A Model of Hysteresis:
Endogenous Rigidity in Wages and Labor Force Participation[1]

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Federal Reserve Board

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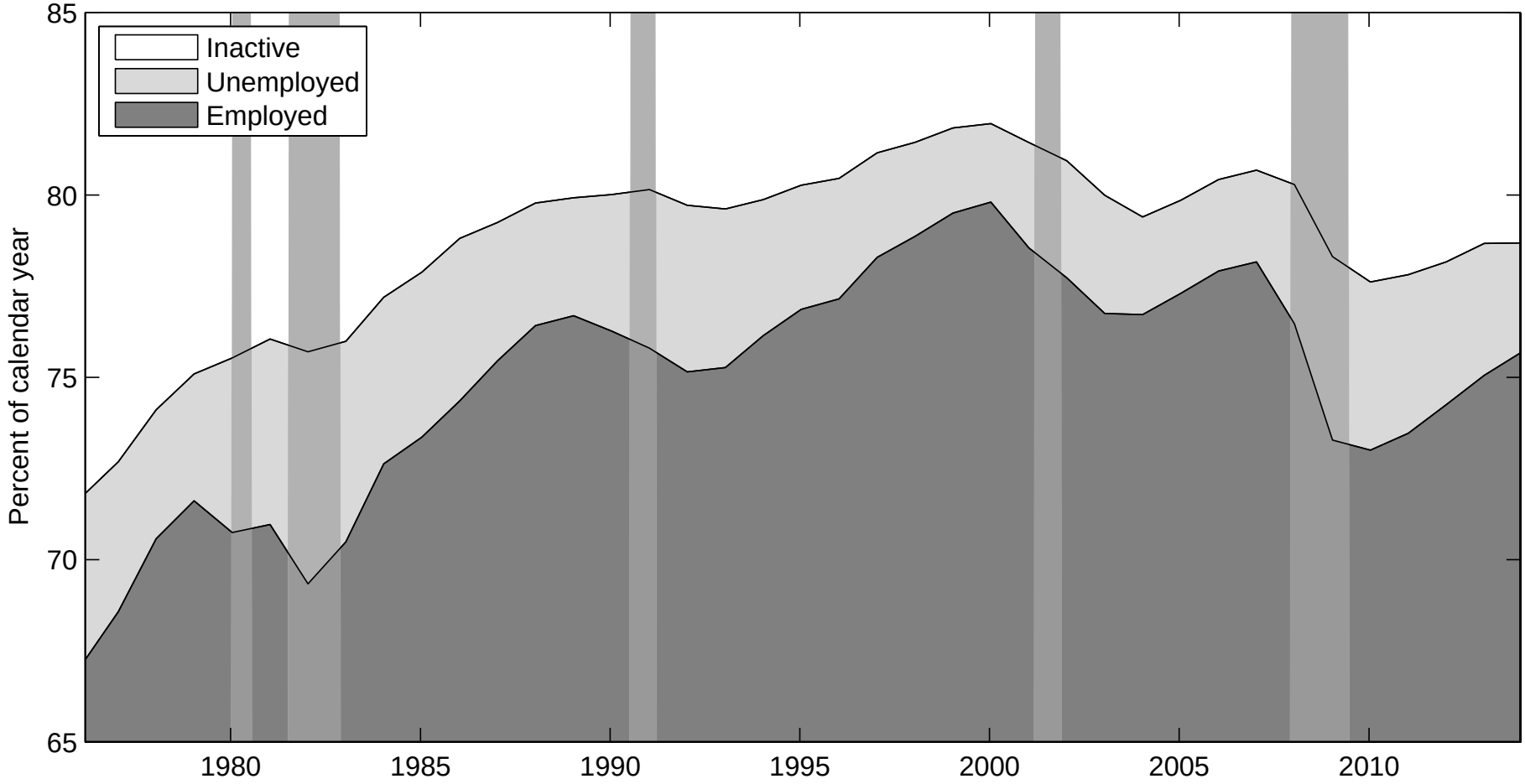
Motivation

- Recessions are often portrayed as short-term events.
- However, a substantial body of empirical literature shows that high unemployment, stalled or falling wages, and reduced economic activity can have long-lasting consequences.
- A recession can lead to scarring: the economy's output shrinks relative to fundamentals.

Literature

- "Fragile" equilibria as in Blanchard and Summers (1986, 1987)
  - Very flat supply and demand curves.
  - "Wrong" sloped supply or demand curves.
- Demand side vs Supply side.
  - Supply side: insider/outsider model. e.g. Blanchard and Summers (1986, 1987)
  - Demand side: strategic complementarities. e.g. Diamond (1982)
- Multiplicity and Dynamics.
  - Global games. e.g. Morris and Shin (2000)
  - Limit cycle. e.g. Beaudry, Galizia, and Portier (2015)
  - Correlated randomization. e.g. Golosov and Menzio (2015)
  - Best-response dynamics. e.g. Vives (1990, 2005); Cooper (1994); Eeckhout and Lindenlaub (2015)

Labor Force Participation



Source: Annual Demographic File of the Current Population Survey following the methodology of Juhn, Murphy, and Topel (1991,2002); Murphy and Topel (1997); Elsby and Shapiro (2012).

Sample: Civilians with 1 to 30 years of potential experience. Individuals who report being students, retired, or ill/disabled are excluded.

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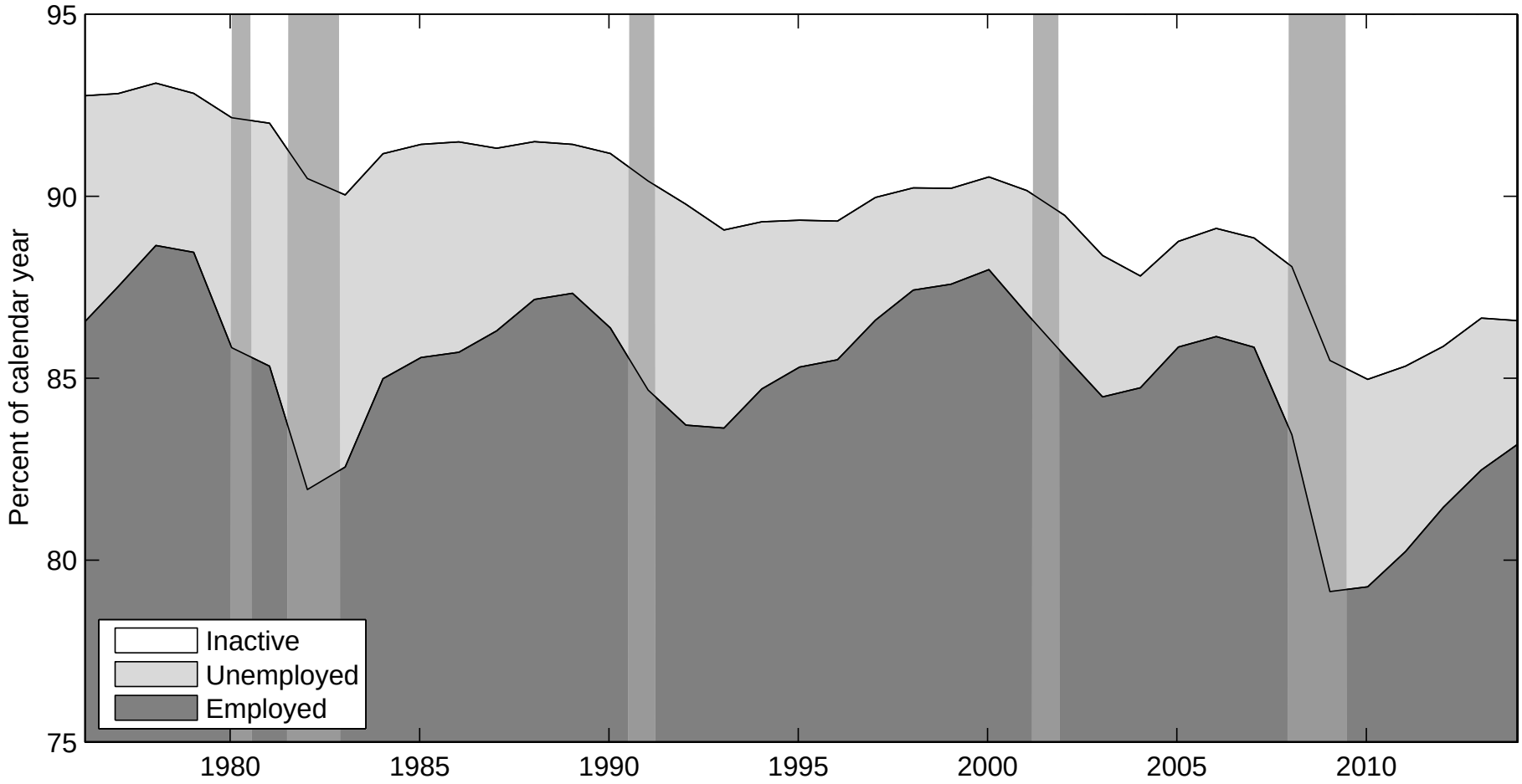

