Spillovers: the role of prudential regulation and monetary policy in small open economies

Paul Castillo, Cesar Carrera, Marco Ortiz, Hugo Vega
Banco Central de Reserva del Perú
January 2015

Abstract

This paper is an extension of the paper prepared as part of the BIS CCA Research Network on “Introducing Financial Stability Considerations into Central Bank policy models”. In this extension, we add nominal rigidities a la Calvo in the non-tradable sector and we introduce monetary policy through a standard Taylor Rule to the two-sector small open economy with borrowing constraints used in the previous version of the paper. Our results show that when it is difficult to enforce debtor to repay their debt unless is secured by a collateral, a productivity shock in the tradable sector generates an increase in asset prices and leverage that spills over the non-tradable sector, generating an appreciation of the real exchange rate and an increase in domestic lending. The spillover effect is larger when borrowing constraints are tighter, and when prices in the non-tradable sector are stickier. Macro-prudential instruments are introduced under the form of cyclical loan-to-value ratios that limit the amount of capital that entrepreneurs can pledge as collateral. Simulations results show that this type of instruments significantly lessen the amplifying effects of borrowing constraints on small open economies and consequently reduce output and asset prices volatility when prices are flexible. Furthermore, we find that the use of only monetary policy cannot achieve output, inflation and asset prices volatility levels as low as those obtained when both macro-prudential instruments (taxes) and monetary policy are used jointly.

Keywords: Collateral, productivity, small open economy.
JEL classification: E21, E23, E32, E44, G01, O11, O16

*We would like to thank Gianluca Benigno, Nobuhiro Kiyotaki, Enrique Mendoza, Kenneth Kuttner, Cédric Tille, Roberto Chang, Horacio Aguiirre, Marco Vega and Ilhyock Shim for helpful comments and suggestions. All remaining errors are our own. The opinions expressed in this paper are not necessarily shared by the institutions with which we are currently affiliated.
1 Introduction

Capital flows constitute one of the main sources of volatility in small open economies.\(^1\) As Calvo (1998) emphasized, a sudden stop of capital flows can trigger a fall in domestic spending and a collapse of the real exchange rate and of asset prices, with long-lasting consequences for the health of the financial system and for the economy in general. Recent papers have addressed this issue by focusing on the phenomenon of over borrowing that typically characterizes sudden stop episodes (see Bianchi (2011), Mendoza (2002), Jeanne and Korinek (2010)). Another branch of the literature has focused on the role that financial development plays in amplifying the externalities that capital flows can generate (Aoki et al. (2009) and Aghion et al. (2004)).

An issue that has attracted less attention in the literature is the spillover effects between the tradable and non-tradable sector, and the impact of these spillovers in overborrowing, asset price volatility and an increased vulnerability to sudden stops. In this paper, we address this issue presenting a small open economy model that generates spillover effects between the tradable and non-tradable sector, altering the allocation of funding between these two sectors, and a larger asset price and real exchange rate volatility than the typical real business cycle models. In particular, the model reproduces the positive correlation between tradable and non-tradable sector, asset prices, firms’ leverage in the non-tradable sector, the real exchange rate and asset price volatility observed in developing countries.\(^2\) Furthermore, we show that loan to value ratios and taxes to non-tradable goods consumption not only can help mitigate the typical distortions observed during persistent periods of capital inflows and outflows but also complement the role of monetary policy in stabilizing output and inflation.

The experience of Latin American economies, previous to the global financial crisis, illustrates the relevance of spillover effects between the tradable and non-tradable sectors. As Table 1 shows, capital flows to the region increased substantially in 2007. Simultaneously, both the gross domestic product (GDP) and credit exhibited high growth rates. More importantly, the growth rates in the tradable and non-tradable sector showed a high degree of positive correlation. An interesting pattern appears after the implementation of the quantitative easing policies by the FED. Over these years, the credit expansion was accompanied by positive growth rates in both sectors. While we would have expected the tradable sector to expand the most, the non-tradable sector registered important growth rates, even larger than the tradable sector ones in countries such as Bolivia, Chile, and Peru. Our model attempts to explain this stylized fact observed in Latin American countries.

Similar to Kiyotaki and Moore (1997), in our model, the dynamics between credit limits and asset prices become a transmission mechanism by which the effects of a shock persist and spill over other sectors. Unlike the existing literature, we explicitly study the interaction between over borrowing, housing prices, and spillovers between tradable and non-tradable sectors. Our results suggest that when it is difficult to enforce debtors to repay their debt unless it is secured by collateral, a productivity shock in the tradable

---

\(^1\)See Converse (2013).

\(^2\)Converse (2013) attributes these correlations to financial constraints and maturity mismatches which impact investment after negative shocks to capital flows. In his setup, this channel also links the volatility of capital flows to output and total factor productivity (TFP) volatility.
Table 1: Capital flows, credit, and output

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Bolivia</td>
<td>14.0</td>
<td>34.8</td>
<td>-0.4</td>
<td>13.8</td>
<td>28.7</td>
<td>8.5</td>
<td>17.5</td>
</tr>
<tr>
<td>Colombia</td>
<td>13.3</td>
<td>20.5</td>
<td>9.7</td>
<td>13.2</td>
<td>16.0</td>
<td>17.9</td>
<td>12.2</td>
</tr>
<tr>
<td>Chile</td>
<td>9.7</td>
<td>19.9</td>
<td>5.3</td>
<td>24.3</td>
<td>23.4</td>
<td>8.2</td>
<td>17.8</td>
</tr>
<tr>
<td>Mexico</td>
<td>16.5</td>
<td>10.7</td>
<td>-7.2</td>
<td>4.6</td>
<td>26.3</td>
<td>-1.0</td>
<td>20.0</td>
</tr>
<tr>
<td>Peru</td>
<td>14.0</td>
<td>5.9</td>
<td>0.1</td>
<td>3.2</td>
<td>6.3</td>
<td>1.7</td>
<td>10.0</td>
</tr>
</tbody>
</table>

International investment position, in percentages

Credit to the private sector growth rates

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Bolivia</td>
<td>Tradable</td>
<td>-1.5</td>
<td>11.7</td>
<td>1.7</td>
<td>2.7</td>
<td>24.1</td>
<td>24.7</td>
<td>17.6</td>
</tr>
<tr>
<td>Bolivia</td>
<td>Non-tradable</td>
<td>9.2</td>
<td>6.6</td>
<td>8.4</td>
<td>10.8</td>
<td>28.4</td>
<td>21.2</td>
<td>17.5</td>
</tr>
<tr>
<td>Colombia</td>
<td>Tradable</td>
<td>32.3</td>
<td>16.2</td>
<td>32.2</td>
<td>-5.0</td>
<td>19.4</td>
<td>8.5</td>
<td>17.4</td>
</tr>
<tr>
<td>Colombia</td>
<td>Non-tradable</td>
<td>44.4</td>
<td>24.0</td>
<td>22.1</td>
<td>0.5</td>
<td>24.3</td>
<td>27.8</td>
<td>9.5</td>
</tr>
<tr>
<td>Chile</td>
<td>Tradable</td>
<td>14.6</td>
<td>17.8</td>
<td>36.2</td>
<td>-18.7</td>
<td>15.0</td>
<td>20.7</td>
<td>4.8</td>
</tr>
<tr>
<td>Chile</td>
<td>Non-tradable</td>
<td>17.6</td>
<td>20.9</td>
<td>19.8</td>
<td>-0.9</td>
<td>10.1</td>
<td>14.5</td>
<td>8.9</td>
</tr>
<tr>
<td>Mexico</td>
<td>Tradable</td>
<td>7.9</td>
<td>10.1</td>
<td>3.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mexico</td>
<td>Non-tradable</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peru</td>
<td>Tradable</td>
<td>8.8</td>
<td>33.4</td>
<td>33.6</td>
<td>-7.1</td>
<td>18.2</td>
<td>12.0</td>
<td>3.2</td>
</tr>
<tr>
<td>Peru</td>
<td>Non-tradable</td>
<td>20.0</td>
<td>31.6</td>
<td>43.3</td>
<td>7.8</td>
<td>20.5</td>
<td>17.0</td>
<td>15.3</td>
</tr>
</tbody>
</table>

GDP, in percentages

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Bolivia</td>
<td>Tradable</td>
<td>5.6</td>
<td>4.1</td>
<td>7.5</td>
<td>3.2</td>
<td>3.1</td>
<td>4.5</td>
<td>4.2</td>
</tr>
<tr>
<td>Bolivia</td>
<td>Non-tradable</td>
<td>-2.6</td>
<td>12.8</td>
<td>4.2</td>
<td>5.2</td>
<td>4.7</td>
<td>5.1</td>
<td>6.4</td>
</tr>
<tr>
<td>Colombia</td>
<td>Tradable</td>
<td>5.8</td>
<td>6.3</td>
<td>2.7</td>
<td>-0.1</td>
<td>4.2</td>
<td>6.6</td>
<td>2.7</td>
</tr>
<tr>
<td>Colombia</td>
<td>Non-tradable</td>
<td>6.6</td>
<td>6.6</td>
<td>4.0</td>
<td>3.1</td>
<td>3.5</td>
<td>5.7</td>
<td>5.0</td>
</tr>
<tr>
<td>Chile</td>
<td>Tradable</td>
<td>4.5</td>
<td>3.3</td>
<td>-0.1</td>
<td>-3.7</td>
<td>4.0</td>
<td>3.8</td>
<td>4.0</td>
</tr>
<tr>
<td>Chile</td>
<td>Non-tradable</td>
<td>6.3</td>
<td>5.7</td>
<td>6.1</td>
<td>1.5</td>
<td>5.8</td>
<td>7.2</td>
<td>6.7</td>
</tr>
<tr>
<td>Mexico</td>
<td>Tradable</td>
<td>3.8</td>
<td>1.6</td>
<td>-1.1</td>
<td>-7.7</td>
<td>6.9</td>
<td>3.5</td>
<td>3.7</td>
</tr>
<tr>
<td>Mexico</td>
<td>Non-tradable</td>
<td>4.8</td>
<td>3.6</td>
<td>3.3</td>
<td>-2.9</td>
<td>5.0</td>
<td>4.1</td>
<td>3.8</td>
</tr>
<tr>
<td>Peru</td>
<td>Tradable</td>
<td>6.8</td>
<td>8.0</td>
<td>9.0</td>
<td>-2.1</td>
<td>8.6</td>
<td>5.6</td>
<td>3.7</td>
</tr>
<tr>
<td>Peru</td>
<td>Non-tradable</td>
<td>8.9</td>
<td>10.1</td>
<td>10.0</td>
<td>4.1</td>
<td>8.4</td>
<td>7.7</td>
<td>8.2</td>
</tr>
</tbody>
</table>

sector generates an increase in asset prices and leverage that spills over to the non-tradable sector, increasing the leverage in this sector and generating a real appreciation. The opposite is observed in response to a negative tradable productivity shock. These dynamics are consistent with the ones observed in emerging market economies during episodes of capital inflows and outflows.

