Unemployment and Gross Credit Flows in a New Keynesian Framework

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Abstract

The Great Recession of 2008-09 was characterized by high and prolonged unemployment and lack of bank lending. The recession was preceded by a housing crisis that quickly spread to the banking and broader financial sectors. In this paper, we attempt to account for the depth and persistence of unemployment during and after the crisis by considering the relationship between credit and firm hiring explicitly. We develop a New Keynesian model with nominal rigidities in wages and prices augmented by a banking sector characterized by search and matching frictions with endogenous credit destruction. We assume that credit contracts are negotiated bilaterally using a Nash bargaining protocol between borrowers and lenders. In the model, financial shocks are propagated and amplified through significant variation over the business cycle in the endogenous component of the total factor productivity (credit inefficiency gap) arising from the existence of search and matching frictions in the credit market. In response to a financial shock, the model economy produces large and persistent increases in credit destruction, declines in credit creation, and overall declines in excess reallocation among banks and firms. The tightening of the credit market results in a sharp rise in the average interest rate spread and the average loan rate. Due to the increase in credit inefficiency that arises from the reduction in firm-bank matches, total factor productivity declines and unemployment increases. TFP and unemployment take at least 12 quarters to return to baseline.

1 Introduction

The Great Recession and slow recovery was characterized domestically by a deep and prolonged decline in new job creation, an increase in financial volatility, and a decline in bank lending. The net decline in bank lending in all loan categories–including to consumers, to firms, and for real estate related reasons–was a novel feature of the Great Recession compared to previous post-Volker recessions, as was the large decline in job creation by small and medium sized firms. Unemployment remained persistently high two years following the onset of the recovery and weak job creation among small and medium sized firms was an important factor.

In this paper, we examine a potential mechanism for financial shocks to impact employment by incumbent as well as new firms indirectly through their impact on bank lending. We model frictions in the loan market using a search and matching framework with wage rigidities in the labor market. Interestingly, we find that frictions in lending impact unemployment through a variety of mechanisms in our model. First, they change the number of firms with profitable projects who can produce by altering the productivity cutoff for loan acquisition. Second, they impact the continuation probability for individual credit contracts. A decline in the continuation probability results in the severance of firm-bank relationships. Third, credit frictions directly affect labor productivity through the cutoff production probability determining the distribution of firms engaging in production.

The mechanisms highlighted in this paper are consistent with recent evidence from papers such as Boustanifar (2014), Chodorow-Reich (2014), Iyer, Peydr, da Rocha-Lopes, and Schoar (2014) that use detailed bank-firm data, banking reforms or carefully matched labor and credit data to consider linkages between financial shocks and bank lending or bank lending and employment. Chodorow-Reich (2014) finds that given the persistence of banking relationships and difficulty of forming new relationship, pre-crisis clients of banks that were more exposed to the financial crisis had a lower probability of receiving a loan and for those firms that did acquire a loan, the interest spread was higher. Chodorow-Reich (2014) also finds that this credit supply channel had differential effects depending on the size of the firm: small to medium sized firms exposed to banks in poor health experienced significant employment declines compared with larger firms. These heterogeneous effects by firm size are consistent with work by Adrian, Colla, and Shin (2012) who demonstrate that banking relationships may be less important for large firms with multiple sources of debt as well as equity finance.

Several recent papers, Craig and Haubrich (2013), Herrera, Kolar, and Minetti (2014), and Contessi, DiCecio, and Francis (2015) consider the empirical relationship between output or employment and bank lending. These papers look at gross lending flows rather than changes in net credit availability using a methodology developed in Davis, Haltiwanger, and Schuh (1996) more commonly used for understanding labor flows, thereby accounting for simultaneous increases and decreases in lending across banks. This literature emphasizes the existence of heterogeneous patterns of credit creation and contraction at any phase of the business cycle and the effects of the reallocation of credit between firms.

The literature linking credit availability to employment includes a number of early contributions, such as Wasmer and Weil (2004) and den Haan, Ramey, and Watson (2003) who were some of the first papers to use matching frictions in credit markets to explain unemployment. More recent papers such as Becsi, Li, and Wang (2013) and Petrovsky-Nadeau and Wasmer (2014) build macroeconomic models incorporating decentralised lending markets characterized by search-and-matching frictions between borrowers and lenders. den Haan, Ramey, and Watson (2003) develop an agency-cost type of model with exogenous matching rates while Becsi, Li, and Wang (2013) introduce credit rationing and asymmetric information into the Nash bargaining protocol. Several recent papers, including Petrosky-Nadeau (2014) and Petrosky-Nadeau and Wasmer (2015), study matching frictions where both credit and labor markets are decentralized using a dual search approach.

In this paper, we incorporate a decentralized lending market into a New Keynesian model characterized by sticky wages and unemployment. Banks obtain funds by raising retail deposits in a competitive market. In order to produce, an individual firm first must obtain external funding by being matched with a bank. Unmatched banks search for lending opportunities while unmatched firms search for funds to finance their wage bill in advance of production. Matched banks and firms decide whether to maintain or sever their credit relationship, depending on the idiosyncratic productivity of the firm' project. If the firm and the bank choose to cooperate, a loan contract is agreed on and Nash bargaining determines how the joint surplus of the match is shared. The conditions of the loan contract are characterized by a match-specific loan principal and a credit interest rate. In equilibrium, there is a productivity threshold (reservation productivity) such that only those firms with an idiosyncratic productivity level above this threshold are able to produce. Thus, aggregate equilibrium is characterized by a distribution of actively producing firms as well as a distribution of match-specific loan rates.

The search and matching friction in the loan market produces an endogenous inefficiency wedge that appears as an additional input in the aggregate production function. This inefficiency wedge depends on the aggregate probability of continuation for a loan contract as well as on the mass of active producing firms. Financial shocks as well as other aggregate shocks are transmitted and amplified by a first order effect in this credit inefficiency gap that affects aggregate labor productivity and technology.

Our model exhibits a cost channel of monetary policy similar to Ravenna and Walsh (2006), in which firms must finance wage payments in advance of production. The standard implication of the cost channel is that the relevant cost of labor is affected by the interest rate firms pay on loans. However, when the loan rate is the outcome of a bargaining process, as it is in our model, the role of the loan rate is to split the surplus between the borrower (the firm) and the lender (the bank). In this context, the loan rate is irrelevant for the firm's employment decision which ultimately is a consequence of the Nash bargaining solution and the conditions of the credit contract. In our model, a cost channel arises but it depends on the opportunity cost of funds to the bank, not the interest rate charged on the loan. Therefore, changes in monetary policy will influence this opportunity cost and affect the real marginal cost of production, employment, and the equilibrium spread between the average rate on bank loans and the policy interest rate. By the same token, the threshold productivity level, below which the firm is unable to obtain financing, depends on monetary policy. In our model, by allowing for entry and exit by banks as well as for active and inactive firms, monetary policy has effects on employment and output on both the extensive and intensive margins. The latter arises as a reduction in the cost of funds for banks makes it optimal for firms with access to credit to expand employment. The former arises because the lower cost of finance will make it profitable for more firms produce.

We model unemployment similarly to Blanchard and Galí (2010), Galí (2011) and Galí, Smets, and Wouters (2012) as a reinterpretation of the labor market in the standard New Keynesian model with staggered wage setting as in Erceg, Henderson, and Levin (2000). In this context, each household member is indexed by the labor type she is specialized in and by a particular labor disutility index that her labor-type generates if she is employed. Therefore, labor is differentiated and each household member has market power to set its wage according to a Calvo pricing scheme. The existence of market power and wage rigidities produce a gap between aggregate labor demand and the labor force. This gap is related to the difference between the prevailing aggregate real wage and the average disutility of labor expressed in terms of consumption and is positively related to the unemployment rate. Credit conditions affect the marginal cost of labor as well as aggregate labor demand by producing an extensive and intensive margin effect (discussed in detail below). The labor force is also affected by credit conditions via the aggregate marginal rate of substitution of the marginal supplier of labor. Thus credit conditions in this framework have direct and indirect effects on employment.

We first provide some motivating empirical evidence on the business cycle properties of gross credit and job flows as well as the relationship between gross credit flows and unemployment as a conditional response to a financial shock. For the latter we use a simple recursive vector auto-regression. We then develop a New Keynesian model with price and wage frictions as well as search and matching frictions in the loan market to match the decline in output and employment following contractionary monetary policy and financial shocks and determine the mechanisms through which shocks affecting credit are propagated to the labor market and productivity.[1]

2 Empirical evidence on gross credit flows, unemployment and financial shocks

In this section we present some empirical regularities on the relationship between job flows and credit. We first discuss a set of unconditional moments and the business cycle properties of gross credit and job flows. Second we look at the conditional responses of gross credit flows, inflation, employment, and unemployment to financial and monetary policy shocks within a recursively identified VAR. We use these two sets of analyses to motivate the theoretical model we develop subsequently.

[1] Appendix E includes an extension of the model to the case where firms can divert loans to unproductive uses.

2.1 Cyclical properties of gross credit flows and gross job flows

Although we primarily consider net job flows in this paper, by focusing on employment and unemployment, the data on gross job flows provides insight into the factors driving unemployment and the relationship between credit flows and unemployment. We use quarterly data on manufacturing job flows from Faberman (2012) updated with data from the Business Economic Dynamics (BED) database. We also use quarterly data on job flows in all non-governmental sectors from the BED. For credit flows, we use a measure of credit availability derived from information in the Reports of Income and Condition. The Reports of Income and Condition, known as the Call Reports, must be filed every quarter by every bank and savings institution overseen by the Federal Reserve (i.e., those who hold a charter with the Federal Reserve). These reports contain a variety of information from banks' income statements and balance sheets. We use quarterly reported total loans and lending to commercial and industrial enterprises to create measures of credit creation and destruction. Quarterly Call Report data is available beginning in 1979Q1. We use an additional 24 quarters of historical data from Craig and Haubrich (2013). Lending data is then linked to data from the National Information Center (NIC) on mergers and acquisitions during this period. Using the M&A data from the NIC we can remove bias that might arise from counting lending activity at both the acquired and acquiring bank (See Contessi and Francis (2013) for a full discussion of how these data are compiled). We then remove seven investment banks and credit card companies that acquired commercial bank charters during the 2008-09 recession and financial crisis.

In order to determine 'gross credit flows' we use a technique first adapted from the labor literature in Dell'Ariccia and Garibaldi (2005) for credit flows. Define $l_{i,t}$: as total loans for bank i in quarter t. Let $g_{i,t}$ be the credit growth rate for bank i between t and $t-1$, adjusted for mergers or acquisitions. Then we can define:

$$POS_t = \sum_{i|g_{i,t}\geq 0}^{N} \alpha_{i,t} g_{i,t}$$

$$NEG_t = \sum_{i|g_{i,t}<0}^{N} \alpha_{i,t} \left| g_{i,t} \right|$$

where

$$\alpha_{i,t} = \left(\frac{0.5(l_{i,t} + l_{i,t-1})}{\sum_{i=1}^{N} l_{i,t-1}} \right)$$

and $\frac{l_{i,t}+l_{i,t-1}}{2}$ is a measure of the average loan portfolio size of bank i between period t and $t-1$ and where $\sum_{i=1}^{N} l_{i,t-1}$ is the loan portfolio of the banking system in the previous period.

Given these measures of credit creation (POS) and credit destruction (NEG), we can define net lending as $NET = POS - NEG$. We use a similar accounting measure for gross job flows using data directly from the BED (1992Q2-2012Q4) and Faberman (2012) (for 1973Q1-1991:Q4).

Table (5) in Appendix A provides the means and standard deviations of lending and job flow variables for our entire sample (1973Q1 to 2012Q4) and three sub-periods–the Great Moderation, 1984Q1 to 2007Q2; the Great Recession, 2007Q3 to 2009Q2; and the post Recession period, 2009Q3 to 2012Q4.[2] Three features of these summary statistics are notable. First, the mean of loan creation plus loan destruction (SUM) is significantly larger during the Great Recession than during other reported sub-periods. This feature is driven by an increase loan destruction. The sum of loan creation and destruction is a measure of 'churning' in the banking sector. Second, the mean value for net loan creation during each subperiod is positive indicating credit growth in each period including the Great Recession. Net loan creation however is very small during the post Great Recession period due to very weak loan creation. Third, excess loan creation (the difference between the SUM and the absolute value of the NET) is the largest during the Great Moderation. Excess

[2] In this paper we have focused on the behavior of credit and job flows during and following the great recession. Please see Contessi, DiCecio, and Francis (2015) for more detailed statistics on credit flows in general.

loan creation (EXC) measures credit reallocation in excess of what is required to accommodate a change in net credit. For example, if credit creation equals one and credit destruction equals zero, then SUM equals one as does NET, thus EXC equals zero meaning no additional reallocation of credit between firms occurred when credit creation increased.

We find that mean job creation is higher during the Great Moderation than any other period, while job destruction is higher during the Great Recession than other sub-periods implying that there was a significant amount of reallocation of workers across firms during this period. Interestingly, the mean of net job creation is negative during both the Great Moderation and the Great Recession though it is significantly more negative during the Great Recession. In general, we find loan flows are more volatile than job flows but both are more volatile than either GDP or unemployment. We also find that volatilities of loan and job flows rise during the Great Recession and fall afterwards.

In Table (6) in Appendix A we provide relative standard deviations of bank lending flows (total loan creation and destruction) and job flows (job creation and job destruction) as well as their correlation with the cyclical component of either real GDP or the unemployment rate for our entire sample period (1973Q1 to 2012Q4) and three sub-periods (detailed above). We use the HP-filtered log-level of each variable in determining their relative standard deviations and correlations.

We find that loan creation and destruction are much more volatile than real GDP and that the relative volatility of loan destruction increased significantly in the post Great Recession period. Job creation and job destruction are much less volatile than unemployment aside from during the Great Recession when job destruction was more volatile.

In terms of correlations, we find that the cyclical component of loan creation is positively correlated with real GDP in all periods (top panel of table 6), though the correlation is only statistically significant during the post-recession period. Loan creation is acyclic during the Great Moderation. Loan destruction, as expected, is (significantly) counter-cyclical, though the counter-cyclicality is primarily driven by the Great Recession and post Great Recession period when it is strongly counter-cyclical. The sum of loan creation and destruction, a measure of general loan turnover, is acyclic and not significant.

Job creation is strongly negatively correlated with unemployment (bottom panel of table 6), during the Great Recession but strongly positively correlated post recession. This last unusual result could be due to the fact that unemployment did not begin a consistent decline until 2010Q4 despite the fact that job creation began to increase, relatively smoothly, beginning in 2009Q2. Job destruction is positively correlated with unemployment during our entire sample and throughout each subsample, though it is most strongly correlated during the Great Recession. The sum of job creation and job destruction, a measure of labor market churn, is positively correlated with unemployment throughout the sample and sub-samples. The pro-cyclicality of labor market churn is strongest during the Great Recession.

Typically during recessions, credit creation slows and credit destruction sharply rises, particularly for commercial lending. Figure 1 displays quarterly net credit flows (credit creation less credit destruction) of commercial and industrial lending following the trough of the last four recessions. The trough of the recession is dated using NBER dates and depicted as zero; movement in net credit following the trough is graphed for eight quarters. We also graph the average of the past four recessions which is strongly influenced by the 2007-09 recession. Each of the past three recessions–beginning in 1990, 2001, and 2007– display a similar pattern with net credit continuing to contract even as GDP and other indicators rise. Net credit flows rebounded much faster following the recession in 1981 and display a distinctly different pattern. The three more recent recessions follow the same pattern although the 2007-09 recession has the largest and most prolonged decline in credit. Figure 2 considers credit flows separately. The top row depicts credit creation (left graph) and credit destruction (right graph). We find that during the Great Recession, credit creation continued to fall after the NBER dated trough (at zero, where credit creation is set to its mean of 3 percent during all four recessions) which is unlike any of the previous four recessions though its recovery looked similar to the recovery during the 1990 recession. Credit destruction, similarly, was significantly larger in the first quarter following the trough of the 2007-09 recession and in that sense much different than any of the three previous recessions. In the bottom row, the left graph shows a measure of credit reallocation which is the sum of credit creation and destruction. The movements in reallocation for the 2007-09 recessions looked somewhat similar to the 1990 and 2001 recessions initially, but then displayed a long and persistent decrease. The right graph in the bottom row provides another measure of credit reallocation–excess reallocation–which is the sum of creation and destruction less the net. Excess reallocation declined significantly through the trough of the 2007-09 recession and then recovered after approximately three quarters.

Similarly, net job flows demonstrate a pro-cyclical pattern, where job creation slows during recessions and job destruction increases to some degree. During the Great Recession, job destruction increased by much more than during previous recessions (percentage wise) and was a much larger contributor to the increase in unemployment than during previous recessions. The job creation rate was also depressed for much longer than usual, which reduced the job finding rate more significantly than in previous recessions. Figure 3 considers net job creation in manufacturing following the trough of the last four recessions. It is clear from the figure, that net job growth during the Great Recession displays a similar pattern particularly compared with the 2001 recession but was significantly deeper. At the trough, unemployment was still rising and recovery in net job creation did not occur until approximately four quarters after the NBER dated the trough of the 2007-09 recession.

Figure 4 provides measures of a set of aggregate variables for the last four recessions: total factor productivity, average labor productivity, real GDP, and the unemployment rate. TFP is shown in percentage change terms, while the log of average labour productivity is smoothed using a Baxter King filter, and the log real GDP is detrended.

2.2 A Simple VAR

In this section, we investigate the relationship among credit flows and macro-aggregates using a simple vector auto-regression framework.

We use the following standard representation of a reduced form VAR:

$$Z_t = A(L)Z_{t-1} + v_t$$

where $A(L) = A_1 + A_2L + ... + A_pL^p, p < \infty$. v_t are the reduced form residuals which are related to the structural shocks ϵ_t via the structural matrix A_0, where $\epsilon_t = A_0 v_t$, and with variance covariance matrix $E[v_t v_t'] = V$. We identify our two shocks of interest, monetary policy and financial, by contemporaneous restrictions. As in Christiano, Eichenbaum, and Evans (2005) variables ordered before the Federal Funds rate in the VAR do not respond contemporaneously to monetary policy shocks. Similarly, using the same identification as Gilchrist and Zakrajšek (2012), variables ordered before the financial shock do not respond contemporaneously to financial shocks.

The data used in our analysis is the following:

$$Z_t = \left[\Delta \ln \frac{Y_t}{N_t}, \Delta \ln N_t, U_t, \pi_t, EBP_t, i_t, GCF_t\right] \tag{1}$$

The variables included are the growth rate of labor productivity $\Delta \ln \frac{Y}{N}$, the growth rate of employment $\Delta \ln N_t$, the unemployment rate U_t, the GDP deflator as a measure of inflation π_t, the excess bond premium EBP_t, the Federal Funds rate,i_t as the measure of the monetary policy stance and a pair of gross credit flows rates (credit creation and credit destruction) denoted by $GCF_t = [POS_t, NEG_t]$. Each data series is seasonally adjusted using the X-12 ARIMA procedure.

We have not explicitly taken into consideration quantitative easing enacted by the Federal Reserve, therefore our measure of the monetary policy stance under-represents the looseness of monetary policy [3].

The excess bond premium (EBP), sometimes referred to as the 'GZ credit spread' is taken from Gilchrist and Zakrajšek (2012). It is constructed from the credit spread of U.S. non-financial corporate bonds over Treasury bills and decomposed into two parts. The first is a component that captures systematic movements in the default risk of individual firms measured using Merton (1974)'s distance to default model. The second is a residual component, the excess bond premium, which is the variation in the average price of bearing exposure to U.S. corporate credit risk that is not otherwise compensated for by the expected default premium. The EBP has been used in many recent papers as a measure of financial risk (for example, Boivin, Kiley, and Mishkin 2010 and Christiano, Motto, and Rostagno 2013). It is a good measure of unanticipated changes in financial markets.

As noted above, the VAR is identified using a recursive ordering such that the last ordered variable responds contemporaneously to all shocks. Given this framework, we need at least 28 restrictions and with the short run restrictions, our model is exactly identified.

Figures **5 and 6 in appendix A** present the impulse responses to a one standard deviation increase in the EBP as the measure of a financial shock. The first figure in this series (figure 5) shows the responses of labor productivity, employment, unemployment and inflation to a one standard deviation increase in the

[3] Since monetary policy is not the focus of this paper, we have not used various measures of the 'shadow' Federal Funds rate (see for example, Krippner (2013) among others)

excess bond premium. The responses are consistent with Gilchrist and Zakrajšek (2012). Employment and unemployment as well as inflation respond sluggishly–and persistently–to the shock, with unemployment peaking at roughly 6 quarters post shock and returning to baseline within 12 to 14 quarters. The next figure, figure 6, depicts the response of credit. There are two notable features of this panel of impulse responses. First, the initial response of credit creation to a financial shock (one standard deviation increase in the EBP), is to *increase*. Note that due to the ordering of the VAR, credit flows can respond contemporaneously to financial and monetary shocks. The increase in credit creation is likely due to firms' and consumers' draw down of credit commitments, such as lines of credit, which creates an initial increase in lending even though few new loans are actually extended. Credit destruction initially falls as well but the drop is not statistically significant. This surprising decline in credit destruction initially is also possibly driven by firms' and consumers' use of credit lines. After the initial impact, credit creation declines quickly and credit destruction rises. Credit creation reaches a trough after approximately four quarters, but rises slowly over the next ten quarters to return to baseline so that a full recovery in lending activity is not observed for three to four years following a financial shock. Similarly, credit destruction rates peak at approximately four quarters but slowly return to baseline after approximately ten quarters.

3 The model economy

The model economy is populated by households, banks, firms, and a central bank. The representative household is composed by a continuum of members that supply differentiated labor to firms, hold cash and bank deposits, and purchase final output in the goods market. Firms seek financing, hire labor financed by bank loans and produce output. Banks accept deposits, sometimes hold reserves with the central bank and finance the wage bill of firms.[4] The central bank pays interest on reserve deposits for those banks not able to extend a loan in a given period. Three aspects of the model are of critical importance. First, it is assumed households cannot lend directly to firms. While this type of market segmentation is taken as exogenous, it could be easily motivated by assuming informational asymmetries under which households do not have access to technology to monitor firms while banks do. This asymmetry also forces firms to make up-front payments to workers to secure labor. Second, lending activity involving firms and banks occurs in a decentralized market characterized by random matching. Third, we assume all payment flows must be settled at the end of each period. At the beginning of each period, aggregate shocks are realized and households deposit funds with a bank. The market for deposits is competitive. In the lending market, firms seek funding to make wage payments. Firms are subject to aggregate and idiosyncratic productivity shocks and these determine whether it is profitable for a firm to operate and, if it is, at what scale. If a firm is not already matched with a bank, it must seek out a new lender. Similarly, banks not already matched with a firm must search for borrowers. After the loan market closes, firms and workers produce and households consume, while unmatched banks deposit their funds with the central bank and receive an interest rate matching the interest rate on deposits. After all markets close all net payment flows are settled. Therefore, loans are not risky and there is no possibility of default. At the end of the period, banks receive repayment from firms and the bank transfers all its profits (positive or negative) to the representative household.

3.1 Households

Each household has a continuum of members. Following Galí (2011), each household member is represented by the unit square and indexed by $(i, j) \in (0, 1)^2$. Where i denotes the type of labor service in which a given household member is specialized and j determines the dis-utility from work for each household member. The dis-utility from work is given by $\chi_t j^{\overline{\varphi}}$ if employed and zero otherwise with χ_t being an exogenous preference shifter. As is standard in the unemployment literature, we assume full risk sharing of consumption among household members (see for example, Andolfatto 1996). Utility from consumption is separable and logarithmic in a CES index of the quantities consumed of the different goods available. Given separability of preferences between consumption and dis-utility from work, full risk sharing implies $C_t(i, j) = C_t \quad \forall i, j$, where $C_t(i, j)$ is the consumption for a household member specialized in labor type i and having dis-utility of work $\chi_t j^{\overline{\varphi}}$.

Each household member has a period utility function given by

$$U(C_t, j) = \log C_t - \mathbf{1}_t(i, j)\, \chi_t j^{\overline{\varphi}}, \tag{2}$$

[4]Banks hold reserves with the central bank when they cannot find a project to fund. The process is explained below.

where $\mathbf{1}_t(i,j)$ is an indicator function taking the value of one if the corresponding household member is employed and zero otherwise. Aggregating across all household members yields the household period utility function denoted by $U(C_t, N_t(i), \chi_t)$ and given by:

$$U(C_t, N_t(i), \chi_t) = \log C_t - \chi_t \int_0^1 \frac{N_t(i)^{\overline{\varphi}+1}}{1+\overline{\varphi}} di$$

where $N_t(i)$ is the fraction of household members specialized in labor type i who are employed during the period. In other words, $N_t(i)$ is the employment rate or aggregate demand during period t among workers specialized in labor type i. Aggregate household consumption is given by the standard CES aggregator:

$$C_t = \left(\int_0^1 C_t(j)^{\frac{\epsilon_p - 1}{\epsilon_p}} dj \right)^{\frac{\epsilon_p}{\epsilon_p - 1}} \tag{3}$$

Optimization over consumption by households implies the following demand schedule for each differentiated good j, $C_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_p} C_t$ such that $\int_0^1 P_t(j) C_t(j) dj = P_t C_t$, where P_t is the final goods price index (i.e, the retail price index) is $P_t = \left(\int_0^1 P_t(j)^{1-\epsilon_p} dj \right)^{\frac{1}{1-\epsilon_p}}$.

The household problem

We assume the household enters the period with money holdings given by M_{t-1} and deposits a fraction of its money holdings, denoted by D_t, in the bank. Each household receives a nominal lump-sum transfer T_t from the government. Employed household members are paid their labor income in advance. The household uses its labor income and money holdings, net of deposits, to buy a continuum of final goods subject to the CIA constraint and the sequence of budget constraints. The representative household problem is given by

$$\max E_t \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t - \chi_t \int_0^1 \frac{N_t(i)^{\overline{\varphi}+1}}{1+\overline{\varphi}} di \right\} \tag{4}$$

subject to the cash in advance constraint

$$P_t C_t \leq M_{t-1} + T_t - D_t + \int_0^1 W_t(i) N_t(i) di \tag{5}$$

and where end of period money holdings are

$$M_t = (1+i_t) D_t + T_t + \Pi_t^b + \Pi_t^I + \Pi_t^f + M_{t-1} - D_t + \int_0^1 W_t(i) N_t(i) di - P_t C_t \tag{6}$$

where i_t is the nominal net interest rate on deposits, Π_t^b, Π_t^f, and Π_t^I are nominal profits transferred respectively by banks, intermediate and final good producers. The household takes as given the distribution of wages $\{W_t(i)\}_{\forall i}$ and employed household members for each labor type $\{N_t(i)\}_{\forall i}$. The household optimality condition with respect to consumption is then given by the standard Euler equation

$$\frac{1}{C_t} = \beta E_t \left\{ \left(\frac{1+i_t}{1+\pi_{t+1}} \right) \frac{1}{C_{t+1}} \right\} \tag{7}$$

where $\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}$ is the net inflation rate. In this case, the marginal utility of consumption is equal to: $\frac{1}{C_t} = \lambda_t + \mu_t$ where λ_t and μ_t are the multipliers associated with the budget constraint and the CIA constraint respectively. The stochastic discount factor is distorted by the nominal interest rate and it is given by $\Delta_{t,t+1} = \beta \left(\frac{1+i_t}{1+\imath_{t+1}} \frac{C_t}{C_{t+1}} \right)$.

Wage setting Workers specialized in a given type of labor, reset their nominal wage with probability $1-\theta_w$ each period. Following (Erceg, Henderson, and Levin, 2000), when re-optimizing wages during period t, workers choose a wage W_t^* in order to maximize their household utility taking as given all aggregate variables. Household workers of type i face a sequence of labor demand schedules of the form: $N_t(i) = \left(\frac{W_t(i)}{W_t}\right)^{-\varepsilon_w} \int_z N_t(z)\, dz$ where W_t denote the aggregate wage index given by $W_t = \left(\int_0^1 W_t(i)^{1-\epsilon_w} di\right)^{\frac{1}{1-\epsilon_w}}$ and $\int_z N_t(z)\, dz$ denotes the aggregate labor demand across all *active* intermediate good producers indexed by z.

The wage setting optimization problem for the household workers of type i is specified as

$$\max_{W_t^*} E_t \sum_{t=0}^{\infty} (\beta\theta_w)^k \left\{ \log C_{t+k} - \chi_{t+k} \int_0^1 \frac{N_{t+k|t}^{\overline{\varphi}+1}}{1+\overline{\varphi}} dz \right\} \tag{8}$$

subject to

$$N_{t+k|t} = \left(\frac{W_t^*}{W_{t+k}}\right)^{-\varepsilon_w} \int_z N_{t+k}(z)\, dz \tag{9}$$

and the CIA and budget constraints.[5] The first order condition for wage setting is

$$E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left\{ \left(\frac{W_t^*}{P_{t+k}} - \frac{\varepsilon_w}{(\varepsilon_w - 1)} MRS_{t+k|t} \right) \left(\frac{N_{t+k|t}}{C_{t+k}} \right) \right\} = 0 \tag{10}$$

where $MRS_{t+k|t}$ denotes the marginal rate of substitution between consumption and employment for a type i worker whose wage is reset during period t, given by $MRS_{t+k|t} = C_{t+k}\chi_{t+k}\left(N_{t+k|t}\right)^{\overline{\varphi}}$. Under Calvo wage setting, the aggregate wage index in real terms is:

$$1 = \theta_w \left(\frac{w_{t-1}}{w_t} \frac{1}{\Pi_t} \right)^{1-\epsilon_w} + (1-\theta_w) \left(\frac{w_t^*}{w_t} \right)^{1-\epsilon_w} \tag{11}$$

The recursive formulation of the wage setting optimality condition, expressed in terms of the real wage, is given by the following set of three equations:

$$f_{1,t} = \left(\frac{\varepsilon_w}{\varepsilon_w - 1} \right) f_{2,t} \tag{12}$$

$$f_{1,t} = (w_t^*)^{1-\varepsilon_w} (w_t)^{\varepsilon_w} \frac{N_t}{C_t \Delta_t^w} + \beta\theta_w E_t \left(\frac{1}{\Pi_{t+1}} \right)^{1-\varepsilon_w} \left(\frac{w_t^*}{w_{t+1}^*} \right)^{1-\varepsilon_w} f_{1,t+1} \tag{13}$$

$$f_{2,t} = \chi_t \left(\frac{w_t^*}{w_t} \right)^{-\varepsilon_w(1+\overline{\varphi})} \left(\frac{N_t}{\Delta_t^w} \right)^{(1+\overline{\varphi})} + (\beta\theta_w) E_t \left(\frac{1}{\Pi_{t+1}} \right)^{-\varepsilon_w(1+\overline{\varphi})} \left(\frac{w_t^*}{w_{t+1}^*} \right)^{-\varepsilon_w(1+\overline{\varphi})} f_{2,t+1} \tag{14}$$

where w_t^* is the optimal real wage, $\Pi_t = 1 + \pi_t$ is the gross inflation rate, $f_{1,t}$ and $f_{2,t}$ are auxiliary variables and Δ_t^W is the wage dispersion index given by $\Delta_t^W = \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w} dj$ which can be written recursively as:

$$\Delta_t^w = \theta_w \left(\frac{w_{t-1}}{w_t} \frac{1}{\Pi_t} \right)^{-\epsilon_w} \Delta_{t-1}^w + (1-\theta_w) \left(\frac{w_t^*}{w_t} \right)^{-\epsilon_w}. \tag{15}$$

[5] Notice that $N_{t+k|t}$ denotes the quantity demanded in period $t+k$ of a labor type whose wage was last reset in period t.

