Endogenous Wage Indexation and Aggregate Shocks
Technical Appendix

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This technical appendix describes the full derivation of the model and provides details about its stochastic steady state.

Contents

A Baseline model

The model consists of 5 types of agents: the final goods producer, intermediate goods producers, consumers, workers and the fiscal and monetary authorities.

A.1 Households and Wage Setting

The economy is inhabited by a continuum of differentiated households, indexed by $i \in [0,1]$. Household i is endowed with a unique labor type, $\ell_{i,t}$, which allows it to set its own wage using its monopolistic power. Household i selects consumption, $c_{i,t}$, one-period-maturity bond holdings, $b_{i,t}$, and a nominal wage, $W_{i,t}$, in order to maximize its expected discounted lifetime utility

$$\mathrm{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \exp\left(\varepsilon_{u,T}\right) \mathcal{U}(c_{i,T}, \ell_{i,T}), \tag{A.1}$$

subject to the sequence of budget constraints

$$c_{i,t} + \frac{b_{i,t}}{R_t \exp\left(\varepsilon_{b,t}\right)} \leq w_{i,t}\ell_{i,t} + \frac{b_{i,t-1}}{1+\pi_t} + \frac{\Upsilon_{i,t}}{P_t}, \tag{A.2}$$

and *no Ponzi* schemes. E_t is the expectation operator conditional on the available information in period t. R_t is the risk-free gross nominal interest rate, P_t denotes the price of the final good, $\pi_t \equiv P_t/P_{t-1}$ is the gross inflation rate, and $\Upsilon_{i,t}$ is a lump sum including net fiscal transfers, Arrow-Debreu state-contingent securities, and profits from monopolistic firms. There are two stochastic disturbances with mean zero: $\varepsilon_{b,t}$ creates a spread between the return on bonds and the risk free rate (cf. risk spread shock in Smets and Wouters, 2007) and $\varepsilon_{u,t}$ is a preference shock. Preferences are separable between consumption and labor:

$$\mathcal{U}(c_{i,t}, \ell_{i,t}) \equiv \frac{\left(c_{i,t} - \gamma^h c_{i,t-1}\right)^{1-\sigma} - 1}{1-\sigma} - \psi \frac{\ell_{i,t}^{1+\omega}}{1+\omega}, \tag{A.3}$$

where γ^h is a parameter controlling external habits, $\sigma^{-1} > 0$ is the inter-temporal elasticity of substitution, ω^{-1} is the Frisch elasticity of labor supply, ψ is a normalizing constant that ensures that labor equals $\frac{1}{3}$ at the deterministic steady-state. We assume for simplicity that households are divided into two units: a consumer and a worker. The former chooses consumption demand and bond holdings, while the latter sets the nominal wage knowing that the elapsed time between wage re-optimizations is a stochastic process. Notice that the presence of state-contingent securities ensures that all households begin a period with the same wealth and therefore choose the same level of consumption. Therefore, we can drop the subscript i in the first order conditions (FOCs) of $b_{i,t}$, and $c_{i,t}$.

A.1.1 Consumption choice

The consumer problem is

$$\max_{c_{i,T}, b_{i,T}} \mathrm{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ \begin{array}{c} \exp\left(\varepsilon_{u,t}\right) \frac{\left(c_{i,T} - \gamma^h c_{i,T-1}\right)^{1-\sigma} - 1}{1-\sigma} \\ + \lambda_T \left[\frac{b_{i,T-1}}{1+\pi_T} + \frac{\Upsilon_{i,T}}{P_T} - c_{i,T} - \frac{b_{i,T}}{R_T \exp\left(\varepsilon_{b,T}\right)} \right] \end{array} \right\}.$$

The FOCs of $b_{i,t}$ and $c_{i,t}$, are respectively given by

$$1 = \beta \mathrm{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \frac{R_t \exp\left(\varepsilon_{b,T}\right)}{1+\pi_{t+1}} \right\}, \tag{A.4}$$

$$\exp\left(\varepsilon_{u,t}\right)\left(c_t - \gamma^h c_{t-1}\right)^{-\sigma} - \beta\gamma^h \mathrm{E}_t \left\{ \exp\left(\varepsilon_{u,t+1}\right)\left(c_{t+1} - \gamma^h c_t\right)^{-\sigma} \right\} = \lambda_t. \tag{A.5}$$

A.1.2 Wage-setting

Labor packer. Following Erceg *et al.* (2000), we assume that a competitive labor intermediary builds a single labor input from a set of differentiated labor types $\ell_{i,t}$, for $i \in [0,1]$, according to the following CES technology

$$\ell_t = \left(\int_0^1 \ell_{i,t}^{(\theta_w - 1)/\theta_w} \mathrm{d}i \right)^{\theta_w/(\theta_w - 1)}, \tag{A.6}$$

where $\theta_w > 1$ is the elasticity of substitution between any two labor types. Profit maximization by the labor intermediary yields the demand for type$-i$ labor

$$\ell_{i,t} = \left(\frac{W_{i,t}}{Wt} \right)^{-\theta_w} \ell_t \ \forall i, \tag{A.7}$$

while the aggregate nominal wage obeys

$$W_t = \left(\int_0^1 W_{i,t}^{1-\theta_w} \mathrm{d}i \right)^{1/(1-\theta_w)}. \tag{A.8}$$

Wage setter. Similar to Calvo (1983), we assume that in each period a worker re-calibrates his labor contract with a probability $1 - \alpha_w$. The re-calibration consists of two steps: first, the worker chooses a wage indexation scheme for updating his wage in non re-optimizing periods; and second, the worker chooses an optimal wage level that maximizes his utility. The two indexation schemes in the economy are δ^1 and δ^2. The former updates wages using the previous period inflation rate while the latter uses current trend inflation. Formally

$$\delta^1_{t,T} = \left(1 + \pi_{T-1}\right) \delta^1_{t,T-1} \qquad \text{and} \qquad \delta^2_{t,T} = \left(1 + \pi^\star_T\right) \delta^2_{t,T-1},$$

$\forall\ T > t$, and $\delta^k_{t,t} = 1$ for $k \in \{1,2\}$; t is the period where the last optimization occurred. For exposition purposes, it is better to first describe the wage-setting problem given the choice of $\delta^k_{t,T}$. The selection of the indexation scheme is described afterwards. Thus, given a $\delta^k_{t,T}$, a worker sets his wage according to (dropping subscript i, as any worker maximizing at t and with a rule $\delta^k_{t,T}$ will choose the same wage)

$$W_t^{k,\star} \in \arg\max_{W_t^k} \mathrm{E}_t \sum_{T=t}^{\infty} \left(\beta\alpha_w\right)^{T-t} \left[\lambda_T \frac{\delta^k_{t,T} W^k_t}{P_T} \ell^k_{t,T} - \psi \exp\left(\varepsilon_{u,T}\right) \frac{\left(\ell^k_{t,T}\right)^{1+\omega}}{1+\omega} \right], \tag{A.9}$$

subject to

$$\ell^k_{t,T} = \left(\frac{\delta^k_{t,T} W^k_t}{W_T} \right)^{-\theta_w} \ell_T. \tag{A.10}$$

The first order condition w.r.t. $W_t^{k,\star}$ is (note that $\frac{\partial \ell_{t,T}^k}{\partial W_t^k} = -\theta_w \frac{\ell_{t,T}^k}{W_t^k}$).

$$rw_t^{k,\star} \equiv \frac{W_t^{k,\star}}{W_t} = \psi \frac{\theta_w}{(\theta_w - 1)} \frac{\mathrm{E}_t \left\{ \sum_{T=t}^{\infty} (\beta\alpha_w)^{T-t} \exp(\varepsilon_{u,T}) \left[\ell_{t,T}^k\right]^{1+\omega} \right\}}{\mathrm{E}_t \left\{ \sum_{T=t}^{\infty} (\beta\alpha_w)^{T-t} \lambda_T \left(\delta_{t,T}^k / \pi_{t,T}^w\right) w_T \ell_{t,T}^k \right\}}$$

where $rw_t^{k,\star}$ is the relative wage of workers using indexation rule k, $\pi_{t,T}^w \equiv W_T/W_t$ and

$$\ell_{t,T}^k = \left(\frac{\delta_{t,T}^{w,k}}{\pi_{t,T}^w} rw_t^{k,\star} \right)^{-\theta_w} \ell_T.$$

Replacing labor-specific demand into the optimal wage-setting equation yields

$$rw_t^{k,\star} \equiv \frac{W_t^{k,\star}}{W_t} = \psi \frac{\theta_w}{(\theta_w - 1)} \frac{\mathrm{E}_t \left\{ \sum_{T=t}^{\infty} (\beta\alpha_w)^{T-t} \exp(\varepsilon_{u,T}) \left(\frac{\pi_{t,T}^w}{\delta_{t,T}^k} \right)^{\theta_w(1+\omega)} \left[rw_t^{k,\star}\right]^{-\theta_w(1+\omega)} [\ell_T]^{1+\omega} \right\}}{\mathrm{E}_t \left\{ \sum_{T=t}^{\infty} (\beta\alpha_w)^{T-t} \lambda_T w_T \left(\frac{\pi_{t,T}^w}{\delta_{t,T}^k} \right)^{\theta_w - 1} \left[rw_t^{k,\star}\right]^{-\theta_w} \ell_T \right\}}, \text{ or}$$

$$\left[rw_t^{k,\star}\right]^{1+\omega\theta_w} = \psi \frac{\theta_w}{(\theta_w - 1)} \frac{\mathrm{E}_t \left\{ \sum_{T=t}^{\infty} (\beta\alpha_w)^{T-t} \exp(\varepsilon_{u,T}) \left(\frac{\pi_{t,T}^w}{\delta_{t,T}^k} \right)^{\theta_w(1+\omega)} [\ell_T]^{1+\omega} \right\}}{\mathrm{E}_t \left\{ \sum_{T=t}^{\infty} (\beta\alpha_w)^{T-t} \lambda_T w_T \left(\frac{\pi_{t,T}^w}{\delta_{t,T}^k} \right)^{\theta_w - 1} \left[rw_t^{k,\star}\right]^{-\theta_w} \ell_T \right\}}.$$

$$\left[rw_t^{k,\star}\right]^{1+\omega\theta_w} = \psi \frac{\theta_w}{(\theta_w - 1)} \frac{\mathrm{num}_{k,t}^{\mathrm{w}}}{\mathrm{den}_{k,t}^{\mathrm{w}}}, \tag{A.11}$$

where

$$\mathrm{num}_{k,t}^{\mathrm{w}} = \exp(\varepsilon_{u,t}) [\ell_t]^{1+\omega} + \beta\alpha_w \mathrm{E}_t \left\{ \begin{array}{c} \left(\frac{1+\pi_{t+1}^w}{\delta_{t,t+1}^k} \right)^{\theta_w(1+\omega)} \times \\ \sum_{T=t+1}^{\infty} (\beta\alpha_w)^{T-t-1} \exp(\varepsilon_{u,T}) \left(\frac{\pi_{t+1,T}^w}{\delta_{t+1,T}^k} \right)^{\theta_w(1+\omega)} [\ell_T]^{1+\omega} \end{array} \right\},$$

or

$$\mathrm{num}_{k,t}^{\mathrm{w}} = \exp(\varepsilon_{u,t}) [\ell_t]^{1+\omega} + \beta\alpha_w \mathrm{E}_t \left\{ \left(\frac{1+\pi_{t+1}^w}{\delta_{t,t+1}^k} \right)^{\theta_w(1+\omega)} \mathrm{num}_{k,t+1}^{\mathrm{w}} \right\}.$$

Similarly,

$$\mathrm{den}_{k,t}^{\mathrm{w}} = \lambda_t w_t \ell_t + \beta\alpha_w \mathrm{E}_t \left\{ \left(\frac{1+\pi_{t+1}^w}{\delta_{t,t+1}^k} \right)^{\theta_w - 1} \mathrm{den}_{k,t+1}^{\mathrm{w}} \right\}. \tag{A.12}$$

$\pi_{t+1}^w = \frac{W_{t+1}}{W_t} - 1$ is wage inflation.[1]

[1] The distortion created by my monopolistic wage setting can be eliminated by subsidizing labor. See Erceg and Levin (2003) for an example.

