conclusions of Morris and Shin (2002) to a dynamic macroeconomic setting, namely, that more precise public information does not necessarily lead to better welfare outcomes. Importantly, this is not predicated on inefficiencies that arise due to poor information available to the central bank. On the contrary, the central bank operates with *full* information on the state of the economy.

[5] The precisions of the public signals are set equal to each other. Similarly, the precisions of the private signals are also set equal to each other. This value is 16 percent (high precision case) and 0.25 percent (low precision case).

[6] This is true at least over the range of values considered for the precision of the public signals.

5.2.3 Impulse Responses

One way to illuminate the dynamic interrelationships among variables in the model is to examine impulse responses to innovations in the fundamental shocks, g_t and a_t. These responses are shown in Figure 4. In each panel, the solid line shows the response of the variable to a one-standard deviation innovation in the demand shock and the dashed line plots the response to a similar innovation in the technology shock. One interesting property of equilibrium in this model evident in the figure is stationarity of the price level (the price level converges back to its mean in the third panel). The basic intuition for this result comes from noticing that neither the pricing rule of firms nor the monetary policy rule introduces any inherent dependence on *past* prices. Since the initial responses of inflation to these shocks are in the direction one would expect — positive for the demand shock, negative for the technology shock — one implication is that inflation must overshoot its mean (see the second panel).

It is illuminating to contrast the responses in Figure 4 with those in an economy that features sticky prices. As an example, consider the standard version of the New Keynesian Phillips Curve given by

$$\pi_t = \beta E_t \pi_{t+1} + \kappa E_t \left(y_t - y_t^n \right) \tag{46}$$

This equation can be derived from either Rotemberg's (1982) model of adjustment costs in price setting or Calvo's (1983) model of staggered price setting.[7] In the Calvo case, the composite parameter κ depends upon, among other things, the average duration prices are held fixed (D) and the elasticity of substitution between goods (ϵ).[8] For the simulations presented here, we set D equal to three quarters.

Analogous to Figure 4, Figure 5 plots impulse responses to innovations in the demand and technology shocks. It is important to recognise that precise

[7] See Woodford (2003) for an extensive treatment of sticky price models of this type.

[8] As in the differential-information model, an increase in competition (larger ϵ) reduces the senstivity of current prices to aggregate demand, i.e. lower κ. Similarly, an increase in D lowers κ.

quantitative comparisons between Figures 4 and 5 are difficult to interpret, as no attempt has been made to choose parameters in the models subject to a common criteria. Nonetheless, it is worthwhile pointing out some features that are likely to be robust under other reasonable parameter values. First, in contrast to the differential-information model, the price level is not stationary and inflation does not overshoot its mean. The behaviour of inflation is also matched by monotonic responses of the interest rate to the shocks, unlike in the differential information model. Second, the responses of output to the shocks are almost identical in the two models, though this is largely due to the fact that we have assumed that households have full common knowledge in both cases.

Concentrating once again on the differential information model, Figure 6 plots impulse responses to shocks to the public signals. The effects on output are trivial due once again to the fact that households have perfect knowledge of the state and the indirect effects of firms' imperfect common knowledge on consumption is minimal. The more interesting behaviour has to do with inflation, and its consequences for interest rates. In the face of a positive shock to the public signal of demand (solid lines), firms initially raise prices, causing inflation to go up. However, since actual demand has not increased (first panel), firms gradually lower their prices back to their original level. This causes price changes to become negative, i.e. inflation overshoots its mean. Meanwhile, anticipating the decline in inflation, the central bank first lowers interest rates. Overall, the initial rise in the short-term real interest rate and its later decline combine to keep the long-term real rate relatively constant, explaining the basically flat response of consumption. Similar, but opposite effects are at work when the public signal of technology is perturbed.

5.2.4 Changes in the Policy Rule

In the last set of simulations we present, we illustrate the impact of changing the coefficients in the policy rule (41). Once again, our interest is in the effects of the relative precision of public and private signals, in this instance when we alter the behaviour of monetary policy. Specifically, we compute the variances of the endogenous variables as a function of α_π, the policy response to expected inflation. The results are displayed in Figure 7. The solid line shows the case with high-precision public signals, the dashed line is the case with low-precision public signals.

From this figure it is evident that this model exhibits the classic trade-off between inflation (or price level) and output stabilisation. As α_π increases, the variance of inflation declines, whereas the variance of output increases. Focusing on inflation (top right panel), the result from Figure 2 that more precise public information can induce greater volatility in inflation is confirmed; the dashed line is always below the solid line. However, the differences in variances between the two cases disappears as policy responds more and more aggresively to inflation. Notice that the more aggressive policy response has no impact on the information that agents receive because firms' signals are not endogenous. A policy that reacts more strongly to expected inflation, and hence price movements, does so by stabilising the output gap — the fundamental that firms are learning about. The more aggressive policy response ends up making the relative precision of public information less relevant in the firm's filtering problem.

6 Conclusions

An economy with diverse private information has features that are not always well captured in representative individual models where all agents share the same information. The most distinctive of these features is the relatively greater impact of common, shared information at the expense of private information. The source of the greater impact of public information lies in the strategic complementarity of the price setting behaviour of firms, and the impact of public information is greater for those economies where price competition is more fierce.

The observation that public signals have a disproportionately large impact in games with coordination elements is not new, but our contribution has been to demonstrate how the theoretical results can be embedded in a standard macroeconomic model that is rich enough to engage in questions of significance for policy purposes. Moreover, our discussion of the theoretical background in section 3 has been motivated by the need to unravel the main mechanisms at work. By developing the argument by means of a series of simple examples, our intention has been to convey the main intuitions, and so show that the results do not rely on senstive ways on specific functional forms or distributional assumptions.