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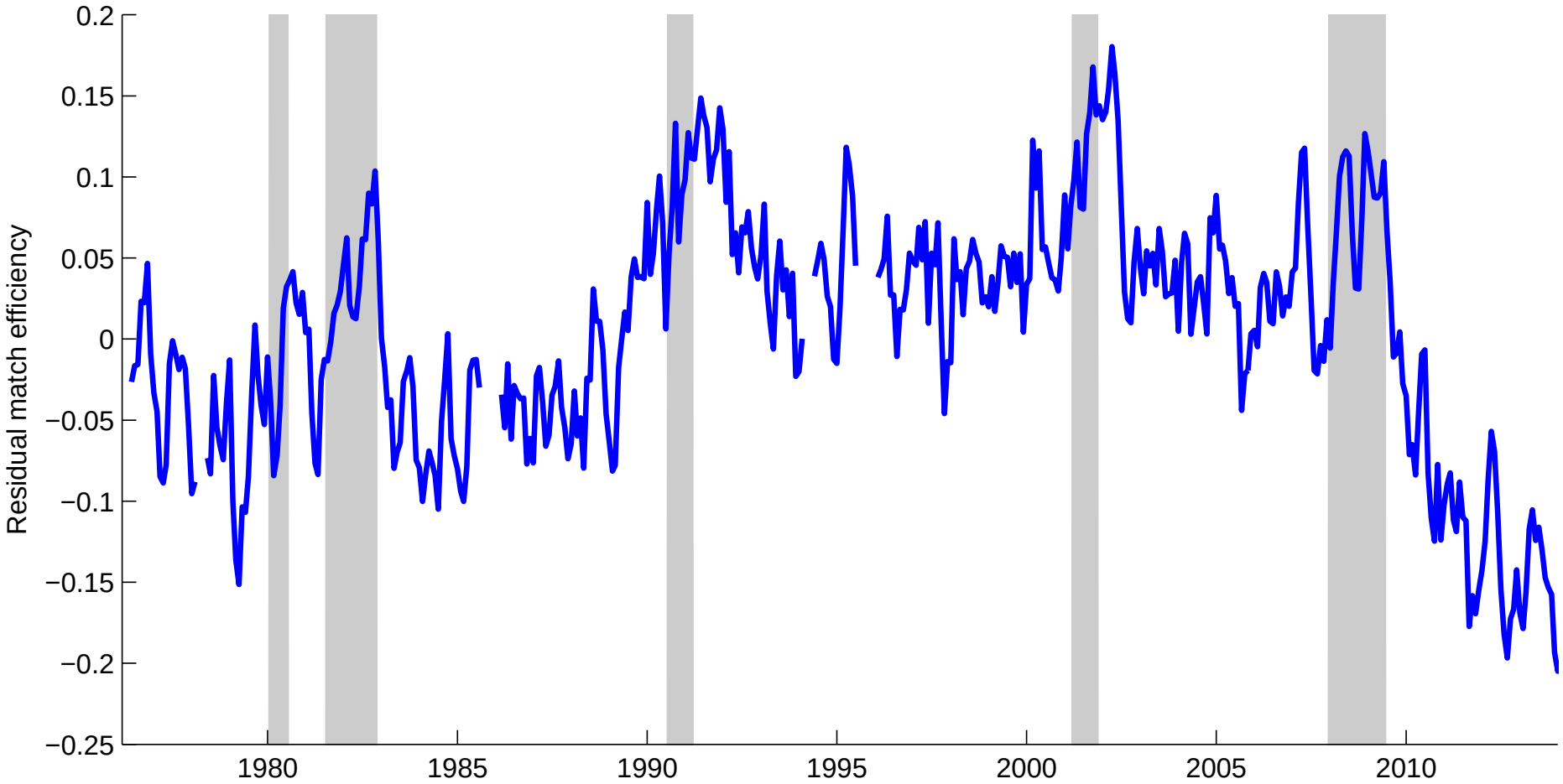
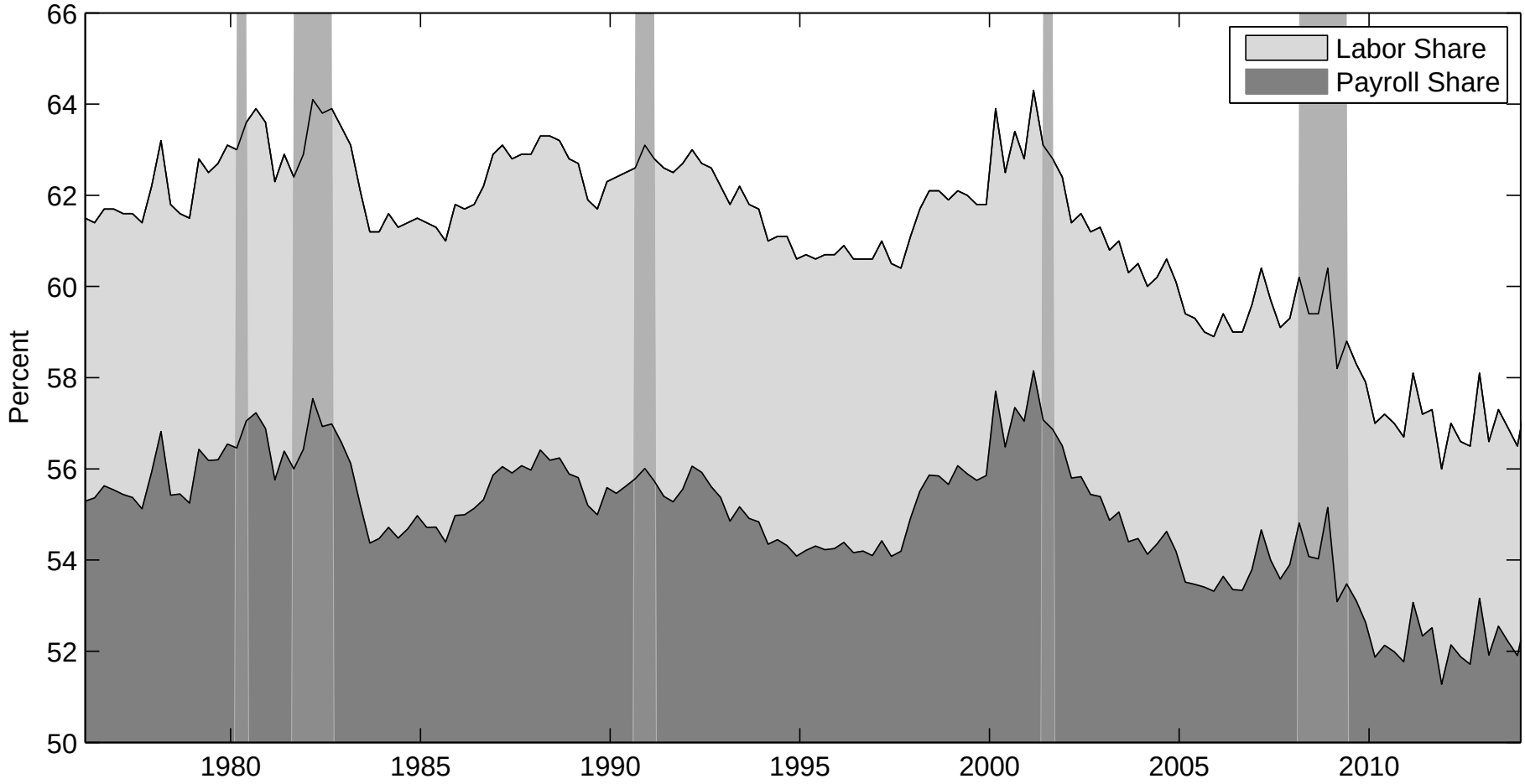
Match Efficiency



Source: Current Population Survey; Help Wanted Index and Help Wanted Online Index; Job Openings and Labor Turnover Survey. Vacancies are constructed from the Help Wanted and Help Wanted Online Index data as in Barnichon (2010) prior to 2001 and follow the Job Opening and Labor Turnover Survey thereafter. The figure is constructed as in Barnichon and Figura (2015).

Labor Share



Source: Bureau of Labor Statistics, Bureau of Economic Analsis. Figure constructed as in Elsby, Hobijn, and Şahin (2013).

A Two Player Game

One worker and one firm may match and produce output p.

- Worker:
  - Draws flow value of non-employment, b, from known distribution $H(b)$.
  - May exit the game with probability i after certain realizations of b.
  - If continuing, meets the firm with probability $(1 - u)$ and forms a match if the wage is acceptable.
- Firm:
  - Posts a non-negotiable wage offer with knowledge of $H(b)$ but not b.
  - Meets with the worker with probability $(1 - v)$ and forms a match if the wage is acceptable.

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Worker's strategy: a reservation participation threshold, r
Firm's strategy: a posted wage, w

Coordination Failures

Cooper and John (1988): Games with **positive spillovers** and **strategic complementarities** may have multiple equilibria. Further these equilibria can be ranked in terms of welfare.

A game exhibits **positive spillovers** for player i when an increase in the other players' actions increases the payoff to player i.

A game exhibits **strategic complementarities** for player i when an increase in the other players' actions increases the best response of player i.

Worker's Problem

Payoff:

$$V^W(r, w_0) = \underbrace{u \int_{\underline{b}}^{r} b dH(b) db + (1-u)\mathbb{I}_{\{w_0 \leq r\}} \int_{w_0}^{r} b dH(b) db + i \int_{r}^{\bar{b}} b dH(b) db}_{unemployed}$$

$$+ \underbrace{(1-u) w_0 H(min\{r, w_0\})}_{employed} + \underbrace{(1-i) \int_{r}^{\bar{b}} b dH(b) db}_{nonparticipant}$$

Best response:

$$b^*(w_0) = \begin{cases} \underline{b} & \text{if } w_0 < \underline{b} \\ w_0 & \text{if } w_0 \in [\underline{b}, \bar{b}] \\ \bar{b} & \text{if } w_0 > \bar{b} \end{cases}$$

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Prop. 1 There are *positive spillovers* and *strategic complementarities* for the worker (the firm's strategy imposes a pecuniary externality).

Firm's Problem

Payoff:

$$V^F(w, r_0) = (1 - v)\Big[\mathbb{I}_{\{w \le r_0\}} \frac{uH(w)}{uH(r_0)) + i(1 - H(r_0))} + (1 - \mathbb{I}_{\{w \le r_0\}}) \frac{uH(r_0) + i[H(w) - H(r_0)]}{uH(r_0)) + i(1 - H(r_0))}\Big](p - w).$$

Best response:

$$w^*(w_0) = \begin{cases} \hat{w} & \text{if } r_0 < w^L \\ r_0 & \text{if } r_0 \in [w^L, w^C] \\ w^C & \text{if } r_0 > w^C \end{cases}$$

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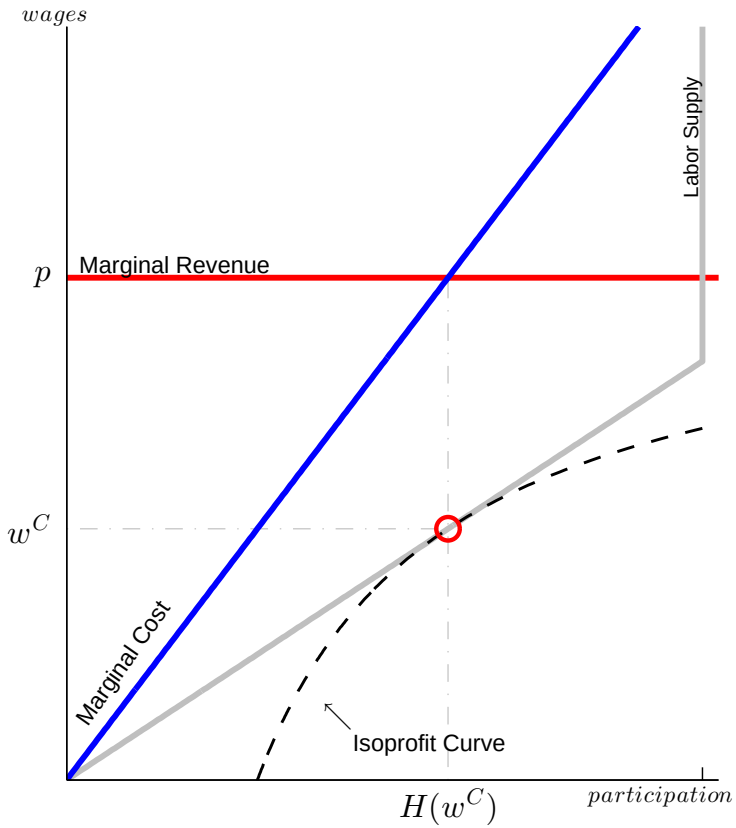
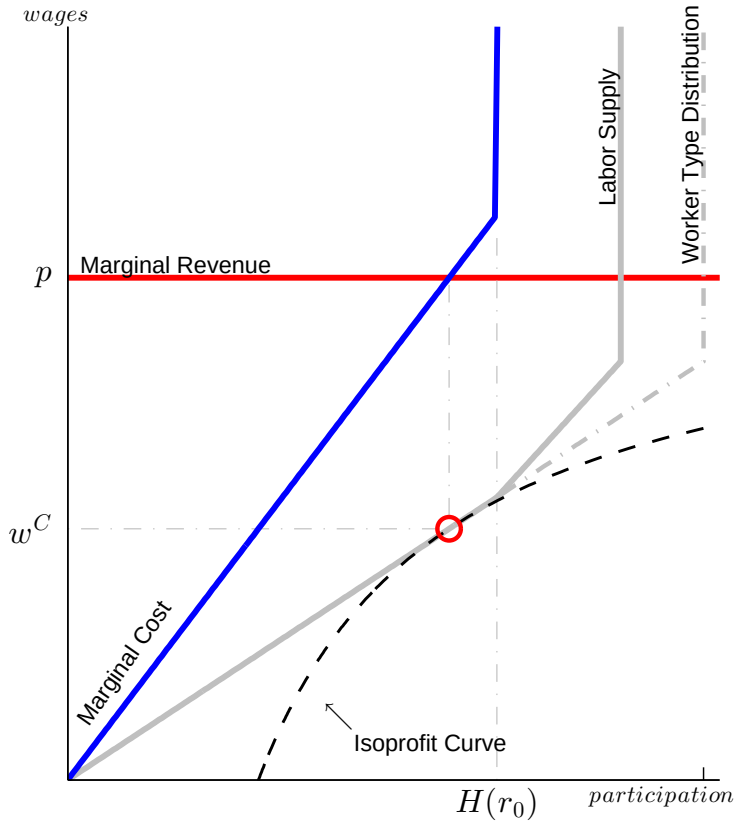
Prop. 2 For r_0 in $[w^L, w^C]$, there are *positive spillovers* and *strategic complementarities* for the firm (the worker's strategy imposes a thick market externality).