Indexation-rule selection. Let ξ_t denote the time t total proportion of workers who have selected past-inflation indexation, independently of their last contract negotiation. In short, ξ_t represents the degree of *aggregate indexation* to past inflation in time t. Furthermore, let Σ_t be an information set describing the economy's markets structure, the distribution of stochastic shocks, and the economic policy rules, i.e., the economic regime in period t. Finally, let the vector Ξ collect present and future levels for aggregate indexation and economic regimes, so $\Xi_t = \mathrm{E}_t\left(\left\{\left[\xi_{t+h},\ \Sigma_{t+h}\right]'\right\}_{h=0}^{\infty}\right)$. We can now formalize workers indexation-rule decision as follows: If worker i can re-negotiate his labor contract in time t, he selects the rule that maximizes his conditional expected utility, i.e.

$$\delta_{i,t}^{\star}\left(\Xi_t\right) \in \underset{\delta_i \in \left\{\delta^{trend}, \delta^{past}\right\}}{\arg\max} \ \mathbb{W}_{i,t}\left(\delta_i, \Xi_t\right) \text{ subject to } \wp\left(\Xi_t\right), \tag{A.13}$$

where

$$\mathbb{W}_{i,t}\left(\delta_i, \Xi_t\right) = \mathrm{E}_t\left(\sum_{T=t}^{\infty}\left(\beta\alpha_w\right)^{T-t}\ \mathcal{U}\left(c_T\left(\xi_T, \Sigma_T\right),\ \ell_{i,T}\left(\delta_i, \xi_T, \Sigma_T\right)\right)\right). \tag{A.14}$$

The term $\wp\left(\Xi_t\right)$ is a system of equations that summarizes all relevant general-equilibrium constraints that determine the allocation of the economy. Notice that $\mathbb{W}_{i,t}$ is constrained by the expected duration of the labor contract (as the effective discount factor is $\beta\alpha_w$). In addition, individual consumption equals the aggregate level, and it does not depend on the individual indexation choice δ_i. It does, however, depend on aggregate indexation ξ_t and the current economic regime Σ_t because, first, all households have the same wealth at the beginning of each period; and second, because a worker's individual indexation-rule choice has a negligible effect on aggregate indexation, given the worker's small size with respect to the aggregate. Thus, for worker i, ξ_t, Σ_t, and c_t are given, and as a consequence, the worker selects the indexation rule δ_i that minimizes this individual expected labor disutility, given by $\Omega\left(\delta_i, \Xi_t\right)$. In formal terms, $\delta_{i,t}^{\star}\left(\Xi_t\right)$ also satisfies the problem

$$\delta_{i,t}^{\star}\left(\Xi_t\right) \in \underset{\delta_i \in \left\{\delta^{trend}, \delta^{past}\right\}}{\arg\min} \ \Omega\left(\delta_i, \Xi_t\right), \text{ subject to } \wp\left(\Xi_t\right),$$

where

$$\Omega\left(\delta_i, \Xi_t\right) = \frac{\psi}{1+\omega}\mathrm{E}_t\left(\sum_{T=t}^{\infty}\left(\beta\alpha_w\right)^{T-t}\left[\ell_{i,T}\left(\delta_i, \xi_T, \Sigma_T\right)\right]^{1+\omega}\right). \tag{A.15}$$

Labor market aggregation. The degree of aggregate indexation ξ_t is determined as follows: each period, only a fraction $1-\alpha_w$ of workers re-optimize their wages. Let χ_t denote the time t proportion of workers from subset $(1-\alpha_w)$ that selects δ^{past}. Accordingly, ξ_t is given by

$$\xi_t = \left(1-\alpha_w\right)\sum_{h=0}^{\infty}\chi_{t-h}\left(\alpha_w\right)^h, \tag{A.16}$$

which recursively can be written as $\xi_t = \left(1-\alpha_w\right)\chi_t + \alpha_w\xi_{t-1}$. Without loss of generality, assume that workers are sorted according to the indexation rule they have chosen, so workers in the interval $i \in I_t^{past} = [0, \xi_t]$ use δ^{past}, while those in the interval $i \in I_t^{trend} = [\xi_t, 1]$ use δ^{trend}. Measures of wage dispersion for each of the two sectors can be computed by adding up total hours worked, which are determined by the set of labor-specific demands. So, we have that $\int_{i\in I_t^k}\ell_{i,t}\mathrm{d}i = \ell_t \mathrm{disp}_{k,t}^{\mathrm{w}}$, where

$\text{disp}^{\text{w}}_{k,t} = \int_{i\in I^k_t} \left(\frac{W_{i,t}}{W_t}\right)^{-\theta_w} \text{d}i$. Recursive expressions for the wage dispersion measures are given by

$$\text{disp}^{\text{w}}_{k,t} = (1-\alpha_w)\,\tilde{\chi}^k_t \left(rw^{k,\star}_t\right)^{-\theta_w} + \alpha_w \left(\frac{1+\pi^w_t}{\delta^k_{t-1,t}}\right)^{\theta_w} \text{disp}^{\text{w}}_{k,t-1}, \tag{A.17}$$

$$\text{where } \tilde{\chi}^k_t = \begin{cases} \chi_t & \text{if } k = past \\ 1-\chi_t & \text{if } k = trend \end{cases}. \tag{A.18}$$

Finally, given the Dixit-Stiglitz technology of the labor intermediary, the aggregate wage level is given by $W^{1-\theta_w}_t = \int_0^1 W^{1-\theta_w}_{i,t}\text{d}i$. This expression can be rewritten in terms of the sum relative wages within each indexation-rule sector, which are given by $\tilde{w}^k_t \equiv \int_{i\in I^k_t}\left(\frac{W_{i,t}}{W_t}\right)^{1-\theta_w} \text{d}i$. Thus, it follows that

$$1 = \tilde{w}^1_t + \tilde{w}^2_t, \text{ and} \tag{A.19}$$

$$\tilde{w}^1_t = (1-\alpha_w)\,\chi_t \left[rw^{1,\star}_t\right]^{1-\theta_w} + \alpha_w \left(\frac{1+\pi^w_t}{\delta^1_{t-1,t}}\right)^{\theta_w-1} \tilde{w}^1_{t-1}, \tag{A.20}$$

$$\tilde{w}^2_t = (1-\alpha_w)\,(1-\chi_t) \left[rw^{2,\star}_t\right]^{1-\theta_w} + \alpha_w \left(\frac{1+\pi^w_t}{\delta^2_{t-1,t}}\right)^{\theta_w-1} \tilde{w}^2_{t-1}. \tag{A.21}$$

Notice that these weights may change over time due to variations in rw^k_t and χ_t. The recursive law of motion of $\tilde{w}^k_t$ is given by

$$\tilde{w}^k_t = (1-\alpha_w)\,\tilde{\chi}^k_t \left[rw^{k,\star}_t\right]^{1-\theta_{w,t}} + \alpha_w \left(\frac{1+\pi^w_t}{\delta^k_{t-1,t}}\right)^{\theta_w-1} \tilde{w}^k_{t-1}. \tag{A.22}$$

The rest of the model is quite standard, so we describe it briefly.

A.2 Firms

A.2.1 Final good producer

A perfectly competitive firm produces a homogenous good, y_t, by combining a continuum of intermediate goods, $y_{j,t}$ for $j \in [0,1]$, using a CES production function

$$y_t = \left(\int_0^1 y_{j,t}^{\frac{\theta_{p,t}-1}{\theta_{p,t}}}\text{d}j\right)^{\frac{\theta_{p,t}}{\theta_{p,t}\;-1}},$$

where $\theta_{p,t} = \theta_{p,t-1} \equiv \exp(\varepsilon_{p,t}) > 1$ is the price elasticity of demand for intermediate good j and $\varepsilon_{p,t}$ is a stochastic disturbance with mean zero. Profit maximization yields the typical set of input-specific demand functions

$$y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\theta_{p,t}} y_t \;\forall j.$$

The aggregate price level compatible with a zero-profit condition and the particular shape of the production function is:

$$P_t = \left(\int_0^1 P^{1-\theta_{p,t}}_{j,t}\text{d}j\right)^{\frac{1}{1-\theta_{p,t}}},$$

where $P_{j,t}$ denotes the price of the type$-j$ intermediate good.

A.2.2 Intermediate-good firms

Each intermediate good $y_{j,t}$ is produced by a single monopolistic firm using the a decreasing returns to scale of the form

$$y_{j,t} = A\exp\left(z_t\right) n_{j,t}^{\phi^{-1}},$$

where $0 < \phi^{-1} < 1$, the rest of variable definitions remain the same. The real cost function is denoted by $S(y_{j,t}) = w_t \left[y_{j,t}/\left(A\exp\left(z_t\right)\right)\right]^{\phi}$ so the marginal cost is $s_{j,t} = \phi w_t \left[y_{j,t}\right]^{\phi-1}\left[A\exp\left(z_t\right)\right]^{-\phi}$.

Profit maximization now reads:

$$P_{j,t}^{\star} \in \arg\max_{P_{j,t}} \hat{\mathrm{E}}_t \sum_{T=t}^{\infty} (\beta\alpha_p)^{T-t} \varphi_{t,T}\left[\frac{\delta_{t,T}^{p} P_{j,t}}{P_T} y_{j,t,T} - S\left(y_{j,t,T}\right)\right],$$

$$\text{subject to } y_{j,t,T} = \left(\frac{\delta_{t,T}^{p} P_{j,t}}{P_T}\right)^{-\theta_{p,t}} y_T$$

The first order condition, in terms of the relative price, is (derivation is simpler by noticing that $y_{j,t,T} = \left(\frac{\delta_{t,T}^{p}}{\pi_{t,T}} p_{j,t}\right)^{-\theta_{p,T}} y_T$ and $\frac{\partial y_{j,t,T}}{\partial p_{j,t}} = -\theta_{p,T}\frac{y_{j,t,T}}{p_{j,t}}$):

$$p_{j,t}^{\star} \equiv \frac{P_{j,t}^{\star}}{P_t} = \frac{\hat{\mathrm{E}}_t \sum_{T=t}^{\infty} (\beta\alpha_p)^{T-t} \theta_{p,t} \varphi_{t,T} s_{j,t,T} y_{j,t,T}}{\hat{\mathrm{E}}_t \sum_{T=t}^{\infty} (\beta\alpha_p)^{T-t} \left(\theta_{p,t}-1\right) \varphi_{t,T} \left(\delta_{t,T}^{p}/\pi_{t,T}\right) y_{j,t,T}}$$

where $\pi_{t,T} \equiv P_T/P_t$. Replacing the input-specific demand into the marginal cost, the latter becomes

$$s_{j,t,T} = \phi \frac{w_t \left[y_T\right]^{\phi-1}}{A^{\phi}\exp\left(\phi z_t\right)} \left(\frac{\delta_{t,T}^{p}}{\pi_{t,T}} p_{j,t}^{\star}\right)^{-\theta_{p,t}(\phi-1)}.$$

Substituting into the optimal relative price yields (j sub-index is dropped)

$$p_t^{\star} \equiv \frac{P_{j,t}^{\star}}{P_t} = \frac{\hat{\mathrm{E}}_t \sum_{T=t}^{\infty} (\beta\alpha_p)^{T-t} \phi\theta_{p,T} \varphi_{t,T} w_T \left(\frac{y_T}{A\exp(z_T)}\right)^{\phi} \left(\frac{\pi_{t,T}}{\delta_{t,T}^{p}}\right)^{\theta_{p,T}\phi} \left[p_t^{\star}\right]^{-\theta_{p,T}\phi}}{\hat{\mathrm{E}}_t \sum_{T=t}^{\infty} (\beta\alpha_p)^{T-t} \left(\theta_{p,T}-1\right) \varphi_{t,T} \left(\frac{\pi_{t,T}}{\delta_{t,T}^{p}}\right)^{\theta_{p,T}-1} \left[p_t^{\star}\right]^{-\theta_{p,T}} y_T}, \text{ or}$$

$$\left[p_t^{\star}\right]^{1+\theta_{p,t}(\phi-1)} = \frac{\mathrm{num}_t^{\mathrm{p}}}{\mathrm{den}_t^{\mathrm{p}}},$$

where

$$\mathrm{num}_t^{\mathrm{p}} = \phi\theta_{p,t} w_t \left(\frac{y_t}{A\exp\left(z_t\right)}\right)^{\phi} + \beta\alpha_p \hat{\mathrm{E}}_t \left\{\frac{\lambda_{t+1}}{\lambda_t}\left(\frac{1+\pi_{t+1}}{\delta_{t,t+1}^{p}}\right)^{\theta_{p,t}\phi} \mathrm{num}_{t+1}^{\mathrm{p}}\right\},$$

$$\mathrm{den}_t^{\mathrm{p}} = \left(\theta_{p,t}-1\right) y_t + \beta\alpha_p \hat{\mathrm{E}}_t \left\{\frac{\lambda_{t+1}}{\lambda_t}\left(\frac{1+\pi_{t+1}}{\delta_{t,t+1}^{p}}\right)^{\theta_{p,t}-1} \mathrm{den}_{t+1}^{\mathrm{p}}\right\}$$

Price dispersion and labor demand Since the aggregate price level equals $P_t = \left(\int_0^1 P_{j,t}^{1-\theta_{p,t}} \mathrm{d}j\right)^{\frac{1}{1-\theta_{p,t}}}$, it follows that

$$1 = (1-\alpha_p)\left(p_t^{\star}\right)^{1-\theta_{p,t}} + \alpha_p \left(\frac{1+\pi_t}{\delta_{t-1,t}^p}\right)^{\theta_{p,t}-1}.$$

The aggregate labor demand is (using the definition of the production function and the input-specific demand)

$$\int_0^1 n_{j,t}\mathrm{d}j = \left(\frac{y_t}{A\exp(z_t)}\right)^{\phi} \mathrm{disp}_t^{\mathrm{p}},$$

where $\mathrm{disp}_t^{\mathrm{p}} = \int_0^1 \left(\frac{P_{j,t}}{P_t}\right)^{-\theta_{p,t}\phi} \mathrm{d}j$. In recursive form, this equation becomes:

$$\mathrm{disp}_t^{\mathrm{p}} = (1-\alpha_p)\left[p_t^{\star}\right]^{-\theta_{p,t}\phi} + \alpha_p \left(\frac{1+\pi_t}{\delta_{t-1,t}^p}\right)^{\theta_{p,t}\phi} \mathrm{disp}_{t-1}^{\mathrm{p}}.$$

Labor market and goods market equilibrium See Erceg *et al.* (2000) and Benigno and Woodford (2005). Aggregate supplied hours equal aggregate labor composite times a the wage dispersion distortion,

$$\int_0^1 \ell_{i,t}\mathrm{d}i = \ell_t \sum_{k\in\{1,2\}} \mathrm{disp}_{k,t}^{\mathrm{w}}.$$

The labor composite is partitioned or distributed among all intermediate firms, according to their labor-specific demand, so

$$\ell_t = \int_0^1 n_{j,t}\mathrm{d}j.$$

Finally, aggregating the labor specific demand across firms yields an output composite times the price dispersion distortion,

$$\ell_t = \left(\frac{y_t}{A\exp(z_t)}\right)^{\phi} \mathrm{disp}_t^{\mathrm{p}}.$$

A.3 Government and Monetary Policy

The government budget constraint is balanced at all times (or there is no role, for debt in the model), and government spending obeys

$$g_t = g\exp(\varepsilon_{g,t})\, y_t \tag{A.23}$$

where $0 < g\exp(\varepsilon_{g,t}) < 1$ is the public-spending-to-GDP ratio and $\varepsilon_{g,t}$ is a stochastic disturbance with mean zero.