The model economy consists of workers and entrepreneurs allocated in the tradable and non-tradable sectors of the economy. Entrepreneurs face borrowing constraints to finance both production and the acquisition of capital. Workers and entrepreneurs consume a basket of tradable and non-tradable goods. We introduce two types of durable goods, houses and capital and both serve as collateral and as production factors. In both cases, due to limited commitment, agents have to pledge collateral in order to borrow. We also consider an asymmetry between domestic and foreign creditors. The foreign creditors only lend to the tradable sector and accept more easily capital as collateral. In contrast, non-tradable entrepreneurs obtain credit exclusively from domestic markets, which accept houses as collateral more easily than capital. This restriction generates a link between the entrepreneurs’ debt limits and the price of collateral. This link, given the collateral pledged by entrepreneurs in the tradable and non-tradable sectors, is the main mechanism that generates co-movements between these two sectors. The rise in the value of collateral triggered by an expected increase in productivity in one sector, also implies that constrained agents in the other sector can benefit from a larger borrowing capacity, which leads to co-movements amongst sectors. As an extension to the model, we additionally consider monopolistic competition and sticky prices a la Calvo in the non-tradable sector, and we introduce monetary policy through a simple Taylor rule.

Although our assumption of two types of collateral is not conventional, it is not new in the literature. Caballero and Krishnamurthy (2001) use a similar assumption to study the interaction between domestic and foreign lending during periods of sudden stops. However, a key difference between our assumption and that of Caballero and Krishnamurthy (2001) is that in our case, international collateral is used only for borrowing in the tradable sector, whereas domestic collateral is used only for borrowing in the non-tradable sector, which is plausible given the empirical evidence that shows that exports play a significant role in generating international collateral, whereas domestic agents prefer domestic collateral, such as real state. For example, as Caballero and Krishnamurthy (2001) highlight, during the 2004-2005 financial crisis Mexico used its oil revenues to back the liquidity package it received.

Our simulation results show that an increase in productivity in the tradable sector generates a rise in output both in the tradable and non-tradable sectors, boosting collateral prices, generating a real appreciation and increasing the leverage of entrepreneurs that operate in the non-tradable sector. During the adjustment process, collateral is transferred from the tradable to the non-tradable sector and viceversa because, after a positive productivity shock, each type of entrepreneur uses relatively less of the collateralizable asset to finance production. In this way, the model can account for the typical stylized facts that precede periods of excess credit growth and capital flows in small open economy models, as periods of transitory increase in productivity in the tradable sector, that spillover to the non-tradable sector, generate exchange rate appreciations, overborrowing in the non-tradable sector and asset price booms. We also find that the spillover effects are amplified by the existence of nominal rigidities in the non-tradable sector.
In the case of a rise in productivity in the non-tradable sector the model generates an increase in non-tradable output, a very mild increase in tradable output, a fall in asset prices and a short lived current account surplus consistent with a real depreciation. The increase in productivity in the non-tradable sector is assimilated in the form of lower prices, generating a significant depreciation. Notice that asset prices and real wages fall in terms of tradable goods, as a consequence of the real depreciation, but increase in terms of the CPI index. As non-tradable entrepreneurs increase their productivity they can produce more using less inputs, their demand for credit falls, pushing real interest rates down.

An increase in the foreign interest rate tightens the borrowing constraint of tradable firms, forcing a fall in tradable output. Lower input demand by tradable firms leads to a fall in the prices of houses and labour. The negative wealth effect on tradable entrepreneurs reduces demand for non-tradable goods, triggering a real depreciation.

Consequently, output in the non-tradable sector falls as well, reducing demand for capital and labour further. The decline in demand for credit from the entrepreneurs pushes domestic interest rates down. As it is usually observed when international interest rates rise, the fall in asset prices and the slowdown in economic activity makes it difficult for lenders to lend since the creditworthiness of borrowers deteriorated. As a result, credit collapses and savings interest rates falls.

Given tighter borrowing constraints, housing is reallocated from the tradable to the non-tradable sector, and capital is reallocated from the non-tradable to the tradable sector. The contraction in domestic output and the depreciation that occurs when this shock hits is consistent with the stylized facts observed in emerging market economies during periods of rising international interest rates.

Interestingly, our results also show that the volatility of the real exchange rate and asset prices is greatly amplified when financial frictions are tighter. Thus, when borrowing constraints are tighter the correlation between the tradable and non-tradable sector output increases and the debt in the non-tradable sector expands more. Asset prices, housing and capital prices also increase more in this case. The contrary is observed when borrowing constraints are less tight. In this case, the model generates smaller spillover effects and both houses and capital prices react less. We also find that sticky prices in the non-tradable sector tend to increase the spillovers effects substantially. Both output in the non-tradable sector and asset prices react more strongly to a positive productivity shock in the tradable sector than in the case of flexible prices. The key channel through which sticky prices generate a large spillover on the non-tradable is by inducing a larger expected appreciation of the real exchange rate. In the case of the model with sticky prices, the real exchange rate appreciates less initially but more and for a longer period after the second quarter of observed the shock, amplifying the effects of the productivity shock on asset prices and on non-tradable output growth.

We also introduce macro-prudential instruments in the form of cyclical loan-to-value ratios that limit the fraction of the value of assets that entrepreneurs can pledge as collateral and cyclical taxes to the consumption of non-tradable goods. Our simulation results show that policies aimed at reducing the volatility of asset prices, and that of the exchange rate, perform well and diminish the cyclical effects of borrowing constraints in small open economies.

Our paper is related to a large body of literature that studies the macroeconomic role
of financial frictions. Bianchi (2011) studies constrained efficient equilibria within a small open economy model with borrowing constraints. In contrast with his work, we study the spillover effects between tradable and non-tradable sectors, asset prices and capital flows in a model that does not rely on occasionally binding constraints.

Mendoza (2002) accounts for the abrupt economic collapses of sudden stops as an atypical phenomenon nested within the smoother co-movements of regular business cycles. In this setting, precautionary savings and state-contingent risk premiums play a key role in driving business cycle dynamics. In particular, he shows that sudden stops can be consistent with the optimal adjustment of a flexible-price economy in response to a suddenly binding credit constraint (occasionally binding credit constraint that limits borrowing). The liquidity constraint requires borrowers to finance a fraction of their current obligations out of their current income.

Caballero and Krishnamurthy (2001) emphasize the interaction between domestic and international collateral constraints for financial crises by constructing a model where firms are subject to liquidity shocks. Since domestic collateral constraints lower the domestic rate of return on saving, agents tend to under-save: “they hold too little spare international borrowing capacity, which makes the economy more vulnerable to adverse shocks.”

Aoki et al. (2009) provide a framework to analyse how the constraints in domestic finance and international finance interact with each other through asset prices. In their model, entrepreneurs combine a fixed asset (land) and working capital to produce output. With some probability, some entrepreneurs are productive while others are not. Here, the fixed asset is a factor of production as well as collateral for loans. The borrower’s credit limit is affected by the price of the fixed asset, while the asset price is affected by credit limits. The interaction between credit limits and asset prices turns out to be a propagation mechanism that may generate large swings in aggregate economic activity. In addition to the fixed asset, some fraction of future output is allowed as collateral for domestic loans. The extent to which future output is usable as collateral depends upon both the technology and the quality of institutions, and proxies for the degree of development of the domestic financial system.

In a related paper, Paasche (2001) studies the spillover effects across countries. The authors extend the model of Kiyotaki and Moore (1997) to a setup of two credit-constrained small open economies which borrow and export differentiated commodities to a third large one. These small countries are only connected through the elasticity of substitution in their exports to the large country. The authors show that spill over effects are present since a negative productivity shock in one of the small countries generates an

---

3Aizenman (2002) questions the findings of Mendoza (2002) and argues that domestic tax policy uncertainty in the presence of exogenous liquidity constraints is a poor description of some countries in the East, such as Korea. Before the crisis, the global market viewed Korea as having a stable and responsible fiscal policy. An alternative interpretation is that an unanticipated tightening of the liquidity constraint would be associated with a very large welfare cost. In that regard, the Korean crisis should be modelled as an economy characterized by erratic access to the international capital market, stable domestic fiscal policies, and a high savings rate in which moral hazard provides the incentive for excessive borrowing. Aizenman (2002) suggests Dooley (2000) for this type of models. Aizenman (2002) also points out that the benchmark model does not consider the investment channel or allow for an endogenous long-run effect of uncertainty on growth. According to Aizenman (2002), sudden stops in Mendoza (2002) are not reflected in long-run business-cycle statistics; they are the outcome of the modelling strategy and may not hold in models in which long-run growth is systematically affected by policy uncertainty and economic volatility.
adverse terms of trade shock on the other, which is amplified through the credit channel.

The rest of the paper is organized as follows. Section 2 presents our theoretical approach. Section 3 presents our results. Finally, section 4 concludes.

2 Baseline Model

In the model, the domestic economy is a small open economy inhabited by a continuum of two types of agents, entrepreneurs and workers. Workers consume a basket of tradable and non-tradable goods; whereas entrepreneurs consume only final goods. We introduce two types of durable goods, houses \( h \) and capital \( k \). Both serve as collateral and as production factors. In both cases, due to limited commitment, agents are required to pledge collateral in order to borrow. Following Aoki et al. (2009) we consider an asymmetry between domestic and foreign creditors. The foreign creditors will only lend to the tradable sector and accept capital as pledgable collateral. In contrast, non-tradable entrepreneurs will obtain credit exclusively from domestic markets, which only accept houses as collateral.

We model workers to be more patient agents than entrepreneurs as in Iacoviello (2005). Workers supply labour to entrepreneurs and do not face borrowing constraints. We restrict the saving possibilities of the workers to the domestic economy. We further introduce macro-prudential instruments into the model by considering that the government can affect the amount entrepreneurs can pledge as collateral when they borrow both in domestic and foreign markets, using loan-to-value ratios as a policy tool and that it can set taxes to the consumption of non-tradable goods.
Figure 1: The model economy
2.1 Workers

Workers maximize a lifetime utility function given by:

$$E_0 \sum_{s=0}^{\infty} \beta^s \left( \ln C_{w,s} - \lambda \frac{(l_s)^{1+\eta}}{1+\eta} \right)$$

where $C_{w,s}$ and $l_s$ represent the worker’s consumption and labour, respectively. The parameter $\lambda$ controls for the importance of labour in the utility function. Parameter $\eta$ pins down the elasticity of substitution between labour and final goods consumption. $E_s$ is the conditional expectation operator set at period $s$ and $\beta$ is the intertemporal discount factor, with $0 < \beta < 1$.

The consumption basket of workers is a composite of tradable and non-tradable goods, aggregated using the following consumption index:

$$C_{w,s} = \left[ (\gamma^T)^{1/\varepsilon} \left( C_{w,s}^T \right)^{\frac{\varepsilon-1}{\varepsilon}} + (1 - \gamma^T)^{1/\varepsilon} \left( C_{w,s}^{NT} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{1}{\varepsilon-1}}$$

where $\varepsilon > 0$ is the elasticity of substitution between tradable ($C_{w,s}^T$) and non-tradable goods ($C_{w,s}^{NT}$), and $\gamma^T$ is the share of tradable goods in the consumption basket of the domestic economy.

