Banking Limits on Foreign Holdings: Disentangling the Portfolio Balance Channel∗

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Abstract

In this paper we analyze the effects of financial constraints on the exchange rate through the portfolio balance channel. Our contribution is twofold: First, we construct a tractable two-period general equilibrium model in which financial constraints inhibit capital flows. Hence, departures from the uncovered interest rate parity condition are used to explain the effects of sterilized foreign exchange intervention. Second, using high frequency data during 2004-2015, we use a sharp policy discontinuity within Colombian regulatory banking limits to empirically test for the portfolio balance channel. Consistent with our model’s postulations, our findings suggest that the effects on the exchange rate are short-lived, and significant only when banking constraints are binding.

JEL Classification: C14, C21, C31, E58, F31

Keywords: Sterilized foreign exchange intervention, portfolio balance channel, uncovered interest rate parity, financial constraints, regression discontinuity design

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1 Introduction

The extense literature on the effectiveness of sterilized foreign exchange intervention identifies two main channels through which the exchange rate can be affected: the signaling channel and the portfolio balance channel. The theoretical underpinnings of these channels are provided in Sarno and Taylor (2001), Evans (2005), Lyons (2006), and Villamizar-Villegas and Perez-Reyna (2015). However, the empirical literature has yet to reach a consensus regarding the effectiveness of foreign exchange intervention.\(^1\) One reason might be that managing the exchange rate while at the same time allowing for free capital flows and having monetary policy autonomy is an impossible trilemma due to arbitrage by foreign investors. In principle, policy effects should be limited.

In this paper we attempt to disentangle the portfolio balance channel by studying the effects of banking limits on the exchange rate. We first construct a tractable two-period general equilibrium model with a representative household and a monetary authority (central bank) that issues domestic debt and holds foreign reserves. The household faces a constraint on its holdings of foreign assets relative to its income. As such, our model shares similar features as Gabaix and Maggiori (2015) and Kuersteiner et al. (2016); namely that the uncovered interest rate parity (\textit{UIP}) condition does not hold due to some market friction. However, in our model the friction results from a financial regulation, construed as lower and upper bounds on households’ foreign exchange positions. In contrast, frictions found in Gabaix and Maggiori and Kuersteiner et al. consist of imperfect intermediation by international financiers due to limited commitment or to taxes on foreign capital, respectively.

Our model yields multiple equilibria: if constraints are not binding, then the UIP condition holds in equilibrium and the household is indifferent between holding domestic or foreign assets. In this case, the exchange rate is constant across any asset composition of the household. Hence, foreign exchange intervention is futile. Alternatively, when constraints are binding, departures from the UIP depend on the households’ relative amount of foreign bonds. This equilibrium creates a wedge on expected returns which affects the exchange rate.

The underlying mechanism of our model centers on how departures from UIP affects

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the income of the household. That is, the government makes a lump-sum transfer each period equivalent to the difference between the return on its foreign assets and its debt, denominated in foreign and domestic currency, respectively. If the UIP condition does not hold, then the income of the household changes. And, since constraints depend on income, the resulting change determines whether constraints are binding.

In order to test for the postulations of our model, we conduct a sharp regression discontinuity design (RDD) so as to fit the description of the data generating process imposed by Colombian regulations on banking limits. Specifically, we compare episodes of exchange rate and portfolio balances, when banks’ foreign exchange exposures reached a binding limit, to episodes in which they barely missed it. Hence, our identifying strategy is based on the fact that the Colombian regulatory framework limits banks’ foreign exposure relative to their capital. When limits are binding, portfolio shifts should, in principle, have an effect on the exchange rate (i.e. shifting from an \textit{impossible trinity} to a \textit{possible binity}). Moreover, effects should be amplified if the central bank conducts sterilized interventions, by issuing or purchasing domestic sovereign debt.

In the empirical application we employ proprietary and high frequency (daily) data from every financial institution in the country during 2004-2015, that include: (i) domestic asset holdings, (ii) loans denominated in domestic and foreign currency, and (iii) foreign exposure (assets minus liabilities in foreign currency). We also employ official intervention data provided by the Central Bank of Colombia (CBoC henceforth) consisting of both the timing and amount of every foreign exchange market transaction conducted within our sample period.

Our findings indicate that the effects of financial restrictions on the exchange rate are short-lived, and significant only when banking limits are binding. Moreover, we find significant effects on portfolio balances (both on loans and foreign exposure) when banks are faced with binding constraints. Finally, we find that exchange rate effects are larger in episodes when the CBoC intervened in the foreign exchange market. These results can be construed as evidence of the portfolio balance channel when the monetary trilemma does not hold (i.e. when financial regulations limit capital flows).

The rest of the paper is organized as follows. Section 2 presents the theoretical model that incorporates financial constraints to study the effectiveness of sterilized foreign exchange intervention. Section 3 describes the data, explains the empirical methodology and highlights the main empirical results. Finally, Section 4 concludes.
Our model incorporates many of the attributes needed to rationalize the effect of banking limits on foreign exchange intervention. To simplify the setup, we abstract from the role of banks in the economy. That is, we exclusively focus on their inability to intermediate foreign currency and the corresponding exchange rate effects when they comply with regulatory constraints on their foreign exchange exposure. In fact, without loss of generality, we abstract from banks altogether and assume that households are subject to constraints on their exposure of foreign currency.