In the spirit of Cogley *et al.* (2010), the central bank chooses the gross nominal interest rate according to the rule

$$R_t = \left[R_{t-1}\right]^{\rho_R}\left[R_t^*\right]^{1-\rho_R}\left[\frac{1+\pi_t}{1+\pi_t^*}\right]^{a_\pi(1-\rho_R)}\left[\frac{y_t}{y_{t-1}}\right]^{a_{\Delta y}(1-\rho_R)} \exp(\varepsilon_{m,t}) \text{ with } R_t^* = \frac{1+\pi_{t+1}^*}{\beta} \tag{A.24}$$

A.4 Equilibrium

The resource constraint is given by

$$y_t = c_t + g_t, \tag{A.25}$$

The equilibrium of this economy is characterized by a set of prices $\{P_t, P_{j,t}, W_t, W_{i,t}, R_t\}$ and a set of quantities $\{y_t, g_t, c_{i,t}, b_{i,t}, n_{j,t}, \ell_t, \ell_{i,t}\}$, for all i and j, such that all markets clear at all times, and agents act consistently according to the maximization of their utility and profits. Notice that in equilibrium $c_t = \int_0^1 c_{i,t}\mathrm{d}i$, $\int_0^1 \ell_{i,t}\mathrm{d}i = \int_0^1 n_{j,t}\mathrm{d}j$, $\int_0^1 b_{i,t}\mathrm{d}i = 0$.

A.5 Summary of non-linear equations

A.5.1 All equations

`Consumption and savings`

$$1 = \beta \hat{\mathrm{E}}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \frac{R_t \exp(\varepsilon_{b,t})}{1+\pi_{t+1}} \right\}, \tag{A.26}$$

$$\exp(\varepsilon_{u,t}) \left(c_t - \gamma^h c_{t-1}\right)^{-\sigma} - \beta\gamma^h \hat{\mathrm{E}}_t \left\{ \exp(\varepsilon_{u,t+1}) \left(c_{t+1} - \gamma^h c_t\right)^{-\sigma} \right\} = \lambda_t. \tag{A.27}$$

`Wage-setting`

$$\left[rw_t^{\star,k}\right]^{1+\omega\theta_w} = \psi \exp(\varepsilon_{u,t}) \frac{\mathrm{num}^{\mathrm{w}}_{k,t}}{\mathrm{den}^{\mathrm{w}}_{k,t}}, \tag{A.28}$$

$$\mathrm{num}^{\mathrm{w}}_{k,t} = \theta_w \left[\ell_t\right]^{1+\omega} + \beta\alpha_w \hat{\mathrm{E}}_t \left\{ \left(\frac{1+\pi^w_{t+1}}{\delta^k_{t,t+1}}\right)^{\theta_w(1+\omega)} \mathrm{num}^{\mathrm{w}}_{k,t+1} \right\} \tag{A.29}$$

$$\mathrm{den}^{\mathrm{w}}_{k,t} = (\theta_w - 1)\lambda_t w_t \ell_t + \beta\alpha_w \hat{\mathrm{E}}_t \left\{ \left(\frac{1+\pi^w_{t+1}}{\delta^k_{t,t+1}}\right)^{\theta_w - 1} \mathrm{den}^{\mathrm{w}}_{k,t+1} \right\}. \tag{A.30}$$

$$1 = \tilde{w}_t^1 + \tilde{w}_t^2 \tag{A.31}$$

$$\tilde{w}_t^1 = (1-\alpha_w)\chi_t \left[rw_t^{\star,1}\right]^{1-\theta_w} + \alpha_w \left(\frac{1+\pi^w_t}{\delta^1_{t-1,t}}\right)^{\theta_w - 1} \tilde{w}^1_{t-1}, \tag{A.32}$$

$$\tilde{w}_t^2 = (1-\alpha_w)(1-\chi_t) \left[rw_t^{\star,2}\right]^{1-\theta_w} + \alpha_w \left(\frac{1+\pi^w_t}{\delta^2_{t-1,t}}\right)^{\theta_w - 1} \tilde{w}^2_{t-1}. \tag{A.33}$$

$$\mathrm{disp}^{\mathrm{w}}_{1,t} = (1-\alpha_w)\chi_t \left(rw_t^{\star,1}\right)^{-\theta_w} + \alpha_w \left(\frac{1+\pi^w_t}{\delta^1_{t-1,t}}\right)^{\theta_w} \mathrm{disp}^{\mathrm{w}}_{1,t-1}, \tag{A.34}$$

$$\mathrm{disp}^{\mathrm{w}}_{2,t} = (1-\alpha_w)(1-\chi_t) \left(rw_t^{\star,2}\right)^{-\theta_w} + \alpha_w \left(\frac{1+\pi^w_t}{\delta^2_{t-1,t}}\right)^{\theta w.t} \mathrm{disp}^{\mathrm{w}}_{2,t-1}. \tag{A.35}$$

$$\delta^1_{t-1,t} = 1 + \pi_{t-1} \qquad \text{and} \qquad \delta^2_{t-1,t} = 1 + \pi^\star_t, \tag{A.36}$$

`Price-setting`

$$[p_t^\star]^{1+\theta_{p,t}(\phi-1)} = \frac{\text{num}_t^\text{p}}{\text{den}_t^\text{p}}, \tag{A.37}$$

$$\text{num}_t^\text{p} = \phi\theta_{p,t} w_t \left(\frac{y_t}{A\exp(z_t)}\right)^\phi + \beta\alpha_p \hat{\text{E}}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{1+\pi_{t+1}}{\delta_{t,T}^p}\right)^{\theta_{p,t}\phi} \text{num}_{t+1}^\text{p} \right\}, \tag{A.38}$$

$$\text{den}_t^\text{p} = (\theta_{p,t}-1)\, y_t + \beta\alpha_p \hat{\text{E}}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{1+\pi_{t+1}}{\delta_{t,T}^p}\right)^{\theta_{p,t}-1} \text{den}_{t+1}^\text{p} \right\} \tag{A.39}$$

$$1 = (1-\alpha_p)\, [p_t^\star]^{1-\theta_{p,t}} + \alpha_p \left(\frac{1+\pi_t}{\delta_{t-1,t}^p}\right)^{\theta_{p,t}-1}, \tag{A.40}$$

$$\text{disp}_t^\text{p} = (1-\alpha_p)\,(p_t^\star)^{-\theta_{p,t}\phi} + \alpha_p \left(\frac{1+\pi_t}{\delta_{t-1,t}^p}\right)^{\theta_{p,t}\phi} \text{disp}_{t-1}^\text{p}. \tag{A.41}$$

`Labor market`

$$1+\pi_t^w = \frac{w_t}{w_{t-1}}(1+\pi_t) \tag{A.42}$$

$$\ell_t = \left(\frac{y_t}{A\exp(z_t)}\right)^\phi \text{disp}_{k,t}^\text{p} \tag{A.43}$$

$$\xi_t = (1-\alpha_w)\,\chi_t + \alpha_w \xi_{t-1} \tag{A.44}$$

`Policy and equilibrium`

$$g_t = g\exp(\varepsilon_{g,t})\, y_t \tag{A.45}$$

$$R_t = [R_{t-1}]^{\rho_R}\, [R_t^*]^{1-\rho_R} \left[\frac{1+\pi_t}{1+\pi_t^*}\right]^{a_\pi(1-\rho_R)} \left[\frac{y_t}{y_{t-1}}\right]^{a_{\Delta y}(1-\rho_R)} \exp(\varepsilon_{m,t}) \text{ with } R_t^* = \frac{1+\pi_{t+1}^*}{\beta} \tag{A.46}$$

$$y_t = c_t + g_t \tag{A.47}$$

`Technology and shocks`

$$\Delta z_t = \rho_z \Delta z_{t-1} + \eta_{z,t}. \tag{A.48}$$

$$(1+\pi_t^*) = (1+\pi^*)\exp(\varepsilon_{\pi,t})$$

$$\varepsilon_{x,t} = \rho_x \varepsilon_{x,t-1} + \eta_{z,t} \text{ for } x \in \{b, w, p, g, \pi, \ell\} \tag{A.49}$$

A.5.2 Detrended economy

The model has an stochastic trend due to the technology shock process. All real trending variables are divided by $\exp(z_t)$.

$$y_t^s \equiv \frac{y_t}{\exp(z_t)}, \quad c_t^s \equiv \frac{c_t}{\exp(z_t)}, \quad g_t^s \equiv \frac{g_t}{\exp(z_t)}, \quad w_t^s \equiv \frac{w_t}{\exp(z_t)}, \quad \lambda_t^s \equiv \lambda_t \exp(z_t).$$

`Consumption and savings`

$$\lambda_t^s / \exp(z_t) = \beta \hat{\mathrm{E}}_t \left\{ \lambda_{t+1}^s / \exp(z_{t+1}) \frac{R_t \exp(\varepsilon_{b,T})}{1+\pi_{t+1}} \right\}, \tag{A.50}$$

$$\lambda_t^s / \exp(z_t) = \mathcal{U}_{c,t} - \beta\gamma^h \hat{\mathrm{E}}_t \mathcal{U}_{c,t+1} \tag{A.51}$$

$$\mathcal{U}_{c,t} = \left(c_t^s \exp(z_t) - \gamma^h c_{t-1}^s \exp(z_{t-1}) \right)^{-\sigma} \tag{A.52}$$

`Wage-setting`

$$\left[rw_t^{\star,k} \right]^{1+\omega\theta_w} = \psi \frac{\mathrm{num}_{k,t}^{\mathrm{w}}}{\mathrm{den}_{k,t}^{\mathrm{w}}}, \tag{A.53}$$

$$\mathrm{num}_{k,t}^{\mathrm{w}} = \theta_w [\ell_t]^{1+\omega} + \beta\alpha_w \hat{\mathrm{E}}_t \left\{ \left(\frac{1+\pi_{t+1}^w}{\delta_{t,t+1}^k} \right)^{\theta_w(1+\omega)} \mathrm{num}_{k,t+1}^{\mathrm{w}} \right\} \tag{A.54}$$

$$\mathrm{den}_{k,t}^{\mathrm{w}} = (\theta_w - 1) \lambda_t^s w_t^s \ell_t + \beta\alpha_w \hat{\mathrm{E}}_t \left\{ \left(\frac{1+\pi_{t+1}^w}{\delta_{t,t+1}^k} \right)^{\theta_w - 1} \mathrm{den}_{k,t+1}^{\mathrm{w}} \right\}. \tag{A.55}$$

$$1 = \tilde{w}_t^1 + \tilde{w}_t^2 \tag{A.56}$$

$$\tilde{w}_t^1 = (1-\alpha_w) \chi_t \left[rw_t^{\star,1} \right]^{1-\theta_w} + \alpha_w \left(\frac{1+\pi_t^w}{\delta_{t-1,t}^1} \right)^{\theta_w - 1} \tilde{w}_{t-1}^1, \tag{A.57}$$

$$\tilde{w}_t^2 = (1-\alpha_w)(1-\chi_t) \left[rw_t^{\star,2} \right]^{1-\theta_w} + \alpha_w \left(\frac{1+\pi_t^w}{\delta_{t-1,t}^2} \right)^{\theta_w - 1} \tilde{w}_{t-1}^2. \tag{A.58}$$

$$\mathrm{disp}_{1,t}^{\mathrm{w}} = (1-\alpha_w) \chi_t \left(rw_t^{\star,1} \right)^{-\theta_w} + \alpha_w \left(\frac{1+\pi_t^w}{\delta_{t-1,t}^1} \right)^{\theta_w} \mathrm{disp}_{1,t-1}^{\mathrm{w}}, \tag{A.59}$$

$$\mathrm{disp}_{2,t}^{\mathrm{w}} = (1-\alpha_w)(1-\chi_t) \left(rw_t^{\star,2} \right)^{-\theta_w} + \alpha_w \left(\frac{1+\pi_t^w}{\delta_{t-1,t}^2} \right)^{\theta_w} \mathrm{disp}_{2,t-1}^{\mathrm{w}}. \tag{A.60}$$

$$\delta_{t-1,t}^1 = 1 + \pi_{t-1} \qquad \text{and} \qquad \delta_{t-1,t}^2 = 1 + \pi_t^\star, \tag{A.61}$$

Price-setting

$$[p_t^\star]^{1+\theta_{p,t}(\phi-1)} = \frac{\mathrm{num}_t^{\mathrm{p}}}{\mathrm{den}_t^{\mathrm{p}}}, \tag{A.62}$$

$$\mathrm{num}_t^{\mathrm{p}} = \phi\theta_{p,t} w_t^s \exp(z_t) \left(\frac{y_t^s}{A}\right)^{\phi} + \beta\alpha_p \hat{\mathrm{E}}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{1+\pi_{t+1}}{\delta_{t,t+1}^p}\right)^{\theta_{p,t}\phi} \mathrm{num}_{t+1}^{\mathrm{p}} \right\}, \tag{A.63}$$

$$\mathrm{den}_t^{\mathrm{p}} = (\theta_{p,t}-1)\, y_t^s \exp(z_t) + \beta\alpha_p \hat{\mathrm{E}}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{1+\pi_{t+1}}{\delta_{t,t+1}^p}\right)^{\theta_{p,t}-1} \mathrm{den}_{t+1}^{\mathrm{p}} \right\} \tag{A.64}$$

$$1 = (1-\alpha_p)\,[p_t^\star]^{1-\theta_{p,t}} + \alpha_p \left(\frac{1+\pi_t}{\delta_{t-1,t}^p}\right)^{\theta_{p,t}-1}, \tag{A.65}$$

$$\mathrm{disp}_t^{\mathrm{p}} = (1-\alpha_p)\,(p_t^\star)^{-\theta_{p,t}\phi} + \alpha_p \left(\frac{1+\pi_t}{\delta_{t-1,t}^p}\right)^{\theta_{p,t}\phi} \mathrm{disp}_{t-1}^{\mathrm{p}}. \tag{A.66}$$

