Financial and Price Stability in Emerging Markets: The Role of the Interest Rate

LORENZO MENNA MARTIN TOBAL*

The Global Financial Crisis opened a debate on whether inflation target regimes must be relaxed and allow for monetary policy to address financial stability concerns. Nonetheless, this debate has focused on the ability of the interest rate to "lean against the wind" and, more generally, on the accumulation of systemic risk arising from the macro-financial challenges faced by advanced economies. This paper extends the debate to the case of emerging markets by borrowing features from the New-Keynesian model with financial frictions of Curdia and Woodford (2016) and the empirical approach of Ajello et al. (2015) and by using these features to develop a small, open economy framework in which domestic credit plays a critical role. In line with the findings of a recent literature on the Global Financial Cycle, in our setup, a large dependence of domestic financial conditions on capital flows diminishes the effectiveness of monetary policy in dampening financial risks. Indeed, after careful calibration for the Mexican economy, we find that capital account openness reduces the optimal policy rate even below the level that would have prevailed in the absence of endogenous financial crisis and systemic risk.

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1. Introduction

The Global Financial Crisis has promoted the notion of systemic risk, according to which the stability of a financial system critically hinges on the relationships maintained by its components. This macroeconomic aspect of financial stability has confronted policy-makers with a new goal and opened a debate on whether inflation targeting regimes must be modified or relaxed and allow for

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monetary policy to complement macro-prudential policies in achieving financial stability. Nonetheless, this debate has focused on advanced economies and, in particular, on the ability of interest rate setting to lean against the wind (see for instance, Borio, Furfine, and Lowe, 2001; Borio and Lowe, 2002; and Van der Ghote, 2016 for studies on this ability). In this context, the present paper explores the link between monetary policy and financial stability for the case of emerging markets economies (EMEs) by borrowing features from the stochastic general equilibrium model with financial frictions of Curdia and Woodford (2016) and from the empirical approach of Ajello et al. (2015) and by using these features to develop a New Keynesian framework of a small, open economy model in which credit plays a relevant role.

In the extended setup, an increase in the interest rate diminishes the output gap and the demand for domestic credit, just as it does in a closed-economy framework. However, in the open economy, the increase in the interest rate also attracts capital flows and, through this channel, raises liquidity and the domestic credit supply. Hence, in our model, the optimal response to a demand shock that inflates credit growth is in general to adjust the interest rate by less than in a closed-economy setup. After calibrating the model for the particular case of Mexico, we find that it is optimal to set a smaller interest rate than in the absence of systemic risk and endogenous financial crisis because openness in the capital account diminishes the ability of monetary policy to lean against the wind. These results suggest that the prescriptions of the leaning against the wind view may not be as well-suited for understanding the link between monetary policy and financial stability in emerging markets as in advanced economies.

Before the most recent crisis, financial stability was mainly viewed through the lens of microeconomic perspective and, accordingly, focused on risk-taking by individual financial institutions (Acharya, 2013; Allen and Carletti, 2013). However, this narrow perspective precluded foreseeing the buildup of imbalances that preceded the crisis of 2007-2008. In response, policy-makers and scholars began to emphasize the notion of systemic risk, according to which financial risk stems not only from individual risk-taking but also from the links maintained by the components of a financial system (BIS, IMF and FSB, 2011). The idea is that financial institutions interact in manners that may not affect financial risk from an individual perspective but, on the other hand, trigger negative externalities that amplify the risk faced by the system as a whole, i.e., systemic risk.

Two types of interactions illustrate the notion of systemic risk. The first boils down to the concept of financial cycle, according to which credit growth and asset prices exhibit a cyclical behavior, generating a cycle along which systemic risk is amplified. For instance, excessive leverage leads financial institutions to deleverage during the downturn. However, the attempt to deleverage massively reduces asset prices and, through this channel, triggers negative externalities by amplifying capital losses (see Adrian and Shin, 2009 for the role of market liquidity over the cycle). The second form of interactions refers to relationships that link financial institutions in a given period of time and, thus, expose them directly or indirectly to idiosyncratic shocks. For example, in the pre-crisis period, the securitization of mortgage loans substantially increased indirect exposure to risk.

From a policy perspective, the new conceptual framework has generated a new goal. Currently, policy-makers must not only address individual risk-taking through micro-prudential policies but also contain systemic risk, for instance, by dampening imbalances over the financial cycle. Thus, being consistent with the Tinbergen principle, the international community has promoted the use of a new instrument, the macro-prudential tools. However, it has been that these tools are insufficient to fully tackle systemic risk given that, for instance, they do not affect activities or institutions lying outside the regulation perimeter. Moreover, it has been argued that the macro-prudential tools foster shadow banking activities, generating risk outside their direct area of influence. In turn, these potential flaws have led some scholars and policy-makers to argue that inflation targeting regimes must be relaxed so that monetary policy can complement macro-prudential tools in achieving financial stability. For instance, the "lean against the wind" view argues that monetary policy must tighten during the upturn of the cycle to avoid the buildup of financial imbalances and accumulation of systemic risk, e.g., excessive leverage and maturity mismatches (see, for instance, Borio and Lowe, 2002 and Woodford, 2012, Svensson, 2012 and 2014 and Smets, 2014 for a review).

However, this debate has concentrated on advanced economies. In particular, the discussion of whether inflation targeting regimes must be modified has focused on systemic risk arising from the two negative externalities mentioned above, which mainly take in place in complex and sophisticated financial systems and are associated with the buildup of imbalances over the financial cycle. Nonetheless, in EMEs, financial systems are not so-well developed and, most importantly, financial risk has been historically based on their sensitivity to external shocks. Indeed, and beyond anecdotal evidence, it is natural to think of capital flows as being an important determinant of financial stability in EMEs given that the size and depth of their financial systems are significantly small relative to the magnitude of the flows they receive (Claessens and Ghosh, 2013).

The importance of capital flows for financial stability in EMEs is also consistent with a recent literature emphasizing the existence of a global financial cycle. This literature argues that domestic financial conditions in EMEs are influenced by capital flows and global liquidity which are, at the same time, strongly determined by push factors, such as the monetary conditions of the main financial centers and the global appetite for financial risk. This literature has documented, for instance, that leverage, credit creation and asset prices in EMEs co-move with capital flows and indices of global risk perceptions, e.g. VIX (see, for instance, Passari and Rey, 2015; Rey, 2015 and Bruno and Shin, 2015). In sum, given that EMEs have different characteristics, they face distinct financial risks, which emphasizes the need of discussing the link between monetary policy and financial stability in these countries. The present paper fulfills this task by applying intuition from the model with financial frictions developed by Curdia and Woodford (2016) and the empirical approach used by Ajello et al. (2015) to the case of a small, open economy with the purpose of studying optimal monetary policy in EMEs.

To build our setup, we take the real side, the labor market and the modelling aspects of nominal rigidities from a basic New-Keynesians framework, and introduce a role for domestic credit by borrowing some of the micro-foundations developed in Curdia and Woodford (2016). Then, in taking the model to data, we modify Curdia and Woodford (2016) in the same two directions as Ajello et al. (2015). First, we augment it with a two-state crisis shock and an endogenously time-varying crisis probability. In this setup, the probability of transitioning to a crisis state is increasing in credit growth, replicating the main ideas of the financial cycle, according to which the excessive leverage generates negative externalities and, thus, amplifies systemic risk. Second, to make the endogenous transition between states computationally feasible, we follow Ajello et al. (2015) in reducing the model to a two-period framework.

Finally, we extend the analysis to a small, open economy by allowing for international trade and for capital flows. In this extended setup, domestic savers acquire financing in global markets and can use it to finance their consumption and to supply domestic credit. Thus, the main financial variable of the model, domestic credit, is strongly influenced by the direction and magnitude of capital flows, replicating the main ideas of the recent literature on the global financial cycle. Importantly, the combination of these modelling aspects imply that, in our setup, a surge of capital inflows raises the probability of a financial crisis by fueling domestic credit growth.

Using the framework outlined above, we consider three different scenarios. First, we consider a closed-economy model with exogenous crisis probability. This scenario represents the world preceding the Global Financial Crisis, in which the concept of systemic risk was not that extended. Second, we consider a closed economy in which the crisis probability responds positively to credit growth. This scenario can be thought of as representing the case of advanced economies, in which the financial cycle is a significant source of financial risk. Finally, we extend this last case to a small, open economy and interpret it as a laboratory for investigating optimal monetary policy in EMEs.

After calibrating the model for Mexico, we find that the optimal interest rate response to a demand shock that inflates credit growth differs across the three scenarios. In the closed economy with exogenous crisis, the policy rate is optimally set to a level that completely stabilizes output and inflation, i.e., a result known as the divine coincidence. However, this result is no longer possible with endogenous crisis since, in this case, the policy-maker must incorporate the effects of policy choices on crisis probability. Indeed, consistently with Ajello et al. (2015), the results show that the optimal interest rate is higher than in the model with exogenous crisis probability. Finally, we show that, in the open economy, the optimal response is to reduce rather than to increase the policy rate with respect to a scenario with no systemic risk. To put it in terms of the literature, the results suggest that the global financial cycle concerns are more important than the financial cycle concerns with respect to monetary policy for the case of a small, open economy, such as most EMEs.

While this last result is particularly relevant from a conceptual point of view, we find indeed that the quantitative differences in terms of optimal interest rate setting among the three scenarios is not significantly big. This outcome is in turn in line with the outcomes obtained by Ajello et al (2015) for the U.S. economy. Thus, we follow their strategy and allow for the possibility that policy-makers have uncertainty on some parameters, notably, the elasticity of crisis probability to credit conditions and the severity of financial crises. Also in line with Ajello et al. (2015), we consider both a Bayesian and a Robust policy-makers and show that uncertainty on some of the model's parameters can significantly increase the quantitative importance of the difference in terms of monetary policy among the three scenarios.

The paper is related to a growing literature that studies optimal interest rate policy in presence of financial stability concerns and, more generally, whether it is optimal to allow for monetary policy to respond to financial stability risks (Borio and Lowe, 2002; Adrian and Shin, 2009; Curdia and Woodford, 2016; Svensson, 2012, 2014; Woodford, 2012; Ajello et al., 2015; Brunnermeier and Sannikov, 2016; Collard et al., forthcoming). As noted above, this literature has based the debate on the characteristics and experiences of advanced economies; and it has not reached a conclusion yet. By contrast, we address this issue in the context of EMEs which, in principle, face financial stability risks of a different nature stemming from the direction and volatility of capital flows. In this sense, the paper also relates to a strand of the literature that argues that monetary conditions of the main financial centers and the global appetite for risk determine capital flows (giving rise to a Global Financial Cycle) and, therefore, affects domestic financial conditions (Obstfeld, 2012; Bruno and Shin, 2015; Passari and Rey, 2015; Rey, 2015). However, we do not provide empirical evidence of the Global Financial Cycle nor study how this cycle determines domestic financial conditions. Instead, we study whether the existence of a Global Financial Cycle further constrains the ability of monetary policy to mitigate the accumulation and amplification of systemic risk.

2. The Model

As mentioned, the model borrows some characteristics from a basic New-Keynesian model, some features from Curdia and Woodford (2016), and some modelling aspects from Ajello et al. (2015). The model is first presented as a standard infinite horizon framework, which allows us to explain how the structural relationships that we use in our empirical analysis can be micro-founded. In particular, we demonstrate that such relationships can be obtained by adding some of the modelling aspects of the Curdia and Woodford (2016) approach to the basic New Keynesian framework, and by extending the resulting model to a small open economy.

More specifically, we consider a small open economy model, in which we include a role for credit. To introduce a role for credit, we borrow from Curdia and Woodford (2016) by considering households of different types. In particular, households differ according to their taste for current consumption relative to future consumption and to their elasticity of intertemporal substitution. Under the assumptions that we explain below, those with a higher valuation of current consumption will borrow and those with a lower valuation will save, which gives rise to domestic credit.[1] In the context of this paper, domestic credit can be thought of as the result of direct exchanges between households of different types, as well as being intermediated by financial intermediaries operating under perfect competition.

Given that the ultimate goal is to take the model to data, we follow Ajello et al. (2015) in introducing two modifications to Curdia and Woodford (2016). First, we assume that the economy can switch from a "normal" to a "crisis" state and the switching probability depends on endogenous variables. In particular, the crisis probability will increase with credit growth, a proxy for leverage and, in turn, an indicator of systemic risk, i.e., the specific functional form of the probability is shown in Section 3. Second, to make the endogenous switching computationally feasible, the model is reduced to a two period setup. In practice, the economy is in the normal state in period one and can switch to a crisis state in period two.

Finally, and to adapt the model to a small open economy setting, we consider a small open economy by allowing for imports and exports and for international capital flows. In particular, we assume that domestic savers can lend and borrow in international financial markets; while borrowers can only borrow on domestic financial markets. Importantly, this feature of the model will generate a mechanism through which capital flows will affect domestic financial conditions and, thus, financial risk: a surge of capital inflows will enhance domestic savers' availability of financial funds and,

[1]As we are interested in studying how credit growth affects crisis probability, and the role of monetary policy therein, and not in how fluctuations in interest rate spreads affect the behavior of the economy, we abstract from the endogenous spread considered by Curdia and Woodford (2016). Actually, the same approach is also the one taken by Ajello et al (2015).

through this channel, fuel domestic credit and leverage in the domestic market. As noted above, higher domestic credit growth will raise the probability of switching to the crisis state.

Households

While the full model is reported in the appendix, here we describe its most important features. As mentioned, we assume that households differ in terms of their utility function and that this function can be summarized as follows:

$$max\tilde{E}_0 \sum_{t=1}^{\infty} \beta^{t-1} \left[\frac{C^{\tau_t^i}}{1-\sigma^{\tau_t^i}} \left(c_t^i\right)^{1-\sigma^{\tau_t^i}} - \frac{\chi^{\tau_t^i}}{2} \left(l_t^i\right)^2 \right], \tag{1}$$

where $\tau_t^i \in \{s, b\}$ indicates household type, c^i is household i's consumption and l^i is household i's worked hours, indexes s and b identify savers and borrowers, respectively, $\beta, C^{\tau_t^i}, \sigma^{\tau_t^i}$ and $\chi^{\tau_t^i}$ are parameters, while $\tilde{E}$ is the expectation operator, which is represented with a tilde to highlight the fact that expectations are not completely rational, as in Ajello et al (2015) and for the reasons explained below. Just as in Curdia and Woodford (2016), some households have a lower taste for current consumption (which is obtained by setting $C^s \leq C^b$) and a lower elasticity of intertemporal substitution ($\frac{1}{\sigma^s} \leq \frac{1}{\sigma^b}$) than others. Taste for leisure is also allowed to differ between agent types.[2] Type $\tau_t = s$ households represent a share $1 - \pi^b$ of the Home population, and type $\tau_t = b$ a share π^b.

The modelization of domestic financial markets strictly follows Curdia and Woodford (2016). Households are assumed to be able to sign an insurance contract that allows them to share all aggregate and idiosyncratic risk, but they can receive transfers from the insurance agency only infrequently. In all other points in time, households can only trade one-period credit contracts. Details of this framework are discussed at length in the appendix. Here it is sufficient to report the budget constraint that they imply for a generic domestic household i:

$$B_t^i + B_t^{fi} = R_{t-1} \frac{B_{t-1}^i}{\pi_t} + S_t^i + \frac{R_{t-1}^f B_{t-1}^{fi} X_t}{X_{t-1}} + w_t l_t^i + D_t - c_t^i - T_t. \tag{2}$$

In the above, R is the nominal domestic interest rate, which we assume to be under the control of the policy-maker, B^i is the real (per-capita) value of domestic credit, π is the inflation rate, S^i is the

[2] As we explain in the appendix, this allows to ensure that the two household types work the same number of hours. A similar approach is also used by Curdia and Woodford (2016).

transfer that the household receives from the insurance agency, R^f is the interest rate on the foreign currency bond, B^f is the real (per-capita) value of the foreign currency bonds, and X is the real exchange rate. S^i is zero for all households that do not receive transfers from the insurance agency at time t, while B^f is constrained to be equal to zero for borrowers, as the latter are assumed not to have access to global financial markets. Further, w is the real wage, D are firm profits, and T are lump-sum taxes. From equation (2), in the appendix, we obtain the average consumption of savers and borrowers:

$$c_t^s + \frac{b_t}{(1-\pi^b)} - \frac{\frac{R_{t-1}}{\pi_t}(b_{t-1})}{(1-\pi^b)} + \frac{b_t^f}{(1-\pi^b)} - \frac{\frac{R_{t-1}^f b_{t-1}^f}{(1-\pi^b)} X_t}{X_{t-1}} = w_t l_t^s + D_t - T_t;$$

and

$$c_t^b = \frac{b_t}{\pi^b} - \frac{\frac{R_{t-1}}{\pi_t}(b_{t-1})}{\pi^b} + w_t l_t^b + D_t - T_t;$$

where b and b^f are respectively the total amounts of domestic credit and of foreign assets in the hands of savers.