We consider a two-period endowment economy with a representative household and an economic authority (Government) that issues domestic debt and holds foreign reserves. The household chooses whether to save in foreign or domestic assets. Domestic assets are denominated in domestic currency, which we call *pesos*, and the payoff of foreign assets is in foreign currency, which we call *dollars*. We assume that the household faces a constraint on the amount of dollar assets she can hold. Assume further that the domestic economy is a small open economy, so that the return on foreign bonds is exogenous.

The household maximizes the utility derived from consuming in the two periods. We assume that utility is time separable, and, in order to get closed form solutions, we assume that the utility of each period is given by $u(c) = \ln c$. We normalize the price of the consumption good in pesos each period to 1. In each period, the household has an endowment in pesos equal to $A_t$. Additionally the government transfers a lump-sum of $\tau_t$.

The household can transfer resources from $t = 0$ to $t = 1$ by saving in assets in pesos, $B$, or in assets in dollars, $B^\ast$. Saving $B$ in pesos allows the household to consume $(1 + r)B$ in the following period. Since the income of the household is in pesos, in order to save in dollars the household must convert its pesos to dollars via the exchange rate, which we denote by $e_t$; namely, the price of one dollar in period $t$ is $e_t$ pesos (i.e. pesos/dollars). Saving $e_0B^\ast$ pesos in $t = 0$ yields $(1 + r^\ast)e_1B^\ast$ pesos one period later.

We next introduce constraints on the households' foreign exposure (i.e. amount of foreign assets). To be exact, constraints are set on the value in pesos of savings in dollars.

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2 We model a real economy. That is, assets denominated in pesos represent assets whose payoff corresponds to the good produced domestically, while assets in dollars allow consumption of the good produced abroad.
relative to the household’s income, as follows:

\[
B \leq \frac{e_0 B^*}{I} \leq B,
\]

where \( I \) is the present value of the household’s income across the two periods,

\[
I = A_0 + \tau_0 + \frac{A_1 + \tau_1}{1 + r}.
\]

There are two reasons for which we analyze this type of constraints (i.e. the two inequalities in equation 1), as opposed to other types of limits: First, our data concerns banking limits on the exposure of foreign currency relative to capital. Therefore, we are interested in constraints that apply to the relative exposure in dollars. Second, by having the constraints depend on equilibrium outcomes \((\tau_t, r)\), they become endogenous. In particular, the exchange rate plays a role in whether the constraints bind or not.

The problem of the household is:

\[
\begin{align*}
\max_{c_0, c_1, B, B^*} & \quad \ln c_0 + \beta \ln c_1 \\
\text{s. t.} & \quad c_0 + B + e_0 B^* = A_0 + \tau_0 \\
& \quad c_1 = (1 + r)B + (1 + r^*)e_1 B^* + A_1 + \tau_1 \\
& \quad B \leq \frac{e_0 B^*}{I} \leq B.
\end{align*}
\]

**Lemma 1.** The solution of (3) is characterized by

\[
\begin{align*}
\frac{1}{c_0} &= \frac{\beta}{c_1} (1 + r) \\
\frac{e_0}{c_0} &= \frac{e_1}{c_1} \beta (1 + r^*) + \frac{e_0}{I} (\lambda - \bar{\lambda}) \\
\bar{\lambda}, \lambda &\geq 0,
\end{align*}
\]

where \( \lambda \) is the Lagrange multiplier of the upper bound on dollar exposition and \( \bar{\lambda} \) is the corresponding one for the lower bound.

Notice that (4) implies the traditional Euler equation, \( c_1 = \beta (1 + r) c_0 \). In turn, equation (5) implies a modified UIP condition for which we make explicit below in Lemma 2.
In this economy, the government is exogenous. Its assets, which we denote as foreign exchange reserves, are in dollars, while its liabilities are in pesos. In $t = 0$ the Government issues debt, $B_G$, in order to finance the acquisition of foreign exchange reserves, $e_0B_G^*$.\(^3\) In $t = 1$ it receives the return on its dollar assets, $(1 + r^*)e_1B_G^*$, and pays back the debt it issued, $(1 + r)B_G$. The budget is balanced through a lump-sum tax transfer on households each period:

$$\tau_0 = B_G - e_0B_G^*$$  \hspace{1cm} (6) \\
$$\tau_1 = (1 + r^*)e_1B_G^* - (1 + r)B_G.$$  \hspace{1cm} (7)

Since this is a real economy, there is no role for monetary policy. In this sense, any change in foreign exchange reserves can be interpreted as a sterilized intervention. Additionally, as will be demonstrated below, the domestic interest rate is constant regardless of the value of foreign exchange reserves.

We close the model by establishing market clearing conditions. Market clearing in the domestic bond markets imply that:

$$B = B_G.$$  \hspace{1cm} (8)

That is, the only domestic asset in this economy is domestic debt issued by the government. Equation (8) allows us to pin down the domestic interest rate $r$. The other prices in the economy are $e_0$ and $e_1$. However, dollar flows in both periods depend on $B^*$ and $B_G^*$. To avoid introducing extra notation, we assume that the country has a balanced current account in both periods. This implies that demand for dollars $(B^* + B_G^*)$ equal zero:

$$B^* + B_G^* = 0$$  \hspace{1cm} (9) \\
$$(1 + r^*)(B^* + B_G^*) = 0.$$  \hspace{1cm} (10)

Therefore, we can only pin down $(e_1/e_0)$, but not the exchange rate in both periods. To simplify the notation further, we assume that $e_0 = 1$.