Unemployment dynamics

Following Galí (2010), using household welfare as a criterion and taking as given current aggregate labor market conditions, the worker indexed by (i,j) is willing to work in period t if and only if the real wage is greater than or equal to the disutility of labor relative to the marginal value of income, λ_t, that is:

$$\frac{W_t(i)}{P_t} \geq \frac{\chi_t j^{\overline{\varphi}}}{\lambda_t} = (1+i_t)C_t\chi_t j^{\overline{\varphi}} \tag{16}$$

since $\lambda_t = \frac{1}{(1+i_t)C_t}$ and the disutility of work for a worker specialized in type i labor is $\chi_t j^{\overline{\varphi}}$. Let $L_t(i)$ be the marginal supplier of type i labor. Notice, the marginal supplier of type i labor satisfies the above equation with equality, since she is indifferent between working or not working.The labor force or aggregate participation condition is obtained by integrating over all the marginal suppliers, $L_t = \int_0^1 L_t(i)di$. Then, the aggregate supply of labor is defined as

$$w_t = (1+i_t)\, C_t\chi_t\,(L_t)^{\overline{\varphi}} \tag{17}$$

where w_t denotes the average real wage of the economy. In the presence of wage rigidities, labor force dynamics are mostly driven by wealth effects, that is, by the inverse of the marginal value of income. The CIA constraint implies a gross nominal interest rate that acts as a consumption tax, affecting the marginal utility of consumption. Therefore, changes in C_t and i_t induce shifts in the labor supply.

The unemployment rate is defined as

$$U_t = 1 - \frac{N_t}{L_t} \tag{18}$$

with N_t being aggregate employment which corresponds to the following index:

$$N_t = \int_z \int_0^1 N_t\,(i, \omega_{z,t})\, didz \tag{19}$$

where as explained below, $N_t(i, \omega_{z,t})$ is the demand for labor type i by the intermediate producer z, who is characterized by idiosyncratic productivity $\omega_{z,t}$.

3.2 Firms

The production side of the model is characterized by a two-sector structure that distinguish between intermediate and final good producers as in Walsh (2005). Firms in the intermediate good sector must have a credit relationship with a bank before production takes place. Only the subset of intermediate good producers that obtain funding will hire workers and produce. The market for intermediate goods is competitive. Each producing firm in the intermediate good sector hires a continuum of workers that includes each type of labor services offered by the household.

Firms in the final good sector purchase the intermediate good and costlessly transform it into a continuum of differentiated final goods sold to the household in a market characterized by monopolistic competition. We assume final goods firms face Calvo pricing restrictions.[6]

3.2.1 Final good producers

We assume there is a continuum of monopolistically competitive firms indexed by j, each producing a differentiated final good. All firms in the final good sector have access to the following technology:

$$Y_t^f(j) = X(j) \tag{20}$$

where $X_t(j)$ is the quantity of the single intermediate good used to produce the final good variety j. Final good producers purchase $X(j)$ from intermediate good producers in a competitive market at the common

[6]The separation between final and intermediate good sectors simplifies the difficulties associated with having a producing firm set its output price and bargain with a bank simultaneously.

price P_t^I and sell their output directly to households as a differentiated final good. Each final good producer faces the following demand schedule obtained from the household decision problem

$$Y_t^f(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_p} C_t \tag{21}$$

where C_t is the aggregate final demand for final or consumption goods.

Price setting Prices for final goods are sticky as in Calvo (1983). Let $1-\theta_p$ be the probability that a firm adjusts its price each period. The nominal total cost for a final good producer of variety j is $TC_t^n(j) = P_t^I X(j)$ with nominal marginal cost $MC_t^n(j) = P_t^I$. As usual, by symmetry, all intermediate good producers who set prices in period t will choose the same price, denoted by P_t^*, since they face an identical problem given by

$$\max_{P_t^*} E_t \sum_{k=0}^{\infty} (\theta_p)^k \Delta_{t,t+k} \left\{ P_t^* Y_{t+k|t}^f - TC_t^n \left(Y_{t+k|t}^f \right) \right\} \tag{22}$$

s.t

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon_p} C_{t+k} \quad \text{for } k = 0, 1, 2, ..., \tag{23}$$

where $\Delta_{t,t+k}$ is the household stochastic discount factor and $Y_{t+k|t}^f$ denotes the demand faced at $t+k$ for a firm that last reset its price in period t, which is consistent with the households' optimality condition with respect to each final good variety. The resulting first order condition to the price setting problem implies,

$$E_t \sum_{k=0}^{\infty} (\theta_p)^k \Delta_{t,t+k} Y_{t+k|t}^f \left\{ \frac{P_t^*}{P_{t+k}} - \frac{\epsilon_p}{\epsilon_p - 1} \frac{P_{t+k|t}^I}{P_{t+k}} \right\} = 0 \tag{24}$$

Final good producers obtain nominal profits at the end of the period of $\Pi_t^f(j) = P_t(j) Y_t^f(j) - P_t^l X_t(j)$. The competitive monopolistic structure together with Calvo nominal price rigidites implies the following aggregate price index P_t for the final good:

$$P_t^{1-\epsilon_p} = \theta_p (P_{t-1})^{1-\epsilon_p} + (1-\theta_p)(P_t^*)^{1-\epsilon_p} \tag{25}$$

The recursive formulation of the optimal price setting equation is

$$g_{1,t} = \left(\frac{\epsilon_p}{\epsilon_p - 1}\right) g_{2,t} \tag{26}$$

$$g_{1,t} = \Delta_{t,t} C_t \Pi_t^* + \theta_p E_t \left(\frac{\Pi_t^*}{\Pi_{t+1}^*}\right) g_{1,t+1} \tag{27}$$

$$g_{2,t} = \Delta_{t,t} \frac{1}{\mu_t^p} C_t + \theta_p E_t g_{2,t+1} \tag{28}$$

where $\Pi_t^* = \frac{P_t^*}{P_t}$ and $g_{1,t}$ and $g_{2,t}$ are auxiliary variables.

3.2.2 Intermediate good producers

We assume intermediate good producers must search for external funding in order to produce. A firm with financing operates a production technology and produces a homogeneous intermediate good indexed by z in a perfectly competitive market. Nominal total costs for an intermediate goods firm includes total labor cost, $R_t^l(j, \omega_{z,t}) W_t N_t(\omega_{z,t})$, plus the fixed cost of production, $P_t^I x^f$, where $R_t^l(j, \omega_{z,t})$ is the gross loan interest rate negotiated bilaterally between bank "j" and firm "z".

In this subsection, we first describe the technology and labor demand for a firm that has obtained financing. In the next section, we describe the loan market and the decisions each intermediate goods producer must take when searching for external funds or after obtaining a bank loan.

Technology and labor demand If an intermediate goods producer is matched with a bank, it is endowed with the following technology:

$$y_t(\omega_{z,t}) = \xi^{pf} A_t \omega_{z,t} N_t(\omega_{z,t})^{\alpha} \tag{29}$$

where ξ^{pf} is a scale technology parameter, A_t is the aggregate productivity level, $\omega_{z,t}$ is a firm-specific idiosyncratic productivity level drawn from a uniform distribution function $G(\omega)$ with support $[\underline{\omega}\ \bar{\omega}]$, and $N_t(\omega_{z,t})$ is the firm's employment index given by

$$N_t(\omega_{z,t}) = \left(\int_0^1 N_t(i, \omega_{z,t})^{\frac{\varepsilon_w - 1}{\varepsilon_w}} di \right)^{\frac{\varepsilon_w}{\varepsilon_w - 1}} \tag{30}$$

where $N_t(i, \omega_z)$ is the demand for labor type i by firm z. Cost minimization, taking wages as given, implies the following demand for labor type i:

$$N_t(i, \omega_{z,t}) = \left(\frac{W_t(i)}{W_t} \right)^{-\varepsilon_w} N_t(\omega_{z,t}) \quad \text{for all } i \tag{31}$$

The aggregate demand for labor type i is obtained by aggregating $N_t(i, \omega_z)$ across all producing firms

$$\begin{aligned} N_t(i) &= \int_z N_t(i, \omega_z)\, dz \\ &= \left(\frac{W_t(i)}{W_t} \right)^{-\varepsilon_w} \int_z N_t(\omega_z)\, dz \end{aligned}$$

where $\int_z N_t(\omega_z)\, dz$ denotes the aggregate labor index of all producing intermediate goods firms during period t and W_t denotes the aggregate wage index. Wages must be paid in advance of production to the household and can only be funded by external bank finance. The nominal loan each intermediate goods producer must obtain is given by their wage bill during period t :

$$L_t(\omega_{z,t}) = \int_0^1 W_t(i)\, N_t(i, \omega_{z,t})\, di = W_t N_t(\omega_{z,t}) \tag{32}$$

We assume loans are paid back to the bank with a gross interest rate $R_t^l(j, \omega_{z,t})$ at the end of the period and no default occurs. End of the period nominal profits during period t, for an intermediate good producer with funding and idiosyncratic productivity, $\omega_{z,t}$, is given by

$$\Pi_t^I(\omega_{z,t}) = P_t^I \left(y_t(\omega_{z,t}) - x^f \right) - R_t^l(j, \omega_{z,t})\, W_t N_t(\omega_{z,t}) \tag{33}$$

3.3 A decentralized loan market

We assume the process of finding a credit partner is costly in terms of time and resources. Intermediate good producers and banks face search and matching frictions that prevent instantaneous trading in the loan market, implying not all market participants will end up matched at a given point in time. Upon a successful match, bilateral Nash bargaining between the parties determines the firm's employment level and the way the match surplus is shared. We allow for both exogenous and endogenous destruction of credit matches, and a matching technology that determines the aggregate flow of new credit relationships over time as a function of the relative number of lenders and borrowers searching for credit partners.

We assume a continuum of banks and firms with the number of banks seeking borrowers varying endogenously and being determined by a free entry condition to the loan market. We assume that banks have a constant returns to scale technology for managing loans so that we can treat each loan as a separate match between a bank and a firm. Each intermediate good producer is endowed with one project and is either searching for funding or involved in an ongoing credit contract with a bank. If a firm is matched with a bank, the bank extends the necessary funds to allow the firm to hire workers and produce.[7]

[7] In appendix E, we introduce the possibility that borrowers may abscond with the funds obtained from banks. Given this,

3.3.1 The matching process

Firms searching for external funds, f_t, are matched to banks seeking for borrowers, b_t^u, according to the following constant returns to scale matching function

$$m_t = \mu f_t^{\nu} (b_t^u)^{1-\nu} \tag{34}$$

The function m_t determines the flow of new credit contracts during date t; μ is a scale parameter that measures the productivity of the matching function and $0 < \nu < 1$ is the elasticity of the match arrival with respect to the mass of searching firms.

Matching rates The variable $\tau_t = f_t/b_t^u$ is the measure of credit market tightness. The probability that an intermediate good producer with an unfunded project is matched with a bank seeking to lend at date t is denoted by p_t^f and is given by

$$p_t^f = \mu \tau_t^{\nu-1} \tag{35}$$

Similarly, the probability that any bank seeking borrowers is matched with an unfunded entrepreneur at time t is denoted by p_t^b and is given by

$$p_t^b = \mu \tau_t^{\nu} \tag{36}$$

Since $\tau_t = p_t^b/p_t^f$, a rise in τ_t implies it is easier for a bank to find a borrower relative to a firm finding a lender and so corresponds to a tighter credit market and reducing the expected time a bank must search for a credit partner, lowering the bank's expected pecuniary search costs. At any date t the number of newly matched banks must equal the number of newly matched firms, or $p_t^b b_t^u = p_t^f f_t$.

Separations and the evolution of loan contracts Credit relationships may end exogenously with probability δ_t whose process is explained below. Contractual parties engaged in a credit relationship that survives this exogenous separation hazard can also decide to dissolve the contract depending on the realization of the productivity of the firm's project. Productivity is taken to be $A_t \omega_{z,t}$, where A_t is the aggregate component common to all firms and ω_{zt} is a firm-specific idiosyncratic productivity shock with distribution $G(\omega_{z,t})$. The decision to endogenously dissolve a credit relationship is characterized by an optimal reservation policy with respect to $\omega_{z,t}$ and denoted by $\tilde{\omega}_t$. If the realization of $\omega_{z,t}$ is above the firm specific productivity cut-off, both parties agree to continue the credit relationship, allowing the entrepreneur to produce conditional on surviving the exogenous separation hazard. On the contrary, if the realization of $\omega_{z,t}$ is below $\tilde{\omega}_t$, both parties choose to dissolve the credit relationship. The probability of endogenous termination of a credit match is $\gamma_t(\tilde{\omega}_t) \equiv prob(\omega_{z,t} \leq \tilde{\omega}_t) = G(\tilde{\omega}_t)$ while the overall separation rate is $\delta_t + (1-\delta_t)\gamma_t(\tilde{\omega}_t)$. Existence and uniqueness of the optimal reservation policy $\tilde{\omega}_t$ are shown in appendix C.

Let f_{t-1}^m be the measure of intermediate good producers that enter period t matched with a bank. Of those, $(1-\delta_t) f_{t-1}^m$ firms survive the exogenous hazard and a fraction γ_t of the survivals receive idiosyncratic productivity shocks that are less than $\tilde{\omega}_t$ and so do not produce. The number of intermediate good producers that actually produce in period t, therefore, is $(1-\delta_t)(1-\gamma_t)f_{t-1}^m$. The number of firms in a credit relationship at the end of period t, denoted by f_t^m, is given by the number of firms producing during time t plus all the new matches formed at time t. Then, the evolution of f_t^m is expressed as

$$f_t^m = \varphi_t(\tilde{\omega}_t) f_{t-1}^m + m_t \tag{37}$$

where $\varphi_t(\tilde{\omega}_t)$ is the overall continuation rate of a credit relationship defined to be:

$$\varphi_t(\tilde{\omega}_t) = (1-\delta_t)(1-\gamma_t(\tilde{\omega}_t)) \tag{38}$$

and $1 - \varphi_t(\tilde{\omega}_t) = \delta_t + (1-\delta_t)\gamma_t(\tilde{\omega}_t)$ denotes the overall separation rate.

We normalize the the total number of intermediate good producers in every period to one and assume that if a credit relationship is exogenously dissolved at time t, both parties start searching immediately during the period. If the credit relationship survives the exogenous separation hazard but then endogenously dissolves,

banks introduce an incentive compatibility constraint in the loan contract. We assume that in the case of absconding the firm is able to produce and generate profits but does not repay the bank. The bank is able to recover an exogenous fraction of the profits made by the intermediate good producer who is then barred from participation in the loan market. The optimal credit contract in this scenario is characterized by credit rationing. A bank will prefer to loan only a fraction of its funds and leave the remainder as excess reserves at the central bank.

both parties must wait until the next period to start searching for a credit partner again. This assumption implies that the number of firms seeking finance during period t, which we have denoted by f_t, is equal to the number of searching firms at the beginning of time t, $(1 - f^m_{t-1})$ plus the number of firms that started the period matched with a bank but were exogenously separated $(\delta_t f^m_{t-1})$. Therefore,

$$f_t = 1 - (1 - \delta_t) f^m_{t-1}. \tag{39}$$

Notice that there are still some firms that have been endogenously separated but cannot search in period t. These firms are unmatched but waiting to search again next period.

Gross credit flows Our timing assumption implies that the fraction $p^f_t \delta_t f^m_{t-1}$ of matched intermediate good producers that were exogenously separated during time t, are able to find a new credit relationship within the same period of time. Credit creation, CC_t, is then defined as equal to the number of newly created credit relationships at the end of time t net of the number of exogenous credit separations that are successfully re-matched in a given period. That is

$$CC_t = m_t - p^f_t \delta_t f^m_{t-1}. \tag{40}$$

The credit creation rate, cc_t is

$$cc_t = \frac{m_t}{f^m_{t-1}} - p^f_t \delta_t. \tag{41}$$

Credit destruction, CD_t, is defined as the total number of credit separations at the end of time t, $(1 - \varphi_t(\tilde{\omega}_t)) f^m_{t-1}$ net of the number of exogenous credit separations that are successfully re-matched in a given period. Thus,

$$CD_t = (1 - \varphi_t(\tilde{\omega}_t)) f^m_{t-1} - p^f_t \delta_t f^m_{t-1} \tag{42}$$

The credit destruction rate, cd_t, is

$$cd_t = (1 - \varphi_t(\tilde{\omega}_t)) - p^f_t \delta_t. \tag{43}$$

The implied credit reallocation rate is defined by

$$cr_t = cc_t + cd_t \tag{44}$$

and net credit growth rate is

$$cg_t = cc_t - cd_t \tag{45}$$

3.3.2 Intermediate good producers and the loan market

In our setting, a credit relationship is a contract between bank j and intermediate good producer z that allows the latter to operate a specific production technology, hire workers and pay their wage bill in advance of production. As long as the credit contract prevails, the firm receives sufficient external funds to pay workers in advance of production in each period. After selling its output to the final goods producers, the firm repays its debt with the bank and transfers all remaining profits to the household. Therefore, as in Fiore and Tristani (2013), we abstract from the endogenous evolution of net worth by assuming firms do not accumulate internal funds after repaying their debt.

Value functions If the intermediate good producer obtains financing its instantaneous real profit flow is

$$\pi^I_t(\omega_{z,t}) = \frac{P^l_t}{P_t}\left(y_t(\omega_{z,t}) - x^f\right) - R^l_t(j, \omega_{z,t}) w_t N_t(\omega_{z,t}) \tag{46}$$

where $\frac{P^l_t}{P_t}$ is the price of the intermediate good expressed in terms of the final good price index and w_t is the real wage index. The loan principle expressed in real terms is the wage bill of the firm given by $l_t(j, \omega_{z,t}) = w_t N_t(\omega_{z,t})$. The loan contract requires the repayment of the total debt with the bank, including interest, $R^l_t(j, \omega_{z,t}) w_t N_t(\omega_{z,t})$ within the same period. It is useful to define the mark-up of final goods over intermediate good price as $\mu^p_t = \frac{P_t}{P^l_t}$ and express $\pi^I_t(\omega_{z,t})$ as

$$\pi^I_t(\omega_{z,t}) = \frac{y_t(\omega_{z,t}) - x^f}{\mu^p_t} - R^l_t(j, \omega_{z,t}) w_t N_t(\omega_{z,t}) \tag{47}$$

Profits depend on the status of the intermediate good producer, that is, whether the firm is searching for external funds or if it is producing. A firm searching for external funds obtains zero real profits since we assume there are no extra search costs when a producer is searching for funding. Under these assumptions, the firm's decision-making is characterized by two value functions: The value of being matched with a bank and able to produce at date t, denoted by $V_t^{FP}(\omega_{z,t})$ and the value of searching for external funds at date t, denoted by V_t^{FN}, both measured in terms of current consumption of the final good. $V_t^{FP}(\omega_{z,t})$ is given by

$$V_t^{FP}(\omega_{z,t}) = \pi_t^I(\omega_{z,t}) + \mathrm{E}_t\Delta_{t,t+1}\left\{\delta_t V_{t+1}^{FN} + (1-\delta_t)\int_{\underline{\omega}}^{\overline{\omega}} \max(V_{t+1}^{FP}(\omega_{z,t+1}), V_{t+1}^{FN})dG(\omega)\right\} \tag{48}$$

where $\Delta_{t,t+1} = \beta\lambda_{t+1}/\lambda_t$ is the household stochastic discount factor. The value of producing is the flow value of current real profits plus the expected continuation value. At the end of the period, the credit relationship is exogenously dissolved with probability δ_t, and the firm must seek new financing. With probability $(1-\delta_t)$, the firm survives the exogenous separation hazard. In the latter case, only those firms receiving an idiosyncratic productivity realization $\omega_{z,t+1} \geq \tilde{\omega}_{t+1}$ will remain matched and produce during next period. Firms with $\omega_{z,t+1} < \tilde{\omega}_{t+1}$ endogenously separate from their bank and obtain V_{t+1}^{FN}.

The value of searching for external funds $\left(V_t^{FN}\right)$ for a firm at date t expressed in terms of current consumption is

$$V_t^{FN} = p_t^f \mathrm{E}_t\Delta_{t,t+1}\left[\delta_t V_{t+1}^{FN} + (1-\delta_t)\int_{\underline{\omega}}^{\overline{\omega}} \max(V_{t+1}^{FP}(\omega_{z,t+1}), V_{t+1}^{FN})dG(\omega)\right] + \left(1-p_t^f\right)V_{t+1}^{FN} \tag{49}$$

where p_t^f is the probability of matching with a bank. Notice that we assume matches made in period t do not produce until $t+1$. With probability $(1-p_t^f)$, the firm does not match and must continue searching for external funds during next period's loan market.

3.3.3 Banks and the loan market

We assume there is an infinite mass of banks indexed by j owned by the representative household. Banks collect deposits from households and invest them in loans with firms. The deposit market is assumed to be a centralized competitive market while the loan market is a decentralized market characterized by search, matching, and bilateral bargaining. Banks decide to enter the loan market to search for potential borrowers until the expected cost of extending a loan is equal to its expected benefit. Due to the decentralized nature of the loan market, some banks may not end up with loans in their portfolio. If this is the case, the bank will deposit its funds with the central bank as excess reserves and receive an interest rate matching the interest rate on deposits, leaving the bank with negative profits due to search costs. All uncertainty is revealed before loans are extended: loans are made and paid back during the same period. At the end of the period, the bank transfers all its profits (positive or negative) to the representative household.

A bank can only form a credit relationship with one firm and vice versa until separation occurs. Bank $j's$ balance sheet expressed in real terms is

$$\boldsymbol{\chi}_t(j)\,l_t(j,\omega_{z,t}) + (1-\boldsymbol{\chi}_t(j))\frac{ER_t(j)}{P_t} = \frac{D_t(j)}{P_t} \tag{50}$$

where $\boldsymbol{\chi}_t(j)$ is an indicator function taking the value of "1" if bank j extends a loan $l_t(j,\omega_{z,t})$ to a firm whose idiosyncratic productivity $\omega_{z,t}$ exceeds a cut-off level and "0" otherwise, $ER_t(j)$ represents nominal excess reserves held with the central bank in case $\boldsymbol{\chi}_t(j) = 0$ and $D_t(j)$ are household deposits. In equilibrium, there will be a measure of banks with positive loans and a measure of banks with excess reserves. Notice that when the bank extends a loan $(\boldsymbol{\chi}_t(j) = 1)$ the bank balance sheet implies that the bank lends out all of its resources $l_t(j,\omega_{z,t}) = \frac{D_t(j)}{P_t}$ which means there is no credit rationing. This is due to the fact there is no default risk.

Bank Profits A bank searching for a borrower will incur a search cost $\frac{P_t^I}{P_t}\kappa$ measured in units of the final good and earn zero profits. The current flow of profits of a bank with household deposits $D_t(j)$ can be written as

$$\pi_t^b(j) = \boldsymbol{\chi}_t(j)\,R_t^l(j,\omega_{z,t})\,l_t(j,\omega_{z,t}) + (1-\boldsymbol{\chi}_t(j))\left(R_t^r\frac{ER_t(j)}{P_t} - \frac{\kappa}{\mu_t^p}\right) - R_t^d\frac{D_t(j)}{P_t} \tag{51}$$

where $R_t^l(j, \omega_{z,t})$ is the bilateral bargained gross loan rate between bank j and firm z, R_t^r is the gross interest rate on excess reserves and R_t^d is the gross deposit rate. The problem of a bank is to maximize its current profits subject to its balance sheet. Optimality with respect to deposits requires that every period $\left(R_t^r - R_t^d\right) D_t(j) = 0$. Since household deposits are always positive in equilibrium, the bank will choose to collect deposits until the gross interest rate on excess reserves is equal to the gross interest rate on deposits, that is $R_t^r = R_t^d = R_t$. Substituting the bank's balance sheet and the optimality conditions with respect to $D_t(j)$ into the profit function yields

$$\pi_t^b(j) = \begin{cases} \pi_t^b(j, \omega_{z,t}) = \left(R_t^l(j, \omega_{z,t}) - R_t\right) l_t(j, \omega_{z,t}) & \text{if extends a loan to firm } \omega_{z,t} \\ -\frac{\kappa}{\mu_t^p} & \text{otherwise} \end{cases} \tag{52}$$

The determination of $R_t^l(j, \omega_{z,t})$ is explained below as the result of Nash bargaining between the bank and the intermediate good producer. The loan size is given by the labor costs of firm z, that is $l_t(j, \omega_{z,t}) = w_t N_t(\omega_{z,t})$.

Bank Value functions Under the assumptions detailed above, the problem of a bank can be characterized by two value functions: The value of lending to a firm at date t, denoted by $V_t^{BL}(\omega_{z,t})$ and the value of searching for a potential borrower at date t, denoted by V_t^{BN}. Both value functions are measured in terms of current consumption of the final good and are given by

$$V_t^{BL}(\omega_{z,t}) = \pi_t^b(\omega_{z,t}) + \mathrm{E}_t \Delta_{t,t+1} \left\{ (1 - \varphi_{t+1}(\tilde{\omega}_{t+1})) V_{t+1}^{BN} + \varphi_{t+1}(\tilde{\omega}_{t+1}) \int_{\tilde{\omega}_{t+1}}^{\overline{\omega}} V_{t+1}^{BL}(\omega_{z,t+1}) \frac{dG(\omega)}{1 - \gamma_{t+1}(\tilde{\omega}_{t+1})} \right\}$$

and

$$V_t^{BN} = -\frac{\kappa}{\mu_t^p} + \mathrm{E}_t \Delta_{t,t+1} \left\{ \begin{array}{c} p_t^b \left[(1 - \varphi_{t+1}(\tilde{\omega}_{t+1})) V_{t+1}^{BN} + \varphi_{t+1}(\tilde{\omega}_{t+1}) \int_{\tilde{\omega}_{t+1}}^{\overline{\omega}} V_{t+1}^{BL}(\omega_{z,t+1}) \frac{dG(\omega)}{1 - \gamma_{t+1}(\tilde{\omega}_{t+1})} \right] \\ + \left(1 - p_t^b\right) V_{t+1}^{BN} \end{array} \right\} \tag{53}$$

The value of extending a loan is the current value of real profits plus the expected continuation value. A bank that extends a loan to a firm with idiosyncratic productivity $\omega_{z,t}$ at date t will continue financing the same firm at time $t+1$ with probability $\varphi_t(\tilde{\omega}_{t+1})$. The credit relationship is severed at time $t+1$ with probability $\delta_t + 1 - \varphi_t(\tilde{\omega}_{t+1})$, in which case the bank obtains a future value of V_{t+1}^{BN}. The value of searching for a borrower at date t is given by the flow value of the search costs plus the continuation value. A searching bank faces a probability $1 - p_t^b$ of not being matched during time t, obtaining a future value of V_{t+1}^{BN} but a probability p_t^b of being matched. If a searching bank ends up being matched with a firm at time t, then at the beginning of period $t+1$ it will face a probability of separation before extending the loan.

Free entry condition In equilibrium, free entry of banks into the loan market ensures that $V_t^{BN} = 0$ for all t. Using this in V_t^{BN}, the free entry condition can be written as

$$\frac{\kappa}{\mu_t^p p_t^b} = \mathrm{E}_t \Delta_{t,t+1} \left\{ \varphi_{t+1}(\tilde{\omega}_{t+1}) \int_{\tilde{\omega}_{t+1}}^{\overline{\omega}} V_{t+1}^{BL}(\omega_{zt+1}) \frac{dG(\omega)}{1 - \gamma_{t+1}(\tilde{\omega}_{t+1})} \right\} \tag{54}$$

Banks will enter the loan market until the expected cost of finding a borrower $\frac{\kappa}{\mu_t^p p_t^b}$ is equal to the expected benefit of extending a loan to a firm with idiosyncratic productivity $\omega_{z,t+1} \geq \tilde{\omega}_{t+1}$. As banks enter the market, the probability a searching bank finds a borrower will fall, up to the point where equality of the above condition is restored. Note that free entry of banks into the loan market modifies the value function $V_t^{BL}(\omega_{z,t})$ as follows

$$V_t^{BL}(\omega_{z,t}) = \pi_t^b(\omega_{z,t}) + \mathrm{E}_t \Delta_{t,t+1} \left\{ \varphi_{t+1}(\tilde{\omega}_{t+1}) \int_{\tilde{\omega}_{t+1}}^{\overline{\omega}} V_{t+1}^{BL}(\omega_{z,t+1}) \frac{dG(\omega)}{1 - \gamma_{t+1}(\tilde{\omega}_{t+1})} \right\} \tag{55}$$

The net surplus for bank extending a loan to a firm with productivity $\omega_{z,t}$ is

$$V_t^{BS}(\omega_{z,t}) = \pi_t^b(\omega_{z,t}) + \frac{\kappa}{\mu_t^p p_t^b} \tag{56}$$

3.3.4 Employment and the loan contract: Nash bargaining

At any point in time, a matched firm and bank that survive the exogenous and endogenous separation hazards engage in bilateral bargaining over the interest rate and loan size to split the joint surplus resulting from the match.[8] This joint surplus is defined as $V_t^{JS}(\omega_{z,t}) = V_t^{FS}(\omega_{z,t}) + V_t^{BS}(\omega_{z,t})$ and it is given by

$$V_t^{JS}(\omega_{z,t}) = \frac{y_t(\omega_{z,t}) - x^f}{\mu_t^p} - w_t R_t N_t(\omega_{z,t}) + \left(1 - p_t^f\right) \mathrm{E}_t \Delta_{t,t+1} \varphi_{t+1}(\tilde{\omega}_{t+1}) \int_{\tilde{\omega}_{t+1}}^{\overline{\omega}} V_{t+1}^{FS}(\omega_{z,t+1}) \frac{dG(\omega)}{1 - \gamma_{t+1}(\tilde{\omega}_{t+1})}. \tag{57}$$

Let $\overline{\eta}$ be the firm's share of the joint surplus. and $1 - \overline{\eta}$ the banks'. The Nash bargaining problem for an active credit relationship is

$$\max_{\left\{R_t^l(j,\omega_{z,t}), l_t(\omega_{z,t})\right\}} \left(V_t^{FS}(\omega_{z,t})\right)^{\overline{\eta}} \left(V_t^{BS}(\omega_{z,t})\right)^{1-\overline{\eta}} \tag{58}$$

where $V_t^{FS}(\omega_{z,t})$ and $V_t^{BS}(\omega_{z,t})$ are defined above. The first order conditions imply the following optimal sharing rule:

$$\overline{\eta} V_t^{BS}(\omega_{z,t}) = (1 - \overline{\eta}) V_t^{FS}(\omega_{z,t}) \tag{59}$$

and an employment condition that sets the marginal product of labor equal to a markup μ_t^p over the marginal cost of labor inclusive of the bank's opportunity cost when extending a loan to an intermediate good producer:

$$\alpha \xi^{pf} A_t \omega_{z,t} N_t^*(\omega_{z,t})^{\alpha - 1} = \mu_t^p w_t R_t \tag{60}$$

Notice that $w_t R_t$ is expressed in terms of the final good and it has to be transformed back in terms of the intermediate good as it is the marginal product of labor.