Households optimize (1) subject to (2). In the appendix, we report the complete list of first order conditions. Here we just show in log-linear form the ones that help us illustrate how equilibrium in domestic and global financial markets is reached, as this can be useful to understand the results of the paper:[3]

$$\hat{R}_t - \tilde{E}_t\pi_{t+1} = \sigma^b\left(\tilde{E}_t\hat{c}_{t+1}^b - \hat{c}_t^b\right) = \sigma^s\left(\tilde{E}_t\hat{c}_{t+1}^s - \hat{c}_t^s\right) \qquad (3)$$

$$\widehat{\mathrm{R}}_t - \widehat{R}_t^f = \tilde{E}_t\hat{X}_{t+1} - \hat{X}_t + \tilde{E}_t\hat{\pi}_{t+1} \qquad (4)$$

Equation (3) states that the real interest rate is equal to the expected growth of the marginal utility of savers and borrowers. Equation (4) is the uncovered interest parity condition that states that the interest rate differential between the Home and the foreign economy is equal to the expected real

[3] In what follows, hats indicate that the variables are in log-deviations from the steady state, while tildes indicate that the variables are in level deviations from the steady state. Additional first order conditions are the ones governing the labor supply of the two agents. Such conditions, which are reported in the appendix, are quite standard.

depreciation plus the inflation rate. The first equation can be thought of as describing equilibrium in domestic financial markets: both household types equalize expected marginal utility growth to the real interest rate. The second equation, instead, describes equilibrium in global financial markets. By combining the budget constraints of the two agent types and by log-linearizing, it is also possible to obtain the following relationship:[4]

$$\frac{1}{(1-\pi^b)\pi^b}\left[\tilde{b}_t - \frac{1}{\beta}\left(\tilde{b}_{t-1} + b\hat{R}_{t-1}\right)\right] = s^b\hat{c}_t^b - s^s\hat{c}_t^s - \frac{b}{\beta(1-\pi^b)\pi^b}\hat{\pi}_t - \frac{1}{1-\pi^b}\left[\tilde{b}_t^f - \frac{1}{\beta}\tilde{b}_{t-1}^f\right] \quad (5)$$

where s^b, s^s, and b are respectively the steady state consumption level of borrowers, the steady state consumption level of savers, and steady state domestic credit. Equation (5) illustrates neatly the channels that affect credit growth in the model. In practice, three channel can be identified, two of which operate in both the closed economy and the open economy model, and one of which is only present in the open economy model. The first channel, which captures the effect of *demand* on credit, is purely due to the interaction between borrowers and savers in domestic financial markets and is captured by the term $s^b\hat{c}_t^b - s^s\hat{c}_t^s$: if borrowers' consumption grows more than savers', this must be financed by additional domestic credit. The second channel is due to a *Fisher* effect and is captured by the term $-\frac{b}{\beta(1-\pi^b)\pi^b}\hat{\pi}_t$: as credit is nominal, inflation reduces its real value. The third effect, i.e. the one that is only present in the open economy model, is captured by the term $-\frac{1}{1-\pi^b}\left[\tilde{b}_t^f - \frac{1}{\beta}\tilde{b}_{t-1}^f\right]$: if the value of foreign assets held by savers fall, i.e. *capital inflows* take place, domestic credit goes up.

Unlike Curdia and Woodford (2016) the consideration of an open economy confronts us with the need of specifying how much of the consumption in the utility function is satisfied with foreign goods and, closely related, how trade balance is determined. Thus, just as in most part of the open economy DSGE literature, we assume that the consumption basket is composed of home and foreign produced consumption goods (imports), respectively defined as c_H and c_F.[5] In particular, the consumption utility aggregator takes the general CES form and can thus be written as follows

$$c_t^i = \left[(1-\gamma)^{\frac{1}{\eta}} {c_{H,t}^i}^{\frac{\eta-1}{\eta}} + \gamma^{\frac{1}{\eta}} {c_{F,t}^i}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},$$

[4] The relationship is obtained under the assumption that steady state foreign credit is zero.
[5] See De Paoli (2009) and Gali and Monacelli (2005), among others.

where γ governs the degree of home bias and η is the elasticity of substitution between home and foreign goods. If domestic households are not interested in consuming foreign goods, which happens for $\gamma = 0$, the model collapses to a closed economy framework. On the contrary, if γ is close to one, the home economy is completely open, in the sense that home produced goods represent a negligible share of the households' consumption basket. In the appendix, we show that imports are decreasing and exports increasing in the real exchange.

Firms

The supply side of the economy is as in the basic New-Keynesian model. Intermediate good firms produce differentiated goods under monopolistic competition, and are subject to Rotemberg quadratic price adjustment costs. Differentiated goods are then aggregated by final good firms operating under perfect competition. The problem of intermediate firm producing good j is to maximize

$$\max \tilde{E}_0 \sum_{t=1}^{\infty} \Omega_{t,t+1}[p_{j,t}y_{j,t} - (1+\tau)w_t l_{j,t} - \frac{\chi_P}{2}\left(\frac{p_{j,t}}{p_{j,t-1}}\pi_t - 1\right)^2 p_{H,t}y_{H,t}]$$

subject to a downward sloping demand function

$$y_{j,t} = \left(\frac{p_{j,t}}{p_{H,t}}\right)^{-\zeta} y_{H,t}$$

and a linear production function

$$y_{j,t} = l_{j,t}.$$

In the above problem, p_j is the real price of good j, p_H is the real price of final domestically produced good, τ is a steady state labor subsidy that is financed through lump-sum taxes on households, y_j and y_H are respectively the production levels of good j and of the final domestically produced good. Ω is a stochastic discount factor, ζ is the elasticity of substitution among differentiated goods, and χ_P is the Rotemberg adjustment cost parameter. In the Rotemberg framework, all firms set the same price and the same production level in equilibrium, i.e. $p_j = p_H$ and $y_j = y_H$. In the appendix, we show that optimization on the part of firms gives rise to a Phillips curve relationship that is presented in linear terms in the following sections.

3. The Log-Linear Model

The model presented in the previous section allows us to obtain the structural relationships that we need to interpret the data. However, some additional steps are need to bring the model to the data, in the same manner as Ajello et al (2015) do for the case of a closed economy. First, i) we need to log-linearize the model, which is what we do in the appendix. Second, ii) we reduce the log-linear model to a two-period framework. Third, iii) we assume that in period two the economy may be subject to a crisis shock, which brings the economy to the crisis state and delivers exogenous and adverse effects on output and inflation; while if such shock does not take place, the economy is in the normal state, which is assumed to imply no deviation of output and inflation from the steady state. We consider both the case in which the crisis shock is exogenous and the case in which it is an increasing function of domestic credit. Fourth, and lastly, iv) we assume that expectations are formed as in Ajello et al (2015) and agents are assumed to give a fixed subjective probability to the realization of a crisis, independently from the actual crisis probability. In fact, and as explained by Ajello et al (2015), if expectations were modeled as being rational, times in which credit growth is high would be associated with reductions of output and inflation, because the increased crisis probability may reduce expected future inflation and output gaps, leading to lower inflation and a lower output gap today. Nonetheless, this result is inconsistent with much empirical evidence suggesting that agents expect good times to continue going forward (see Shiller, 2005).

To present our results, we consider three scenarios: (i) a closed economy in which financial crises are exogenous and; (ii) a closed economy in which this probability increases with credit growth; and (iii) an open economy in which capital flows have an influence on domestic financial conditions.

Closed economy model with exogenous crisis probability

When the model of section 2 is log-linearized under the assumption that $\gamma = 0$ (that is the economy is closed) and that crisis probability is independent of credit, the conditions that determine equilibrium are written as follows:

$$\hat{y}_1 = \tilde{E}_1 \hat{y}_2 - \bar{\sigma}\left(\hat{R}_1 - \tilde{E}_1 \hat{\pi}_2\right) + \epsilon_1, \tag{6}$$

$$\hat{\pi}_1 = \varphi \frac{\bar{\sigma} + 1}{\bar{\sigma}} \hat{y}_1 + \beta \tilde{E}_1 \hat{\pi}_2; \tag{7}$$

where ϵ_1 is a normally distributed demand shock. As can be seen from equations (6) and (7), when the economy is closed and crisis probability is exogenous, the model collapses to a standard New-Keynesian model. In this model, Equation (6) is the IS curve according to which output responds positively to expectations about output tomorrow and negatively to the real interest rate, and equation (7) is the Phillips Curve according to which inflation depends positively on expectations of inflation tomorrow and on output today.

Before proceeding, it is useful to express some of the parameters in equations (6) and (7) in terms of deep fundamentals of the model given that this will ease the task of explaining the parametrization in what follows. Hence, we consider the following definitions:

$$\varphi = \frac{\zeta - 1}{\chi_P}$$

and

$$\bar{\sigma} = [\frac{\pi^b s^b}{\sigma^b} + \frac{(1-\pi^b)s^s}{\sigma^s}].$$

Closed economy model with endogenous crisis probability

This subsection augments the model summarized in Equations (6) and (7) to make endogenous the probability of transitioning from a normal state to a crisis state. For this purpose, we follow Ajello et al (2015) in defining the transition probability and the growth rate of credit in real terms as follows

$$P(\tilde{b}_1) = \frac{e^{p+\kappa\tilde{b}_1}}{1 + e^{p+\kappa\tilde{b}_1}} \tag{8}$$

where

$$\tilde{b}_1 - \frac{1}{\beta}\left(\tilde{b}_0 + b\hat{R}_0^d - b\hat{\pi}_1\right) = s_{\aleph}\hat{y}_1; \tag{9}$$

and p and κ are parameters determining the average crisis probability and the elasticity of crisis probability to credit, respectively. Furthermore, we define parameter $s_{\aleph}$, an indicator of credit growth sensitivity to output, as follows:

$$s_{\aleph} = \frac{\pi^b(1-\pi^b)\left[\frac{s^b}{\sigma^b} - \frac{s^s}{\sigma^s}\right]}{\bar{\sigma}}.$$

Equation (8) states that the transition probability increases with $\tilde{b}_1$, i.e. domestic credit in real terms and equation (9) defines credit as being an increasing function of output and a decreasing function of inflation. With the exception of the specific functional form chosen by Ajello et al (2015) for the transition probability, the remaining aspects of equations (8) and (9) can be theoretically and empirically founded.

Equation (8) finds empirical support in Schularick and Taylor (2012) and Ajello et al (2015). Assuming that the crisis probability follows a logistic function, they find that real credit growth is a critical predictor of financial crises. Furthermore, this equation is consistent with the idea of systemic risk developed by Borio and Lowe (2002), according to which it is precisely the build-up of imbalances, and particularly the accumulation of leverage during the upturn of the financial cycle, which prompts a financial turmoil. Furthermore, we estimate the equation for a group of Latin American countries in the appendix and use the estimates to calibrate parameters p and κ in our numerical analysis.

As for equation (9), it is derived from the closed economy version of the model of section 2; thus, this equation is theoretically supported by the micro-foundations of the model. To be more precise, equation (9) is another way to illustrate the factors that influence credit accumulation, alternative to equation (5). The term $s_{\aleph}\hat{y}_1$ in equation (9) captures the same effect as term $s^b \hat{c}_t^b - s^s \hat{c}_t^s$ in equation (5). In fact, a positive shock to income has a stronger effect on the expenditure decisions of borrowers than on those of savers, and, therefore, makes the former more willing to borrow and the latter willing to lend, thereby raising credit. The Fisher effect is similarly captured by term $\frac{b}{\beta}\hat{\pi}_t$.

Open economy model with endogenous crisis probability

To extend the analysis to the case of a small, open economy, this subsection considers the full model presented in section 2 and derives the following equilibrium conditions:

$$\hat{y}_1 = \tilde{E}_1\hat{y}_2 - \frac{\gamma(\nu-\gamma)}{1-\gamma}\left(\tilde{E}_1\hat{X}_2 - \hat{X}_1\right) - (1-\gamma)\bar{\sigma}(\hat{R}_1 - \tilde{E}_1\hat{\pi}_2) + \epsilon_1, \tag{10}$$

$$\hat{R}_1 - \hat{R}_1^f = \tilde{E}_1\hat{X}_2 - \hat{X}_1 + \tilde{E}_1\hat{\pi}_2, \tag{11}$$

$$\begin{aligned}\hat{\pi}_1 = &-\frac{\gamma}{1-\gamma}\hat{X}_0 + \frac{\gamma}{1-\gamma}\left(1+\beta+\frac{\zeta-1}{\chi_P}\left(2-\frac{(\nu-\gamma)}{(1-\gamma)\bar{\sigma}}\right)\right)\hat{X}_1 \\ &+\frac{\zeta-1}{\chi_P}\frac{(1-\gamma)\bar{\sigma}+1}{(1-\gamma)\bar{\sigma}}\hat{y}_1 + \beta\left[\tilde{E}_1\hat{\pi}_2 - \frac{\gamma}{1-\gamma}\tilde{E}_1\hat{X}_2\right],\end{aligned} \tag{12}$$

$$\begin{aligned}&\tilde{b}_1 - \frac{1}{\beta}\left(\tilde{b}_0 + b\hat{R}_0 - b\hat{\pi}_1\right) \\ &\quad = \frac{(1-\gamma)s_{\aleph} + \pi^b\gamma}{1-\gamma}\hat{y}_1 - \frac{\gamma(\nu-\gamma)}{1-\gamma}\frac{(1-\gamma)s_{\aleph}+\pi^b}{1-\gamma}\frac{s_{\aleph}+\pi^b}{1-\gamma}\hat{X}_1;\end{aligned} \tag{13}$$

Equation (10) is the open economy intertemporal IS equation. Just as in the closed economy model, current output gap, $\hat{y}$, depends on its future expectations, and on the real interest rate. In the open economy, current output also depends on the real exchange rate: a real devaluation tends to increase it through higher net exports. Equation (12), the Phillips curve, shows that inflation depends on output and on expected future inflation, as in the closed economy model; while the real exchange rate enters the equation due to openness. In particular, current depreciations tend to increase inflation, while past and expected future depreciations tend to reduce it.[6] It is important to highlight the fact that the modifications to the IS curve and to the Phillips curve that are due to openness affect the benchmark to which the model with endogenous systemic risk is compared, but not the basic result of this paper. In other words, the fact that the IS curve and the Phillips curve are different in an open economy changes the optimal interest rate compared to a closed economy also when systemic risk is exogenous. However, as we show below, results when adding endogenous systemic risk are compared to the different benchmark: in the closed economy model, the interest rate is increased with respect to the closed economy benchmark with exogenous systemic risk, while in the open economy model, the interest rate is reduced, with respect to the different benchmark.

Equation (13) is also an alternative way to equation (5) to describe credit accumulation in the open economy model. As in the closed economy case, the Fisher effect is captured by $\frac{b}{\beta}\hat{\pi}_t$ and the output term plays the same role as term $s^b\hat{c}_t^b - s^s\hat{c}_t^s$. However, in the open economy credit is more sensitive to output gap, i.e., $\frac{\pi^b\gamma}{1-\gamma}$ is greater than zero. Finally, the term $-\frac{\gamma(\nu-\gamma)}{1-\gamma}\frac{(1-\gamma)s_{\aleph}+\pi^b}{1-\gamma}\frac{s_{\aleph}+\pi^b}{1-\gamma}\hat{X}_1$ plays the same role as $-\frac{1}{1-\pi^b}\left[\tilde{b}_t^f - \frac{1}{\beta}\tilde{b}_{t-1}^f\right]$ in equation (5). In fact, capital inflows cause currency appreciation, i.e. a fall in $\hat{X}$, and higher domestic credit.

The fact that credit growth depends on the real exchange rate (and capital flows) has a relevant implication: a raise in the policy rate is more likely to increase credit rather than reduce it, compared to a closed economy model. In both the open economy and the closed economy model a contractionary monetary policy tends to reduce output and inflation. The fall in output tends to reduce credit, due to the demand effect, and to raise it through the Fisher effect. In our numerical exercise, we will show that higher rates reduce credit in the closed economy, which implies that the demand channel prevails. In the open economy, however, an interest rate hike also generates capital inflows and an appreciating effect on the real exchange rate. Capital inflows, in turn, increase credit. Hence, the effectiveness of monetary policy in controlling credit growth is clearly diminished in the open economy and, thus, interest rate setting is less suited to avoid excessive leverage and to dampen financial risks.

[6] Current depreciations tend to increase inflation due to several mechanisms. It is not fundamental to discuss all of them here, as such effects are not necessary to understand the results. However, one might recall that depreciations increase inflation due to the pass-through of higher import prices to the CPI. Past depreciations are correlated to lower inflation simply due to the fact that they increase past prices, thereby widening the base over which today's inflation is computed. Expected future depreciations reduce current inflation because price adjustment costs only affect domestic prices and so firms do not take account of the future changes in the real exchange rate when setting today's prices.

Notice that when parameterizing equation (13), and more generally the model, we depart from Ajello et al (2015) who uses U.S. data to estimate a reduced form relation between credit growth and output. More precisely, we obtain (13) by using the structural model. As a robustness check, the appendix estimates a reduced form relation between domestic credit, output and the real exchange rate for Mexico and shows that actually domestic credit is positively influenced by output growth and negatively influenced by real depreciations.

Parameter Values

In computing optimal monetary policy, we are confronted with the need of choosing a loss function that the policy-maker wants to minimize. Thus, in order to keep consistency with Ajello et al. (2015), the period loss function is assumed take the following form:

$$L(\hat{\pi}_t, \hat{y}_t) = \frac{1}{2}\left(\phi_\pi \hat{\pi}_t^2 + \phi_y \hat{y}_t^2\right),$$

and inflation and output are given the same weight, i.e. $\phi_\pi = \phi_y$. Also in the manner of Ajello et al. (2015), the period two loss is adjusted to take into account that crises can last more than one period in the following manner:

$$L(\hat{\pi}_2(C), \hat{y}_2(C)) = \frac{\frac{1}{2}\left(\phi_\pi \hat{\pi}_2^2(C) + \phi_y \hat{y}_2^2(C)\right)}{1 - \beta\tau}.$$

where the τ parameter can be adjusted to increase or decrease the crisis duration and C identifies the crisis state. Following Ajello et al (2015), it has been assumed that in the normal state (N) no variable deviates from its approximation point, hence $\hat{\pi}_2(N) = \hat{y}_2(N) = \hat{X}_2(N) = \hat{\aleph}_2(N) = 0$. Furthermore, it is assumed that the private sector disregards the possibility of a crisis, given that, as discussed by Ajello et al (2015) and Shiller (2005), credit booms are accompanied by private sector expectations that "good times" will continue going forward.

