Optimal Unconditional Monetary Policy, Trend Inflation and the Zero Lower Bound

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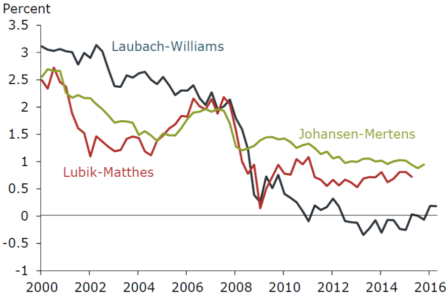
8th BIS CCA Research Conference
Low Interest Rates, Monetary Policy and International Spillovers
Washington, DC (May 2017)

Outline

- Introduction/Contribution
- Model
- Precautionary Optimal Policy
- IRF's and Welfare Analysis
- Conclusions

Introduction

- The long-run real rate of interest has been showing a decreasing path during the last decades, recently hitting estimated levels as low as 1% or even smaller (see e.g. Laubach and Williams (FRBSF 2015), Bauer and Rudebusch (FRBSF 2016) and Yi and Zhang (FRBM 2016)).



Source: Bauer and Rudebusch (FRBSF 2016)

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- The optimal targeting rule holds its precautionary behavior even under (log)linear approximations. It prescribes price level targeting as $\beta \to 1$ and $\bar{\pi} \to 0$. For larger levels of $\bar{\pi}$, the rule is more entangled.
- Even under precautionary optimal policy, and occasionally bind ZLB constraints, the optimal level of trend inflation is still slightly above zero...

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- Simulations indicate that, under occasionally binding ZLB constraints, the precautionary optimal policy welfare-dominates the standard one.
- IRFs after negative demand shocks: policy rate does not reduce as much on spot, making room for more policy effectiveness. After the shock ceases, precautionary optimal policy keeps the rate at lower levels for much longer.

The model

Households

$$u_t = \overbrace{\epsilon_t}^{shock} \frac{C_t^{1-\sigma}}{(1-\sigma)} \qquad v_t \equiv \int_0^1 v_t(z)\, dz \qquad v_t(z) \equiv \chi \frac{h_t(z)^{1+\nu}}{(1+\nu)}$$

$$C_t^{\frac{\theta-1}{\theta}} = \int_0^1 c_t(z)^{\frac{\theta-1}{\theta}}\, dz \qquad c_t(z) = C_t \left(\frac{p_t(z)}{P_t}\right)^{-\theta}$$

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Firms

$$y_t(z) = \overbrace{\mathcal{A}_t}^{shock} h_t(z)^{\varepsilon} \qquad z \in (0,1)$$

Calvo: $\alpha \in (0,1)$ Indexation: $\Pi_t^{ind} = \Pi_{t-1}^{\gamma_\pi}$

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$$\mathcal{A}_t = \mathcal{A}_{t-1}^{\rho_{\mathfrak{a}}} \epsilon_{\mathfrak{a},t}, \text{ where } \epsilon_{\mathfrak{a},t} \overset{iid}{\sim} LN\left(0, \mathfrak{s}_{\mathfrak{a}}^2\right)$$

Policy trade-off as Alves (JME 2014)

GNKPC curve:

$$\left(\hat{\pi}_t - \hat{\pi}_t^{ind}\right) = \beta E_t\left(\hat{\pi}_{t+1} - \hat{\pi}_{t+1}^{ind}\right) + \bar{\kappa}\hat{x}_t$$

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$$\widehat{\mathfrak{u}}_t = \bar{\alpha}\vartheta\beta E_t\widehat{\mathfrak{u}}_{t+1} + (\vartheta - 1)\,\beta E_t\hat{\xi}_{t+1}$$

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$$\bar{\kappa} \equiv \frac{(1-\bar{\alpha})(1-\bar{\alpha}\beta\vartheta)}{\bar{\alpha}}\frac{(\omega+\sigma)}{(1+\theta\omega)} \quad \bar{\kappa}_{\varpi} \equiv \frac{(1-\bar{\alpha})}{(1+\theta\omega)} \quad \omega \equiv \frac{(1+\nu)}{\varepsilon} - 1$$

Probability of hitting the ZLB

- Definition: $\mathfrak{p}_{\mathfrak{o},t} \equiv \mathbb{P}\left(i_t \leq 0 | \mathfrak{I}_{t-1}\right)$