\(^3\)Specifically we interpret $B_G^*$ as net assets in dollars of the government. A negative value of $B_G^*$ means that debt in dollars is greater than foreign reserves. We assume that the debt in dollars of the government is constant so that any changes in $B_G^*$ are due to changes in foreign reserves.
We now define a competitive equilibrium in this economy:

**Definition 1.** A competitive equilibrium in this economy consists of prices $P = \{e_1, r\}$, allocations $X = \{c_0, c_1, B, B^*\}$ and government policies $G = \{B_G, B^*_G\}$ such that:

1. given $P$, $X$ satisfies equations (4) and (5);
2. markets clear: equations (8) and (9) are satisfied.

We now solve the model. Our main result is that there is multiplicity of equilibria. To see why, we first derive a modified uncovered interest rate parity condition:

**Lemma 2.** In equilibrium the following equation holds true:

$$1 + r = e_1(1 + r^*) - \frac{\bar{\lambda}}{\beta I} c_1.$$  \hspace{1cm} (11)

**Proof.** The proof follows directly from equations (4) and (5). \hfill \Box

Suppose that the household considers a situation where the UIP condition holds; that is, the household considers that $1 + r = e_1(1 + r^*)$. Then, she will be indifferent between holding assets denominated in pesos or dollars. In particular, the household will be indifferent across any value of $B^*$ that satisfies equation (1) with strict inequalities, as long as total savings (in pesos and dollars) are of a certain amount. It follows that $\bar{\lambda} = \bar{\lambda} = 0$, so, from equation (11), UIP will hold in equilibrium.

On the other hand, assume that the household considers a situation where UIP is violated. For example, assume that the household considers a situation where $1 + r < e_1(1 + r^*)$. In this case, the household will strictly prefer to save in dollars since the return of dollar assets is strictly greater than saving in pesos. Since the household is subject to constraints, this means that the upper bound in equation (1) will be reached, which implies that $\bar{\lambda} > 0$. Equation (11) shows that $1 + r < e_1(1 + r^*)$ will, in fact, be an equilibrium. The departure from UIP is hence measured by

$$\rho \equiv 1 + r - e_1(1 + r^*).$$  \hspace{1cm} (12)

In particular, when $\rho = 0$, then UIP holds. Before proving that there are multiple equilibria, we first prove that in any equilibrium the household has the same welfare. In fact, the equilibrium interest rate, $r$, is constant. If constraints in equation (1) do not bind, then UIP holds. When constraints are binding, there is a wedge that is counteracted by the
exchange rate, $e_1$. In these scenarios a foreign exchange intervention (i.e. a change in $B^*_G$) has an effect on $e_1$.

**Proposition 1.** *In any equilibrium the following conditions hold:*

\[ 1 + r = \frac{A_1}{\beta A_0} \]
\[ c_0 = A_0 \]
\[ c_1 = A_1. \]

Moreover, *in equilibria where constraints in equation (1) do not bind, then*

\[ e_1 = \frac{1 + r}{1 + r^*} = \frac{A_1}{\beta A_0} \frac{1}{1 + r^*}. \]

Alternately, *when constraints bind, i.e. $\frac{B^*}{\bar{r}} = \bar{B}$ for $\bar{B} \in \{ \bar{B}, B \}$, then*

\[ e_1 = \frac{1 + r}{1 + r^*} \left( 1 - \frac{1}{\bar{B}} - \frac{(1 + \beta)A_0}{B^*_G} \right) = \frac{A_1}{\beta A_0} \frac{1}{1 + r^*} \left( 1 - \frac{1}{\bar{B}} - \frac{(1 + \beta)A_0}{B^*_G} \right). \]

*Proof.** The proof is in Appendix A. \qed

A consequence of Proposition 1 is that, regardless of the equilibrium, the household’s welfare and interest rate remain unchanged. However, as Lemma 2 shows, there is a wedge when either of the constraints binds, which is offset by $e_1$. Furthermore, Proposition 1 shows that when constraints bind, a foreign exchange intervention has an effect on $e_1$. Nonetheless, the intervention will only help drive the exchange rate towards a value that would have otherwise been met in an equilibrium without a wedge, i.e. where constraints do not bind. In the limit, $e_1$ reaches this value, as stated in the following Corollary.

**Corollary 1.** *If $\bar{B} \to \infty$, then an intervention where the government buys dollars from the market, i.e. $B^*_G$ increases, helps the exchange rate $e_1$ reach $\frac{1 + r}{1 + r^*}$. Specifically, $e_1 \to \frac{1 + r}{1 + r^*}$ as $B^*_G \to \infty$. *

Now consider Figure 1. For clarification purposes we use parameter values such that $e_1 = 1$ in the equilibrium where constraints do not bind. The red dotted line shows $e_1$ in the other equilibrium. We prove below that these are, in fact, the two equilibria, but for now assume that when $B^*_G < 0$ the upper constraint in equation (1) binds whereas the lower
constraint binds when $B^*_G > 0$.

Consider the case where $B^*_G > 0$. From Proposition 1, we know that $e_1$ depends on $B^*_G$. So an intervention, i.e. a change in $B^*_G$, has an effect on the exchange rate. Furthermore, if $B < 0$, an increase in $B^*_G$, which occurs when a Central Bank buys dollars from the market, causes $e_1$ to increase. However, notice that $e_1$ is below unity. This implies that an intervention intended to raise the exchange rate causes $e_1$ to move closer to an equilibrium value without a wedge. Nonetheless, there is no impact on welfare. The opposite occurs when $B^*_G$ decreases.