Labor market

$$1+\pi_t^w = \frac{w_t^s \exp(z_t)}{w_{t-1}^s \exp(z_{t-1})}(1+\pi_t) \tag{A.67}$$

$$\ell_t = \left(\frac{y_t}{A\exp(z_t)}\right)^{\phi} \mathrm{disp}_{k,t}^{\mathrm{p}} \tag{A.68}$$

$$\wp_{1,t}^w = (1-\alpha_w)\,\chi_t + \alpha_w \wp_{1,t-1}^w \tag{A.69}$$

$$\wp_{2,t}^w = (1-\alpha_w)\,(1-\chi_t) + \alpha_w \wp_{2,t-1}^w. \tag{A.70}$$

Policy and equilibrium

$$g_t^s = g\exp(\varepsilon_{g,t})\, y_t^s \tag{A.71}$$

$$R_t = [R_{t-1}]^{\rho_R}\,[R_t^*]^{1-\rho_R} \left[\frac{1+\pi_t}{1+\pi_t^*}\right]^{a_\pi(1-\rho_R)} \left[\frac{y_t^s\exp(z_t)}{y_{t-1}^s\exp(z_{t-1})}\right]^{a_{\Delta y}(1-\rho_R)} \exp(\varepsilon_{m,t}) \text{ with } R_t^* = \frac{1+\pi_{t+1}^*}{\beta} \tag{A.72}$$

$$y_t^s = c_t^s + g_t^s \tag{A.73}$$

Technology and shocks

$$\Delta z_t = \rho_z \Delta z_{t-1} + \eta_{z,t}. \tag{A.74}$$

$$(1+\pi_t^*) = (1+\pi^*)\exp(\varepsilon_{\pi,t}) \tag{A.75}$$

$$\varepsilon_{x,t} = \rho_x \varepsilon_{x,t-1} + \eta_{z,t} \text{ for } x \in \{b,w,p,g,m\} \tag{A.76}$$

A.5.3 Deterministic steady state

`Consumption and savings`

$$R = \frac{1+\pi^*}{\beta} \tag{A.77}$$

$$c^s \left(1-\gamma^h\right)^{-\sigma} \left(1-\beta\gamma^h\right) = \lambda \tag{A.78}$$

`Wage-setting`

$$\text{den}^{\text{w}} = \psi \mu_w \text{num}^{\text{w}}, \text{ with } \mu_w = \frac{\theta_w}{\theta_w - 1}. \tag{A.79}$$

$$\text{num}^{\text{w}} = \frac{[\ell]^{1+\omega}}{1-\beta\alpha_w} \tag{A.80}$$

$$\text{den}^{\text{w}} = \frac{\lambda w \ell}{1-\beta\alpha_w} \tag{A.81}$$

$$\psi = \frac{1}{\mu_w}\frac{\lambda w}{\ell^{\omega}} \tag{A.82}$$

$$1 = \tilde{w}_t^1 + \tilde{w}_t^2 \tag{A.83}$$

$$\tilde{w}^1 = \chi \tag{A.84}$$

$$\tilde{w}_t^2 = (1-\chi) \tag{A.85}$$

$$\text{disp}_1^{\text{w}} = \chi \tag{A.86}$$

$$\text{disp}_2^{\text{w}} = 1-\chi \tag{A.87}$$

`Price-setting`

$$p^\star = 1 \tag{A.88}$$

$$\text{num}^{\text{p}} = \frac{1}{1-\beta\alpha_p}\phi\theta_p w^s \left(\frac{1}{A}\right)^{\phi}, \tag{A.89}$$

$$\text{den}^{\text{p}} = \frac{1}{1-\beta\alpha_p}\left(\theta_p - 1\right) \tag{A.90}$$

$$\text{disp}^{\text{p}} = 1 \tag{A.91}$$

`Firms profits`

$$\text{Profits} = \frac{1}{1-\beta\alpha_p} y \left[1 - \frac{w}{A^{\phi}}\right]$$

`Labor market`

$$A = \frac{y^s}{\ell} \tag{A.92}$$

$$\wp_1 = \chi \tag{A.93}$$

$$\wp_2 = 1 - \chi \tag{A.94}$$

$$\chi = \xi^\star \tag{A.95}$$

$\xi^\star$ the one at which workers have no incentives to change indexation rule.

`Policy and resource constraint`

$$g = \frac{g^s}{y^s} \tag{A.96}$$

$$R = \frac{1+\pi^*}{\beta} \tag{A.97}$$

$$1 - g^s = \frac{c^s}{y^s} \tag{A.98}$$

A.5.4 Flexible price/wage economy

`Consumption and savings`

$$\lambda_t^{s,f} / \exp(z_t) = \beta \hat{\mathrm{E}}_t \left\{ \lambda_{t+1}^{s,f} / \exp(z_{t+1})\, r_t^f \exp(\varepsilon_{b,T}) \right\}, \tag{A.99}$$

$$\lambda_t^s / \exp(z_t) \quad = \quad u_{c,t}^f - \beta\gamma^h \hat{\mathrm{E}}_t u_{c,t+1}^f \tag{A.100}$$

$$u_{c,t}^f \quad = \quad \left(c_t^{s,f} \exp(z_t) - \gamma^h c_{t-1}^{s,f} \exp(z_{t-1}) \right)^{-\sigma} \tag{A.101}$$

`Wage-setting`

$$(\theta_w - 1)\, \lambda_t^{s,f} w_t^{s,f} = \psi \theta_w \left[\ell_t^f \right]^\omega \tag{A.102}$$

`Price-setting`

$$1 = \frac{\phi \theta_{p,t} w_t^{s,f} \left(y_t^{s,f} \right)^{\phi-1} \left(\frac{1}{A} \right)^\phi}{(\theta_{p,t} - 1)}, \tag{A.103}$$

`Labor market`

$$\ell_t^f = \left(\frac{y_t^{s,f}}{A} \right)^\phi \tag{A.104}$$

`Policy and resource constraint`

$$g_t^{s,f} = g \exp(\varepsilon_{g,t})\, y_t^{s,f} \tag{A.105}$$

$$y_t^{s,f} = c_t^{s,f} + g_t^{s,f} \tag{A.106}$$

B Deterministic *vs.* Stochastic Steady State

This section explains the differences between the deterministic and stochastic steady states. We can verify that the model has been correctly solved by comparing the suitable analytical expressions with Dynare output. We show that this is particularly important for forward-looking variables. We also deliver the analytical expressions for wage dispersion that are used in the main document.

B.1 Main differences.

Consider the household first order condition for consumption (stochastic shocks have been removed to simplify the exposition):

$$\lambda_t = \left(c_t - \gamma^h c_{t-1}\right)^{-\sigma} - \beta\gamma^h \mathrm{E}_t \left\{ \left(c_{t+1} - \gamma^h c_t\right)^{-\sigma} \right\},$$

In the absence of external habits ($\gamma^h = 0$), this condition establishes a contemporaneous relation between λ and c. But the existence of habits makes this relationship inter-temporal, with a forward-looking term for consumption.

Define the steady state level of variable x_t as its unconditional expectation, so $x_{ss} \equiv \mathrm{E}\left(x_t\right),$ which, by definition, is time invariant. Now, suppose that all variables in the economy equal their steady state level for periods t and $t-1$, and that all stochastic shock processes equal zero. Applying the unconditional expectation operator to the last expression and re-arranging, we have

$$\lambda_{ss} = \left(c_{ss}\left(1-\gamma^h\right)\right)^{-\sigma} - \beta\gamma^h \mathrm{E}\left\{ \left(c_{t+1} - \gamma^h c_{ss}\right)^{-\sigma} \right\}, \tag{B.1}$$

The expectation term gauges the impact that future shocks will have on future variables.[2] If there is at least one shock with a strictly positive probability of realization (i.e., its variance is positive), the expectation term takes it into account and so $\mathrm{E}\left\{ \left(c_{t+1} - \gamma^h c_{ss}\right)^{-\sigma} \right\} \neq \left(c_{ss}\left(1-\gamma^h\right)\right)^{-\sigma}$. To see why, consider the second order Taylor approximation to the term $\left(c_{t+1} - \gamma^h c_{ss}\right)^{-\sigma}$ around the point c_{ss} :

$$\begin{aligned} \left(c_{t+1} - \gamma^h c_{ss}\right)^{-\sigma} \simeq & \left(c_{ss}\left(1-\gamma^h\right)\right)^{-\sigma} + \left(c_{t+1} - c_{ss}\right)\left(-\sigma\left(c_{ss}\left(1-\gamma^h\right)\right)^{-\sigma-1}\right) \\ & \ldots + \frac{1}{2}\left(c_{t+1} - c_{ss}\right)^2 \left(\sigma\left(\sigma+1\right)\left(c_{ss}\left(1-\gamma^h\right)\right)^{-\sigma-2}\right). \end{aligned}$$

or

$$\left(c_{t+1} - \gamma^h c_{ss}\right)^{-\sigma} \simeq \left(c_{ss}\left(1-\gamma^h\right)\right)^{-\sigma} \left\{ 1 - \frac{\sigma\left(c_{t+1} - c_{ss}\right)}{c_{ss}\left(1-\gamma^h\right)} + \frac{1}{2}\frac{\sigma\left(\sigma+1\right)\left(c_{t+1} - c_{ss}\right)^2}{\left(c_{ss}\left(1-\gamma^h\right)\right)^2} \right\}.$$

It follows that the unconditional expectation of this term is affected by the second moment of the random variable c_{t+1}:

$$\mathrm{E}\left\{ \left(c_{t+1} - \gamma^h c_{ss}\right)^{-\sigma} \right\} \simeq \left(c_{ss}\left(1-\gamma^h\right)\right)^{-\sigma} + \frac{1}{2}\frac{\sigma\left(\sigma+1\right)}{\left(c_{ss}\left(1-\gamma^h\right)\right)^2} \mathrm{var}\left\{c_{t+1}\right\},.$$

[2]The expectation operator does not apply to variables in t and $t-1$ because, once their value is known, as it is assumed in the example, they are no longer random variables.

where $\text{var}\{c_{t+1}\} = \text{E}\left\{(c_{t+1} - c_{ss})^2\right\}$. When any stochastic shock has a positive probability of realization in $t+1$, the variance term $\text{var}\{c_{t+1}\}$ needs to be added to the equation determining λ_{ss}, which, up to the second-order approximation, becomes:

$$\lambda_{ss} \simeq \left(c_{ss}\left(1-\gamma^h\right)\right)^{-\sigma}\left(1-\beta\gamma^h\right) - \beta\gamma^h \frac{\sigma(\sigma+1)}{2\left(c_{ss}\left(1-\gamma^h\right)\right)^2}\text{var}\{c_{t+1}\}. \tag{B.2}$$

In the **deterministic** steady state, as by definition no shocks exist or are expected, the variance term collapses to zero. *It thus follows that in general the stochastic steady state will differ from its deterministic counterpart.*

Finally, notice that in the absence of habits, λ_{ss} would not depend on $\text{var}\{c_{t+1}\}$. In other words, correcting 'variance' terms will appear in all equations that have forward-looking variables, or future values of control variables. In contrast, equations that describe a contemporaneous relation or depend on past variables (e.g., for state variables) will not have correcting variance terms, because no conditional expectations operator (i.e., $\text{E}_t\{\cdot\}$) is present in such equations. Some examples are provided below.

B.2 Correction terms for the labor market equilibrium

Output and other real variables are lower in the stochastic steady state than in the deterministic steady state because the possibility that shocks may happen entails welfare losses that agents would like to prevent. Their decisions are thus affected by the risky character of their environment (see Schmitt-Grohé and Uribe, 2004, 2007) From last subsection, we also know that the decision rules of forward-looking variables are affected by their expected future variance. For instance, Amano *et al.* (2007) show that the long-run stochastic mean of inflation differs from trend inflation (i.e., the central bank's target). Their results support the notion that, when computing the stochastic steady state levels of relative wages, aggregate wages, and wage dispersion, we should take into account the long-run differences in the levels of inflation and trend inflation, and the expected variances of forward-looking terms.

Before showing some partial results, define the differences between π, π^w, and $\pi^\star$ as:

$$\begin{aligned} 1+\digamma_t &= \frac{1+\pi_t^w}{\delta_{t-1,t}^1} = \frac{1+\pi_t^w}{1+\pi_{t-1}}, \text{ and} \\ 1+\Delta_t &= \frac{1+\pi_t^w}{\delta_{t-1,t}^2} = \frac{1+\pi_t^w}{1+\pi_t^\star}. \end{aligned}$$

If the expected growth rate of productivity is zero, we have that $\|\Delta_{ss}\| > \|\digamma_{ss}\| \simeq 0$ at the stochastic steady state. This implies that $\|\text{E}(\pi_t^w) - \text{E}(\pi_t^\star)\| > 0$ while $\|\text{E}(\pi_t^w) - \text{E}(\pi_t)\| \simeq 0$.[3]

B.2.1 Relative wages (heuristic).