Firm's Problem:

The Wage Choice with Coordination



A Wage Choice without Coordination



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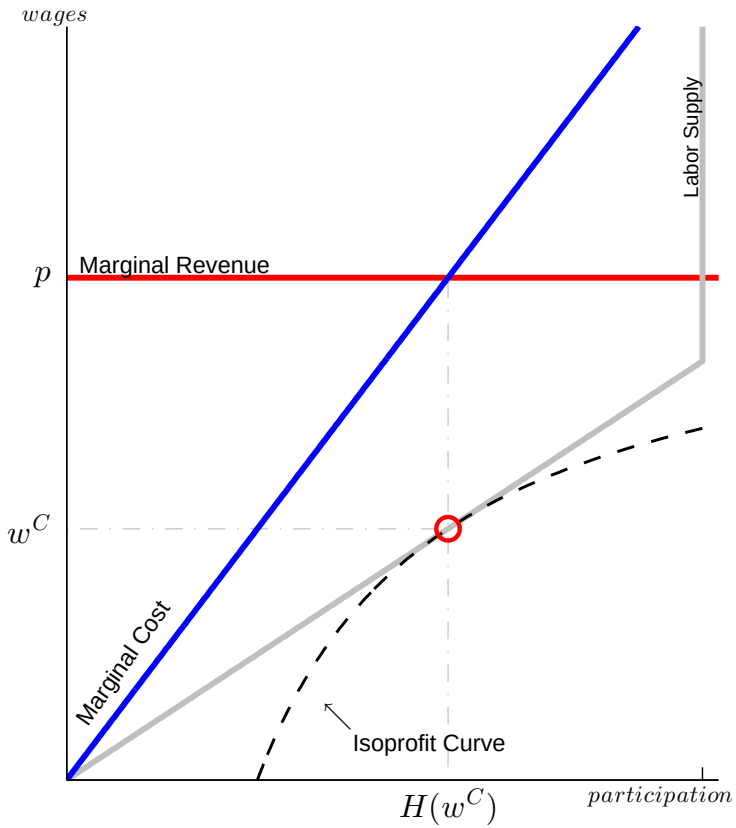

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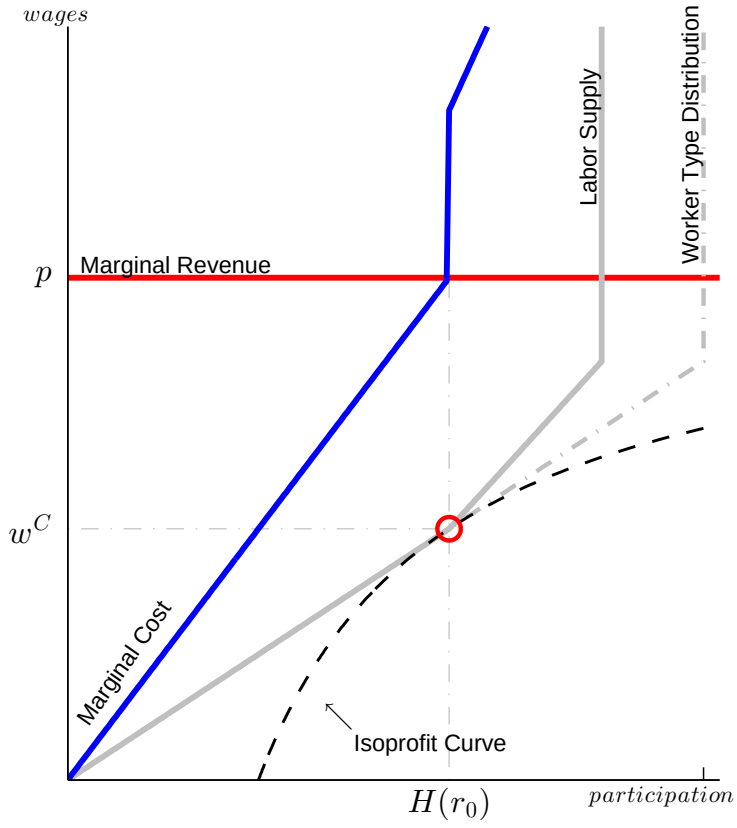

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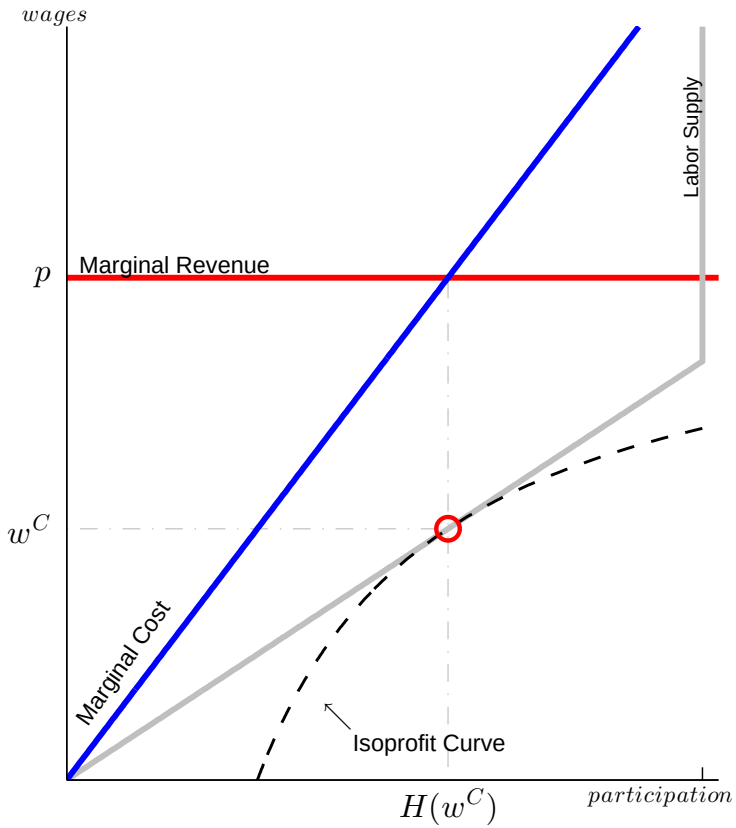

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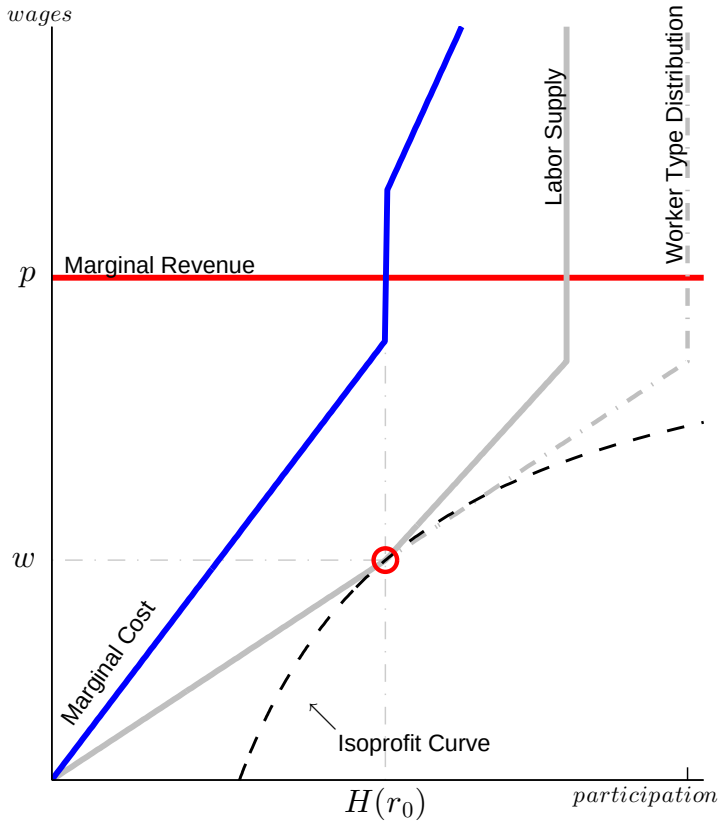

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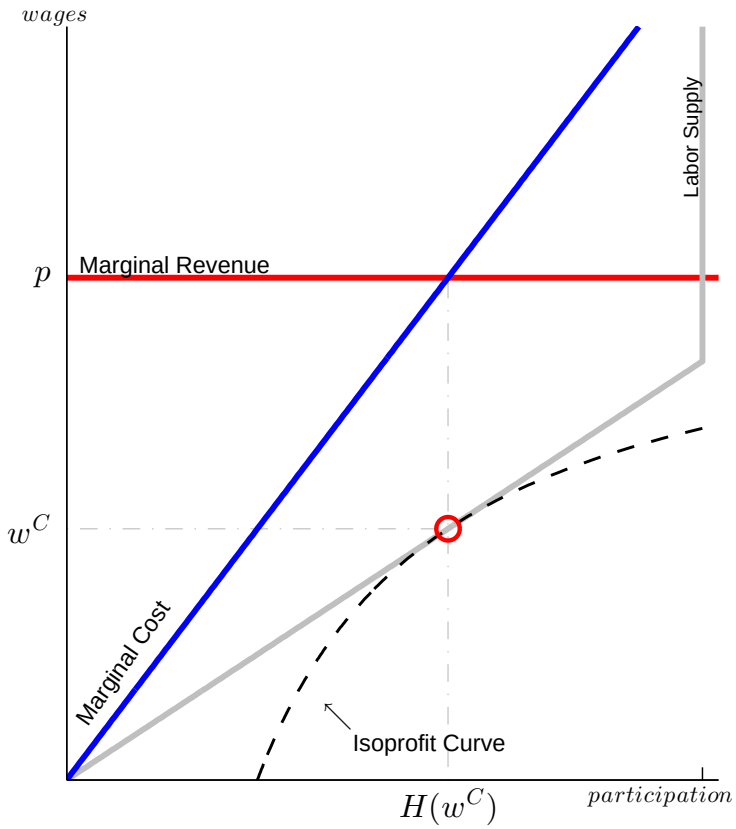

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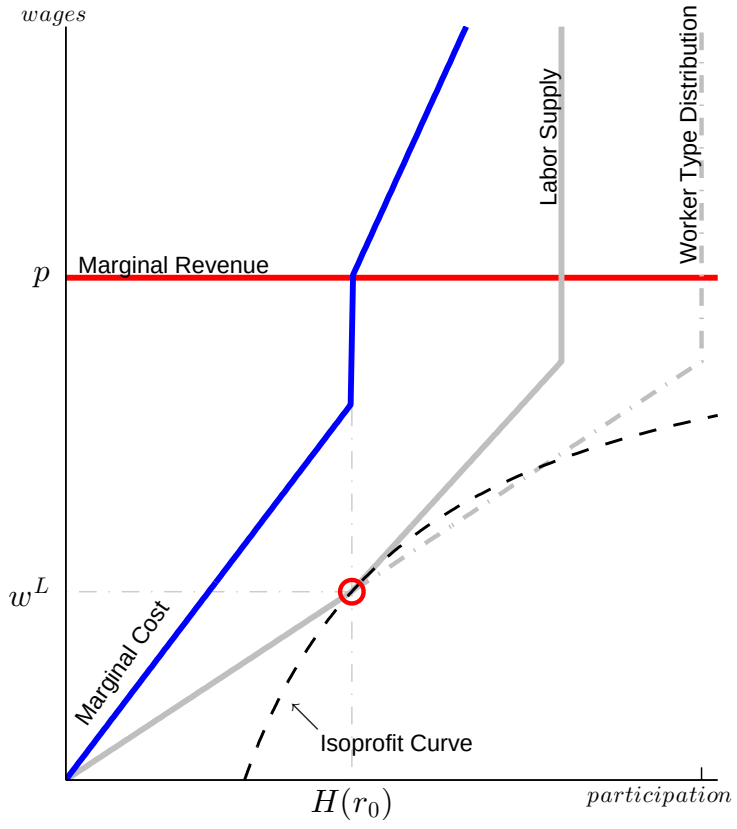

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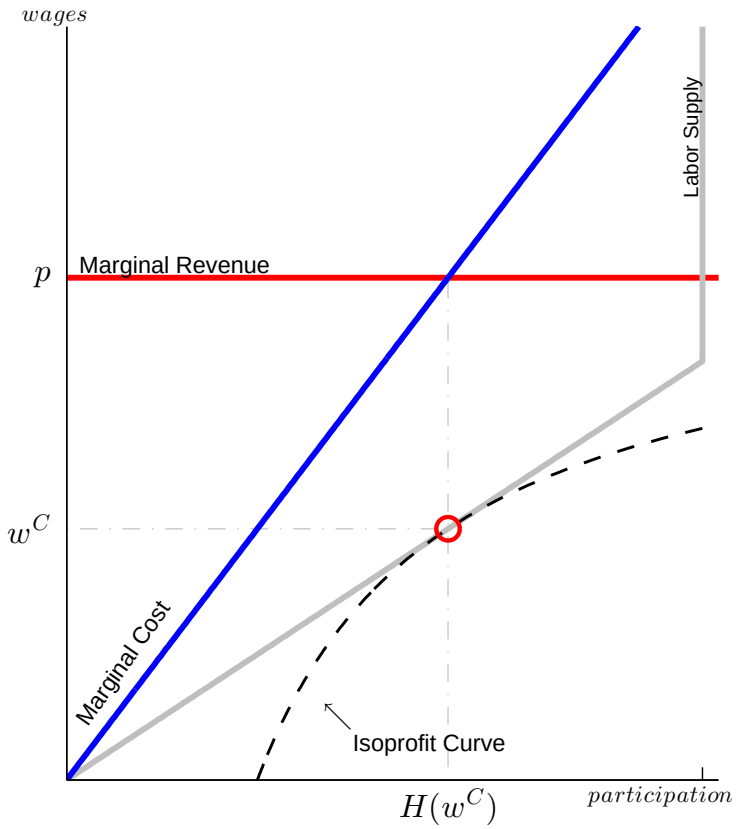