![Figure 1: $e_1$ for different values of $B^*_G$.](image)

The continuous line depicts the equilibrium exchange rate when constraints do not bind. The dotted line depicts the equilibrium exchange rate when constraints bind. $A_0 = A_1 = 0.1$, $\beta = 0.95$, $r^* = \frac{1}{\beta} - 1$, $\overline{B} = -\overline{B} = 15$ and $B_G = 0.9$.

Before further illustrating this point, recall that by definition of $\rho$, as stated in equation (12):

$$e_1 = \frac{1 + r}{1 + r^*} - \frac{\rho}{1 + r^*}.$$ 

Since both $r$ and $r^*$ depend on parameter values, understanding how $e_1$ changes depending on $B^*_G$ is equivalent to understanding how $\rho$ changes in response to changes in $B^*_G$. In the following Lemma we state the relationship between $\rho$ and the Lagrange multipliers of the upper and lower bounds found in equation (1):

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4If $\overline{B} < 0$ or $\overline{B} > 0$ we still get multiple equilibria and similar results.
Lemma 3.

\[ \bar{\lambda} = \max \left\{ -\frac{\beta I}{A_1} \rho, 0 \right\} \]
\[ \lambda = \max \left\{ \frac{\beta I}{A_1} \rho, 0 \right\}. \]

Proof. The proof is in Appendix A.

Lemma 3 explicitly states the relation between departures from UIP and binding constraints. If \( \rho > 0 \), then the return on dollar assets is lower than the one in pesos. Therefore, the household will want to short-dollar assets, which drives them to the lower bound on dollar holdings.

Figure 2 shows \( \rho \) for different values of \( B_G^* \). We have not yet proven that these are in fact equilibria, but assume for now that the lower constraint binds when \( B_G^* > 0 \) and the upper bound is reached when \( B_G^* < 0 \). The red dotted line shows the resulting wedge: \( \rho > 0 \) when the lower bound is reached and \( \rho < 0 \) when the upper limit binds. When \( B_G^* \) is further away from 0, the wedge becomes smaller in absolute terms. The reason for this lies in the constraints that the household faces. Consider the case when \( B_G^* > 0 \). If the government has higher savings, then equilibrium conditions imply that the household has more debt denominated in dollars. In this case, the lower bound in equation (1) will bind. Since \( B \) is fixed, then \( I \) must increase for the constraint to hold.

![Figure 2: \( \rho \) for different values of \( B_G^* \)](image)

The continuous line depicts the equilibrium exchange rate when constraints do not bind. The dotted line depicts the equilibrium exchange rate when constraints bind. \( A_1 = A_2 = 0.1, \beta = 0.95, r^* = \frac{1}{\sigma} - 1, \overline{B} = -\underline{B} = 15 \) and \( B_G = 0.9 \).
To further clarify, we can rewrite income as $I = A_0 + \frac{A_1}{1+r} - \frac{\rho B_G}{1+r}$, so a higher $I$ implies a lower $\rho$. In sum, if the government holds more foreign exchange reserves and the lower constraint binds, then the income of the household must increase. But it will only increase if the wedge is lower.

We now analyze the equilibria by only changing $B_G^*$ while keeping the other government policy constant. As mentioned, this is equivalent to analyzing the equilibria for different levels of foreign exchange intervention.

**Proposition 2.** Assume that $A_0, A_1, \beta, 1+r^* > 0$. Furthermore assume that $\overline{B} < 0 < \underline{B}$.

Let $\underline{B}_G^* = -\overline{B}(1+\beta)A_0$ and $\overline{B}_G^* = -\underline{B}(1+\beta)A_0$.

Then the model has two equilibria as long as $B_G^* \in (\underline{B}_G^*, \overline{B}_G^*)$. The equilibria are characterized as follows:

1. The constraints in (1) do not bind.
2. The constraints in (1) bind and
   • For $B_G^* \in (\underline{B}_G^*, 0)$ the upper constraint in (1) binds.
   • For $B_G^* \in (0, \overline{B}_G^*)$ the upper constraint in (1) binds.

**Proof.** The proof is in Appendix A. □

## 3 Empirical Methodology and Results

In order to test for the postulations of our model, we estimate the causal effect of banking limits on the exchange rate by using a sharp policy discontinuity within the Colombian regulatory framework. In essence, we compare observations of outcome variables in the immediate neighborhood of a given threshold, dictated by the existing financial regulations on foreign holdings. Intuitively, the cutoff creates a natural experiment in which financial institutions arbitrarily face binding constraints (i.e. treatment group) as long as they are in close proximity to the required limit. Alternatively, institutions which barely miss the threshold (i.e. control group) represent ideal counterfactuals to financially constrained institutions, had the constraint not been binding.
3.1 Data

Our running variable (foreign exchange exposure) is defined as the difference between assets and liabilities denominated in foreign currency relative to total capital, without including positions in derivatives (see Mora-Arbeláez et al. (2015)). By regulation, banks cannot have a foreign exposure that exceeds 50% of their capital and it cannot be negative. The lower limit of 0% was initially introduced to control for speculative attacks that would intensify the appreciation of the Colombian peso. That is, speculators could sell forward contracts (denominated in US dollars) to the financial system while financial intermediaries borrowed abroad to hedge their position. The resulting monetization of external debt could then exert appreciation pressures over the Peso.