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$$\hat{\mathfrak{p}}^n_{\mathfrak{o},t} \approx -\phi_\epsilon\left[\frac{\omega\rho_{\mathfrak{u}}(1-\rho_{\mathfrak{u}})}{(\omega+\sigma)}\hat{\epsilon}_{t-1} - \frac{\sigma(1+\omega)\rho_{\mathfrak{a}}(1-\rho_{\mathfrak{a}})}{(\omega+\sigma)}\widehat{\mathcal{A}}_{t-1}\right]$$

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- Sticky Prices: $\bar{\mathfrak{p}}_{\mathfrak{o}} = \bar{\mathfrak{p}}_{\mathfrak{o}}^{n}$

$$\hat{\mathfrak{p}}_{\mathfrak{o},t} \approx -\phi_{\epsilon}E_t\left[\sigma\left(\hat{Y}_{t+1} - \hat{Y}_t\right) + \hat{\pi}_{t+1}\right] - \phi_{\epsilon}\rho_{\mathfrak{u}}\left(1-\rho_{\mathfrak{u}}\right)\hat{\epsilon}_{t-1}$$

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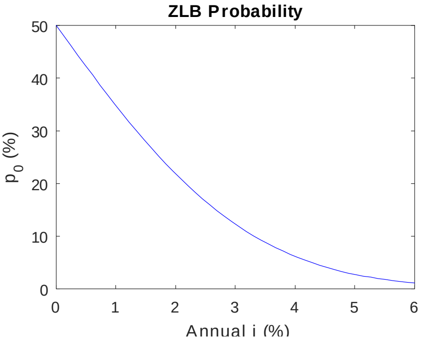

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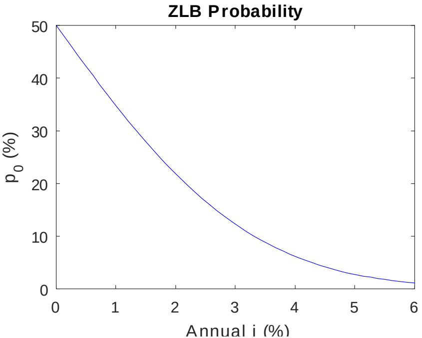

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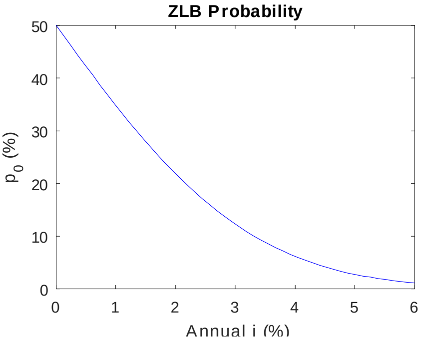

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Simulated: $\bar{\mathfrak{p}}_{\mathfrak{o}} = 32\%$, $E\mathfrak{p}_{\mathfrak{o},t} \in (43\%, 44\%)$

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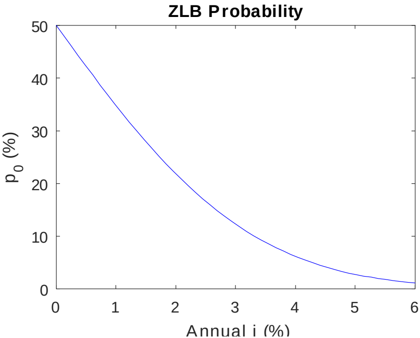

$\bar{\mathfrak{p}}_0 \neq E\mathfrak{p}_{0,t}$:

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1985Q1-2016Q4: Freq(i<0.15)=14%, mean(i)=3.73 (Fed Funds)

Simulated: $\bar{\mathfrak{p}}_0 = 7.5\%$, $E\mathfrak{p}_{0,t} \in (14\%, 16\%)$

Optimal Policy under unconditionally commitment

and occasionally binding ZLB constraints

- Trend inflation welfare-based loss function, Alves (JME 2014):

$$\mathcal{L}_t = \left(\hat{\pi}_t - \hat{\pi}_t^{ind} + \bar{\phi}_\pi\right)^2 + \mathcal{X}\left(\hat{x}_t - \bar{\phi}_x\right)^2$$

$$W_t \approx \bar{W} + \frac{\bar{\mathcal{V}}}{2}\sum_{j\geq 0}\beta^j \mathcal{L}_{t+j} \quad ; \; \mathcal{X} \equiv \frac{(1-\bar{\alpha})}{(1-\bar{\alpha}\vartheta)}\frac{\bar{\kappa}}{\theta}$$

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- As in Damjanovic et al. (JME 2008), minimize unconditional expectation, taking $\mathfrak{p}_{\mathfrak{o},t}$ under consideration.
- It is time-consistent! In addition, it does not depend on transition probabilities into and from ZLB states.