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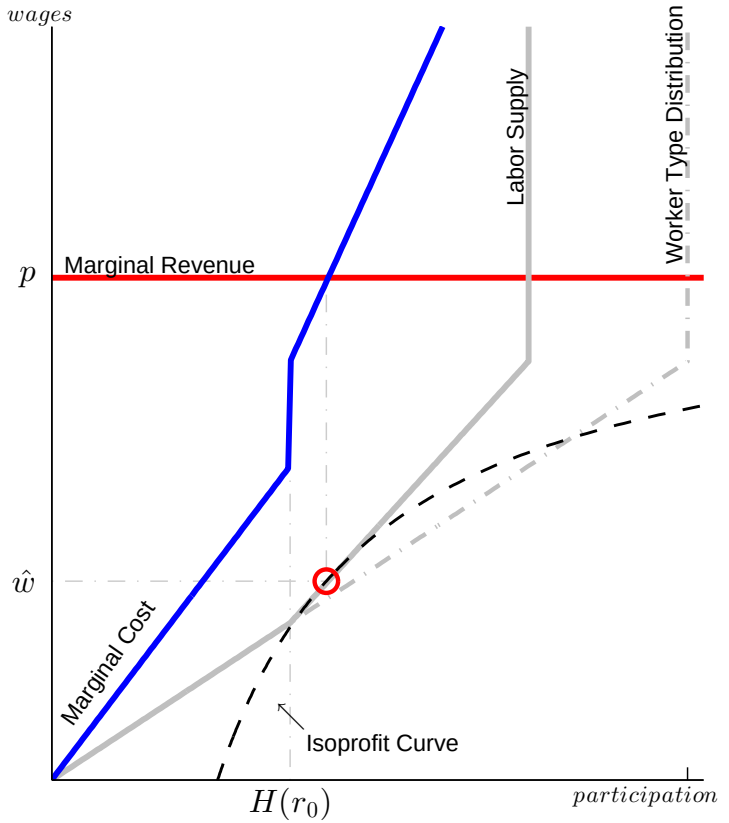


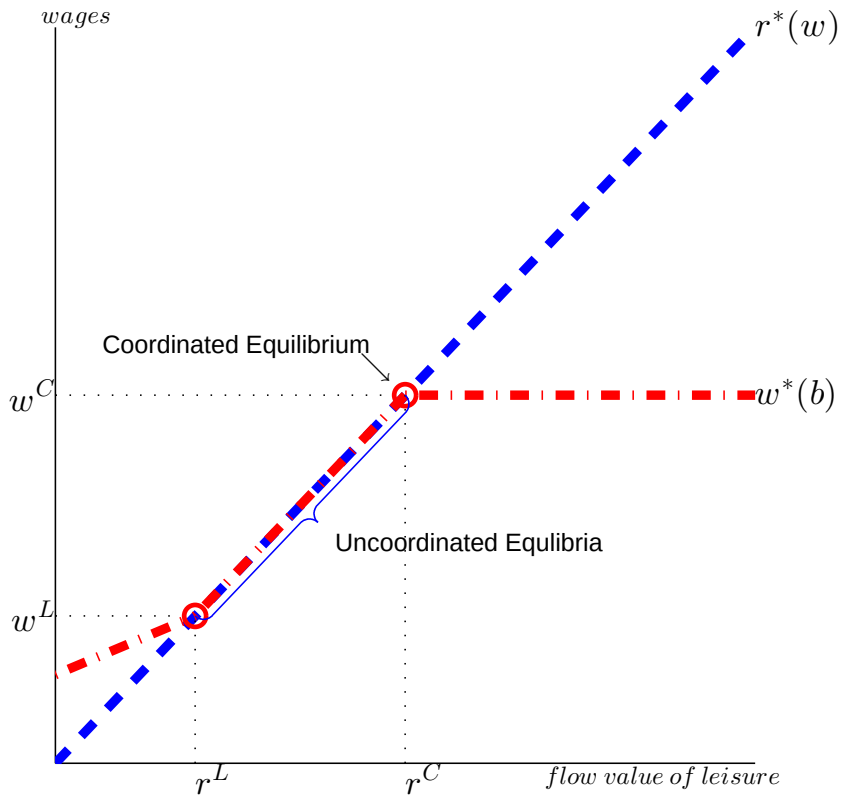
Equilibria

Definition An equilibrium of the two-player game is a double – **wage level**, **participation threshold** – such that the wage level of the firm and participation threshold of the worker are mutual best responses.

Prop. 3 A continuum of equilibria exist – wage levels in the interval $[w^L, w^C]$ – with higher welfare for higher wage levels.

Note: In every equilibrium $w_0 = r_0$.

Mutual Best Response



▸ Multiple uncoordinated *and* coordinated Equilibria

A Two Sided Frictional Labor Market

- Pairwise random matching
- Atomistic agents
- Worker:
  - Heterogeneous w.r.t. the flow value of non-employment, $b \sim H(b)$.
  - Can flexibly move in and out of the labor force.
  - Aware of the average wage level.
- Firm:
  - Post a non-negotiable wage offer.
  - Aware of $H(b)$ and the average participation threshold.

No single worker or single firm can alter the average wage or average participation threshold through unilateral deviation.

Worker's strategy: reservation participation threshold, r. (as before)
Firm's strategy: a posted wage, w. (as before)

Matching Technology and Congestion

Standard CRS matching function:

$$matches = m(U, V); \qquad job\ finding = f(\theta) \equiv \frac{m}{U};$$

U mass unemp.; V mass vac.; $\theta = \frac{V}{U}$ market tightnesss; $\frac{df(\theta)}{d\theta} > 0$; and $\frac{df^2(\theta)}{d\theta^2} < 0$.

Random search, unemployment, and congestion:

- Mass of unemployed: $U = uH(w_0) + i(1 - H(w_0))$
- Vacancy filling rate: $\frac{uH(w^*)}{uH(w^*)+i(1-H(w^*))} \frac{M}{V} = \Lambda(w^*)q(\theta)$

No single worker or single firm can alter the job finding or vacancy filling rates through unilateral deviation.

Production, Free Entry, and Job Creation Condition

- Production is linear in labor.
- Free entry into vacancy creation at flow cost c.

Definition An equilibrium of the two-sided game is a triple – **wage level**, **participation threshold**, **labor market tightness** – such that the wage level of each firm and participation threshold of workers are mutual best responses and tightness satisfies the job creation condition:

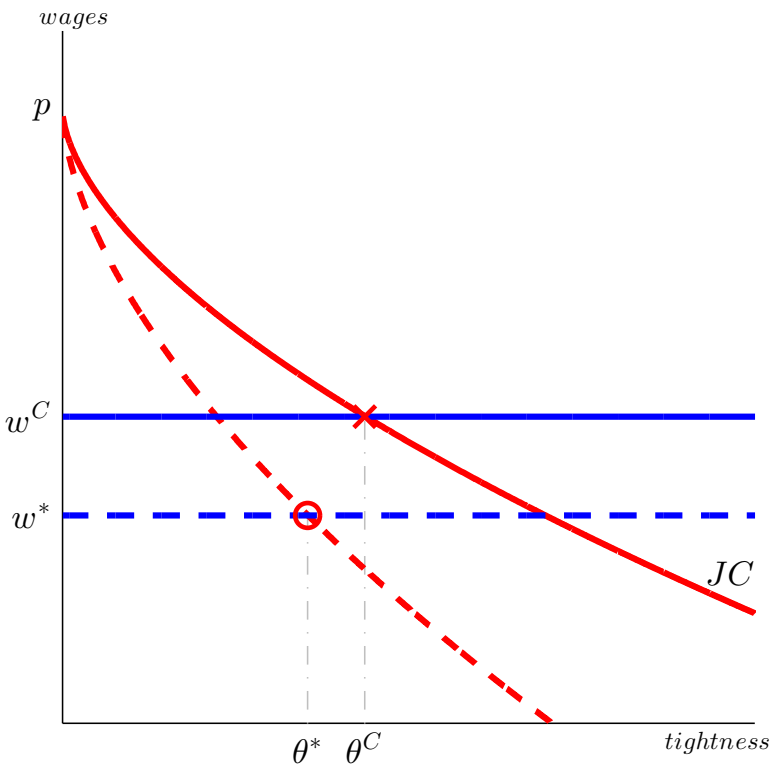
$$\frac{c}{q(\theta(w^*))\Lambda(w^*)} = \frac{p - w^*}{\rho + \delta}.$$

Prop. 4 A continuum of equilibria exist – wage levels in the interval $[w^L, w^C]$ – with higher welfare for higher wage levels.

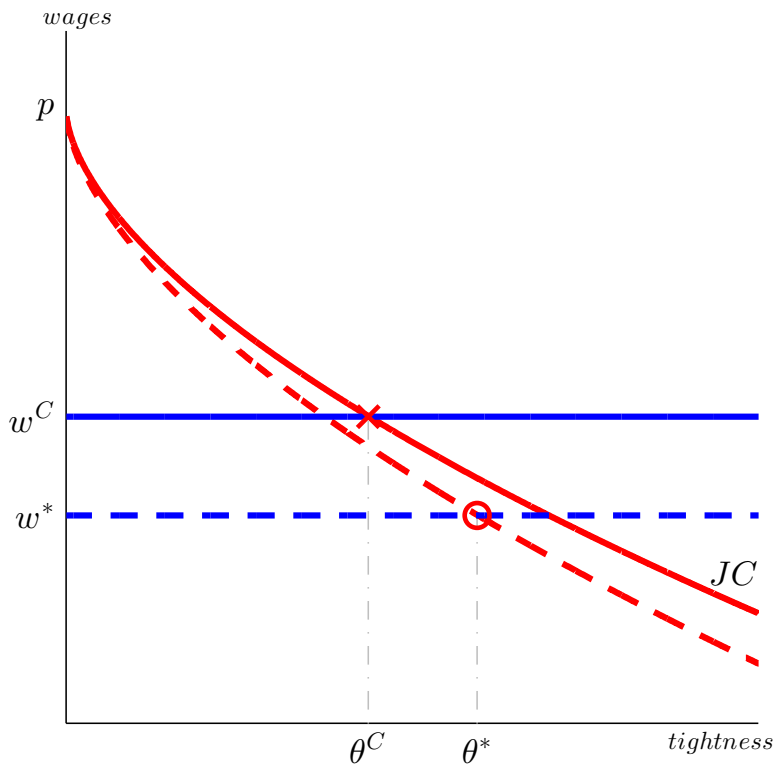
Congestion Effects and Wage Effects

Job creation condition: $\frac{c}{q(\theta(w^*))\Lambda(w^*)} = \frac{p-w^*}{\rho+\delta}$.

Congestion Effect Dominates



Wage Effect Dominates



Shocks and Dynamic Best-Response

Stochastic Aggregate Productivity:

- The productivity level of firms, p, is a martingale.

Assumption 1: Unilaterally Optimal Deviation.

The aggregate wage level changes only if deviation from the old wage level to the new wage level would be unilaterally optimal for each firm.

(Vives, 1990, 2005; Cooper, 1994)

- In other words, the new wage level is each firm's best response to all other firms setting wages at the old wage level accompanied by the corresponding reservation wage policy of workers.

Endogenous Rigidity in Wages and Participation

Prop 5:

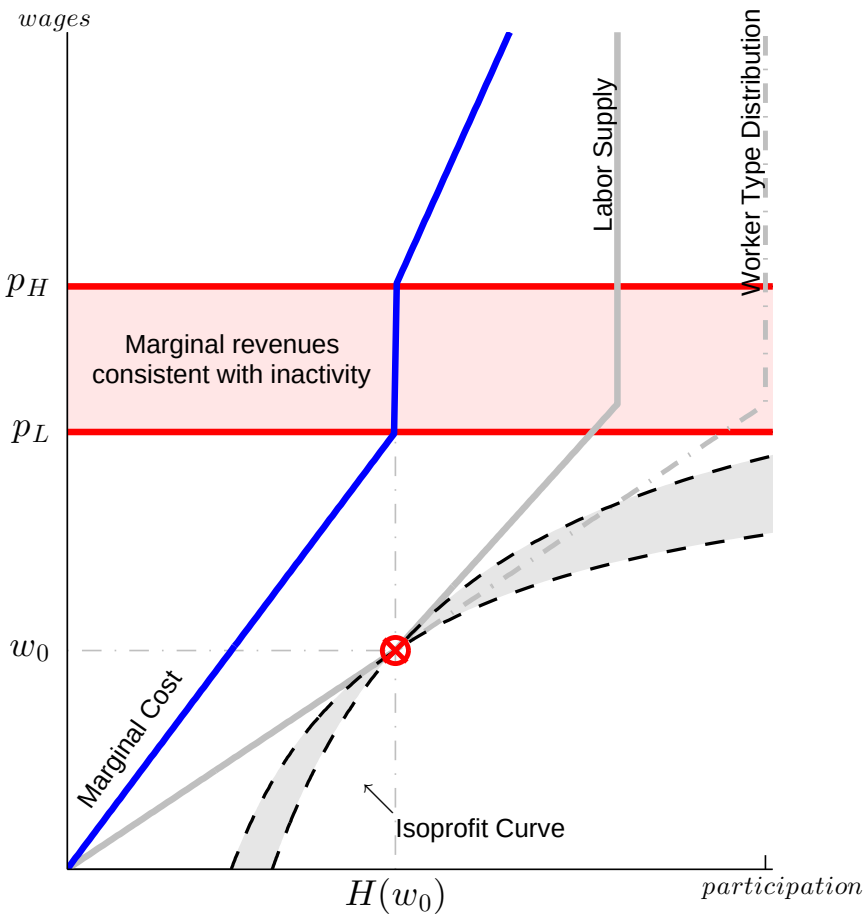
For each wage and participation threshold pair, $\{w_0, r_0\}$:

