Optimal Unconditional Monetary Policy, Trend Inflation and the Zero Lower Bound

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The views expressed here are those of the authors and not necessarily those of the Banco Central do Brasil
Outline

- Introduction/Contribution
- Model
- Precautionary Optimal Policy
- IRF’s and Welfare Analysis
- Conclusions
Introduction

- The long-run real rate of interest has been showing a decreasing path during the last decades, recently hitting estimated levels as low as 1% or even smaller (see e.g. Laubach and Williams (FRBSF 2015), Bauer and Rudebusch (FRBSF 2016) and Yi and Zhang (FRBM 2016)).

Source: Bauer and Rudebusch (FRBSF 2016)
Contribution

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- The optimal targeting rule holds its precautionary behavior even under (log)linear approximations. It prescribes price level targeting as $\beta \to 1$ and $\bar{\pi} \to 0$. For larger levels of $\bar{\pi}$, the rule is more entangled.

- Even under precautionary optimal policy, and occasionally bind ZLB constraints, the optimal level of trend inflation is still slightly above zero...
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- Simulations indicate that, under occasionally binding ZLB constraints, the precautionary optimal policy welfare-dominates the standard one.

- IRFs after negative demand shocks: policy rate does not reduce as much on spot, making room for more policy effectiveness. After the shock ceases, precautionary optimal policy keeps the rate at lower levels for much longer.
The model

Households

\[ u_t = \begin{cases} \varepsilon_t \frac{C_t^{1-\sigma}}{(1-\sigma)} & \text{shock} \\ C_t^{\theta-1} & \end{cases} \]

\[ v_t \equiv \int_0^1 v_t(z) \, dz \quad v_t(z) \equiv \chi \frac{h_t(z)^{1+\nu}}{(1+\nu)} \]

\[ C_t^{\theta-1} = \int_0^1 c_t(z)^{\frac{\theta-1}{\theta}} \, dz \quad c_t(z) = C_t \left( \frac{p_t(z)}{P_t} \right)^{-\theta} \]
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\[ \epsilon_t = \epsilon_{t-1}^u \epsilon_{u,t}, \text{ where } \epsilon_{u,t} \overset{iid}{\sim} LN(0, \sigma_u^2) \]
The model

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Firms

\[ y_t(z) = \mathcal{A}_t h_t(z)^\alpha \quad z \in (0, 1) \]

Calvo: \( \alpha \in (0, 1) \) \quad Indexation: \( \Pi_t^{ind} = \Pi_t^{\gamma_{PI}} \)
The model

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\[ y_t(z) = A_t h_t(z) \]
\[ A_t = A_{t-1}^{\rho_a} \epsilon_{a,t}, \text{ where } \epsilon_{a,t} \overset{iid}{\sim} LN(0, \sigma_a^2) \]

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Policy trade-off as Alves (JME 2014)

GNKPC curve:

$$\left( \hat{\pi}_t - \hat{\pi}^{ind}_t \right) = \beta E_t \left( \hat{\pi}_{t+1} - \hat{\pi}^{ind}_{t+1} \right) + \bar{\kappa} \hat{x}_t$$
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+ (\vartheta - 1) \bar{\kappa} \omega \beta E_t \hat{\omega}_{t+1} + \hat{u}_t
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\[
\hat{\omega}_t = \bar{\alpha} \vartheta \beta E_t \hat{\omega}_{t+1} + \theta (1 + \omega) \left( \hat{\pi}_t - \hat{\pi}_t^{ind} \right) \\
+ (1 - \bar{\alpha} \vartheta \beta) (\omega + \sigma) \hat{x}_t + (1 - \sigma) (\hat{x}_t - \hat{x}_{t-1})
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\]

\[
\hat{u}_t = \bar{\alpha} \vartheta \beta E_t \hat{u}_{t+1} + (\vartheta - 1) \beta E_t \hat{\xi}_{t+1}
\]

\[
\hat{\xi}_t = \bar{\kappa} \omega \frac{(1+\omega)}{(\omega+\sigma)} \left[ (1 - \sigma) \left( \hat{A}_t - \hat{A}_{t-1} \right) + (\hat{e}_t - \hat{e}_{t-1}) \right]
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\]

\[
\bar{\kappa} \equiv \frac{(1-\bar{\alpha})(1-\bar{\alpha} \beta \theta)}{\bar{\alpha}} \frac{(\omega+\sigma)}{(1+\theta \omega)} \quad \bar{\kappa}_\omega \equiv \frac{(1-\bar{\alpha})}{(1+\theta \omega)} \quad \omega \equiv \frac{(1+\nu)}{\epsilon} - 1
\]
Probability of hitting the ZLB