Table 1 reports the calibration of the model. The calibration is based on Mexican data when possible, while in all other instances we run a robustness analysis to test the sensitivity of the results. Those parameters that govern the welfare loss due to the realization of the crisis state are set to capture the effects of the Mexican financial crisis of 1994-1995 on macroeconomic variables. On the basis of information provided by the OECD recession dummy for Mexico, it is assumed that the crisis begins on 1994-Q4 and ends on 1995-Q3.[7],[8] Over this period, the output gap and inflation averaged -2.4%

[7] This data set is obtained from the FRED website.
[8] This amounts to delaying the crisis by one month.

and 9.1% respectively on a quarterly basis.[9] To set the duration of the crisis to four quarters, τ is calibrated to 0.7537.[10] The discount factor, β, is set to 0.99 to obtain an annualized quarterly steady state real interest rate of 4%, as is standard in the macroeconomic literature. Different values of β however do not change significantly the results.

The baseline value of parameter γ, which governs the degree of home bias, is set to 0.3 to obtain a degree of trade openness, computed as the ratio between the sum of imports and exports and GDP, equal to 60%, consistently with the Mexican figure.11 However, as the main focus of the analysis concerns the effect of openness on optimal monetary policy, we also consider several other values of γ. The elasticity of exports to the terms of trade, ν, is set to 1, as common in the literature (see Gali and Monacelli, 2005). To assess the robustness of results with respect to this assumption, we also consider values of ν between zero and six.

The ζ and χ_P parameters are set to 6 and 77, respectively, in order to obtain a mark-up of prices over marginal costs of 20% and prices that, if a Calvo model was used instead of a Rotemberg model, would adjust every 4.5 quarters.12 These two parameters govern the steepness of the Phillips curve and hence the relative elasticity of inflation and output to monetary policy. Given the uncertainty surrounding these values for Mexico, we also run robustness checks on them. Following Curdia and Woodford (2016), we set the share of borrowers, π^b, to 0.5 and the ratio between the inverse elasticity of intertemporal substitution of the two types, σ^s/σ^b, to five. Also in this case, several other values are considered as a robustness check. Furthermore, the absolute values of σ^s and σ^b are set such that the slope of the IS curve with respect to the real interest rate, $\bar{\sigma}$, is one. This value is what would be obtained in a representative agent model under log-utility. The share of borrowers' consumption, $\pi^b s^b$ is conventionally set to 0.7, but other values are also considered. We calibrate the steady state domestic credit to output ratio, b to 1.17 consistently with the average Mexican figure.13

The p and κ parameters, which govern the relation between domestic credit and crisis probability, are set to -4.1137 and 1.1625, respectively. As explained in the appendix, these values are obtained

[9] Data on the output gap and inflation are obtained from Banco de Mexico. Potential output is set equal to actual output on 1994-Q3 and is increased at a constant growth rate, calculated as the average growth rate of the Mexican economy between 1993-Q1 and 2015-Q3. The output gap is computed as the difference between actual output and potential output.

[10] Given that $L\left(\hat{\pi}_2(C), \hat{y}_2(C), \hat{\aleph}_2(C)\right)$ is the one period loss, τ can be obtained solving the equation $\frac{L\left(\hat{\pi}_2(C), \hat{y}_2(C), \hat{\aleph}_2(C)\right)}{1-\beta\tau} = L\left(\hat{\pi}_2(C), \hat{y}_2(C), \hat{\aleph}_2(C)\right)[1+\beta+\beta^2+\beta^3]$.

[11] Data on exports and imports are obtained from FRED.

[12] These values are taken from De Paoli *et al* (2010). Keen and Wang (2007) show how to convert a Rotemberg parameter to a Calvo frequency of adjustment.

[13] b is computed as the ratio between total credit to the non-financial sector and GDP. Data are taken from Banco de Mexico and cover the period 1994-Q4 to 2015-Q3.

by running a logistic regression with country fixed effects of four years domestic bank credit growth on crisis probability for a group of Latin American countries. Crisis years are identified using the dataset of Laeven and Valencia (2012). This procedure is the same as the one employed by Ajello et al (2015) to calibrate the parameters governing crisis probability in their paper. In practice, crisis probability is 6.28% on an annual basis on average, higher than the one obtained by Ajello et al (2015) (equal to 3.24%), but its elasticity to credit is lower than the one estimated by Ajello et al (2015) (which is 1.88). In addition, time zero credit and real exchange rate deviation from average are set to zero. However, as crises often strike in periods of high indebtness, we consider several other values for inherited credit.

	Parameter Value	Description
$\hat{y}_2(C)$	-2.4% (-9.6% annualized)	Crisis State Output Gap
$\hat{\pi}_2(C)$	9.1% (36.4% annualized)	Crisis State Inflation
ζ	6	Elasticity of Substitution Among Home Goods
χ_P	77	Rotemberg Parameter
γ	0.3	Home Bias
ν	0-6	Elasticity of Exports
π^b	0.5	Share of Borrowers
β	0.99	Discount Factor
σ^b	Set to obtain $\bar{\sigma} = 1$	Inverse Intertemporal Elasticity of Subs. of Borrowers
σ^s	$5\sigma^b$	Inverse Intertemporal Elasticity of Subs. of Savers
s^b	1.4	Expenditure Share of Borrowers
s^s	0.6	Expenditure Share of Savers
b	1.17	Steady State Debt
p	-4.1137	Coefficient on Crisis Probability
κ	1.1625	Coefficient on Crisis Probability

ϕ_y	1/2	Loss Function Coefficient on Output Gap
ϕ_π	1/2	Loss Function Coefficient on Inflation
τ	0.7537	Coefficient to Set Crisis Duration

Table 1 – Parameter calibration.

4. Results

Key Intertemporal Trade Off

Using the parametrization presented in Section 0, we explore the effects of a 1% positive aggregate demand shock in period 1. In the standard New Keynesian framework, monetary policy can fully stabilize output and inflation in response to a demand shock, i.e., this result is frequently referred to as the divine coincidence; thus, this is the type of shock that most clearly exposes how the introduction of financial stability concerns moves the central bank away from its traditional objectives. Moreover, considering a demand shock allows us to follow closely the approach taken by Ajello et al. (2015). Figure 1 shows the output gap, inflation, loss in period one, the continuation loss (i.e. the loss in period two), the total loss and the crisis probability as a function of the policy rate for the closed economy model with exogenous crisis probability (dots), for the closed economy model with endogenous crisis probability (dot and lines) and for the open economy model (full line). The optimal policy rate is respectively identified with blue, green and red circles.

The comparison between the two closed economies reveals that the results are similar to those obtained by Ajello et al (2015). While the crisis probability is independent of the policy rate in the exogenous crisis model, a raise in this rate reduces output, credit, leverage and therefore, the probability of a crisis in the endogenous crisis model. The implication is that in the latter scenario it is no longer optimal to fully stabilize output and inflation: taking into account the effect of her policy choices on crisis probability, the policy-maker optimally sets a somewhat higher interest rate than in the model with exogenous crisis. However, just as Ajello et al. (2015), we find that the difference in terms of optimal policy between the scenarios is small (around three basis points on an annual basis).14

Continuing with the case of a closed economy with endogenous crisis, Figure 2 studies the sensitivity of optimal monetary policy to differences lagged credit conditions, $b_0 \in [0,0.5]$. Note that

[14] Quantitative differences between our results and the results in Ajello et al (2015) are due to differences in the calibration. However, the differences in the results are minor

a higher level of inherited credit is associated with a higher probability of crisis. This implies that, due to the convexity of the logit function, monetary policy is more efficient in reducing the risk of a crisis and, as a result, the optimal policy rate is higher. Nonetheless, we find again small differences between varying scenarios: for credit levels that are 50% higher than their normal level, the policy rate is increased around 3 basis points on an annual basis.

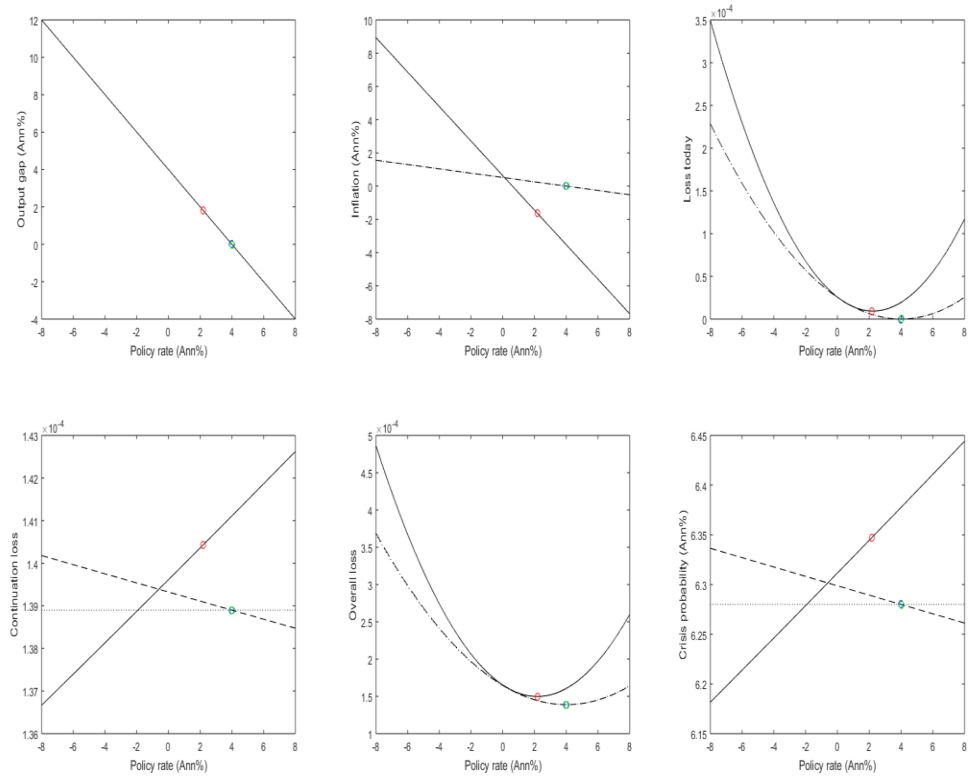

Figure 1 - Full line: open economy model. Lines and dots: closed economy model with endogenous crisis. Dots: closed economy model with exogenous crisis. Red circles: optimal values in the open economy model. Green circles: optimal values in the closed economy model with endogenous crisis. Blue circles: optimal values in the closed economy model with exogenous crisis.

As for the open economy model, openness in the balance-of-payment generates additional channels through which the interest rate affects the traditional objectives of a central bank, as well as the crisis probability. Besides affecting inflation and output through its impact on aggregate demand, a rise in the interest rate affects its traditional objectives through its effect on the real exchange rate. For instance, a rise in interest rate produces a reevaluation of the exchange rate, which further compresses exports and, at the same time, diminishes inflation by reducing the domestic prices of imported goods.

The fact that inflation is more sensitive to monetary policy implies that, even in the absence of financial stability considerations, the optimal interest rate is smaller in the open economy.

Furthermore, the inclusion of financial stability concerns provides additional incentives for setting a lower interest rate. Just as in the closed economy with endogenous crisis, an increase in the policy rate has a direct and diminishing impact on credit and, therefore, on the crisis probability. Nonetheless, in the open economy, a rise in the interest rate attracts capital flows and, through this channel, fuels domestic credit. The second panel in Figure 1 show that the latter effect overpowers the former effect, so that credit, the crisis probability and the continuation loss are increasing in the policy rate. In other words, to reduce the probability of a financial crisis, the central bank in a small, open economy must reduce the interest rate below the level that would prevail in the absence of systemic risk. This is precisely the opposite policy perspective from the one supported by the "leaning against the wind" approach proposed for the case of advanced economies.

Taking into account the considerations we have just made, we compute optimal monetary policy rate for the open economy model, with and without endogenous financial crisis. The optimal rates are equal to 2.18% and 2.24% on an annual basis, respectively, and in both cases smaller than in the corresponding closed economy case. Note that, just as in the closed economy and as in Ajello et al. (2015), the adjustment due to endogenous financial crisis is small. Our main point here, however, is not quantitative, but qualitative. Introducing endogenous crises in the closed economy model implies that interest rates must react more strongly to demand shocks, while the opposite is true in an open economy. Equally important, the next section shows that the effect is quantitatively stronger when the policy-maker is uncertain about the value of some parameters, i.e., mimicking the result obtained by Ajello et al. (2015) for the case of a closed economy.

Figure 3 investigates the sensitivity of optimal monetary policy to lagged credit conditions in the open economy, $b_0 \in [0,0.5]$. Unlike in the closed economy model, higher inherited credit levels reduce the optimal interest rate in the open economy model. With credit 50% above its normal level, the policy rate is now set around 5 basis points lower on an annual basis. The lower policy rate implies that the policy maker in an open economy has to accept higher output and inflation to reduce crisis probability when credit levels are relatively high.

The Appendix reports additional figures exploring the sensitivity of optimal monetary policy to different values of openness γ, the elasticity of exports ν, the ratio between the elasticities of intertemporal substitution of the two agent types σ^s/σ^b, price stickiness χ_p and the elasticity of substitution among home goods, ζ. For instance, these figures show that the optimal interest rate falls with γ, emphasizing the intuition that openness generates channels through which the incentives for increasing this rate are reduced, i.e., when $\gamma = 0.3$, as in the baseline calibration, the optimal policy rate is close to 2% and it falls to less than 1% for degrees of openness larger than 100%, which are obtained for $\gamma > 0.5$ (see Appendix for the remaining exercises).[15]

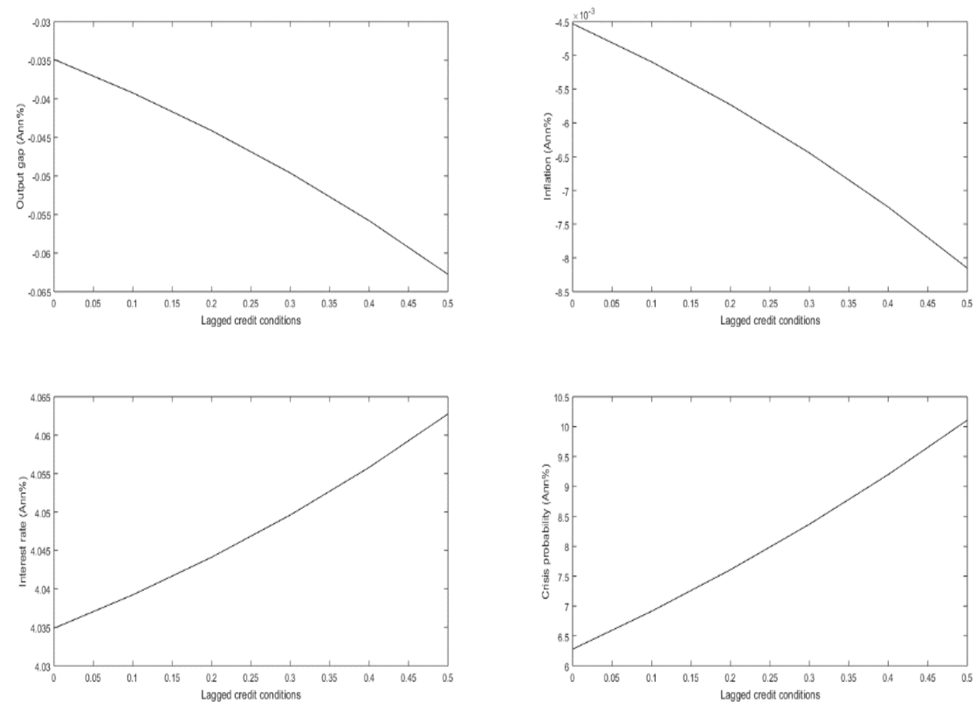

Figure 2 – Optimal values as a function of lagged credit conditions in the closed economy model

[15] Different values of χ_p change the slope of the Phillips curve. As is evident from Figure 10, apart from more flexible prices and a close to vertical Phillips curve, results are almost unaffected by χ_p. Even for very small values of χ_p the optimal policy rate is smaller than in the closed economy model and, as we verified in a separate experiment, smaller than in the open economy model with exogenous systemic risk. A higher ζ corresponds to a lower mark-up and lower profits for domestic firms. From Figure 11 – Optimal values for different values of ζ, it is easy to see that this is almost irrelevant for results. Also the effect of different ratios between the elasticities of intertemporal substitution of the two agent types on optimal policy is negligible (Figure 9).