In order to operationalise our model for the purpose of numerical simulations, we have had to make a number of simplifiying assumptions, such as the fact that consumers are fully informed, and that the central bank is also fully informed. Nevertheless, our simulation results reveal that the impact of public information is large, and shifts in the precision of public information has significant effects on observable variables that enter into calculations of welfare. At the cost of some additional complexity, it is possible to extend our model to contexts where agents not only have noisy information of the underlying fundamentals of the economy, but also of the endogenous variables. Also, it is possible to inject more realism into the numerical simulations by allowing the central bank to have less than perfect information of the fundamentals. In both cases, we conjecture that the central bank could actually do more harm by adopting a vigorous response to inflation or its inflation forecast through further degradation of the informational content of signals observed by firms. We are currently at work on these two extensions.

The results we have obtained here are suggestive, and invite further investigation. The policy conclusions that flow from these results also merit further consideration.

A Solving for General Equilibrium

A.1 Proof of Lemma 4

Recall that X_t is defined as

$$X_t \equiv \begin{bmatrix} \theta_t \\ \psi_t \end{bmatrix} \tag{47}$$

where θ_t is a vector of variables that are exogenous with respect to p_t, y_t and R_t, and ψ_t is defined as

$$\psi_t \equiv \sum_{k=1}^{\infty} \xi \left(1-\xi\right)^{k-1} \bar{E}_t^k \left(\theta_t\right) \tag{48}$$

θ_t is governed by the process

$$\theta_t = B\theta_{t-1} + bu_t \tag{49}$$

for known matrices B and b and where $u_t \sim N(0, \Omega_u)$ is a vector of *iid* random variables.

The state-space model is completed by specifying the observation equation. Denoting by $y_t^{sig}(i)$ the vector of variables observed by firm i at date t, the observation equation in general can be written as

$$y_t^{sig}(i) = ZX_t + zv_t(i)$$

for known matrices $Z \equiv [Z_1 \quad Z_1]$ and z, and where $v_t(i) \sim N(0, \Omega_v)$ is a vector of random variables that are independently and identically distributed across time and firms. These assumptions, and the law of large numbers, imply that $\int_0^1 v_t(i)di = 0$.

For now assume (to be confirmed later) that the state, X_t, follows a process given by

$$X_t = MX_{t-1} + mu_t \tag{50}$$

where

$$M \equiv \begin{bmatrix} B & 0_n \\ G & H \end{bmatrix}, m \equiv \begin{bmatrix} b \\ h \end{bmatrix}$$

and the matrices G, H and h are yet to be determined. When there is no ambiguity, the subscript will be omitted from I_n and 0_n.

Now consider the firm's problem of estimating the state, X_t, using the Kalman filter. Given the assumptions made so far, the Kalman filter produces minimum mean squared error estimates of the state for the log-linearised version of the model. Assume that a time-invariant filter exists that is also independent of i, with the Kalman gain denoted by K. Let $X_{t|s}(i) \equiv E_s^i X_t$. Combining the prediction and updating equations from the Kalman filter for firm i gives

$$X_{t|t}(i) = MX_{t-1|t-1}(i) + K\left(y_t^{sig}(i) - ZMX_{t-1|t-1}(i)\right) \tag{51}$$

Averaging across i and rearranging gives

$$\begin{aligned} X_{t|t} &= (I - KZ)\, MX_{t-1|t-1} + KZX_t \\ &= (I - KZ)\, MX_{t-1|t-1} + KZMX_{t-1} + KZmu_t \end{aligned}$$

Defining $\Xi \equiv [\xi I \quad (1-\xi)I]$ and $\hat{K} \equiv \Xi K$, first notice that $\psi_t = \Xi X_{t|t}$, and thus $(1-\xi)\psi_{t-1|t-1} = \psi_{t-1} - \xi\theta_{t-1|t-1}$. This implies

$$\psi_t = (\Xi - \hat{K}Z)MX_{t-1|t-1} + \hat{K}ZMX_{t-1} + \hat{K}Zmu_t \tag{52}$$

and

$$X_{t-1|t-1} = \varphi_1\psi_{t-1} + \varphi_2\theta_{t-1|t-1} \tag{53}$$

where $\varphi_1 \equiv [0 \quad \frac{1}{1-\xi}I]'$ and $\varphi_2 \equiv [I \quad -\frac{\xi}{1-\xi}I]'$. Substituting (53) into (52) and expanding gives

$$\begin{aligned} \psi_t &= \left[\hat{K}Z_1B + \hat{K}Z_2G\right]\theta_{t-1} + \left[\frac{1}{(1-\xi)}\hat{\Xi}_2 + \hat{K}Z_2H\right]\psi_{t-1} \\ &\quad + \left[\hat{\Xi}_1 - \frac{\xi}{(1-\xi)}\hat{\Xi}_2\right]\theta_{t-1|t-1} + \left[\hat{K}Z_1b + \hat{K}Z_2h\right]u_t \end{aligned}$$

If X_t is governed by (50), then it must be the case that

$$\begin{aligned} G &= \hat{K}Z_1B + \hat{K}Z_2G && (54) \\ H &= \frac{1}{1-\xi}\hat{\Xi}_2 + \hat{K}Z_2H && (55) \\ h &= \hat{K}Z_1b + \hat{K}Z_2h && (56) \\ \hat{\Xi}_1 &= \frac{\xi}{1-\xi}\hat{\Xi}_2 && (57) \end{aligned}$$

Solving for G and h directly, and substituting (57) and the result for G into (55), requires that

$$\begin{aligned} G &= \left(I - \hat{K}Z_2\right)^{-1} \hat{K}Z_1 B \\ H &= \frac{1}{\xi}\left(I - \hat{K}Z_2\right)^{-1} \{\left(\xi I - \hat{K}Z_1\right) \\ &\quad + \left((1-\xi)I - \hat{K}Z_2\right)\left(I - \hat{K}Z_2\right)^{-1} \hat{K}Z_1\} B \\ h &= \left(I - \hat{K}Z_2\right)^{-1} \hat{K}Z_1 b \end{aligned}$$

provided that $\left(I - \hat{K}Z_2\right)$ is nonsingular. Noting that $Z_2 = 0_4$, these expressions can be simplified to yield

$$\begin{aligned} G &= \hat{K}Z_1 B & (58) \\ H &= \left(I - \hat{K}Z_1\right) B & (59) \\ h &= \hat{K}Z_1 b & (60) \end{aligned}$$