The worker’s optimal demands for tradable and non-tradable consumption are given by:

$$c_{w,s}^T = \gamma^T \left( \frac{1}{P_s} \right)^{-\varepsilon} C_{w,s}$$

$$c_{w,s}^{NT} = (1 - \gamma^T) \left( \frac{P_{NT}^s}{P_s} \right)^{-\varepsilon} C_{w,s}$$

This set of demand functions is obtained by minimizing the total expenditure in consumption $P_s C_{w,s}$ where $P_s$ stands for the worker’s consumer price index in terms of tradable goods. Notice that the consumption of each type of good is increasing in the total consumption level, and decreasing in their corresponding relative price. Also, it is easy to show that under these preference assumptions, the consumer price index is determined by the following condition:

$$P_s = \left[ \gamma^T + (1 - \gamma^T) \left( p_{NT}^s \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}$$

where $p_{NT}^s$ denotes the relative price level of non-tradable goods.

We assume workers do not have access to the international financial market, therefore they can only save and lend in the domestic financial system. Their flow of funds is given by:

$$C_{w,s} + \frac{b_{w,s}^{NT}}{P_s} = R_{s-1} \frac{b_{s-1}^{NT}}{P_s} + \frac{w_s}{P_s} l_s$$

where $w_s$ is the nominal wage and $R_s$ the domestic interest factor (expressed in tradable units of account). $b_{w,s}^{NT}$ represents the amount that workers lend to entrepreneurs in the non-tradable sector.
Solving the first order conditions and assuming workers ignore the (potential) constraints on savings we get:

\[ \frac{1}{R_s} = \beta E_s \left[ \frac{C_{w,s}}{C_{w,s+1}} \frac{P_s}{P_{s+1}} \right] \]  

(7)

Equation (7) corresponds to the Euler equation that determines the optimal path of consumption for unconstrained households in the home economy, equalizing the marginal benefits of savings to its corresponding marginal costs.

Also from the first-order conditions, we obtain the labour supply function:

\[ \frac{w_s}{P_s} = C_{w,s} \lambda (l_s)^\eta \]  

(8)

where \( \frac{w_s}{P_s} \) denotes real wages. In a competitive labour market, the marginal rate of substitution equals the real wage, as in equation (8).

2.2 Entrepreneurs

There are two types of entrepreneurs in the economy, which differ in the goods they produce and in their access to financial markets. The first type of entrepreneur produces tradable goods (which can be sold in the international markets) while the second produces non tradable goods, which can be traded only in the domestic market (non-tradable entrepreneur). Also, we restrict access to the international financial markets to tradable entrepreneurs.

2.2.1 Non-tradable entrepreneurs

Non-tradable entrepreneurs produce the non-tradable good \( y_{NT}^s \) using housing \( h_{NT}^s \), capital \( k_{NT}^s \), and labour \( l_{NT}^s \). They can only obtain financing from the domestic market, where they face a credit constraint based on their housing asset. Since they hire factors in period \( t \) and receive output in period \( t+1 \), they will have to pay the factors in advance.

This type of entrepreneur has access to the following production technology:

\[ y_{NT}^s = \zeta (h_{NT}^s)^\alpha (k_{NT}^s)^\kappa (l_{NT}^s)^{1-\alpha-\kappa} \]  

(9)

where \( \zeta \) is the total factor productivity of the non-tradable sector and \( \alpha \), and \( \kappa \) are the housing and capital output shares, respectively. We assume non-tradable entrepreneurs extract utility from the consumption good only. Their objective is to maximize the following utility function:

\[ E_0 \sum_{s=0}^{\infty} \gamma^s \ln C_{nt,s} \]  

(10)

where \( \gamma \) is the time discount rate of the non-tradable entrepreneur. \( \gamma < \beta \) is a necessary condition to guarantee that the borrowing constraint for these entrepreneurs is binding.

Iacoviello (2005) does not introduce these constraints in the patient household problem either. This could be justified assuming the households are atomistic, while firms are not. Thus, households do not take into account their impact on the total funds available to lend. Still, it seems somewhat implausible that all households will be constrained in equilibrium and still don’t incorporate these restrictions in their optimization programme.
in the steady state. Hence, our entrepreneurs are relatively impatient with respect to workers, in line with Iacoviello (2005). This assumption helps us define the role of entrepreneurs as borrowers and workers as creditors in the domestic market. Their incomes and expenses (flow of funds) are captured in the following expression:

\[
\frac{P_s^N T \Delta h_s^N T}{P_s} + \frac{b_s^N T}{P_s} = C_{nt,s} + \frac{q_s^h}{P_s} \Delta h_s^N T + \frac{q_s^k}{P_s} \Delta k_s^N T + R_{s-1} \frac{b_{s-1}^N T}{P_s} + \frac{w_s}{P_s} t_{s}^N T,
\]

(11)

where \( C_{nt,s} \) is a bundle of tradable and non-tradable goods. \( \Delta h_s^N T \) and \( \Delta k_s^N T \) are the changes in the non-tradable entrepreneur’s holding of houses and capital, respectively. \( q_s^k \) is the price of capital and \( q_s^h \) is the price of housing.

In the domestic financial market firms borrow from domestic agents using housing as collateral in line with Aoki et al. (2009). Domestic lenders accept this collateral. The domestic credit restriction is given by:

\[
R_s \frac{b_s^N T}{P_{s+1}} \leq \theta_s^N T \frac{q_s^h}{P_{s+1}} + \frac{q_s^k}{P_{s+1}} h_s^N T
\]

(12)

where \( \theta_s^N T \) represents the fraction of the value of the collateral that the non-tradable entrepreneur can effectively pledge on his real state holdings. We set the Lagrangian that summarizes the non-tradable entrepreneur’s problem. The first order conditions set optimality in the choice of entrepreneurs regarding consumption, labour, housing, capital and debt. Out of these conditions we single out the demand for factors, given by the following expressions:

\[
q_s^h = \gamma E_s \left[ \frac{C_{nt,s}}{C_{nt,s+1}} \frac{P_s}{P_{s+1}} \left( q_s^h + p_{s+1} \frac{\partial y_{s+1}^N T}{\partial h_s^N T} \right) \right] + \left( \frac{1}{R_s} - \gamma E_s \left[ \frac{C_{nt,s}}{C_{nt,s+1}} \frac{P_s}{P_{s+1}} \right] \right) \theta_s^N T E_s \left[ q_s^h \right]
\]

(13)

\[
q_s^k = \gamma E_s \left[ \frac{C_{nt,s}}{C_{nt,s+1}} \frac{P_s}{P_{s+1}} \left( q_s^k + p_{s+1} \frac{\partial y_{s+1}^N T}{\partial k_s^N T} \right) \right]
\]

(14)

\[
w_s = \gamma E_s \left[ \frac{C_{nt,s}}{C_{nt,s+1}} \frac{P_s}{P_{s+1}} \left( p_{s+1} \frac{\partial y_{s+1}^N T}{\partial l_s^N T} \right) \right]
\]

(15)

Equations (13) and (14) represent the non-tradable entrepreneur’s demand for housing and capital respectively, while equation (15) represents their demand for labor. The optimal allocation of consumption between tradable and non-tradable goods is determined by equating the rate of substitution of these two types of goods to their corresponding relative price.

By comparing equations (13) with (14) we can notice that the first order condition for housing involves an additional term, given by the second expression of the left-hand side. This expression represents the gains that entrepreneurs obtain by holding an asset that allows them access to credit. This benefit is proportional to the difference between the interest rate and their stochastic discount factor, and plays a very important role in the model dynamics.

\[\text{This expression is constructed in exactly the same way as that of the worker and thus is valued at the same price index the workers face, } P_s.\]
2.2.2 Tradable entrepreneurs

This case parallels the one of non-tradable entrepreneurs. The production technology is a Cobb-Douglas similar in structure to that of the tradable entrepreneur:

\[ y^T_s = A_{s-1} \left( h^T_{s-1} \right)^\nu \left( k^T_{s-1} \right)^\psi \left( l^T_{s-1} \right)^{1-\nu-\psi} \]  

(16)

where \( A \) is the total factor productivity of the tradable sector. \( \nu \), and \( \psi \) are the housing and capital tradable output shares, respectively. The tradable entrepreneurs extract utility only from consumption goods. They maximize the following utility function:

\[ E_0 \sum_{s=0}^{\infty} \gamma^s \ln C_{t,s} \]  

(17)

where \( \gamma < \beta \). As in the case of the typical non-tradable entrepreneur, this guarantees the borrowing constraint will be binding in equilibrium. The producer of tradable goods has the following flow of funds:

\[ \frac{1}{P_s} y^T_s + \frac{1}{P_s} b^T_s = C_{t,s} + \frac{q^h_s}{P_s} \Delta h^T_s + \frac{q^k_s}{P_s} \Delta k^T_s + R^*_s \frac{1}{P_{s-1}} b^T_{s-1} + \frac{w_s}{P_s} \]  

(18)

where \( C_{t,s} \) is a bundle of tradable and non-tradable goods constructed in exactly the same way as that of the worker and thus is valued at the same price index the workers use, \( P_s \). \( b^T_s \) represents the debt of tradable entrepreneurs, while \( R^*_s \) is the foreign interest rate.

The tradable entrepreneur faces the following financial constraint,

\[ R^*_s \frac{1}{P_{s+1}} b^T_s \leq \theta^*_s \frac{q^{k+1}_{s+1}}{P_{s+1}} k^T_s \]  

(19)

where \( \theta^*_s \) represents the fraction of collateral that can be used against a loan. We assume that tradable entrepreneurs only access foreign credit markets, where the only asset accepted as collateral is capital. In this case, the Lagrangian for the tradable entrepreneur mirrors the one of the non-tradable entrepreneur. Then, similar to the first order conditions of the non-tradable entrepreneur, tradable entrepreneur’s demand for factors can be described as:

\[ q^h_s = \gamma E_s \left[ \frac{C_{t,s}}{C_{t,s+1}} \frac{P_s}{P_{s+1}} \left( q^{h+1}_{s+1} + \frac{\partial y^T_{s+1}}{\partial h^T_s} \right) \right] \]  

(20)

\[ q^k_s = \gamma E_s \left[ \frac{C_{t,s}}{C_{t,s+1}} \frac{P_s}{P_{s+1}} \left( q^{k+1}_{s+1} + \frac{\partial y^T_{s+1}}{\partial k^T_s} \right) \right] + \left( \frac{1}{R^*_s} - \gamma E_s \left[ \frac{C_{t,s}}{C_{t,s+1}} \frac{P_s}{P_{s+1}} \right] \right) \theta^*_s E_s \left[ q^{k+1}_{s+1} \right] \]  

(21)

\[ w_s = \gamma E_s \left[ \frac{C_{t,s}}{C_{t,s+1}} \frac{P_s}{P_{s+1}} \frac{\partial y^T_{s+1}}{\partial l^T_s} \right] \]  

(22)