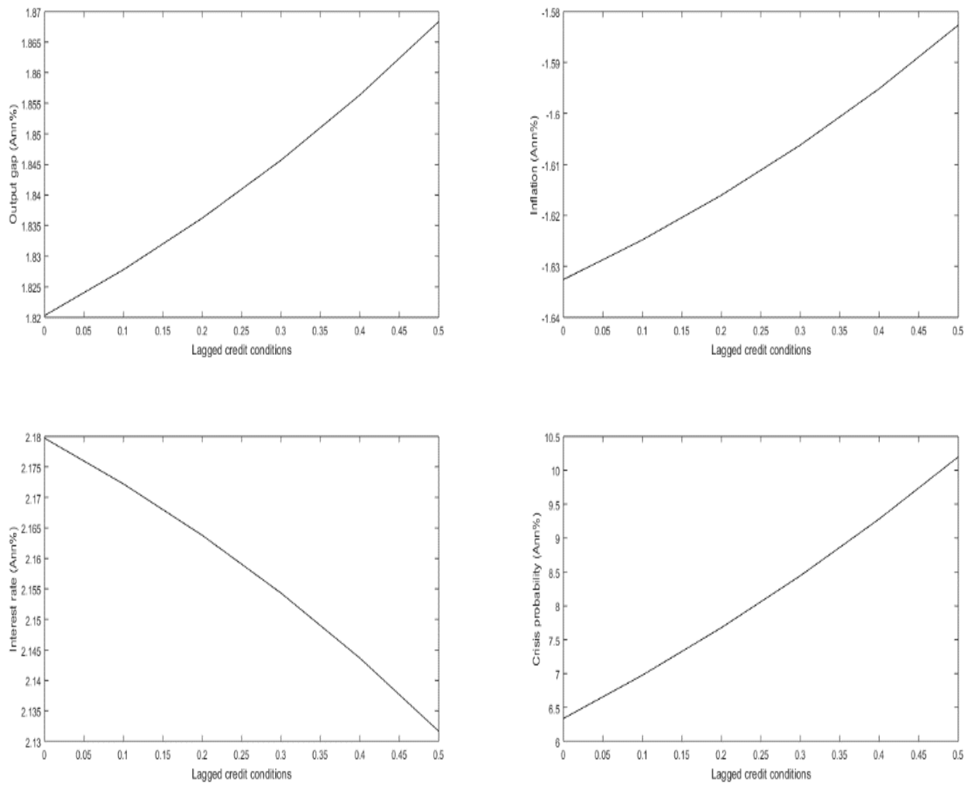

Figure 3 - Optimal values as a function of lagged credit conditions in the open economy model

5. Optimal Policy under Parameter Uncertainty

This section follows Ajello et al (2015) and highlights how the results are affected by the uncertainty of the policy-maker on the value of some of parameters. In particular, we follow two approaches. We first assume that the policy-maker is Bayesian as in Brainard (1967) and then consider the optimal policy as decided by a robust policy-maker, defined as in Hansen and Sargent (2008).

We consider uncertainty about the elasticity of crisis probability to credit conditions, κ, the severity of the crisis, that is the increase in inflation and the fall in output that take place in the crisis state, the export elasticity, ν and (half) the degree of openness, γ. We also follow Ajello et al (2015) in the modelling of uncertainty about these parameters. For each generic parameter h subject to uncertainty, if the policy-maker is Bayesian we assume that the parameter follows a discrete uniform distribution that takes values $h_{\min}, h_{base}$ and $h_{\max}$. h_{base} is at the same time the expected value of h and the value of the parameter in the baseline calibration. When the policy-maker is robust he considers h in the closed interval $[h_{\min}, h_{max}]$. In the case of the uncertain severity of the crisis, uncertainty about output and inflation is jointly analyzed and structured as just described.

Parameter	Value	Probability
Uncertain elasticity to credit conditions		
κ_{min}	0.0225	1/3
κ_{base}	1.1625	1/3
κ_{max}	2.3025	1/3
Uncertain severity of the crisis		
$\hat{y}_2(C)_{min}$	-3.4% (-13.6% annualized)	1/3
$\hat{y}_2(C)_{base}$	-2.4% (-9.6% annualized)	1/3
$\hat{y}_2(C)_{max}$	-1.4% (-5.6% annualized)	1/3
$\hat{\pi}_2(C)_{min}$	8.1% (32.4% annualized)	1/3
$\hat{\pi}_2(C)_{base}$	9.1% (36.4% annualized)	1/3
$\hat{\pi}_2(C)_{max}$	10.1% (40.4% annualized)	1/3
Uncertain degree of openness		
γ_{min}	0.1	1/3
γ_{base}	0.3	1/3
γ_{max}	0.5	1/3

Table 2 – Distribution of parameter values

Bayesian policy-maker

The loss function of the Bayesian policy-maker can be written as follows:

$$L = E_{1,h}[L_1(\hat{\pi}_1, \hat{y}_1) + \beta L_2(\hat{\pi}_2, \hat{y}_2)]. \tag{14}$$

The notation $E_{1,h}$ denotes that the Bayesian policy-maker minimizes the expectations of the loss function taken with respect to the joint distribution of states and of the subset of uncertain parameters. Since the loss function is convex, even though the expected fall in output and increase in inflation are the same as under the model without uncertainty, the expected loss they entail is higher. In practice, the policy-maker is risk averse and this influences the optimal policy.

Robust policy-maker

The loss function of the robust policy-maker can be written as follows:

$$L = \min\left[max_{h\in[h_{min},h_{max}]}\left(L_1(\hat{\pi}_1,\hat{y}_1) + \beta L_2(\hat{\pi}_2,\hat{y}_2)\right)\right] \tag{15}$$

The usual interpretation of the min-max formulation is that there is an evil agent inside the head of the policy-maker that chooses parameter h in the interval $[h_{min}, h_{max}]$ to maximize the loss. The policy-maker then chooses the policy rate that minimizes the loss when the parameter value is the one chosen by the evil agent., i.e., the value under which welfare is minimized.

5.1 Uncertain elasticity of crisis probability to credit conditions

Figure 4 shows that uncertainty on the elasticity of the crisis probability to credit conditions, κ reduces the optimal policy rate for both the Bayesian and the robust policy-maker and for all initial credit conditions. When the policy-maker is uncertain on the value of κ, she fears that the effect of credit on crisis probability may be higher than in her baseline guess.[16] The reduction of the optimal policy rate is greater when the policy-maker is robust because in that case the worst case scenario of the highest possible elasticity of crisis probability to credit is taken into account. With credit 50% above its average level, the robust policy-maker reduces the policy rate by around 10 basis points annually and the Bayesian policy maker by around six basis points with respect to the model without parameter uncertainty. Notice that the result we obtain is opposite to that of Ajello et al (2015). In a closed economy in fact, uncertainty about the elasticity of crisis probability to credit induces the policy-maker to increase the interest rate, rather than reduce it.

[16] In fact, the policy-maker is risk averse. In the Bayesian case, the policy-maker wants to minimize the expected value of a convex (quadratic) function. In the robust case, she simply takes into account the worst case scenario.

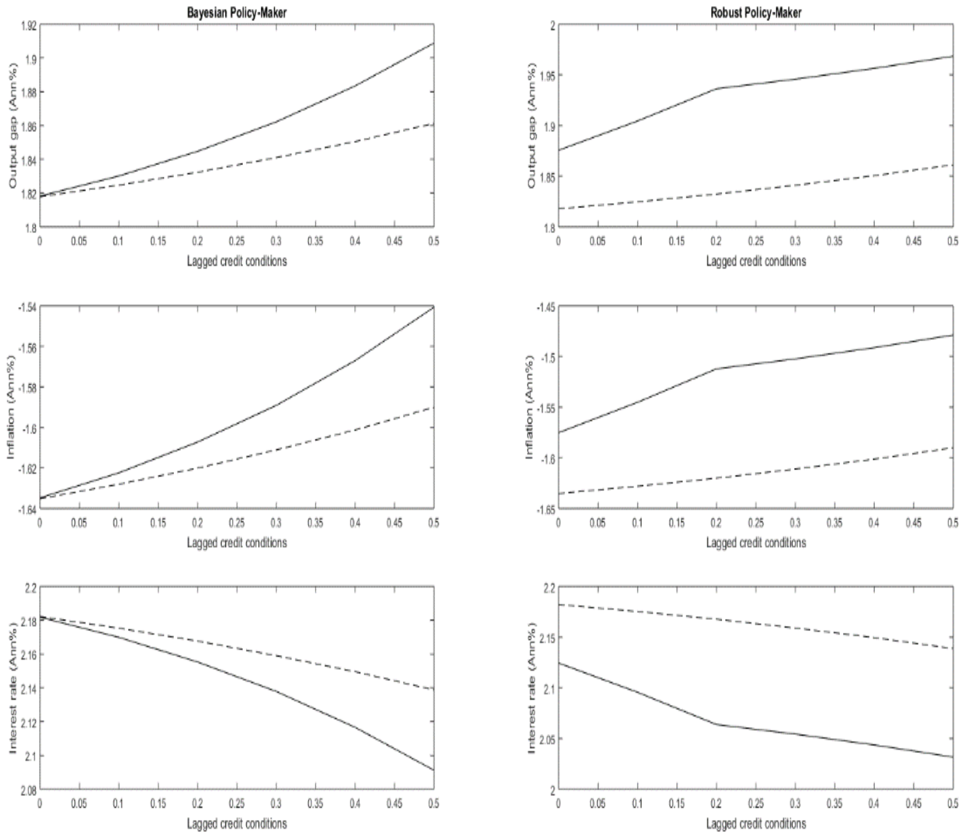

Figure 4 – Uncertain elasticity to credit conditions. Full line: optimal values under uncertainty. Dotted line: optimal values without uncertainty

5.2 Uncertain Severity of the Crisis

Figure 5 shows that, in the presence of an uncertain severity of the crisis, the results are similar to those obtained for uncertain elasticities of crisis probability to credit. Also in this case, both the Bayesian and the robust policy-maker set the policy rate lower than what they do in the absence of uncertainty because they fear the crisis could be worse than expected. The robust policy-maker reduces the policy rate more than what does the Bayesian policy-maker: the first reduces it by around 10 basis points and the second by around 8 basis points.

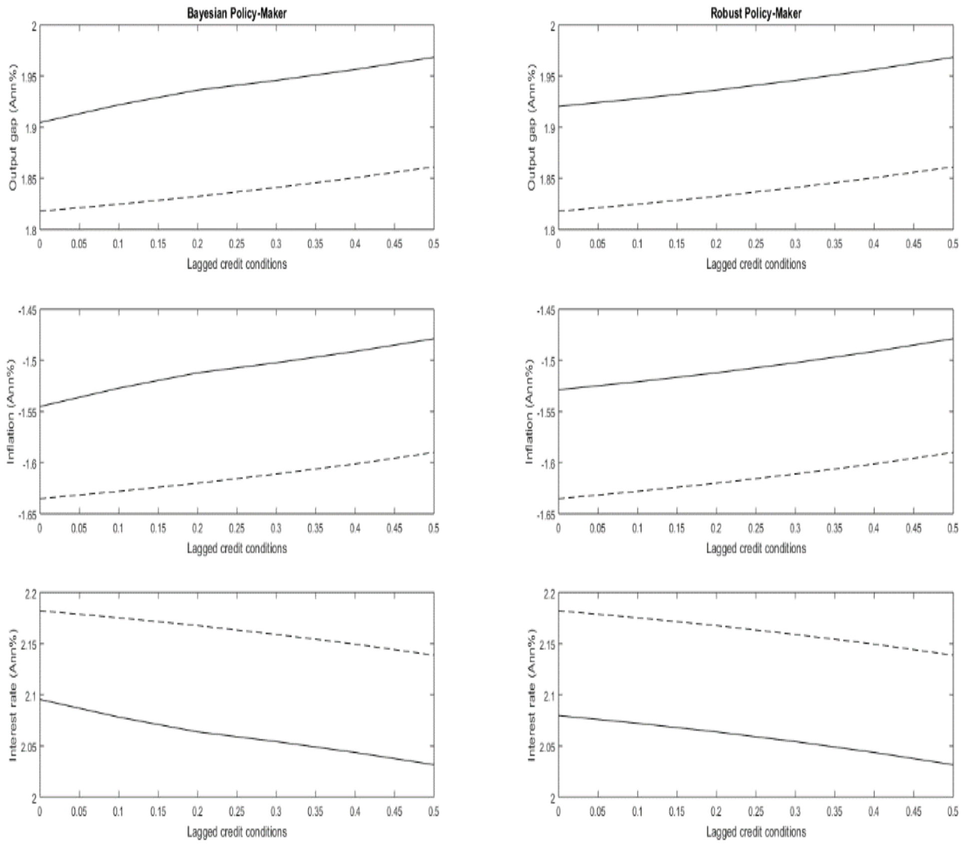

Figure 5 - Uncertain severity of the crisis. Full line: optimal values under uncertainty. Dotted line: optimal values without uncertainty.

6. Conclusions

The financial turmoil of 2007-2008 has renewed policy-makers' and scholars' interest in the relationship between monetary policy and financial stability. According to the "lean against the wind" view, central banks should take into account financial stability concerns when taking monetary policy decisions. In particular, central banks should increase interest rates in the expansionary phases of the financial cycle to reduce the accumulation of financial imbalances and to dampen systemic risk. Ajello et al (2015) show that this is the case in a formal model, even though the adjustment of monetary policy due to financial concerns is quantitatively small.

Most of the literature on the relationship between monetary policy and financial stability has concentrated on advanced economies. However, there are good reasons to think that such relationship is quite different in small open economies subject to important fluctuations in capital flows. In fact, higher interest rates can end up attracting foreign capital and, through this channel, increasing credit availability, leverage and systemic risk. In this paper, we have extended the framework analyzed by

Ajello (2015) to a small open economy and, calibrating the model for Mexico, we have obtained that central banks in such economies should reduce rather than increase interest rates during the expansionary phase of the financial cycle. Our results suggest that a “leaning against the wind” policy is not suited for small open economies and may end up worsening rather than improving financial stability.

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Appendix

The model

In this section of the appendix, we describe the model more in detail. The world is divided in two countries, Home and Foreign. The population size of country Home is n, while the population size of country Foreign is $1-n$. n is assumed to be close to zero, so that country Home is a small open economy.

The Home economy is modelled following Curdia and Woodford (2016). There are two types of domestic financial assets in which Home households can trade. First, households can sign state-contingent contracts that insure them against both aggregate and idiosyncratic risk. However, they can only receive transfers from the insurance agency intermittently, with probability $1-\delta$. At all other points in time, i.e. with probability δ, they can only trade one-period credit contracts. In this sense, domestic financial markets are incomplete.

A market for credit contracts exists because households in the Home economy are of two types according to the parameterization of their utility function, which allows to ensure that some households want to lend and others want to borrow in domestic financial markets, in a way that is better specified below. Following Curdia and Woodford (2016), we assume that only when they have access to insurance markets, households face a positive probability of switching type, i.e. they can pass from being a borrower to being a saver, or viceversa. Curdia and Woodford (2016) show that under the assumption that initial wealth levels are the same for all households, the optimal insurance contract ensures that all households able to receive transfers from the insurance company at time t will begin time $t+1$ with the same wealth level. This allows to limit heterogeneity to two household types, without tracking the whole wealth distribution. Notice that, contrary to Curdia and Woodford (2016), we assume that there is no friction in domestic financial intermediation, i.e. the interest rate that borrowers pay on the credit contract is equal to the interest rate that savers receive.

The type of household i at time t is identified by the symbol τ_t^i which can be either s, for savers, or b for borrowers. In country Home, borrowers represent a share π^b of the population and savers a share $1-\pi^b$. Apart from being able to lend in domestic financial markets, savers also have access to global financial markets. Such markets are also assumed to be incomplete, in that a single bond denominated in foreign currency is traded. Borrowers are assumed to have no access to global financial markets.

Households supply labor and receive profits from domestic intermediate firms.[17] Using these incomes, and inherited wealth, households make their consumption and saving decisions. Both household types choose how to allocate consumption among domestic and foreign goods. Savers choose how to allocate their savings among domestic financial markets, in which they lend to borrowers, and global financial markets, in which they lend or borrow by trading the foreign currency bond.

In order to illustrate the maximization problem of domestic households it is useful to first show formally how their wealth evolves over time. Beginning of period real wealth for a generic household i, W_t^i, is:

$$W_t^i = R_{t-1}\frac{B_{t-1}^i}{\pi_t} + S_t^i + \frac{R_{t-1}^f B_{t-1}^{fi} X_t}{X_{t-1}},$$

where R is the nominal domestic interest rate, B^i is the real value of domestic credit inherited from the past, π is the inflation rate, S^i is the transfer that the household receives from the insurance agency, R^f is the interest rate on the foreign currency bond, B^f is the real value of the foreign currency bonds inherited from the past, and X is the real exchange rate. Of course, both B^i and S^i can be either positive or negative. However, S^i is equal to zero for all households that do not access insurance markets at time t, while it is different from zero for the ones that access them. B^f is equal to zero for households that were borrowers in period $t-1$. Household i's end of period assets, $B^i + B^f$ is:

$$B_t^i + B_t^{fi} = W_t^i + w_t l_t^i + D_t - c_t^i - T_t.$$

where w is the real wage, l^i are worked hours, D are real (per capita) firm profits, c^i is consumption, and T are (real) lump-sum taxes.