The last step is to determine the value of K. Under the above assumptions, we have (see Harvey (1989))

$$K = \Sigma Z' \left(Z\Sigma Z' + z\Omega_v z'\right)^{-1}$$

where

$$\begin{aligned} \Sigma &= MVM' + m\Omega_u m' & (61) \\ V &= \Sigma - \Sigma Z' \left(Z\Sigma Z' + z\Omega_v z'\right)^{-1} Z\Sigma & (62) \end{aligned}$$

Substituting (62) into (61), we obtain a Riccati equation:

$$\Sigma = M\left(\Sigma - \Sigma Z' \left(Z\Sigma Z' + z\Omega_v z'\right)^{-1} Z\Sigma\right) M' + m\Omega_u m' \qquad (63)$$

The equation (63) can be solved for Σ by finding the fixed point using numerical techniques.

A.2 Proof of Theorem 5

We wish to determine equilibrium processes for p_t, y_t and r_t as a function of the state, X_t. First, substitute out for r_t in (24) using (41) and rewrite in terms of nominal output, q_t. This gives

$$\begin{aligned} q_t &= E_t q_{t+1} - \left[\sigma^{-1}(\alpha_\pi - 1) + 1\right] E_t(p_{t+1} - p_t) + \sigma^{-1}\alpha_y(p_t - q_t) + (1 - \rho_G)g_t \\ &= \mu_1 E_t q_{t+1} + (\mu_1\mu_2 + 1)p_t - \mu_1(\mu_2 + 1)E_t p_{t+1} + \mu_1(1 - \rho_G)g_t \end{aligned} \tag{64}$$

where

$$\mu_1 \equiv \frac{1}{\sigma^{-1}\alpha_y + 1}, \quad \mu_2 \equiv \sigma^{-1}(\alpha_\pi - 1) \tag{65}$$

Next, suppose (to be confirmed later) that the price level can be written as

$$p_t = \lambda' X_t \tag{66}$$

for some vector, λ, to be determined. Substituting (66) into (64), we obtain

$$q_t = \mu_1 E_t q_{t+1} + \delta' X_t \tag{67}$$

where

$$\delta \equiv ((\mu_1\mu_2 + 1)I - \mu_1(\mu_2 + 1)M')\,\lambda + \begin{bmatrix} I \\ 0 \end{bmatrix} \beta \tag{68}$$

and $\beta \equiv \begin{bmatrix} 0 & \mu_1(1-\rho_G) & 0 & 0 \end{bmatrix}'$. Assuming that $\alpha_y > 0$, so that $\mu_1 < 1$, we can solve (67) forward to get

$$\begin{aligned} q_t &= \delta' \sum_{i=0}^{\infty} \mu_1^i E_t X_{t+i} \\ &= \delta' \sum_{i=0}^{\infty} (\mu_1 M)^i X_t \\ &= \delta'(I - \mu_1 M)^{-1} X_t \end{aligned} \tag{69}$$

It remains to be determined whether there exists a unique value for λ, and hence δ, such that (66) holds. Recall that p_t is given by

$$p_t = \sum_{k=1}^{\infty} \xi\,(1-\xi)^{k-1}\,\bar{E}_t^k\,(q_t - y_t^n) \tag{70}$$

Define $N \equiv (I - \mu_1 M)^{-1}$ and partition N and δ conformably with X_t according to

$$N \equiv \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix}, \delta \equiv \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix}$$

The natural rate of output is a function solely of exogenous variables, and thus can be expressed as $y_t^n = \gamma'\theta_t$, for some vector, γ. Substituting this expression for y_t^n and (69) into (70) yields

$$p_t = \sum_{k=1}^{\infty} \xi \left(1-\xi\right)^{k-1} \bar{E}_t^k \left(\left[N'_{11}\delta_1 + N'_{21}\delta_2 - \gamma\right]' \theta_t + \left[N'_{12}\delta_1 + N'_{22}\delta_2\right]' \psi_t \right)$$

If the price level has the form $p_t = \lambda' X_t$, as conjectured in (66), then it must be the case

$$\lambda \equiv \begin{bmatrix} 0 \\ \lambda_2 \end{bmatrix} \tag{71}$$

$$\lambda_2 = N'_{11}\delta_1 + N'_{21}\delta_2 - \gamma \tag{72}$$

$$\delta_2 = -N'^{-1}_{22} N'_{12}\delta_1 \tag{73}$$

assuming that N_{22} is nonsingular. Substituting out for δ_2 in (72) using (73) and rearranging implies

$$\lambda_2 = \left[N_{11} - N_{12}N_{22}^{-1}N_{21}\right]' \delta_1 - \gamma \tag{74}$$

By (68), and noting (71), we have

$$\delta_1 = \beta - \mu_1(\mu_2 + 1)G'\lambda_2 \tag{75}$$

Define

$$\Lambda \equiv I + \mu_1(\mu_2+1)\left(N_{11} - N_{12}N_{22}^{-1}N_{21}\right)' G',$$

Thus, substituting (75) into (74), and assuming that Λ is nonsingular, a solution for λ_2 in terms of known matrices is given by

$$\lambda_2 = \Lambda^{-1}\left[\left(N_{11} - N_{12}N_{22}^{-1}N_{21}\right)\beta - \gamma\right] \tag{76}$$

The solution for q_t, and hence y_t, in terms of known matrices can also be obtained by substituting the expression for λ into (69). Similarly, the solution for r_t is obtained by noting that

$$E_t(p_{t+1} - p_t) = \lambda'(M - I)X_t$$

Therefore, by (41), we get

$$r_t = \alpha' X_t$$

where

$$\alpha \equiv \alpha_\pi (M' - I)\lambda + \alpha_y (I - \mu_1 M')^{-1}\delta \tag{77}$$

Finally, it remains to be shown under which conditions N_{22} and Λ are non-singular. However, in general, an analytical derivation of the necessary and sufficient conditions is unwieldy. The rank of these matrices can be checked on a case-by-case basis in numerical implementations.