Optimal relative wages, rw_t^k, at the stochastic steady state are distorted by the variances of the terms $1+\digamma_{t+1}$ and $1+\Delta_{t+1}$ and their covariances with other forward-looking terms. To see why, recall that

$$\left[rw_t^{k,\star}\right]^{1+\omega\theta_w} = \psi\mu_w \frac{\text{num}_{k,t}^{\text{w}}}{\text{den}_{k,t}^{\text{w}}}, \text{ for } k \in \{1,2\} \tag{B.3}$$

[3] These differences hold in Dynare. For a formal proof, one needs to compute the unconditional expectations for inflation and wage inflation, and then to show that they are indeed the same. This exercise is beyond the scope of this appendix.

with

$$\begin{aligned} \text{num}^{\text{w}}_{k,t} &= [\ell_t]^{1+\omega} + \beta\alpha_w \text{E}_t \left\{ (1+\Lambda_{t+1})^{\theta_w(1+\omega)} \text{num}^{\text{w}}_{k,t+1} \right\}, \\ \text{den}^{\text{w}}_{k,t} &= \lambda_t w_t \ell_t + \beta\alpha_w \text{E}_t \left\{ (1+\Lambda_{t+1})^{\theta_w - 1} \text{den}^{\text{w}}_{k,t+1} \right\}, \end{aligned}$$

where $1 + \Lambda_t = 1 + \Gamma_t$ for $\delta^1_{t-1,t}$, and $1 + \Lambda_t = 1 + \Delta_t$ for $\delta^2_{t-1,t}$. A second-order approximation of the term $(1 + \Lambda_{t+1})^{\theta_w(1+\omega)} \text{num}^{\text{w}}_{k,t+1}$ is

$$(1+\Lambda_{t+1})^{\theta_w(1+\omega)} \text{num}^{\text{w}}_{k,t+1} \simeq (1+\Lambda_{ss})^{\theta_w(1+\omega)} \text{num}^{\text{w}}_{k,ss} \left\{ \begin{array}{c} 1 + \theta_w (1+\omega) \hat{\Lambda}_{t+1} + \widehat{\text{num}}^{\text{w}}_{k,t+1} + \\ \frac{1}{2} \left[\begin{array}{c} \theta_w (1+\omega)(\theta_w (1+\omega) - 1) \left(\hat{\Lambda}_{t+1}\right)^2 \\ 2\theta_w (1+\omega) \left(\hat{\Lambda}_{t+1}\right) \left(\widehat{\text{num}}^{\text{w}}_{k,t+1}\right) \end{array} \right] \end{array} \right\},$$

where $\hat{x}_t = \frac{x_t - x_{ss}}{x_{ss}}$. It follows that the unconditional expectation of this term is equal to

$$\text{E}\left\{ (1+\Lambda_{t+1})^{\theta_w(1+\omega)} \text{num}^{\text{w}}_{k,t+1} \right\} \simeq (1+\Lambda_{ss})^{\theta_w(1+\omega)} \text{num}^{\text{w}}_{k,ss} + G_n\left(\Lambda_{t+1}, \text{num}^{\text{w}}_{k,t+1}\right),$$

where $G_n\left(\Lambda_{t+1}, \text{num}^{\text{w}}_{k,t+1}\right) \equiv \frac{1}{2}\left[\theta_w (1+\omega)(\theta_w(1+\omega)-1) \text{var}\{\Lambda_{t+1}\} + 2\theta_w(1+\omega) \text{cov}\left\{\Lambda_{t+1}, \text{num}^{\text{w}}_{k,t+1}\right\}\right]$. The term $\text{num}^{\text{w}}_{k,t}$ at the stochastic steady state is thus equal to:

$$\text{num}^{\text{w}}_{k,ss} = \frac{1}{1-\beta\alpha_w}\left([\ell_{ss}]^{1+\omega} + G_n\left(\Lambda_{t+1}, \text{num}^{\text{w}}_{k,t+1}\right)\right).$$

A similar reasoning leads to an analytical expression for the term $\text{den}^{\text{w}}_{k,ss}$,

$$\text{den}^{\text{w}}_{k,ss} = \frac{1}{1-\beta\alpha_w}\left(\lambda_{ss} w_{ss} \ell_{ss} + G_d\left(\Lambda_{t+1}, \text{den}^{\text{w}}_{k,t+1}\right)\right).$$

Notice that in the deterministic steady state, we have that $\psi\mu_w \text{num}^{\text{w}}_{k,ss} = \text{den}^{\text{w}}_{k,ss}$ while $G_n(\cdot) = G_d(\cdot) = 0$. However, in the stochastic steady state these equalities might not hold and the optimal wage $rw^{k,\star}_{ss}$ might be above or below 1.

B.2.2 Aggregate wages.

Now, to have analytical expressions for the aggregation of wages within each sector, $\tilde{w}^k_t \equiv \int_{i \in IR_{k,t}} \left[\frac{W_{i,t}}{W_t}\right]^{1-\theta_w} \text{d}i$ for $k = 1, 2$, notice that this a state variable and it depends on past and present values of state and control variables. Therefore, no correcting terms related to the expected variance are needed. At the steady state, equations A.20 and A.21 become:

$$\tilde{w}^1_{ss} = (1-\alpha_w)\,\xi \left[rw^{1,\star}_{ss}\right]^{1-\theta_w} + \alpha_w \tilde{w}^1_{ss}, \text{ and} \tag{B.4}$$

$$\tilde{w}^2_{ss} = (1-\alpha_w)(1-\xi)\left[rw^{2,\star}_{ss}\right]^{1-\theta_w} + \alpha_w (1+\Delta)^{\theta_w - 1} \tilde{w}^2_{ss}. \tag{B.5}$$

solving these equations, we have that

$$\tilde{w}^1_{ss} = \xi \times \left(rw^{1,\star}_{ss}\right)^{1-\theta_w}, \text{ and} \tag{B.6}$$

$$\tilde{w}^2_{ss} = \left(\frac{1-\alpha_w}{1-\alpha_w(1+\Delta)^{\theta_w-1}}\right)(1-\xi) \times \left(rw^{2,\star}_{ss}\right)^{1-\theta_w}. \tag{B.7}$$

These analytical expressions hold in Dynare.

B.2.3 Wage dispersion and optimal wages.

The wage dispersion for each sector is also a state variable, so no correcting variance terms are needed here. Following similar steps than for aggregate wages, we have that (has been checked with Dynare)

$$\text{disp}^{\text{w}}_{1,ss} = \xi \times \left(rw^{1,\star}_{ss}\right)^{-\theta_w}, \text{ and} \tag{B.8}$$

$$\text{disp}^{\text{w}}_{2,ss} = \left(\frac{1-\alpha_w}{1-\alpha_w\left(1+\Delta\right)^{\theta_w}}\right)(1-\xi)\times\left(rw^{2,\star}_{ss}\right)^{-\theta_w}. \tag{B.9}$$

Comparing equations B.6 to B.9, it is apparent that

$$\begin{aligned}
\text{disp}^{\text{w}}_{1,ss} &< \tilde{w}^1_{ss} < \xi, \text{ when } rw^{1,\star}_{ss} > 1, \\
\text{disp}^{\text{w}}_{1,ss} &> \tilde{w}^1_{ss} > \xi, \text{ when } rw^{1,\star}_{ss} < 1, \text{ and} \\
\text{disp}^{\text{w}}_{1,ss} &= \tilde{w}^1_{ss} = \xi, \text{ when } rw^{1,\star}_{ss} = 1.
\end{aligned}$$

If Δ is small enough, symmetry between $\tilde{w}^1_{ss}$ and $\tilde{w}^2_{ss}$ (the zero-profit condition of the labor intermediary, $1=\int_0^1\left(\frac{W_{i,t}}{W_t}\right)^{1-\theta_w}\mathrm{d}i$, imposes the symmetry) will lead to similar conclusions between $\text{disp}^{\text{w}}_{2,ss}$, $\tilde{w}^2_{ss}$, and $1-\xi$. In Dynare, we frequently observe that $\parallel \text{disp}^{\text{w}}_{k,ss}-\tilde{\xi} \parallel \geq \parallel \tilde{w}^k_{ss}-\tilde{\xi}\parallel$, for $\tilde{\xi}=\{\xi,1-\xi\}$ for $\#k=1,2$ respectively.

C Welfare

We make a distinction between an individual's conditional welfare, which depends on the duration on his labor contract, and social welfare. We describe the latter first.

C.1 Social welfare

Define $\mathcal{W}_t$ as the un-weighted sum of instantaneous household utilities:

$$\begin{aligned}
\mathcal{W}_t &= \int_i \exp\left(\varepsilon_{u,T}\right)\mathcal{U}(c_{i,t},\ell_{i,t})\mathrm{d}i \\
&= \exp\left(\varepsilon_{u,T}\right)\int_0^1\left(\frac{\left(c_{i,T}-\gamma^h c_{i,T-1}\right)^{1-\sigma}-1}{1-\sigma}-\psi\frac{\ell^{1+\omega}_{i,T}}{1+\omega}\right)\mathrm{d}i \\
&= \exp\left(\varepsilon_{u,T}\right)\frac{\left(c_T-\gamma^h c_{T-1}\right)^{1-\sigma}-1}{1-\sigma}-\psi\exp\left(\varepsilon_{u,T}\right)\int_0^1\frac{\ell^{1+\omega}_{i,T}}{1+\omega}\mathrm{d}i.
\end{aligned} \tag{C.1}$$

The last line follows from the fact that consumption is equal across households under the model assumptions.

Expected social welfare is then defined as:

$$\begin{aligned}
\mathcal{SW}_t &= \mathrm{E}_t\left\{\sum_{T=t}^{\infty}\beta_T^{T-t}\mathcal{W}_T\right\} \\
&= \mathcal{W}_t+\beta\mathrm{E}_t\left\{\mathcal{SW}_{t+1}\right\}.
\end{aligned}$$

At the steady state, social welfare equals its **unconditional expectation**:

$$\begin{aligned}\mathrm{E}\left\{\mathcal{SW}_t\right\} &= \mathrm{E}\left\{\sum_{T=t}^{\infty}\beta^{T-t}\mathcal{W}_T\right\}\\ &= \frac{1}{1-\beta}\mathrm{E}\left\{\mathcal{W}_t\right\},\ \text{since } \mathrm{E}\left\{\mathcal{W}_T\right\}=\mathrm{E}\left\{\mathcal{W}_t\right\}\ \forall T.\end{aligned}$$

Notice that in equation $(C.1)$ aggregate labor disutility can be decomposed into

$$\int_0^1 \frac{\ell_{i,t}^{1+\omega}}{1+\omega}\mathrm{d}i = \int_{i\in IR_{1,t}^w}\frac{\ell_{i,t}^{1+\omega}}{1+\omega}\mathrm{d}i + \int_{i\in IR_{2,t}^w}\frac{\ell_{i,t}^{1+\omega}}{1+\omega}\mathrm{d}i,$$

where

$$\int_{i\in IR_{k,t}^w}\frac{\ell_{i,t}^{1+\omega}}{1+\omega}\mathrm{d}i = \frac{\psi}{1+\omega}\left[\ell_T\right]^{1+\omega}\mathrm{dispV}_{k,t}\mathrm{d}i$$

with $\mathrm{dispV}_{k,t} = \int_{i\in IR_{k,t}^w}\left(\frac{W_{i,t}}{W_t}\right)^{-\theta_w(1+\omega)}\mathrm{d}i$. Dispersion in wages entails welfare costs, just as price dispersion. Following the approach for the $\mathrm{disp}_{k,t}^{\mathrm{w}}$ formulations in A.17, one can show that:

$$\mathrm{dispV}_{1,t} = (1-\alpha_w)\,\chi_t\left(rw_t^{1,\star}\right)^{-\theta_w(1+\omega)} + \alpha_w\left(\frac{1+\pi_t^w}{\delta_{t-1,t}^1}\right)^{\theta_w(1+\omega)}\mathrm{dispV}_{1,t-1}, \tag{C.2}$$

$$\mathrm{dispV}_{2,t} = (1-\alpha_w)\,(1-\chi_t)\left(rw_t^{2,\star}\right)^{-\theta_w(1+\omega)} + \alpha_w\left(\frac{1+\pi_t^w}{\delta_{t-1,t}^2}\right)^{\theta_w(1+\omega)}\mathrm{dispV}_{2,t-1}. \tag{C.3}$$

C.2 Private welfare

The relevant welfare criterion for workers drawn to choose a new indexation rule is their own expected lifetime utility conditional on the duration of the labor contract:

$$\mathbb{W}_{i,t}\left(\delta_i,\Xi_t\right) = \mathrm{E}_t\left(\sum_{T=t}^{\infty}\left(\beta\alpha_w\right)^{T-t}\ \mathcal{U}\left(c_T\left(\xi_T,\Sigma_T\right),\ \ell_{i,T}\left(\delta_i,\xi_T,\Sigma_T\right)\right)\right). \tag{C.4}$$

Notice that the discount factor takes into account that the labor contract might end each period with probability $1-\alpha_w$. Since individual consumption is equal to aggregate consumption, we can decompose the welfare criterion into two terms, one related to consumption and the other to labor disutility: $\mathbb{W}_{i,t}\left(\delta_i,\Xi_t\right) = \Gamma_t - \Omega_{i,t}\left(\delta,\Xi_t\right)$ where

$$\begin{aligned}\Gamma_t &= \mathrm{E}_t\left\{\sum_{T=t}^{\infty}\left(\beta\alpha_w\right)^{T-t}\exp\left(\varepsilon_{u,T}\right)\frac{\left(c_T-\gamma^h c_{T-1}\right)^{1-\sigma}-1}{1-\sigma}\right\},\ \text{and}\\ \Omega_{i,t}\left(\delta,\Xi_t\right) &= \mathrm{E}_t\left\{\sum_{T=t}^{\infty}(\beta\alpha_w)^{T-t}\exp\left(\varepsilon_{u,T}\right)\frac{\psi}{1+\omega}\ell_{i,T}^{1+\omega}\right\}.\end{aligned}$$