In this case, the term representing the benefit from accessing credit is present in the first order condition related to capital (21). Once again this will be a function of the difference between the loan rate and the stochastic discount factor of entrepreneurs.
2.3 Market equilibrium conditions

The model comprises five markets: (i) the labour market, (ii) the housing market, (iii) the capital market, and final goods markets, (iv) tradable and (v) non-tradable. Labour is homogeneous and it is used as a production factor by both the tradable and non-tradable sectors. Houses are demanded by both types of entrepreneurs in the economy as a production factor. Non-tradable entrepreneurs use them as well as collateral for borrowing. Capital is used by both entrepreneurs as a production factor, and as collateral only by the tradable sector entrepreneurs. In the economy there is no investment, which implies that the total supply of housing and capital is fixed at $H$ and $K$, respectively. The corresponding equilibrium conditions of the labour, housing and capital markets are given by the following three equations:

$$ l_s = l_s^{NT} + l_s^T \tag{23} $$

$$ H = h_s^{NT} + h_s^T \tag{24} $$

$$ K = k_s^T + k_s^{NT} \tag{25} $$

Non-tradable goods are consumed by workers and entrepreneurs. Aggregate demand of non-tradable goods depends on its relative price and the total demand for consumption, as the following equation describes:

$$ y_s^{NT} = \left(1 - \gamma^T\right) \left(\frac{p_s^{NT}}{p_s}\right)^{-\varepsilon} \left(C_{w,s} + C_{t,s} + C_{nt,s}\right) \tag{26} $$

Only entrepreneurs in the tradable sector have access to international capital markets. In contrast, non-tradable entrepreneurs and workers operate exclusively in the domestic financial system. Therefore, the debt of non-tradable entrepreneurs alone will affect the dynamics of the balance of payments.

$$ y_s^T - \gamma^T \left(\frac{1}{p_s}\right)^{-\varepsilon} \left(C_{w,s} + C_{t,s} + C_{nt,s}\right) - (R^*_s - 1) b_s^T = -\left(b_s^T - b_{s-1}^T\right) \tag{27} $$

2.4 Policy instruments

The presence of borrowing constraints in our model is a structural one. In other words, the values for $\theta^T$ and $\theta^{NT}$ should be treated either as deep parameters or an endogenous response of agents to the frictions present in credit markets. For instance, Kiyotaki and Moore (1997) base the use of collateral in the imperfect enforceability model of Hart and Moore (1994), in which human capital is inalienable. This pushes lenders to demand collateral as a way to protect themselves against the risk of default. For this reason, an authority that employs LTV ratios as a policy instrument faces an upper bound, as it is not possible to force lenders to accept less collateral than the one they privately deem adequate.

---

6 The literature presents several reasons for the use of collateral: moral hazard concerns (Holmstrom and Tirole (1997)), limited contract enforceability (Cooley et al. (2004), Kehoe and Levine (1993), Hart and Moore (1994); costly state verification (Townsend (1979)), and private information (Stiglitz and Weiss (1981), Wette (1983)), among others.
The diagram shows how loan to value rules should be designed. $\theta^{priv}$ stands for the deep parameter that acts as an upper bound for the macroprudential authority. LTV rules involve reducing the average LTV ($\theta^{int}$), in effect making the constraint more binding. As a counterpart, the macroprudential authority will be able to introduce a time-varying component ($\tau$) into its rule.

For this reason, the application of time varying LTV rules must involve two steps. First, the policy value of $\theta$ ($\theta^{int}$) must be set below the private one ($\theta^{priv}$). After that, it is possible to add an additional component ($\tau$), which can be an effective instrument to reduce spill over patterns. Figure 2 displays these family of rules. In the present paper, we focus only on the second-order effects on welfare, which are associated with the aforementioned boom and bust patterns. We consider the following cyclical LTV rule, where the LTV changes with the cyclical evolution of firm’s debt.\(^7\)

\[
\frac{\theta_{T^*,int}^{s}}{\theta_{T^*,int}^{NT, int}} = \frac{\theta_{NT, int}^{s}}{\theta_{NT, int}^{NT}} = \left( \frac{b_s}{b} \right)^{-\phi} \tag{28}
\]

where, $b_s = b_{s,T^*} + b_{s,NT}$.

3 Results

3.1 Baseline calibration

In this section we describe the calibration of the model and assess its quantitative implications. The values assigned to all model’s parameters are listed in Table 2. The discount factor for workers is set to 0.99, which implies an annual interest rate of 4 percent, whereas for the case of entrepreneurs this parameter is set to 0.98, consistent with the assumption that entrepreneurs are more impatient agents than workers in the model. The inverse of the Frisch labour elasticity, $\frac{1}{\eta}$ is set to 1, in line with the microeconomic studies\(^7\) Note that given our assumption of tradable entrepreneurs borrowing from abroad, adjusting $\theta_{T^*}^{s}$ is akin to imposing capital controls.
<table>
<thead>
<tr>
<th>Preferences</th>
<th>Technologies</th>
<th>Collateral constraint</th>
<th>Open economy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta = 0.99$</td>
<td>$\alpha = 0.3$</td>
<td>$\theta^T = 0.6$</td>
<td>$R^* = 1.005$</td>
</tr>
<tr>
<td>$\gamma = 0.98$</td>
<td>$\kappa = 0.3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda = 1$</td>
<td>$\nu = 0.3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\eta = 1$</td>
<td>$\rho_A = 0.7$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma^T = 0.3$</td>
<td>$\rho_S = 0.7$</td>
<td>$\rho_{R^*} = 0.7$</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon = 0.5$</td>
<td>$\psi = 0.3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_{\zeta} = 0.7$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| Showing this parameter should be relatively small. The classification between tradables and non-tradables follows that of Stockman and Tesar (1995). We set $\gamma^T = 0.3$. The share of labor factor is calibrated in 0.4 for the tradable and non-tradable sector. This value is consistent with those in Bernanke and Gurkaynak (2002) who document a range between 0.22 and 0.73 for different emerging economies. In order to estimate the remaining parameters for the production function, we consider the input-output table for the Peruvian economy, and we follow the approach of intermediate and final demand for both sectors. As for the share of housing, we use the participation of construction in the final demand for capital formation process. The remainder is assigned to the capital share. Regarding the collateral constraints, legal limits impose a maximum that ranges from 65 to 90 percent of collateralized debt (this rate depends on the type of asset used as collateral). We set $\theta^T = 0.6$. Productivity shocks in both sectors are assumed to follow first order autoregressive process, with relatively low persistence. Besides the productivity shocks, we consider a foreign interest rate shock which also follows an AR(1) process.  

### 3.2 The dynamics of the model

Figures 3 to 5 show the impulse response functions of the main variables of the model to productivity shocks and a foreign interest rate shock.

In the model, an increase in productivity in the tradable sector (Figure 3) generates an expansion in output both in the tradable and non-tradable sectors and boosts the price of both assets used as collateral. The productivity shock increases the tradable sector’s demand for inputs, increasing the price of housing and labour. Given our assumptions, the positive wealth effect experienced by tradable entrepreneurs increases demand for non-tradable goods, pushing up their price. This generates an appreciation of the real exchange rate.

The real appreciation generates an expansion in the non-tradable sector. This sector now demands more inputs as well, pushing up further the price of capital and labour.

---

King and Rebelo (1999) assume a value of 4 for $\eta$. 

---

15
Note that before the shock, the borrowing constraint implied that the tradable sector needed to hold more capital than necessary from a pure production perspective, because of its usefulness as collateral. When the price of capital increases, the tradable sector’s borrowing constraint is relaxed and its demand for capital relative to housing decreases.

Given the increase in housing prices, the borrowing constraint of the non-tradable sector is relaxed as well, inducing non-tradable firms to increase their leverage. The increase in housing prices also increases the cost of using housing as a production input, inducing entrepreneurs to substitute housing for capital. This decrease in non-tradable firms’ demand for housing is not big enough to outweigh the impact of higher housing prices on non-tradable entrepreneurs net worth and consequently, borrowing by entrepreneurs that operate in the non-tradable sector expands. During the adjustment process, collateral assets are therefore exchanged between the non-tradable and the tradable sector. Non-tradable firms use less housing and the excess is absorbed by tradable firms. Tradable firms liberate some of the capital they were using and it is acquired by their non-tradable counterparts.

Workers experience a positive wealth effect because of the temporary increase in real wages, which induces them to work more and to increase their supply of savings. However, the demand for credit from non-tradable entrepreneurs increases more pushing real domestic interest rates up. The appreciation of the real exchange rate also induces a transitory current account deficit, which is financed by an increase in leverage by the tradable sector entrepreneurs. Positive wealth effects induced by the productivity shock stimulates savings, reducing the domestic interest rate and reverting the initial current account deficit.

In the case of a rise in productivity in the non-tradable sector (Figure 4), the model generates an increase in non-tradable output, a very mild increase in tradable output, a fall in asset prices and a short lived current account surplus consistent with a real depreciation. The key difference between the impact of the non-tradable productivity shock and the tradable productivity shock is that the price of tradable goods is fixed by arbitrage with the (not explicitly modelled) foreign sector while the price of non-tradable goods is determined domestically under perfect competition. Thus, the increase in productivity in the non-tradable sector is assimilated in the form of lower prices, generating a significant depreciation. Notice that asset prices and real wages fall in terms of tradable goods, as consequence of the real depreciation, but increases in terms of the CPI index. As non-tradable entrepreneurs increase their productivity they can produce more using less inputs, their demand for credit falls, pushing real interest rates down.

An increase in the foreign interest rate (Figure 5) tightens the borrowing constraint of tradable firms, forcing a fall in tradable output. Lower input demand by tradable firms leads to a fall in the prices of houses and labour. The negative wealth effect on tradable entrepreneurs reduces demand for non-tradable goods, triggering a real depreciation.

As result, output in the non-tradable sector falls as well, reducing demand for capital and labour further. The decline in demand for credit from the entrepreneurs pushes domestic interest rates down. As it is usually observed when international interest rates rise, the fall in asset prices and the slowdown in economic activity makes difficult for lenders to lend since the creditworthiness of borrowers deteriorated. As a result, credit collapses and savings interest rates fall.

Given tighter borrowing constraints, housing is reallocated from the tradable to the
non-tradable sector, and capital is reallocated from the non-tradable to the tradable sector. The contraction in domestic output and the depreciation that occurs when this shock hits is consistent with the stylized facts observed in emerging market economies during periods of rise in the international interest rate.

Note that this shock is basically the opposite of the tradable productivity shock. The implication is that a fall in the foreign interest rate would produce the same response as that shown in Figure 3: higher asset prices, real depreciation and a boom in the non-tradable sector coupled with higher debt.

3.3 The role of borrowing constraints

In order to illustrate the role that borrowing constraints play in the model, Figure 6 shows the dynamics of the model considering different values for $\theta$. A larger $\theta$ implies that borrowing constraints are less restrictive for entrepreneurs’ decisions, consequently, spillover effects should be less important. As this figure shows, when $\theta$ is relative large ($\theta = 0.95$), the model generate less spillover effects. Output in the non-tradable sector increases less and asset prices’ responses to a positive productivity shock in the tradable sector are milder.