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Table 1

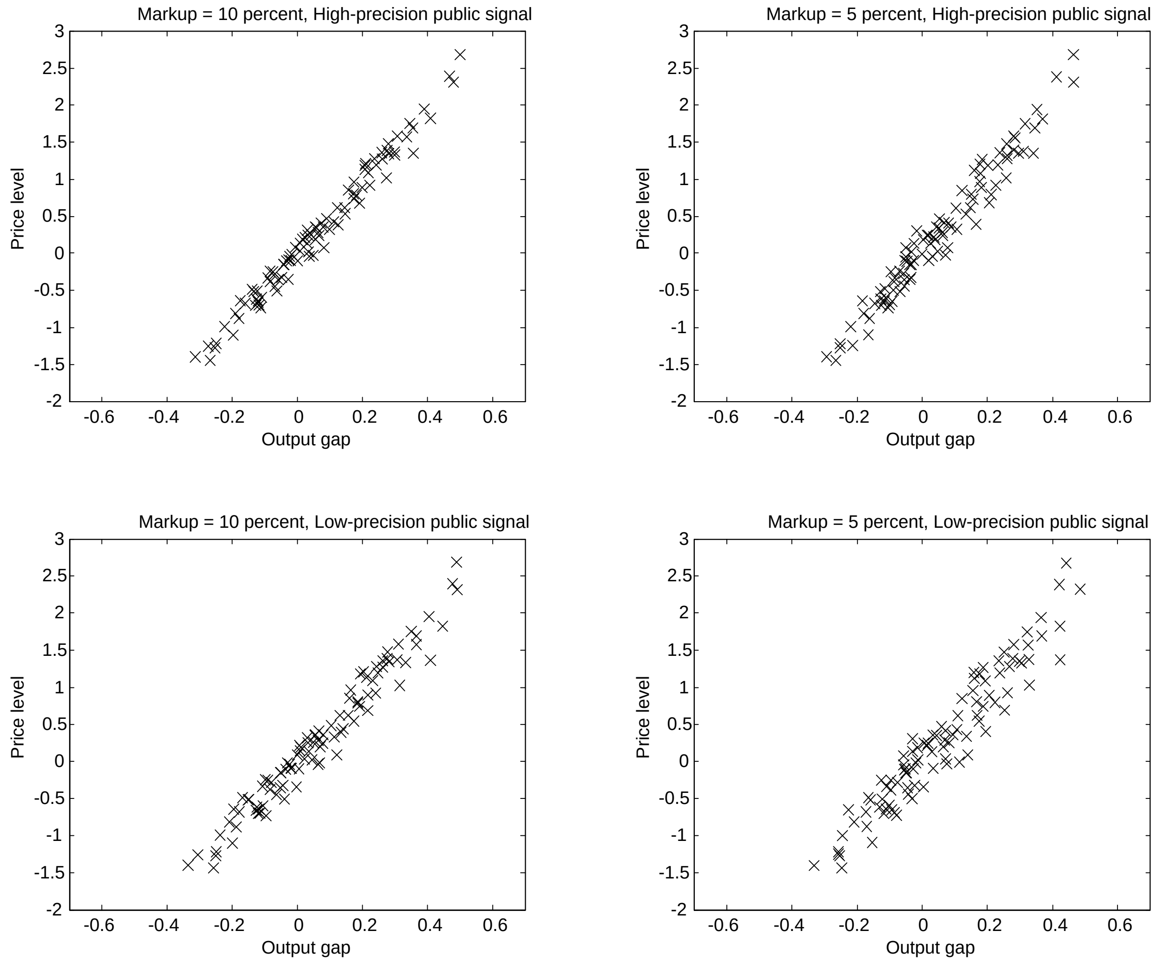
Baseline Calibrated Parameters

Preferences		Technology	
σ	2	ζ	0.3
ν	2	ϵ	11
Demand Shock		Technology Shock	
ρ_g	0.8	ρ_g	0.8
$\sigma^2_{\varepsilon,g}$	1%/quarter	$\sigma^2_{\varepsilon,a}$	1%/quarter
Private Signals		Public Signals	
$\sigma^2_{e,g}$	1%/quarter	$\sigma^2_{\eta,g}$	1%/quarter
$\sigma^2_{e,a}$	1%/quarter	$\sigma^2_{\eta,a}$	1%/quarter
Monetary Policy			
α_π	1.5	α_y	0.5