- Definition: $p_{o,t} \equiv P(i_t \leq 0 | \mathcal{I}_{t-1})$
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- Flex Prices: $\bar{p}_0^n = \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{-1}{\sqrt{2} s_{ua}} \frac{i}{\sigma} \right) \right] \quad i \equiv \log (\bar{I})$

$$\hat{p}_{o,t}^n \approx -\phi \epsilon \left[ \frac{\omega \rho_u (1-\rho_u)}{(\omega+\sigma)} \hat{e}_{t-1} - \frac{\sigma (1+\omega) \rho_a (1-\rho_a)}{(\omega+\sigma)} \hat{A}_{t-1} \right]$$
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\]

- **Sticky Prices:** \( \bar{p}_o = \bar{p}_o^n \)

\[
\hat{p}_{o,t} \approx -\phi \epsilon E_t \left[ \sigma (\hat{Y}_{t+1} - \hat{Y}_t) + \hat{\pi}_{t+1} \right] - \phi \epsilon \rho_u (1-\rho_u) \hat{e}_{t-1}
\]
Probability of hitting the ZLB

\[ \bar{p}_0 = \mathbb{E} p_0, t : 2006Q1-2016Q4: \text{Freq}(i < 0.15) = 41\%, \text{mean}(i) = 1.19 \text{ (Fed Funds)} \]

Simulated: \[ \bar{p}_0 = 32.5\% \]

\[ \text{Simulated: } \bar{p}_0 = 7.5\%, \mathbb{E} p_0, t^2 (14\%, 16\%) \]

\[ 1985Q1-2016Q4: \text{Freq}(i < 0.15) = 14\%, \text{mean}(i) = 3.73 \text{ (Fed Funds)} \]

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- **2006Q1-2016Q4**: \( \text{Freq}(i<0.15)=41\% \), \( \text{mean}(i)=1.19 \) (Fed Funds)
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- **1985Q1-2016Q4**: \( \text{Freq}(i<0.15)=14\% \), \( \text{mean}(i)=3.73 \) (Fed Funds)
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Optimal Policy under unconditionally commitment
and occasionally binding ZLB constraints

- Trend inflation welfare-based loss function, Alves (JME 2014):

\[
\mathcal{L}_t = \left( \hat{\pi}_t - \hat{\pi}_t^{ind} + \bar{\phi}_\pi \right)^2 + \mathcal{X} \left( \hat{x}_t - \bar{\phi}_x \right)^2
\]

\[
\mathcal{W}_t \approx \bar{\mathcal{W}} + \frac{\bar{\gamma}}{2} \sum_{j \geq 0} \beta^j \mathcal{L}_{t+j} ; \quad \mathcal{X} \equiv \frac{(1-\bar{\alpha})}{(1-\bar{\alpha}\bar{\vartheta})} \bar{\kappa}
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\mathcal{W}_t \approx \bar{\mathcal{W}} + \frac{\bar{\nu}}{2} \sum_{j \geq 0} \beta^j \mathcal{L}_{t+j} \quad ; \quad \mathcal{X} \equiv \frac{(1-\bar{\alpha})}{(1-\bar{\alpha}\bar{\theta})} \bar{\kappa}
\]

- As in Damjanovic et al. (JME 2008), minimize unconditional expectation, taking \( p_{o,t} \) under consideration.
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- As in Damjanovic et al. (JME 2008), minimize unconditional expectation, taking \( p_{o,t} \) under consideration.

- It is time-consistent!
Optimal Policy under unconditionally commitment
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- Trend inflation welfare-based loss function, Alves (JME 2014):

\[ \mathcal{L}_t = \left( \hat{\pi}_t - \hat{\pi}^{ind}_t + \bar{\phi}_\pi \right)^2 + \lambda \left( \hat{x}_t - \bar{\phi}_x \right)^2 \]

\[ \mathcal{W}_t \approx \bar{\mathcal{W}} + \frac{\bar{\psi}}{2} \sum_{j \geq 0} \beta^j \mathcal{L}_{t+j} \quad ; \quad \lambda \equiv \frac{(1-\bar{\alpha})}{(1-\bar{\alpha} \bar{\theta})} \bar{\kappa} \]

- As in Damjanovic et al. (JME 2008), minimize unconditional expectation, taking \( p_{o,t} \) under consideration.