Given the description of wealth accumulation described above, we can write the problem of domestic household i as follows:

$$max\tilde{E}_0 \sum_{t=1}^{\infty} \beta^{t-1}\left[\frac{C^{\tau_t^i}}{1-\sigma^{\tau_t^i}}\left(c_t^i\right)^{1-\sigma^{\tau_t^i}} - \frac{\chi^{\tau_t^i}}{2}\left(l_t^i\right)^2\right],$$

subject to

[17] We assume that labor can be supplied only to domestic firms, i.e. it is immobile across countries. Further, domestic firms are owned by domestic households, i.e. stock markets are country specific and there is no cross-country ownership.

$$B_t^i + B_t^{fi} = R_{t-1}\frac{B_{t-1}^i}{\pi_t} + S_t^i + \frac{R_{t-1}^f B_{t-1}^{fi} X_t}{X_{t-1}} + w_t l_t^i + D_t - c_t^i - T_t. \tag{16}$$

Borrowers, i.e. type $\tau_t = b$ households, also have to respect the following constraint:

$$B_t^{fi} = 0.$$

In the above, utility is increasing in consumption, c, and decreasing in worked hours, l. Parameter $\sigma^{\tau_t^i}$ is the inverse of the intertemporal elasticity of substitution, $C^{\tau_t^i}$ is a parameter that affects taste for current consumption, and $\chi^{\tau_t^i}$ governs taste for leisure. Borrowers are assumed to have a higher taste for current consumption and a higher elasticity of intertemporal substitution than savers. Taste for leisure is assumed to differ between agent types only to ensure that they work the same amount of time in steady state, as in Curdia and Woodford (2016). Indexation by τ_t indicates that these are the parameters that can change value over time for each agent i.

It is important at this point to define total domestic credit, b, as follows:

$$b_t = \int_{\pi^b}^{1} B_t^i di = -\int_0^{\pi^b} B_t^i di\,; \tag{17}$$

and the total amount of foreign assets held by savers as:

$$b_t^f = \int_{\pi^b}^{1} B_t^{fi} di. \tag{18}$$

Given the ability of households to sign insurance contracts, Curdia and Woodford (2016) show that all savers hold the same amount of wealth and all borrowers issue the same amount of debt, which implies that:

$$\frac{b_t}{1-\pi^b} = B_t^s; \frac{b_t}{\pi^b} = -B_t^b;$$

and

$$\frac{b_t^f}{1-\pi^b} = B_t^{fs}.$$

The optimality conditions of the problem of households are the following:

$$\lambda_t^i = C^{\tau_t^i} {c_1^i}^{-\sigma^{\tau_t^i}};$$

$$w_t = \frac{\chi^{\tau_t^i} l_t^i}{\lambda_t^i};$$

$$\frac{1}{R_t} = \beta \tilde{E}_t \frac{\lambda_{t+1}^i}{\lambda_t^i \pi_{t+1}}. \tag{19}$$

Savers also optimize with respect to foreign asset holdings, which gives rise to the following additional first order condition:

$$\frac{1}{R_t^f} = \beta \tilde{E}_t \left(\frac{\frac{\lambda_{t+1}^i}{\lambda_t^s} X_{t+1}}{X_t} \right). \tag{20}$$

Under the assumption that initial wealth levels are the same for all households, Curdia and Woodford (2016) show that

$$\tilde{E}_t \lambda_{t+1}^i = \tilde{E}_t \{ [\delta + (1-\delta)\pi^b] \lambda_{t+1}^b + (1-\delta)(1-\pi^b) \lambda_{t+1}^s \}$$

for households that are borrowers at time t and

$$\tilde{E}_t \lambda_{t+1}^i = \tilde{E}_t \{ [\delta + (1-\delta)(1-\pi^b)] \lambda_{t+1}^s + (1-\delta)\pi^b \lambda_{t+1}^b \},$$

for households that are savers at time t. Hence, equation (19) can be rewritten as

$$\frac{1}{R_t} = \beta \tilde{E}_t \frac{[\delta + (1-\delta)\pi^b] \lambda_{t+1}^b + (1-\delta)(1-\pi^b) \lambda_{t+1}^s}{\lambda_t^b \pi_{t+1}} \tag{21}$$

for borrowers and as

$$\frac{1}{R_t} = \beta \tilde{E}_t \frac{[\delta + (1-\delta)(1-\pi^b)] \lambda_{t+1}^s + (1-\delta)\pi^b \lambda_{t+1}^b}{\lambda_t^s \pi_{t+1}} \tag{22}$$

for savers. Equation (20) becomes:

$$\frac{1}{R_t^f} = \beta \tilde{E}_t \frac{[\delta + (1-\delta)(1-\pi^b)]\lambda_{t+1}^s + (1-\delta)\pi^b \lambda_{t+1}^b}{\lambda_t^s} X_{t+1}/X_t. \quad (23)$$

Equations (21) and (22) clarify why it is necessary to have time-varying types. As $C^s \leq C^b$ and $\frac{1}{\sigma^s} \leq \frac{1}{\sigma^b}$, type $\tau_{\mathrm{t}} = b$ agents know that they may like consumption less in the future, while type $\tau_{\mathrm{t}} = s$ know that they may like it more.[18] Due to this, in equilibrium, type $\tau_{\mathrm{t}} = b$ agents are borrowers, and type $\tau_{\mathrm{t}} = s$ are savers at time one.

Households choose how to allocate consumption between domestic and foreign goods. The consumption basket for all domestic households is the following:

$$c_t^i = \left[(1-\gamma)^{\frac{1}{\eta}} {c_{H,t}^i}^{\frac{\eta-1}{\eta}} + \gamma^{\frac{1}{\eta}} {c_{F,t}^i}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}.$$

Notice that $\gamma = (1-n)\alpha$, where α is the degree of home bias. In a small open economy n is close to zero, so $\gamma = \alpha$ in the limit. If domestic households are not interested in consuming foreign goods, which happens for $\gamma = 0$, the model collapses to a closed economy framework. On the contrary, if γ is close to one, the home economy is completely open, in the sense that home produced goods represent a negligible share of the households' consumption basket. η is the elasticity of substitution between domestic and foreign goods and it is set to one in the main text, which implies that the consumption basket is Cobb-Douglas. Total expenditure has to satisfy the following equation:

$$c_t^i = p_{H,t} c_{H,t}^i + X_t c_{F,t}^i \quad (24)$$

and the optimal allocation implies the following conditions:

$$c_{H,t}^i = (1-\gamma)\left(p_{H,t}\right)^{-\eta} c_t^i \quad (25)$$

and

$$c_{F,t}^i = \gamma (X_t)^{-\eta} c_t^i; \quad (26)$$

where p_H is the real price of the domestic good, i.e. the ratio between the price of the domestically produced good and the price of the consumption basket.

[18] Notice that a different taste for current consumption would be sufficient to ensure this result even if all types had the same elasticity of intertemporal substitution. We allow for different elasticities of intertemporal substitution because, as explained by Curdia and Woodford (2016), this allows to capture the fact that reductions in the interest rate imply higher credit levels.

Exports are assumed to be governed by the following equation:

$$EXP_t = \gamma \left(\frac{p_{H,t}}{X_t}\right)^{-\nu}; \tag{27}$$

where ν is the elasticity of exports to the terms of trade. Equation (27) can be obtained by assuming that also the foreign consumption basket is CES.

The firm sector is as in the standard New Keynesian model. There are intermediate firms that produce differentiated goods, indexed by j, under monopolistic competition and final good firms that aggregate such goods to produce the domestic final consumption good, y_H. As is well known, this is equivalent to assuming that households consume a basket of differentiated goods directly. Owners of domestic firms are borrowers and savers in equal shares, which implies that a share $1-\pi^b$ of firms is owned by savers and a share π^b is owned by borrowers.

Intermediate firms choose the price of their good, labor demand, and production levels. Final good firms choose the amount of each intermediate good to buy and the amount of final good to produce. They take the price of the final good as given, due to perfect competition.

The production function of final good firms is a standard Dixit-Stiglitz aggregator:

$$y_{H,t} = \left[\int_0^1 y_{j,t}^{\frac{\zeta-1}{\zeta}} dj\right]^{\frac{\zeta}{\zeta-1}}$$

where ζ is the elasticity of substitution among differentiated goods. Profit maximization on the part of final good firms gives rise to the following demand for the differentiated good j

$$y_{j,t} = \left(\frac{p_{j,t}}{p_{H,t}}\right)^{-\zeta} y_{H,t} \tag{28}$$

where p_j is the real price of good j and p_H was defined above. The problem of intermediate good firm j is to choose $p_{j,t}$, $y_{j,t}$ and $l_{j,t}$ to:

$$\max \tilde{E}_0 \sum_{t=1}^{\infty} \Omega_{1,t}\left[p_{j,t}y_{j,t} - (1+\tau)w_t l_{j,t} - \frac{\chi_P}{2}\left(\frac{p_{j,t}}{p_{j,t-1}}\pi_t - 1\right)^2 p_{H,t}y_{H,t}\right] \tag{29}$$

subject to

$$y_{j,t} = l_{j,t} \tag{30}$$

and to equation (28). The term $\frac{\chi_P}{2}\left(\frac{p_{j,t}}{p_{j,t-1}}\pi_t - 1\right)^2 p_{H,t}y_{H,t}$ represents a quadratic Rotemberg price adjustment cost, expressed in real terms. Ω is a stochastic discount factor, whose form is assumed to be $\Omega_{t,t+1} = \frac{\beta\left((1-\pi^b)\lambda_{t+1}^s+\pi^b\lambda_{t+1}^b\right)}{(1-\pi^b)\lambda_t^s+\pi^b\lambda_t^b}$: as firms are owned by both savers and borrowers in shares equal to their proportion over the Home population, an average of their marginal utilities is employed to discount profits. We assume that firms receive an employment subsidy, whose rate is τ, which is financed through lump-sum taxes on households and which eliminates the distortionary effect of monopolistic competition on production at the approximation point.[19]

After substituting equation (28) in (29) and in (30), and optimizing with respect to $p_{j,t}$ and to $l_{j,t}$ one gets:

$$(1-\zeta)\frac{p_{j,t}^{1-\zeta}}{p_{H,t}^{-\zeta}} + \mu_t\zeta\frac{p_{j,t}^{-\zeta}}{p_{H,t}^{-\zeta}} - \chi_p\left(\frac{p_{j,t}}{p_{j,t-1}}\pi_t - 1\right)\frac{p_{j,t}}{p_{j,t-1}}\pi_t \\ + \tilde{E}_t\frac{\Omega_{t,t+1}\left(\frac{p_{j,t+1}}{p_{j,t}}\pi_{t+1} - 1\right)\frac{p_{j,t+1}}{p_{j,t}}\pi_{t+1}\left(p_{H,t+1}y_{H,t+1}\right)}{p_{H,t}y_{H,t}} = 0$$

and

$$(1+\tau)w_t = \mu_t;$$

where μ is the Lagrange multiplier on constraint (30). In a symmetric equilibrium, all firms set the same price and hire the same amount of labor. Combining the two optimality conditions, and taking into account that the subsidy is set in a way that eliminates the effect of monopolistic competition on labor demand, i.e. $\tau = \frac{\zeta-1}{\zeta} - 1$, one obtains:

$$w_t = p_{H,t} + \frac{\chi_P}{\zeta-1}(\pi_{H,t}-1)\pi_{H,t} \\ - \frac{\chi_P}{\zeta-1}\tilde{E}_t\Omega_{t,t+1}\left[\frac{(\pi_{H,t+1}-1)\pi_{H,t+1}p_{H,t+1}y_{H,t+1}}{p_{H,t}y_{H,t}}\right] \tag{31}$$

[19] This is commonly assumed in the New-Keynesian literature, as for example in Rotemberg and Woodford (1998). This assumption allows to obtain an efficient steady state in which the adverse effect of monopolistic competition on employment is absent. The latter result helps when computing the optimal policy with a micro-founded loss function. Even if we do not need this assumption here, we keep it to remain closer to the literature on optimal monetary policy.

In equation (31), $\pi_{H,t} = p_{H,t}/p_{H,t-1}\pi_t$ is domestic price inflation. In the absence of sticky prices, (31) would collapse to $w_t = p_{H,t}$, i.e. the real wage (which is also equal to real marginal costs) is equal to the real price. When prices are sticky, a positive shock to demand, which puts upward pressure on output and inflation, increases the real wage above the real price, because firms find it difficult to raise their price. Instead, if they expect higher demand, and hence higher inflation, tomorrow, they start increasing their price today above the real wage, in order to smooth costly price adjustment over time. Aggregation also implies that:

$$y_{H,t} = l_t = (1-\pi^b)l_t^s + \pi^b l_t^b. \tag{32}$$

Total firm profits are

$$D_t = p_{H,t}y_{H,t} - (1+\tau)w_t l_t - \frac{\chi_P}{2}\left(\pi_{H,t}-1\right)^2 p_{H,t}y_{H,t}. \tag{33}$$

In both periods, the home final good market clearing condition implies that:

$$y_{H,t} = \pi^b c_{H,t}^b + (1-\pi^b)c_{H,t}^s + EXP_t + \frac{\chi_P}{2}\left(\pi_{H,t}-1\right)^2 y_{H,t}. \tag{34}$$

Multiplying equation (34) on both sides by p_H and taking into account equation (24) one gets

$$\begin{aligned} p_{H,t}y_{H,t} = \pi^b c_t^b + (1-\pi^b)c_t^s + p_{H,t}EXP_t - X_t\left[\pi^b c_{F,t}^b + (1-\pi^b)c_{F,t}^s\right] \\ + \frac{\chi_P}{2}\left(\pi_{H,t}-1\right)^2 p_{H,t}y_{H,t}. \end{aligned} \tag{35}$$

Equation (35) is the standard aggregate resource constraint that states that home gross domestic product, $p_H y_H$, is equal to consumption, $\pi^b c^b + \left(1-\pi^b\right)c^s$, plus exports, $p_H EXP$, minus imports, $X\left[\pi^b c_F^b + \left(1-\pi^b\right)c_F^s\right]$, all expressed in real terms. The equation is corrected for the presence of price adjustment costs. For later use, we define real GDP, y, as follows:

$$y_t = p_{H,t}y_{H,t}.$$

Using equations (16), (17), (18), and (35) one can show that:

$$p_{H,t}EXP_t - X_t\left[\pi^b c_{F,t}^b + (1-\pi^b)c_{F,t}^s\right] + \frac{\left(R_{t-1}^f - 1\right)X_t}{X_{t-1}} b_{t-1}^f \\ = b_t^f - X_t/X_{t-1} b_{t-1}^f. \tag{36}$$

Equation (36) states that the current account, given by the sum of the trade balance, $p_{H,t}EXP_t - X_t\left[\pi^b c_{F,t}^b + \left(1-\pi^b\right)c_{F,t}^s\right]$, and net interest income from abroad, $\frac{\left(R_{t-1}^f - 1\right)X_t}{X_{t-1}} b_{t-1}^f$, must equal inverse capital flows, given by the change in the value of international bonds held by domestic households, $b_t^f - X_t/X_{t-1} b_{t-1}^f$.

Now, we describe the aggregate demand block of the economy, which will be used to obtain the log-linear IS curve. It is composed of the following equations:

$$\lambda_t^b = C^b c_t^{b^{-\sigma^b}};$$

$$\frac{1}{R_t} = \beta \tilde{E}_t \frac{[\delta + (1-\delta)\pi^b]\lambda_{t+1}^b + (1-\delta)(1-\pi^b)\lambda_{t+1}^s}{\lambda_t^b \pi_{t+1}};$$

$$\lambda_t^s = C^s c_t^{s^{-\sigma^s}}$$

$$\frac{1}{R_t} = \beta \tilde{E}_t \frac{[\delta + (1-\delta)(1-\pi^b)]\lambda_{t+1}^s + (1-\delta)\pi^b \lambda_{t+1}^b}{\lambda_t^s \pi_{t+1}};$$

which are present in both the general (open economy) version of the model and in the closed economy version and which determine how much households value current versus future consumption. The following equations instead are present only in the open economy model:

$$c_{H,t}^i = (1-\gamma)\left(p_{H,t}\right)^{-\eta} c_t^i$$

$$c_{F,t}^i = \gamma (X_t)^{-\eta} c_t^i;$$

$$EXP_t = \gamma \left(\frac{p_{H,t}}{X_t}\right)^{-\nu};$$

and allow to obtain the net-exports contribution to aggregate demand. Finally, the aggregate demand block of the economy is closed by the aggregate resource constraint:

$$p_{H,t}y_{H,t} = \pi^b c_t^b + (1-\pi^b)c_t^s + p_{H,t}EXP_t - X_t\left[\pi^b c_{F,t}^b + (1-\pi^b)c_{F,t}^s\right] \\ + \frac{\chi_P}{2}\left(\pi_{H,t} - 1\right)^2 p_{H,t}y_{H,t};$$

which in a closed economy takes the form $y_t = \pi^b c_t^b + \left(1-\pi^b\right)c_t^s + \frac{\chi_P}{2}(\pi_t - 1)^2 y_t$.

The aggregate supply block of the economy, and as a consequence the Phillips curve, can be obtained from the following equations:

$$w_t = \frac{\chi^b l_t^b}{\lambda_t^b} \tag{37}$$

$$w_t = \frac{\chi^s l_t^s}{\lambda_t^s} \tag{38}$$

$$y_{H,t} = l_t = \left(1-\pi^b\right)l_t^s + \pi^b l_t^b$$

which determine the labor supply of households and hence output, and by

$$w_t = p_{H,t} + \frac{\chi_P}{\zeta-1}\left(\pi_{H,t}-1\right)\pi_{H,t} - \frac{\chi_P}{\zeta-1}\tilde{E}_t\Omega_{t,t+1}\left[\frac{\left(\pi_{H,t+1}-1\right)\pi_{H,t+1}p_{H,t+1}y_{H,t+1}}{p_{H,t}y_{H,t}}\right],$$

which governs the evolution of price mark-ups. In a closed economy the latter equation becomes:

$$w_t = 1 + \frac{\chi_P}{\zeta-1}(\pi_t - 1)\pi_t - \frac{\chi_P}{\zeta-1}\tilde{E}_t\Omega_{t,t+1}\left[\frac{(\pi_{t+1}-1)\pi_{t+1}y_{t+1}}{y_t}\right].$$

In an open economy, it is necessary to add a global financial market block, which is:

$$\frac{1}{R_t^f} = \beta\tilde{E}_t\frac{[\delta + (1-\delta)(1-\pi^b)]\lambda_{t+1}^s + (1-\delta)\pi^b\lambda_{t+1}^b}{\lambda_t^s}X_{t+1}/X_t$$

and which will give rise to an uncovered interest parity condition when log-linearized.