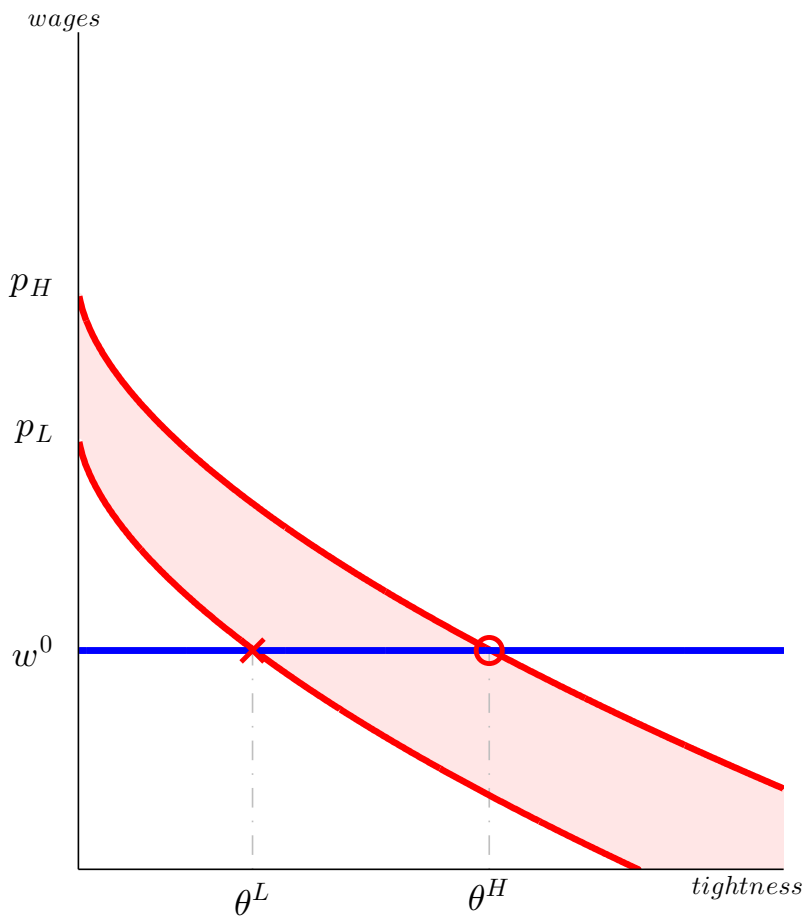
- There is a consistent interval of productivity levels – (p^L, p^H) – for which no uncoordinated firm wishes to unilaterally deviate to a different wage:

$$p^L = w_0 + \frac{H(w_0)}{h(w_0)} \qquad p^H = w_0 + \frac{H(w_0)}{\frac{i}{u}h(w_0)}.$$

- There is a consistent interval of labor market tightness – (θ^L, θ^H) – where the lower bound solves the free entry condition, for p^L and the upper bound for p^H.

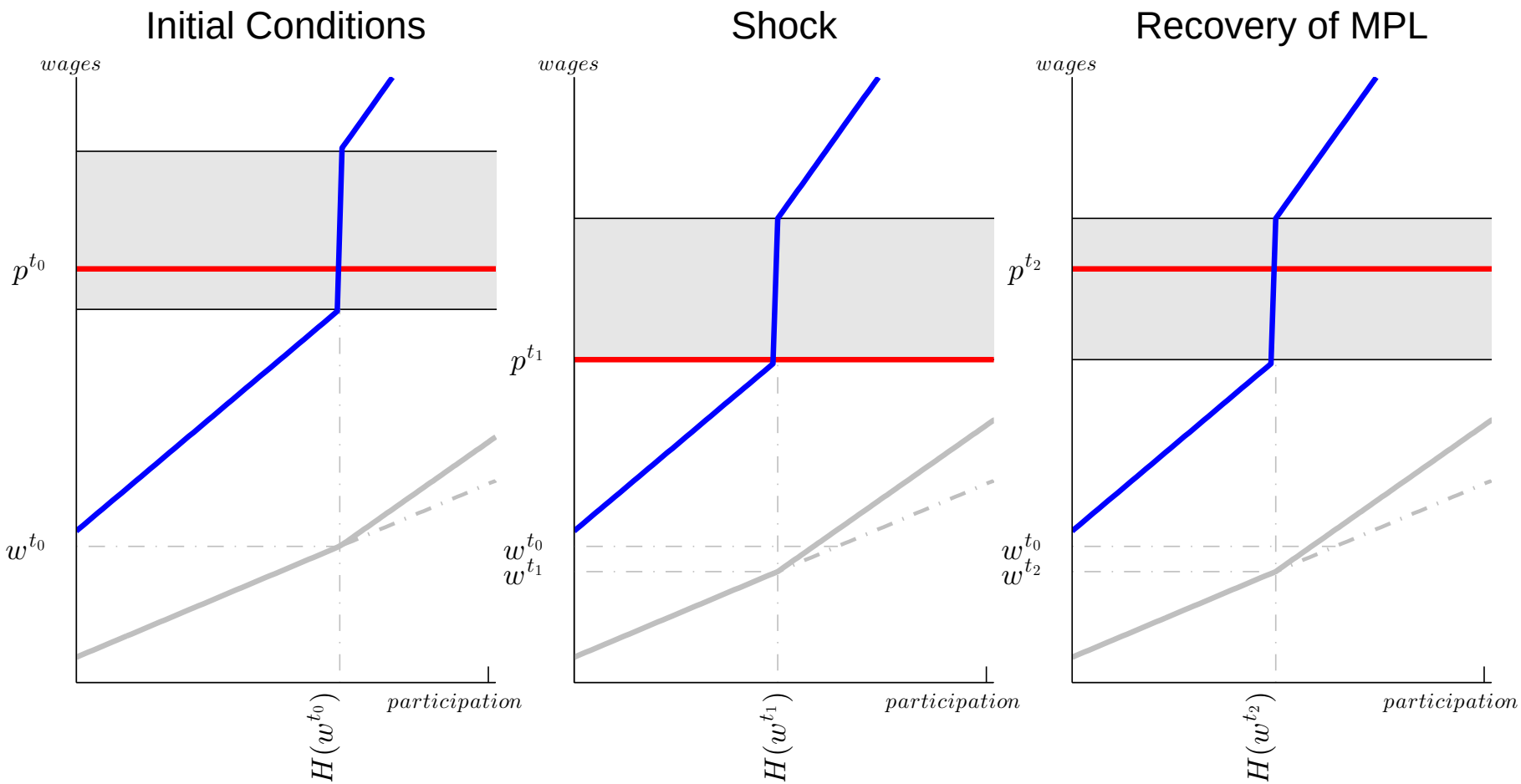
Endogenous Rigidity in Wages and Participation





Hysteresis

A contraction arrives at t_1 and labor productivity recovers at time t_2.

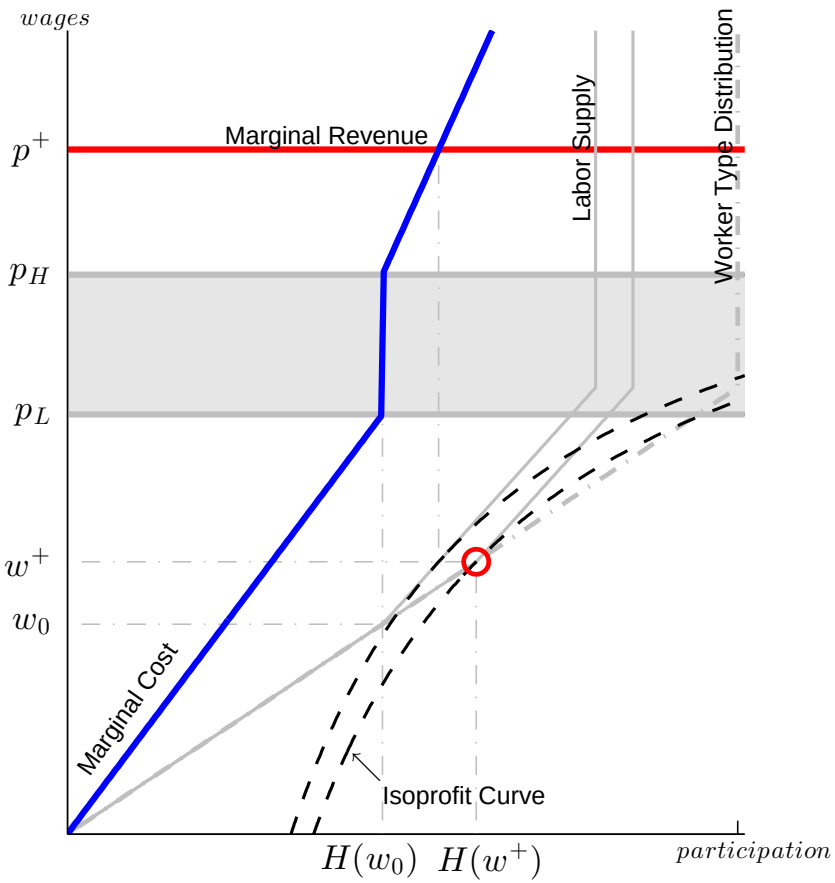


Inefficient Expansions

Prop 6:

For $\frac{i}{u} < 1$, employment and total output are inefficiently low even after an arbitrarily long or steep expansion,

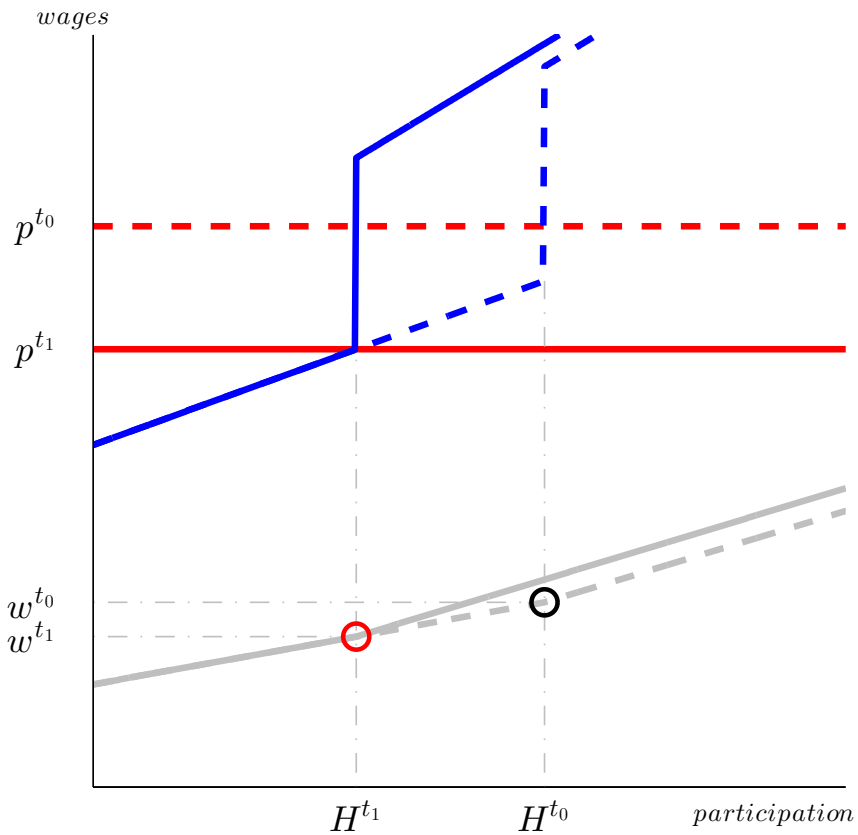
- if the distribution of worker types is unbounded above.
  or
- except in the trivial case where wages are high enough that all workers participate.