- It is time-consistent! In addition, it does not depend on transition probabilities into and from ZLB states.
Optimal Policy under unconditionally commitment and occasionally binding ZLB constraints

- Trend inflation welfare-based loss function, Alves (JME 2014):

\[
\min \frac{1}{1-\beta} \frac{\hat{\nu}}{2} E \left[ \left( \hat{\pi}_t - \hat{\pi}_t^{ind} + \bar{\phi}_\pi \right)^2 + (1 - p_{o,t}) \lambda \left( \hat{x}_t - \bar{\phi}_x \right)^2 \right. \\
\left. + p_{o,t} \lambda \left( \hat{x}_{t+1} + \frac{1}{\sigma} \hat{\pi}_{t+1} + \frac{1}{\sigma} i + \frac{1}{\sigma} \hat{\pi}^n - \bar{\phi}_x \right)^2 \right]
\]

- If $i$ is of order $O(1)$, the equation on $\hat{p}_{o,t}$ is not binding, and so everything works as if considering only the effects of $\bar{p}_o$. 
Optimal Policy under unconditionally commitment and occasionally binding ZLB constraints

- Targeting Rule:

\[
0 = \left( \hat{\pi}_t - \hat{\pi}^{ind}_t \right) + \left( 1 - \bar{p}_o \right) \frac{1}{c_1} \frac{\bar{X}}{\bar{K}} \left[ \hat{x}_t - \beta \hat{x}_{t-1} - (c_2 - c_1) \hat{x}_{1,t-1} \right] \\
+ \bar{p}_o \frac{\bar{X}}{\bar{K}} \left( \frac{\bar{K}}{\bar{\sigma}} \hat{d}_{1,t} + \hat{d}_{2,t} \right)
\]
Optimal Policy under unconditionally commitment
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- Where

\[ \hat{x}_t = \frac{c_4}{c_1} \hat{x}_{t-1} + \frac{c_3}{c_1} \hat{x}_t - \frac{\beta}{c_1} \left( 1 - \bar{\alpha} \beta \bar{\theta} \right) \hat{x}_{t-1} \]
\[ \hat{d}_{1,t} = \gamma_{\pi} E_t \hat{d}_{1,t+1} + \left( \hat{x}_t + \frac{1}{\bar{\sigma}} \hat{i}_t \right) \]
\[ \hat{d}_{2,t} = \frac{c_4}{c_1} \hat{d}_{2,t-1} + \frac{1}{c_1} \left( \hat{x}_{t-1} + \frac{1}{\bar{\sigma}} \hat{i}_{t-1} \right) \]
\[ - \frac{1}{c_1} \left[ \left( 1 + \bar{\alpha} \bar{\theta} \right) \beta + \bar{\kappa} \theta \left( c_2 - c_1 \right) \right] \left( \hat{x}_{t-2} + \frac{1}{\bar{\sigma}} \hat{i}_{t-2} \right) \]
\[ + \frac{1}{c_1} \bar{\alpha} \bar{\theta} \beta^2 \left( \hat{x}_{t-3} + \frac{1}{\bar{\sigma}} \hat{i}_{t-3} \right) \]
Optimal Policy under unconditionally commitment
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- Targeting Rule, if $\beta \rightarrow 1$, $\bar{\pi} \rightarrow 0$ and $\bar{\kappa}/\sigma$ is small (as empirical evidence supports):

$$0 \approx (\hat{p}_t - \hat{p}_t^{ind}) + (1 - \bar{p}_o) \frac{\bar{X}}{\bar{k}} \hat{x}_t + \bar{p}_o \frac{\bar{X}}{\bar{k}} (\hat{x}_{t-1} + \frac{1}{\sigma} \hat{i}_{t-1})$$

$$\hat{\pi}_t = (1 - L) \hat{p}_t, \quad \hat{\pi}_t^{ind} = (1 - L) \hat{p}_t^{ind}, \quad \hat{p}_t^{ind} = \gamma_\pi \hat{p}_{t-1}$$
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- Therefore: **price level targeting** if $\beta \rightarrow 1$ and $\bar{\pi} \rightarrow 0$. 
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- Targeting Rule, if $\beta \rightarrow 1$, $\bar{\pi} \rightarrow 0$ and $\frac{\bar{K}}{\bar{\sigma}}$ is small (as empirical evidence supports):

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- Therefore: price level targeting if $\beta \rightarrow 1$ and $\bar{\pi} \rightarrow 0$.