However, even though regulation states that the lower limit on foreign exchange exposure for banks is 0%, we notice that the actual limit, relevant for banking operations, is a threshold of 1%. The main reason for this concern is the penalty involved when banks violate the imposed lower bound. Given that banks face unexpected changes in their daily exposures, they take preemptive measures to avoid being penalized. As it turns out, the total (daily) change in banks foreign exposure relative to their capital during 2004-2015 was, on average, 1% (see Appendix B). Consequently, financial institutions generally require a capital buffer of at least 1% in order to avoid monetary sanctions. We thus proceed with the effective lower bound of 1% in the estimations that follow.

Figure 3 depicts foreign exposure (in dollars) relative to total capital of the entire financial system, where the two horizontal lines denote the upper (50%) and lower (1%) bounds. As observed, the upper limit was never binding and foreign exposure relative to capital oscillated between -0.6% (Jan 15, 2004) and 19.68% (June 10, 2005). We restrict our sample to the period of January 01, 2004 up until October 15, 2015, given that regulations for the lower limit changed to -20% on October 16, 2015.

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5 Specifically, this ratio is computed as the 3-day average liquid foreign exchange exposure relative to total capital.
6 The 0% limit was established on January 23, 2004. Banks that initially had a negative foreign exposure had to adjust it by March 31, 2004.
7 We estimated the daily average change of banks foreign exposure relative to their capital using different windows. This ratio was, on average, equal to 1% for the total sample.
8 Stationary properties are reported in Appendix C.
Finally, we control for episodes of foreign exchange intervention (Figure 4) in order to avoid potential confounding factors between the proximity of banks’ foreign exposure to regulatory limits and the decision of the CBoC to intervene in the foreign exchange market. Nonetheless, if banks “even while having some influence are unable to precisely manipulate the assignment variable, a consequence of this is that the variation in treatment near the threshold is randomized as though from a randomized experiment.”

Figure 4: Official Foreign Exchange Intervention

Positive (negative) values correspond to purchases (sales) of foreign currency.

9Lee and Lemieux (2010), page 3.
3.2 RDD Setup

RDD roots back to the early work of Thistlethwaite and Campbell (1960) and lay dormant for almost 40 years. Since its resurgence in the early 2000’s, RDD has been applied to a variety of fields including health, labor, and education.\textsuperscript{10} Most of the RDD framework was formalized during this time (see Hahn et al. (2001); Porter (2003); McCrary (2008); and Imbens and Kalyanaraman (2012)) and useful surveys include the works of Imbens and Lemieux (2008), Lee and Lemieux (2010), Jacob et al. (2012) and Villamizar-Villegas et al. (2016). However, RDD studies have seen limited applications to macroeconomic questions and none, besides Kuersteiner et al. (2016), have been applied to study the effects of monetary policy.

In the standard \textbf{Sharp} RDD setup, the assignment of treatment, $D_t$, is completely determined by a cutoff rule based on an observable running variable, $X_t$,

$$D_t = \mathbb{I}\{X_t \geq x_0\}, \quad (13)$$

where $\mathbb{I}\{\cdot\}$ is an indicator function. The discontinuity arises because no matter how close $X_t$ gets to the cutoff value, the treatment is unchanged until $X_t = x_0$. If the treatment has an effect, then it should be measured by comparing the conditional means of the outcome variable at the limit on either side of the discontinuity point:

$$\text{Average Treatment Effect} = E(Y_{1t} - Y_{0t} \mid X_t = x_0) \nonumber$$

$$= E(Y_{1t} \mid X_t = x_0) - E(Y_{0t} \mid X_t = x_0) \nonumber$$

$$= \lim_{\epsilon \downarrow 0} E(Y_t \mid X_t = x_0 + \epsilon) - \lim_{\epsilon \uparrow 0} E(Y_t \mid X_t = x_0 + \epsilon), \quad (14)$$

where $Y_{1t}$ and $Y_{0t}$ denote potential outcomes with and without exposure to treatment and the final equality holds as long as the conditional distributions of potential outcomes, $\Pr(Y_{1t} \leq y \mid X_t = x)$ and $\Pr(Y_{0t} \leq y \mid X_t = x)$, are continuous at $X_t = x_0$.\textsuperscript{11}

\textsuperscript{10} See, for example, Hahn et al. (1999), Angrist and Lavy (1999), van der Klaauw (2002), Lemieux and Milligan (2008), Carpenter and Dobkin (2009), Cellini et al. (2010), and Lee (2008), among many others.

\textsuperscript{11} In our case, $x_0 = 0.01$, since, as shown in Figure 3, the upper bound was never binding.
3.2.1 Testable Implications

Even though the continuity and unconfoundedness assumptions required in RDD analysis cannot be empirically tested, they do have some testable implications. In particular, evidence of a discontinuity at the threshold of the running variable would suggest that observations are not randomly allocated (i.e. evidence of “manipulation”, as presented in Lee and Lemieux (2010)). This is opposite of what is wanted for when considering outcome variables, given that a discontinuity in this case would suggest a significant effect of treatment.