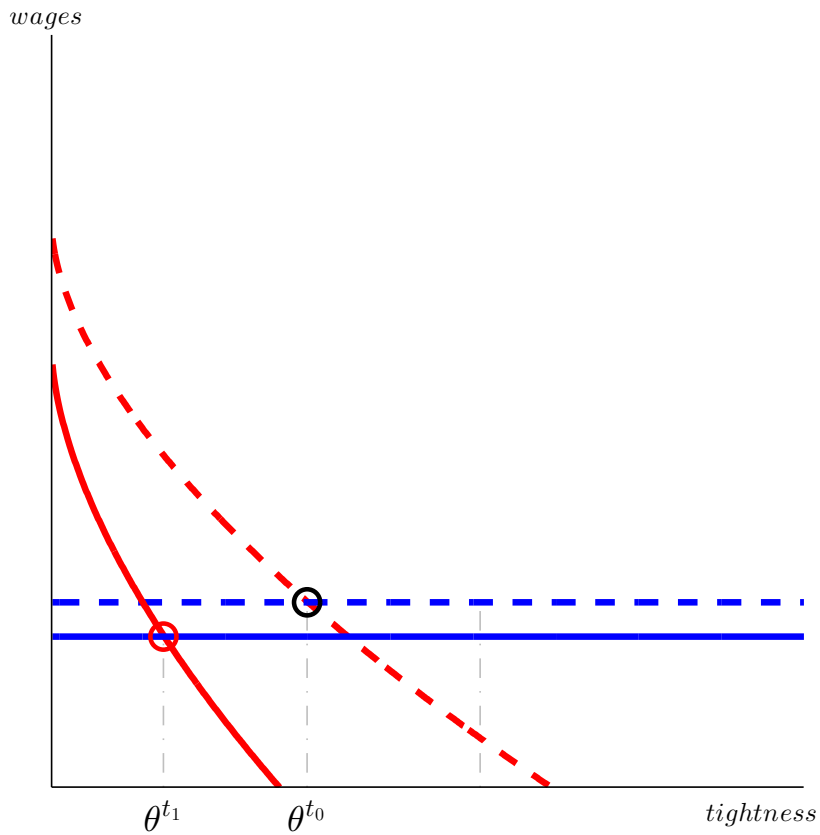


"Jobless" Recovery

When congestion is severe enough unemployment is persistently high after MPL recovers.

▸ wage effect dominates under low congestion.





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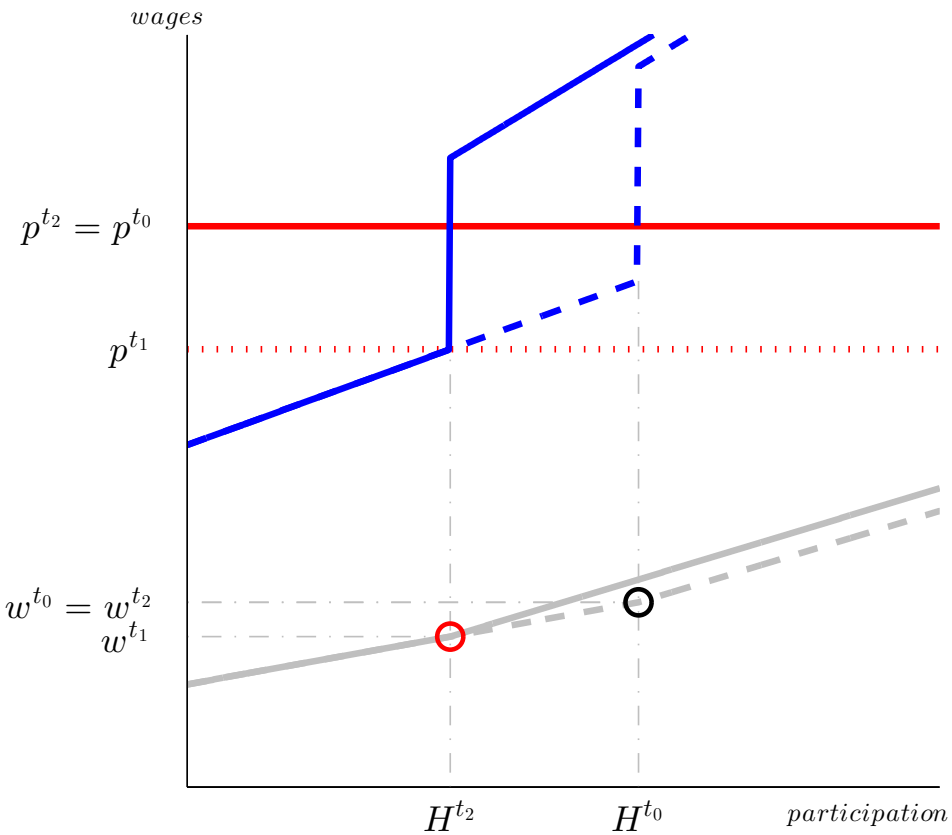
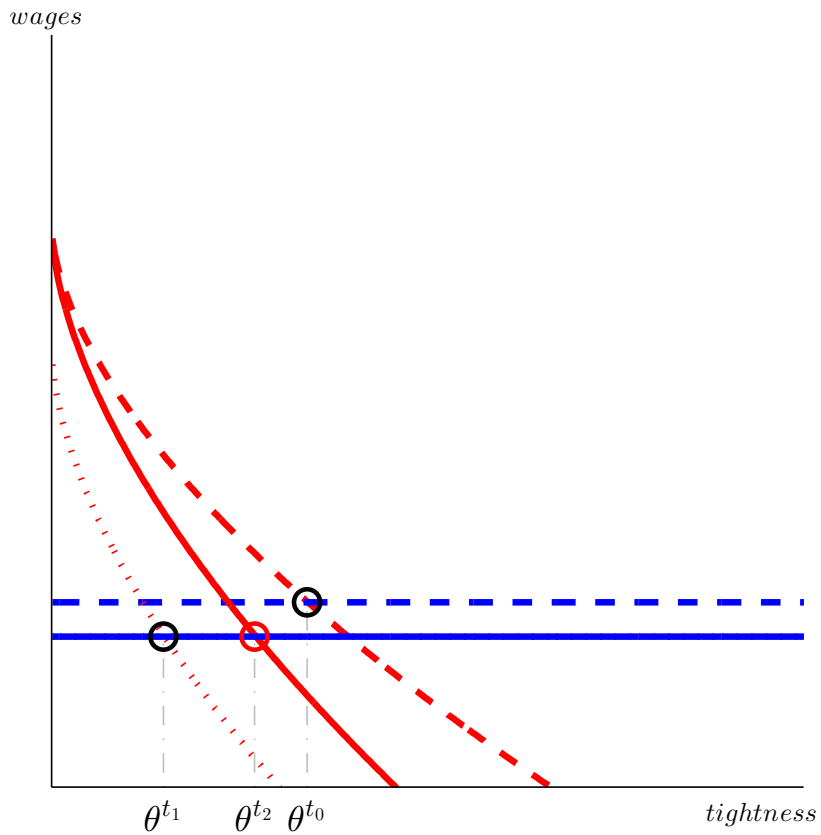
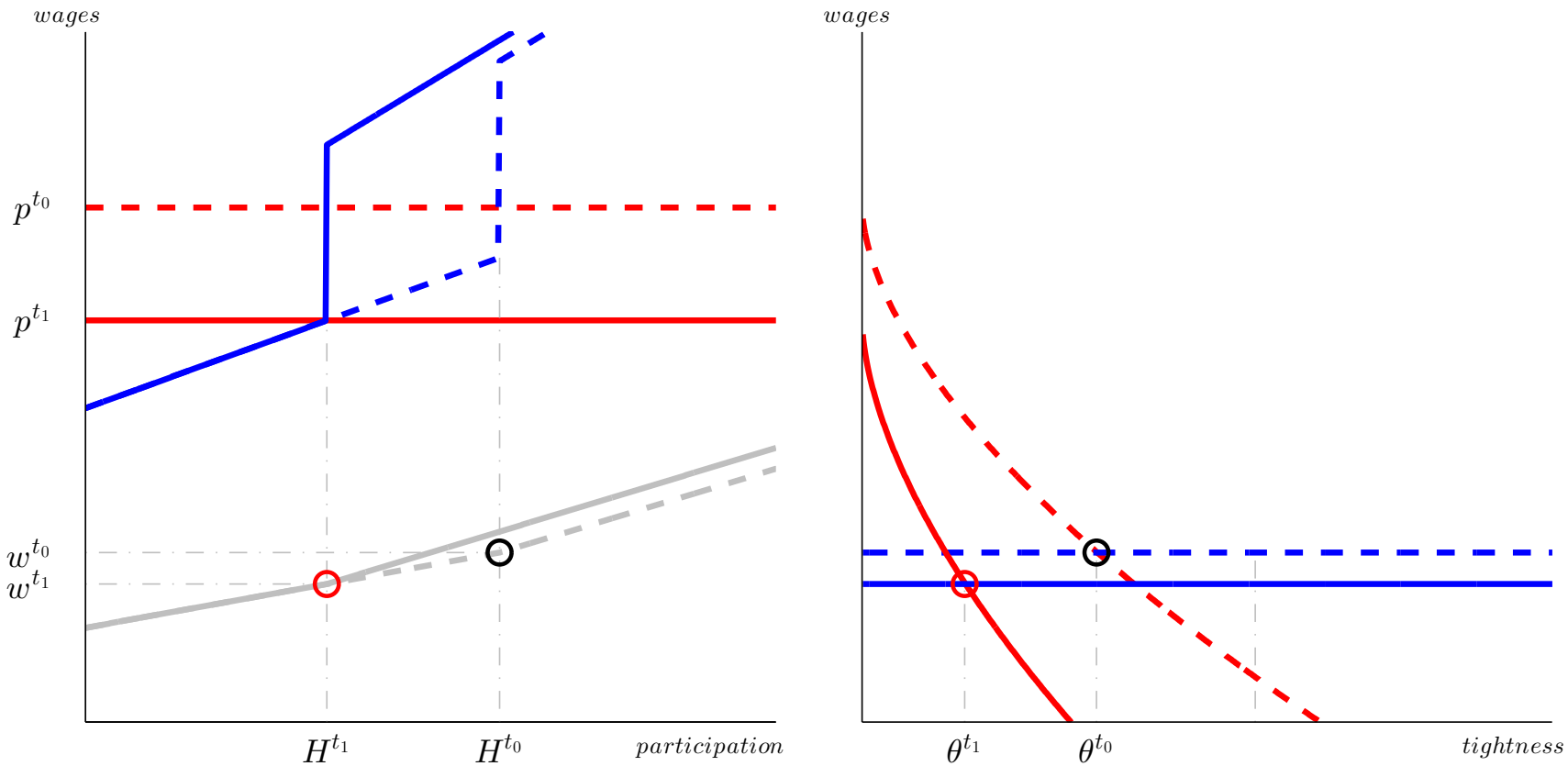




"Wageless" Recovery

Unemployment falls to unprecedentedly low levels before wages fully recover.



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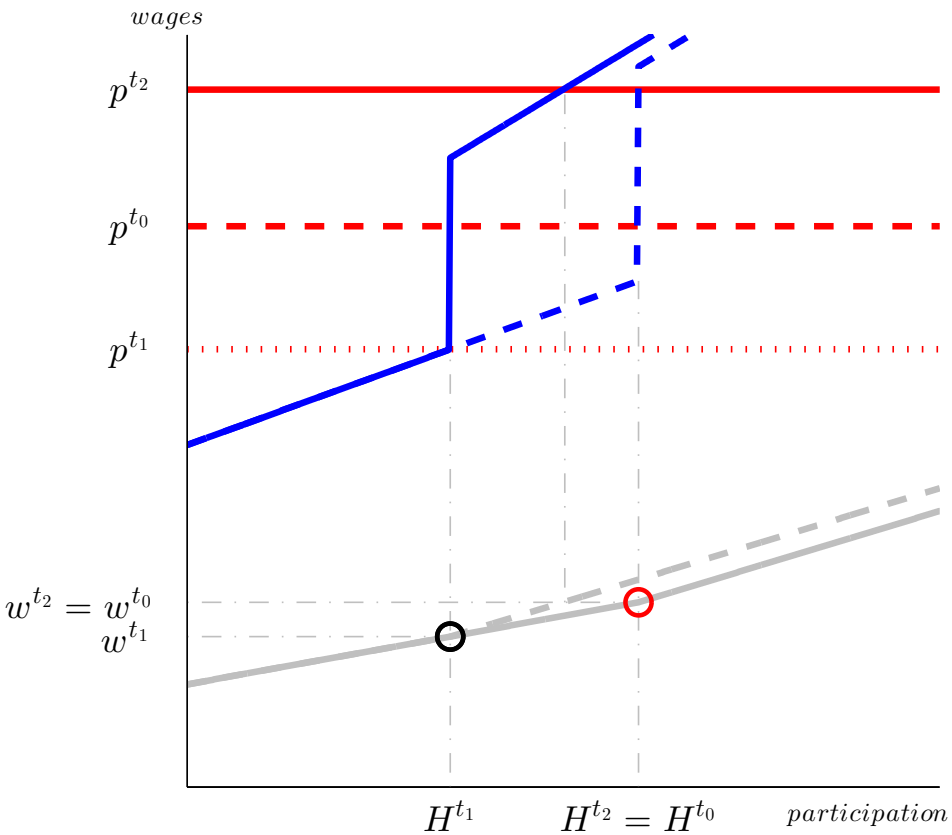
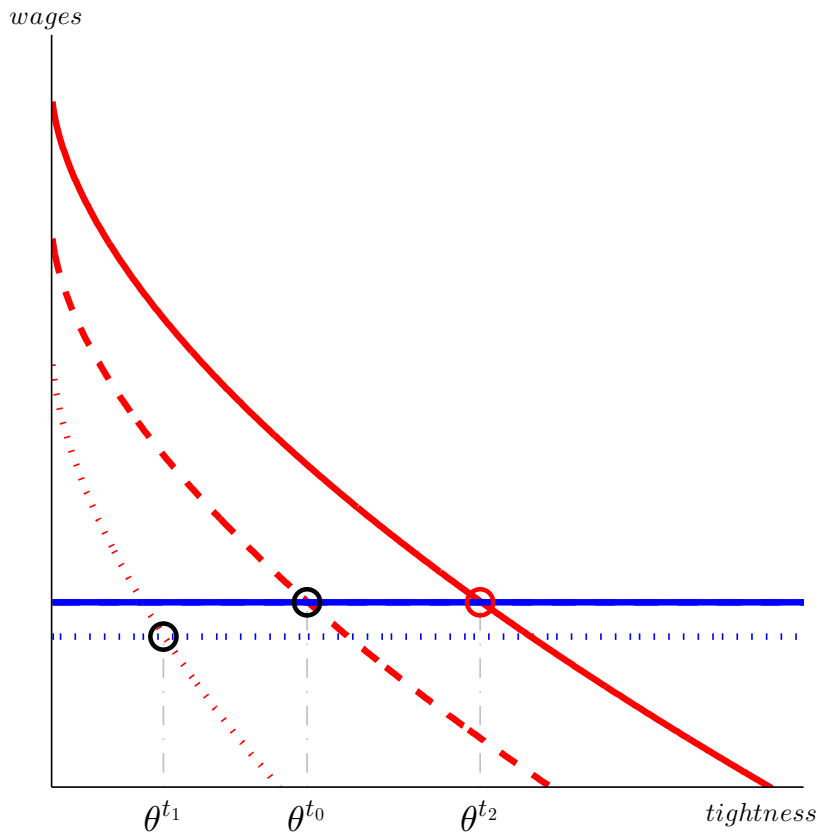