Figure 1

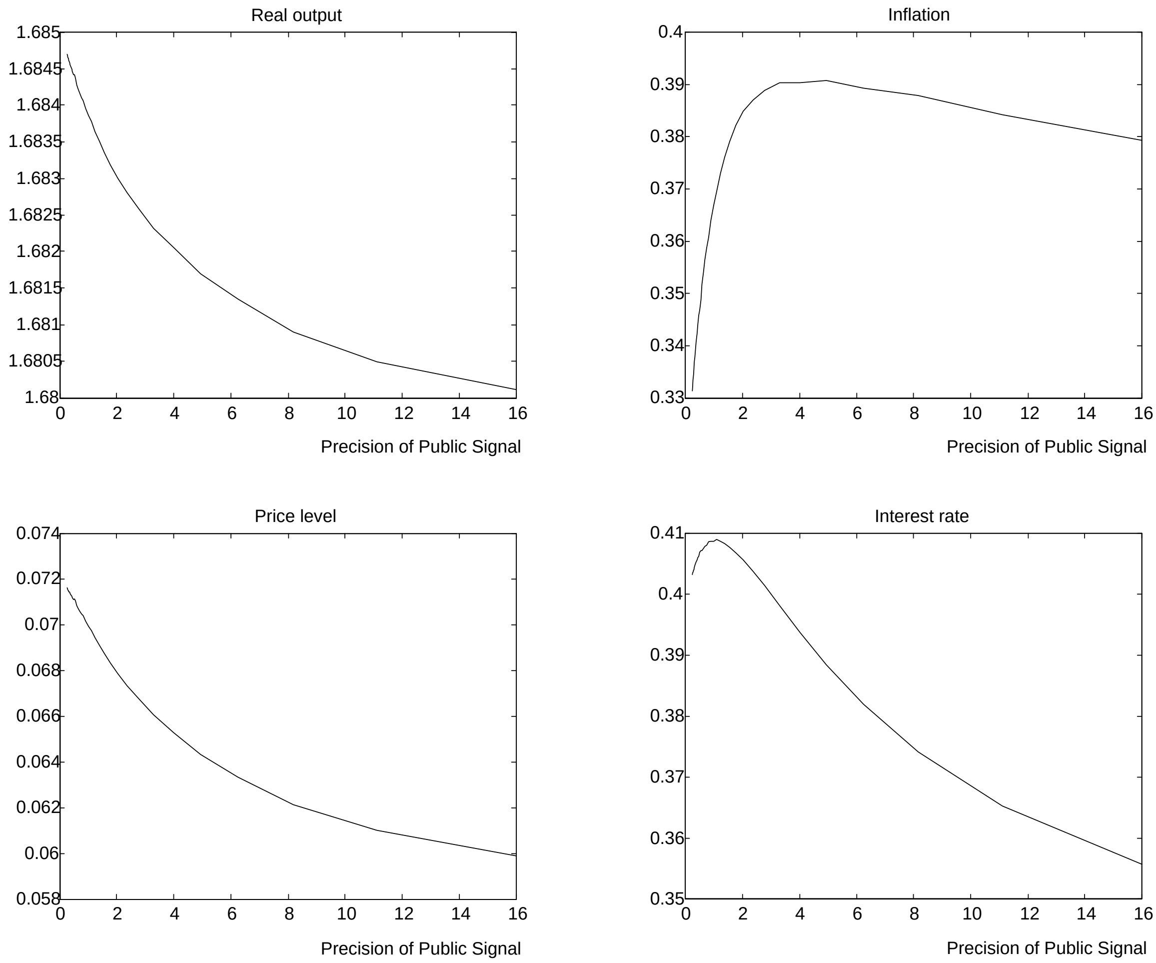
Effects of Changing the Markup and Precision of Public Signals

Sample Realisation of the Output Gap and Price Level



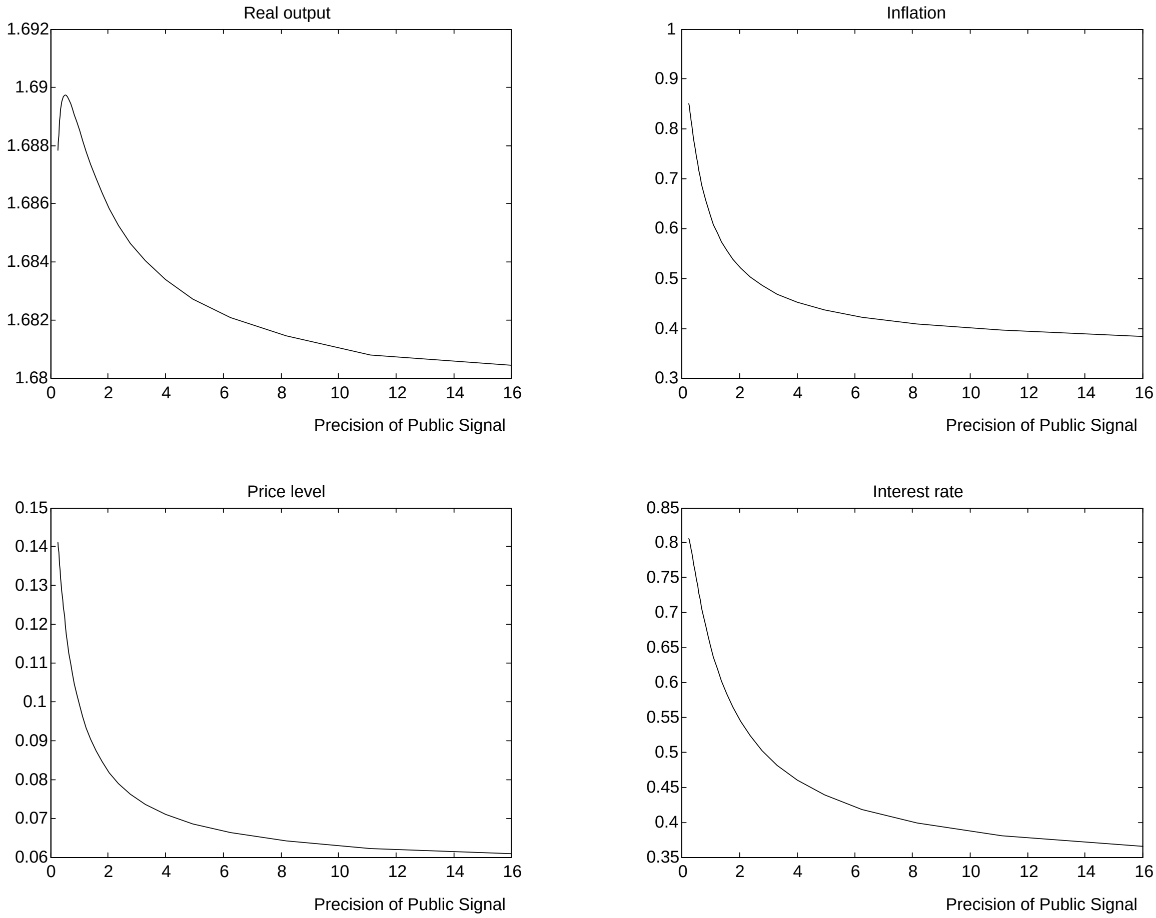
Notes: Each panel plots one sample realisation of the price level against the output gap. The same sample of randomly drawn shocks is used in each panel when simulating the time-paths of the endogenous variables. Data is constructed for 120 periods, but the first 20 observations are dropped to minimise the influence of initial values. The price level and output gap are in percentages.