The optimal loan size negotiated between credit partners is

$$l_t^*(j, \omega_{z,t}) = \left(\frac{\alpha \xi^{pf} A_t \omega_{z,t}}{\mu_t^p w_t^{\alpha} R_t}\right)^{\frac{1}{1-\alpha}} \tag{61}$$

with an equilibrium loan interest rate

$$R_t^l(j, \omega_{z,t}) = \frac{1}{l_t^*(j, \omega_{z,t})} \left((1 - \overline{\eta}) \left(\frac{y_t^*(\omega_{z,t}) - x^f}{\mu_t^p}\right) + \overline{\eta} \left(R_t w_t N_t^*(\omega_{z,t}) - \frac{\kappa p_t^f}{\mu_t^p p_t^b}\right)\right) \tag{62}$$

The above conditions imply that firm z will produce $y_t^*(\omega_{z,t})$ units of the intermediate good and employ $N_t^*(\omega_{z,t})$ workers, given by:

$$y_t^*(\omega_{z,t}) = \left(\xi^{pf} A_t \omega_{z,t}\right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{\mu_t^p w_t R_t}\right)^{\frac{\alpha}{1-\alpha}} \tag{63}$$

$$N_t^*(\omega_{z,t}) = \left(\frac{\alpha \xi^{pf} A_t \omega_{z,t}}{\mu_t^p w_t R_t}\right)^{\frac{1}{1-\alpha}} \tag{64}$$

The effect of the nominal interest rate on the cost of labor is generally referred to as the 'cost channel' of monetary policy (see Ravenna and Walsh 2006). Normally, the relevant interest rate is taken to be the interest rate the firm pays on loans to finance wage payments. Here, the loan interest rate simply ensures the joint surplus generated by a credit relationship is divided optimally between the firm and the bank, with the relevant interest rate capturing the cost channel being R_t, the bank's opportunity cost of funds. Even though firms will face different interest rates on bank loans, since the loan rate depends on the firms' idiosyncratic productivity realization, the interest cost relevant for labor demand is the same for all firms.

[8] See Appendix C for derivations.

3.3.5 The optimal reservation policy: Endogenous separations

The joint surplus of a credit relationship can be written explicitly as a function of the idiosyncratic productivity shock $\omega_{z,t}$ in order to facilitate the characterization of the loan market equilibrium as follows

$$V_t^{JS}(\omega_{z,t}) = \frac{1}{\mu_t^p}\left((1-\alpha)\left(\xi^{pf}A_t\omega_{z,t}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{\mu_t^p w_t R_t}\right)^{\frac{\alpha}{1-\alpha}} - x^f + \frac{\kappa}{p_t^b}\left(\frac{1-\overline{\eta}p_t^f}{1-\overline{\eta}}\right)\right) \tag{65}$$

The optimal reservation policy with respect to the idiosyncratic productivity shock implies that

$$\begin{aligned} &if \quad \omega_{i,t} \leq \tilde{\omega}_{i,t} && \Longrightarrow V_t^{JS}(\omega_{i,t}) \leq 0 \\ &if \quad \omega_{i,t} > \tilde{\omega}_{i,t} && \Longrightarrow V_t^{JS}(\omega_{i,t}) > 0. \end{aligned}$$

Since the joint surplus is increasing in the firm's idiosyncratic productivity, there is an unique threshold level $\tilde{\omega}_t$ defined by

$$V_t^{JS}\left(\tilde{\omega}_t\right) = 0 \tag{66}$$

such that the joint surplus is negative for any firm facing an idiosyncratic productivity $\omega_{i,t} < \tilde{\omega}_t$. The optimal threshold level $\tilde{\omega}_t$ is

$$\widetilde{\omega}_t = \left(\frac{1}{\alpha^\alpha\left(1-\alpha\right)^{1-\alpha}}\frac{\left(\mu_t^p w_t R_t\right)^\alpha}{\xi^{pf}A_t}\right)\left[x^f - \left(\frac{1-\overline{\eta}p_t^f}{1-\overline{\eta}}\right)\frac{\kappa}{p_t^b}\right]^{1-\alpha} \tag{67}$$

Since $\tilde{\omega}_t$ is independent of i, the cutoff value is the same for all firms and banks. Moreover, it is decreasing in aggregate productivity A_t so that a positive aggregate productivity shock means the number of credit matches that separate endogenously falls and more matched firms produce. The cutoff value is increasing in the cost of labor ($w_t R_t$), the firm's fixed cost (x^f) and the markup of final good over intermediate good prices (μ_t^p).

The bank's opportunity cost of funds R_t influences the level of economic activity at both the extensive and intensive margins. A rise in R_t increases the threshold level of the idiosyncratic productivity of firms that generate a positive joint surplus. As a consequence, fewer firms obtain financing and produce. This is the extensive margin effect. Conditional on producing, firms equate the marginal product of labor to $w_t R_t$, so that an increase in R_t reduces labor demand at each level of the real wage. This is the intensive margin effect. Both channels work to reduce aggregate output as R_t rises. In addition, credit market conditions reflected in p_t^f (probability of a firm matching with a bank) and p_t^b (the probability of a bank matching with a firm) directly affect the extensive margin; a rise in τ_t (a credit tightening) increases $\tilde{\omega}_t$ and fewer firms obtain credit. Both interest costs measured by R and credit conditions measured by τ matter for employment and output.[9]

Finally, the evolution of credit market tightness is obtained by using the free entry condition, the Nash bargaining sharing rule, and the definition of the joint surplus of a credit relationship, and it is given by the following equation:

$$\frac{\kappa}{\mu_t^p\mu\tau_t^\varphi} - \mathrm{E}_t\Delta_{t,t+1}\varphi_{t+1}\left(\tilde{\omega}_{t+1}\right)\left(1-\overline{\eta}\mu\tau_{t+1}^{\varphi-1}\right)\frac{\kappa}{\mu_{t+1}^p\mu\tau_{t+1}^\varphi} = \left(1-\overline{\eta}\right)\mathrm{E}_t\Delta_{t,t+1}\frac{1}{\mu_{t+1}^p}\left(\left(1-\alpha\right)\frac{Y_{t+1}^I}{f_t^m} - \varphi_{t+1}\left(\tilde{\omega}_{t+1}\right)x^f\right) \tag{68}$$

The aggregate dynamics for τ_t are determined by the point where the average current cost of searching for productive borrowers is equal to the average expected benefit of extending a loan. The latter has two components: (i) The average expected output produced by all active intermediate firms net of the average fixed cost of production and (ii) The expected average savings in search costs in $t+1$ conditional on surviving the credit separation hazards.

3.4 Government

Central bank budget constraint There are no government bonds in this economy but the central bank pays the same interest rate as the banks' deposit rate on excess reserves. Therefore, the central bank's budget constraint is given by:

[9]See Appendix C for derivations of the loan market equilibrium.

$$i_t ER_t + RCB_t = M_t - M_{t-1} \tag{69}$$

where RCB_t denotes the central bank transfers to the treasury and M_t is the money supply. Aggregate excess reserves, ER_t, are obtained by integrating across the measure of banks not able to extend loans to intermediate good producers within the period, that is

$$ER_t = \int_j (1 - \boldsymbol{\chi}_t(j)) \frac{ER_t(j)}{P_t} dj \tag{70}$$

where as explained above, $\boldsymbol{\chi}_t(j)$ is an indicator function taking the value of 1 if the bank extends a loan and 0 if the bank maintains its funds as excess reserves with the central bank.

Consolidated government budget constraint Combining the above two constraints for the government sector yields the following consolidated budget constraint:

$$M_t - M_{t-1} = P_t T_t + i_t ER_t \tag{71}$$

where the treasury budget constraint is defined as $RCB_t = P_t T_t$.

Monetary policy We assume that the central bank follows an exogenous growth rate for the nominal supply of money given by

$$M_t = (1 + \theta_t) M_{t-1} \tag{72}$$

where θ_t denotes the nominal money growth given by

$$\left(\frac{\theta_t}{\theta}\right) = \left(\frac{\theta_{t-1}}{\theta}\right)^{\rho_\theta} \exp\left(\in_t^\theta\right) \tag{73}$$

Notice that in this case, the nominal interest rate on deposits, R_t, will be an endogenous variable clearing the market for real money balances.[10]

3.5 Aggregation and Market Clearing

Final goods sector Market clearing in the final goods market requires demand to equal supply for each final good which implies:

$$C_t(j) = Y_t^f(j) \quad \text{for all } j \tag{74}$$

Using the same CES aggregator for final consumption goods as the one used for final goods yields the following aggregate equilibrium condition:

$$C_t = Y_t^f \tag{75}$$

Aggregating individual production functions across all final good producers and taking into account that the demand schedule must be consistent with household optimization, gives us the following condition:

$$C_t \Delta_t^p = X_t \tag{76}$$

where $X_t = \int X(j) \, dj$ and $\Delta_t^p = \int \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_p} dj$ measures price dispersion.

[10]In appendix D, we develop the case of a cashless economy where monetary policy follows a Taylor rule.

Labor market The aggregate labor demand is a downward sloped schedule in the real wage-employment/labor force space since it directly depends on the marginal cost of labor which in turn depends on the real wage.

$$\begin{aligned} N_t &= \Delta_t^w \left(\int_z N_t \left(\omega_{z,t} \right) dz \right) \\ &= \left(1-\delta_t\right) \left(\frac{\alpha \xi^{pf} A_t}{\mu_t^p w_t R_t} \right)^{\frac{1}{1-\alpha}} \left(\frac{\left(\overline{\omega}\right)^k - \left(\widetilde{\omega}_t\right)^k}{k\left(\overline{\omega}-\underline{\omega}\right)} \right) f_{t-1}^m \Delta_t^w \end{aligned}$$

The labor supply or aggregate labor force schedule is obtained by aggregating over all marginal suppliers of each labor type and it is given by

$$w_t = R_t C_t \chi_t (L_t)^{\overline{\varphi}}$$

The labor supply is a positively sloped schedule due to the presence of the aggregate labor market participation condition. As in Gali (2010) the unemployment rate corresponds to the horizontal gap between the labor supply and the labor demand schedules at the level of the prevailing average real wage. In this model, the position of the labor demand and supply schedules depend directly on R_t due to the presence of a working capital channel. More importantly, the position of the labor demand schedule directly depends on the cutoff productivity value $\widetilde{\omega}_t$ as well as on the measure of active intermediate good producers f_{t-1}^m reflecting search frictions in the loan market.

Intermediate good sector Recall that firms and goods in the intermediate good sector are indexed by the idiosyncratic productivity of each active producer, $\omega_{z,t}$. The equilibrium condition in this market is given by

$$X_t\left(\omega_{z,t}\right) = y_t\left(\omega_{z,t}\right) \quad \text{for all } z \tag{77}$$

where the demand for each intermediate good is denoted by $X_t\left(\omega_{z,t}\right)$ and comes from the final good producers. Aggregating across each of the z producers yields the following market clearing condition

$$\begin{aligned} X_t &= \int_z y_t\left(\omega_{z,t}\right) dz \\ &\equiv Y_t^l \end{aligned}$$

where Y_t^l denotes the aggregate supply of intermediate goods and is given by the total number of producing firms $\left(1-\delta_t\right)\left(1-\gamma_t\left(\widetilde{\omega}_t\right)\right) f_{t-1}^m$ times their average output, that is

$$Y_t^I = \left(1-\delta_t\right)\left(1-\gamma_t\left(\widetilde{\omega}_t\right)\right) f_{t-1}^m E\left[y_t^*(\omega_{z,t}) \mid \omega_{z,t} \geq \widetilde{\omega}_t\right] \tag{78}$$

where $E\left[y_t^*(\omega_{z,t}) \mid \omega_{z,t} \geq \widetilde{\omega}_t\right] = \int_{\widetilde{\omega}_t}^{\overline{\omega}} y_t^*(\omega_{z,t}) \frac{g(\omega)d\omega}{(1-\gamma_t)}$ is average output. Assuming that $g\left(\omega\right)$ is a uniform distribution allows us to explicitly calculate the truncated expectation of $y_t^*(\omega_{z,t})$ and compute Y_t^I as

$$Y_t^I = (1-\delta_t)\alpha^{\frac{\alpha}{1-\alpha}} \left(\frac{\xi^{pf} A_t}{\left(\mu_t^p w_t R_t\right)^{\alpha}} \right)^{\frac{1}{1-\alpha}} \left(\frac{\left(\overline{\omega}\right)^k - \left(\widetilde{\omega}_t\right)^k}{k\left(\overline{\omega}-\underline{\omega}\right)} \right) f_{t-1}^m \tag{79}$$

where $k = \frac{2-\alpha}{1-\alpha}$. Notice that Y_t^I depends directly on the measure of firms matched with a bank at the beginning of the period f_{t-1}^m, on the probability that a credit contract survives during the period but scaled by k: $\left(1-\delta_t\right)\left(\frac{(\overline{\omega})^k-(\widetilde{\omega}_t)^k}{k(\overline{\omega}-\underline{\omega})}\right)$, and on the aggregate productivity shock, A_t. Moreover, Y_t^I depends inversely on the gross interest rate on deposits,R_t, the real wage,w_t, and the price-markup, μ_t^p, that is, Y_t^I depends inversely on the real marginal cost of labor expressed in terms of the intermediate good.

Aggregate technology and TFP for the intermediate good sector Combining the above equations for Y_t^I and N_t and letting $F_t = (1-\delta_t)\left(\frac{(\overline{\omega})^k - (\tilde{\omega}_t)^k}{k(\overline{\omega}-\underline{\omega})}\right) f_{t-1}^m$, yields the following expression for the aggregate production function in the intermediate good sector:

$$Y_t^I = \xi^{pf} A_t (F_t)^{1-\alpha} \left(\frac{N_t}{\Delta_t^w}\right)^{\alpha} \tag{80}$$

where

$$N_t = \left(\frac{\alpha \xi^{pf} A_t}{\mu_t^p w_t R_t}\right)^{\frac{1}{1-\alpha}} F_t \Delta_t^w \tag{81}$$

Notice that F_t is the endogenous component of TFP for the aggregate technology of the intermediate good sector. F_t depends on the exogenous separation rate, the measure of producing firms and on credit conditions that are reflected on the reservation productivity. The assumption that $\omega_{z,t}$ follows a uniform distribution with support $[\underline{\omega}, \overline{\omega}]$ implies a total continuation rate given by $\varphi_t(\tilde{\omega}_t) = (1-\delta_t)\left(\frac{\overline{\omega}-\tilde{\omega}_t}{\overline{\omega}-\underline{\omega}}\right)$.

Deposit and loan markets The deposit market equilibrium implies that households have deposits in all active banks, therefore in the aggregate equilibrium $D_t = \int_j D_t(j)\, dj$ must hold. Since all active intermediate good producers take loans to cover their wage bill, market clearing in the loan market requires

$$l_t^*(j, \omega_{z,t}) = w_t N_t^*(\omega_{z,t}) \quad \text{for all } z \tag{82}$$

Aggregating the above condition across all active intermediate good producers and taking into account the wage heterogeneity due to the wage rigidity assumption, yields the following expression for aggregate loans:

$$l_t = \frac{w_t N_t}{\Delta_t^w} \tag{83}$$

Aggregating $l_t(j, \omega_{z,t}) = d_t(j)$, where $d_t(j)$ denotes real deposits at bank j, across all active intermediate good producers and all lending banks yields an aggregate relation between loans and deposits:

$$l_t = \varphi_t(\tilde{\omega}_t) f_{t-1}^m d_t \tag{84}$$

Thus in the aggregate, loans are a fraction of deposits. The specific fraction is endogenous and given by the measure of active credit contracts during period t which is $\varphi_t(\tilde{\omega}_t) f_{t-1}^m$. By the same token, the aggregate level of excess reserves is the fraction of real deposits banks were not able to lend out to firms, that is

$$er_t = \left(1 - \varphi_t(\tilde{\omega}_t) f_{t-1}^m\right) d_t \tag{85}$$

Equilibrium in this economy also takes into consideration the aggregate balance sheet for banks

$$l_t + er_t + \xi^{bs} = d_t \tag{86}$$

where ξ^{bs} is a fixed residual that represents, in steady state, all assets of the banking sector that are not loans or excess reserves with the central bank.

Household Intermediate good producers, final good producers, and banks transfer their profits to the household at the end of each period. The aggregate real transfer received by the household from banks and each type of firm is given by

$$\pi_t^b = \left(R_t^l - R_t\right) l_t - b_t^u \kappa \tag{87}$$

$$\pi_t^I = \frac{Y_t^I}{\mu_t^p} - R_t^l l_t - \varphi_t(\tilde{\omega}_t) f_{t-1}^m x^f \tag{88}$$

$$\pi_t^f = C_t \left(1 - \frac{\Delta_t^p}{\mu_t^p}\right) \tag{89}$$

Goods and Money Markets Taking into account all of the aggregate equilibrium conditions and budget constraints, the aggregate resource constraint in this economy is characterized by

$$C_t = Y_t^f = Y_t^I - \left(b_t^u \kappa + \varphi_t\left(\tilde{\omega}_t\right) f_{t-1}^m x^f\right) \tag{90}$$

Therefore equilibrium in the final goods market requires that consumption equals aggregate household income which, in turn, is equal to aggregate production of the intermediate good net of aggregate search and fixed costs. On the other hand, aggregating the CIA constraint, together with the government budget constraint, the aggregate balance sheet of banks as well as the aggregate equilibrium condition in the loan market, yields the following equilibrium condition for the real money balances market:

$$C_t = m_t - (1 + i_t)\, er_t \tag{91}$$

The above equilibrium condition implies the aggregate supply of real money balances is allocated to aggregate consumption as well as repaying excess reserves holdings.

Finally, we define the average spread of interest rates (average credit spread) as the difference between the average loan rate and the bank's opportunity cost of funds, given by the deposit rate:

$$\frac{R_t^l l_t - R_t l_t}{l_t} = \frac{1}{l_t}\left[\left((1-\overline{\eta})(1-\alpha)\right)\frac{1}{\mu_t^p}\frac{Y_t^I}{\varphi_t\left(\tilde{\omega}_t\right) f_{t-1}^m} - \left((1-\overline{\eta})\frac{x^f}{\mu_t^p} + \overline{\eta}\frac{\kappa}{\mu_t^p \tau_t}\right)\right] \tag{92}$$

where the terms $R_t^l l_t$ and $R_t l_t$ are obtained by computing the following conditional expectations:

$$R_t^l l_t = E\left[R_t^l\left(j, \omega_{z,t}\right) l_t^*(\omega_{z,t}) \mid \omega_{it} \geq \widetilde{\omega}_t\right]$$

and

$$R_t l_t = E\left[R_t l_t^*(\omega_{it}) \mid \omega_{it} \geq \widetilde{\omega}_t\right]$$

4 Computation and simulations

The non-linear system of equations that characterizes the dynamic equilibrium of the model is summarized in the non-stochastic steady state section below and Appendix C. The vector of endogenous variables X_t is given by the following 41 variables classified according to the following groups:

1. Prices and real wages (11 variables):
$$X_{1,t} = \left[\Pi_t, \Pi_t^*, w_t, w_t^*, \widetilde{g}_{1,t}, \widetilde{g}_{2,t}, f_t^1, f_t^2, \mu_t^p, \Delta_t^p, \Delta_t^w\right] \tag{93}$$
2. Real variables (7 variables):
$$X_{2,t} = \left[Y_t^I, Y_t^f, C_t, N_t, U_t, L_t, \Delta_{t,t+1}\right] \tag{94}$$
3. "Monetary policy" variables (3 variables):
$$X_{3,t} = [m_t, er_t, R_t] \tag{95}$$
4. Credit market variables (15 variables):
$$X_{4,t} = \left[l_t, d_t, \tau_t, p_t^b, p_t^f, b_t^u, f_t^m, f_t, F_t, \widetilde{\omega}_t, \varphi_t\left(\widetilde{\omega}_t\right), cd_t, cc_t, cg_t, cr_t\right] \tag{96}$$
5. Auxiliary definitions for calibration purposes (4 variables):
$$X_{5,t} = [LS_t, FCS_t, \widehat{l}_t, \widehat{er}_t] \tag{97}$$
where LS_t denotes the labor share of GDP, FCS_t is the fixed cost of production share of GDP, $\widehat{l}_t$ is aggregate loans as a fraction of total deposits and $\widehat{er}_t$ is aggregate excess reserves as a fraction of total deposits.

The vector of endogenous variables is summarized as:

$$X_t = [X_{1,t}, X_{2,t}, X_{3,t}, X_{4,t}, X_{5,t}] \tag{98}$$

We solve the model using a standard perturbation method applied to a first order approximation around the non-stochastic steady-state of the model. Next we explain the computation of the steady-state as well as the calibration procedure for the unknown parameters of the model.

4.1 The non-stochastic steady state

We assume in steady state that the growth rate of real money balances is zero. This assumption together with the Euler equation evaluated at the steady state implies a gross inflation rate of $\Pi = 1$ and a gross nominal interest rate of $R = \frac{1}{\beta}$.[11] Thus the price and wage index equations together with the price and wage dispersion equations evaluated at the steady state with zero net inflation imply no relative price and wage distortions. This implies $\Pi^* = 1, \Delta^p = 1$, $w^* = w$ and $\Delta^w = 1$. Similarly, the optimal price setting equation evaluated at the steady state yields the following constant markup of final goods over intermediate goods prices:

$$\mu^p = \frac{\epsilon_p}{\epsilon_p - 1} \tag{SS1}$$

By the same token, the wage setting equation evaluated at the steady state yields a constant markup of the real wage over the aggregate marginal rate of substitution:

$$w = \left(\frac{\varepsilon_w}{\varepsilon_w - 1}\right) MRS \tag{SS2}$$

where $MRS = C\chi N^{\overline{\varphi}}$ is the aggregate marginal rate of substitution in steady state.

Equation SS2 together with the aggregate labor force equation and the unemployment rate definition evaluated at the steady state, imply the following relationship between the unemployment rate and the elasticity among labor types ε_w:

$$\left(\frac{1}{1-U}\right)^{\overline{\varphi}} = \left(\frac{\varepsilon_w}{\varepsilon_w - 1}\right) \tag{SS3}$$

Notice that if we parameterize $\overline{\varphi}$ and target a particular value for the unemployment rate at the steady-state, we obtain a value for the ε_w parameter. Combining the resource constraint together with the aggregate CIA constraint implies: $Y^f = C = m - R\,(er)$ where Y^f is given by

$$Y^f = \left(A\xi^{pf}\right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{\mu^p w R}\right)^{\frac{\alpha}{1-\alpha}} (1-\delta) \left(\frac{(\overline{\omega})^k - (\tilde{\omega})^k}{k\,(\overline{\omega} - \underline{\omega})}\right) f^m - \left(\left(\frac{1-(1-\delta)\,f^m}{\tau}\right)\kappa + \varphi\,(\tilde{\omega})\,f^m x^f\right) \tag{SS4}$$

Aggregate labor demand, together with the aggregate 'credit' input denoted by F, implies that the following equation must hold at the steady state:

$$N = (1-\delta)\left(\frac{\alpha A\xi^{pf}}{\mu^p w R}\right)^{\frac{1}{1-\alpha}} \left(\frac{(\overline{\omega})^k - (\tilde{\omega})^k}{k\,(\overline{\omega} - \underline{\omega})}\right) f^m \tag{SS5}$$

Finally, the equilibrium in the loan market, evaluated at the steady-state, can be reduced to the following set of equations:

$$\left[\alpha^\alpha (1-\alpha)^{1-\alpha} A\xi^{pf}\tilde{\omega}\right]^{\frac{1}{1-\alpha}} = (\mu^p w R)^{\frac{\alpha}{1-\alpha}} \left[x^f - \left(\frac{1-\overline{\eta}\mu\tau^{\nu-1}}{1-\overline{\eta}}\right)\frac{\kappa}{\mu\tau^\nu}\right] \tag{SS6}$$

$$\left(1 - \beta\varphi\,(\tilde{\omega})\left(1-\overline{\eta}\mu\tau^{\nu-1}\right)\right)\frac{\kappa}{\mu\tau^\nu} \tag{SS7}$$

$$= (1-\overline{\eta})\,\beta\left((1-\alpha)\frac{\left(A\xi^{pf}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{\mu^p w R}\right)^{\frac{\alpha}{1-\alpha}}(1-\delta)\left(\frac{(\overline{\omega})^k - (\tilde{\omega})^k}{k(\overline{\omega}-\underline{\omega})}\right) f^m}{f^m} - \varphi\,(\tilde{\omega})\,x^f\right) \tag{99}$$

$$\left(1 - \varphi\,(\tilde{\omega}) + (1-\delta)\,\mu\tau^{\nu-1}\right) f^m = \mu\tau^{\nu-1} \tag{SS8}$$

The steady-state of the model can be partitioned in two blocks. The first block of equations can be solved recursively and consists of equations D1-D20 evaluated at the steady-state (See Appendix C for the list of

[11] If the model is closed with a Taylor rule instead of a money growth rule then at the steady state the gross nominal interest rate is given by $R = \left(\frac{1}{\beta}\right)(\Pi)^{\phi_\pi}$ while the Euler equation implies $R = \frac{\Pi}{\beta}$. Since $\phi_\pi > 1$ then at the steady state $\Pi = 1$ and $R = \frac{1}{\beta}$., see appendix D for a version of the model as a cashless economy and a Taylor rule

equations). The second block of equations constitute a simultaneous system of equations that incorporate equations SS4-SS8 together with equations D27 and D34 evaluated at the steady-state. We calibrate the following subset of nine parameters: $x^f, \kappa, \mu, \xi^{pf}, \delta, \xi^{bs}, \chi, \alpha$ and ϵ_w to be consistent with specified targets for the following endogenous variables: $U, N, Y^f, \varphi(\tilde{\omega}), cd, FCS, LS, \frac{l}{d}$ and $\frac{er}{d}$. The strategy is explained in more detail in the next section.

4.2 Calibration

In order to compute the model's equilibrium, we must assign values to the following list of parameters:

- Preferences: $\beta, \overline{\varphi}, \chi$
- Technology: $A, \xi^{pf}, \alpha, x^f, [\underline{\omega}, \overline{\omega}]$
- Search technology and the loan market: $\mu, \nu, \delta, \kappa, \xi^{bs}, \overline{\eta}$
- Price and wage stickiness: $\theta_p, \theta_w, \epsilon_p, \epsilon_w$
- Monetary policy: θ_{ss}

We set the following subset of parameters according to convention: The subjective discount factor is set to $\beta = 0.99$ consistent with a steady-state real interest rate of 1 percent per quarter. We normalize the baseline level of technology to be $A = 1$ as well as the support for the idiosyncratic productivity to be $[\underline{\omega}, \overline{\omega}] = [0, 1]$. The parameters determining the degree of price and wage stickiness are set to imply average duration of one year, that is $\theta_p = \theta_w = 0.75$. The latter is set consistent with much of the microeconomic evidence on wage and price setting [12] The elasticity of substitution among final goods is set to be $\epsilon_p = 9$, implying a steady sate price markup of $\mu^p = 1.125$ or 12.5% and the inverse of the Frisch labor supply elasticity is set to be $\overline{\varphi} = 5$ which corresponds to a Frisch elasticity of 0.2.

Additionally, we fix values for two of the six parameters related to the loan market: The firm's share in the Nash bargain problem is assumed to be $\overline{\eta} = 0.3$ which is nearly close to the 0.32 value used by Petrosky-Nadeau and Wasmer (2015). Petrosky-Nadeau and Wasmer (2015) calibrate the bank's share, $1 - \overline{\eta}$, by calculating the financial sector's share of aggregate value-added using data and the corresponding value added definition from their model. We assume that that the Hosios condition does not hold in steady state and set the elasticity of the matching function to $\nu = 0.7$. which is two times the firm's bargaining parameter. [13]

In the next table we summarize the parameter values described above:

Parameter	Description	Value
β	Discount rate	0.99
A	Baseline Technology	1.0
$[\underline{\omega}, \overline{\omega}]$	Support for idiosyncratic productivity	[0,1]
θ_p	Calvo parameter for price setting	0.75
θ_w	Calvo parameter for wage setting	0.75
ϵ_p	Elasticity of substitution among final goods	9.0
$\overline{\varphi}$	Inverse of Frisch Elasticity	5
$\overline{\eta}$	Firm's Nash bargaining share	0.3
ν	Matching function elasticity	0.7

Table 1: Parameters taken from the data and conventional values from the literature

A total of nine parameters of the model are calibrated to be consistent with a set of nine endogenous targets that we specify below. These parameters are classified as follows:

[12] See, for example,Nakamura and Steinsson 2008.