- Even at its full form, it generates precautionary behavior:
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- Therefore: **price level targeting** if $\beta \to 1$ and $\bar{\pi} \to 0$.

- Even at its full form, it generates precautionary behavior:

  - Reacting to $\hat{x}_{t-1}$, it takes does not reduce the rate as much on spot after a negative demand shock.
Optimal Policy under unconditionally commitment
and occasionally binding ZLB constraints

- Targeting Rule, if $\beta \to 1$, $\bar{\pi} \to 0$ and $\frac{\bar{\kappa}}{\sigma}$ is small (as empirical evidence supports):

$$0 \approx (\hat{p}_t - \hat{p}^{ind}_t) + (1 - \bar{p}_o) \frac{\bar{\kappa}}{\bar{\kappa}} \hat{x}_t + \bar{p}_o \frac{\bar{\kappa}}{\bar{\kappa}} (\hat{x}_{t-1} + \frac{1}{\sigma} \hat{\sigma}t-1)$$

$$\hat{\pi}_t = (1 - L) \hat{p}_t, \quad \hat{\pi}^{ind}_t = (1 - L) \hat{p}^{ind}_t, \quad \hat{p}^{ind}_t = \gamma_\pi \hat{p}_{t-1}$$

- Therefore: **price level targeting** if $\beta \to 1$ and $\bar{\pi} \to 0$.

- Even at its full form, it generates precautionary behavior:
  - Reacting to $\hat{x}_{t-1}$, it takes does not reduce the rate as much on spot after a negative demand shock.
  - Reacting to $\hat{\sigma}t-1$, it takes longer to increase the rate after the shock dissipates.
Simulations

- Calibration: Smets and R. Wouters (AER 2007). $s_u^2$ estimated w/ Great Moderation sample and $\bar{\pi} = 3\%$. 
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- First I show IRF’s after two-periods negative demand shocks w/ $\bar{\pi} = 2\%$ and $\epsilon_{u,t} = - (0.5) s_u$ to $\epsilon_{u,t} = - (5.0) s_u$. 

We must take in consideration that, under the ZLB, $E\hat{\pi}_t = 0$ and $E\hat{x}_t = 0$. And so this must taken in consideration when computing $E\hat{\pi}^2_t$ and $E\hat{x}^2_t$. 
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• Next I show simulated welfare losses w/ $\bar{r} = 1\% \quad (\beta = 0.9975)$. Simulated sample w/ $T = 10,000$ w/ Occbin, by Guerrieri and Iacoviello (JME 2015) - fixed seeds.
Simulations

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- Next I show simulated welfare losses w/ \( \bar{\rho} = 1\% \) (\( \beta = 0.9975 \)). Simulated sample w/ \( T = 10,000 \) w/ Occbin, by Guerrieri and Iacoviello (JME 2015) - fixed seeds.

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\]
## Welfare Analysis

### A) Under ZLB constraints

<table>
<thead>
<tr>
<th>Steady States $\bar{r}=1%$</th>
<th>PrOP Rates (%)</th>
<th>StOP Rates (%)</th>
<th>TayR Rates (%)</th>
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</thead>
<tbody>
<tr>
<td>$\bar{\pi}$ $\bar{t}$ $\bar{\bar{p}}_0$</td>
<td>$\lambda$ $E_{\bar{p}_0,t}$</td>
<td>$\lambda$ $E_{\bar{p}_0,t}$</td>
<td>$\lambda$ $E_{\bar{p}_0,t}$</td>
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<tr>
<td>2 3 12.1</td>
<td>2.56 18.9</td>
<td>2.65 19.0</td>
<td>2.78 0.0</td>
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### B) No ZLB constraints

<table>
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<th>Steady States</th>
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<th>TayR Rates (%)</th>
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</table>
Main Results

• I derive optimal precautionary policy under occasionally binding ZLB constraints, when the central bank directly internalize its role in affecting the $p_{o,t}$ by means of the expectations channel.
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- Even at its full form, it generates precautionary behavior, by reacting to lagged output-gaps and policy rates.

- It dominates standard (commitment) optimal policy under occasionally binding ZLB constraints.