Figure 2
Precision of Public Information and the Variances of Endogenous Variables
Case I. High-Precision Private Signal



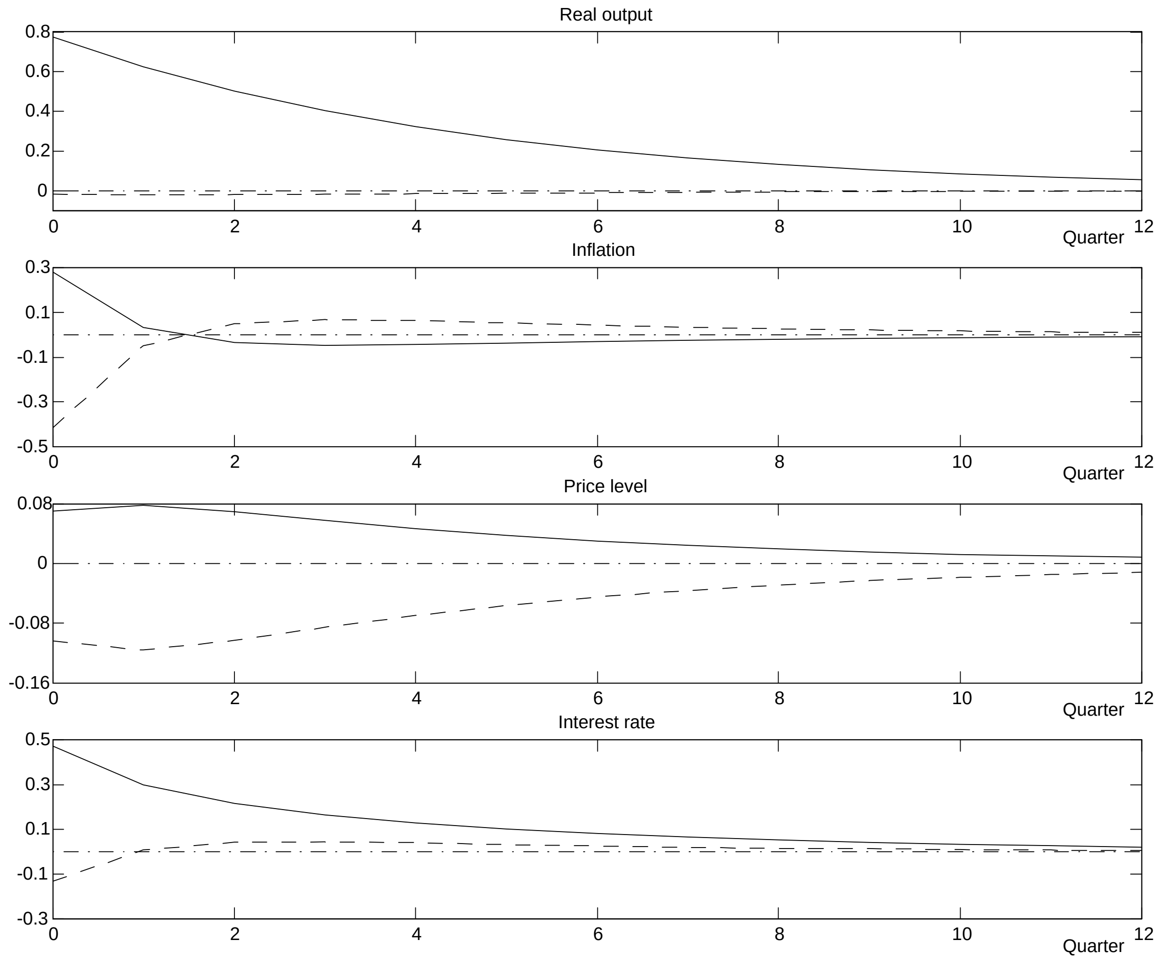
Notes: The figure plots the variances of endogenous variables with respect to the precision of the (innovation in) public signal. The precision of the private signal is set equal to 16.0 percent. Inflation and the interest rate are expressed in annualised percentages, while the price level, output and precision of signal innovations are in percentages.

Figure 3
Precision of Public Information and the Variances of Endogenous Variables
Case II. Low-Precision Private Signal