Following McCrary (2008), Figure 5 shows whether the densities of the running variables (i.e. foreign exposure relative to capital of the financial system and of the five largest banks in the country) exhibit a discontinuity at the cutoff point. In essence, the test separately estimates the density of the running variable on either side of $x_0 = 0.01$ and provides a Wald estimate in which the null corresponds to the non-existence of a discontinuity at the cutoff. As it turns out, the null is accepted in all cases with a p-value of: 0.12, 0.24, 0.17, 0.11, 0.14 and 0.12, respectively.
Figure 5: McCrary’s Test for Different Running Variables

(a) Financial System

(b) Bank 1

(c) Bank 2

(d) Bank 3

(e) Bank 4

(f) Bank 5
3.2.2 Impulse Response Functions

We estimate the effects of financial regulatory limits by using both a parametric and a non-parametric approach. The parametric (global) approach consists of the following specification:

\[ \Delta e_t = \beta_0 + \beta_1 D_t + \varphi_0(X_{t-1} - x_0) + \varphi_1(X_{t-1} - x_0) * D_t + \epsilon_t \]  

(15)

where \( \Delta e_t \) denotes the exchange rate change (in logs) and \( \varphi_0 \) and \( \varphi_1 \) are polynomial functions of the running variable.\footnote{We report polynomials of order 2. The reason for this is that while estimating regressions with large polynomials yields consistent estimates of treatment, they can be influenced by data far from \( x_0 \).} The treatment effect is then captured by \( \beta_1 \) when evaluating the conditional mean of outcome at the discontinuity point, comparing values above and below the threshold.

Our non-parametric approach consists of minimizing the following two objective functions, each for windows above and below the discontinuity point:

\[ \sum_{x_0-w<X_t<x_0}(\Delta e_t - \{\beta_0 + \beta_1(X_t - x_0)\})^2, \quad \sum_{x_0<X_t<x_0+w}(\Delta e_t - \{\alpha_0 + \alpha_1(X_t - x_0)\})^2 \]  

(16)

where \( w \) is the window size. The treatment effect in this case is captured by \( \alpha_0 - \beta_0 \).

Finally, we follow Jorda (2005) method of local projections to estimate the implied impulse response functions (IRF’s), using the treatment effects obtained from equations (15) and (16). Essentially, we estimate sequential regressions in which the endogenous variable (i.e. exchange rate change or portfolio balance) is shifted at each forecasting period.

3.3 Results

3.3.1 Exchange rate Changes

Figure 6 depicts the IRF’s of exchange rate changes in response to financial constraints imposed on the entire financial sector. Estimates using a polynomial global regression (panel a) and using a sharp regression discontinuity (panel b) show that the effects on exchange rate
changes are short-lived. Namely, positive effects on the exchange rate are significant only during the first week.

Figure 7 further analyzes the effects of banking limits on the exchange rate, in episodes of foreign exchange intervention (panel a) and in episodes of no intervention (panel b). As shown, effects are significant only in episodes in which the CBoC intervened in the foreign exchange market. While these last results should be read with caution (especially since intervention dates account for 52% of our total sample), they do suggest that capital restrictions (i.e. banking limits) enable foreign exchange intervention to be effective. In other words, they transition the impossible trinity to a possible binity.

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13 We present results using only the RDD methodology but results are similar when using polynomial regressions.

14 A caveat however, is that results vary depending on the sample period. For instance, effects on the exchange generally last for less than a week if the sample starts in March 2004 rather than in January 2004. In some cases, there can even be a reversal (negative effects on the exchange rate after the first week). Nonetheless, the immediate positive effect on the exchange rate is robust accross all sub-samples considered.
3.3.2 Portfolio Shifts

We next consider the effects of banking limits on portfolio balances measured as:

- \( \frac{(A_t^* - L_t^*)e_t}{A_t} \): Assets minus Liabilities in dollars (converted to pesos) as a share of domestic assets
- \( \frac{L_t^* e_t}{L_t} \): Loans denominated in dollars (converted to pesos) as a share of domestic loans

Figures 8 and 9 show the IRFs for \( \frac{(A_t^* - L_t^*)e_t}{A_t} \) and \( \frac{L_t^* e_t}{L_t} \), respectively, when considering the five largest commercial banks in the country.\(^{15}\) Results show that for all cases (except for panel (b) of Figure 8), there is a significant portfolio re-composition when banking limits bind. This confirms the portfolio channel’s *modus operandi*. In other words, when banks’ foreign exposures narrow in on the established regulatory limit, they immediately shift their assets and liabilities (denominated in dollars) so as to move away from the threshold. We sustain that it is mostly through this recomposition that the exchange rate is affected, although additional research on the expectations’ channel is warranted.

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\(^{15}\)In accordance to law no. 1266 of 2008 ("Habeas Data"), we do not disclose the names of these banks.
4 Concluding remarks

We analyze the portfolio balance channel by studying the effects of banking limits established by financial regulations. We contribute to the literature in two important ways. First, we construct a two-period general equilibrium model with a representative household and a central bank that issues domestic debt and holds foreign reserves. Second, we empirically test these predictions for the Colombian case during 2004-2014. We use a sharp regression discontinuity design to fit the description of the data generating process imposed by banking limits on foreign exchange exposure.