When considering the model versions with endogenous crisis probability, it is also necessary to add a credit accumulation block. The latter is described by

$$c_t^s + \frac{b_t}{(1-\pi^b)} - \delta\frac{\frac{R_{t-1}}{\pi_t}b_{t-1}}{(1-\pi^b)} + \frac{b_t^f}{(1-\pi^b)} - \frac{1-\pi^b+\delta\pi^b}{1-\pi^b}\frac{R_{t-1}^f b_{t-1}^f X_t}{X_{t-1}} = w_t l_t^s + D_t - T_t,$$

which is obtained by summing equation (16) over the interval $(\pi^b, 1]$ and where c^s is the average consumption of savers, and by

$$p_{H,t}EXP_t - X_t\left[\pi^b c_{F,t}^b + \left(1-\pi^b\right)c_{F,t}^s\right] + \frac{\left(R_{t-1}^f - 1\right)X_t}{X_{t-1}} b_{t-1}^f = b_t^f - \frac{X_t}{X_{t-1}} b_{t-1}^f.$$

These equations describe how credit evolves using the average consumption of savers and equality between the current account and capital flows. In a closed economy, the average consumption of savers is sufficient, and has the following form:

$$c_t^s + \frac{b_t}{(1-\pi^b)} - \delta \frac{\frac{R_{t-1}}{\pi_t} b_{t-1}}{(1-\pi^b)} = w_t l_t^s + D_t - T_t.$$

Steady state

The model described in the previous section is non-linear. In order to log-linearize it and obtain a form for the open economy model that is analogous to that employed by Ajello et al (2015), we first have to compute its steady state.

As we show below, the values of the parameters governing taste for leisure, χ^b and χ^s, can be set in such a way to make sure that at the approximation point, $\bar{l}_1^b = \bar{l}_1^s = 1$.[20] From the production function (32), we obtain that

$$\bar{y}_H = \bar{l}^s = \bar{l}^b = 1. \tag{39}$$

Assuming that $\bar{b}^f = 0$, i.e. that steady state foreign assets are equal to zero, and using equation (36), one can show that

$$\bar{p}_H \overline{EXP} = \bar{X}\bar{c}_F = \bar{X}\left(\pi^b \bar{c}_F^b + \left(1-\pi^b\right)\bar{c}_F^s\right). \tag{40}$$

where $\bar{c}_F = \pi^b \bar{c}_F^b + \left(1-\pi^b\right)\bar{c}_F^s$ is the total amount of foreign good imported from abroad. We assume that inflation is zero at the steady state, i.e. $\bar{\pi} = 1$. As domestic price inflation is defined as $\pi_{H,t} = p_{H,t}/p_{H,t-1}\pi_t$, in the steady state also $\bar{\pi}_{H,1} = 1$. So, using (40) and equation (35), one can show that

$$\bar{p}_H = \bar{c} = \pi^b \bar{c}^b + \left(1-\pi^b\right)\bar{c}^s. \tag{41}$$

[20] From now on, variables at the approximation point are represented with a bar.

where $\bar{c} = \pi^b \bar{c}^b + (1 - \pi^b)\bar{c}^s$ is total consumption. Now, it is possible to combine (41) with (40), which after using (26) and (27) allows to obtain

$$\bar{X} = \bar{p}_H^{-\frac{\nu(1-\eta)}{1-\eta-\nu}}. \tag{42}$$

Substituting (25) and (26) in (24), one gets another relationship between the real exchange rate and the real domestic price:

$$1 = (1-\gamma)\bar{p}_H^{1-\eta} + \gamma \bar{X}^{1-\eta}. \tag{43}$$

The solution of the system of equations (42) and (43) is of course $\bar{p}_H = \bar{X} = 1$. Using equation (41) one obtains that

$$\bar{c} = 1. \tag{44}$$

In the main text, s^s and s^b are respectively defined as the ratios between the consumption of savers and aggregate consumption, and the ratio between the consumption of borrowers and aggregate consumption. Given (44), we get:

$$\bar{c}^s = s^s$$

and

$$\bar{c}^b = s^b.$$

Similarly, from equations (25), (26) and (27), one can obtain:

$$\bar{c}_H^s = (1-\gamma)s^s;$$

$$\bar{c}_F^s = \gamma s^s;$$

$$\bar{c}_H^b = (1-\gamma)s^b;$$

$$\bar{c}_F^b = \gamma s^b;$$

and

$$\overline{EXP} = \gamma.$$

Equation (31) can be used to obtain

$$\bar{w} = \bar{p}_H = 1.$$

As we calibrate the steady state value of domestic credit to $\bar{b}$, we then use the following system of equations

$$\frac{1}{\bar{R}} = \beta \frac{[\delta + (1-\delta)\pi^b]\bar{\lambda}^b + (1-\delta)\left(1-\pi^b\right)\bar{\lambda}^s}{\bar{\lambda}^b}$$

$$\frac{1}{\bar{\bar{R}}} = \beta \frac{[\delta + (1-\delta)(1-\pi^b)]\bar{\lambda}^s + (1-\delta)\pi^b\bar{\lambda}^b}{\bar{\lambda}^s};$$

to solve for $\bar{R}$ and for one parameter among C^b and C^s, while setting the other to one. In fact, only the ratio between the tastes for current consumption of borrowers and of savers matters for the determination of domestic credit levels. In practice, such ratio is set to ensure that the consumption levels of the two household types are equal to the parameterization given by s^s and s^b.

At this point, equations (37) and (38) can be used to obtain

$$\chi^b = C^b {s^b}^{-\sigma^b}$$

and

$$\chi^s = C^s {s^s}^{-\sigma^b}.$$

Log-Linearization

We log-linearize the model under the assumption that δ is close to one, as this allows to simplify the derivation of the log-linear system. Furthermore, given that our objective is only to use the micro-foundations as a means useful to build an open economy version of the framework employed by Ajello et al (2015), we want to capture the basic structural framework behind the open economy model and not take account of all its details. In fact, δ is calibrated to a number close to one in Curdia and Woodford (2016), which suggests that savers and borrowers switch types infrequently. Ignoring this switch in our two period version of the log-linear model is unlikely to be costly from the point of view of the numerical results and buys us a lot of clarity from the point of view of the intuition behind our log-linear relation.

The first step of the log-linearization is obtaining the IS curve from the aggregate demand block presented in the previous sections. To do so, we first consider equations

$$\lambda_t^b = C^b c_t^{b^{-\sigma^b}};$$

$$\lambda_t^s = C^s c_t^{s^{-\sigma^s}}.$$

Taking logs on both sides and subtracting the log of each variable at the approximation point, on obtains:

$$\hat{\lambda}_t^i = -\sigma^i \hat{c}_t^i, \tag{45}$$

where $i = b, s$ and where for any variable x we have $\hat{x} = \log(\frac{x}{\bar{x}})$. Then, consider the equations

$$\frac{1}{R_t} = \beta \tilde{E}_t \frac{[\delta + (1-\delta)(1-\pi^b)]\lambda_{t+1}^s + (1-\delta)\pi^b \lambda_{t+1}^b}{\lambda_t^s \pi_{t+1}}$$

$$\frac{1}{R_t} = \beta \tilde{E}_t \frac{[\delta + (1-\delta)\pi^b]\lambda_{t+1}^b + (1-\delta)\left(1-\pi^b\right)\lambda_{t+1}^s}{\lambda_t^b \pi_{t+1}};$$

Their log-linear versions, under the assumption that $\delta \to 1$, are:

$$-\hat{R}_t = \tilde{E}_1 \hat{\lambda}_{t+1}^s - \hat{\lambda}_t^s - \tilde{E}_t \hat{\pi}_{t+1}. \tag{46}$$

And

$$-\hat{R}_t = \tilde{E}_t \hat{\lambda}_{t+1}^b - \hat{\lambda}_t^b - \tilde{E}_1 \hat{\pi}_{t+1}. \tag{47}$$

Under the assumption that initial wealth levels are equal for all agents, equations (46) and (47) imply that the marginal utilities of the two agent types are always equal:

$$\hat{\lambda}_{\mathrm{t}}^{\mathrm{b}} = \hat{\lambda}_{\mathrm{t}}^{\mathrm{s}}. \tag{48}$$

Hence, we can drop the type index, and simply refer to $\hat{\lambda}$ as marginal utility. Given the latter fact, equation (46) can be re-written as follows:

$$\hat{\lambda}_t = \tilde{E}_t \hat{\lambda}_{t+1} + \hat{R}_t - \tilde{E}_t \hat{\pi}_{t+1}. \tag{49}$$

The next step is the log-linearization of the aggregate resource constraint (35), which after combining it with equations (26) and (27), can be re-written as:

$$y_t = (1-\pi^b)c_t^s + \pi^b c_t^b + \gamma p_{H,t}^{1-\nu} X_t^\nu - \gamma X_t^{1-\eta}\left((1-\pi^b)c_t^s + \pi^b c_t^b\right) + \frac{\chi_P}{2}(\pi_{H,t}-1)^2 y_t;$$

Now, we assume that the domestic consumption basket is Cobb-Douglas, i.e. $\eta = 1$, which allows to obtain:

$$y_t = (1-\gamma)\left((1-\pi^b)c_t^s + \pi^b c_t^b\right) + \gamma p_{H,t}^{1-\nu} X_t^\nu + \frac{\chi_P}{2}(\pi_{H,t}-1)^2 y_t.$$

The above equation can be log-linearized to obtain:

$$\hat{y}_t = (1-\gamma)\left(\pi^b s^b \hat{c}_t^b + (1-\pi^b)s^s \hat{c}_t^s\right) + \gamma\left((1-\nu)\hat{p}_{H,t} + \nu \hat{X}_t\right).$$

Consider now equation (43), which in log-linear terms can be written as $\hat{p}_{H,t} = -\frac{\gamma}{1-\gamma}\hat{X}_t$. Given this, the above equation can be written as follows

$$\hat{y}_t = (1-\gamma)\left(\pi^b s^b \hat{c}_t^b + (1-\pi^b)s^s \hat{c}_t^s\right) + \frac{\gamma(\nu-\gamma)}{1-\gamma}\hat{X}_t.$$

Then, using equations (45) and (48), we can write:

$$y_t = -(1-\gamma)\left(\frac{\pi^b s^b}{\sigma^b} + \frac{(1-\pi^b)s^s}{\sigma^s}\right)\hat{\lambda}_t + \frac{\gamma(\nu-\gamma)}{1-\gamma}\hat{X}_t. \tag{50}$$

So, defining the additional parameter:

$$\bar{\sigma} = \left[\frac{\pi^b s^b}{\sigma^b} + \frac{(1-\pi^b)s^s}{\sigma^s}\right],$$

equation (50) can be written as:

$$\hat{\lambda}_t = -\frac{1}{(1-\gamma)\bar{\sigma}}\hat{y}_t + \frac{1}{(1-\gamma)\bar{\sigma}}\frac{\gamma(\nu-\gamma)}{1-\gamma}\hat{X}_t. \tag{51}$$

Combining the above equation with (49), delivers

$$-\frac{1}{(1-\gamma)\bar{\sigma}}\hat{y}_t + \frac{1}{(1-\gamma)\bar{\sigma}}\frac{\gamma(\nu-\gamma)}{1-\gamma}\hat{X}_t$$
$$= \tilde{E}_1\left[-\frac{1}{(1-\gamma)\bar{\sigma}}\hat{y}_{t+1} + \frac{1}{(1-\gamma)\bar{\sigma}}\frac{\gamma(\nu-\gamma)}{1-\gamma}\hat{X}_{t+1}\right] + \hat{R}_t - \tilde{E}_1\hat{\pi}_{t+1}$$

Rearranging the equation above, we obtain the IS curve:

$$\hat{y}_t = \tilde{E}_1 \hat{y}_{t+1} - \frac{\gamma(\nu-\gamma)}{1-\gamma}\left(\tilde{E}_1 \hat{X}_{t+1} - \hat{X}_t\right) - (1-\gamma)\bar{\sigma}\left(\hat{R}_t - \tilde{E}_1 \hat{\pi}_{t+1}\right). \tag{52}$$

To obtain the closed economy version of (52), it is sufficient to set $\gamma = 0$:

$$\hat{y}_t = \tilde{E}_1 \hat{y}_{t+1} - \bar{\sigma}\left(\hat{R}_t - \tilde{E}_1 \hat{\pi}_{t+1}\right).$$

To obtain the Phillips curve, we now switch to the aggregate supply block of the model. It is useful to begin with the labor supplies:

$$w_t = \frac{\chi^b l_t^b}{\lambda_t^b}$$

$$w_t = \frac{\chi^s l_t^s}{\lambda_t^s}$$

which in log-linear terms are:

$$\hat{l}_t^i = \hat{\lambda}_t^i + \hat{w}_t; \tag{53}$$

where $i = b, s$. Next, consider the production function

$$y_{H,t} = l_t = \left(1-\pi^b\right) l_t^s + \pi^b l_t^b;$$

which in log-linear terms is:

$$\hat{y}_{H,t} = \hat{l}_t = \left(1-\pi^b\right)\hat{l}_t^s + \pi^b \hat{l}_t^b. \tag{54}$$

From equation (53) and (48), it is easy to see that $\hat{l}_t^b = \hat{l}_t^s$, and so we will drop the type index and refer to labor supply simply as $\hat{l}_t$. The following step is the log-linearization of the pricing equation:

$$w_t = p_{H,t} + \frac{\chi_P}{\zeta-1}\left(\pi_{H,t}-1\right)\pi_{H,t} - \frac{\chi_P}{\zeta-1}\tilde{E}_1 \Omega_{t,t+1}\left[\frac{\left(\pi_{H,t+1}-1\right)\pi_{H,t+1} p_{H,t+1} y_{H,t+1}}{p_{H,t} y_{H,t}}\right].$$

The above equation in log-linear terms is:

$$\hat{w}_t = \hat{p}_{H,t} + \frac{\chi_P}{\zeta-1}\hat{\pi}_{H,t} - \beta\frac{\chi_P}{\zeta-1}\tilde{E}_t \hat{\pi}_{H,t+1}.$$

Using (53) and (54), the latter equation can be re-written as:

$$\hat{y}_{H,t} - \hat{\lambda}_t = \hat{p}_{H,t} + \frac{\chi_P}{\zeta - 1}\hat{\pi}_{H,t} - \beta\frac{\chi_P}{\zeta - 1}\tilde{E}_t\hat{\pi}_{H,t+1}.$$

Using (51), adding $\hat{p}_{H,t}$ on both sides and taking into account that $\hat{p}_{H,t} = -\frac{\gamma}{1-\gamma}\hat{X}_t$, the above equation becomes:

$$\hat{y}_t + \frac{1}{(1-\gamma)\bar{\sigma}}\hat{y}_t - \frac{1}{(1-\gamma)\bar{\sigma}}\frac{\gamma(\nu-\gamma)}{1-\gamma}\hat{X}_t = -2\frac{\gamma}{1-\gamma}\hat{X}_t + \frac{\chi_P}{\zeta - 1}\hat{\pi}_{H,t} - \beta\frac{\chi_P}{\zeta - 1}\tilde{E}_t\hat{\pi}_{H,t+1}.$$

Rearranging it, one obtains:

$$\hat{\pi}_{H,t} = \frac{\zeta - 1}{\chi_P}\frac{\gamma}{1-\gamma}\left(2 - \frac{(\nu-\gamma)}{(1-\gamma)\bar{\sigma}}\right)\hat{X}_t + \frac{\zeta - 1}{\chi_P}\frac{(1-\gamma)\bar{\sigma}+1}{(1-\gamma)\bar{\sigma}}\hat{y}_t + \beta\tilde{E}_t\hat{\pi}_{H,t+1} \qquad (55)$$

Now, recall that the relationship between inflation, the real domestic price and domestic price inflation is $\pi_{H,t} = p_{H,t}/p_{H,t-1}\pi_t$, which in log-linear terms becomes $\hat{\pi}_{H,t} = \hat{p}_{H,t} - \hat{p}_{H,t-1} + \hat{\pi}_t$. Using this relationship and recalling that $\hat{p}_{H,t} = -\frac{\gamma}{1-\gamma}\hat{X}_t$, equation (55) becomes

$$\hat{\pi}_t = \frac{\gamma}{1-\gamma}\left(\hat{X}_t - \hat{X}_{t-1}\right) + \frac{\zeta - 1}{\chi_P}\frac{\gamma}{1-\gamma}\left(2 - \frac{(\nu-\gamma)}{(1-\gamma)\bar{\sigma}}\right)\hat{X}_t + \frac{\zeta - 1}{\chi_P}\frac{(1-\gamma)\bar{\sigma}+1}{(1-\gamma)\bar{\sigma}}\hat{y}_t + \beta\left[\tilde{E}_t\hat{\pi}_{t+1} - \frac{\gamma}{1-\gamma}\left(\tilde{E}_t\hat{X}_{t+1} - \hat{X}_t\right)\right].$$

Rearranging the latter equation, and defining $\varphi = \frac{\zeta-1}{\chi_P}$ delivers the Phillips curve:

$$\hat{\pi}_t = -\frac{\gamma}{1-\gamma}\hat{X}_{t-1} + \frac{\gamma}{1-\gamma}\left(1 + \beta + \varphi\left(2 - \frac{(\nu-\gamma)}{(1-\gamma)\bar{\sigma}}\right)\right)\hat{X}_t + \varphi\frac{(1-\gamma)\bar{\sigma}+1}{(1-\gamma)\bar{\sigma}}\hat{y}_t + \beta\left[\tilde{E}_t\hat{\pi}_{t+1} - \frac{\gamma}{1-\gamma}\tilde{E}_t\hat{X}_{t+1}\right]. \qquad (56)$$

In a closed economy, (56) collapses to:

$$\hat{\pi}_t = \varphi\frac{\bar{\sigma}+1}{\bar{\sigma}}\hat{y}_t + \beta[\tilde{E}_t\hat{\pi}_{t+1}].$$

The financial block of the economy can be obtained using the following equation:

$$\frac{1}{R_t^f} = \beta\tilde{E}_t\frac{[\delta + (1-\delta)(1-\pi^b)]\lambda_{t+1}^s + (1-\delta)\pi^b\lambda_{t+1}^b}{\lambda_t^s}X_{t+1}/X_t;$$

combining it (23), and recalling that $\delta \to 1$, one gets:

$$\frac{R_t}{R_t^f} = \tilde{E}_1\left(\frac{X_{t+1}}{X_t \pi_{t+1}}\right).$$

Log-linearizing the latter equation gives:

$$\widehat{\mathrm{R}}_t - \hat{R}_t^f = \tilde{E}_t \hat{X}_{t+1} - \hat{X}_t + \tilde{E}_t \hat{\pi}_{t+1}; \tag{57}$$

which is the uncovered interest parity condition, in which the interest rate differential between the domestic and the foreign economy is equal to the expected nominal depreciation or, equivalently, to the expected real depreciation plus the expected inflation rate.