[13] We checked the model's robustness to the following range of values: $v \in [0.6, 0.8]$ and $\overline{\eta} \in [0.5, 0.8]$ for two reasons. First, when $v < 0.6$ the linear approximation of the dynamic equations of the model does not satisfy the Blanchard and Kahn 1980 rank condition. But second, if $\overline{\eta} < 0.5$, solving for the non-linear steady state yields imaginary roots. Both restrictions on the rage of values for ν and $\overline{\eta}$ may be an indication that there is no equilibrium in the loan market or that the loan market collapses such as in Becsi, Li, and Wang 2005.

- Calibrated loan market parameters: The search cost faced by a bank κ, the scale parameter of the aggregate matching function μ, the exogenous probability of credit destruction δ and the residual term on the aggregate banks' balance sheet ξ^{bs}.
- Calibrated technology parameters: The elasticity of labor and the scale parameter in the aggregate production function for intermediate goods α and ξ^{pf} respectively as well as the fixed cost of producing the intermediate good x^f.
- Calibrated preference parameters: The preference shifter χ and the elasticity among labor types, ϵ_w.

In the next table, we report the steady state targets that we use to calibrate the above subset of parameters:

Parameter	Description	Value
U	Unemployment rate	0.05
N	Employment	0.59
Y^f	GDP	1
$\varphi(\tilde{\omega})$	Overall continuation rate	0.7
cd	Credit destruction rate	0.029
$\frac{\varphi(\tilde{\omega})f^m x^f}{Y^f}$	Fixed cost share of GDP	0.35
$\frac{wN}{Y^f}$	Labor share of GDP	2/3
$\frac{l}{d}$	Loan to deposits ratio	0.63
$\frac{er}{d}$	Excess reserves to deposits ratio	0.015

Table 2: Steady state targets

Following Galí (2011) we target an unemployment rate of $U = 0.05$ and aggregate employment of $N = 0.59$ at the steady-state. Given this, equation SS3 above,implies an elasticity of substitution among labor types of $\varepsilon_w = 4.4205$ which in turn is associated with an average wage markup of 29 percent. We assume a 35 percent share of the fixed cost of production in GDP, $FCS = 0.35$, as in Christiano, Eichenbaum and Trabandt (2015). The steady-state overall continuation rate for a credit relationship is set at 70 percent,$\varphi(\tilde{\omega}) = 0.7.$, which is consistent with findings reported in Chodorow-Reich (2014) for banking relationships in the U.S. syndicated loan market. Specifically, Chodorow-Reich (2014) finds that after controlling for a bank's average market share, a bank that served as the prior lead lender of a private borrower has a 71 percent point greater likelihood of serving as the new lead lender in the same loan contract. Given that the scale technology parameter, ξ^{pf}, is chosen in order to normalize the steady-state level of GDP to unity $\left(i.e, Y^f = 1 \text{ in equation SS4}\right)$, we can solve for x^f to be consistent with the steady-state target imposed on the fixed cost share of GDP, which is given by

$$\frac{\varphi(\tilde{\omega}) f^m x^f}{Y^f} = 0.35 \tag{SS9}$$

We target a steady-state loan to deposit ratio of $\frac{l}{d} = 0.63$ by using quarterly data on commercial and industrial loans as well as saving deposits for all U.S commercial banks during the great moderation period which is assumed to be between 1985 and 2007. The steady-state target for the loan to deposit ratio $\frac{l}{d}$, together with the steady-state target for $\varphi(\tilde{\omega})$ explained above, allow us to obtain the steady-state level for the measure of firms in a credit relationship (f^m) by using the relationship between loans and deposits that arises when aggregating the balance sheet of those banks that are able to lend out their available funds to intermediate good producers. This condition at the steady-state is given by:

$$\frac{l}{d} = \varphi(\tilde{\omega}) f^m \tag{SS10}$$

Clearly, equations SS9 and SS10 together with the specified steady-state targets for $FCS, \varphi(\tilde{\omega}), \frac{l}{d}$ and Y^f are consistent with $f^m = 0.9$ and $x^f = 0.56$. Therefore, the steady-state of the model implies that 90 percent of producing firms (intermediate good producers) are in a credit contract with a bank. The parameter $x^f = 0.56$ is consistent with a 35 percent fixed cost of production share of GDP, a 70 percent probability of overall continuation for a credit relationship and a 90 percent measure of firms in a credit relationship.

We target a labor share of GDP at the steady-state of 2/3,that is $LS = \frac{wN}{Y^f} = 2/3$. The latter definition together with the equilibrium condition in the loan market evaluated at the steady-state, $l = wN$ yields a steady-state value for the real wage equal to $w = 1.13$ and aggregate real loans of $l = 2/3$.Then, given the steady-state target on the loan to deposit ratio, we obtain the steady-state value of aggregate real deposits to be $d = 1.0582$. We use the average of all reserve balances with federal reserve banks during the great moderation period and the average of all saving deposits at U.S commercial banks during the same period of time in order to set the ratio of aggregate excess reserves to aggregate deposits to be 1.5 percent. The aggregate level of reserves consistent with the specified target for $\frac{er}{d} = 0.015$ and the steady-state level of aggregate deposits obtained before is $er = 0.0159$. The resource constraint of the economy implies consumption at the steady-state to be $Y^f = C = 1$ while the aggregate CIA constraint can be solved for the steady-sate level of real money balances m given our parametrization of $R = 1.0101$ and the steady-state level of aggregate excess reserves that we have already obtained. Thus, at the steady-state, the supply of real money balances must be allocated into consumption and interest rate payments on excess reserves,

$$m = C + (R)\, er \tag{SS11}$$

with $m = 1.016$.

The labor force equation evaluated at the steady-sate implies:

$$w = C\chi\,(L)^{\overline{\varphi}} \tag{SS12}$$

given the parameterization of $\overline{\varphi}$ and the steady-state values for w,C and L obtained above, we can solve consistently for the preference shift parameter to be $\chi = 12.2297$.Notice that the calibration of χ is also consistent with the optimal price setting equation evaluated at the steady state, equation SS2 above, and therefore, it is consistent with the steady-state level of employment that we are targeting $(N = 0.59)$. The stochastic discount factor evaluated at the steady-sate yields $\Delta = \beta = 0.99$.

The aggregate balance sheet of banks evaluated at the steady-state allow us to obtain the residual term as a fraction of deposits as $\xi^{bs} = 1 - \dfrac{l}{d} - \dfrac{er}{d} = 0.3550$

Following (Contessi and Francis, 2013), we target an average quarterly credit destruction rate of 2.9 percent during the great moderation period. The credit destruction rate implied by the model and evaluated at the steady-state is

$$cd = 0.029 = 1 - \varphi\,(\tilde{\omega}) - \mu\tau^{\nu}\delta \tag{SS13}$$

Given the above targets and parameter calibration, equations SS4-SS8 together with equations SS13 and SS14 can be solved for the following set of parameters $\kappa, \mu, \delta, \xi^{pf}, \alpha$ as well as for the corresponding steady-state values for $\widetilde{\omega}, \tau$. Equation SS14 is given by the steady-state probability of continuation for a credit contract:

$$\varphi\,(\tilde{\omega}) = (1 - \delta)\left(\frac{\overline{\omega} - \tilde{\omega}}{\overline{\omega} - \underline{\omega}}\right) \tag{SS14}$$

Solving the system of equations formed by equations SS4-SS8 and SS13-SS14 yields the baseline calibration for the remaining parameters of the model: κ, μ, δ, ξ^{pf} and α. The next table summarizes the calibrated parameters of the model that are solved to be consistent with the steady state targets specified above.

Parameter	Description	Value
κ	Bank's search costs	0.6697
μ	Matching function scale parameter	1.0564
δ	Exogenous probability of separation	0.2029
ξ^{pf}	Production function scale parameter	3.9482
α	Labor elasticity of production function	0.51
ξ^{bs}	Residual term on aggregate bank's balance sheet	0.3550
x^f	Fixed cost of production	0.5556
χ	Preference parameter for dis-utility of labor	12.2297
ϵ_w	Elasticity of substitution among labor types	4.4205

Table 3: Calibrated parameters to be consistent with steady state targets

The above results imply that $k = \frac{2-\alpha}{1-\alpha} = 3.0409$. The steady state values for a group of endogenous variables of the model are summarized in the following table:

Parameter	Value	Parameter	Value
Π	1	Π^*	1
Δ^w	1	Δ^p	1
Y^I	1.4853	Y^f	1
F	0.2356	C	1
b^u	0.2021	m	1.0160
f	0.2826	R	1.0101
p^f	0.9554	w	1.1299
p^b	1.3356	μ^p	1.1250
U	0.0500	L	0.6210
N	0.5900	$\varphi(\tilde{\omega})$	0.7000
f^m	0.9	$\tilde{\omega}$	0.1219
l	0.6667	d	1.0582
τ	1.3979	er	0.0159

Table 4: Steady state values

4.3 Equilibrium dynamics: Monetary policy and financial shocks

We interpret the dynamic responses to different shocks focusing on two main equations.The optimal hiring rule for all active credit matches, given by:

$$\alpha \xi^{pf} A_t \omega_{z,t} N_t^* \left(\omega_{z,t}\right)^{\alpha-1} = \mu_t^p w_t R_t ; \forall \omega_{z,t} > \widetilde{\omega}_t \tag{100}$$

and the reservation productivity level written in terms of the real marginal cost MC_t and the credit market tightness τ_t:

$$\widetilde{\omega}_t = \frac{1}{\alpha^\alpha \left(1-\alpha\right)^{1-\alpha}} \frac{\left(MC_t\right)^\alpha}{A_t} \left[x^f - \frac{\kappa}{1-\overline{\eta}} \left(\mu \tau_t^{1-v} - \overline{\eta} \mu \tau_t \right) \right]^{1-\alpha} \tag{101}$$

Notice that the first equation is the result of the Nash bargaining protocol over the joint surplus generated by a credit contract. Therefore, conditional on surviving, each credit contract will determine the loan size and hire workers consistent to the point where the marginal product of labor $MPL_t\left(\omega_{z,t}\right) = \alpha \xi^{pf} A_t \omega_{z,t} N_t^* \left(\omega_{z,t}\right)^{\alpha-1}$ is equal to the real marginal cost of labor expressed in terms of the intermediate good $MC_t = \mu_t^p w_t R_t$. Notice that the term $w_t R_t$ is expressed in terms of the final good thus in order to express the real marginal cost in terms of the intermediate good, the term $w_t R_t$ have to be multiplied by the mark-up, $\mu_t^p = \frac{P_t}{P_t^I}$ where P_t^I is the intermediate good price index. Changes in this equation generate an intensive margin effect since it holds only for those credit matches that have survived the exogenous as well as endogenous separation hazards.

The second equation is obtained by setting the joint surplus for a credit contract to zero. The reservation productivity, $\widetilde{\omega}_t$, that results,is a productivity threshold that select the subset of firms that are able to obtain funds, hire workers and produce during the period. This threshold productivity generates an extensive margin effect whenever it responds to aggregate macroeconomic shocks. Notice that $\widetilde{\omega}_t$ has two main determinants: MC_t and τ_t. In a partial equilibrium setting,an increase in MC_t will raise the reservation productivity taking as given credit market tightness, τ_t. By the same token, our benchmark calibration implies that given MC_t constant, an increase in τ_t will produce an increase in the reservation productivity. The main transmission mechanism of aggregate shocks in this economy goes through changes in $\widetilde{\omega}_t$ which ultimately is a consequence of movements in the joint surplus of a credit match, $V_t^{JS}(\omega_{z,t})$. Fluctuations in MC_t and τ_t affect $V_t^{JS}(\omega_{z,t})$: An increase in MC_t given τ_t constant reduces the joint surplus of an active credit contract while a tighter credit market (higher τ_t), given MC_t constant will also end up reducing this joint surplus. General equilibrium effects will determine simultaneously all the variables.

4.3.1 Credit inefficiency wedge and labor productivity

Recall that, F_t is the endogenous component of technology and that it depends on credit market conditions as well as on the reservation productivity level. This term is given by:

$$F_t = \left(1-\delta_t\right) \left(\frac{\left(\overline{\omega}\right)^k - \left(\widetilde{\omega}_t\right)^k}{k \left(\overline{\omega} - \underline{\omega}\right)} \right) f_{t-1}^m \tag{102}$$

Through this section, we define the credit inefficiency wedge as the endogenous component of technology that is not related to employment, given by: $(F_t)^{1-\alpha}$. Notice that without credit market frictions, $F_t = 1$ and aggregate output in the intermediate sector would be: $Y_t^I = \xi^{pf} A_t (\frac{N_t}{\Delta_t^w})^\alpha$. In this latter case, the only inefficiency that appears after aggregation is the one related to presence of wage rigidities. But, under the assumption of search and matching frictions in the loan market, this inefficiency wedge depends on the aggregate probability of continuation of a credit contract as well as on the mass of active credit contracts. Both, depending ultimately on the common reservation productivity threshold. Clearly, in this model, credit conditions affect this inefficiency wedge, generating amplification effects to any shock that hit the economy.

On the other hand, labor productivity in the intermediate good sector is given by:

$$LP_t = \frac{Y_t^I}{N_t} = \frac{\xi^{pf} A_t}{(\Delta_t^w)^\alpha} \left(\frac{F_t}{N_t}\right)^{1-\alpha} \tag{103}$$

In our model, labor productivity is also affected by the credit wedge. If the loan market is a Walrasian centralized market then $F_t = 1$ and credit conditions do not affect labor productivity. Credit market frictions in the form of search and matching frictions generate inefficient fluctuations of labor productivity, employment and intermediate output as well as final output. This of course, translates into inefficient fluctuations in the unemployment rate given the interaction of wage rigidities, market power, and the labor force participation condition that characterize the labor market. Financial shocks are propagated and amplified by the endogenous response of this credit inefficiency 'input' term.

Finally, we can define total factor productivity, TFP_t, as all the terms in the aggregate production function that are not associated with the labor input, that is:

$$TFP_t = \xi^{pf} A_t \left(F_t\right)^{1-\alpha} \tag{104}$$

The inefficiency associated with the presence of credit frictions also affects the evolution of total factor productivity by making it responsive to aggregate shocks as long as credit conditions, summarized by the term F_t, also respond to those same shocks. Therefore, total factor productivity is also subject to inefficient endogenous fluctuations that then propagate and amplify aggregate shocks.

4.3.2 The effects of a monetary policy shock

Figures **7-10, in Appendix A** present the equilibrium responses of several variables of interest to an expansionary monetary policy shock under the assumption the central bank follows an exogenous money growth rule.[14] An expansionary monetary policy shock corresponds to a 0.25 percent quarterly increase, on impact, in the rate of nominal money growth. On impact, the nominal interest rate increases slightly (20 basis points) since the model does not engender a liquidity effect as we assume a CIA constraint and a logarithmic period utility function. The existence of money demand in our model is a consequence of the cash-in-advance structure assumed.

Monetary policy is transmitted through the standard interest rate channel as well as through impacts on the intensive and extensive margin associated with the interaction among the working capital channel, search and matching frictions in the loan market, and the presence of a match-specific productivity level. The standard interest rate channel of monetary policy works through changes the long run real interest rate and its consequent effect on consumption, output employment, unemployment and inflation. Although the nominal interest rate increases on impact, it falls quickly below zero and remains negative for almost 10 quarters, due to the presence of nominal rigidities and their influence on inflation expectations.

Recall that conditional on surviving, a credit contract requires the firm to equalize the marginal product of labor to the real marginal cost of labor expressed in terms of the intermediate good. Monetary policy shocks alter this optimality condition generating an intensive margin effect. On the other hand, the extensive margin effect is generated by the persistent decline of reservation productivity, $\widetilde{\omega}_t$, which expands the measure of active firms. The aggregation of both margin effects is summarized in the evolution of the credit inefficiency wedge given by the term F_t that appears in the aggregate production function for the intermediate good sector as well as on the aggregate labor demand. The interaction of monetary policy and credit frictions together with heterogeneous productivity at the firm level, generates a very persistent increase in F_t that lasts for approximately 16 quarters.

This additional monetary policy transmission mechanism, through F_t, reinforces the standard interest rate channel since it creates better credit conditions for firms and banks by raising the joint surplus of a credit

[14] In appendix D we present results for a cashless economy version of the model that is closed with a Taylor rule.

match thus generating a persistent decline in the measure of credit market tightness, τ_t , the average spread of interest rates, and the average loan interest rate. Expansionary monetary policy reduces the real marginal cost of hiring a new worker for all producing firms, raising aggregate labor demand at each level of the real wage, and thus employment and production for all active production units (the intensive margin effect of monetary policy). As noted earlier, an expansionary monetary policy does not generate a liquidity effect–this implies the nominal interest rate will increase initially. However the marginal cost of labor expressed in terms of the intermediate good will decrease because the price mark-up (μ_t) exhibits a strong decline that overpowers the initial increase in R_t as well as the persistent increase in w_t. Notice that the effect over the real wage occurs due to the increase in aggregate labor demand and is also present in the standard New Keynesian model with wage rigidities. As noted earlier, the expansion of real money balances also reduces the reservation productivity level for those credit matches generating non-negative joint surpluses. As a consequence, more firms are able to obtain external funds, hire workers and produce (the extensive margin of monetary policy) so there will be fewer firms searching for external funds (f_t). Thus the firm finding rate increases.

Free entry of banks to the loan market implies more banks will enter, inducing a temporary increase in the measure of banks searching for projects/borrowers, b_t^u and decreasing the bank finding rate on impact and for two subsequent quarters. As the reservation productivity, $\tilde{\omega}_t$, declines the overall continuation rate for credit contracts increases by almost 20 basis points on impact and remains persistently high for more than six quarters. Therefore credit conditions improve for currently active firms as well as for potential firms–those with profitable projects–as can be seen through the persistent decline in τ_t and corresponding increase in the measure of active credit contracts, f_t^m.

These new credit conditions translate into a persistent decline in the credit destruction rate as well as a persistent increase in the credit creation rate which together create a net increase in aggregate loans and deposits. The decline in credit destruction is larger and more persistent than the increase in credit creation on impact. This well-known feature of bank credit is generated by the relative ease with which banks can moderate their current contracts compared to negotiating new contracts. The increase in the probability of credit contract continuation impacts credit destruction immediately while in order for new credit to be negotiated, banks need to enter.

Another feature of the modeling framework is that the expansion in economic activity as well as the improved credit conditions in the loan market induce the central bank to automatically reduce its loans to the banking sector. In this model, central bank lending to the banking sector corresponds to excess reserves, er_t, that banks hold with the central bank. The decline in er_t is a consequence of the reduction in the overall continuation rate of credit contracts and the free entry of banks into the loan market. That is, in equilibrium, there will be fewer banks requiring loans from the central bank to cover the interest rate on deposits.

Nominal price and wage rigidities also contribute to generating the persistent decline in the unemployment rate and an expansionary effect over all aggregate macroeconomic variables (consumption, employment, and output). In **figures 31-34 of Appendix F**, the role played by nominal rigidities in shaping the responses of the economy to a monetary policy shock is analyzed. Please see Appendix F for a discussion.

4.3.3 The effects of a financial shock

Figures **11-14, in appendix A** illustrate the dynamics responses of a number of aggregate variables to a negative (bad) financial shock. In our modeling context, a financial shock is defined as an unexpected persistent increase in the exogenous separation rate for credit contracts, δ_t. Recall, the overall continuation rate of a credit contract is defined as $\varphi_t(\tilde{\omega}_t) = (1-\delta_t)(1-\gamma_t(\tilde{\omega}_t))$ where its exogenous component δ_t follows an AR(1) process given by:

$$\delta_t - \delta = \rho_\delta (\delta_{t-1} - \delta) + \varepsilon_t^\delta \tag{105}$$

Our calibration procedure is consistent with a steady-state value for δ of 0.2163.

In our model, a financial shock implies that a fraction of existing credit contracts are exogenously terminated due to the decline in the overall continuation rate of credit relationships. The impact of such an increase in the separation rate qualitatively matches the impact of a rise in the excess bond premium in our VAR results discussed in the motivation section above, where we use a one standard deviation in crease in the excess bond premium as a measure of a financial shock. After a negative financial shock, there will be a larger mass of intermediate good producers searching for funds, f_t as well as a larger mass of banks searching for profitable projects to fund, denoted by b_t^u. From the point of view of a bank, the expected value of searching for a project/borrower turns out to be temporally negative following a negative financial shock, inducing banks to exit the loan market until the expected value of searching for borrowers increases. Since banks are able to exit the loan market, the measure of firms searching for funds after a financial shock will be larger than the measure of banks searching for borrowers inducing an increase in the measure of credit market tightness, τt. Moreover, the mass of intermediate good producers separated from their previous credit contract, are not able to exit the market as is the case of banks, but are only able to search for external funding in order to attempt production in the future. Therefore, credit market conditions tighten from the point of view of borrowers, exhibited by a decline in the firm's finding rate, p_t^f.The bank's finding rate, p_t^b increases because banks are able to exit the market when its surplus from looking for firms becomes negative. As a result of the increase in the bank finding rate but decline in the firm finding rate, the overall loan market tightens from the perspective of borrowers.

Notice that when banks are separated and exit the loan market, the central bank must automatically increase excess reserves, er_t, to compensate for the fact that banks have no remaining assets to pay interest on households' deposits.

The transmission of the financial shock is reinforced by a decline in the joint surplus to a credit match which is a consequence of the large persistent raise in the reservation productivity level $\widetilde{\omega}_t$. The latter, induces an even more pronounced and persistent fall on the overall continuation rate,$\varphi_t\left(\tilde{\omega}_t\right)$, that adds to the one generated by the initial shock to the exogenous separation rate, δ_t. The mass of firms and banks that start the period in a credit contract but also survive the higher separation rate that occurs after a financial shock, decide to raise their reservation productivity threshold due to the fall in the joint surplus of a credit relationship and the consequent tighter credit market (higher τt). The dynamics associated to the reservation productivity level produce an extensive margin effect associated to a selection effect that reduces the subset of firms able to obtain external funds, hire workers and produce. The tighter credit conditions that occur after a negative financial shock are also reflected in a in a persistent decline of the mass of firms engaged in a credit contract,f_{t-1}^m and a significant reduction on the aggregate amount of loans, l_t. These new credit conditions translates to a persistent raise of the average spread of interest rates as well as a raise in the average loan rate.

However, the financial shock generates an intensive margin effect that partially off sets the extensive margin effect. This intensive margin effect is related to those credit contracts -firm and bank pairs- that survive the financial shock but adjust their existing loan contract by changing the conditions that characterize their bilateral bargaining protocol. Specifically, a financial shock reduces the real marginal cost of labor expressed in terms of the intermediate good, $\mu_t^p w_t R_t$ for all active intermediate good producers, inducing a small recovery on the aggregate labor demand that is not enough to off set the negative extensive margin effect of a financial shock.

At the aggregate level, a persistent negative financial shock generates a negative response on employment, labor productivity and total factor productivity. Such responses are a consequence of aggregating the intensive and extensive margin effects described above. The deep and prolonged recession that occurs after a financial shock is characterized by a persistent decline of the unemployment rate which is explained by the interaction between price and wage rigidities, the labor force condition together with the search and matching frictions of the loan market. By the same token, aggregate loans decline even though households increase their deposits with banks as a response to the tightening of credit conditions. In this sense, a financial crises also generates an increase in savings as well as a decline in aggregate expenditures and bank lending. Thus,a financial shock, modeled as an exogenous increase in the separation rate of credit contracts, induces a persistent recession in terms of GDP, the unemployment rate, consumption, labor productivity and total factor productivity. In this model, the propagation of the financial shock goes through persistent changes in the economy wide reservation productivity, the overall continuation rate as well as changes in the marginal cost of labor expressed in terms of the intermediate good. All of these changes produce a significant tightening of aggregate credit conditions that are finally reflected in a persistent fall of the endogenous component of the aggregate total factor productivity, labor productivity and a persistent raise of the average interest rate spread.

Finally, an exogenous increase in the credit separation rate, leads to negative response of the inflation rate which is associated to the persistent fall in the real marginal cost. The real supply of money increases causing the nominal interest rate to fall below zero for the first two quarters. After that, the fall in the real money demand (fall in consumption) more than compensate the increase in the real money supply and the nominal interest rate raises.

In appendix F, the role of nominal rigidities in shaping the economy's response to a financial shock is analyzed. **Figures, 27-30 of appendix F**, displays the simulated responses to a financial shock under complete price and wage flexibility as well as under stocky prices but flexible wages. Those responses are compared to the benchmark case presented here.

5 Conclusion

The Great Recession and slow recovery was characterized by high and persistent unemployment and a decline in overall bank lending. The net decline in bank lending across all loan types was a novel feature of the Great Recession as it had not occurred in any previous post-Volker recession. These characteristics of the recession are suggestive of a relationship between bank credit and unemployment. But the mechanism linking credit tightness to employment is not clear particularly since previous research has highlighted that bank lending is primarily meaningful for small firms.

In this paper, we link credit flows to employment indirectly through search and matching frictions in the market for credit, embedded in an otherwise standard New Keynesian framework with wage and pricing frictions. We allow for endogenous credit destruction which then permits us to calculate movements in gross flows match them to empirical credit behavior. One of the interesting features of our model is the 'credit inefficiency wedge' it generates. It arises as the endogenous component of aggregate technology that is unrelated to employment and acts like a productivity wedge. If we shut off credit market frictions, the inefficiency disappears. But in the presence of credit frictions the inefficiency wedge depends on the aggregate probability that credit contracts are not broken as well as on the number of active firms. Both of factors depend ultimately on the reservation productivity threshold that separates producing from non-producing firms. Thus, in our model, credit tightness serves to amplify the effects of financial and monetary shocks through the inefficiency wedge.

Although our paper provides an important step in understanding the impacts of links between credit tightness and employment, there are several ways in which we could extend it to capture additional features of the economy. The first is to consider a more realistic monetary policy rule, such as a Taylor rule where the central bank cares about future inflation and the output gap. Secondly, we could consider the case where the firm could default on repayment of its loans. Undoubtedly part of the reluctance of banks to lend was related to increases in default risk particularly given the rise in volatility that accompanied the Great Recession. These issues are left for future work.

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6 Appendix A: Tables and Figures

Table 5: Descriptive statistics for credit and employment

Period	1973Q1-2012Q4		1984Q1-2007Q2		2007q3-2009q2		2009q3-2012q4	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Total loan creation	3.78	1.27	4.04	1.13	3.92	1.95	1.95	0.98
Total loan destruction	1.59	0.96	1.92	0.73	2.17	1.18	1.74	1.46
Sum total lending	5.37	1.44	5.95	1.10	6.09	1.88	3.69	1.54
Net total lending	2.19	1.74	2.12	1.55	1.76	2.61	0.21	1.96
Exc total lending	2.98	1.65	3.75	1.37	3.44	1.79	2.21	0.91
Unemployment rate	6.45	1.60	5.71	1.05	6.51	1.74	9.00	0.70
Average labor productivity	0.39	0.61	0.45	0.55	0.81	0.71	0.18	0.57
Job creation	4.83	0.84	4.76	0.57	3.18	0.40	3.76	0.22
Job destruction	5.24	1.07	5.14	0.69	5.40	1.51	3.64	0.53
Sum JC + JD	10.08	1.54	9.91	1.01	8.56	1.18	7.39	0.59
Net JC-JD	-0.41	1.17	-0.39	0.77	-2.23	1.87	0.12	0.57
Exc job creation	9.24	1.42	9.30	1.01	6.35	0.80	6.91	0.52

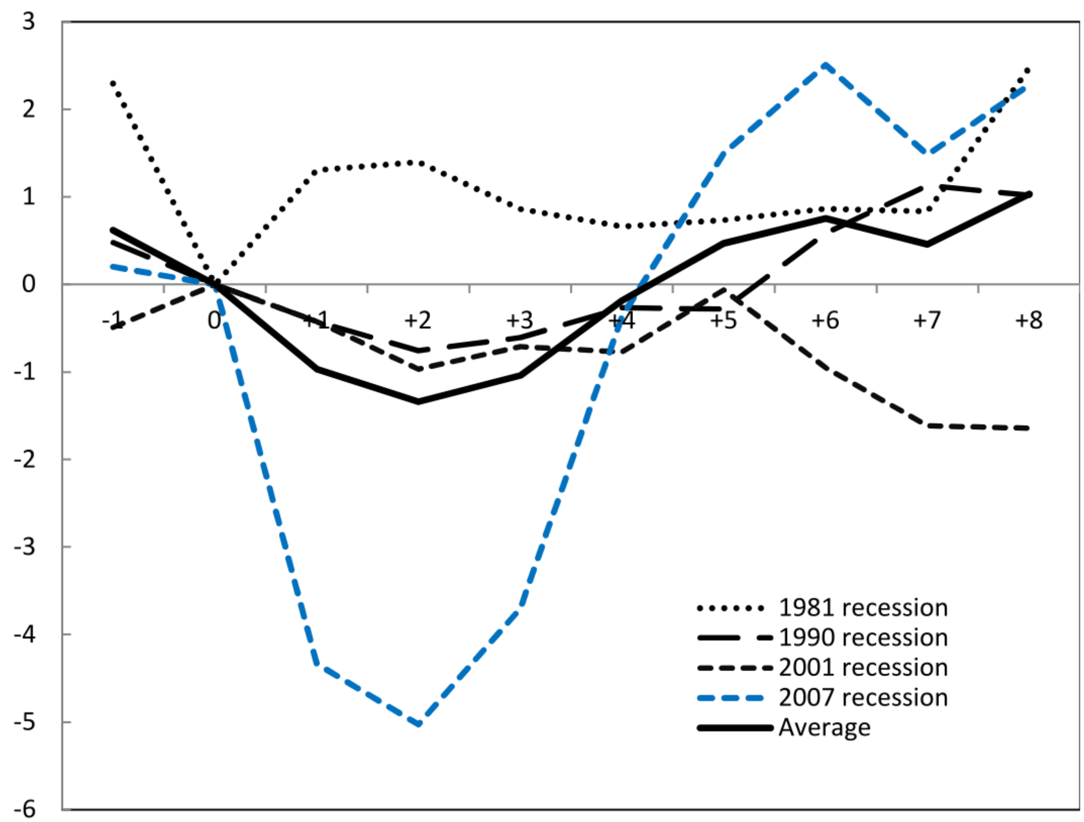
Note: Lending data is based on Reports of Income and Condition and calculations provided in **?**. Job flows data are taken from Faberman (2012) and updated with the 2012 Business Employment Dynamics Survey. The means and standard deviations of creation or destruction of credit or jobs are expressed in rates. The unemployment rate is seasonally adjusted and downloaded from the FRED data repository at the Federal Reserve, St Louis. Average Labor productivity is calculated from real GDP (seasonally adjusted) and hours of non-farm business employees. These data are also downloaded from FRED.