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- Trend inflation welfare-based loss function, Alves (JME 2014):

$$\min \frac{1}{1-\beta}\frac{\bar{\mathcal{V}}}{2}E\left[\left(\hat{\pi}_t - \hat{\pi}_t^{ind} + \bar{\phi}_\pi\right)^2 + \left(1-\mathfrak{p}_{\mathfrak{o},t}\right)\mathcal{X}\left(\hat{x}_t - \bar{\phi}_x\right)^2 \right.$$
$$\left. + \mathfrak{p}_{\mathfrak{o},t}\mathcal{X}\left(\hat{x}_{t+1} + \frac{1}{\sigma}\hat{\pi}_{t+1} + \frac{1}{\sigma}\mathring{i} + \frac{1}{\sigma}\hat{r}_t^n - \bar{\phi}_x\right)^2\right]$$

- If $\mathring{i}$ is of order $\mathcal{O}(1)$, the equation on $\hat{\mathfrak{p}}_{\mathfrak{o},t}$ is not binding, and so everything works as if considering only the effects of $\bar{\mathfrak{p}}_{\mathfrak{o}}$.

Optimal Policy under unconditionally commitment

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- Targeting Rule:

$$0 = \left(\hat{\pi}_t - \hat{\pi}_t^{ind}\right) + \left(1 - \bar{\mathfrak{p}}_{\mathfrak{o}}\right) \frac{1}{\mathfrak{c}_1} \frac{\bar{\mathcal{X}}}{\bar{\kappa}} \left[\hat{x}_t - \beta \hat{x}_{t-1} - \left(\mathfrak{c}_2 - \mathfrak{c}_1\right) \hat{\varkappa}_{1,t-1}\right] + \bar{\mathfrak{p}}_{\mathfrak{o}} \frac{\bar{\mathcal{X}}}{\bar{\kappa}} \left(\frac{\bar{\kappa}}{\sigma} \hat{\partial}_{1,t} + \hat{\partial}_{2,t}\right)$$

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- Where

$$\hat{\varkappa}_t = \frac{\mathfrak{c}_4}{\mathfrak{c}_1} \hat{\varkappa}_{t-1} + \frac{\mathfrak{c}_3}{\mathfrak{c}_1} \hat{x}_t - \frac{\beta}{\mathfrak{c}_1} \left(1 - \bar{\alpha} \beta \bar{\vartheta}\right) \hat{x}_{t-1}$$

$$\hat{\eth}_{1,t} = \gamma_\pi E_t \hat{\eth}_{1,t+1} + \left(\hat{x}_t + \frac{1}{\sigma} \hat{\imath}_t\right)$$

$$\hat{\eth}_{2,t} = \frac{\mathfrak{c}_4}{\mathfrak{c}_1} \hat{\eth}_{2,t-1} + \frac{1}{\mathfrak{c}_1} \left(\hat{x}_{t-1} + \frac{1}{\sigma} \hat{\imath}_{t-1}\right) - \frac{1}{\mathfrak{c}_1} \left[\left(1 + \bar{\alpha} \bar{\vartheta}\right) \beta + \bar{\kappa} \theta \left(\mathfrak{c}_2 - \mathfrak{c}_1\right)\right] \left(\hat{x}_{t-2} + \frac{1}{\sigma} \hat{\imath}_{t-2}\right) + \frac{1}{\mathfrak{c}_1} \bar{\alpha} \bar{\vartheta} \beta^2 \left(\hat{x}_{t-3} + \frac{1}{\sigma} \hat{\imath}_{t-3}\right)$$