In our theoretical model, we find multiplicity of equilibria. One equilibrium consists of non-binding constraints which supports the uncovered interest rate parity (UIP) condition. The other equilibrium consists of departures from the UIP, which depend on households’ relative foreign asset holdings and binding constraints. These postulations are consistent with our empirical findings which indicate that the effects of financial restrictions on the exchange rate are significant (albeit short-lived), only when banking limits bind. We also find significant effects on portfolio balances which can be construed as evidence of the portfolio balance channel when the monetary trilemma does not hold.
5 Bibliography


Appendix A  Selected proofs

Appendix A.1  Proof of Proposition 1

From the budget constraint of the household in period 1 and (11) we have

$$(1 + r)(B + B^*) = \left(1 - \frac{\bar{\lambda} - \lambda}{\beta I} B^*\right) c_1 - (A_1 + \tau_1). \quad (A1)$$

Plugging (A1) into the budget constraint of period 0 we get

$$c_0 + \frac{c_1}{1 + r} \left(1 - \frac{\bar{\lambda} - \lambda}{\beta I} B^*\right) = A_0 + \tau_0 + \frac{A_1 + \tau_1}{1 + r}. \quad (A2)$$

Since $c_1 = \beta(1 + r)c_0$, we use (A2) to get

$$c_0 = \frac{I}{1 + \beta - (\bar{\lambda} - \lambda) \frac{B^*}{I}},$$

$$c_1 = \frac{\beta I(1 + r)}{1 + \beta - (\bar{\lambda} - \lambda) \frac{B^*}{I}},$$

$$B + B^* = \frac{\beta - (\bar{\lambda} - \lambda) \frac{B^*}{I}}{1 + \beta - (\bar{\lambda} - \lambda) \frac{B^*}{I}} (A_0 + \tau_0) - \frac{1}{1 + \beta - (\bar{\lambda} - \lambda) \frac{B^*}{I}} \left(\frac{A_1 + \tau_1}{1 + r}\right). \quad (A3)$$

Additionally we can express $I$ as

$$I = A_0 + \frac{A_1}{1 + r} - \frac{\rho B^*_G}{1 + r}. \quad (A4)$$

To prove the main result we consider three cases: 1) Constraints in (1) don’t bind; 2) the upper constraint in (1) binds; and 3) the lower constraint in (1) binds.

Consider first Case 1, which occurs when $\bar{\lambda} = \bar{\lambda} = 0$. From (A3) we have

$$c_0 = \frac{I}{1 + \beta},$$

$$c_1 = \frac{\beta I(1 + r)}{1 + \beta},$$

$$B + B^* = \frac{\beta}{1 + \beta} (A_0 + \tau_0) - \frac{1}{1 + \beta} \frac{A_1 + \tau_1}{1 + r},$$

and from (11) we conclude that $\rho = 0$ in this case. Due to this, we have that $\tau_1 = -(1 + r)\tau_0$. Moreover, in equilibrium $B^* = -B^*_G$ and $B = B_G$. Plugging this equilibrium conditions into
the third equation, and recalling that \( \tau_0 = B_G - B_G^* \) in (A5) yields

\[
1 + r = \frac{A_1}{\beta A_0}.
\]

(A4) implies

\[
I = A_0 + \frac{A_1}{1 + r} = (1 + \beta)A_0.
\]

From the other two equations in (A5) we get

\[
c_0 = A_0 \\
c_1 = A_1.
\]

Finally, we have

\[
e_1 = \frac{1 + r}{1 + r^*} = \frac{A_1}{\beta A_0} \frac{1}{1 + r^*}.
\]

Now consider Case 2, where \( \lambda > 0, \lambda = 0 \) and \( B^* = BI \). From (9) and (A4) we have

\[
\overline{B} \left[ A_0 - B_G^* + \frac{1}{1 + r} (A_1 + e_1(1 + r^*)B_G^*) \right] = -B_G^*,
\]

which can be reorganized to be written as

\[
e_1 = \frac{B_G^* \left( 1 - \frac{1}{\overline{B}} \right) - A_0 - \frac{A_1}{1 + r} (1 + r)}{(1 + r^*)B_G^*} \] (A6)

Additionally, from (11) and (A3) we get

\[
\bar{\lambda} = \frac{(1 + \beta)\rho}{\overline{B}\rho - (1 + r)} \\
c_0 = \left( 1 - \frac{\overline{B}\rho}{1 + r} \right) \frac{I}{1 + \beta} \\
c_1 = \left( 1 - \frac{\overline{B}\rho}{1 + r} \right) \beta I (1 + r) \] (A7)

\[
B = \frac{\beta}{1 + \beta} (A_0 + \tau_0) - \frac{1}{1 + \beta} \frac{A_1 + \tau_1}{1 + r} + B^* \left( \frac{1}{1 + \beta} \frac{\rho}{1 + r} - 1 \right).
\]

We can now use (A6) to write

\[
\tau_1 = \left( B_G^* \left( 1 - \frac{1}{\overline{B}} \right) - B_G - A_0 - \frac{A_1}{1 + r} \right) (1 + r).
\]

We now use (8) and the last equation in (A7) to solve for \( r \), which yields

\[
1 + r = \frac{A_1}{\beta A_0}.
\]
Plugging this into (A6) results in
\[ e_1 = \frac{1 + r}{1 + r^*} \left( 1 - \frac{1}{B} - \frac{(1 + \beta)A_0}{B_G} \right) = \frac{A_1}{\beta A_0} \frac{1}{1 + r^*} \left( 1 - \frac{1}{B} - \frac{(1 + \beta)A_0}{B_G} \right). \]

Finally, using the fact that \( I = -\frac{B_G}{B} \) we have
\[ c_0 = A_0 \]
\[ c_1 = A_1. \]

Finally consider Case 3, where \( \lambda = 0, \lambda > 0 \) and \( B^* = BI \). From (9) and (A4) we have
\[ B \left[ A_0 - B^*_G + \frac{1}{1 + r} (A_1 + e_1(1 + r^*)B^*_G) \right] = -B^*_G, \]
which can be reorganized to be written as
\[ e_1 = \frac{B^*_G \left( 1 - \frac{1}{B} \right) - A_0 - \frac{A_1}{1 + r}}{(1 + r^*)B^*_G} (1 + r) \] (A8)