Interestingly, the real exchange rate appreciates much less in this case and the positive correlation between tradable and non-tradable sector that the model generates also falls as $\theta$ rises. The opposite is observed when $\theta$ is relative low; the real exchange rate appreciates substantially, the correlation between the tradable and non-tradable sector output increases and the debt in the non-tradable sector expands more. Asset prices, housing and capital prices also increase more when $\theta$ is smaller.

In the case of the non-tradable productivity shock, Figure 7, asset prices and non-tradable debt are also much less responsive to this shock when $\theta$ is large ($\theta = 0.95$) than when this parameter is small ($\theta = 0.25$). This result highlights the role of borrowing constraints in explaining asset price volatility and fluctuations in borrowing in the non-tradable sector that the model generates.

In the case of the foreign interest shock, we observe the opposite, most of the variables of the economy are less responsive to this shock when borrowing constraints are tighter, which also can be interpreted as the case of an economy less integrated with international financial markets. As Figure 8 shows, output both in the tradable and non-tradable sector and asset prices fall less when $\theta$ is small. This is also the case for the real exchange rate that experiments a milder depreciation in this case.

3.4 Robustness checks

Figures 9, 10, 11 and 12 show robustness exercises to changes in three key parameter values, $\beta$, the degree of impatience of workers, $\gamma$, the degree of impatience of entrepreneurs, $\varepsilon$, the elasticity of substitution between tradable and non-tradable goods and $\gamma^T$, the participation of tradable goods in the consumption basket.

For the first parameter we take as a low value, $\beta = 0.985$, and as a high value, $\beta = 0.995$. A similar set of values is considered for $\gamma$, whereas for $\varepsilon$ we consider as a low value, $\varepsilon = 0.1$ and as a high value, $\varepsilon = 4$. For $\gamma^T$ the following two alternate values are used in the simulation: $\gamma^T = 0.1$ and $\gamma^T = 0.7$. The simulation results show that
the spillover effect is robust for most of the parameter values explored. It tends to be lower when tradable and non-tradable goods exhibit a higher degree of substitutability, however the spillover effects still prevail.

The other parameters do not affect significantly the magnitude of the spillover effects, they only change the relative response of the demand for houses and capital of entrepreneurs across the tradable and non-tradable sectors.

### 3.5 The role of macro-prudential policies

Figure 3 shows the effect that the loan-to-value policy rule defined in equation (28) would have on the dynamics that our model generates in response to a tradable productivity shock.

An LTV rule targeting aggregate credit does a good job dampening the spillover from the tradable to the non-tradable sector in the aftermath of a tradable productivity shock. Aggregate output is barely affected, but there is a sizeable dampening on asset prices and to a lower extent on the real exchange rate. Tighter LTV ratios imposed on the economy manage to curtail the expansion in debt in both sectors but the effect is bigger on non-tradable firms. Borrowing taken by these firms increases less, forcing non-tradable entrepreneurs to hold on to their houses.

For further examination of the model under LTV rules, we solve the model using a second order approximation around the non-stochastic steady state. We simulate the paths for a series of key variables. Focusing on output, the countercyclical rule reduces its volatility. Table 3 reports model generated coefficients of variation for aggregate, tradable and non-tradable output under different assumptions regarding which shocks hit the economy. Except for the case of an economy subject to only non-tradable shocks, the introduction of rules reduces volatility for all indicators. In the case of the tradable output under non-tradable shocks, the former is barely affected by non-tradable shocks.

Table 3 also shows the second order effects on welfare. This measure is the difference between the mean welfare measure and its non-stochastic steady-state value. We use this measure since we are interested in the effects that policy has through the reduction of spillover effects. The parameter of the rule is chosen optimally to i) maximize an aggregate measure of welfare that weights the welfare of the three agents in the economy based on their share of aggregate consumption and ii) minimize aggregate output volatility. Results show that the introduction of a countercyclical macroprudential policy rule generates strong redistribution effects. Namely, its use produces welfare increases for a subset of agents in the economy, while the rest suffer a reversal. Which agents are favoured by the rule depends on the source of the shocks and how limiting the borrowing constraints are, captured by $\theta$. For example, when all shocks are taken into account, imposing the countercyclical rule on an economy makes the entrepreneurs better off and the workers worse off. This outcome is reversed when $\theta$ is high. The intuition is that at low values of $\theta$ the entrepreneurs are very constrained and shocks generate high domestic interest rate fluctuations which disappear at high levels of $\theta$.

Table 3 also shows the impact of the LTV rule on output volatility and on the correlation between tradable and non-tradable output. In all cases, the optimal LTV rule reduces the volatility of aggregate output and the correlation between tradable and non-tradable output, which indicates that LTV reduces aggregate volatility by reducing the spillover
effects that the existence of borrowing constraints generate in the model.

3.6 Other policy instruments

We also consider taxes to the consumption of non-tradable good as an alternative policy tool aimed to reduce spillover effects. In particular, we evaluate the performance of a tax rule that responds to changes in the relative price of the non-tradable goods (the inverse of the real exchange rate), as follows:\(^9\).

\[
\frac{\tau_s}{\gamma} = \left(\frac{p_s^{NT}}{P_s} \right)^\phi \tag{29}
\]

We further assume that the income from the collection of this tax is reverted to entrepreneurs and workers as lump sum transfers. In order to avoid wealth effects associated with the implementation of the tax policy, entrepreneurs and workers receive as transfers the same amount they paid as taxes, as follows:

\[
P_s^{NT} y_s^{NT} + b_s^{NT} + T_s^{NT} = C_{nt,s} + \frac{q_h^s}{P_s} \Delta h_s^{NT} + \frac{q_k^s}{P_s} \Delta k_s^{NT} + R_{s-1} \frac{b_s^{NT}}{P_{s-1}} + \frac{w_s}{P_s} t_s^{NT}, \tag{30}
\]

\[
T_s^{NT} = \tau_s p_s^{NT} \left(1 - \gamma^T\right) \left(1 + \tau_s \frac{p_s^{NT}}{P_s}\right)^{-\varepsilon} C_{nt,s} \tag{31}
\]

\[
\frac{1}{P_s} y_s^T + \frac{1}{P_s} b_s^T + T_s^T = C_{l,s} + \frac{q_h^s}{P_s} \Delta h_s^T + \frac{q_k^s}{P_s} \Delta k_s^T + R_s^* \frac{b_s^T}{P_{s-1}} + \frac{w_s}{P_s} t_s^T, \tag{32}
\]

\[
T_s^T = \tau_s p_s^{NT} \left(1 - \gamma^T\right) \left(1 + \tau_s \frac{p_s^{NT}}{P_s}\right)^{-\varepsilon} C_{l,s} \tag{33}
\]

\[
C_{w,s} + \frac{b_s^{NT}}{P_s} = R_{s-1} \frac{b_s^{NT}}{P_{s-1}} + \frac{w_s}{P_s} t_s + T_s^W \tag{34}
\]

\[
T_s^W = \tau_s p_s^{NT} \left(1 - \gamma^T\right) \left(1 + \tau_s \frac{p_s^{NT}}{P_s}\right)^{-\varepsilon} C_{w,s} \tag{35}
\]

The tax instrument introduced also affects the definition of CPI and the aggregate demand of non-tradable goods, as follows:

\[
P_s \equiv \left[\gamma^T + (1 - \gamma^T) \left(1 + \tau_s \frac{p_s^{NT}}{P_s}\right)^{-\varepsilon}\right]^{1/\gamma} \tag{36}
\]

\[
y_s^{NT} = \left(1 - \gamma^T\right) \left(1 + \tau_s \frac{p_s^{NT}}{P_s}\right)^{-\varepsilon} \left(C_{w,s} + C_{l,s} + C_{nt,s}\right) \tag{37}
\]

The results of the simulations are presented in Figures 3, 4 and 5 that show the dynamics of the main variables of the model in comparison with the baseline model and with the LTV rule.

The simulation is conducted for the three shocks previously considered. As these figures show, this particular tax policy dampens the spillover effect and significantly

\(^9\)We consider this to be a form of price support policy in the spirit of Benigno et al. (2013) and
reduces the volatility of the real exchange rate and that of asset prices in the case of the three shocks analyzed. The key mechanism that generates this result is the effect that this tax rule has on the demand of non-tradable goods. As taxes for the consumption of non-tradable goods increase when the price of this goods increases, the demand for these goods increases less in response to an increase in the productivity shock in the tradable sector or a fall in the foreign interest rate. A minor increase in the demand for non-tradable goods also reduces the pressure over the inputs demand of non-tradable entrepreneurs, which generates a lower increase in asset prices and in the real exchange rate.

In the case of the foreign interest rate shock, the tax rule also generates a minor contraction in output and in asset prices. Interestingly, non-tradable debt also falls less when the tax rule is in place in comparison with the baseline case. Also, Table 4 shows the impact of this rule in the volatility of output and welfare for the case of the optimal value for the policy rule. The results of the simulation show that this tax rule reduces the volatility of aggregate output for the cases of the foreign interest rate and tradable productivity shocks. Also, the correlation between output in the tradable and non-tradable sectors changes from positive, which indicates spillovers effects, to negative. Notice that the tax rule considered improves equilibrium allocations by reducing the volatility of the exchange rate, and through this mechanism by reducing the volatility of asset prices and the positive correlation between the tradable and non-tradable sector that borrowing constraints generate in the baseline model.

In terms of welfare, the rule increases the welfare of non-tradable entrepreneurs and workers but it reduces the welfare of tradable entrepreneurs. This is because the tax rule reduces the volatility of non-tradable output but increases that of output in the tradable sector.

4 Introducing nominal rigidities

In order to study the interaction of monetary and macroprudential policy in our model, we introduce nominal rigidities to the baseline set up. To ensure results are comparable, we flesh out a non-tradable goods retailer (NTR) sector subject to imperfect competition that sets prices a la Calvo. Furthermore, we impose a lump-sum tax on workers to subsidize the acquisition of “wholesale” non-tradable goods by the retailer from the producers/entrepreneurs, restoring the perfect competition allocation.

When tapped by the Calvo fairy - with probability \((1-\omega)\) - retailers set price \(\tilde{P}_{s}^{NTR}(z)\) to maximize the expected present discounted value of real profits defined as follows:

\[
\Pi_{s}^{NTR} = \mathbb{E}_{s} \left[ \sum_{k=0}^{\infty} (\omega)^{k} \Lambda_{s+k}^{W} \frac{P_{s+k}^{NTR} \tilde{P}_{s}^{NTR}(z)}{P_{s+k}^{NTR}} \left( \left( \frac{\tilde{P}_{s}^{NTR}(z)}{P_{s}^{NTR}} ight) F_{NT,s+k} - \left(1 - \tau\right) mc_{s+k} \right) c_{NT,s+k}^{NT}(z) \right] 
\]

where:

\[
F_{NT,s+k} = \frac{P_{s+k}^{NTR}}{P_{s+k}^{NTR}}
\]
is the inverse of accumulated retailer non-tradable good inflation between \( s + k \) and \( s \),

\[ \Lambda_{s+k}^W = \beta^k \Lambda_{s+k}^W \]

is the workers’ real stochastic discount factor,

\[ m_{c_{s+k}} = \frac{P_{s+k}^{NT}}{P_{s+k}^{NTR}} \]

is marginal cost expressed in retailer non-tradable goods,

\[ c_{s+k}^{NT}(z) = \left( \frac{P_{s+k}^{NTR}(z)}{P_{s+k}^{NTR}} F_{NT,s+k}^{1-\xi} c_{s+k}^{NT} \right)^{-\xi} \]

is the demand of retailer \( z \) and \( \tau \) is a subsidy per unit.