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- Targeting Rule, if $\beta \longrightarrow 1$, $\bar{\pi} \longrightarrow 0$ and $\frac{\bar{\kappa}}{\sigma}$ is small (as empirical evidence supports):

$$0 \approx \left(\hat{p}_t - \hat{p}_t^{ind}\right) + \left(1 - \bar{\mathfrak{p}}_{\mathfrak{o}}\right) \frac{\bar{\mathcal{X}}}{\bar{\kappa}} \hat{x}_t + \bar{\mathfrak{p}}_{\mathfrak{o}} \frac{\bar{\mathcal{X}}}{\bar{\kappa}} \left(\hat{x}_{t-1} + \frac{1}{\sigma} \hat{\imath}_{t-1}\right)$$

$$\hat{\pi}_t = \left(1 - L\right) \hat{p}_t, \quad \hat{\pi}_t^{ind} = \left(1 - L\right) \hat{p}_t^{ind}, \quad \hat{p}_t^{ind} = \gamma_{\pi} \hat{p}_{t-1}$$

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- Therefore: **price level targeting** if $\beta \longrightarrow 1$ and $\bar{\pi} \longrightarrow 0$.

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- Therefore: **price level targeting** if $\beta \to 1$ and $\bar{\pi} \to 0$.
- Even at its full form, it generates precautionary behavior:

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$$0 \approx \left(\hat{p}_t - \hat{p}_t^{ind}\right) + \left(1 - \bar{\mathfrak{p}}_{\mathfrak{o}}\right) \frac{\bar{\mathcal{X}}}{\bar{\kappa}} \hat{x}_t + \bar{\mathfrak{p}}_{\mathfrak{o}} \frac{\bar{\mathcal{X}}}{\bar{\kappa}} \left(\hat{x}_{t-1} + \frac{1}{\sigma} \hat{\imath}_{t-1}\right)$$

$$\hat{\pi}_t = (1 - L)\,\hat{p}_t, \quad \hat{\pi}_t^{ind} = (1 - L)\,\hat{p}_t^{ind}, \quad \hat{p}_t^{ind} = \gamma_\pi \hat{p}_{t-1}$$

- Therefore: **price level targeting** if $\beta \to 1$ and $\bar{\pi} \to 0$.
- Even at its full form, it generates precautionary behavior:
    - Reacting to $\hat{x}_{t-1}$, it takes does not reduce the rate as much on spot after a negative demand shock.

Optimal Policy under unconditionally commitment

and occasionally binding ZLB constraints

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- Therefore: **price level targeting** if $\beta \to 1$ and $\bar{\pi} \to 0$.
- Even at its full form, it generates precautionary behavior:
  - Reacting to $\hat{x}_{t-1}$, it takes does not reduce the rate as much on spot after a negative demand shock.
  - Reacting to $\hat{\imath}_{t-1}$, it takes longer to increase the rate after the shock dissipates.

Simulations

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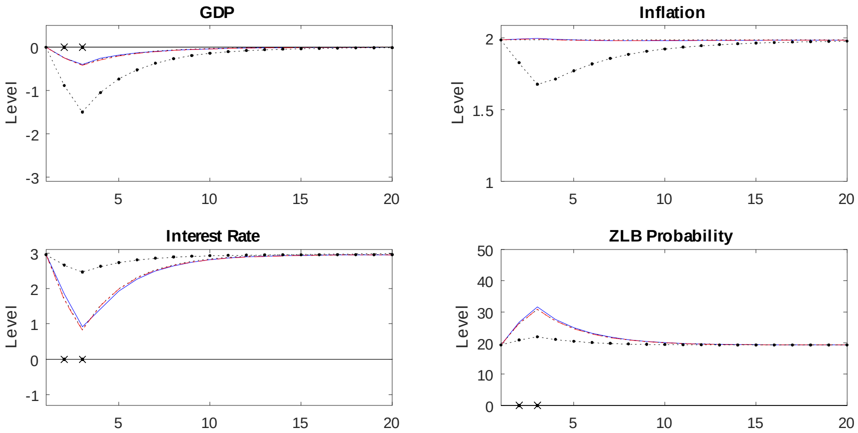
- We must take in consideration that, under the ZLB, $E\hat{\pi}_t \neq 0$ and $E\hat{x}_t \neq 0$. And so this must taken in consideration when computing $E\hat{\pi}_t^2$ and $E\hat{x}_t^2$.