Table 6: Standard Deviations and Correlations

Period	1973q1-2012q4		1984Q1-2007Q2		2007q3-2009q2		2009q3-2012q4	
	σ/σ^{rgdp}	$\rho(.,rgdp)$	σ/σ^{rgdp}	$\rho(.,rgdp)$	σ/σ^{rgdp}	$\rho(.,rgdp)$	σ/σ^{rgdp}	$\rho(.,rgdp)$
Total loan creation	20.48	**0.311**	27.91	0.06	22.90	0.55	41.47	**0.47**
Total loan destruction	27.24	**-0.46**	32.47	**-0.28**	31.95	**-0.63**	42.24	**-0.82**
Sum total lending	13.50	0.09	18.32	-0.16	13.62	-0.04	27.19	-0.14
	σ/σ^{unemp}	$\rho(.,unemp)$	σ/σ^{unemp}	$\rho(.,unemp)$	σ/σ^{unemp}	$\rho(.,unemp)$	σ/σ^{unemp}	$\rho(.,unemp)$
Job creation	0.64	0.11	0.62	-0.05	0.70	**-0.80**	0.45	**0.66**
Job destruction	1.11	**0.41**	0.97	**0.32**	1.48	**0.99**	0.67	0.04
Sum JC + JD	0.53	**0.56**	0.50	**0.32**	0.75	**0.98**	0.32	0.38
	SD		SD		SD		SD	
Real GDP	0.02	–	0.01	–	0.022	–	0.01	–
Unemployment	0.12	–	0.10	–	0.19	–	0.15	–
Number of Observations	160		94		8		14	

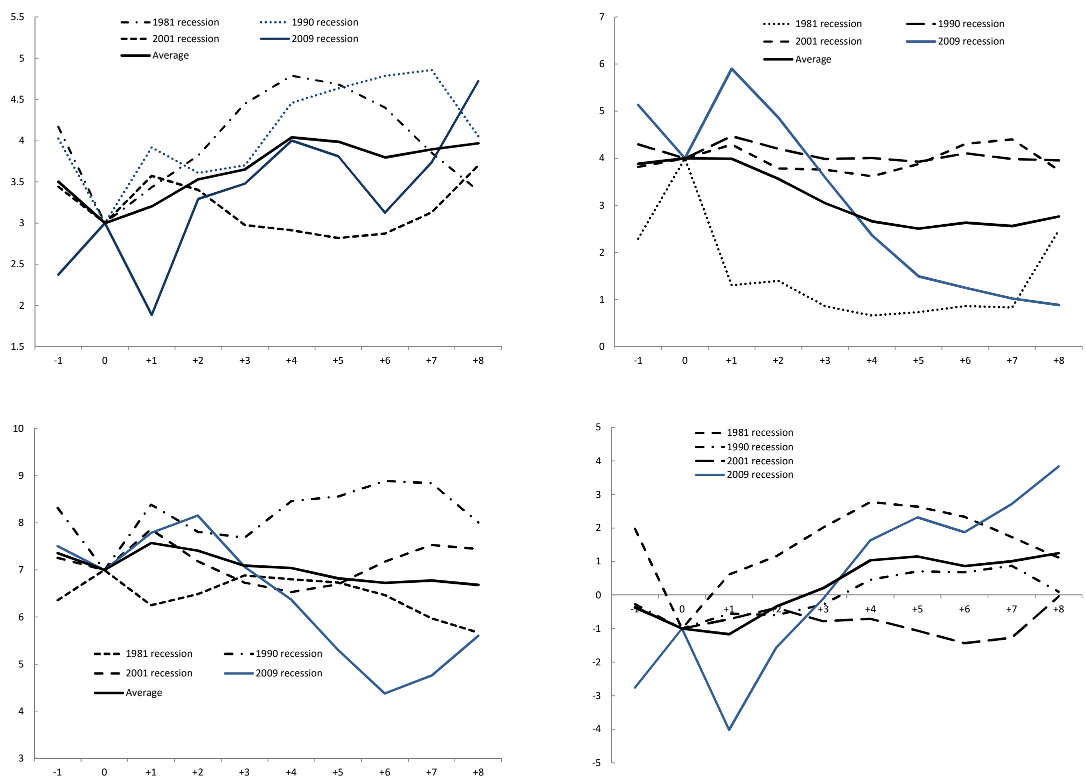
Note: Lending data is based on Reports of Income and Condition and calculations provided in Contessi, DiCecio, and Francis (2015). Job flows data are taken from Faberman (2012) and updated with the 2012 Business Employment Dynamics Survey. Standard deviations and correlations are calculated using the cyclical component of the HP-filtered log-level of the variable. Boldface numbers indicate statistical significance at the 5% confidence level or better. Unemployment and RGDP are the HP filtered cyclical components of the log series of the unemployment rate and seasonally adjusted real GDP respectively.

Figure 1: Net credit flows: Commercial lending



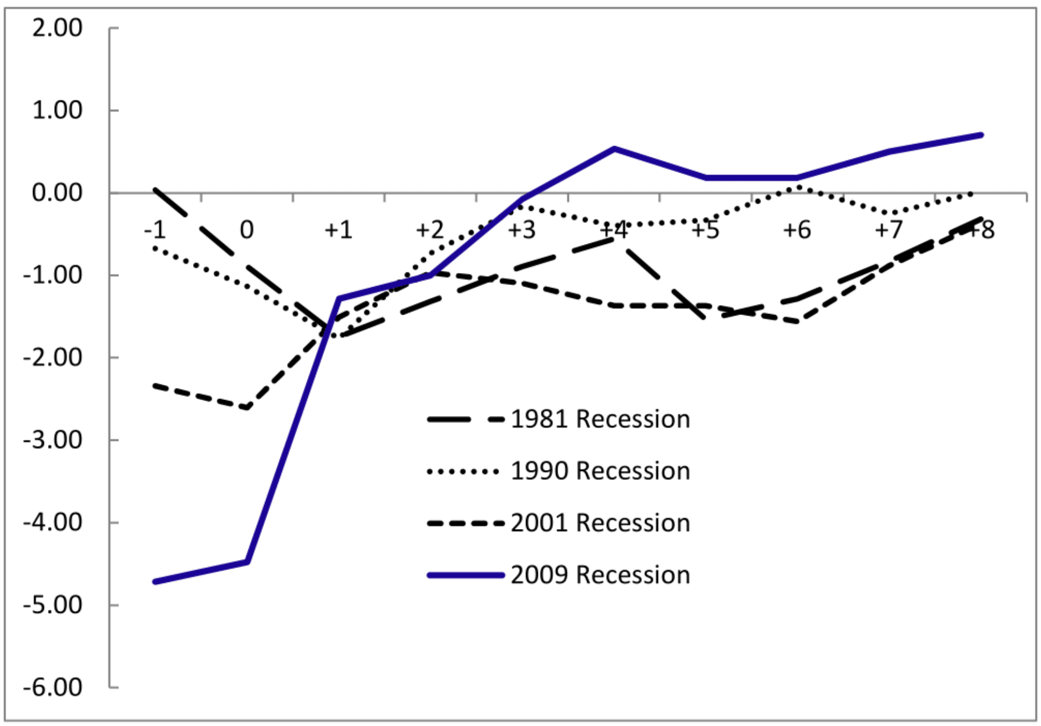
In percent; quarters from trough (located at 0). Net credit creation in commercial and industrial lending. Net credit creation is seasonally adjusted and smoothed with a moving average process. Authors' calculations based on Reports of Income and Condition.

Figure 2: Credit flows and reallocation: Commercial lending



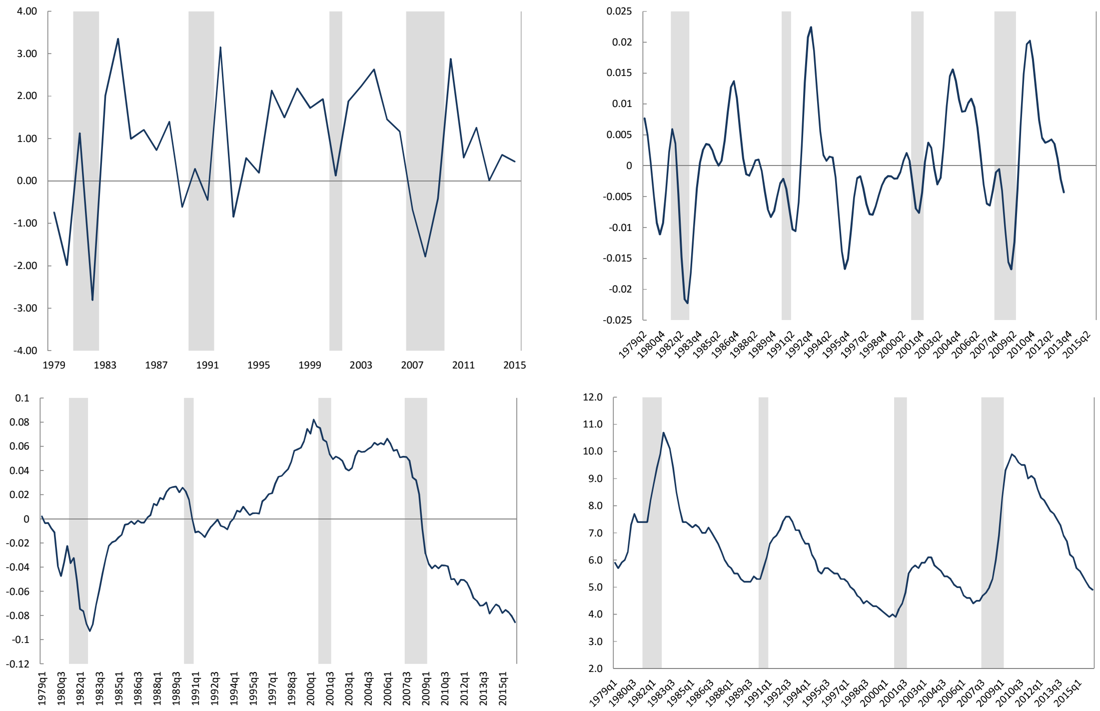
Top row: credit creation (NBER dated recession trough is at mean equal to 3 percent) and credit destruction (trough is at mean equal to 4 percent) in commercial and industrial lending. Bottom row: credit reallocation (which equals the sum of credit creation and destruction; trough is at mean equal to 7 percent) and excess credit reallocation (which equals the sum of credit creation and destruction less the absolute value of net credit creation; trough is at mean equal to -1 percent). All data are reported in percent, seasonally adjusted and smoothed with a moving average process. Authors' calculations based on Reports of Income and Condition.

Figure 3: Net job creation last two recessions



In percent; quarters from trough (located at 0). Net job creation in manufacturing. Authors' calculations based on data from Faberman (2012) and updated with recent Business Employment Dynamics data from the U.S. Census.

Figure 4: Aggregate data



Top row, from left, Total Factor Productivity (Based on data from Fernald (2012) update downloaded from http://www.frbsf.org/economic-research/indicators-data/total-factor-productivity-tfp/. Annual percentage change reported for business sector TFP.) and seasonally adjusted quarterly average labor productivity filtered with a Baxter King filter. Bottom row, from left, detrended logged seasonally adjusted quarterly real GDP and the unemployment rate. All data from the Federal Reserve, St Louis FRED data repository except for TFP as noted.