The first order condition of this problem is given by:

\[
E_s \left[ \sum_{k=0}^{\infty} (\omega)^k \Lambda_{s+k}^W \frac{P_{s+k}^{NTR}}{P_{s+k}^{NTR}} \left( \frac{P_{s+k}^{NTR}(z)}{P_{s+k}^{NTR}} F_{NT,s+k}^{1-\xi} c_{s+k}^{NT} \right) \right] = 0
\]

which can be written as follows:

\[
\tilde{P}_{s}^{NTR} = \mu (1 - \tau) \frac{E_s \left[ \sum_{k=0}^{\infty} (\omega)^k \Lambda_{s+k}^W \frac{P_{s+k}^{NTR}}{P_{s+k}^{NT}} F_{NT,s+k}^{1-\xi} c_{s+k}^{NT} \right]}{E_s \left[ \sum_{k=0}^{\infty} (\omega)^k \Lambda_{s+k}^W \frac{P_{s+k}^{NTR}}{P_{s+k}^{NT}} F_{NT,s+k}^{1-\xi} c_{s+k}^{NT} \right]} \equiv \mu (1 - \tau) \frac{V_s^N}{V_s^D}
\]

where:

\[ \mu = \frac{\xi}{(\xi - 1)} \]

and \( V_s^N, V_s^D \) can be defined recursively as follows:

\[
V_s^N = \frac{P_{s}^{NTR}}{P_s} m_{c_{s}}^{NT} + \omega \beta E_s \left[ \frac{\Lambda_{s+1}^W}{\Lambda_s^W} (\Pi_{NT,s+1})^\xi V_{s+1}^N \right]
\]

\[
V_s^D = \frac{P_{s}^{NTR}}{P_s} c_{s}^{NT} + \omega \beta E_s \left[ \frac{\Lambda_{s+1}^W}{\Lambda_s^W} (\Pi_{NT,s+1})^{\xi-1} V_{s+1}^D \right]
\]

Using the law of large numbers, the price level of retailer non-tradables can be written as follows (note that the optimal price is the same for all \( z \) that change price in time \( s \)):

\[
P_{s}^{NTR} = \left( 1 - \omega \right) \left( \tilde{P}_{s}^{NTR} \right)^{1-\xi} + \omega \left( P_{s-1}^{NTR} \right)^{1-\xi}\]

The optimal subsidy is the one that eliminates distortion imposed by monopolistic competition (the markup): \( 1 - \tau = \frac{\xi-1}{\xi} \) which implies \( \tau = \frac{1}{\xi} \).
The retailer’s aggregate supply is

\[ \int_0^1 c_s^{NT}(z) \, dz = \left( \frac{\hat{P}_s^{NTR}}{P_s^{NTR}} \right)^{-\xi} c_s^{NT} \]  

(41)

where

\[ \hat{P}_s^{NTR} = \left( (1 - \omega) \left( \tilde{P}_s^{NTR} \right)^{-\xi} + \omega \left( \hat{P}_{s-1}^{NTR} \right)^{-\xi} \right)^{-\frac{1}{\xi}} \]

(42)

is an alternative price index.

Thus, equilibrium in the wholesale market requires:

\[ \int_0^1 c_s^{NT}(z) \, dz = y_s^{NT} \]

implying

\[ \left( \frac{\hat{P}_s^{NTR}}{P_s^{NTR}} \right)^{-\xi} c_s^{NT} = y_s^{NT} \]

(43)

Aggregate retailers’ real profits \( (\Xi_s^{NT}) \) are

\[ \frac{\Xi_s^{NTR}}{P_s} = \frac{P_s^{NTR}}{P_s} c_s^{NT} - (1 - \tau) \frac{P_s^{NT}}{P_s} y_s^{NT} \]

Thus, the (real) lump-sum tax we need to levy from households is

\[ T_s = \tau \frac{P_s^{NT}}{P_s} y_s^{NT} \]

Finally, closing the model now requires a Taylor rule to pin down the domestic nominal interest rate:

\[ \frac{R_s}{\bar{R}} = \left( \frac{R_s - 1}{\bar{R}} \right) \left( \frac{\Pi_s}{\Pi} \right) \phi_s^{1-\rho_R} \exp(\epsilon_s^{MON}) \]

In order to maintain consistency, some of the other equations in the model require adjustments as well. Those are shown in Appendix B.

4.1 Nominal rigidities and spillover effects

Figure 13, compares the dynamic response of the model to a productivity shock in the tradable sector for two models, the baseline model with flexible prices and the model with nominal rigidities in the non-tradable sector.

The introduction of sticky prices in the non-tradable sector dampens the response of the price of non-tradable goods to the tradable productivity shock. This has a series of implications:

• House prices no longer jump as a result of the shock. The reason is that house prices can be interpreted as a present discounted value of present and future marginal
product of housing in the non-tradable sector (see the equation that characterizes housing demand in the non-tradable sector). The marginal product of housing in that sector is valued using the price of wholesale non-tradable goods and those no longer increase as much given that some of the stickiness in retail prices translates partially to wholesale prices.

- As a result, demand for housing in the non-tradable sector falls further (compared to the flexible price case) and thus, domestic debt in the non-tradable sector can no longer expand (these agents now hold less collateral and its value has not increased much).

- Output in the non-tradable sector decreases initially because of lower credit availability but quickly starts responding to demand for non-tradable goods. The positive effect on said demand of the tradable productivity shock is amplified when there are nominal rigidities because there is no big increase in non-tradable good prices to compensate the surge in demand that is generated by the wealth effect on tradable entrepreneurs.

- Capital prices no longer increase because their value as collateral has decreased. In the model with flexible prices, the relative price of tradable goods does not change when the tradable productivity shock hits. In the nominal version of the model, however, appreciation has a negative impact on tradable entrepreneurs’ demand for capital. The value of the marginal product is lower in the future and given that capital is valued in foreign currency in its role as collateral, that component of demand decreases as well.

Figure 14 explores how the degree of nominal rigidity affects the spillover effect. When prices are stickier (higher \( \omega \)), the spillover from the tradable to the non-tradable sector increases. This is in line with the idea that stickier prices imply a stronger demand for non-tradable goods as a result of the wealth effect generated by the tradable productivity shock.

Figure 15 shows the effect of the degree of “hawkishness” of the central bank. A lower inflation coefficient on the Taylor rule reduces the spillover effect. Note that this implies a lower commitment of the central bank to price stability which can be interpreted by the agents as a stronger response of non-tradable good prices to the tradable productivity shock.

Finally, figure 16 shows the interaction of monetary policy and our macroprudential rules. The LTV rule is the same one used in the model with flexible prices and, counter-intuitively, amplifies the spillover effect. The reason is that tighter LTVs in response to the tradable productivity shock amplify the effect of the shock on asset prices (particularly housing), allowing the non-tradable producer to substitute housing and capital for labour without reducing his debt levels.

The tax rule depicted in figure 16 is slightly different to the one described in the flexible price model. This time, when the wholesale price of non-tradable goods increases, we impose a tax on them and return the proceeds lump-sum to the non-tradable entrepreneurs. The result is a dampened spillover effect. Thus, imposing this macroprudential rule allows the central bank to pursue price stability more aggressively (become more “hawkish”) without generating a bigger (and potentially inefficient) spillover effect.
5 Conclusions

This paper investigates the interaction between over borrowing, housing prices, and spillovers between tradable and non-tradable sectors within a general equilibrium framework. The key contribution of the paper is to show that when it is difficult to enforce debtors to repay their debt unless it is secured by collateral, a productivity shock in the tradable sector generates an increase in asset prices and leverage that spills over to the non-tradable sector and appreciates the real exchange rate. The appreciation of the exchange rate and the increase in housing prices further reinforces this mechanism by increasing the ability of non-tradable firms to increase their leverage. As a result, the economy experiences a large increase in leverage and credit in the non-tradable sector. All these effects are consistent with the empirical evidence.

In the model, the aforementioned dynamic response of the economy to a positive productivity shock in the tradable sector is similar to the response that an increase in commodity prices would generate for economies where the tradable sector production is mostly commodities. Therefore, the model simulations can also be interpreted as showing a positive correlation between credit boom in the non-tradable sector and terms of trade, a stylized fact observed in many commodity producer economies, such as Chile, Peru and Canada.

In the case of a rise in productivity in the non-tradable sector, the model generates an increase in non-tradable output, which spills over to the tradable sector, a fall in asset prices and lower demand for credit in the non-tradable sector. The fall in the demand for credit in the non-tradable sector firms in turn pushes domestic interest rates down.

On the other hand, an increase in the foreign interest rate tightens the borrowing constraint of tradable firms, forcing a fall in tradable output. Lower input demand by tradable firms leads to a fall in the price of houses and labour. The negative wealth effect on tradable entrepreneurs reduces demand for non-tradable goods, triggering a real depreciation.

On the policy side, we show that macro-prudential instruments under the form of cyclical loan-to-value ratios that limit the amount of capital that entrepreneurs can pledge as collateral can dampen the effects of borrowing constraints.

We also consider taxes as an alternative policy tool to reduce the externalities that borrowing constraints generate in a small open economy, this particular tax policy reduces significantly the volatility of the real exchange rate and that of asset prices, limiting the spillover effects between the tradable and non-tradable sector. The simulation results are also consistent with the welfare calculations under this tax policy that show potential welfare gains of reducing asset price and exchange rate volatility in economies with tight borrowing constraints, particularly for non-tradable entrepreneurs and workers.
References


A Model Equations

In this appendix we present a compiled list of all the equations in the model in non-linear form.