Dynamic Congestion skip

- At Poisson hazard d workers get relocated.
- After relocation workers have Knightian uncertainty about $b - w_0$.
- The value of $b - w_0$ is revealed upon matching.

Prop 7: If:

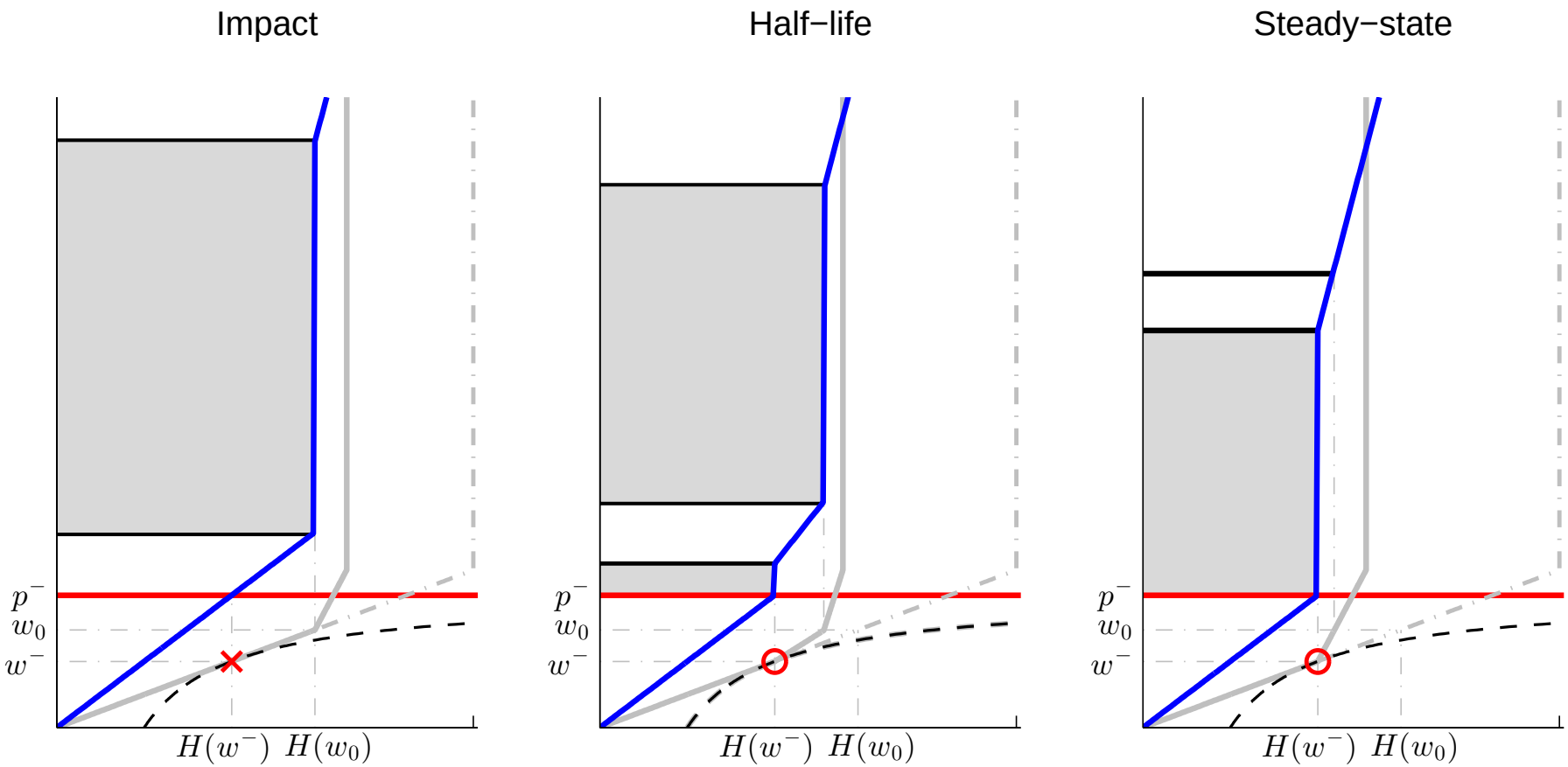
- knowledge of $b - w_0$ is retained once learned, and
- search costs are null,

then $db_0/dw_0 = 1$ on $[\underline{b}, \bar{b}]$ and the enriched model exhibits a continuum of equilibrium on $[\underline{b}, w^C(p)]$ for every realization of $p > \underline{b}$.

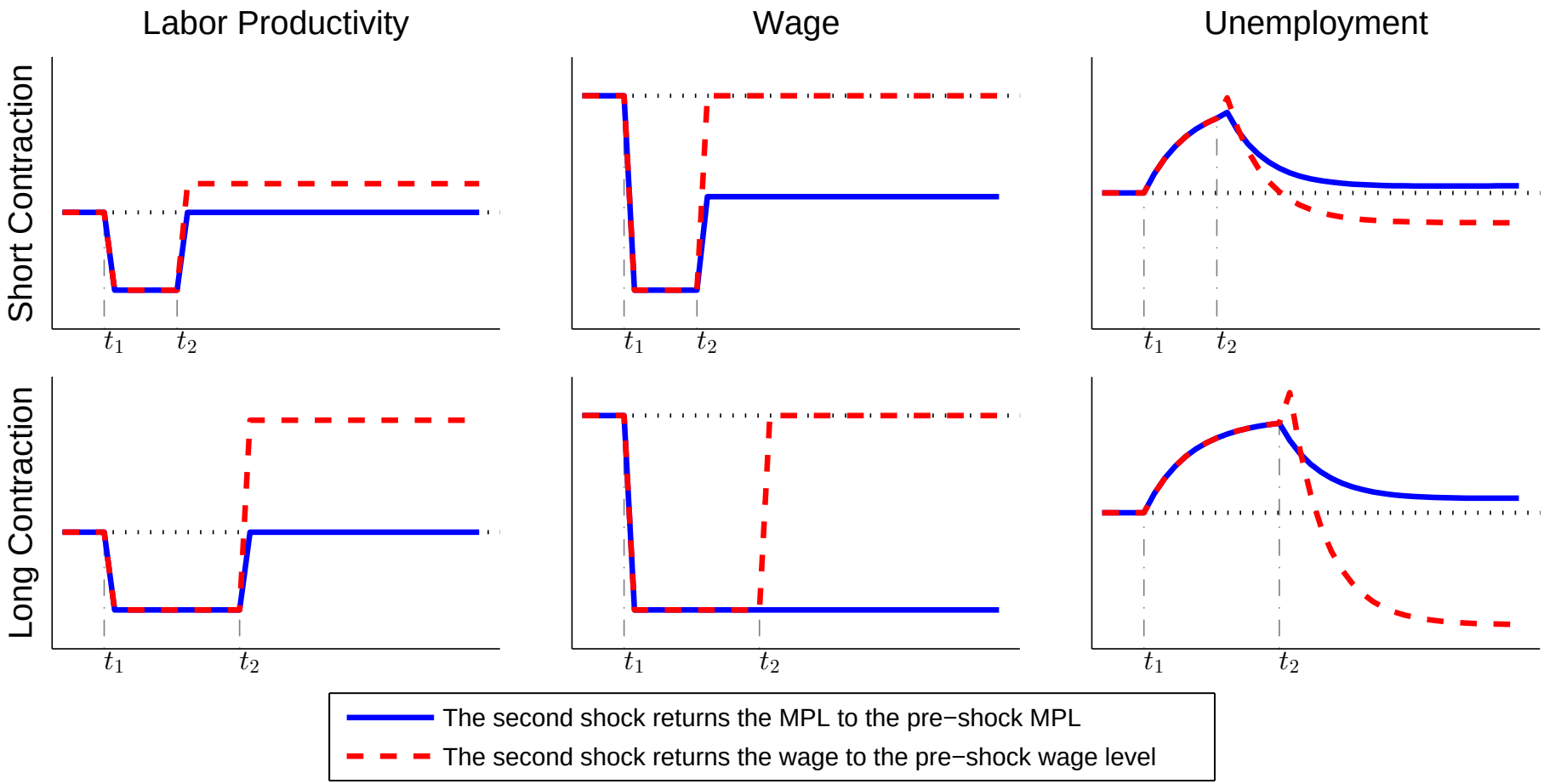
In steady state $\frac{i}{u} = \frac{\frac{d}{d+f(\theta)}}{\frac{\delta}{\delta+f(\theta)}} \approx \frac{d}{\delta}$

Following contraction $\frac{i}{u} \approx 1$ in the neighborhood of the equilibrium.