Figure 5: Conditional responses to a financial (EBP) shock

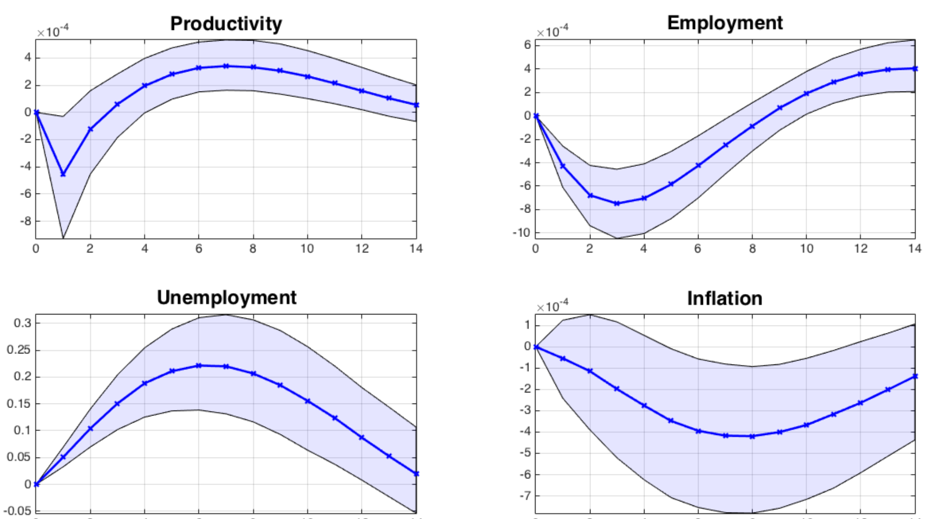


Figure 6: Conditional responses to a financial (EBP) shock

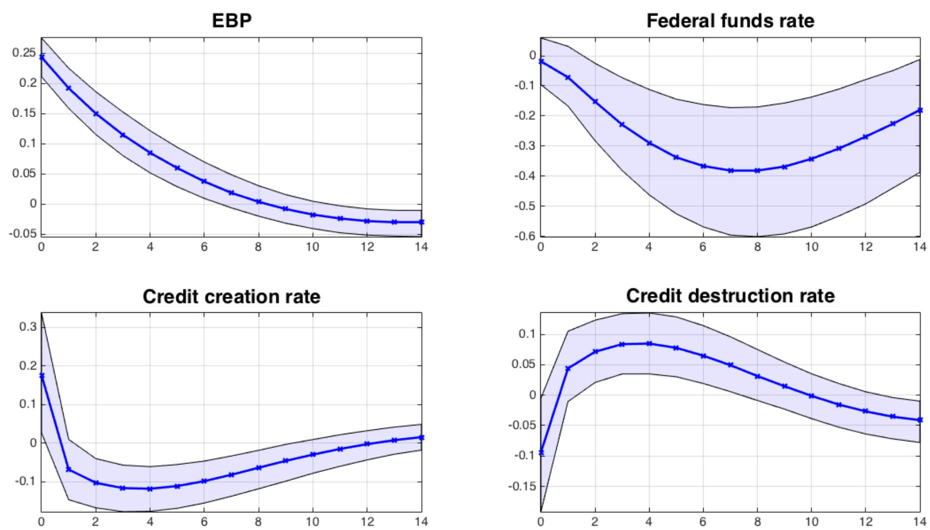


Figure 7: Model responses to a monetary policy shock: panel 1

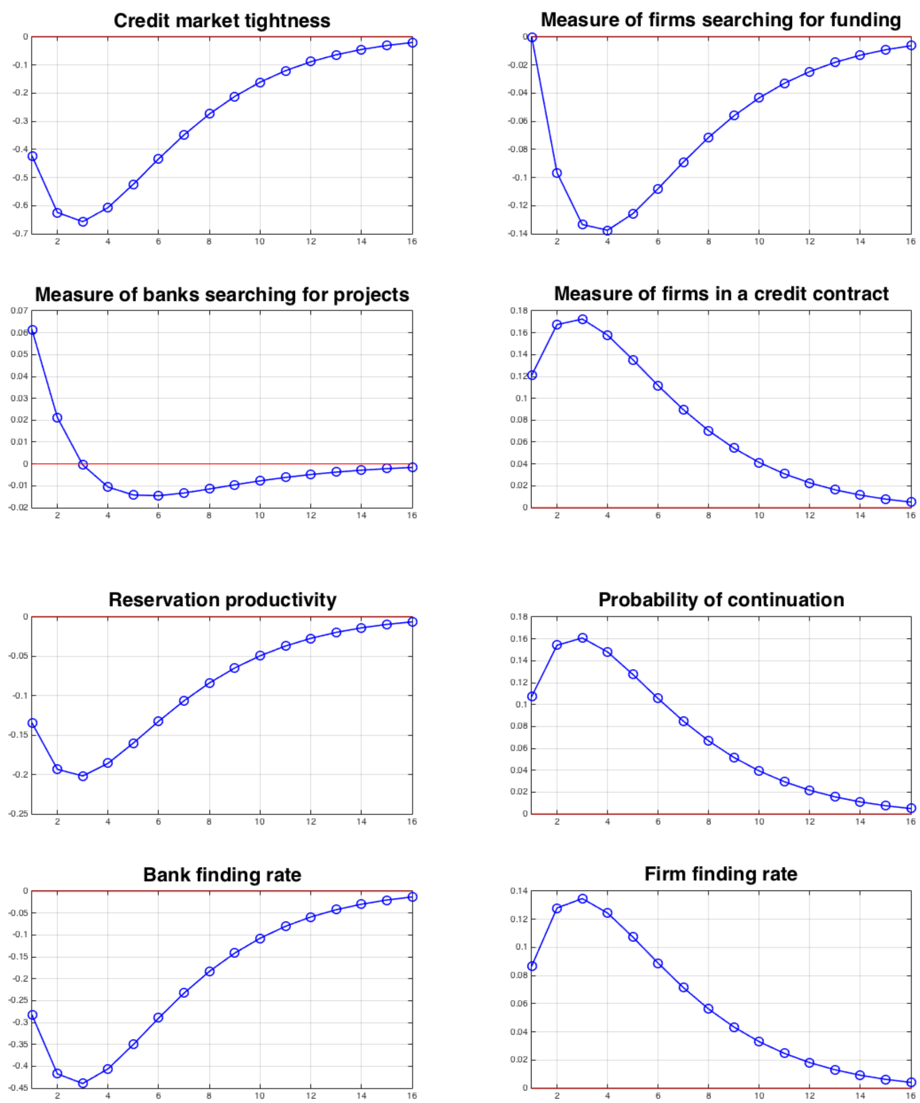


Figure 8: Model responses to a monetary policy shock: panel 2

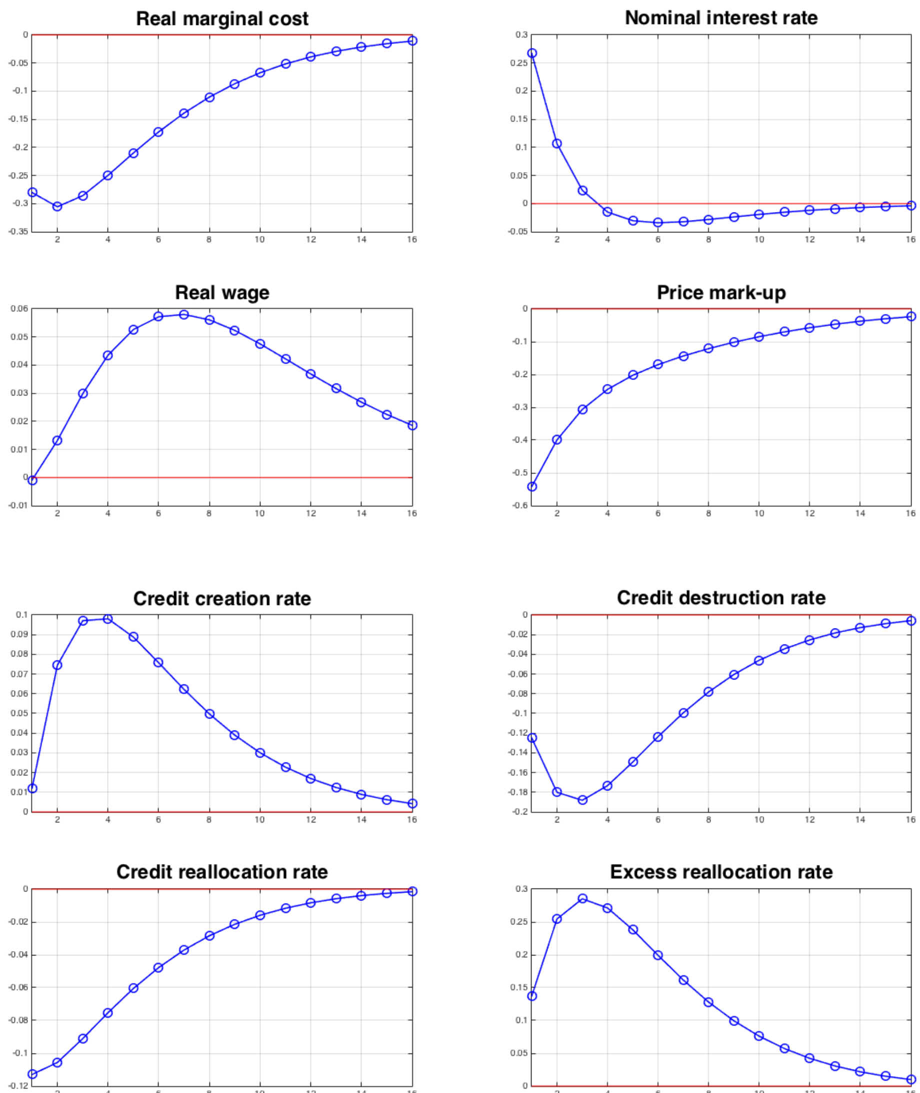


Figure 9: Model responses to a monetary policy shock: panel 3

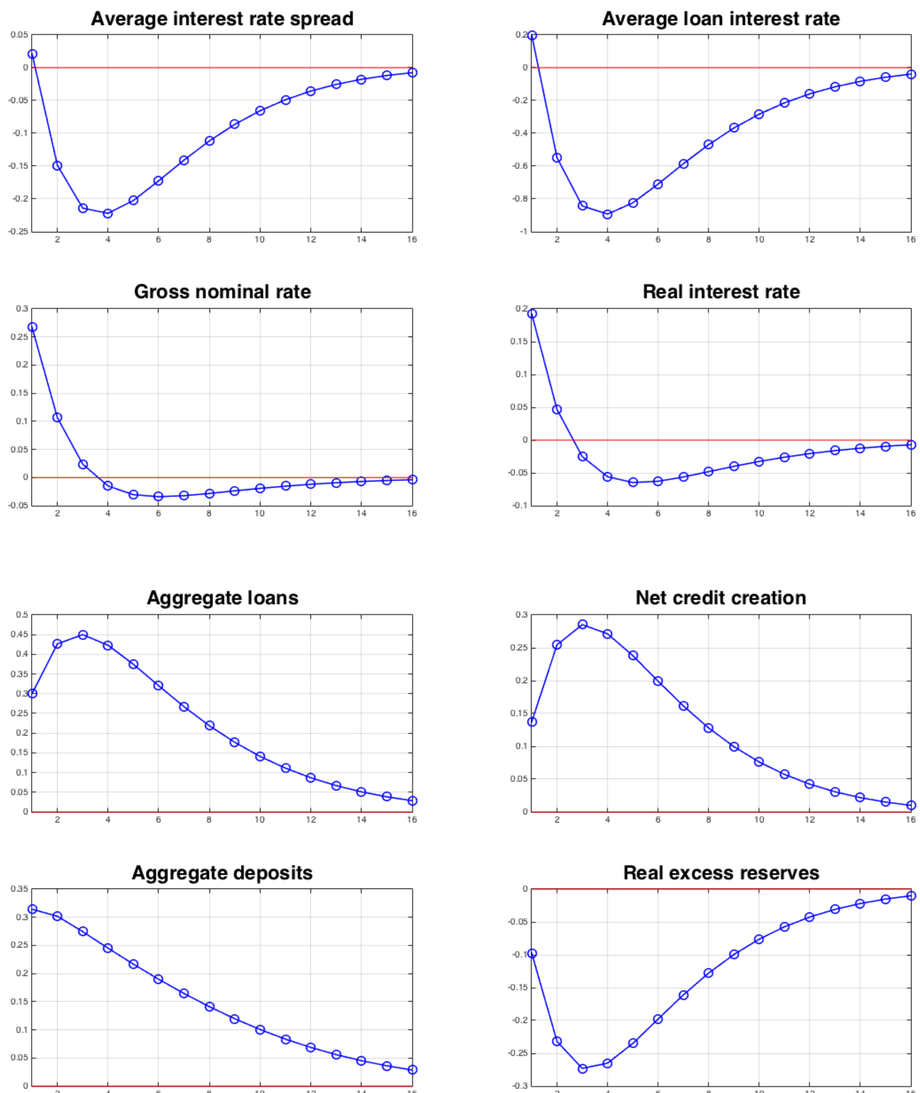


Figure 10: Model responses to a monetary policy shock: panel 4

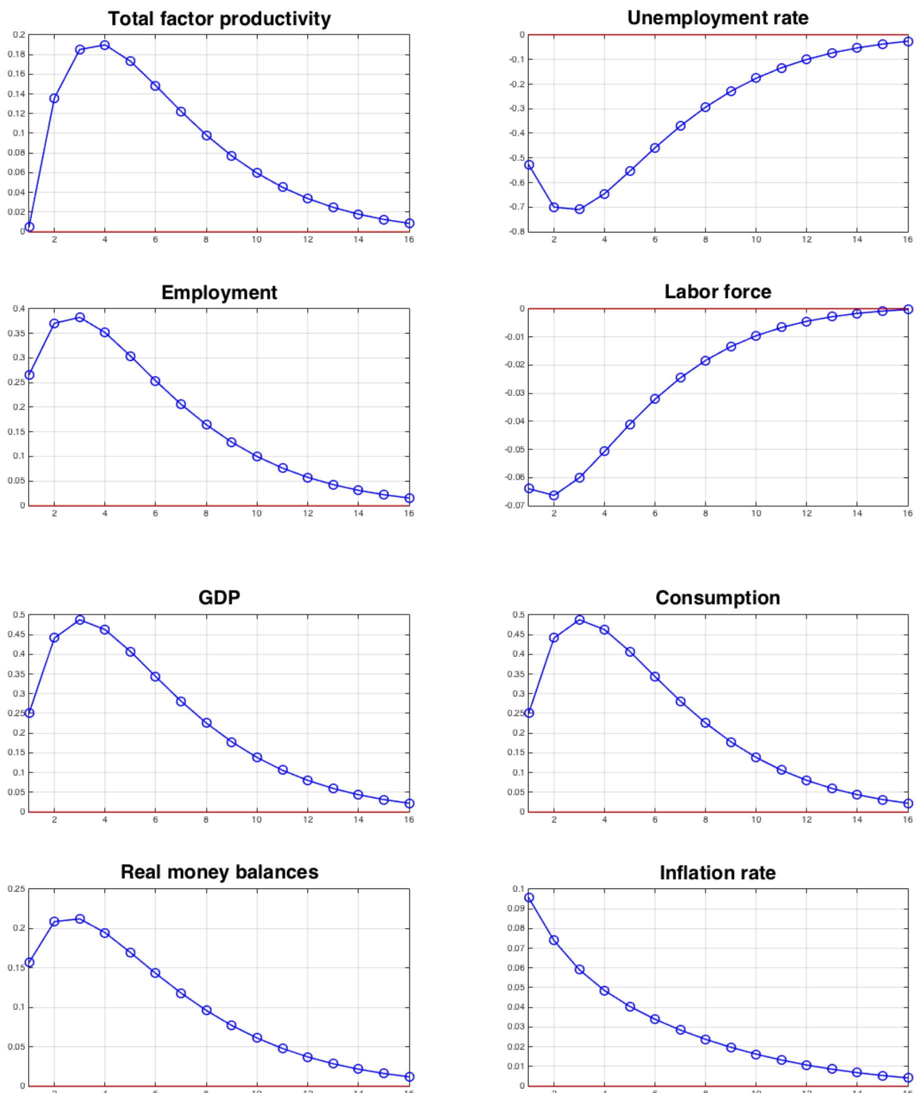


Figure 11: Model responses to a financial shock: panel 1

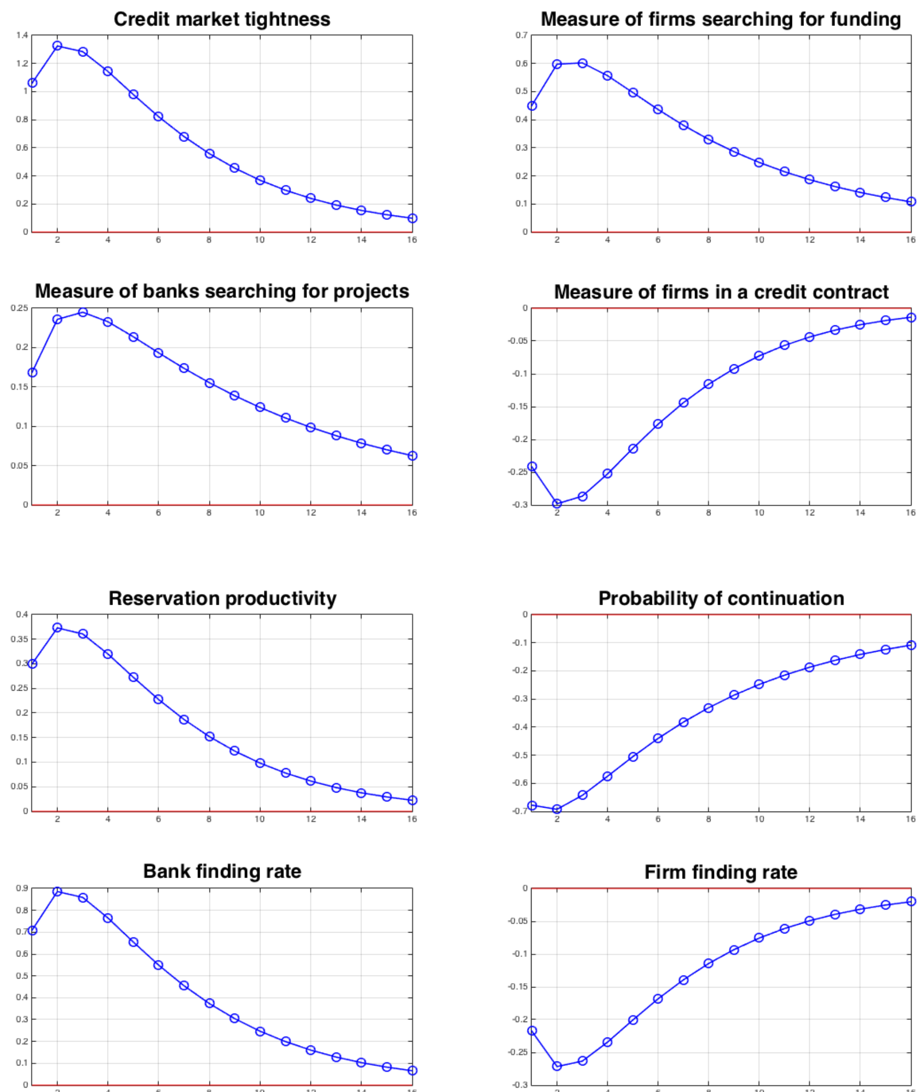


Figure 12: Model responses to a financial shock: panel 2

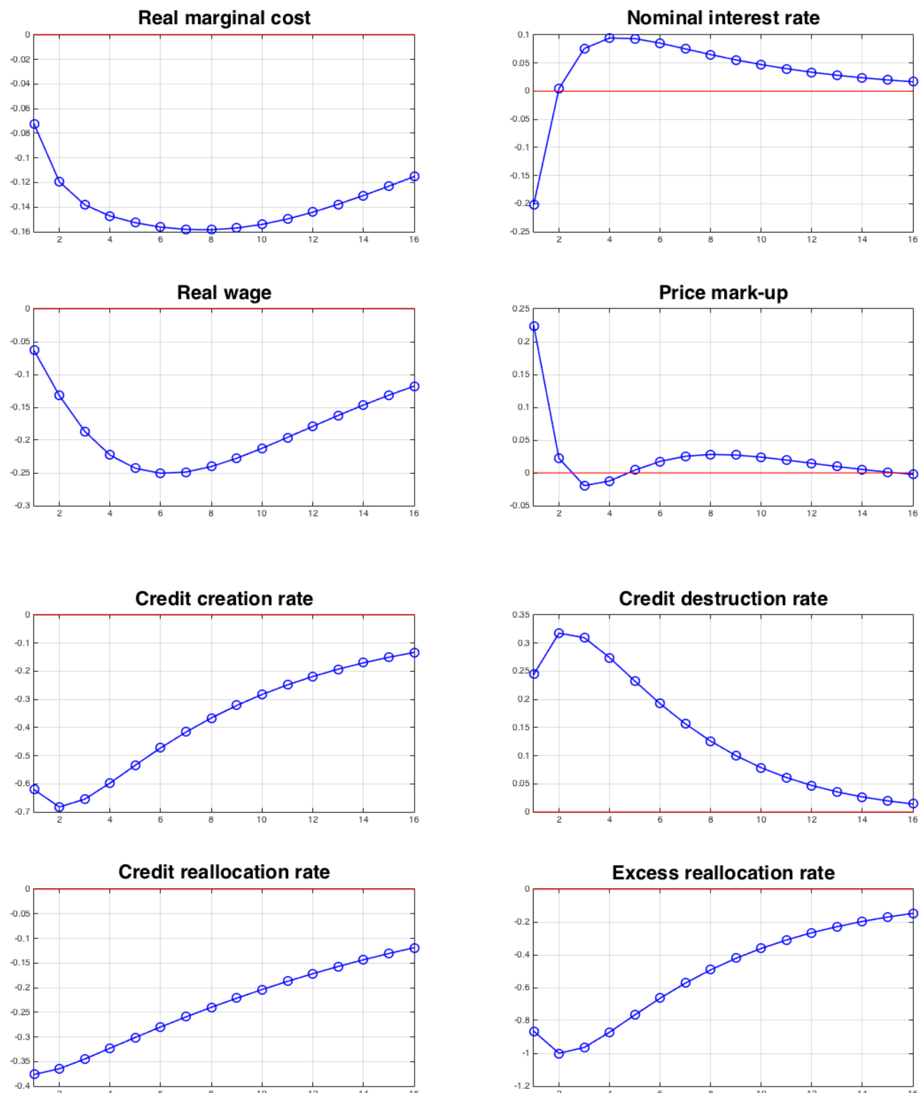


Figure 13: Model responses to a financial shock panel 3

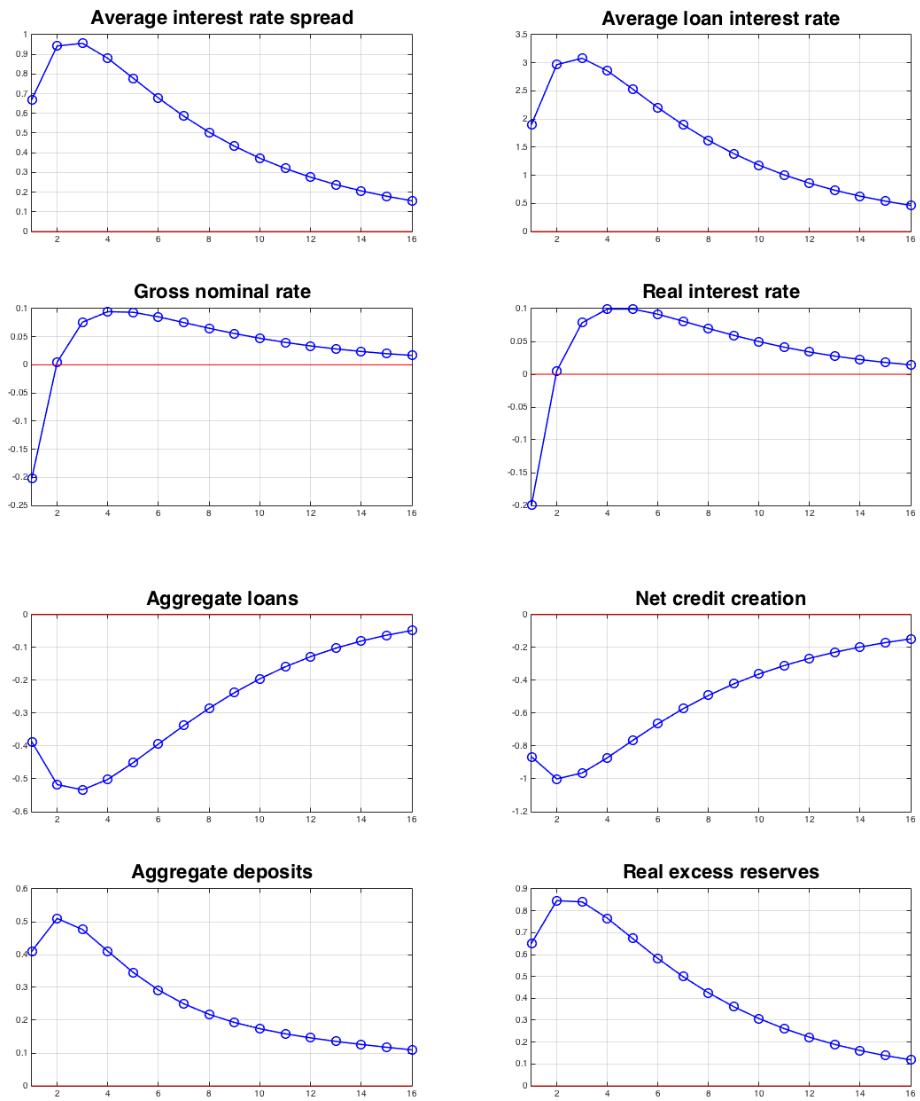
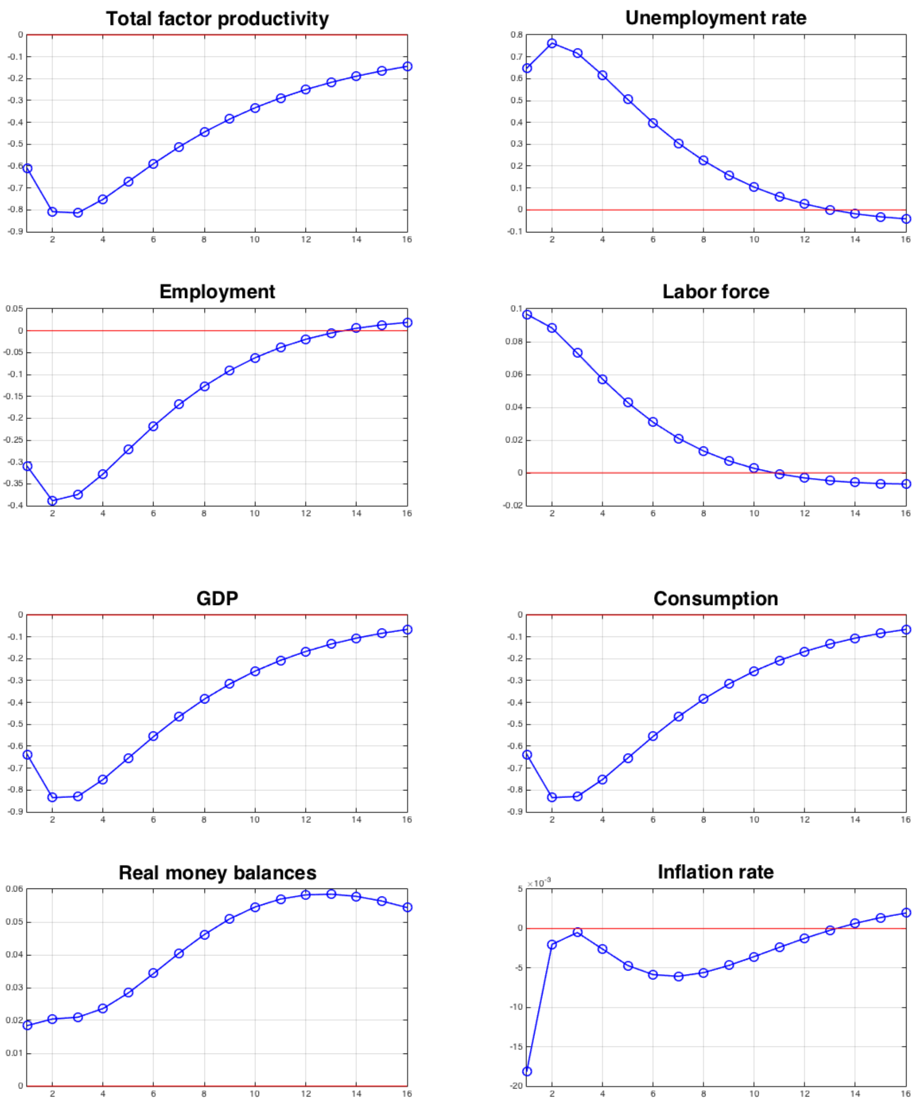


Figure 14: Model responses to a financial shock panel 4



7 Appendix B: Data sources

The macroeconomic data used in our analysis is available from public sources. Real GDP Y, the GDP deflator, the measure of employment (which is defined as the log of workers in the non-farm business sector and denoted by $\ln N$), average labor productivity calculated as $\ln \frac{Y}{N}$, the unemployment rate and the federal funds rate FFR, are all downloaded from the FRED repository at the Federal Reserve Bank of St. Louis. Quarterly gross job flows from 1992-2012 are downloaded from the Business Economic Dynamics database maintained by the Bureau of Labor Statistics and augmented with historical data (1980-1991) from Faberman (2012).

8 Appendix C: Technical details

8.1 Value functions for intermediate good producers

Under Nash bargaining, the reservation productivity level $\tilde{\omega}_t$ that triggers endogenous separation is determined by the point at which the joint surplus of the match is equal to zero. The probability of endogenous separation is $\gamma_{t+1}(\tilde{\omega}_{t+1}) = G(\tilde{\omega}_{t+1}) = prob(\omega_{z,t+1} \leq \tilde{\omega}_{t+1})$. Given the existence and uniqueness of $\tilde{\omega}_{t+1}$, the integral term on the expected continuation value is

$$\int_{\underline{\omega}}^{\overline{\omega}} \max(V_{t+1}^{FP}(\omega_{z,t+1}), V_{t+1}^{FN}) dG(\omega) = \gamma_{t+1} V_{t+1}^{FN} + (1-\gamma_{t+1}(\tilde{\omega}_{t+1})) \int_{\tilde{\omega}_{t+1}}^{\overline{\omega}} V_{t+1}^{FP}(\omega_{z,t+1}) \frac{dG(\omega)}{1-\gamma_{t+1}(\tilde{\omega}_{t+1})}. \quad (106)$$

Therefore, the firm value functions can be written as

$$V_t^{FP}(\omega_{z,t}) = \pi_t^I(\omega_{z,t}) + \mathrm{E}_t \Delta_{t,t+1} \left\{ (1-\varphi_t(\tilde{\omega}_{t+1})) V_{t+1}^{FN} + \varphi_t(\tilde{\omega}_{t+1}) \int_{\tilde{\omega}_{t+1}}^{\overline{\omega}} V_{t+1}^{FP}(\omega_{z,t+1}) \frac{dG(\omega)}{1-\gamma_{t+1}(\tilde{\omega}_{t+1})} \right\}$$

and

$$V_t^{FN} = \mathrm{E}_t \Delta_{t,t+1} \left\{ p_t^f \left[(1-\varphi_t(\tilde{\omega}_{t+1})) V_{t+1}^{FN} + \varphi_t(\tilde{\omega}_{t+1}) \int_{\tilde{\omega}_{t+1}}^{\overline{\omega}} V_{t+1}^{FP}(\omega_{z,t+1}) \frac{dG(\omega)}{1-\gamma_{t+1}(\tilde{\omega}_{t+1})} \right] + \left(1-p_t^f\right) V_{t+1}^{FN} \right\} \quad (107)$$

Let the surplus to a producing firm be defined as $V_t^{FS}(\omega_{z,t}) = V_t^{FP}(\omega_{z,t}) - V_t^{FN}$, then the intermediate producer surplus of being in a credit relationship can be written as

$$V_t^{FS}(\omega_{z,t}) = \pi_t^I(\omega_{z,t}) + \left(1-p_t^f\right) \mathrm{E}_t \Delta_{t,t+1} \varphi_t(\tilde{\omega}_{t+1}) \int_{\tilde{\omega}_{t+1}}^{\overline{\omega}} V_{t+1}^{FS}(\omega_{z,t+1}) \frac{dG(\omega)}{1-\gamma_{t+1}(\tilde{\omega}_{t+1})} \quad (108)$$

Characterizing loan market equilibrium Partial equilibrium in the loan market can be characterized by a system of two equations in two unknowns: credit market tightness τ_t and the reservation productivity level $\widetilde{\omega}_t$. The evolution of credit market tightness is obtained by using the free entry condition, the Nash bargaining sharing rule, and the definition of the joint surplus of a credit relationship, and it is given by the following equation:

$$\frac{\kappa}{\mu_t^p \mu \tau_t^{\varphi}} - \mathrm{E}_t \Delta_{t,t+1} \varphi_{t+1}(\tilde{\omega}_{t+1}) \left(1-\overline{\eta}\mu\tau_{t+1}^{\varphi-1}\right) \frac{\kappa}{\mu_{t+1}^p \mu \tau_{t+1}^{\varphi}} = (1-\overline{\eta}) \mathrm{E}_t \Delta_{t,t+1} \frac{1}{\mu_{t+1}^p} \left((1-\alpha) \frac{Y_{t+1}^I}{f_t^m} - \varphi_{t+1}(\tilde{\omega}_{t+1}) x^f \right) \quad (109)$$

The second equation is given by the optimal reservation productivity level, $\widetilde{\omega}_t$ written as a function of τ_t:

$$\left[\alpha^{\alpha}(1-\alpha)^{1-\alpha} \xi^{pf} A_t \widetilde{\omega}_t\right]^{\frac{1}{1-\alpha}} = (\mu_t^p w_t R_t)^{\frac{\alpha}{1-\alpha}} \left[x^f - \left(\frac{1-\overline{\eta}\mu\tau_t^{\nu-1}}{1-\overline{\eta}} \right) \frac{\kappa}{\mu\tau_t^{\nu}} \right] \quad (110)$$

At the steady state, the equations for τ_t and $\widetilde{\omega}_t$ become

$$\kappa\left(1-\beta\varphi(\tilde{\omega})\left(1-\overline{\eta}\mu\tau^{\nu-1}\right)\right) = \mu\tau^{\nu}(1-\overline{\eta})\beta\left((1-\alpha)\frac{Y^I}{f^m} - \varphi(\tilde{\omega}) x^f\right) \quad (111)$$

and

$$\left[\alpha^{\alpha}(1-\alpha)^{1-\alpha} \xi^{pf} A \widetilde{\omega}\right]^{\frac{1}{1-\alpha}} = (\mu^p w R)^{\frac{\alpha}{1-\alpha}} \left[x^f - \frac{\kappa}{1-\overline{\eta}} \left(\frac{1-\overline{\eta}\mu\tau^{\nu-1}}{\mu\tau^{\nu}} \right) \right] \quad (112)$$

respectively.

8.2 Summarizing the non-linear equilibrium conditions

The system of non-linear equations that characterize the aggregate equilibrium of the model economy is:

- **Monetary policy:**
- Money growth rule:

$$m_t = \left(\frac{1+\theta_t}{\Pi_t}\right) m_{t-1} \tag{D1}$$

where

$$\theta_t - \theta^{ss} = \rho\left(\theta_{t-1} - \theta^{ss}\right) + \xi_t \tag{113}$$

- **Euler equation:**

$$\frac{1}{C_t} = \beta E_t\left\{\left(\frac{R_t}{\Pi_{t+1}}\right)\frac{1}{C_{t+1}}\right\} \tag{D2}$$

- **CIA constraint:**

$$C_t = m_t - R_t er_t \tag{D3}$$

- **Wage setting equation:**

$$f_{1,t} = \left(\frac{\varepsilon_w}{\varepsilon_w - 1}\right) f_{2,t} \tag{D4}$$

$$f_{1,t} = \left(w_t^*\right)^{1-\varepsilon_w}\left(w_t\right)^{\varepsilon_w}\frac{N_t}{C_t\Delta_t^w} + \beta\theta_w E_t\left(\frac{1}{\Pi_{t+1}}\right)^{1-\varepsilon_w}\left(\frac{w_t^*}{w_{t+1}^*}\right)^{1-\varepsilon_w} f_{1,t+1} \tag{D5}$$

$$f_{2,t} = \chi_t\left(\frac{w_t^*}{w_t}\right)^{-\varepsilon_w(1+\overline{\varphi})}\left(\frac{N_t}{\Delta_t^w}\right)^{(1+\overline{\varphi})} + \left(\beta\theta_w\right)E_t\left(\frac{1}{\Pi_{t+1}}\right)^{-\varepsilon_w(1+\overline{\varphi})}\left(\frac{w_t^*}{w_{t+1}^*}\right)^{-\varepsilon_w(1+\overline{\varphi})} f_{2,t+1} \tag{D6}$$

- **Aggregate wage index in real terms:**

$$1 = \theta_w\left(\frac{w_{t-1}}{w_t}\frac{1}{\Pi_t}\right)^{1-\epsilon_w} + \left(1-\theta_w\right)\left(\frac{w_t^*}{w_t}\right)^{1-\epsilon_w} \tag{D7}$$

- **Wage dispersion:**

$$\Delta_t^w = \theta_w\left(\frac{w_{t-1}}{w_t}\frac{1}{\Pi_t}\right)^{-\epsilon_w}\Delta_{t-1}^w + \left(1-\theta_w\right)\left(\frac{w_t^*}{w_t}\right)^{-\epsilon_w} \tag{D8}$$

- **Price setting equation:**

$$g_{1,t} = \beta C_t\Pi_t^* + \theta_p E_t\left(\frac{\Pi_t^*}{\Pi_{t+1}^*}\right) g_{1,t+1} \tag{D9}$$

$$g_{2,t} = \beta\frac{1}{\mu_t^p}C_t + \theta_p E_t g_{2,t+1} \tag{D10}$$

$$g_{1,t} = \left(\frac{\epsilon_p}{\epsilon_p - 1}\right) g_{2,t} \tag{D11}$$

- **Aggregate price index in terms of inflation rates:**

$$1 = \theta_p\left(\frac{1}{\Pi_t}\right)^{1-\epsilon_p} + \left(1-\theta_p\right)\left(\Pi_t^*\right)^{1-\epsilon_p} \tag{D12}$$

- **Price dispersion:**

$$\Delta_t^p = \theta_p\left(\frac{1}{\Pi_t}\right)^{-\epsilon_p}\Delta_{t-1}^p + \left(1-\theta_p\right)\left(\Pi_t^*\right)^{-\epsilon_p} \tag{D13}$$

- **Unemployment:**

$$U_t = 1 - \frac{N_t}{L_t} \tag{D14}$$

- **Aggregate labor supply:**

$$w_t = R_t C_t \chi_t \left(L_t\right)^{\overline{\varphi}} \tag{D15}$$

- **Resource constraint:**

$$\Delta_t^p C_t = Y_t^f \tag{D16}$$

- **Aggregate final good :**

$$Y_t^f = Y_t^I - \left(b_t^u \kappa + \varphi_t\left(\tilde{\omega}_t\right) f_{t-1}^m x^f\right) \tag{D17}$$

- **Aggregate bank's balance sheet:**

$$l_t + er_t + \xi = d_t \tag{D18}$$

- **Consistency of aggregate loans and deposits:**

$$l_t = \varphi_t\left(\tilde{\omega}_t\right) f_{t-1}^m d_t \tag{D19}$$

- **Aggregate equilibrium in the loan market:**

$$l_t = \frac{w_t N_t}{\Delta_t^w} \tag{D20}$$

- **Aggregate production function for the intermediate good sector:**

$$Y_t^I = A_t \xi^{pf} \left(F_t\right)^{1-\alpha} \left(\frac{N_t}{\Delta_t^w}\right)^{\alpha} \tag{D21}$$

- **Aggregate employment:**

$$N_t = \left(\frac{\alpha A_t \xi^{pf}}{\mu_t^p w_t R_t}\right)^{\frac{1}{1-\alpha}} F_t \Delta_t^w \tag{D22}$$

- **Credit friction input (credit miss-allocation "input")** F_t :

$$F_t = \left(1-\delta_t\right)\left(\frac{\left(\overline{\omega}\right)^k - \left(\widetilde{\omega}_t\right)^k}{k\left(\overline{\omega}-\underline{\omega}\right)}\right) f_{t-1}^m \tag{D23}$$

- **Credit market tightness:**

$$\tau_t = \frac{f_t}{b_t^u} \tag{D24}$$

- **Measure of firms in a credit relationship:**

$$f_t^m = \varphi_t\left(\tilde{\omega}_t\right) f_{t-1}^m + p_t^f f_t \tag{D25}$$

- **Measure of firms searching for credit:**

$$f_t = 1 - \left(1-\delta_t\right) f_{t-1}^m \tag{D26}$$

- **Overall continuation rate for a credit contract:**

$$\varphi_t\left(\tilde{\omega}_t\right) = \left(1-\delta_t\right)\left(\frac{\overline{\omega}-\tilde{\omega}_t}{\overline{\omega}-\underline{\omega}}\right) \tag{D27}$$

- **Reservation productivity:**

$$\left[\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha} A_t \xi^{pf} \widetilde{\omega}_t\right]^{\frac{1}{1-\alpha}} = \left(\mu_t^p w_t R_t\right)^{\frac{\alpha}{1-\alpha}} \left[x^f - \left(\frac{1-\overline{\eta} p_t^f}{1-\overline{\eta}}\right) \frac{\kappa}{p_t^b}\right] \tag{D28}$$

- **Evolution of credit market tightness:**

$$\frac{\kappa}{\mu_t^p p_t^b} - \mathrm{E}_t \Delta_{t,t+1} \varphi_{t+1} (\tilde{\omega}_{t+1}) \left(1 - \overline{\eta} p_{t+1}^f\right) \frac{\kappa}{\mu_{t+1}^p p_{t+1}^b} = (1-\overline{\eta}) \mathrm{E}_t \Delta_{t,t+1} \frac{1}{\mu_{t+1}^p} \left((1-\alpha) \frac{Y_{t+1}^I}{f_t^m} - \varphi_{t+1} (\tilde{\omega}_{t+1}) x^f\right) \tag{D29}$$

- **Stochastic discount factor:**

$$\Delta_{t,t+1} = \beta \left(\frac{R_t}{R_{t+1}} \frac{C_t}{C_{t+1}}\right) \tag{D30}$$

- **Matching rate for firms:**

$$p_t^f = \mu \tau_t^{\nu-1} \tag{D31}$$

- **Matching rate for banks:**

$$p_t^b = \mu \tau_t^{\nu} \tag{D32}$$

- **Gross real interest rate:**

$$1 + r_t = \frac{R_t}{\mathrm{E}_t \Pi_{t+1}} \tag{D33}$$

- **Credit destruction rate:**

$$cd_t = 1 - \varphi_t (\tilde{\omega}_t) - p_t^b \delta_t \tag{D34}$$

- **Credit creation rate:**

$$cc_t = \frac{m_t}{f_{t-1}^m} - p_t^f \delta_t \tag{D35}$$

- **Labor share of GDP:**

$$LS_t = \frac{w_t N_t}{Y_t^f} \tag{D36}$$

- **Fixed cost of production share of GDP:**

$$FCS_t = \frac{\varphi_t (\tilde{\omega}_t) f_{t-1}^m x^f}{Y_t^f} \tag{D37}$$

- **Definition of aggregate loans as a fraction of deposits:**

$$\widehat{l}_t = \frac{l}{d} \tag{D38}$$

- **Definition of aggregate excess reserves as a fraction of deposits:**

$$\widehat{er}_t = \frac{er}{d} \tag{D39}$$

9 Appendix D: A cashless economy with a Taylor Rule for Monetary Policy

Ideally, we would like to solve our model with a Taylor rule but at this point, we have regions of indeterminacy for sensible parameter values when we attempt it.[15] We can solve the model with a Taylor rule in the case where the cash-in-advance constraint is removed as in a cashless economy. We set the Taylor rule coefficient on current inflation to 2, increase the persistence of the credit separation shock (financial shock) to 0.95. We present the impulse responses to a contractionary monetary shock, financial shock, and technology shock under a Taylor rule below. We find that the responses to a monetary shock are nearly identical under either the money growth rule or the Taylor rule. We find different responses when we consider a financial shock. We find the following differences: a much larger increase in unemployment and larger decrease in GDP and TFP. These are likely caused by significantly more credit tightness with a larger decrease in credit creation

[15]Link to updated paper

and increase in credit destruction. The inflation rate raises in the cashless economy while it falls in the cash in advance economy after a financial shock occurs. This result is explained by the different response of the price mark-up observed in each model. In both economies, the price mark-up is inversely related to the inflation rate but in the cashless economy, the price mark-up falls, producing a persistent raise in the inflation rate. Therefore, the central bank raises the interest rate following the Taylor rule. On the contrary, in the cash in advance economy, the inflation rate falls producing a persistent raise in real money balances and the consequent fall in the nominal interest rate. The latter occurs since nominal money balances growth at a constant and exogenous rate.

For this version of the model, we assume a cashless economy where firms need to secure funds for paying its wage bill in advance of production. For this, firms take loans from banks before attempting to produce but do not use those funds until production takes place and it is time to pay for labor at the end of the period. In other words, banks will place those funds in the firm's account as a secured deposit. Then, the firm will be able to produce and hire workers. At the end of the period firms are able to use those funds to pay for labor services. Under these timing assumption, workers do not receive their payments in advance as in the benchmark model.

The household problem without the CIA constraint is given by

$$\max \log C_t - \chi_t \int_0^1 \frac{N_t(i)^{\varphi+1}}{1+\varphi} di$$

s.t

$$\left(1+i_t^d\right) D_t + \Pi_t^b + \Pi_t^f - D_{t+1} + \int_0^1 W_t(i) N_t(i) di - P_t C_t$$

and the first order conditions are characterized by the following Euler equation:

$$\frac{1}{C_t} = \beta E_t \left\{ \frac{1}{C_{t+1}} \left(1 + r_{t+1}\right) \right\}$$

where

$$1 + r_{t+1} = \frac{1 + i_{t+1}}{\Pi_{t+1}}$$

In this setting, the marginal utility of consumption is the same as the marginal benefit of income (shadow price of income). On the other hand, the wage setting problem does not take into account the CIA constraint and the first order condition that characterizes the solution of this problem can be written in the same way as in the model that incorporates the CIA constraint with the only difference being that the multiplier that affects the first order condition is the marginal utility of consumption.

The model is closed assuming that the central bank follows a Taylor rule of the form:

$$R_t = (R_t)^{\phi_R} \left(\left(\frac{1}{\beta}\right) (\Pi_t)^{\phi_\pi} \left(\frac{Y_t^f}{Y^f}\right)^{\phi_Y} \right)^{(1-\phi_R)} \exp\left(\epsilon_t^i\right)$$

We calibrate the above monetary policy rule as $\phi_R = 0.8, \phi_\pi = 2$ and $\phi_Y = 0.$.

In this case, since the marginal value of income (the shadow price of income) is the same as the marginal utility of consumption then aggregate labor supply does not depend on R and it is defined to be

$$w_t = C_t \chi_t (L_t)^{\overline{\varphi}}$$

Finally, the consolidated budget constraint takes the form

$$0 = T_t + i_t e r_t$$

since we are assuming a cashless economy. Notice that in this case, excess reserves are related to fiscal policy funded by lump-sum taxation. The rest of the equations remain the same as before.