A.1 Worker problem

\[ \max L = E_0 \sum_{s=0}^{\infty} \beta^s \left[ \ln C_{w,s} - \lambda \frac{(l_s)^{1+\eta}}{1+\eta} + \lambda W_s \left( -C_{w,s} - b_{NT} \frac{P_s}{P_s} + R_{s-1} \frac{b_{NT}}{P_s} + w_s l_s \right) \right] \]

where \( \lambda_w \) is the current value lagrangian multiplier associated to the worker’s budget constraint. The first order conditions of this problem are:

\[ C_{w,s} + b_{NT} \frac{P_s}{P_s} = R_s - b_{NT} \frac{P_s}{P_s} + w_s l_s \]  
(A.1)

\[ \lambda_w = \frac{1}{C_{w,s}} \]  
(A.2)

\[ w_s \frac{1}{P_s C_{w,s}} = \lambda (l_s)^\eta \]  
(A.3)

\[ \frac{1}{R_s} = \beta E_s \left[ \frac{\lambda_{w+1}}{\lambda_w} \frac{P_s}{P_{s+1}} \right] \]  
(A.4)

Where the price level, \( P_s \), is defined as:

\[ P_s \equiv \left[ \gamma^T + (1 - \gamma^T) (p_{NT}^{NT})^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \]  
(A.5)

A.2 Non-tradable entrepreneur’s problem

\[ \mathcal{L} = E_0 \sum_{s=0}^{\infty} \gamma^s \left[ -C_{nt,s} - q_h \frac{P_s}{P_s} \Delta h_{sNT} - q_k \frac{P_s}{P_s} \Delta k_{sNT} - R_{s-1} \frac{b_{NT}}{P_s} + \frac{b_{NT} P_s}{P_{s+1}} + b_{NT} \frac{P_{s+1}}{P_s} l_{sNT} \right] \]

\[ + \lambda_{NT} \left( -C_{nt,s} - q_h \frac{P_s}{P_s} \Delta h_{sNT} - q_k \frac{P_s}{P_s} \Delta k_{sNT} - R_{s-1} \frac{b_{NT}}{P_s} + \frac{b_{NT} P_s}{P_{s+1}} l_{sNT} \right) \]

\[ + \mu_{NT} \left( -R_s b_{NT} \frac{P_{s+1}}{P_{s+1}} + \theta_s^{NT} q_{s+1}^{NT} h_{sNT} \right) \]

where \( \lambda_{NT} \) and \( \mu_{NT} \) are the current value lagrangian multipliers associated to the non-tradable entrepreneur’s budget and borrowing constraint, respectively. The first order conditions of this problem are:

\[ \frac{P_s}{P_s} y_{sNT} + b_{NT} \frac{P_s}{P_s} = C_{nt,s} + q_h \frac{P_s}{P_s} \Delta h_{sNT} + q_k \frac{P_s}{P_s} \Delta k_{sNT} + R_{s-1} \frac{b_{NT}}{P_s} + \frac{b_{NT} P_s}{P_{s+1}} l_{sNT} \]  
(A.6)

\[ R_s b_{NT} \frac{P_{s+1}}{P_{s+1}} \leq \theta^{NT} q_{s+1}^{NT} h_{sNT} \]  
(A.7)
where

The non-tradable entrepreneur’s production function is:

\[
P = \frac{1}{C_{nt,s}} (A.8)
\]

\[
q_s^h = \gamma E_s \left[ \frac{\lambda_{NT}^s q_s^{h+1}}{\lambda_s^T P_{s+1}} \right] + \gamma E_s \left[ \frac{\lambda_{NT}^s p_{s+1}^{NT} \partial y_s^{NT}}{\lambda_s^T P_{s+1} \partial h_s^{NT}} \right] + \mu_{NT}^s \theta_{NT}^s E_s \left[ \frac{q_s^{h+1}}{P_{s+1}} \right] (A.9)
\]

\[
q_s^k = \gamma E_s \left[ \frac{\lambda_{NT}^s q_s^{k+1}}{\lambda_s^T P_{s+1}} \right] + \gamma E_s \left[ \frac{\lambda_{NT}^s p_{s+1}^{NT} \partial y_s^{NT}}{\lambda_s^T P_{s+1} \partial k_s^{NT}} \right] (A.10)
\]

\[
w_s^h = \gamma E_s \left[ \frac{\lambda_{NT}^s q_s^{h+1} \partial y_s^{NT}}{\lambda_s^T P_{s+1} \partial h_s^{NT}} \right] (A.11)
\]

\[
1 \frac{R_s^T}{P_s} = \gamma E_s \left[ \frac{\lambda_{NT}^s p_s^{NT} \partial y_s^{NT}}{\lambda_s^T P_{s+1} \partial \lambda_k^{NT}} \right] + \mu_{NT}^s \theta_{NT}^s E_s \left[ \frac{P_s}{P_{s+1}} \right] (A.12)
\]

The non-tradable entrepreneur’s production function is:

\[
y_s^{NT} = \zeta_{s-1} \left( h_{s-1}^{NT} \right)^{\alpha} \left( k_{s-1}^{NT} \right)^{\kappa} \left( \gamma_{s-1}^{NT} \right)^{1-\alpha-\kappa} (A.13)
\]

A.3 Tradable entrepreneur’s problem

\[
\max \mathcal{L} = E_0 \sum_{s=0}^{\infty} \gamma^s \left[ +\lambda_s^T \left( -C_{t,s} - \frac{\phi_T^T}{\psi_s} \Delta h_s^T - \frac{\phi_T^T}{\psi_s} \Delta k_s^T - R_s^T \frac{1}{P_s} b_{s-1}^T + \frac{w_s^T}{P_s} \right) + \mu_s^T \left( \frac{R_s^T}{P_s} \frac{1}{P_s} b_{s-1}^T + \theta_s^T \frac{q_s^{h+1}}{P_s} \frac{1}{P_s} k_{s+1}^T \right) \right]
\]

where \( \lambda_s^T \) and \( \mu_s^T \) are the current value lagrangian multipliers associated to the tradable entrepreneur’s budget and borrowing constraint, respectively. The first order conditions of this problem are:

\[
1 \frac{1}{P_s} y_s^T + 1 \frac{1}{P_s} b_{s-1}^T = C_{t,s} + \frac{q_s^h}{P_s} \Delta h_s^T + \frac{q_s^k}{P_s} \Delta k_s^T + R_s^T \frac{1}{P_s} b_{s-1}^T + \frac{w_s^T}{P_s} (A.14)
\]

\[
R_s^T \frac{1}{P_s} b_{s+1}^T \leq \theta_s^T \frac{q_s^{h+1}}{P_s} \frac{1}{P_s} k_{s+1}^T (A.15)
\]

\[
\lambda_s^T = \frac{1}{C_{t,s}} (A.16)
\]

\[
q_s^h = \gamma E_s \left[ \frac{\lambda_{s+1}^T q_{s+1}^h}{\lambda_s^T P_{s+1}} \right] + \gamma E_s \left[ \frac{\lambda_{s+1}^T p_{s+1}^{NT} \partial y_{s+1}^{NT}}{\lambda_s^T P_{s+1} \partial h_{s+1}^{NT}} \right] (A.17)
\]

\[
q_s^k = \gamma E_s \left[ \frac{\lambda_{s+1}^T q_{s+1}^k}{\lambda_s^T P_{s+1}} \right] + \gamma E_s \left[ \frac{\lambda_{s+1}^T p_{s+1}^{NT} \partial y_{s+1}^{NT}}{\lambda_s^T P_{s+1} \partial k_{s+1}^{NT}} \right] + \mu_s^T \psi_s^T \theta_s^T \psi_s E_s \left[ \frac{q_{s+1}^h}{P_{s+1}} \right] (A.18)
\]

\[
w_s^h = \gamma E_s \left[ \frac{\lambda_{s+1}^T \psi_{s+1} \partial y_{s+1}^{NT}}{\lambda_s^T P_{s+1} \partial \psi_s} \right] (A.19)
\]

\[
1 \frac{R_s^T}{P_s} = \gamma E_s \left[ \frac{\lambda_{s+1}^T P_s}{\lambda_s^T P_{s+1}} \right] + \mu_s^T \psi_s^T \psi_s E_s \left[ \frac{P_s}{P_{s+1}} \right] (A.20)
\]

The tradable entrepreneur’s production function is:

\[
y_s^T = A_{s-1} \left( h_{s-1}^T \right)^{\nu} \left( k_{s-1}^T \right)^{\psi} \left( \gamma_{s-1}^T \right)^{1-\nu-\psi} (A.21)
\]
A.4 Market equilibrium

\[ l_s = l_s^{NT} + l_s^T \]  
\[ y_s^{NT} = (1 - \gamma^s) \left( \frac{P_s^{NT}}{P_s} \right)^{-\epsilon} (C_{w,s} + C_{t,s} + C_{nt,s}) \]  
\[ H = h_s^{NT} + h_s^T \]  
\[ K = k_s^T + k_s^{NT} \]

Equations (A.1) to (A.25) describe a system in 28 endogenous variables\(^{10}\). Additionally, we consider 6 exogenous processes:

\[ R_s^* = \overline{R}^* + \rho_R \left( R_{s-1}^* - \overline{R}^* \right) + \epsilon_s^R \]  
\[ \log \zeta_s = \rho_\zeta \left( \log \zeta_{s-1} \right) + \epsilon_s^\zeta \]  
\[ \log A_s = \rho_A \left( \log A_{s-1} \right) + \epsilon_s^A \]

For \( \theta^{NT} \) and \( \theta^{T*} \), AR(1) processes and several different rules are considered.

---

\(^{10}\)These are: \( P, C_w, y^T, p^{NT}, w, t, q^k, q^h, h^W, y^{NT}, C_{nt}, h^{NT}, l^{NT}, l^T, h^T, k^T, h^{NT}, m^T, m^{NT}, C_t, R, b^{NT}, b^{T*}, \lambda^W, \lambda^T, \lambda^{NT}, \mu^{NT} \) and \( \mu^{T*} \).
B Required changes to incorporate nominal rigidities to the model

B.1 Worker problem

The new price level, $P_s$, is defined implicitly according to:

$$1 \equiv \left[ \gamma^T \left( \frac{P_T^s}{P_s} \right)^{1-\varepsilon} + \left( 1 - \gamma^T \right) \left( \frac{P_{NTR}^s}{P_s} \right)^{1-\varepsilon} \right]^\frac{1}{1-\varepsilon}$$

where

$$P_T^s = NER_s P_T^*$$
$$P_{NTR}^s = P_s^*$$

 Tradable prices in domestic currency, $P_T^s$, are determined according to purchasing power parity and the foreign price of tradables ($P_T^*$) is assumed identical to the foreign price index ($P_s^*$). $NER$ denotes the nominal exchange rate. Note that the real exchange rate, $RER$, becomes

$$RER_s = \frac{NER_s P_s^*}{P_s} = \frac{P_T^s}{P_s}$$

The consumption basket of final goods is a composite of domestic and foreign goods, aggregated using the following consumption index:

$$C_{w,s} \equiv \left[ \left( \gamma^T \right)^{\frac{1}{\varepsilon}} \left( C_{w,s}^T \right)^{\frac{\varepsilon-1}{\varepsilon}} + \left( 1 - \gamma^T \right)^{\frac{1}{\varepsilon}} \left( C_{w,s}^{NT} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^\frac{\varepsilon}{\varepsilon-1}, \quad (B.1)$$

where $\varepsilon$ is the elasticity of substitution between tradable ($C_{w,s}^T$) and non-tradable goods ($C_{w,s}^{NT}$), and $\gamma^T$ is the share of tradable goods in the consumption basket of the domestic economy. The worker’s optimal demands for tradable and non-tradable consumption are given by:

$$C_{w,s}^T = \gamma^T \left( \frac{P_T^s}{P_s} \right)^{-\varepsilon} C_{w,s} \quad (B.2)$$
$$C_{w,s}^{NT} = \left( 1 - \gamma^T \right) \left( \frac{P_{NTR}^s}{P_s} \right)^{-\varepsilon} C_{w,s} \quad (B.3)$$