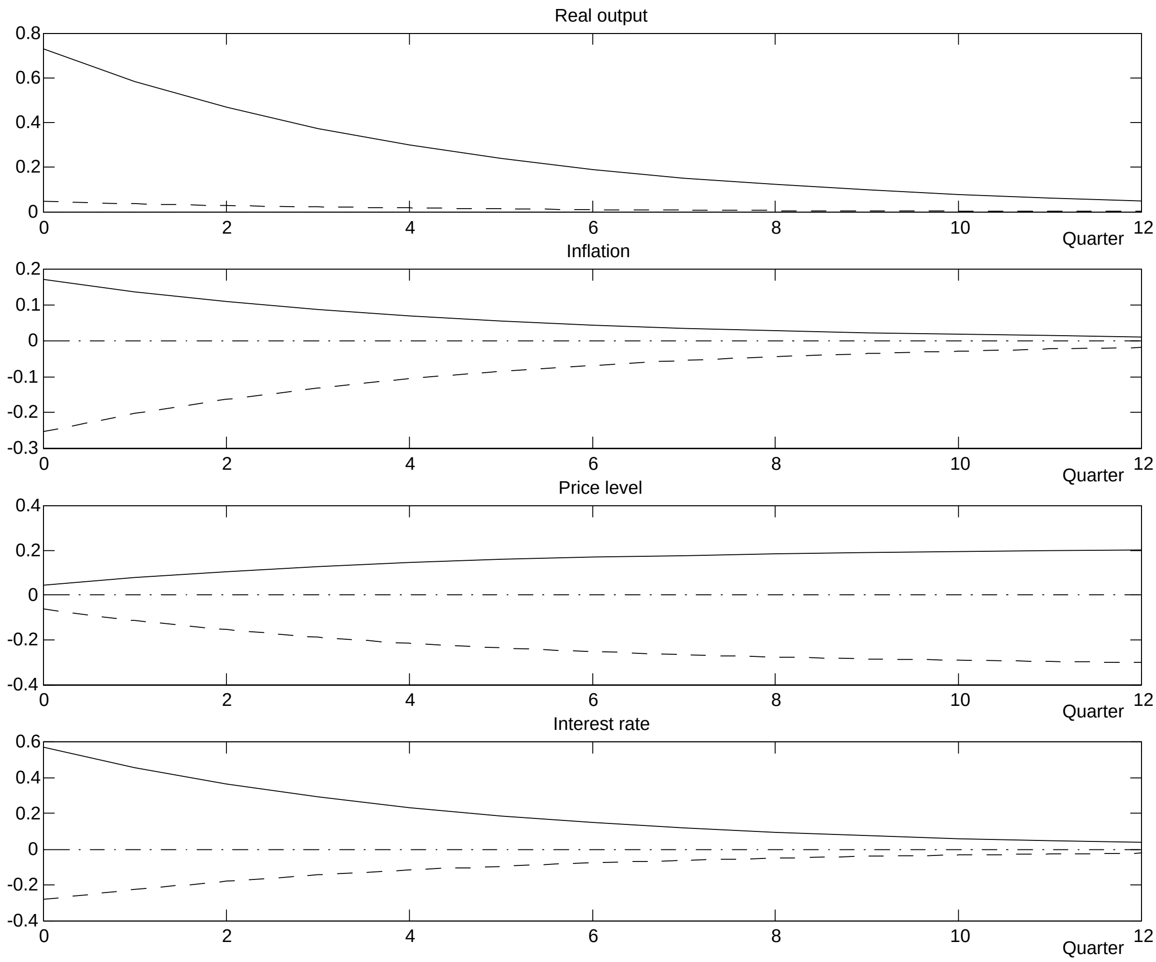
Notes: The figure plots the variances of endogenous variables with respect to the precision of the (innovation in) public signal. The precision of the private signal is set equal to 0.25 percent. Inflation and the interest rate are expressed in annualized percentages, while the price level, output and precision of signal innovations are in percentages.

Figure 4
Impulse Responses to Demand and Technology Shocks



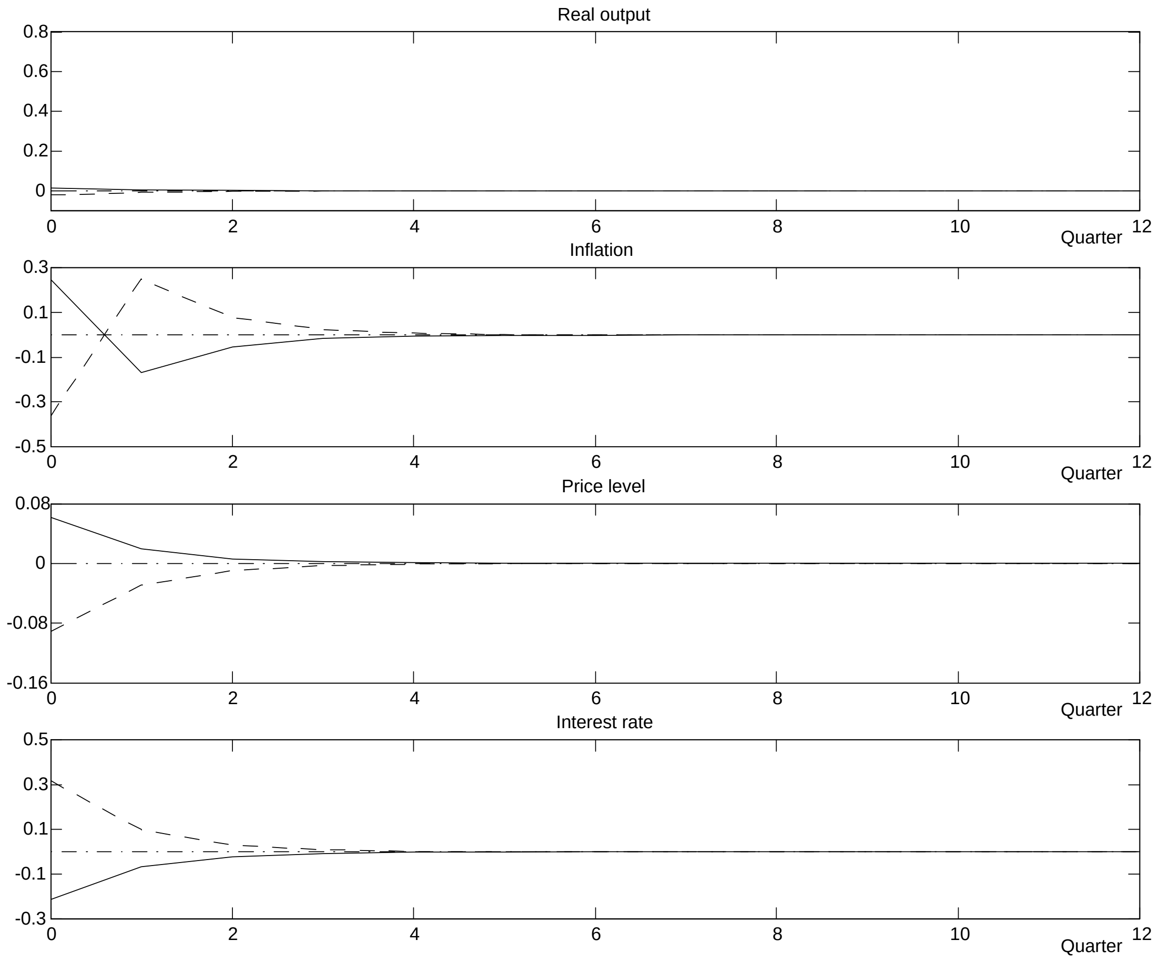
Notes: The figure shows the impulse responses of the endogenous variables in the differential-information model to a one-standard deviation innovation in the demand shock (solid) and technology shock (dash). The zero-line is represented by the dash-dot line. One period is equal to a quarter. Inflation and the interest rate are expressed in annualized percentages, while the price level and output are in percentages.