Figure 15: Model responses to a contractionary monetary policy shock under Taylor rule: panel 1

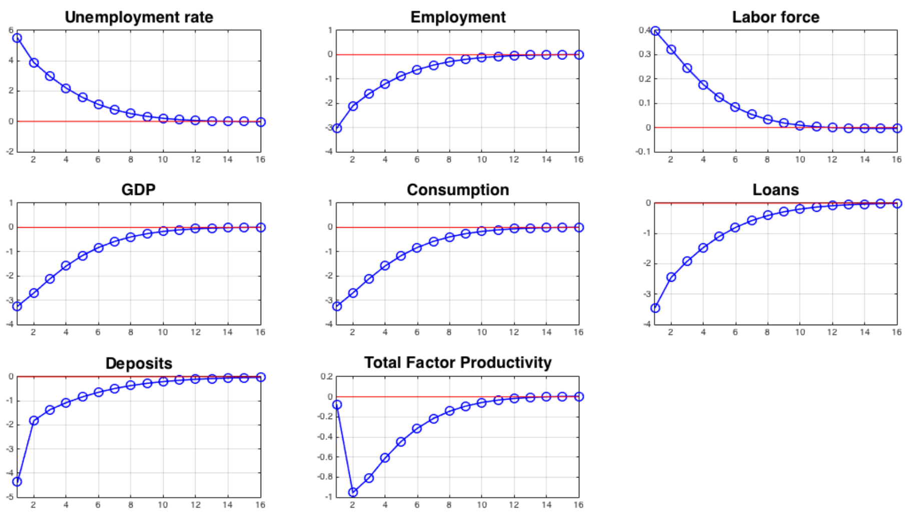


Figure 16: Model responses to a contractionary monetary policy shock under Taylor rule: panel 2

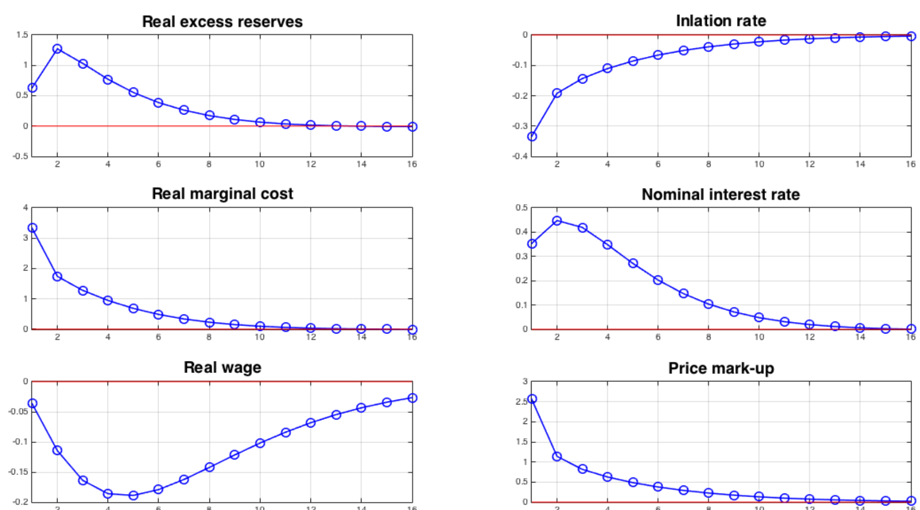


Figure 17: Model responses to a contractionary monetary policy shock under Taylor rule: panel 3

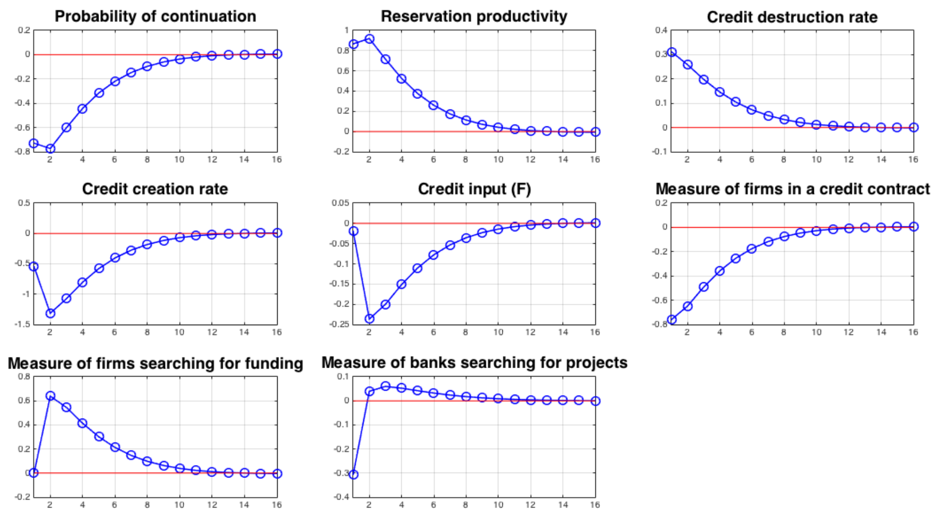


Figure 18: Model responses to a contractionary monetary policy shock under Taylor rule: panel 4

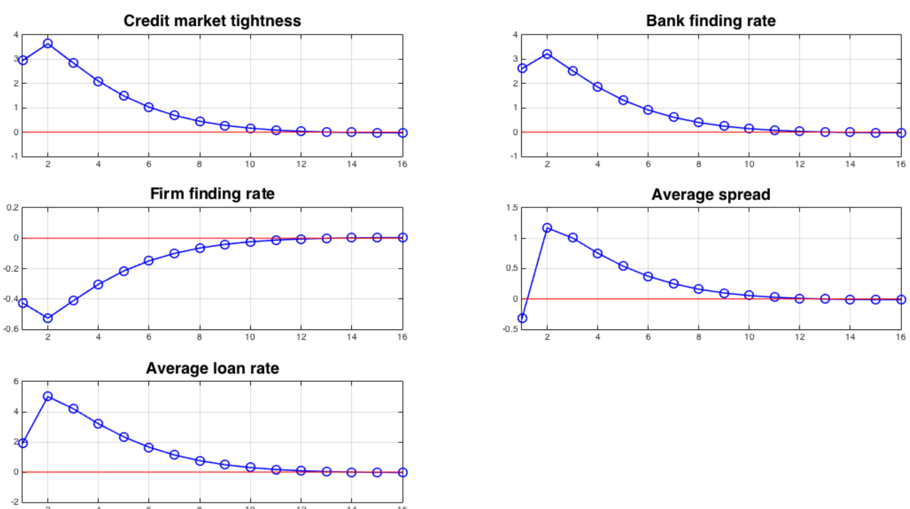


Figure 19: Model responses to a financial shock under Taylor rule: panel 1

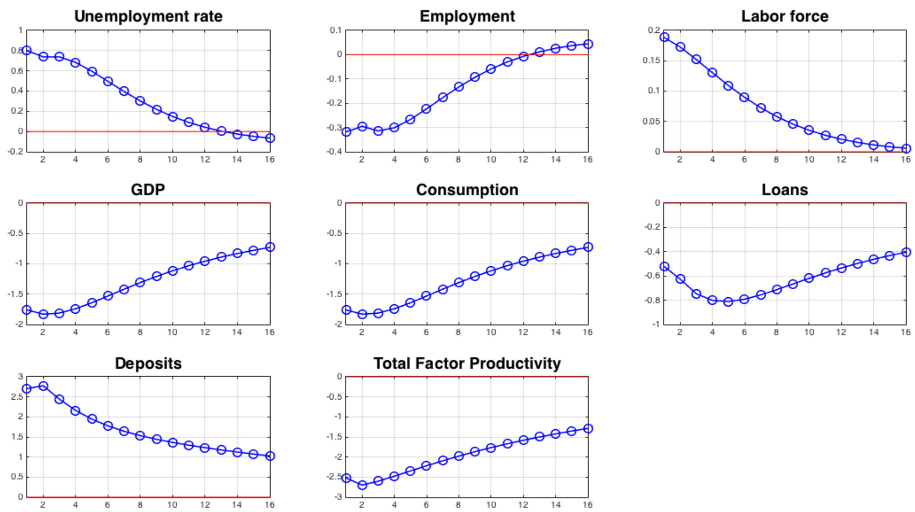


Figure 20: Model responses to a financial shock under Taylor rule: panel 2

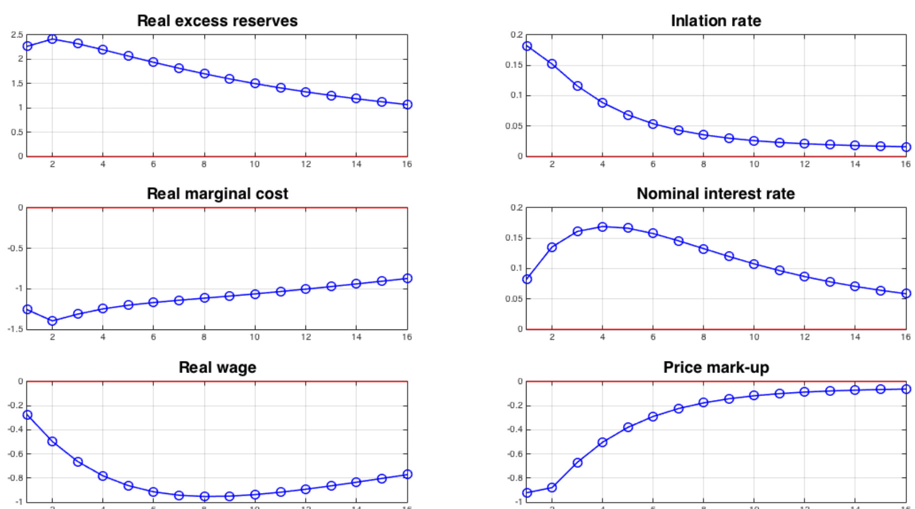


Figure 21: Model responses to a financial shock under Taylor rule: panel 3

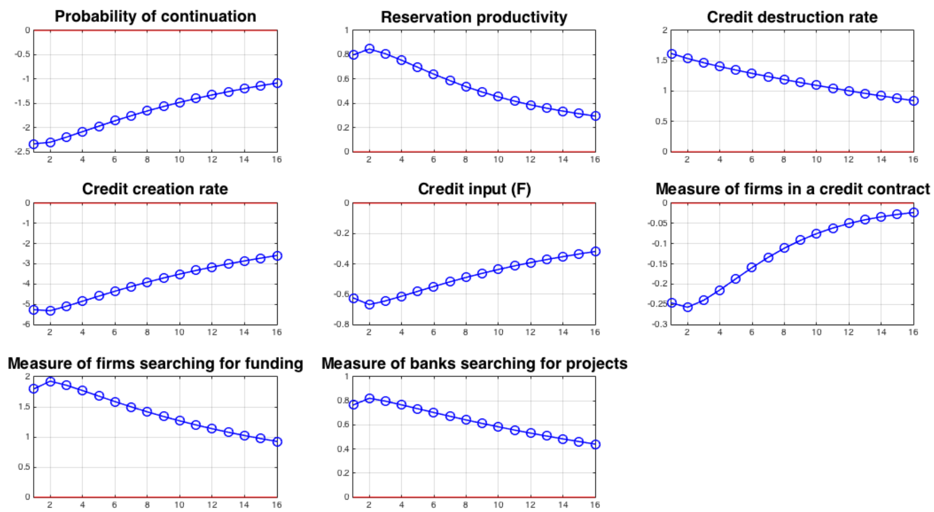


Figure 22: Model responses to a financial shock under Taylor rule: panel 4

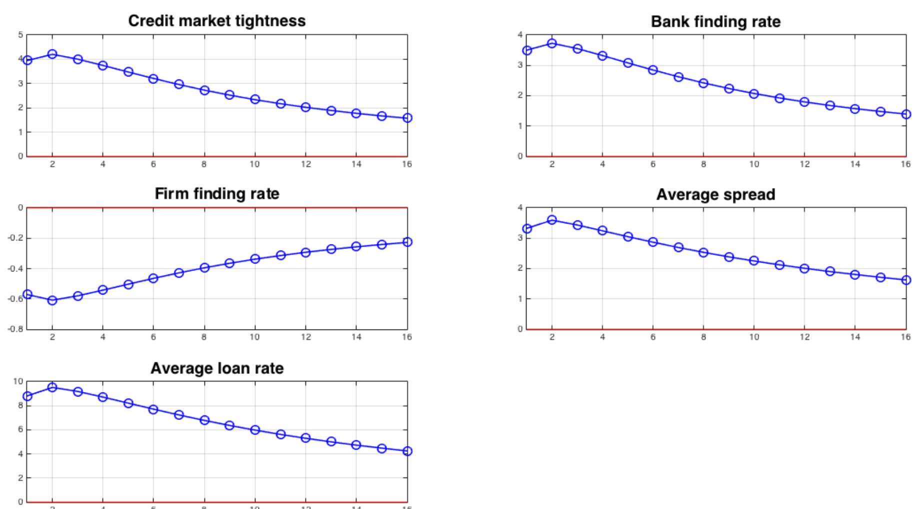


Figure 23: Model responses to a technology shock under Taylor rule: panel 1

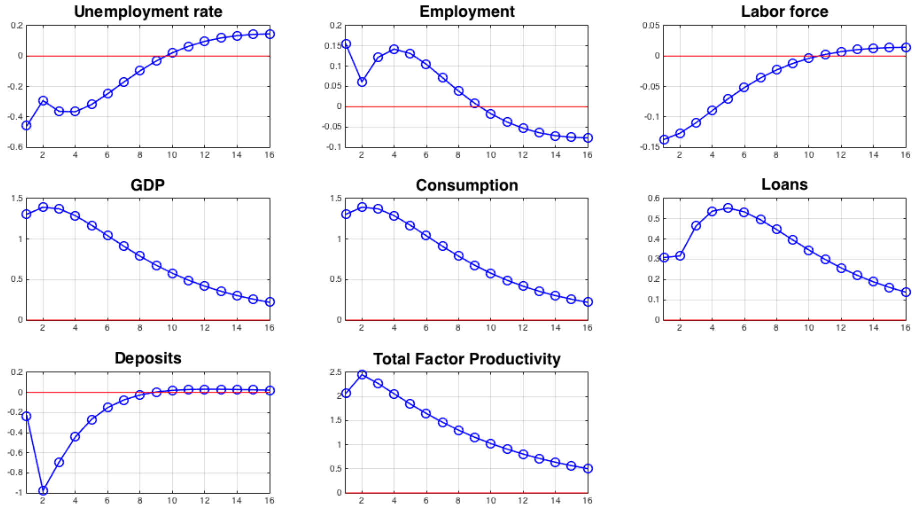


Figure 24: Model responses to a technology shock under Taylor rule: panel 2

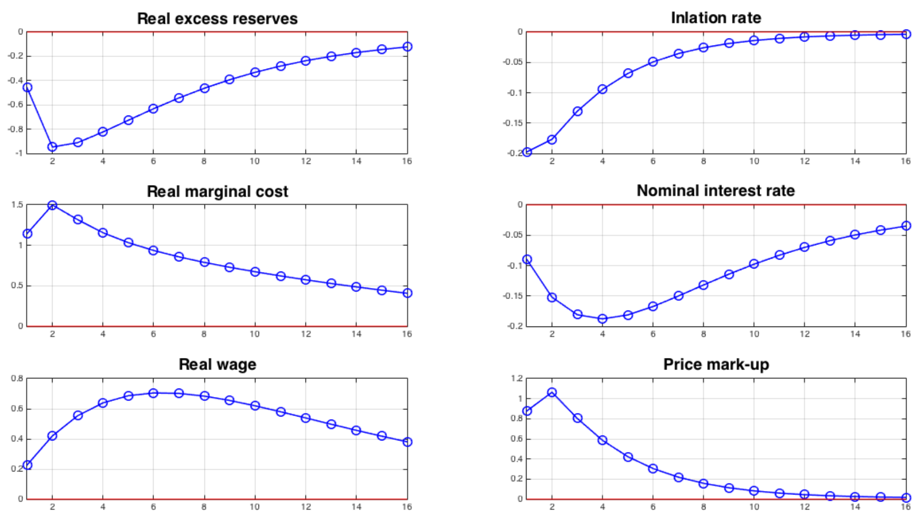


Figure 25: Model responses to a technology shock under Taylor rule: panel 3

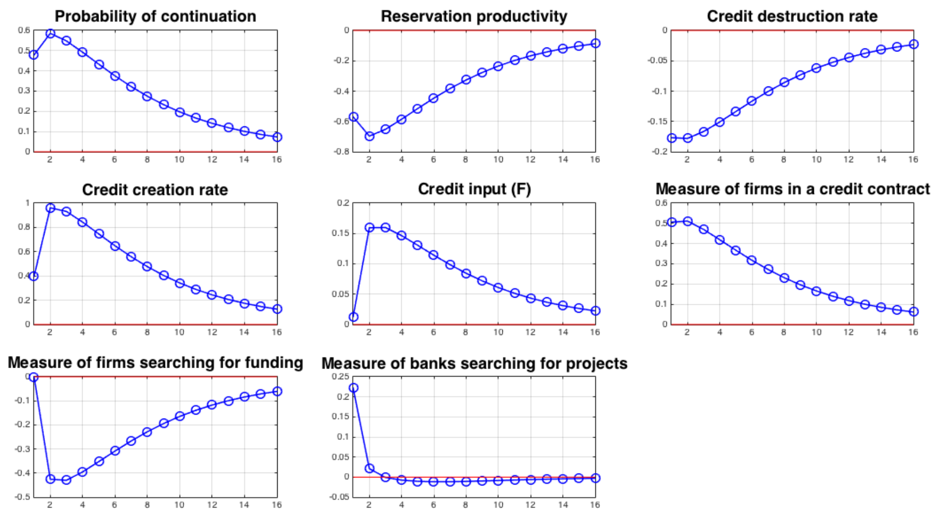
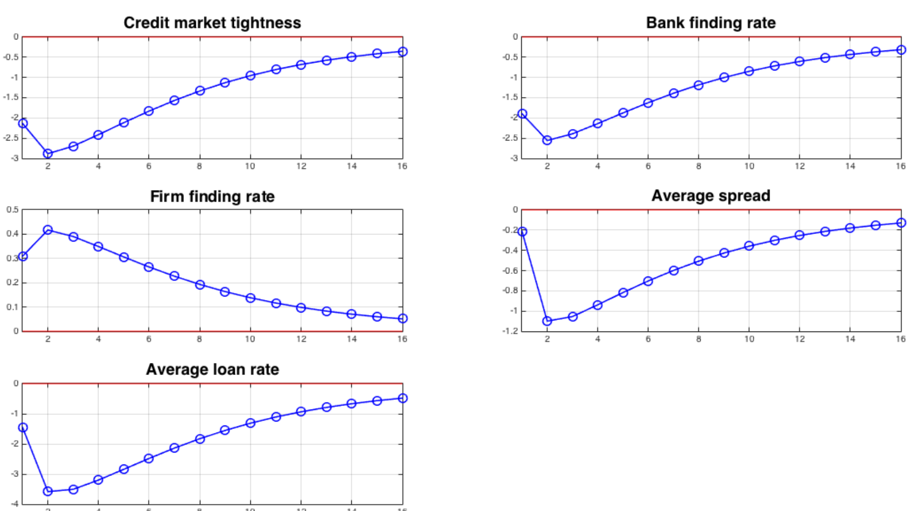


Figure 26: Model responses to a technology shock under Taylor rule: panel 4



10 Appendix E: Introducing absconding and credit rationing into the loan contract

In this section we consider the possibility that borrowers may abscond with the funds obtained from banks. Following Becsi, Li, and Wang (2013) banks introduce an incentive compatibility constraint into the loan contract such that the value of the net surplus for an intermediate good producer in a credit contract is greater or equal to the value of absconding. We assume that in the case of absconding the firm is able to produce and generate profits but does not repay the bank. The bank is able to recover an exogenous fraction θ_t of the profits made by the intermediate good producer who is then barred from the loan market permanently. Therefore, if the producer decides to abscond, then it obtains $(1-\theta_t)\left(\frac{y_t(\omega_{z,t})-x^f}{\mu_t^p}-\widetilde{l}_t\left(j,\omega_{z,t}\right)\right)$ where $\widetilde{l}_t\left(j,\omega_{z,t}\right)=w_t N_t\left(\omega_{z,t}\right)$ denotes the loan extended by bank j to producer z in this new context. The optimal credit contract in this scenario is characterized by credit rationing. A bank will prefer to loan only a fraction $q_t(j,\omega_{z,t})$ of its deposits and leave the rest of its deposits as excess reserves at the central bank.

Banks' balance sheet

Under the possibility that the borrower may abscond with the funds obtained from a loan, the balance sheet for bank j is

$$\boldsymbol{\chi}_t\left(j\right)\left(\widetilde{l}_t\left(j,\omega_{z,t}\right)+\frac{\widetilde{ER}_t\left(j\right)}{P_t}\right)+\left(1-\boldsymbol{\chi}_t\left(j\right)\right)\frac{ER_t\left(j\right)}{P_t}=\frac{D_t\left(j\right)}{P_t} \tag{114}$$

such that if $\boldsymbol{\chi}_t\left(j\right)=1$ the bank extends a loan to an intermediate good producer characterized by $\omega_{z,t}$ and

$$\widetilde{l}_t\left(j,\omega_{z,t}\right)=q_t(j,\omega_{z,t})\frac{D_t\left(j\right)}{P_t} \tag{115}$$

and

$$\frac{\widetilde{ER}_t\left(j\right)}{P_t}=\left(1-q_t(j,\omega_{z,t})\right)\frac{D_t\left(j\right)}{P_t} \tag{116}$$

with $q_t(j,\omega_{z,t})\leq 1$. On the other hand, if $\boldsymbol{\chi}_t\left(j\right)=0$, the bank sets $\frac{ER_t(j)}{P_t}=\frac{D_t(j)}{P_t}$. Notice that if $q_t(j,\omega_{z,t})=1$ then we are back to the standard case since whenever $\boldsymbol{\chi}_t\left(j\right)=1$ then $l_t\left(j,\omega_{z,t}\right)=\frac{D_t(j)}{P_t}$ and $\frac{ER_t(j)}{P_t}=0$. The possibility of credit rationing means that if the bank extends a loan, then it will not use all its available funds, that is $\widetilde{l}_t\left(j,\omega_{z,t}\right)\leq\frac{D_t(j)}{P_t}$. Under this circumstances, individual bank profits expressed in real terms are given by

$$\pi_t^b\left(j\right)=\boldsymbol{\chi}_t\left(j\right)\left(R_t^l\left(j,\omega_{z,t}\right)\widetilde{l}_t\left(j,\omega_{z,t}\right)+R_t^r\frac{\widetilde{ER}_t\left(j\right)}{P_t}\right)+\left(1-\boldsymbol{\chi}_t\left(j\right)\right)\left(R_t^r\frac{ER_t\left(j\right)}{P_t}-\frac{\kappa}{\mu_t^p}\right)-R_t^d\frac{D_t\left(j\right)}{P_t} \tag{117}$$

Substituting out the term $\left(1-\boldsymbol{\chi}_t\left(j\right)\right)\frac{ER_t(j)}{P_t}$ from bank j balance sheet into $\pi_t^b\left(j\right)$ yields

$$\pi_t^b\left(j\right)=\left(R_t^l\left(j,\omega_{z,t}\right)-R_t^r\right)\boldsymbol{\chi}_t\left(j\right)\widetilde{l}_t\left(j,\omega_{z,t}\right)-\left(1-\boldsymbol{\chi}_t\left(j\right)\right)\frac{\kappa}{\mu_t^p}+\left(R_t^r-R_t^d\right)\frac{D_t\left(j\right)}{P_t} \tag{118}$$

As before, optimality with respect to deposits requires $\left(R_t^r-R_t^d\right)D_t\left(j\right)=0$ every period.Since household deposits are always positive in equilibrium, the bank will choose to collect deposits until the gross interest rate on excess reserves is equal to the gross interest rate on deposits, that is $R_t^r=R_t^d=R_t$. Substituting the optimality condition with respect to $D_t\left(j\right)$ into the profit function yields the flow value of a bank

$$\pi_t^b\left(j\right)=\left(R_t^l\left(j,\omega_{z,t}\right)-R_t\right)\boldsymbol{\chi}_t\left(j\right)\widetilde{l}_t\left(j,\omega_{z,t}\right)-\left(1-\boldsymbol{\chi}_t\left(j\right)\right)\frac{\kappa}{\mu_t^p} \tag{119}$$

Bank profits can be written as

$$\pi_t^b\left(j\right)=\begin{cases}\pi_t^b\left(j,\omega_i\right)=\left(R_t^l\left(j,\omega_{z,t}\right)-R_t\right)\boldsymbol{\chi}_t\left(j\right)\widetilde{l}_t\left(j,\omega_{z,t}\right) & \text{if extends a loan to firm } \omega_z\\ -\frac{\kappa}{\mu_t^p} & \text{otherwise}\end{cases} \tag{120}$$

The net surplus to a bank of being in an active credit contract is given by

$$V_t^{BS}(\omega_{z,t}) = \pi_t^b (j, \omega_i) + \frac{\kappa}{\mu_t^p p_t^b} \tag{121}$$

where $\frac{\kappa}{\mu_t^p p_t^b}$ is equal to the continuation value due to free entry of banks. The surplus to a firm is

$$V_t^{FS}(\omega_{z,t}) = \pi_t^I (\omega_{z,t}) + \mathrm{E}_t \Delta_{t,t+1} \left(1 - p_t^f\right) \varphi_t (\tilde{\omega}_{t+1}) \int_{\tilde{\omega}_{t+1}}^{\overline{\omega}} V_{t+1}^{FS}(\omega_{z,t+1}) \frac{dG(\omega)}{1 - \gamma_{t+1}} \tag{122}$$

where

$$\pi_t^I (\omega_{z,t}) = \frac{y_t (\omega_{z,t}) - x^f}{\mu_t^p} - R_t^l (j, \omega_{z,t}) \widetilde{l}_t (j, \omega_{z,t}) \tag{123}$$

10.1 The loan contract

If the borrower has the possibility of absconding, the lender will modify the loan contract to ensure that absconding does not arise in equilibrium. The loan contract will be such that the borrower have incentives to abide the terms of the contract and repay the principal and interest rate to the lender at the end of the period. We assume the borrower is able to produce in the case of absconding, the contract must offer the household at least a fraction of the profits it obtains when not repaying to the bank. The loan contract solves the following Nash bargaining problem:

$$\max_{\left\{R_t^l (j, \omega_{z,t}), \widetilde{l}_t (j, \omega_{z,t})\right\}} \left(V_t^{FS}(\omega_{z,t})\right)^{\overline{\eta}} \left(V_t^{BS}(\omega_{z,t})\right)^{1-\overline{\eta}} \tag{124}$$

subject to the following **incentive compatibility constraint**

$$V_t^{FS}(\omega_{z,t}) \geq (1 - \theta_t) \left(\frac{y_t (\omega_{z,t}) - x^f}{\mu_t^p} - \widetilde{l}_t (j, \omega_{z,t})\right) \tag{125}$$

where

$$\widetilde{l}_t (j, \omega_{z,t}) = q_t (j, \omega_{z,t}) \frac{D_t (j)}{P_t} \tag{126}$$

The first order condition with respect to $R_t^l (j, \omega_{z,t})$ yields the following sharing rule for the surplus generated in a credit relationship:

$$(1 - \overline{\eta}) \left(\frac{V_t^{FS}(\omega_{z,t})}{V_t^{BS}(\omega_{z,t})}\right)^{\overline{\eta}} = \overline{\eta} \left(\frac{V_t^{BS}(\omega_{z,t})}{V_t^{FS}(\omega_{z,t})}\right)^{1-\overline{\eta}} + \lambda_t (\omega_{z,t}) \tag{127}$$

where $\lambda_t (\omega_{z,t})$ is the Lagrange multiplier associated with the incentive compatibility constraint. If the constraint is not binding, $\lambda_t (\omega_{z,t}) = 0$, we obtain the standard sharing rule for a Nash bargaining problem: $(1 - \overline{\eta}) V_t^{FS}(\omega_{z,t}) = \overline{\eta} V_t^{BS}(\omega_{z,t})$ as in the case with no possibility of absconding. Let $MPL_t (\omega_{z,t})$ denote the marginal product of labor, $MPL_t (\omega_{z,t}) = \alpha A_t \omega_{z,t} N_t^* (\omega_{z,t})^{\alpha - 1}$, and recall the loan extended by bank j is used by the intermediate producer to cover its labor costs $\widetilde{l}_t (j, \omega_{z,t}) = w_t N_t^* (\omega_{z,t})$ .Then,the first order condition with respect to $\widetilde{l}_t (j, \omega_{z,t})$ implies:

$$\begin{aligned} &\overline{\eta} \left(\frac{V_t^{BS}(\omega_{z,t})}{V_t^{FS}(\omega_{z,t})}\right)^{1-\overline{\eta}} \left(\frac{MPL_t (\omega_{z,t})}{\mu_t^p w_t} - R_t^l (j, \omega_{z,t})\right) + (1 - \overline{\eta}) \left(\frac{V_t^{FS}(\omega_{z,t})}{V_t^{BS}(\omega_{z,t})}\right)^{\overline{\eta}} \left(\left(R_t^l (j, \omega_{z,t}) - R_t\right)\right) \\ &= \lambda_t (\omega_{z,t}) \left[(1 - \theta_t) \left(\frac{MPL_t (\omega_{z,t})}{\mu_t^p w_t} - 1\right) - \left(\frac{MPL_t (\omega_{z,t})}{\mu_t^p w_t} - R_t^l (j, \omega_{z,t})\right)\right] \end{aligned}$$

Notice that the term $\frac{MPL_t(\omega_{z,t})}{\mu_t^p w_t} - R_t^l (j, \omega_{z,t})$ is the marginal profit for an intermediate producer that decide to finish the project and not abscond. By the same token, the term $R_t^l (j, \omega_{z,t}) - R_t$ is the marginal benefit for a bank that extends a loan to a non absconding producer. Those two marginal profits are weighted by $\overline{\eta} \left(\frac{V_t^{BS}(\omega_{z,t})}{V_t^{FS}(\omega_{z,t})}\right)^{1-\overline{\eta}}$ and $(1 - \overline{\eta}) \left(\frac{V_t^{FS}(\omega_{z,t})}{V_t^{BS}(\omega_{z,t})}\right)^{\overline{\eta}}$. If the bank knows with certainty that the borrower will always pay back the loan, then $\lambda_t (\omega_{z,t}) = 0$ and the first order condition with respect to $\widetilde{l}_t (j, \omega_{z,t})$ would imply $MPL_t(\omega_{z,t}) = \mu_t^p w_t R_t$. If the bank knows that there is a possibility that the borrower will abscond, the weighted average of marginal profits do not cancel out as in the standard case but are equal to the marginal opportunity cost of absconding which is given by the term $(1-\theta_t)\left(\frac{MPL_t(\omega_{z,t})}{\mu_t^p w_t} - 1\right) - \left(\frac{MPL_t(\omega_{z,t})}{\mu_t^p w_t} - R_t^l(j,\omega_{z,t})\right)$.