C.2.1 Recursive expressions

A recursive expression for $\Gamma_t(\delta)$ is simply:

$$\Gamma_t = \exp(\varepsilon_{u,t}) \frac{\left(c_t - \gamma^h c_{t-1}\right)^{1-\sigma} - 1}{1-\sigma} + \beta\alpha_w \mathrm{E}_t \Gamma_{t+1}. \tag{C.5}$$

In turn, for the the discounted conditional expected disutility of labor, assume that $\Xi_t = \Xi$ for all t. Then, simplify notation to $\Omega_t^k = \Omega(\delta_{i,t}, \Xi)$ for δ^k and make use of the labor-specific demand $\ell_{i,t,T}^k = \left(\frac{\delta_{t,T}^k}{\pi_{t,T}^w} rw_t^k\right)^{-\theta_w} \ell_T$ to obtain

$$\Omega_t^k = \frac{\psi}{1+\omega} \mathrm{E}_t \left(\sum_{T=t}^{\infty} (\beta\alpha_w)^{T-t} \exp(\varepsilon_{u,T}) \left[\left(\frac{\delta_{t,T}^k}{\pi_{t,T}^w} rw_t^k \right)^{-\theta_w} \ell_T \right]^{1+\omega} \right).$$

Next, expand the expression and factorize common terms:

$$\Omega_t^k = \frac{\psi}{1+\omega} \mathrm{E}_t \left\{ \left(rw_t^k\right)^{-\theta_w(1+\omega)} \left(\begin{array}{c} \exp(\varepsilon_{u,t}) \left[\ell_t\right]^{1+\omega} + \\ \beta\alpha_w \exp(\varepsilon_{u,t+1}) \left[\left(\frac{\delta_{t,t+1}^k}{\pi_{t,t+1}^w} \right)^{-\theta_w} \ell_{t+1} \right]^{1+\omega} + \\ (\beta\alpha_w)^2 \exp(\varepsilon_{u,t+2}) \left[\left(\frac{\delta_{t,t+2}^k}{\pi_{t,t+2}^w} \right)^{-\theta_w} \ell_{t+2} \right]^{1+\omega} + ... \end{array} \right) \right\}.$$

Notice that Ω_{t+1}^k is:

$$\Omega_{t+1}^k = \frac{\psi}{1+\omega} \mathrm{E}_t \left\{ \left(rw_{t+1}^k\right)^{-\theta_w(1+\omega)} \left(\begin{array}{c} \exp(\varepsilon_{u,t+1}) \left[\ell_{t+1}\right]^{1+\omega} + \\ \beta\alpha_w \exp(\varepsilon_{u,t+2}) \left[\left(\frac{\delta_{t+1,t+2}^k}{\pi_{t+1,t+2}^w} \right)^{-\theta_w} \ell_{t+2} \right]^{1+\omega} + \\ (\beta\alpha_w)^2 \exp(\varepsilon_{u,t+3}) \left[\left(\frac{\delta_{t+1,t+3}^k}{\pi_{t+1,t+3}^w} \right)^{-\theta_w} \ell_{t+3} \right]^{1+\omega} + ... \end{array} \right) \right\}.$$

So we can write Ω_t^k as :

$$\Omega_t^k = \frac{\psi}{1+\omega} \exp(\varepsilon_{u,t}) \left[\left(rw_t^k\right)^{-\theta_w} \ell_t \right]^{1+\omega} + \beta\alpha_w \mathrm{E}_t \left\{ \left(\frac{\pi_{t,t+1}^w}{\delta_{t,t+1}^k} \frac{rw_{t+1}^k}{rw_t^k} \right)^{\theta_w(1+\omega)} \Omega_{t+1}^k \right\}.$$

(trick 1: write $\frac{\delta_{t,t+T}^k}{\pi_{t,t+T}^w} = \frac{\delta_{t,t+1}^k}{\pi_{t,t+1}^w} \frac{\delta_{t+1,t+T}^k}{\pi_{t+1,t+T}^w}$; trick 2: multiply by $\left(\frac{rw_{t+1}^k}{rw_{t+1}^k}\right)^{-\theta_w(1+\omega)}$ and re-arrange).

C.2.2 A second-order approximation to labor disutility

The indexation criterion prompts workers to choose the contract associated with the lowest labor disutility at the stochastic steady steady state. For this subsection, we depart from the definition of labor disutility to obtain a second-order approximation, i.e.,

$$\Omega_t^k = \frac{\psi}{1+\omega} \mathrm{E}_t \left(\sum_{T=t}^{\infty} (\beta\alpha_w)^{T-t} \exp(\varepsilon_{u,T}) \left[\ell_{t,T}^k\right]^{1+\omega} \right).$$

Notice that at the steady state, we have that $\ell^k_{ss,l} = \left(\frac{\delta^k_{ss,l}}{\pi^w_{ss,l}} rw^k_{ss}\right)^{-\theta_w} \ell_{ss}$, where $l \in \{0,1,2,3,...\}$ is the number of periods since the last re-optimization. If we assume that $\frac{\delta^k_{ss,l}}{\pi^w_{ss,l}} \approx 1$ for all l (recall that, because of the stochastic steady state, there might be small differences between inflation and target inflation; see Section B), then we have that hours in sector k do not depend on the periods since last re-optimization but only on the relative optimal wages of each labor contract, i.e.

$$\ell^k_{ss} = \left(rw^k_{ss}\right)^{-\theta_w} \ell_{ss}.$$

Now, assume that $\varepsilon_{u,T} = 0 \; \forall \; T$. A second-order approximation of term $\left[\ell^k_{t,T}\right]^{1+\omega}$ around its steady state reads

$$\left[\ell^k_{t,T}\right]^{1+\omega} \approx \left[\ell^k_{ss}\right]^{1+\omega} + \left(\ell^k_{t,T} - \ell^k_{ss}\right)(1+\omega)\left[\ell^k_{ss}\right]^{\omega} + \frac{1}{2}\left(\ell^k_{t,T} - \ell^k_{ss}\right)^2 \omega(1+\omega)\left[\ell^k_{ss}\right]^{\omega-1}.$$

It follows that Ω^k_t can be expressed as:

$$\begin{aligned} \Omega^k_t &\approx \frac{\psi}{1+\omega} \mathrm{E}_t \left\{ \sum_{T=t}^{\infty} (\beta\alpha_w)^{T-t} \left\{ \begin{array}{c} \left[\ell^k_{ss}\right]^{1+\omega} + \left(\ell^k_{t,T} - \ell^k_{ss}\right)(1+\omega)\left[\ell^k_{ss}\right]^{\omega} \\ +\frac{1}{2}\left(\ell^k_{t,T} - \ell^k_{ss}\right)^2 \omega(1+\omega)\left[\ell^k_{ss}\right]^{\omega-1} \end{array} \right\} \right\}, \\ &\approx \frac{\psi}{1+\omega} \frac{\left[\ell^k_{ss}\right]^{1+\omega}}{1-\beta\alpha_w} + \psi \mathrm{E}_t \left\{ \sum_{T=t}^{\infty} (\beta\alpha_w)^{T-t} \left\{ \begin{array}{c} \left(\ell^k_{t,T} - \ell^k_{ss}\right)\left[\ell^k_{ss}\right]^{\omega} \\ +\frac{1}{2}\left(\ell^k_{t,T} - \ell^k_{ss}\right)^2 \omega \left[\ell^k_{ss}\right]^{\omega-1} \end{array} \right\} \right\}. \end{aligned}$$

In order to find the steady-state value Ω^k_{ss}, we need to apply the unconditional expectation operator to each side of the last expression, which leads to:

$$\mathrm{E}\left\{\Omega^k_t\right\} \equiv \Omega^k_{ss} \approx \mathrm{E}\left\{ \frac{\psi}{1+\omega} \frac{\left[\ell^k_{ss}\right]^{1+\omega}}{1-\beta\alpha_w} + \psi \sum_{T=t}^{\infty} (\beta\alpha_w)^{T-t} \left\{ \begin{array}{c} \left(\ell^k_{t,T} - \ell^k_{ss}\right)\left[\ell^k_{ss}\right]^{\omega} \\ +\frac{1}{2}\left(\ell^k_{t,T} - \ell^k_{ss}\right)^2 \omega \left[\ell^k_{ss}\right]^{\omega-1} \end{array} \right\} \right\},$$

since $\mathrm{E}\left\{\left(\ell^k_{t,T} - \ell^k_{ss}\right)\right\} = 0$ and $\mathrm{var}\left(\ell^k_{t,T}\right) = \mathrm{E}\left\{\left(\ell^k_{t,T} - \ell^k_{ss}\right)^2\right\}$, it follows that

$$\Omega^k_{ss} \approx \frac{\psi}{1-\beta\alpha_w} \frac{\left[\ell^k_{ss}\right]^{1+\omega}}{1+\omega} + \frac{\psi}{2}\omega\left[\ell^k_{ss}\right]^{\omega-1} \sum_{T=t}^{\infty} (\beta\alpha_w)^{T-t} \mathrm{var}\left(\ell^k_{t,T}\right),$$

From this expression, it is evident that labor disutility at the stochastic steady state is composed by two terms related to the expected level of hours worked as well as its variance. If we maintain the assumption that $\frac{\delta^k_{ss,l}}{\pi^w_{ss,l}} \approx 1$, then the second term can be simplified into $\frac{\psi}{(1-\beta\alpha_w)} \frac{\omega}{2} \left[\ell^k_{ss}\right]^{\omega-1} \mathrm{var}\left(\ell^k_t\right)$, where the subscript T has being removed. And Ω^k_{ss} can be rewritten as

$$\begin{aligned} \Omega^k_{ss} &\approx \frac{\psi}{1-\beta\alpha_w}\left(R^k_{ss} + V^k_{ss}\right), \text{ where} \\ R^k_{ss} &= \frac{\left[\ell^k_{ss}\right]^{1+\omega}}{1+\omega}, \\ V^k_{ss} &= \frac{\omega}{2}\left[\ell^k_{ss}\right]^{\omega-1} \mathrm{var}\left(\ell^k_t\right) \end{aligned}$$

Now, notice that we can use the labor-specific demand to substitute $\ell_{ss}^{k} = \left(rw_{ss}^{k}\right)^{-\theta_w}\ell_{ss}$ from the expressions above. Further, notice that at the stochastic steady state, relative wages and wage dispersion are altered as follows

$$\begin{aligned}\frac{\mathrm{disp}_{1,ss}^{\mathrm{w}}}{\xi} &= \left(rw_{ss}^{1,\star}\right)^{-\theta_w}, \text{ and} \\ \frac{\mathrm{disp}_{2,ss}^{\mathrm{w}}}{1-\xi} &\approx \left(rw_{ss}^{2,\star}\right)^{-\theta_w}\end{aligned}$$

given that $\frac{\delta_{ss,l}^{k}}{\pi_{ss,l}^{w}} \approx 1.$

C.2.3 Wage dispersion and the expected variance of hours worked.

The optimal policy literature usually describes the price and wage dispersion measures in terms of the volatility of wages using a second-order approximation (see Rotemberg and Woodford, 1998; Erceg and Levin, 2003; Galí and Monacelli, 2004) To build up intuition, we will compute similar expressions for our labor market. Notice that, since we have two sectors in the labor market (one for each indexation rule), some extra terms are added to the otherwise typical expressions for dispersion found in the aforementioned papers.

The first step is to approximate the term $\left(\frac{W_{i,t}}{W_t}\right)^{1-\theta_w}$ using the transformation $X = \exp\left(\ln\left(X\right)\right)$, and define $\mathtt{w}_t\left(i\right) \equiv \ln W_{i,t,}$, $\mathtt{w}_t \equiv \ln W_t$, and $\hat{\mathtt{w}}_t\left(i\right) \equiv \mathtt{w}_t\left(i\right) - \mathtt{w}_t$. The approximation is centered around 1 (or, equivalently, $\hat{\mathtt{w}}_0 = 0$). This number is arbitrarily chosen, but it helps greatly to simplify the computations (another candidate point would be $\hat{\mathtt{w}}_{ss}\left(i\right) = \mathtt{w}_{ss}\left(i\right) - \mathtt{w}_{ss}$, in case $\hat{\mathtt{w}}_{ss}\left(i\right)$ is not unique due to wage dispersion and the distortions caused by the stochastic steady state, but the computations become cumbersome). So, the transformation and then Taylor expansion of $\left(\frac{W_{i,t}}{W_t}\right)^{1-\theta_w}$ is given by

$$\begin{aligned}\left(\frac{W_{i,t}}{W_t}\right)^{1-\theta_w} &= \exp\left(\left(1-\theta_w\right)\hat{\mathtt{w}}_t\left(i\right)\right) \\ &= \exp\left(\left(1-\theta_w\right)\hat{\mathtt{w}}_0\right)\left(1+\left(1-\theta_w\right)\hat{\mathtt{w}}_t\left(i\right)+\frac{\left(1-\theta_w\right)^2}{2}\hat{\mathtt{w}}_t^2\left(i\right)\right)+o\left(\|a^3\|\right) \\ &\simeq 1+\left(1-\theta_w\right)\hat{\mathtt{w}}_t\left(i\right)+\frac{\left(1-\theta_w\right)^2}{2}\hat{\mathtt{w}}_t^2\left(i\right) \quad \text{(up to the second order).}\end{aligned}$$