Additionally, from (11) and (A3) we get
\[ \lambda = \frac{(1 + \beta)\rho}{I((1 + r) - B\rho)} \]
\[ c_0 = \left( 1 - \frac{B\rho}{1 + r} \right) \frac{I}{1 + \beta} \]
\[ c_1 = \left( 1 - \frac{B\rho}{1 + r} \right) \beta I(1 + r) \]
\[ B = \frac{\beta}{1 + \beta} (A_0 + \tau_0) - \frac{1}{1 + \beta} \frac{A_1 + \tau_1}{1 + r} + B^* \left( \frac{1}{1 + \beta} \frac{\rho}{1 + r} - 1 \right). \]

We can now use (A8) to write
\[ \tau_1 = \left( B^*_G \left( 1 - \frac{1}{B} \right) - B_G - A_0 - \frac{A_1}{1 + r} \right) (1 + r). \]

We now use (8) and the last equation in (A5) to solve for \( r \), which yields
\[ 1 + r = \frac{A_1}{\beta A_0}. \]

Plugging this into (A8) results in
\[ e_1 = \frac{1 + r}{1 + r^*} \left( 1 - \frac{1}{B} - \frac{(1 + \beta)A_0}{B_G^*} \right) = \frac{A_1}{\beta A_0} \frac{1}{1 + r^*} \left( 1 - \frac{1}{B} - \frac{(1 + \beta)A_0}{B_G^*} \right). \]
Finally, using the fact that $I = -\frac{B^*_G}{B}$ we have

$$
c_0 = A_0 \\
c_1 = A_1.
$$

**Appendix A.2 Proof of Lemma 3**

The proof of this Lemma follows from the fact that $1 + r = \frac{A_1}{\beta A_0}$ in equilibrium and from (A7) and (A9).

**Appendix A.3 Proof of Proposition 2**

To prove the result we consider three cases: 1) Constraints in (1) don’t bind; 2) the upper constraint in (1) binds; and 3) the lower constraint in (1) binds. We begin with Case 1. As we have seen, in this case $\rho = 0$, so (A4) implies $I = (1 + \beta)A_0$. Using (9) we can write the constraints in (1) as

$$
B \leq -\frac{B^*_G}{(1 + \beta)A_0} \leq \overline{B}.
$$

Since $\underline{B} < 0 < \overline{B}$, the result follows.

Now consider Case 2. This case occurs when $\overline{\lambda} > 0$. By Lemma 3 this case occurs when $\rho < 0$. From Proposition 1 we have

$$
\rho = 1 + r - e_1(1 + r^*) \\
= \frac{A_1}{\beta A_0} - \frac{A_1}{\beta A_0} \left( 1 - \frac{1}{B} - \frac{(1 + \beta)A_0}{B^*_G} \right) \\
= \frac{A_1}{\beta A_0} \left( \frac{1}{B} + \frac{(1 + \beta)A_0}{B^*_G} \right).
$$

Since $\underline{B} > 0$, $B^*_G > 0$ implies that $\rho > 0$. Therefore, in order for Case 2 to occur, $B^*_G < 0$. The result then follows.

Finally consider Case 3. This case occurs when $\underline{\lambda} > 0$. By Lemma 3 this case occurs when $\rho > 0$. From Proposition 1 we have

$$
\rho = 1 + r - e_1(1 + r^*) \\
= \frac{A_1}{\beta A_0} - \frac{A_1}{\beta A_0} \left( 1 - \frac{1}{B} - \frac{(1 + \beta)A_0}{B^*_G} \right) \\
= \frac{A_1}{\beta A_0} \left( \frac{1}{B} + \frac{(1 + \beta)A_0}{B^*_G} \right).
$$

Since $\overline{B} < 0$, $B^*_G < 0$ implies that $\rho < 0$. Therefore, in order for Case 3 to occur, $B^*_G > 0$. The result then follows.
Appendix B  Banks Foreign Exposure

Figure B.1: Daily Average Change of Banks Foreign Exposure as % of Capital

(a) 500-Days Moving Average
(b) 1000-Days Moving Average
### Appendix C  Stationarity Properties

Table 1: Elliott-Rothenberg-Stock Test for Unit Root

<table>
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<tr>
<th>Variable (up to 28 lags)</th>
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<th>1% critical value</th>
<th>10% critical value</th>
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</thead>
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<td>-3.480</td>
<td>-2.570</td>
</tr>
<tr>
<td>Financial System</td>
<td>-5.064</td>
<td>-3.480</td>
<td>-2.570</td>
</tr>
<tr>
<td>Five Largest Banks</td>
<td>-7.756</td>
<td>-3.480</td>
<td>-2.570</td>
</tr>
<tr>
<td>Exchange rate Change</td>
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<td>-3.480</td>
<td>-2.570</td>
</tr>
<tr>
<td>Foreign Exchange Interventions</td>
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<td>-3.480</td>
<td>-2.570</td>
</tr>
<tr>
<td>Assets-Liabilities (USD) as % of Domestic Assets</td>
<td>-23.976</td>
<td>-3.480</td>
<td>-2.570</td>
</tr>
</tbody>
</table>

Authors’ calculations. The minimum lag is determined using the modified akaike’s information criterion (MAIC).