When crisis probability is endogenous, it is necessary to obtain the credit accumulation equation. To do this, start from the average consumption of savers, in which we substitute for equation (33). Recalling again that $\delta \to 1$:

$$c_t^s + \frac{b_t}{(1-\pi^b)} - \frac{\frac{R_{t-1}}{\pi_t}(b_{t-1})}{(1-\pi^b)} + \frac{b_t^f}{(1-\pi^b)} - \frac{\frac{R_{t-1}^f b_{t-1}^f}{(1-\pi^b)} X_t}{X_{t-1}}$$
$$= w_t l_t^s + y_t - (1+\tau_t) w_t l_t - \frac{\chi_P}{2}\left(\pi_{H,t} - 1\right)^2 y_t - T_t.$$

Using the fact that the subsidy is financed with taxes, i.e. $T_t = -\tau_t w_t l_t$, the above equation can be written as

$$c_t^s + \frac{b_t}{(1-\pi^b)} - \frac{\frac{R_{t-1}}{\pi_t}(b_{t-1})}{(1-\pi^b)} + \frac{b_t^f}{(1-\pi^b)} - \frac{\frac{R_{t-1}^f b_{t-1}^f}{(1-\pi^b)} X_t}{X_{t-1}}$$
$$= w_t l_t^s + y_t - w_t l_t - \frac{\chi_P}{2}\left(\pi_{H,t} - 1\right)^2 y_t. \tag{58}$$

Using the current account equals inverse capital flows condition, i.e.

$$p_{H,t} EXP_t - X_t\left[\pi^b c_{F,t}^b + \left(1-\pi^b\right) c_{F,t}^s\right] + \frac{\left(R_{t-1}^f - 1\right) X_t}{X_{t-1}} b_{t-1}^f = b_t^f - \frac{X_t}{X_{t-1}} b_{t-1}^f$$

and equation (35), equation (58) becomes

$$c_t^s + \frac{b_t}{(1-\pi^b)} - \frac{\frac{R_{t-1}}{\pi_t}(b_{t-1})}{(1-\pi^b)} + \frac{1}{1-\pi^b}(y_t - c_t) = w_t l_t^s + y_t - w_t l_t - \frac{\chi_P}{2}\left(\pi_{H,t} - 1\right)^2 y_t.$$

After log-linearization, the latter equation becomes:

$$s^s \hat{c}_t^s + \frac{1}{1-\pi^b}\left(\tilde{b}_t - \frac{1}{\beta}\left(\hat{b}_{t-1} + b\hat{R}_{t-1} - b\hat{\pi}_t\right) + \hat{y}_t - \hat{c}_t\right) = \hat{y}_t; \tag{59}$$

Where we used the fact that $\hat{l}_t^b = \hat{l}_t^s = \hat{l}_t$. Further, we defined $b = \bar{b}$ and used the fact that $\bar{R} = 1/\beta$. Notice that domestic credit $\tilde{b}$ is represented with a tilde instead that with a hat because it is reported in deviations rather than log-deviations from the approximation point.

Rearranging equation (59), one can get:

$$\tilde{b}_t - \frac{1}{\beta}\left(\tilde{b}_{t-1} + b\hat{R}_{t-1} - b\hat{\pi}_t\right) = -\pi^b(\hat{y}_t - \hat{c}_t) - \left(1-\pi^b\right)(s^s\hat{c}_t^s - \hat{c}_t).$$

Using the fact that $\hat{c}_t = \left(1-\pi^b\right)s^s\hat{c}_t^s + \pi^b s^b \hat{c}_t^b$, the above equation can be rewritten as follows:

$$\begin{aligned}\tilde{b}_t - \frac{1}{\beta}&\left(\tilde{b}_{t-1} + b\hat{R}_{t-1} - b\hat{\pi}_t\right) \\ &= -\pi^b\left(\hat{y}_t - \left(1-\pi^b\right)s^s\hat{c}_t^s - \pi^b s^b\hat{c}_t^b\right) - \pi^b\left(1-\pi^b\right)\left(s^s\hat{c}_t^s - s^b\hat{c}_t^b\right).\end{aligned}$$

Using (45), the latter equation becomes:

$$\tilde{b}_t - \frac{1}{\beta}\left(\tilde{b}_{t-1} + b\hat{R}_{t-1} - b\hat{\pi}_t\right) = -\pi^b\left(\hat{y}_t + \bar{\sigma}\hat{\lambda}_t\right) - (1-\pi^b)\pi^b\left(\frac{s^b}{\sigma^b} - \frac{s^s}{\sigma^s}\right)\hat{\lambda}_t. \tag{60}$$

Now, one can use equation (51) to write:

$$\begin{aligned}\tilde{b}_t - \frac{1}{\beta}\left(\tilde{b}_{t-1} + b\hat{R}_{t-1} - b\hat{\pi}_t\right) &= -\pi^b\hat{y}_1 + \pi^b\bar{\sigma}\left(\frac{1}{(1-\gamma)\bar{\sigma}}\hat{y}_1 - \frac{1}{(1-\gamma)\bar{\sigma}}\frac{\gamma(\nu-\gamma)}{1-\gamma}\hat{X}_1\right) \\ &+ (1-\pi^b)\pi^b\left(\frac{s^b}{\sigma^b} - \frac{s^s}{\sigma^s}\right)\left(\frac{1}{(1-\gamma)\bar{\sigma}}\hat{y}_1 - \frac{1}{(1-\gamma)\bar{\sigma}}\frac{\gamma(\nu-\gamma)}{1-\gamma}\hat{X}_1\right).\end{aligned}$$

Rearranging, and defining the parameter

$$s_{\aleph} = \frac{\pi^b\left(1-\pi^b\right)\left[\frac{s^b}{\sigma^b} - \frac{s^s}{\sigma^s}\right]}{\bar{\sigma}};$$

we obtain:

$$
\begin{aligned}
&\tilde{b}_t - \frac{1}{\beta}\left(\tilde{b}_{t-1} + b\hat{R}_{t-1} - b\hat{\pi}_t\right) \\
&\qquad\qquad = \frac{(1-\gamma)s_{\aleph} + \pi^b\gamma}{1-\gamma}\hat{y}_t - \frac{\gamma(\nu-\gamma)}{1-\gamma}\frac{(1-\gamma)s_{\aleph} + \pi^b}{1-\gamma}\hat{X}_t.
\end{aligned}
\tag{61}
$$

The closed economy version of the credit accumulation equation (61) is obtained by setting $\gamma = 0$:

$$
\tilde{b}_t - \frac{1}{\beta}\left(b\hat{R}_{t-1} + \tilde{b}_{t-1} - b\hat{\pi}_t\right) = s_{\aleph}\hat{y}_t.
$$

Optimization problem

As we explain in the main text, before solving the optimal policy problem, we reduce the model to a two-period framework. Hence, the policy-maker solves the following minimization problem:

$$
\min \frac{1}{2}\left(\phi_y\hat{y}_1^2 + \phi_\pi\hat{\pi}_1^2\right) + \frac{1}{2}\beta P\left(\tilde{b}_1\right)\left(\phi_y\hat{y}_2(C)^2 + \phi_\pi\hat{\pi}_2(C)^2\right)/(1-\beta\tau)
$$

subject to

$$
\hat{y}_1 = \tilde{P}\left(\tilde{b}_1\right)\hat{y}_2(C) - \frac{\gamma(\nu-\gamma)}{1-\gamma}\left[\tilde{P}\left(\tilde{b}_1\right)\hat{X}_2(C) - \hat{X}_1\right] - (1-\gamma)\bar{\sigma}\left[\tilde{P}\left(\tilde{b}_1\right)\hat{X}_2(C) - \hat{X}_1\right] + \epsilon_1,
$$

$$
\begin{aligned}
\hat{\pi}_1 = -\frac{\gamma}{1-\gamma}\hat{X}_0 + \frac{\gamma}{1-\gamma}\left(1 + \beta + \varphi\left(2 - \frac{(\nu-\gamma)}{(1-\gamma)\bar{\sigma}}\right)\right)\hat{X}_1 + \varphi\frac{(1-\gamma)\bar{\sigma} + 1}{(1-\gamma)\bar{\sigma}}\hat{y}_1 \\
+ \beta\left[\tilde{P}\left(\tilde{b}_1\right)\hat{\pi}_2(C) - \frac{\gamma}{1-\gamma}\tilde{P}\left(\tilde{b}_1\right)\hat{X}_2(C)\right],
\end{aligned}
$$

$$
\tilde{b}_1 - \frac{1}{\beta}\left(\tilde{b}_0 + b\hat{R}_0 - b\hat{\pi}_1\right) = \frac{(1-\gamma)s_{\aleph} + \pi^b\gamma}{1-\gamma}\hat{y}_1 - \frac{\gamma(\nu-\gamma)}{1-\gamma}\frac{(1-\gamma)s_{\aleph} + \pi^b}{1-\gamma}\hat{X}_1.
$$

Notice that the first constraint is obtained after substituting the uncovered interest parity condition (57) in the IS curve (52). The second and third constraint are respectively the Phillips curve (56) and the credit accumulation equation (61). Given our assumptions about expectations of second period variables, for any variable x_2, $E_1x_2 = P\left(\tilde{b}_1\right)x_2(C) + \left(1 - P\left(\tilde{b}_1\right)\right)x_2(N) = P\left(\tilde{b}_1\right)x_2(C)$.

While the policy maker is assumed to know the real probability of the realization of the crisis state, $P\left(\tilde{b}_1\right)$; the private sector has a distorted perception of such probability, $\tilde{P}\left(\tilde{b}_1\right)$. In the main text we assume that $\tilde{P}\left(\tilde{b}_1\right) = 0$, which allows us to rewrite the problem of the policy-maker as follows:

$$
\min \frac{1}{2}\left(\phi_y\hat{y}_1^2 + \phi_\pi\hat{\pi}_1^2\right) + \frac{1}{2}\beta P\left(\tilde{b}_1\right)\left(\phi_y\hat{y}_2(C)^2 + \phi_\pi\hat{\pi}_2(C)^2\right)/(1-\beta\tau)
$$

subject to

$$\hat{y}_1 = \left(\frac{\gamma(\nu-\gamma)}{1-\gamma} + (1-\gamma)\bar{\sigma}\right)\hat{X}_1 + \epsilon_1,$$

$$\hat{\pi}_1 = -\frac{\gamma}{1-\gamma}\hat{X}_0 + \frac{\gamma}{1-\gamma}\left(1+\beta+\varphi\left(2-\frac{(\nu-\gamma)}{(1-\gamma)\bar{\sigma}}\right)\right)\hat{X}_1 + \varphi\frac{(1-\gamma)\bar{\sigma}+1}{(1-\gamma)\bar{\sigma}}\hat{y}_1$$

$$\tilde{b}_1 - \frac{1}{\beta}\left(\tilde{b}_0 + b\hat{R}_0 - b\hat{\pi}_1\right) = \frac{(1-\gamma)s_{\aleph} + \pi^b\gamma}{1-\gamma}\hat{y}_1 - \frac{\gamma(\nu-\gamma)}{1-\gamma}\frac{(1-\gamma)s_{\aleph} + \pi^b}{1-\gamma}\hat{X}_1.$$

Parameter τ takes values between zero and one, and is meant to capture the effect of the duration of financial crises on the welfare loss. When τ is equal to zero, the welfare loss is equivalent to that that would be obtained when the crisis lasts only one period; while when τ is one, the welfare loss is equivalent to that that would be obtained when the crisis lasts forever. Intermediate values allows to obtain crises whose durations are higher than one period but not infinite.

The Lagrangian of the problem is:

$$\begin{aligned}
L = -\frac{1}{2}\left(\phi_y\hat{y}_1^2 + \phi_\pi\hat{\pi}_1^2\right) - \frac{1}{2}\beta P\left(\tilde{b}_1\right)\left(\phi_y\hat{y}_2(C)^2 + \phi_\pi\hat{\pi}_2(C)^2\right)/(1-\beta\tau) \\
+ \lambda_1\left\{\left(\frac{\gamma(\nu-\gamma)}{1-\gamma} + (1-\gamma)\bar{\sigma}\right)\hat{X}_1 - \hat{y}_1 + \epsilon_1\right\} \\
+ \lambda_2\left\{-\frac{\gamma}{1-\gamma}\hat{X}_0 + \frac{\gamma}{1-\gamma}\left(1+\beta+\varphi\left(2-\frac{(\nu-\gamma)}{(1-\gamma)\bar{\sigma}}\right)\right)\hat{X}_1 + \varphi\frac{(1-\gamma)\bar{\sigma}+1}{(1-\gamma)\bar{\sigma}}\hat{y}_1 \right. \\
\left. - \hat{\pi}_1\right\} \\
+ \lambda_3\left\{\frac{(1-\gamma)s_{\aleph} + \pi^b\gamma}{1-\gamma}\hat{y}_1 - \frac{\gamma(\nu-\gamma)}{1-\gamma}\frac{(1-\gamma)s_{\aleph} + \pi^b}{1-\gamma}\hat{X}_1 - \tilde{b}_1 \right. \\
\left. + \frac{1}{\beta}\left(\tilde{b}_0 + b\hat{R}_0^d - b\hat{\pi}_1\right)\right\}
\end{aligned}$$

The first order conditions with respect to output, inflation, the real exchange rate and domestic credit are, respectively:

$$-\phi_y\hat{y}_1 - \lambda_1 + \varphi\frac{(1-\gamma)\bar{\sigma}+1}{(1-\gamma)\bar{\sigma}}\lambda_2 + \frac{(1-\gamma)s_{\aleph} + \pi^b\gamma}{1-\gamma}\lambda_3 = 0, \tag{62}$$

$$-\phi_\pi\hat{\pi}_1 - \lambda_2 - \frac{b}{\beta}\lambda_3 = 0, \tag{63}$$

$$\left[\frac{\gamma(\nu-\gamma)}{1-\gamma}+(1-\gamma)\bar{\sigma}\right]\lambda_1+\frac{\gamma}{1-\gamma}\left(1+\beta+\varphi\left(2-\frac{(\nu-\gamma)}{(1-\gamma)\bar{\sigma}}\right)\right)\lambda_2$$
$$-\frac{\gamma(\nu-\gamma)}{1-\gamma}\frac{(1-\gamma)s_{\aleph}+\pi^b}{1-\gamma}\lambda_3=0 \tag{64}$$

and

$$-\frac{\frac{1}{2}\beta\left(\phi_y\hat{y}_2(C)^2+\phi_\pi\hat{\pi}_2(C)^2\right)P_b\left(\tilde{b}_1\right)}{1-\beta\tau}-\lambda_3=0; \tag{65}$$

where $P_b\left(\tilde{b}_1\right)$ the derivative of crisis probability with respect to credit. Equations (62)-(65) represent the general case, i.e. the open economy model with endogenous crisis probability. The closed economy model with exogenous crises can be obtained by setting $\gamma=0$ and $P_b\left(\tilde{b}_1\right)=0$, which deliver:

$$-\phi_y\hat{y}_1+\varphi\frac{\bar{\sigma}+1}{\bar{\sigma}}\lambda_2=0,$$

$$-\phi_\pi\hat{\pi}_1-\lambda_2=0,$$

$$\lambda_1=0$$

and

$$\lambda_3=0.$$

The closed economy with endogenous crisis probability instead deliver the following optimality conditions:

$$-\phi_y\hat{y}_1+\varphi\frac{(1-\gamma)\bar{\sigma}+1}{(1-\gamma)\bar{\sigma}}\lambda_2+\frac{(1-\gamma)s_{\aleph}+\pi^b\gamma}{1-\gamma}\lambda_3=0$$

$$-\phi_\pi\hat{\pi}_1-\lambda_2-\frac{b}{\beta}\lambda_3=0,$$

$$\lambda_1=0,$$

and

$$-\frac{\frac{1}{2}\beta\left(\phi_y \hat{y}_2(C)^2 + \phi_\pi \hat{\pi}_2(C)^2\right)P_b\left(\tilde{b}_1\right)}{1-\beta\tau} - \lambda_3 = 0.$$

Parameters governing the effect of credit on crisis probability

Following Schularick and Taylor (2012) and Ajello et al (2015), we model crisis probability for country i as a logistic function of predictor X_t^i:

$$P\left(X_t^i\right) = \frac{e^{X_t^i}}{1+e^{X_t^i}}.$$

The predictor X_t is assumed to be a function of the four year cumulated credit growth and of country fixed effects:

$$X_t^i = h_0 + h_i + h_1 L_t$$

To obtain h_0 and h_1, that in fact correspond to p and κ in the model, we follow an approach similar to that used by Ajello et al (2015) and apply it to a group of Latin American countries. In practice, we take IFS data on bank credit and the CPI index for Brazil, Colombia, Costa Rica, Ecuador, Mexico, Peru and Uruguay over the period 1980:Q1-2008:Q4. Given these data, we compute the cumulative four years growth of real bank credit for each country i:

$$L_t^i = \sum_{s=0}^{4}\left(\log\left(\frac{B_{t-s}^i}{P_{t-s}^i}\right) - \log\left(\frac{B_{t-s-1}^i}{P_{t-s-1}^i}\right)\right).$$

Crisis years for each country are identified using the dataset of Laeven and Valencia. We set the country fixed effect for Mexico to zero for identification purposes and run a logistic regression. The results of the regression are reported in Table 3.