Integrating the latter over the interval $[0,\xi]$ leaves

$$\tilde{w}_t^1 \equiv \int_0^{\xi}\left(\frac{W_{i,t}}{W_t}\right)^{1-\theta_w}\mathrm{d}i \simeq \xi + \left(1-\theta_w\right)\mathrm{E}_0^{\xi}\left\{\hat{\mathtt{w}}_t\left(i\right)\right\} + \frac{\left(1-\theta_w\right)^2}{2}\mathrm{E}_0^{\xi}\left\{\hat{\mathtt{w}}_t^2\left(i\right)\right\},$$

where $\mathrm{E}_0^{\xi}\left\{x\left(i\right)\right\} \equiv \int_0^{\xi} x\left(i\right)\mathrm{d}i$ is the partial expectation operator on the interval $[0,\xi]$. Solving this equation for $\mathrm{E}_0^{\xi}\left\{\hat{\mathtt{w}}_t\left(i\right)\right\}$ yields

$$\mathrm{E}_0^{\xi}\left\{\hat{\mathtt{w}}_t\left(i\right)\right\} \simeq \frac{1}{\theta_w - 1}\left(\xi - \tilde{w}_t^1\right) + \frac{\theta_w - 1}{2}\mathrm{E}_0^{\xi}\left\{\hat{\mathtt{w}}_t^2\left(i\right)\right\}$$

The second step to approximate the term $\left(\frac{W_{i,t}}{W_t}\right)^{-\theta_w}$ using the same methodology as before, so

$$\left(\frac{W_{i,t}}{W_t}\right)^{-\theta_w} \simeq 1-\theta_w \hat{\mathtt{w}}_t(i)+\frac{\theta_w^2}{2}\hat{\mathtt{w}}_t^2(i) \quad \text{(up to the second order).}$$

Again, integrating over the interval $[0,\xi]$ yields

$$\text{disp}_{1,t}^{\text{w}} \equiv \int_0^{\xi}\left(\frac{W_{i,t}}{W_t}\right)^{-\theta_w}\mathrm{d}i \simeq \xi-\theta_w \mathrm{E}_0^{\xi}\left\{\hat{\mathtt{w}}_t(i)\right\}+\frac{\theta_w^2}{2}\mathrm{E}_0^{\xi}\left\{\hat{\mathtt{w}}_t^2(i)\right\}.$$

Now, substituting $\mathrm{E}_0^{\xi}\left\{\hat{\mathtt{w}}_t(i)\right\}$ yields:

$$\text{disp}_{1,t}^{\text{w}} \equiv \int_0^{\xi}\left(\frac{W_{i,t}}{W_t}\right)^{-\theta_w}\mathrm{d}i \simeq \xi+\frac{\theta_w}{2}\mathrm{E}_0^{\xi}\left\{\hat{\mathtt{w}}_t^2(i)\right\}-\mu_w\left(\xi-\tilde{w}_t^1\right),$$

Relative wage dispersion $\frac{\text{disp}_{1,t}^{\text{w}}}{\xi}$ is thus

$$\frac{\text{disp}_{1,t}^{\text{w}}}{\xi} \simeq 1+\frac{\theta_w}{2\xi}\mathrm{E}_0^{\xi}\left\{\hat{\mathtt{w}}_t^2(i)\right\}-\mu_w\left(\frac{\xi-\tilde{w}_t^1}{\xi}\right).$$

It proves convenient to restate the term $\mathrm{E}_0^{\xi}\left\{\hat{\mathtt{w}}_t^2(i)\right\}$ as a function related to the distance of the sectorial wages $\mathtt{w}_t(i)=\ln W_{i,t}$ from the the aggregate wage level $w_t=\ln W_t$. Let vector $\bar{w}_t^k$ contain all the (log of) relatives wages $\hat{\mathtt{w}}_t(i)=\ln W_{i,t}-\ln W_t$ for labor contract k, such as $\bar{w}_t^k=\left\{\hat{\mathtt{w}}_t(i): i\in IR_k\right\}$. Now, define function $\mathrm{D_k}\left(\bar{w}_t^k\right)=\frac{1}{\tilde{\xi}}\int_{i\in IR_k}\left(\ln W_{i,t}-\ln W_t\right)^2\mathrm{d}i$, where $\tilde{\xi}=\begin{cases}\xi & \text{if } k=1\\ 1-\xi & \text{if } k=2\end{cases}$. Function $\mathrm{D_k}$ is proportional to the square of the Euclidean norm of vector $\bar{w}_t^k$, i.e. $\mathrm{D_k}\left(\bar{w}_t^k\right)=\frac{1}{\tilde{\xi}}\left\|\bar{w}_t^k\ell\right\|^2$. Function $\mathrm{D_k}$ effectively measures the square of the distance between all the wages perceived in sector k and the aggreate wage level, normalized by the proportion of workers choosing indexation rule k. In other words, the larger is $\mathrm{D_k}$, the farther away are the sectorial wages from the economy average. Now we can write the relative dispersion measure as:

$$\frac{\text{disp}_{1,t}^{\text{w}}}{\xi} \simeq 1+\frac{\theta_w}{2}\mathrm{D}_1\left(\bar{w}_t^1\right)-\mu_w\left(\frac{\xi-\tilde{w}_t^1}{\xi}\right).$$

At the stochastic steady state, this expression becomes

$$\mathrm{E}\left\{\frac{\text{disp}_{1,t}^{\text{w}}}{\xi}\right\} \equiv \frac{\text{disp}_{1,ss}^{\text{w}}}{\xi} \simeq 1+\frac{\theta_w}{2}\mathrm{E}\left\{\mathrm{D}_1\left(\bar{w}_t^1\right)\right\}-\mu_w\left(\frac{\xi-\mathrm{E}\left\{\tilde{w}_t^1\right\}}{\xi}\right) \gtrless 1.$$

The term $-\mu_w\left(\frac{\xi-\mathrm{E}\left\{\tilde{w}_t^1\right\}}{\xi}\right)$ appears because in the steady state $\tilde{w}_{ss}^1$ might differ from ξ (see above).

A similar expression can be found for $k=2$. First, notice that

$$\mathrm{E}_{\xi}^1\left\{\hat{\mathtt{w}}_t(i)\right\} \simeq \frac{1}{\theta_w-1}\left((1-\xi)-\tilde{w}_t^2\right)+\frac{\theta_w-1}{2}\mathrm{D}_2\left(\bar{w}_t^2\right)$$

where $\text{var}_2\left\{\ln W_{i,t}\right\}=\frac{1}{1-\xi}\int_{\xi}^1\left(\ln W_{i,t}-\ln W_t\right)^2\mathrm{d}i$. For relative dispersion, at the stochastic steady state, we have that

$$\mathrm{E}\left\{\frac{\text{disp}_{2,t}^{\text{w}}}{1-\xi}\right\} \equiv \frac{\text{disp}_{2,ss}^{\text{w}}}{1-\xi} \simeq 1+\frac{\theta_w}{2}\mathrm{E}\left\{\mathrm{D}_2\left(\bar{w}_t^2\right)\right\}-\mu_w\left(\frac{(1-\xi)-\mathrm{E}\left\{\tilde{w}_t^2\right\}}{1-\xi}\right) \gtrless 1.$$

To conclude, notice that labor demand implies that

$$\begin{aligned}\frac{\ell_{i,t}}{\ell_t} &= \left(\frac{W_{i,t}}{W_t}\right)^{-\theta_w}, \text{ which implies} \\ -\left(\ln \ell_{i,t} - \ln \ell_t\right)\frac{1}{\theta_w} &= \left(\ln W_{i,t} - \ln W_t\right),\end{aligned}$$

and so, we can write $\mathrm{D_k}\left(\bar{w}_t^k\right) = \frac{1}{\tilde{\xi}}\int_{i\in IR_k}\left(-\left(\ln \ell_{i,t}^k - \ln \ell_t\right)\frac{1}{\theta_w}\right)^2 \mathrm{d}i = \frac{1}{\theta_w^2}\mathrm{D_k}\left(\bar{\ell}_t^k\right)$, where $\bar{\ell}_t^k = \left\{\ln \ell_{i,t}^k - \ln \ell_t : i \in IR_k\right\}$. The dispersion measures could be written as:

$$\begin{aligned}\frac{\mathrm{disp}_{k,ss}^{\mathrm{w}}}{\tilde{\xi}} &\simeq 1 + \frac{1}{2\theta_w}\mathrm{E}\left\{\mathrm{D_k}\left(\bar{\ell}_t^k\right)\right\} - \mu_w\left(\frac{\tilde{\xi} - \tilde{w}_{ss}^k}{\tilde{\xi}}\right), \qquad \text{(C.6)} \\ \text{where } \tilde{\xi} &= \begin{cases}\xi & \text{if } k = 1 \\ 1 - \xi & \text{if } k = 2\end{cases}.\end{aligned}$$

It follows that the farther away are the expected hours worked in sector k from the economy average, relative wage dispersion increases, and so does labor disutility.

C.3 Welfare costs measures

We quantify welfare costs due to the presence of stochastic shocks with respect to a benchmark welfare level. Schmitt-Grohé and Uribe (2007) measure welfare costs in terms of the deterministic steady state consumption. We choose to measure them in terms of the deterministic steady state labor, thus highlighting the differences across labor contracts (although we reach exactly the same conclusions with the consumption-based measure). We assume in what follows that utility in consumption is logarithmic ($\sigma = 1$).

Social welfare in the deterministic steady state is given by:

$$\begin{aligned}\mathcal{SW}\left(c_d, \ell_d\right) &= \frac{1}{1-\beta}\mathcal{U}(c_d, \ell_d), \\ &= \frac{1}{1-\beta}\left\{\log\left(c_d\left(1-\gamma^h\right)\right) - \frac{\psi}{1+\omega}\left[\ell_d\right]^{1+\omega}\right\},\end{aligned}$$

where Notice that this level does not depend on ξ. In contrast, at the stochastic steady state, social welfare will vary with ξ and the structure of shocks, and the current policy practices.

$$\mathcal{SW}_{ss} \equiv \mathrm{E}\left\{\sum_{T=t}^{\infty}\beta_T^{T-t}\int_0^1 \exp\left(\varepsilon_{u,T}\right)\mathcal{U}(c_{i,t}, \ell_{i,t})\mathrm{d}i\right\}.$$

For reasons explained above, $\mathcal{SW}_{ss} \leq \mathcal{SW}\left(c_d, \ell_d\right)$ when at least one of the shock variances is strictly positive. For expressing welfare costs in terms of deterministic consumption, let the fraction λ^c represent the proportional cost in c_d that makes the social planner indifferent between the deterministic environment and the stochastic one, i.e.

$$\begin{aligned}\mathcal{SW}_{ss} &= \mathcal{SW}\left(\left(1-\lambda^c\right)c_d, \ell_d\right) \\ &= \frac{1}{1-\beta}\left\{\log\left(\left(1-\lambda^c\right)c_d\left(1-\gamma^h\right)\right) - \frac{\psi}{1+\omega}\left[\ell_d\right]^{1+\omega}\right\}.\end{aligned}$$

Solving for λ^c yields

$$\begin{aligned}
\mathcal{SW}_{ss}(1-\beta) &= \log\left(c_d\left(1-\gamma^h\right)\right) - \frac{\psi}{1+\omega}[\ell_d]^{1+\omega} + \log(1-\lambda^c) \\
\log(1-\lambda^c) &= \mathcal{SW}_{ss}(1-\beta) - \mathcal{SW}(c_d,\ell_d) \\
\lambda^c &= 1-\exp\{\mathcal{SW}_{ss}(1-\beta) - \mathcal{SW}(c_d,\ell_d)\}.
\end{aligned}$$

As an alternative, we propose to measure these costs in terms of leisure. Let λ^ℓ denote the percentage increase in deterministic steady state labor that makes the social planner indifferent between the deterministic environment and the stochastic one. Formally, λ^ℓ is defined as

$$\begin{aligned}
\mathcal{SW}_{ss} &= \mathcal{SW}\left(c_d,\left(1+\lambda^\ell\right)\ell_d\right) \\
&= \frac{1}{1-\beta}\left\{\log\left(c_d\left(1-\gamma^h\right)\right) - \frac{\psi}{1+\omega}\left[\left(1+\lambda^\ell\right)\ell_d\right]^{1+\omega}\right\}.
\end{aligned}$$

Solving for λ^ℓ yields

$$\lambda^\ell = \left[\frac{\mathcal{SW}_{ss}(1-\beta) - \log\left(c_d\left(1-\gamma^h\right)\right)}{-\frac{\psi}{1+\omega}[\ell_d]^{1+\omega}}\right]^{\frac{1}{1+\omega}} - 1.$$

Notice that $-\frac{\psi}{1+\omega}[\ell_d]^{1+\omega} = \mathcal{SW}(c_d,\ell_d)(1-\beta) - \log\left(c_d\left(1-\gamma^h\right)\right)$, so we can rewrite the above expression as

$$\lambda^\ell = \left[\frac{\mathcal{SW}_{ss}(1-\beta) - \log\left(c_d\left(1-\gamma^h\right)\right)}{\mathcal{SW}(c_d,\ell_d)(1-\beta) - \log\left(c_d\left(1-\gamma^h\right)\right)}\right]^{\frac{1}{1+\omega}} - 1.$$

For individual workers, the benchmark welfare is the one that is conditional on the expected duration of the labor contract, and which value is maximum at the stochastic steady state, i.e.,

$$\mathbb{W}(c_d,\ell_d) = \frac{1}{1-\beta\alpha_w}\mathcal{U}(c_d,\ell_d).$$

Notice that this benchmark is equal amid agents, and does not depend on ξ neither. Similarly to social welfare, in the stochastic long-run equilibrium, individual welfare will depend on ξ, on the structure of shocks, on current policy practices, and on the indexation rule a worker decides upon.

$$\mathbb{W}_{ss}\left(\delta^k\right) = \mathrm{E}\left\{\sum_{T=t}^{\infty}(\beta\alpha_w)^{T-t}\exp(\varepsilon_{u,T})\,\mathcal{U}(c_{i,t},\ell_{i,t})\right\}.$$