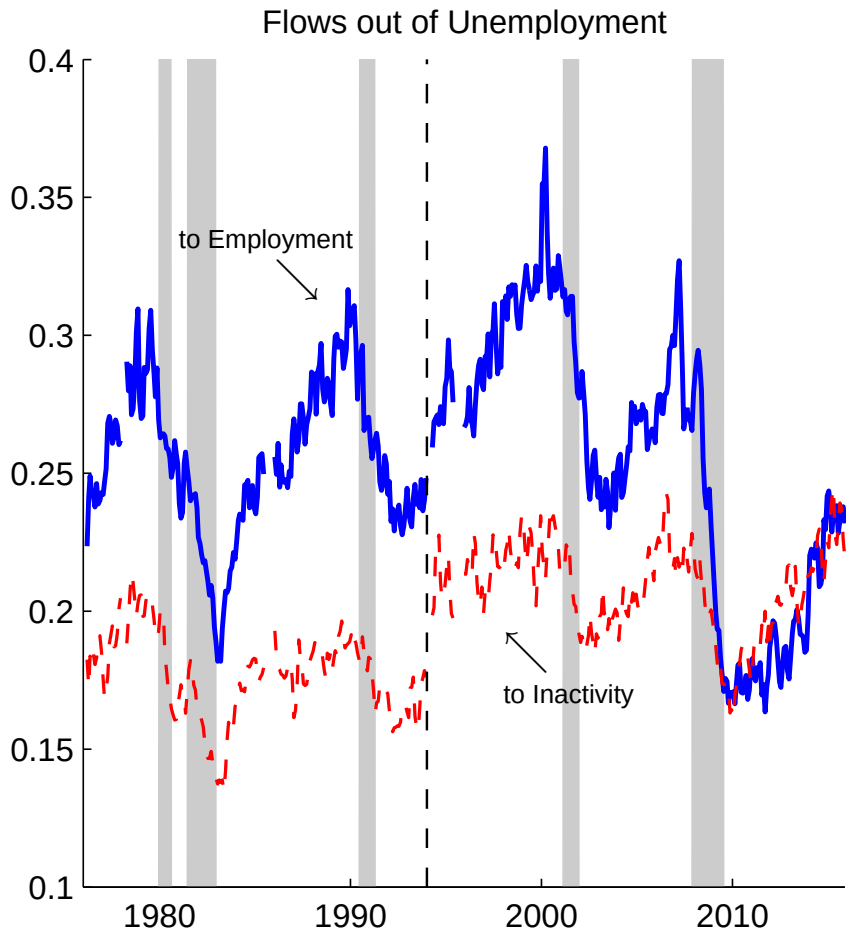
Recovery After Prolonged Contraction

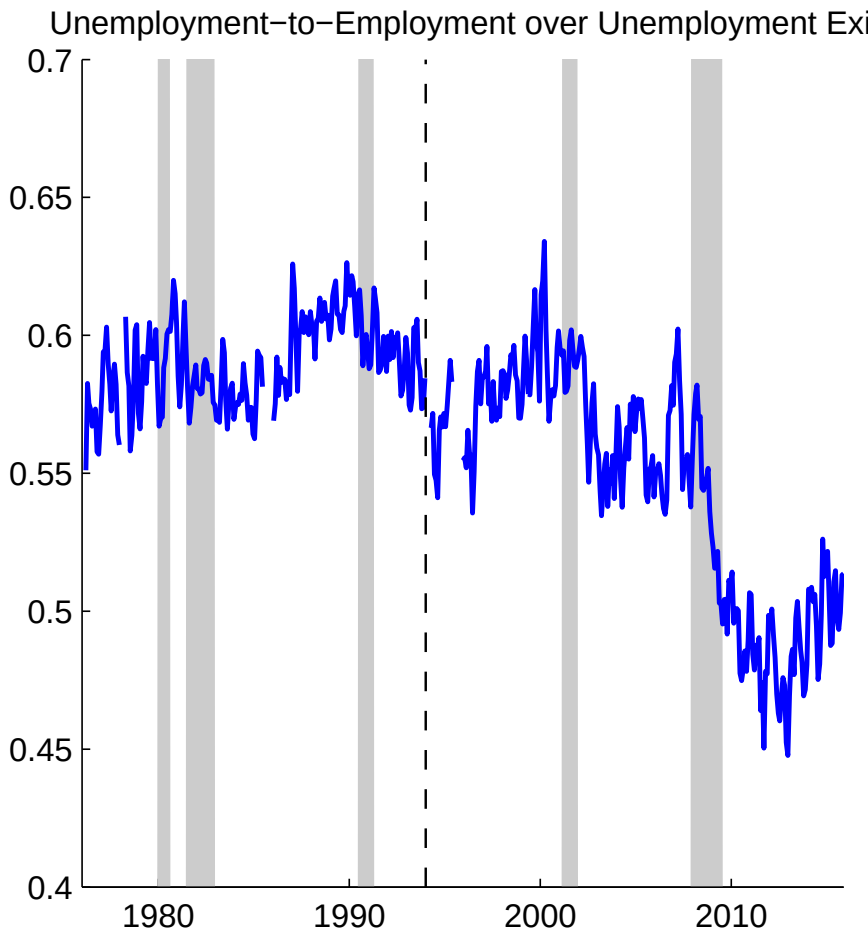


Recession duration exacerbates "Joblessness" and "Wagelessness"



Labor Flows





Source: Authors' calculations based on matched monthly Current Population Survey as in Shimer (2012).

Sample: Non-institutionalized, civilians with 1 to 30 years of potential experience.

Note: The 1994 redesign of the Current Population Survey is indicated by the vertical black hashed line. Importantly the resign include a switch to dependent interviewing.

Test: Congestion and Match Efficiency

skip

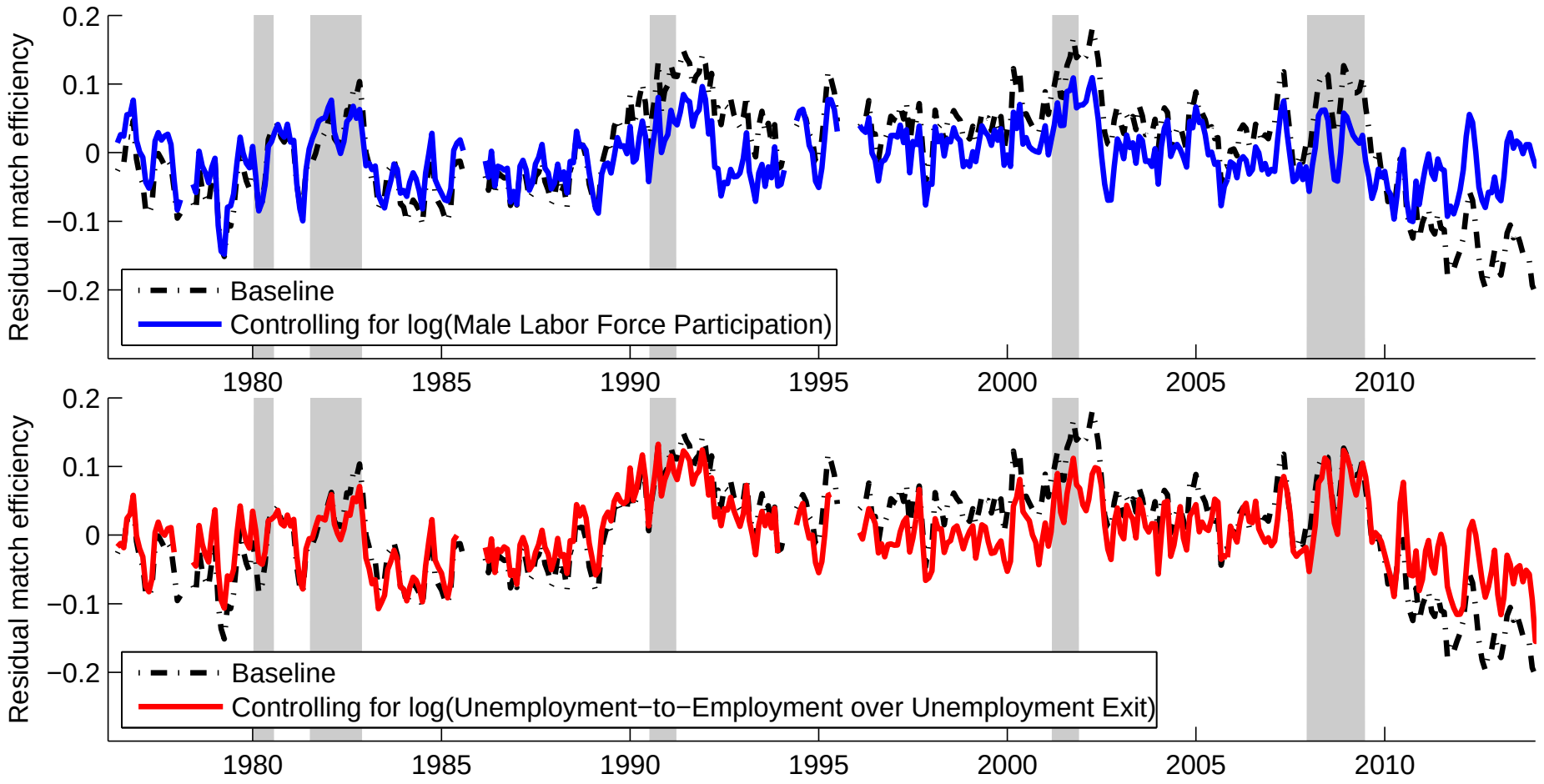
A Cobb-Douglass Approximation: $\ln f_t = \ln(A) + \ln(\Lambda_t) + (1-\eta)\ln\theta_t + \varepsilon_t$

$\ln(A)$	-1.08***	0.37***	-0.97***
	(.008)	(0.080)	(0.090)
$(1-\eta)$	0.34***	0.26***	0.27***
	(.010)	(0.009)	(0.012)
$\ln(male\ LFP)$		6.673***	
		(0.366)	
$\ln(\frac{Unemp.toEmp.}{Unemp.Exit})$			0.28
			(0.146)
$\ln(\frac{Unemp.toEmp.}{Unemp.Exit})x\mathbb{I}_{t>=1994}$			0.59***
			(0.166)
$\mathbb{I}_{t>=1994}$			0.43***
			(0.096)
R^2	0.77	0.86	0.80
R^2, 1976-2014	0.74	0.84	0.80
Sample (monthly frequency)	1967-2014	1967-2014	1976-2014

Note: All regressions control for a linear time trend. Standard errors in parentheses.

Source: Data Description

Residual Match Efficiency. (Sample Period: 1976–2015)



Policy: Pro-Cyclical Minimum Wage

Prop 8:

The policy maker can attain higher levels of output by imposing minimum wages, $\underline{w} \in (w_0, w^C(p)]$ during the expansion.

- Rasing the minimum wage above the average wage level as productivity rises induces labor force participation to rise more quickly than firm's unilateral wage revisions.
- Keeping minimum wage weakly below w^C ensures that gain to firms from greater participation exceeds the cost of higher wages.
- New matches must compensate (firms in) existing matches.

Policy: Counter-Cyclical Stimulus

- Counter-Cyclical stimulus could reduce scarring effect to the extent that the duration of contractions can be shortened.
- Pro-Cyclical minimum wage policy remains effective at inducing output nearer to the coordinated equilibrium.
- Unemployment is an inconsistent measure of slack.

Conclusion

Recap:

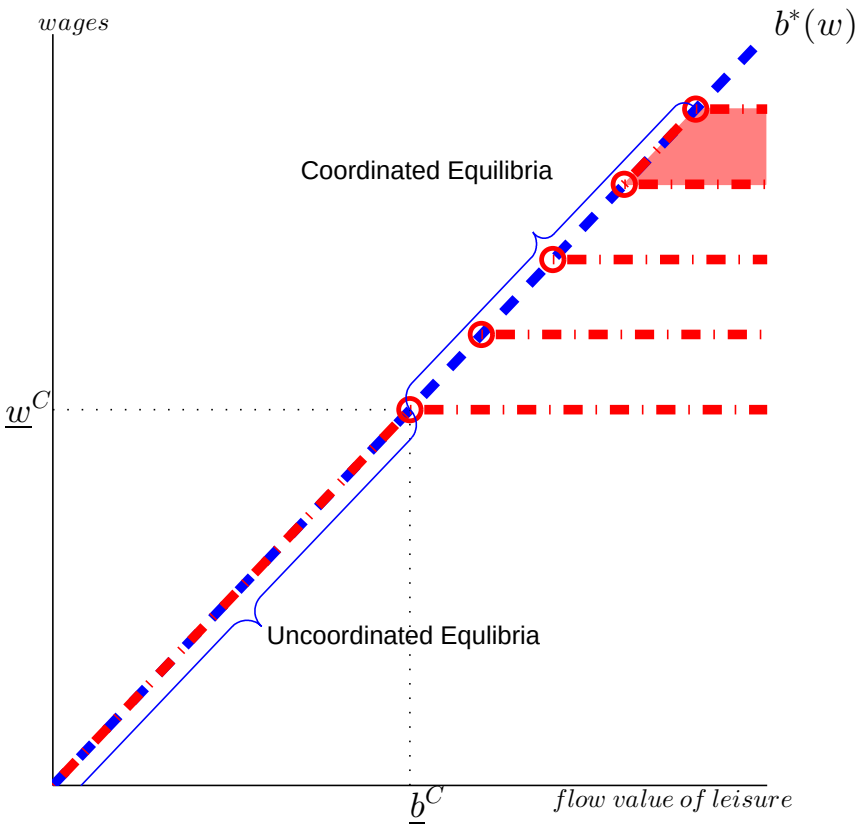
- A model of "fragile" equilibria due to strategic complementarities.
- Scarring effects / hysteresis in response to productivity shocks.
- Insufficiently robust expansions.
- "Wageless" recoveries and possibly "jobless" recoveries.
- Scarring / hysteresis are more severe following prolonged contractions.

Policy:

- Pro-Cyclical Minimum Wage
- Counter-Cyclical Stimulus (aimed at shortening contractions)

Multiple Coordinated Equilibria

Mutual Best Response



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Data Description

Bureau of Labor Statistics, Current Population Survey; Conference Board, Help Wanted Index and Help Wanted Online Index; Bureau of Labor Statistics, Job Openings and Labor Turnover Survey. Vacancies are constructed from the Conference Board data as in Barnichon (2010) prior to 2001 and follow the Job Opening and Labor Turnover Survey thereafter. Male labor force participation is calculated from the Current Population Survey restricted to males between 25 and 55. Labor flows are calculated from the Current Population Survey monthly data using the matching procedure of Shimer 2012.

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"Jobless" Recovery

When congestion is severe enough unemployment is persistently high even after MPL recovers.