Combining the first order conditions yields an equation for $\lambda_t(\omega_{z,t})$ given by:

$$\lambda_t(\omega_{z,t}) = (1-\overline{\eta})\left(\frac{V_t^{FS}(\omega_{z,t})}{V_t^{BS}(\omega_{z,t})}\right)^{\overline{\eta}}\left(\frac{\frac{MPL_t(\omega_{z,t})}{\mu_t^p w_t} - R_t}{(1-\theta_t)\left(\frac{MPL_t(\omega_{z,t})}{\mu_t^p w_t} - 1\right)}\right) \tag{128}$$

Notice that since $\lambda_t(\omega_{z,t}) \geq 0$ then it must be the case that $\frac{MPL_t(\omega_{z,t})}{\mu_t^p w_t} \geq R_t$ and $\frac{MPL_t(\omega_{z,t})}{\mu_t^p w_t} \geq 1$. If $\frac{MPL_t(\omega_{z,t})}{\mu_t^p w_t} \leq R_t$ the firm and the bank are not willing to participate in the credit relationship. Substituting the solution for $\lambda_t(\omega_{z,t})$ into the sharing rule yields the optimal hiring condition for an intermediate good producer in a credit relationship:

$$(1-\overline{\eta})V_t^{FS}(\omega_{z,t})\left(R_t - 1 - \theta_t\left(\frac{MPL_t(\omega_{z,t})}{\mu_t^p w_t} - 1\right)\right) = \overline{\eta}V_t^{BS}(\omega_{z,t})(1-\theta_t)\left(\frac{MPL_t(\omega_{z,t})}{\mu_t^p w_t} - 1\right) \tag{129}$$

The above equation clearly shows that when $(1-\overline{\eta})V_t^{FS}(\omega_{z,t}) = \overline{\eta}V_t^{BS}(\omega_{z,t})$ which is equivalent to having $\lambda_t(\omega_{z,t}) = 0$, the intermediate good producer will hire workers up to the point where $MPL_t(\omega_{z,t}) = \mu_t^p w_t R_t$. When $\lambda_t(\omega_{z,t}) > 0$ then it must be the case that $MPL_t(\omega_{z,t}) > \mu_t^p w_t R_t$ and the intermediate good producer will hire an inefficiently low number of workers due to the existence of equilibrium credit rationing. In order to focus on the implications of credit rationing, we assume that the incentive compatibility constraint is always binding, that is

$$V_t^{FS}(\omega_{z,t}) = (1-\theta_t)\left(\frac{y_t(\omega_{z,t}) - x^f}{\mu_t^p} - w_t N_t(\omega_{z,t})\right) \tag{130}$$

since $V_t^{BS}(\omega_{z,t}) = \pi_t^b(j,\omega_i) + \frac{\kappa}{\mu_t^p p_t^b}$ and $\pi_t^b(j,\omega_i) = \left(R_t^l(j,\omega_{z,t}) - R_t\right)w_t N_t(j,\omega_{z,t})$ then the ratio $\frac{V_t^{FS}(\omega_{z,t})}{V_t^{BS}(\omega_{z,t})}$ is given by

$$\frac{V_t^{FS}(\omega_{z,t})}{V_t^{BS}(\omega_{z,t})} = (1-\theta_t)\left(\frac{\frac{y_t(\omega_{z,t})-x^f}{\mu_t^p} - w_t N_t(j,\omega_{z,t})}{\left(R_t^l(j,\omega_{z,t}) - R_t\right)w_t N_t(j,\omega_{z,t}) + \frac{\kappa}{\mu_t^p p_t^b}}\right) \tag{131}$$

substituting $\frac{V_t^{FS}(\omega_{z,t})}{V_t^{BS}(\omega_{z,t})}$ into the optimal sharing rule yields the equilibrium loan interest rate equation:

$$\left(R_t^l(j,\omega_{z,t}) - R_t\right)\widetilde{l}_t(j,\omega_{z,t}) = \left(\frac{1-\overline{\eta}}{\overline{\eta}}\right)\left(\frac{y_t(\omega_{z,t}) - x^f}{\mu_t^p} - w_t N_t(\omega_{z,t})\right)\left(\frac{R_t - 1 - \theta_t\left(\frac{MPL_t(\omega_{z,t})}{\mu_t^p w_t} - 1\right)}{\frac{MPL_t(\omega_{z,t})}{\mu_t^p w_t} - 1}\right) - \frac{\kappa}{\mu_t^p p_t^b} \tag{132}$$

The joint surplus of a credit relationship under the possibility of absconding is

$$V_t^{JS}(\omega_{z,t}) = (1-\theta_t)\left(\frac{y_t(\omega_{z,t}) - x^f}{\mu_t^p} - w_t N_t(\omega_{z,t})\right) + \left(R_t^l(j,\omega_{z,t}) - R_t\right)w_t N_t(\omega_{z,t}) + \frac{\kappa}{\mu_t^p p_t^b} \tag{133}$$

The optimal level of employment is obtained by maximizing the joint surplus of a credit relationship with respect to $N_t(\omega_{z,t})$:

$$MPL_t(\omega_{z,t}) = \mu_t^p w_t + \frac{\mu_t^p w_t R_t}{1-\theta_t} - \frac{\mu_t^p w_t R_t^l(j,\omega_{z,t})}{1-\theta_t} \tag{134}$$

and the cut-off productivity level solves $V_t^{JS}(\widetilde{\omega}_{z,t}) = 0$ which implies:

$$0 = (1-\theta_t)\left(\frac{y_t(\widetilde{\omega}_{z,t}) - x^f}{\mu_t^p} - w_t N_t(\widetilde{\omega}_{z,t})\right) + \left(R_t^l(j,\widetilde{\omega}_{z,t}) - R_t\right)w_t N_t(\widetilde{\omega}_{z,t}) + \frac{\kappa}{\mu_t^p p_t^b} \tag{135}$$

This extension of the credit contract has several implications. First, the loan interest rate $R_t^l(j,\widetilde{\omega}_{z,t})$ affects the joint surplus of a credit relationship and the cut-off $\widetilde{\omega}_{z,t}$. This occurs only when the incentive compatibility constraint is binding since the net surplus that a borrower obtains from an active credit match has to be equal to its absconding value. By definition, the absconding value for an intermediate good producer does not take into account $R_t^l(j, \widetilde{\omega}_{z,t})$ which implies that the joint surplus will depend on it. Under this assumptions is more difficult to compute the optimal equilibrium values $y_t^*(\omega_{z,t}), N_t^*(\omega_{z,t})$ and $\widetilde{l}_t(j, \omega_{z,t})$ since we need to solve a non linear system of equations.

11 Appendix F: The role of nominal rigidities

In this appendix we compare the impulse responses of a financial shock in our benchmark model with the case of flexible prices and wages as well as with the case of flexible wages but sticky prices. Recall, the benchmark model considers a CIA constraint and a nominal money growth rule for monetary policy. We compute the model responses under two alternative cases: 1) The model under complete price and wage flexibility and 2) The model under sticky prices but flexible wages. We compare the responses to a monetary policy shock for these two cases with the responses of the benchmark model presented in the main body of the paper. The results suggest that sticky wages are crucial for obtaining a persistent and significant response of unemployment to a financial shock. Under wage flexibility, a financial shock produces a persistent recession that is reflected only in GDP, employment and consumption. Notice that the endogenous component of TFP also declines persistently under the two alternatives but the decline is stronger under nominal rigidities. These findings are explained by the marginal cost of labor response under the three cases. Notice that after a financial shock, the fall in marginal cost of labor is more pronounced in the case of wage and price stickness. Therefore, the intensive margin effect is stronger in the case of price and/or wage flexibility. Recall that a financial shock produces an extensive margin (selection) and an intensive margin effect over employment that go in opposite directions. If wages and/or prices are flexible, the intensive margin effect is stronger and may cancel the extensive margin effect.

On the other hand, the opposite applies to the case of a monetary policy shock. Under nominal flexibility, the intensive margin effect is weaker. That is, the marginal cost of labor does not fall as much as in the case of sticky prices and wages reducing the expansionary effect of monetary policy.

Figure 27: Model responses to a financial shock: The role of nominal rigidities: panel 1

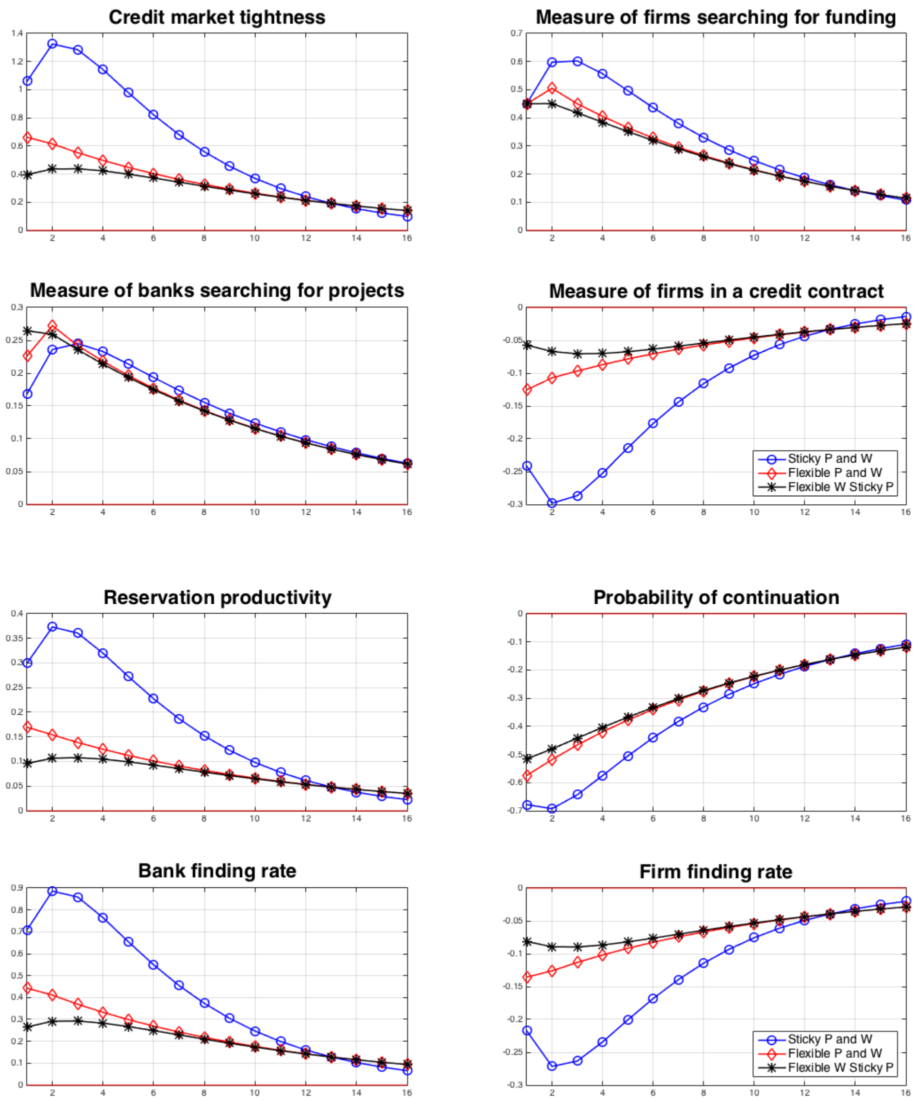


Figure 28: Model responses to a financial shock: The role of nominal rigidities: panel 2

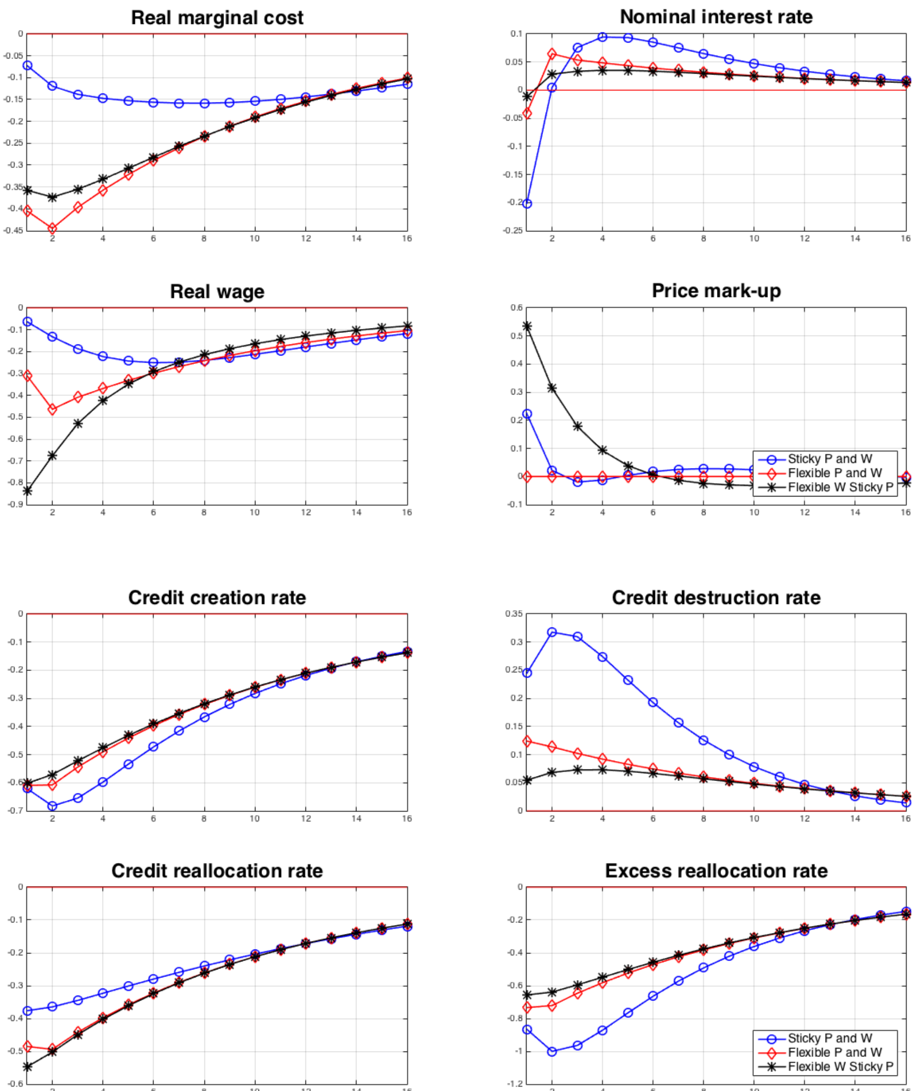


Figure 29: Model responses to a financial shock: The role of nominal rigidities: panel 3

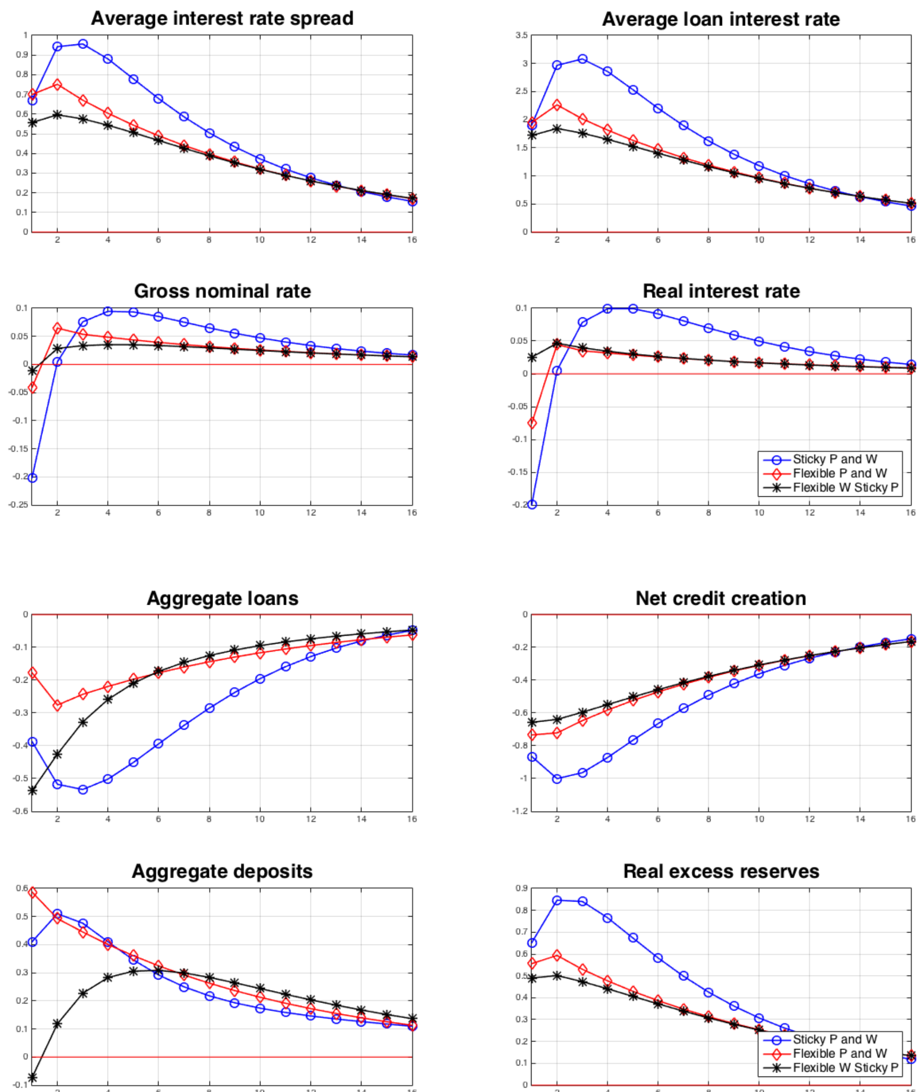


Figure 30: Model responses to a financial shock: The role of nominal rigidities: panel 4

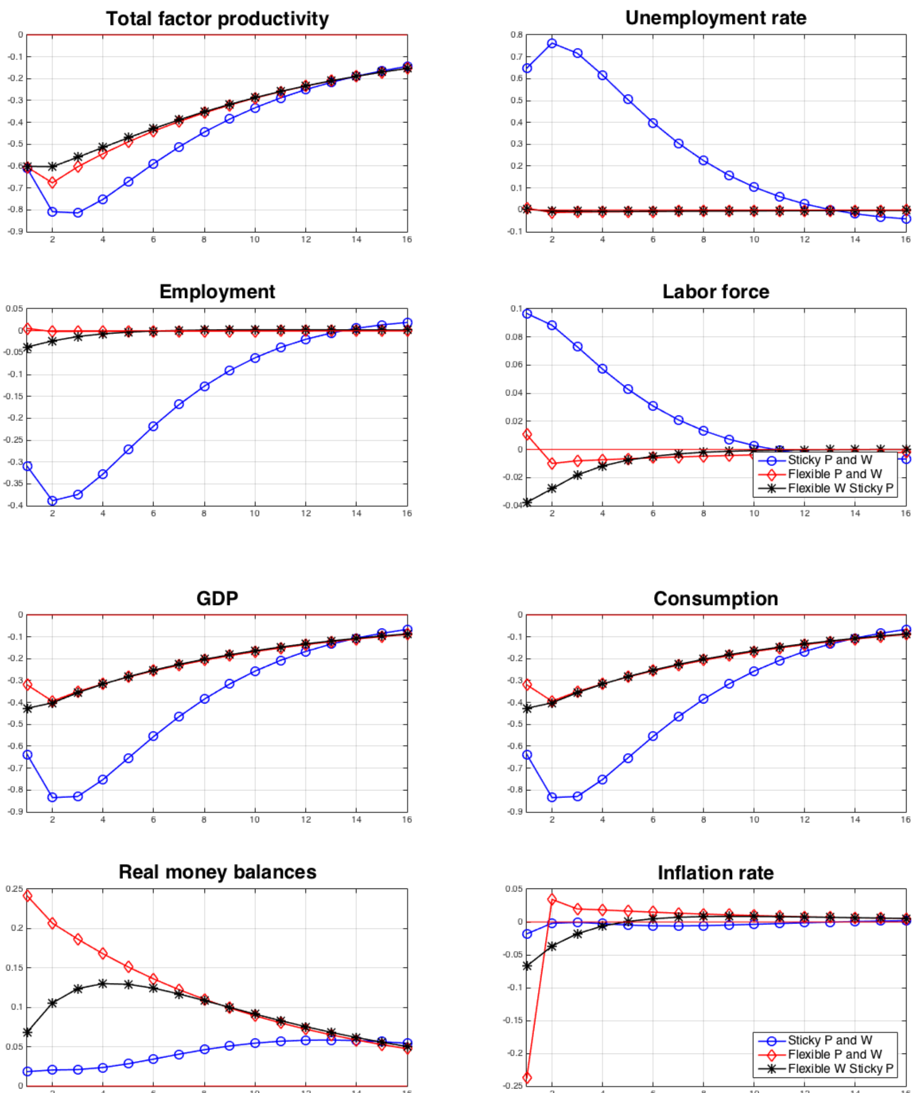


Figure 31: Model responses to a monetary policy shock: The role of nominal rigidities: panel 1

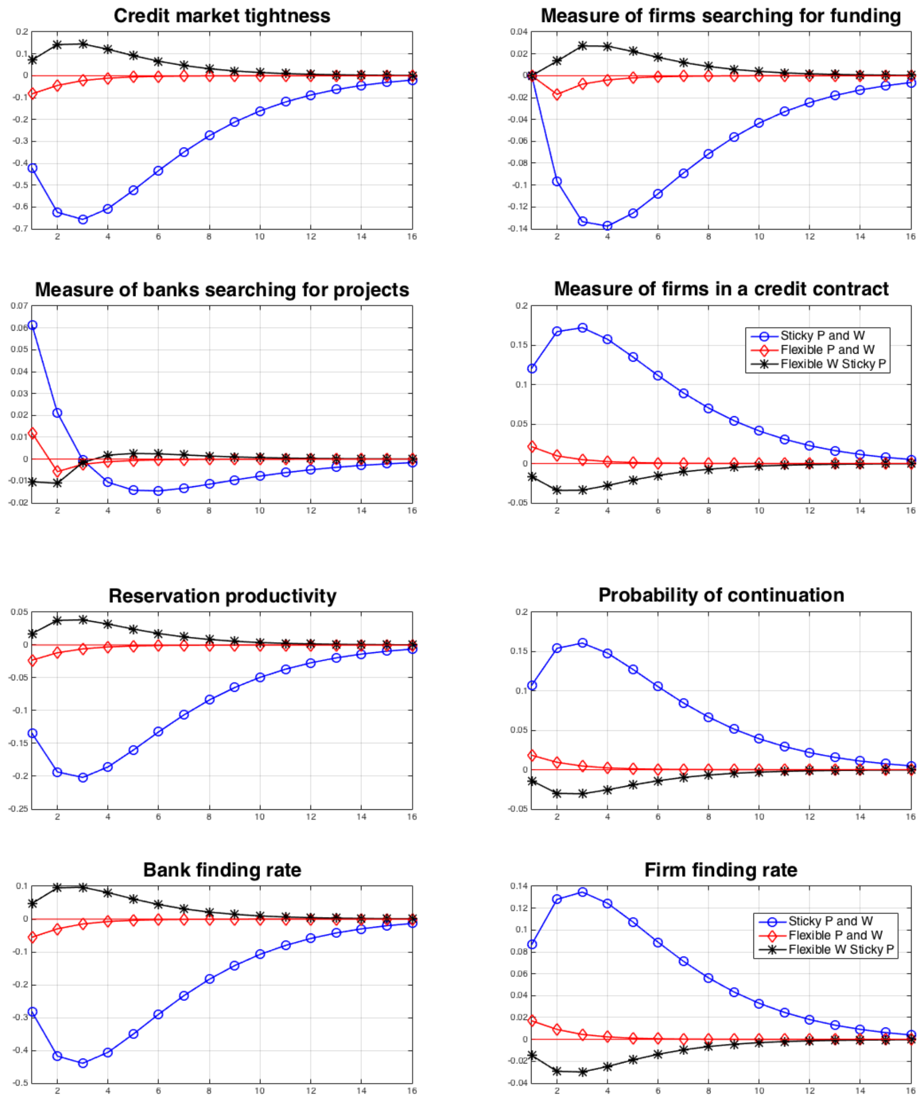


Figure 32: Model responses to a monetary policy shock: The role of nominal rigidities: panel 2

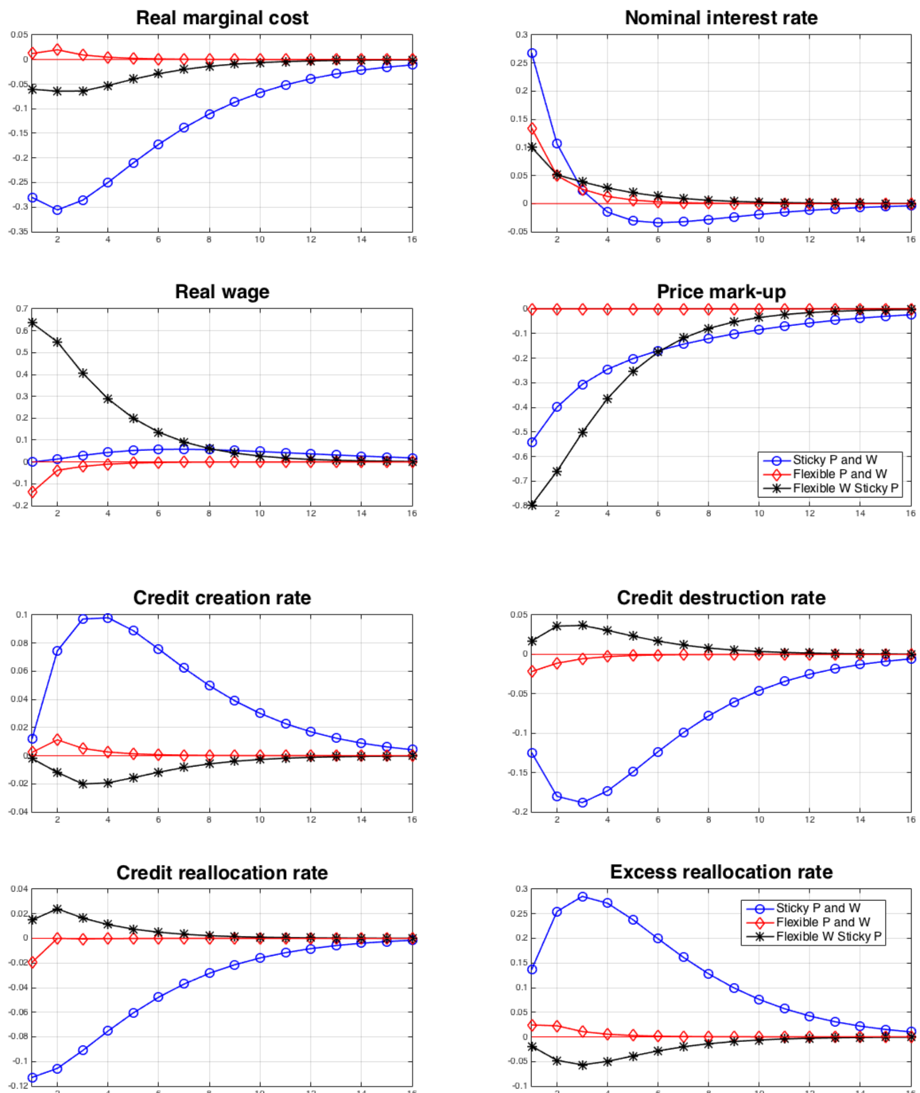


Figure 33: Model responses to a monetary policy shock: The role of nominal rigidities: panel 3

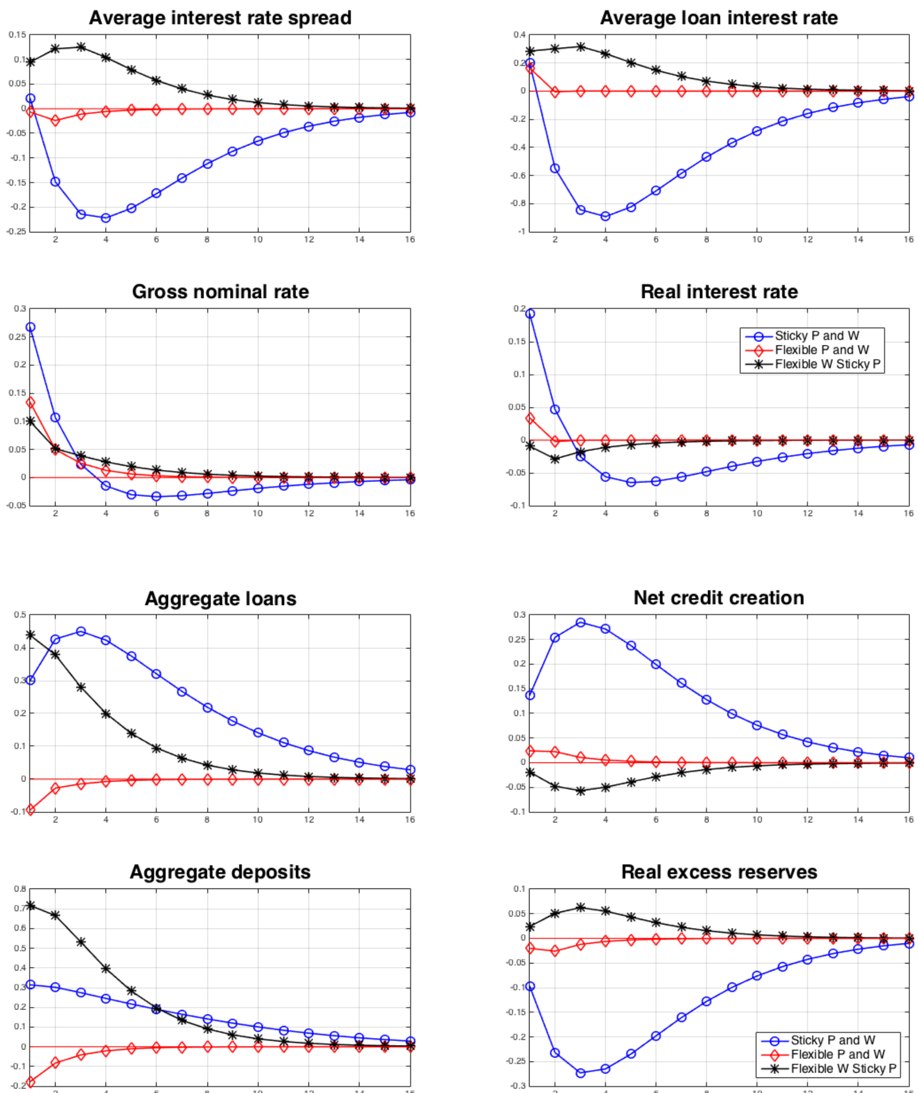


Figure 34: Model responses to a monetary policy shock: The role of nominal rigidities: panel 4