Let $\lambda^{\ell,k}$ denote the percentage increase in deterministic steady state labor that makes an worker with indexation rule k indifferent between the deterministic and the stochastic environment. Formally, $\lambda^{\ell,k}$ is implicitly given by

$$\begin{aligned}
\mathbb{W}_{ss}\left(\delta^k\right) &= \mathbb{W}\left(c_d,\left(1+\lambda^{\ell,k}\right)\ell_d\right). \\
&= \frac{1}{1-\beta\alpha_w}\left\{\log\left(c_d\left(1-\gamma^h\right)\right) - \frac{\psi}{1+\omega}\left[\left(1+\lambda^{\ell,k}\right)\ell_d\right]^{1+\omega}\right\}.
\end{aligned}$$

Solving for $\lambda^{\ell,k}$ yields

$$\lambda^{\ell,k} = \left[\frac{\mathbb{W}_{ss}\left(\delta^k\right)(1-\beta\alpha_w) - \log\left(c_d\left(1-\gamma^h\right)\right)}{\mathbb{W}\left(c_d,\ell_d\right)(1-\beta\alpha_w) - \log\left(c_d\left(1-\gamma^h\right)\right)}\right]^{\frac{1}{1+\omega}} - 1.$$

D New-Keynesian Wage Phillips Curve with fixed indexation coefficients

Proposition 1 *If the proportion of workers indexing their wages to past inflation is exogenous and fixed, so $\xi = \gamma^w \in [0,1]$, then the log-linearized equation for wage inflation collapses to a typical New-Keynesian wage Phillips curve of the form*

$$\hat{\pi}^w_t - \gamma^w\hat{\pi}_{t-1} - (1-\gamma^w)\hat{\pi}^\star_t = \frac{(1-\beta\alpha_w)(1-\alpha_w)}{\alpha_w}\kappa^w_0 m^w_t + \beta\mathrm{E}_t\left(\hat{\pi}^w_{t+1} - \gamma^w\hat{\pi}_t - (1-\gamma^w)\hat{\pi}^\star_{t+1}\right),$$

where $\kappa^w_0 = (1+\omega_w\theta_w)^{-1}$ and $m^w_t = \omega_w\hat{\ell}_t - \hat{\lambda}_t - \hat{w}_t$.

Proof. First, we need to log-linearize the equations for the optimal relative wages (eq. A.28, A.29, and A.30), and the equations for labor-market aggregating equations (eq. A.31, A.32, and A.33), which give

$$\widehat{rw}^{\star,1}_t = (1-\beta\alpha_w)\kappa^w_0 m^w_t + \beta\alpha_w\hat{\mathrm{E}}_t\left(\hat{\pi}^w_{t+1} - \hat{\pi}_t\right) + \beta\alpha_w\hat{\mathrm{E}}_t\widehat{rw}^{\star,1}_{t+1}, \tag{D.1}$$

$$\widehat{rw}^{\star,2}_t = (1-\beta\alpha_w)\kappa^w_0 m^w_t + \beta\alpha_w\hat{\mathrm{E}}_t\left(\hat{\pi}^w_{t+1} - \hat{\pi}^\star_{t,t+1}\right) + \beta\alpha_w\hat{\mathrm{E}}_t\widehat{rw}^{\star,2}_{t+1} \tag{D.2}$$

$$0 = \gamma^w\left(\widehat{\tilde{w}}^1_t\right) + (1-\gamma^w)\left(\widehat{\tilde{w}}^2_t\right) \tag{D.3}$$

$$\widehat{\tilde{w}}^1_t = (1-\alpha_w)\left((1-\theta_w)\widehat{rw}^{\star,1}_t\right) + \alpha_w\left((\theta_w-1)\left(\hat{\pi}^w_t - \hat{\pi}_{t-1}\right) + \widehat{\tilde{w}}^1_{t-1}\right) \tag{D.4}$$

$$\widehat{\tilde{w}}^2_t = (1-\alpha_w)(1-\theta_w)\widehat{rw}^{\star,2}_t + \alpha_w\left((\theta_w-1)\left(\hat{\pi}^w_t - \hat{\pi}^\star_t\right) + \widehat{\tilde{w}}^2_{t-1}\right) \tag{D.5}$$

Substituting eq. D.4 and D.5 into D.3 results in

$$\begin{aligned} : \quad & -\gamma^w(1-\alpha_w)(1-\theta_w)\widehat{rw}^{\star,1}_t - \gamma^w\alpha_w\left((\theta_w-1)\left(\hat{\pi}^w_t - \hat{\pi}_{t-1}\right) + \widehat{\tilde{w}}^1_{t-1}\right) = \\ & (1-\gamma^w)(1-\alpha_w)(1-\theta_w)\widehat{rw}^{\star,2}_t + (1-\gamma^w)\alpha_w\left((\theta_w-1)\left(\hat{\pi}^w_t - \hat{\pi}^\star_t\right) + \widehat{\tilde{w}}^2_{t-1}\right). \end{aligned} \tag{D.6}$$

After simplifying eq. D.6 by using eq. D.3 in order to eliminate the $\widehat{\tilde{w}}^1_{t-1}$ and $\widehat{\tilde{w}}^2_{t-1}$ terms we obtain

$$\hat{\pi}^w_t - (1-\gamma^w)\hat{\pi}^\star_t - \gamma^w\hat{\pi}_{t-1} = \frac{1-\alpha_w}{\alpha_w}\left(\gamma^w\widehat{rw}^{\star,1}_t + (1-\gamma^w)\widehat{rw}^{\star,2}_t\right). \tag{D.7}$$

Substituting eq. D.1 and D.2 into the above expression gives

$$\begin{aligned} \left(\hat{\pi}^w_t - (1-\gamma^w)\hat{\pi}^\star_t - \gamma^w\hat{\pi}_{t-1}\right) \quad & \frac{\alpha_w}{1-\alpha_w} = \\ & (1-\beta\alpha_w)\kappa^w_0 m^w_t + \beta\alpha_w\mathrm{E}_t\left(\hat{\pi}^w_{t+1} - (1-\gamma^w)\hat{\pi}^*_{t,t+1} - \gamma^w\hat{\pi}_t\right) + \\ & \beta\alpha_w\mathrm{E}_t\left(\chi\widehat{rw}^{\star,1}_{t+1} + (1-\chi)\widehat{rw}^{\star,2}_{t+1}\right). \end{aligned} \tag{D.8}$$

After iterating one period forward eq. D.7 and substituting it into the last term on the right hand side of eq. D.8, plus some manipulations, we obtain that

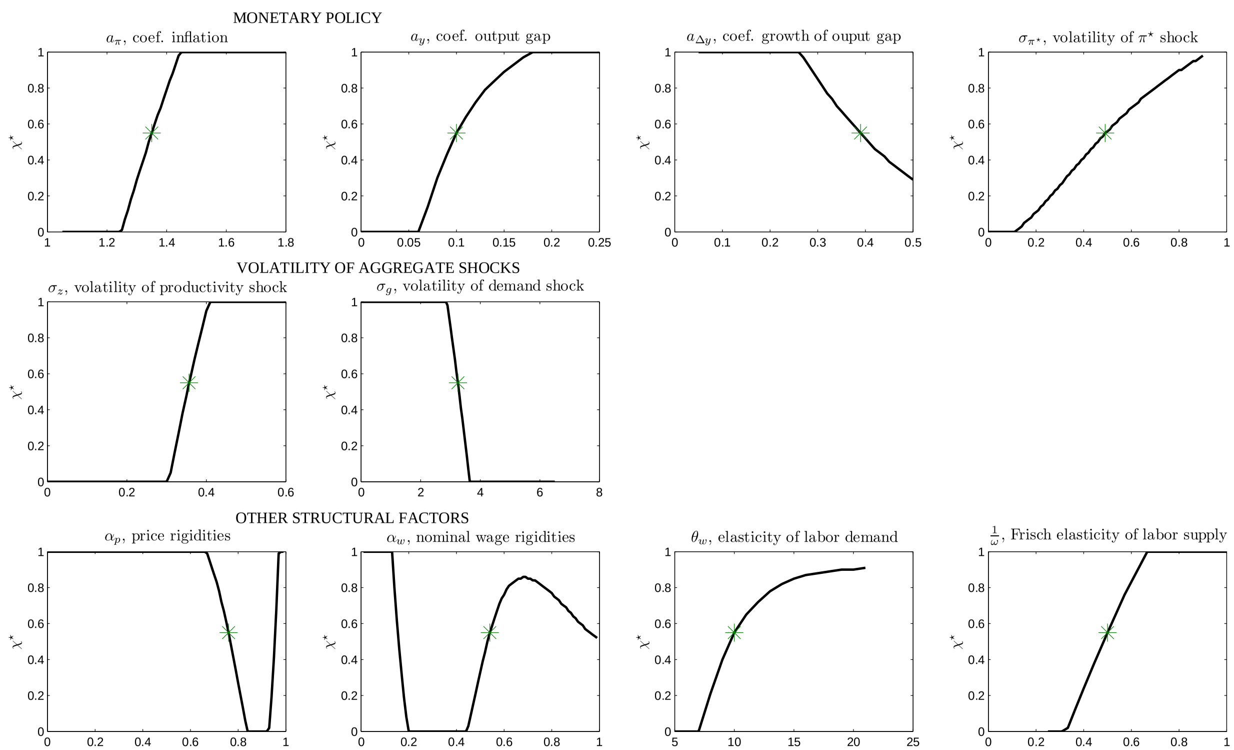
$$\hat{\pi}_t^w - \gamma^w \hat{\pi}_{t-1} - (1-\gamma^w)\hat{\pi}_t^\star = \frac{(1-\beta\alpha_w)(1-\alpha_w)}{\alpha_w}\kappa_0^w m_t^w + \beta \mathrm{E}_t \left(\hat{\pi}_{t+1}^w - \gamma^w \hat{\pi}_t - (1-\gamma^w)\hat{\pi}_{t+1}^\star\right).$$

■

E Sensitivity analysis

Figure (1) illustrates some cases where parameter changes affect equilibrium indexation in a nonlinear fashion. We start from the 2000:Q1 calibration and increase the volatility of the technology shock by 15%. This benchmark calibration results in an aggregate indexation level of $\xi^\star = .55$ and serves as a reference point. This point is marked by a star in each box. Each panel shows how different values for a structural parameter, given on the x-axis and holding all other structural parameters at their benchmark values, results in different levels of aggregate indexation $\xi^\star$ (on the y-axis). It is apparent that shifting the structural parameters away from the reference point can result in some pronounced nonlinear effects on $\xi^\star$, a result that also comes to attention in Section 4.2 in the main paper.

Figure 1: Sensitivity analysis



Note: The responses for selected variables are shown after shocks in productivity, government spending (demand shock), and the inflation target. For productivity, it is assumed that output rises 1 percent in the long-run. For demand shock, government spending rise at impact by 1 percent. Finally, the inflation-target shock rises by 2 percent at impact; in the first case, the rise is temporary while in the second one, the change is permanent.

F Validation exercise with a monetary policy shock

This section complements Section 4.1 of the main paper. We vary the standard deviation of monetary policy shocks and check the implications for equilibrium indexation. As mentioned in the paper, Gray (1976) and Fischer (1977) state that the relative importance of nominal shocks in explaining output fluctuations is a determinant of the degree of wage indexation to past inflation. Since there is evidence that the variance of shocks to the interest rate equation has changed over time (see e.g. Boivin and Giannoni, 2006, Canova and Gambetti, 2009), it could be that changes in the non-systematic component of monetary policy was an important driver of the changes in U.S. wage indexation. Table (1) shows the equilibrium indexation levels after adding a monetary policy shock to the model. As Hofmann *et al.* (2012) do not include a monetary shock to their model, we take the values from the estimation of Cogley *et al.* (2010) as measures for the standard deviation of monetary policy shocks. It follows that introducing the monetary policy shock to the model has only a negligible influence on equilibrium indexation $\xi^{\star}$ in the 1974 calibration and no effect at all in the 2000 calibration. Compared with the other shocks in the model, it is not an important contributor to the business cycle. We conclude that changes in this parameter cannot explain the changes in wage indexation, which is why we leave it out of the main text.

Table 1. Validation exercise with monetary policy shock

		Great Moderation 2000 (benchmark)	Great Inflation 1974
	Common parameters		
β	Subj. discount factor	.99	.99
σ	Intertemp. elasticity of subst.	1	1
ϕ^{-1}	Labor share	1	1
ω^{-1}	Frisch elast. of labor supply	2	2
θ_w	Elast. labor demand	10	10
θ_p	Elast. input demand	10	10
	Specific parameters		
γ^h	Habit formation	.37	.71
γ^p	Inflation inertia	.17	.8
α_p	Calvo-price rigidity	.78	.84
α_w	Calvo-wage rigidity	.54	.64
a_π	Taylor Rule: inflation	1.35	1.11
a_y	Taylor Rule: output gap	.1	.11
$a_{\Delta y}$	Taylor Rule: output gap growth	.39	.5
ρ_R	Taylor Rule: smoothing	.78	.69
σ_z	Std. dev. Tech. shock	.31	1.02
σ_g	Std. dev. Dem. shock	3.25	4.73
σ_m	**Std. dev. Mon pol shock**	**.07**	**.16**
ρ^g	Autocorr. Dem. shock	.91	.89
$\hat{\xi}$	Estimated indexation by HPS	.17	.91
	Case 1: $\sigma_{\pi^\star} = 0$		
$\xi^\star$	Implied equilibrium indexation	0	.86
ξ^S	Implied social optimum	1	0
	Case 2: $\sigma_{\pi^\star} > 0$		
σ_{π^*}	Std. dev. inflation target	.049	.081
$\xi^\star$	Implied equilibrium indexation	0	.87
ξ^S	Implied social optimum	1	0

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