Figure 5
Sticky Price Model
Impulse Responses to Demand and Technology Shocks



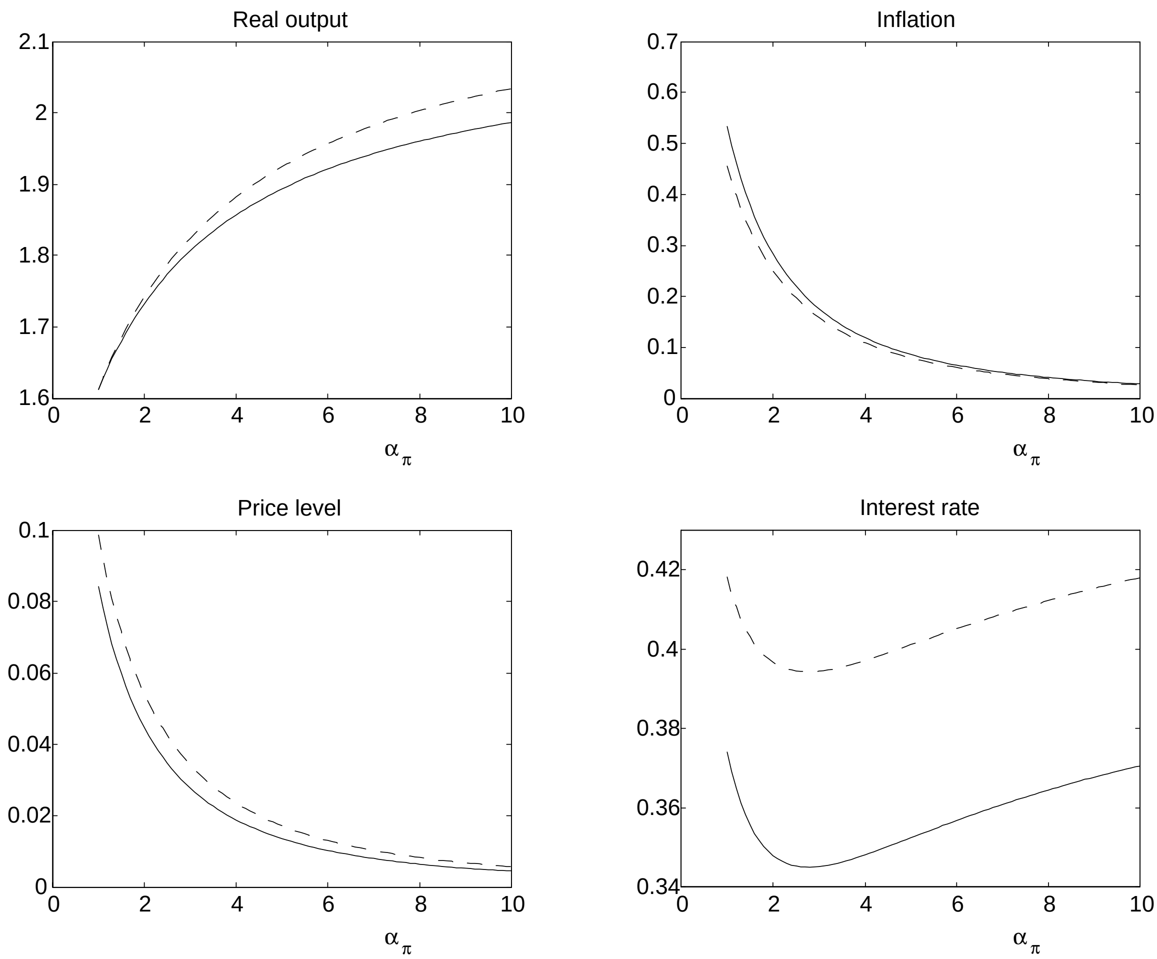
Notes: The figure shows the impulse responses of the endogenous variables in the sticky-price model to a one-standard deviation innovation in the demand shock (solid) and technology shock (dash). The zero-line is represented by the dash-dot line. One period is equal to a quarter. Inflation and the interest rate are expressed in annualized percentages, while the price level and output are in percentages.

Figure 6
Impulse Responses to Shocks of Public Signals



Notes: The figure shows the impulse responses of the endogenous variables to a one-standard deviation innovation to the public signals of demand shocks (solid) and technology shocks (dash). One period is equal to a quarter. Inflation and the interest rate are expressed in annualized percentages, while the price level and output are in percentages.

Figure 7
Effects of Changing Policy Response to Expected Inflation (α_π)



Notes: The figure plots the variances of endogenous variables with respect to α_π, the coefficient on $E_t\pi_{t+1}$ in (41). Two cases are shown: high-precision public signals (solid), low-precision public signals (dash). In both cases, private signals have high precision. Inflation and the interest rate are expressed in annualised percentages, while the price level and output are in percentages.