Equation	Variables
h_1	1.1625 * (0.6409)
h_0	-2.7032*** (0.7655)
Observations	168

Table 3 – Logistic regression results

The estimation reported in Table 3 implies that when $L_t = 0$, the probability of a crisis is 6.28% on an annual basis. The annual crisis probability is converted to a quarterly crisis probability using the following equation:

$$P_Q = 1 - (1 - P_A)^{\frac{1}{4}},$$

where P_Q and P_A are respectively the quarterly and the annual crisis probability. We obtain $P_Q =$ 1.61%. Then the value of parameter p is obtained as follows:

$$p = \log\left(\frac{P_Q}{1 - P_Q}\right).$$

Parameter h_1 governs the response of crisis probability to credit.

A reduced form credit accumulation equation for Mexico

As mentioned in section 0, differently from Ajello et al (2015) we obtain the credit accumulation equation (61) from the structural model. In this section of the appendix, we estimate a reduced form credit accumulation equation for Mexico relating domestic bank credit to output and to the real exchange rate. Real domestic bank credit is obtained from IFS data dividing nominal credit by the CPI index. Real output and the real exchange rate are obtained from Banco de Mexico and are available beginning in 1993:Q1. We run the regression of bank credit quarterly growth rate on the log-deviations of output and the real exchange rate from a HP trend over the period 1993:Q1-2008:Q4:

$$\Delta Credit_t = \beta_0 + \beta_1 Output_t + \beta_2 RealExchangeRate_t + \epsilon_t$$

Results are reported in Table 4. Credit growth is significantly positively correlated to output and negatively to a depreciation of the real exchange rate, similarly to what is obtained in the structural model.

Variables	$\Delta Credit$
β_0	0.00228 (0.0075)
β_1	0.79454** (0.37389)
β_2	-0.1944* (0.10707)
Observations	63

R-squared	0.242

Table 4 – Credit accumulation regression results

Details of optimal policy under uncertainty

a) Bayesian policy-maker

The optimization problem of the Bayesian policymaker is solved numerically, using the same approach employed by Ajello et al (2015). For each b_0, we evaluate the welfare loss on 1001 grid points of the interest-rate on the interval $[R - \frac{0.1}{400}; R + \frac{0.1}{400}]$ where R is the optimal policy rate in the absence of uncertainty, and choose the policy rate that minimize the welfare loss.

b) Robust policy-maker

Also the optimization problem of the robust policymaker is solved numerically. For each b_0, we evaluate the welfare loss on 1001 grid points of the interest-rate on the interval $[R - \frac{0.1}{400}; R + \frac{0.1}{400}]$ where R is the optimal policy rate in the absence of uncertainty, and choose the policy rate that minimizes the welfare loss. The optimization of the evil agent is also done numerically, obtaining the value of the parameter(s) that maximizes the loss for each interest rate. In particular, when only one parameter is uncertain, we compute the objective function on 21 grid points on the interval $[h_{min}, h_{max}]$ and choose the parameter value that maximizes the welfare loss. When two parameters are uncertain, we compute the objective function on 21-by-21 grid points on the interval $[(h_{1min}, h_{1max}), (h_{2min}, h_{2max})]$ where h_1 and h_2 are the two parameters under consideration, and choose the combination of parameter values that maximizes the welfare loss.

Additional figures

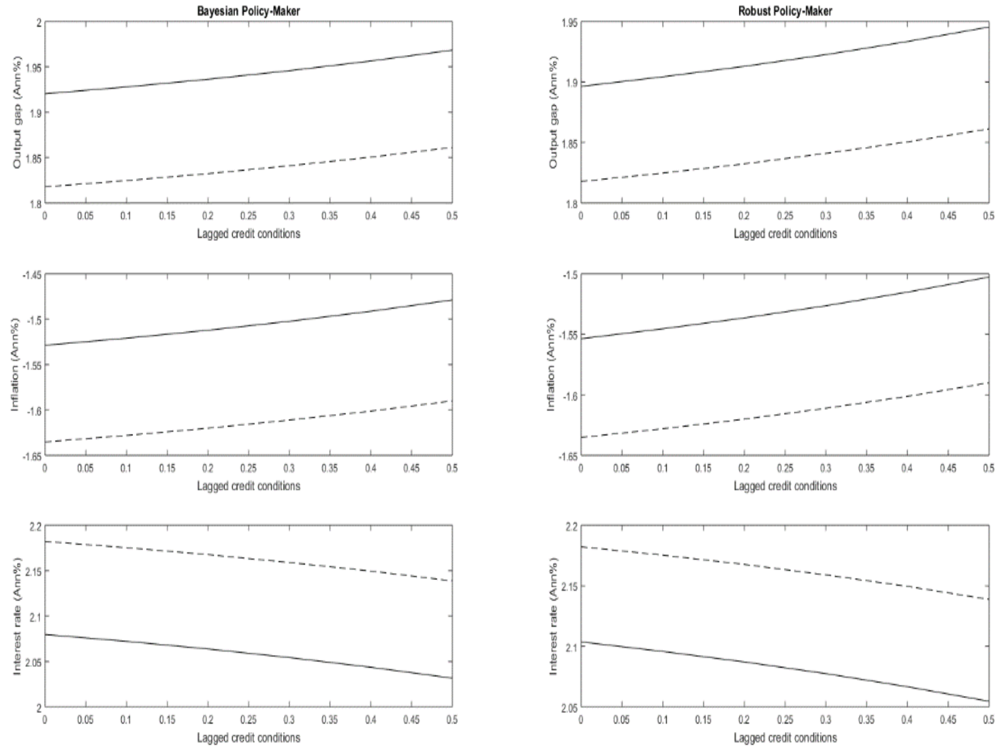

Figure 6 - Uncertain export elasticity. Full line: optimal values under uncertainty. Dotted line: optimal values without uncertainty.

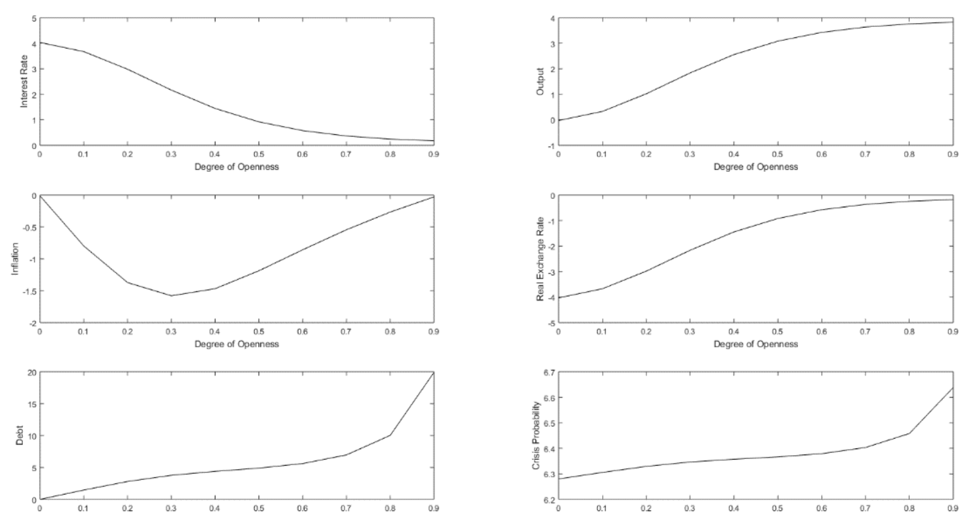

Figure 7 - Optimal values for different values of γ.

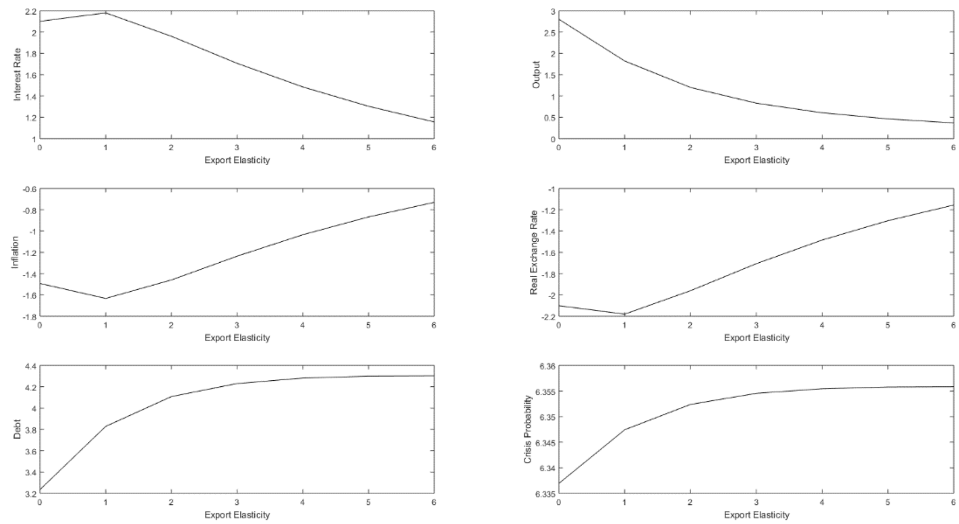

Figure 8 - Optimal values for different values of ν.

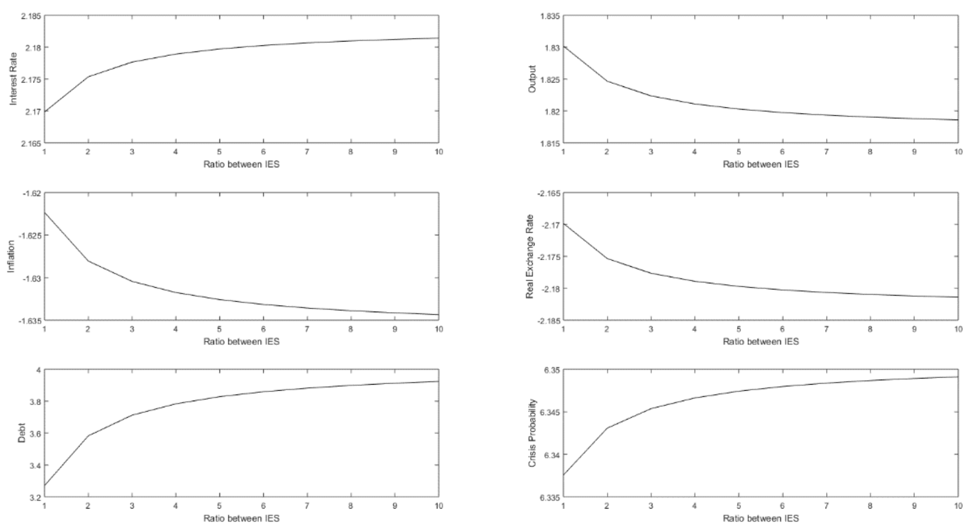

Figure 9 - Optimal values for different values of $\frac{\sigma^s}{\sigma^b}$.

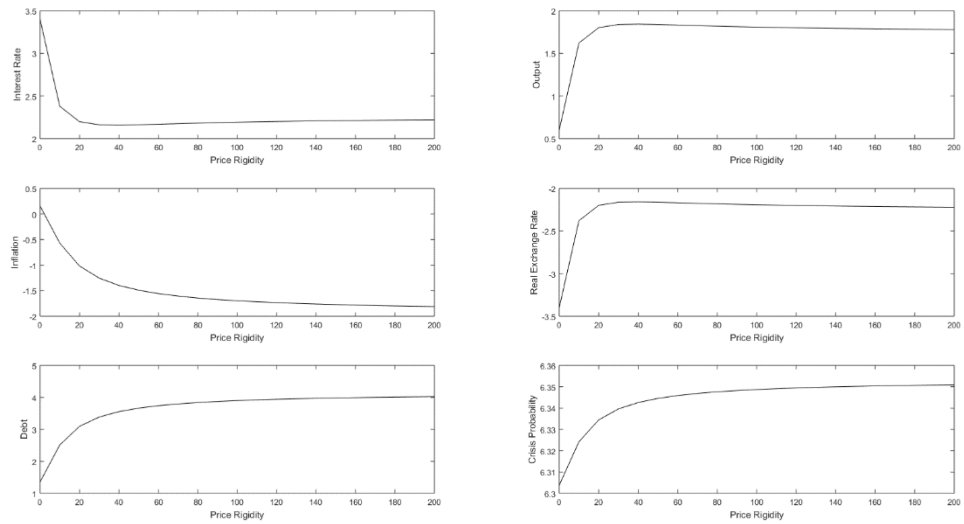

Figure 10 – Optimal values for different values of χ_P

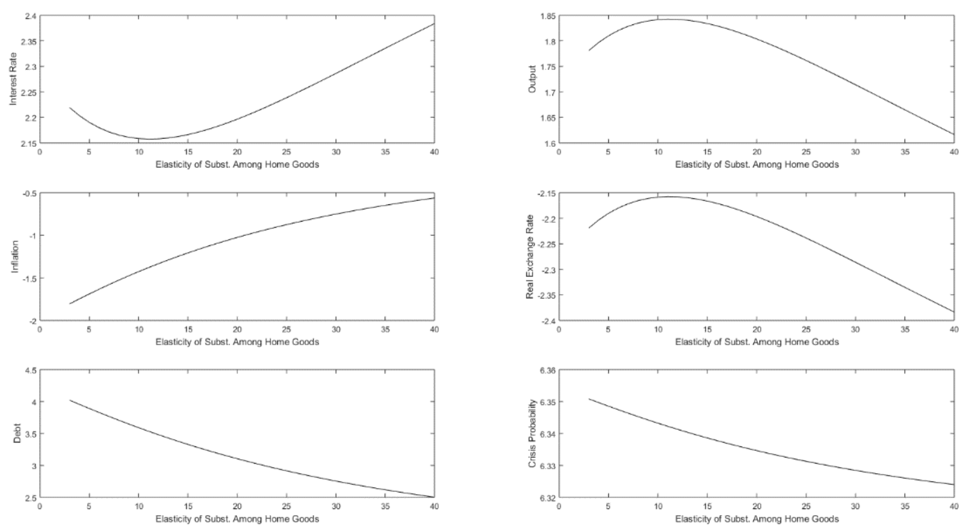

Figure 11 – Optimal values for different values of ζ

Figure 8 considers the effect of higher export elasticities on the optimal interest rate (and the other macroeconomic variables). The higher is the elasticity of exports, the lower the optimal interest rate. In fact, higher export elasticities are reflected into higher elasticities of capital flows to the interest rate differential between the home and the foreign economy. It is easy to see that the higher is ν the more important is the real exchange rate in influencing credit accumulation. A reduction of the nominal interest rate produces a stronger capital outflow when export is very elastic because foreign households are eager to exploit the possibility of borrowing cheaper (or lending less because of the lower interest rate) from (to) domestic households to increase their consumption of the domestic good. The strong capital outflow reduces domestic credit more, making interest rate cuts more effective at diminishing crisis probability. Going from $\nu = 1$ to $\nu = 6$, the optimal policy rate falls from around 2.2% to around 1.7%, a reduction of fifty basis points. Notice that also in this case the crisis probability is higher the higher is ν. Here again the high elasticity of capital flows makes sure that a small increase in the interest rate attracts a great amount of additional foreign credit which in turn increases domestic credit. When $\nu = 6$, domestic credit increases by around 8%; compared to around 3% when $\nu = 1$, even though the policy rate is raised more in the latter case.

Figure 10 shows the effect of modifying the price stickiness parameter χ_p and the elasticity of substitution among home goods, ζ on optimal policy. Modifying χ_p allows to change the slope of the Phillips curve. As is evident from Figure 10, apart from very flexible prices and a close to vertical Phillips curve, results are almost unaffected by χ_p. Even for very small values of χ_p the optimal policy rate is smaller than in the closed economy model and, as we verified in a separate experiment, smaller than in the open economy model with exogenous systemic risk. A higher ζ corresponds to a lower mark-up and lower profits for domestic firms. It is easy to see that this is almost irrelevant for results.