IRF

$$\bar{r} = 1\%,\ \bar{\pi} = 2\%,\ \epsilon_{\mathfrak{u},t} = -(0.5)\mathfrak{s}_{\mathfrak{u}}$$



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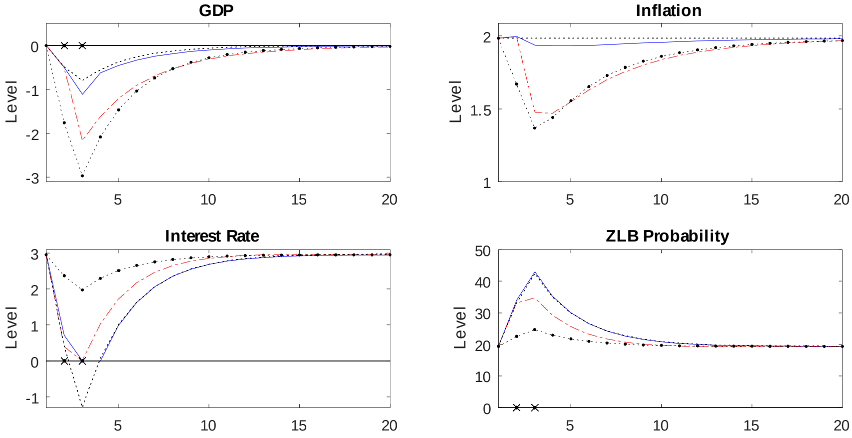

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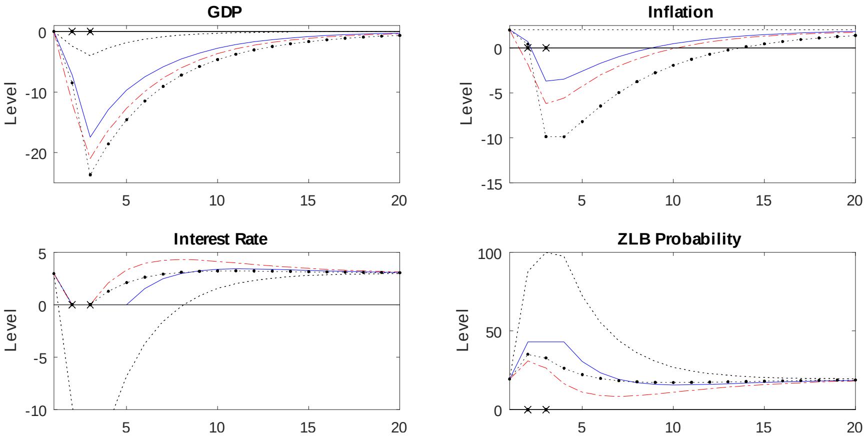

Welfare Analysis

A) Under ZLB constraints

Steady States $\bar{r}$=1%			PrOP *Rates* (%)		StOP *Rates* (%)		TayR *Rates* (%)	
$\bar{\pi}$	$\bar{\imath}$	$\bar{\mathfrak{p}}_{\mathfrak{o}}$	λ	$E\mathfrak{p}_{\mathfrak{o},t}$	λ	$E\mathfrak{p}_{\mathfrak{o},t}$	λ	$E\mathfrak{p}_{\mathfrak{o},t}$
0	1	34.8	0.12	52.3	0.13	49.9	0.96	8.9
1	2	21.7	0.59	31.2	0.59	25.0	0.91	0.8
2	3	12.1	2.56	18.9	2.65	19.0	2.78	0.0

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1	2	21.7	0.59	31.2	0.59	25.0	0.91	0.8
2	3	12.1	2.56	18.9	2.65	19.0	2.78	0.0

B) No ZLB constraints

Steady States $\bar{r}=1\%$			PrOP *Rates* (%)		StOP *Rates* (%)		TayR *Rates* (%)	
$\bar{\pi}$	$\bar{\imath}$	$\bar{p}_{\mathfrak{o}}$	λ	$Ep_{\mathfrak{o},t}$	λ	$Ep_{\mathfrak{o},t}$	λ	$Ep_{\mathfrak{o},t}$
0	1	34.8	0.00	37.2	0.00	38.1	0.35	11.3
1	2	21.7	0.48	26.8	0.48	28.0	0.84	0.9
2	3	12.1	2.42	18.9	2.42	19.7	2.78	0.0

Main Results

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- I derive optimal precautionary policy under occasionally binding ZLB constraints, when the central bank directly internalize its role in affecting the $\mathfrak{p}_{\mathfrak{o},t}$ by means of the expectations channel.
- The precautionary targeting rule resembles price level targeting when $\beta \longrightarrow 1$ and $\bar{\pi} \longrightarrow 0$.
- Even at its full form, it generates precautionary behavior, by reacting to lagged output-gaps and policy rates.
- It dominates standard (commitment) optimal policy under occasionally binding ZLB constraints.