Demand for a particular variety of non-tradable good ($C_{w,s}^{NT} (z)$) is given by

$$C_{w,s}^{NT} (z) = \left( \frac{P_{NTR}^s (z)}{P_s} \right)^{-\xi} C_{w,s}^{NT}$$

where $\xi$ is the elasticity of substitution between varieties.
B.2 Non-tradable entrepreneur’s problem

We only need to replace this agent’s demand for housing with:

\[
\frac{Q_h}{P_s} = \gamma E_s \left[ \frac{\lambda^{NT}_{s+1} Q_h^{NT}}{\lambda^{NT}_{s} P_{s+1}} \right] + \gamma E_s \left[ \frac{\lambda^{NT}_{s+1} P^{NT}_{s+1} \partial y^{NT}_{s+1}}{\lambda^{NT}_{s} P_{s+1} \partial h^{NT}_{s}} \right] + \frac{\mu^{NT}_{s+1} \bar{\theta}^{NT}_{s} E_s}{\lambda^{NT}_{s}} \left[ \frac{Q_h^{NT}}{P_{s+1}} (\Pi_{s+1}) \right] \tag{B.4}
\]

where \( \Pi_s \) denotes gross domestic inflation:

\[
\Pi_s = \frac{P_s}{P_{s-1}}
\]

B.3 Tradable entrepreneur’s problem

Changes to the tradable entrepreneur’s budget constraint arise because the foreign price of tradables is assumed fixed implying that fluctuations in the nominal exchange rate have an effect on the domestic price of tradables, \( P^*_T \). Foreign debt, \( B^{T*}_s \), is now assumed denominated in foreign currency. The budget constraint becomes:

\[
RER_s y^T_s + \frac{RER_s}{P^*_s} B^{T*}_s = C_{t,s} + \frac{Q^h}{P^*_s} \Delta h^T_s + \frac{Q^k}{P^*_s} \Delta k^T_s + \frac{RER_s}{P^*_s} R^{T*}_{s-1} B^{T*}_{s-1} + \frac{W_s}{P^*_s} \theta^T_s
\]

while the borrowing constraint is now expressed as

\[
R^*_s \frac{RER_s}{P^*_s} B^{T*}_s \leq \theta^{T*}_s E_s \left[ \frac{Q^k}{P_{s+1}^{NT}} \frac{RER_s}{P_{s+1}^{NT} R_{s+1}^{NT}} (\Pi^{NT}_{s+1}) \right] \theta^{T*}_s
\]

where \( \Pi^*_s \) denotes gross foreign inflation. Tradable entrepreneur’s factor demands change to incorporate the effects of depreciation:

\[
\frac{Q^h}{P^*_s} = \gamma E_s \left[ \frac{\lambda^{T}_{s+1} Q^h_{s+1}}{\lambda^{T}_{s} P_{s+1}} \right] + \gamma E_s \left[ \frac{\lambda^{T}_{s+1} RER_{s+1} \partial y^{T}_{s+1}}{\lambda^{T}_{s} \partial h^{T}_{s}} \right]
\]

\[
\frac{Q^k}{P^*_s} = \gamma E_s \left[ \frac{\lambda^{T}_{s+1} Q^k_{s+1}}{\lambda^{T}_{s} P_{s+1}} \right] + \gamma E_s \left[ \frac{\lambda^{T}_{s+1} RER_{s+1} \partial y^{T}_{s+1}}{\lambda^{T}_{s} \partial k^{T}_{s}} \right] + \frac{\mu^{T*}_{s} \bar{\theta}^{T*}_{s} E_s}{\lambda^{T}_{s}} \left[ \frac{Q^k}{P_{s+1}^{NT}} \frac{RER_s}{P_{s+1}^{NT} R_{s+1}^{NT}} (\Pi^{NT}_{s+1}) \right] \tag{B.5}
\]

His Euler equation can now be expressed as,

\[
\frac{1}{R^*_s} = \gamma E_s \left[ \frac{\lambda^{T}_{s+1} RER_{s+1} \partial y^{T}_{s+1}}{\lambda^{T}_{s} \partial \theta^{T*}_s} \right] + \mu^{T*}_{s} \bar{\theta}^{T*}_{s}
\]

B.4 Market equilibrium

Tradable goods market equilibrium now requires

\[
\left( \frac{\hat{P}^{NT}_{s}}{P^{NT}_{s}} \right)^{\xi} y^T_s = (1 - \gamma^T) \left( \frac{P^{NT}_s}{P_s} \right)^{-\varepsilon} (C_w,s + C_{t,s} + C_{nt,s}) \tag{B.6}
\]
### C Figures and Tables

#### Table 3: Results: LTV rule responding to credit

<table>
<thead>
<tr>
<th>Variable</th>
<th>All shocks</th>
<th>Trad. prod. shock</th>
<th>For. interest rate shock</th>
<th>Non – Trad. prod. shock</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\phi = 0$</td>
<td>$\phi_{opt,1}$</td>
<td>$\phi_{opt,2}$</td>
<td>$\phi = 0$</td>
</tr>
<tr>
<td>Worker</td>
<td>-0.100</td>
<td>-0.100</td>
<td>-1.741</td>
<td>-0.095</td>
</tr>
<tr>
<td>Non – Tradable</td>
<td>-0.004</td>
<td>-0.004</td>
<td>1.612</td>
<td>0.003</td>
</tr>
<tr>
<td>Tradable</td>
<td>0.139</td>
<td>0.139</td>
<td>1.064</td>
<td>0.003</td>
</tr>
<tr>
<td>Welfare</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Worker</td>
<td>-0.100</td>
<td>-0.095</td>
<td>-1.767</td>
<td>-0.001</td>
</tr>
<tr>
<td>Non – Tradable</td>
<td>-0.004</td>
<td>0.007</td>
<td>1.638</td>
<td>-0.001</td>
</tr>
<tr>
<td>Tradable</td>
<td>0.136</td>
<td>0.136</td>
<td>1.081</td>
<td>-0.000</td>
</tr>
<tr>
<td>Coef. of variability</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Y^T$</td>
<td>0.101</td>
<td>0.096</td>
<td>0.094</td>
<td>0.028</td>
</tr>
<tr>
<td>$Y^{NT}$</td>
<td>0.020</td>
<td>0.004</td>
<td>0.003</td>
<td>0.014</td>
</tr>
<tr>
<td>$Y$</td>
<td>0.011</td>
<td>0.005</td>
<td>0.005</td>
<td>0.007</td>
</tr>
<tr>
<td>Memo</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>corr($Y^T,Y^{NT}$)</td>
<td>0.164</td>
<td>0.334</td>
<td>0.123</td>
<td>0.422</td>
</tr>
<tr>
<td>min($\mu^{NT}$)</td>
<td>-0.258</td>
<td>-0.236</td>
<td>-0.209</td>
<td>0.033</td>
</tr>
<tr>
<td>min($\mu^T$)</td>
<td>-0.058</td>
<td>-0.058</td>
<td>-0.051</td>
<td>-0.270</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.000</td>
<td>0.000</td>
<td>0.300</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Note:** Section “Welfare” shows the difference between the mean and the non-stochastic steady-state value for welfare for each agent type. $W^w,W^{NT},W^T$ stand for welfare measures for the worker, non-tradable and tradable sector entrepreneurs, respectively. Section “Coef. of variability” reports the coefficients of variability calculated from simulations of the model. $Y,Y^{NT},Y^T$ stand for aggregate, non-tradable and tradable output, respectively. We report the correlation coefficient between tradable and non-tradable output and the minimum value taken by the Lagrange multipliers associated to the credit constraints at the 16 period after the shock horizon. A value lower than $-1$ represents that the credit constraint stops being binding. Results are reported for different configurations of shocks hitting the economy and for three different intensities for the reaction rule: (a) no intervention, (b) intensity that maximizes an aggregate welfare measure, and (c) intensity that minimizes the volatility of aggregate output.
Table 4: Results: Tax rule responding to non-tradable price

<table>
<thead>
<tr>
<th>Variable</th>
<th>All shocks</th>
<th>Trad. prod. shock</th>
<th>For. interest rate shock</th>
<th>Non – Trad. prod. shock</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\phi = 0)</td>
<td>(\phi^{opt,1})</td>
<td>(\phi^{opt,2})</td>
<td>(\phi = 0)</td>
</tr>
<tr>
<td>Worker</td>
<td>-0.100</td>
<td>0.108</td>
<td>-0.026</td>
<td>-0.004</td>
</tr>
<tr>
<td>Non – Tradable</td>
<td>-0.004</td>
<td>0.128</td>
<td>0.019</td>
<td>0.003</td>
</tr>
<tr>
<td>Tradable</td>
<td>0.139</td>
<td>-0.160</td>
<td>0.016</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coef. of variability</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Y_T)</td>
<td>0.101</td>
<td>0.299</td>
<td>0.177</td>
<td>0.096</td>
</tr>
<tr>
<td>(Y_{NT})</td>
<td>0.020</td>
<td>0.034</td>
<td>0.023</td>
<td>0.004</td>
</tr>
<tr>
<td>(Y)</td>
<td>0.011</td>
<td>0.010</td>
<td>0.010</td>
<td>0.005</td>
</tr>
<tr>
<td>Memo</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(corr(Y_T,Y_{NT}))</td>
<td>0.164</td>
<td>-0.771</td>
<td>-0.438</td>
<td>0.334</td>
</tr>
<tr>
<td>(min(\mu_{NT}))</td>
<td>-0.258</td>
<td>-0.129</td>
<td>-0.148</td>
<td>-0.236</td>
</tr>
<tr>
<td>(min(\mu_T))</td>
<td>-0.058</td>
<td>-0.029</td>
<td>-0.033</td>
<td>-0.058</td>
</tr>
<tr>
<td>(\phi)</td>
<td>0.000</td>
<td>1.000</td>
<td>0.300</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Note: Section “Welfare” shows the difference between the mean and the non-stochastic steady-state value for welfare for each agent type. \(W^W, W^{NT}, W^T\) stand for welfare measures for the worker, non-tradable and tradable sector entrepreneurs, respectively. Section “Coeff. of variability” reports the coefficients of variability calculated from simulations of the model. \(Y, Y_{NT}, Y_T\) stand for aggregate, non-tradable and tradable output, respectively. We report the correlation coefficient between tradable and non-tradable output and the minimum value taken by the Lagrange multipliers associated to the credit constraints at the 16 period after the shock horizon. A value lower than \(-1\) represents that the credit constraint stops being binding. Results are reported for different configurations of shocks hitting the economy and for three different intensities for the reaction rule: (a) no intervention, (b) intensity that maximizes an aggregate welfare measure, and (c) intensity that minimizes the volatility of aggregate output.
Figure 3: Tradable Productivity Shock
Figure 4: Non Tradable Productivity Shock
Figure 5: Foreign Interest Rate Shock
Figure 6: Financial development: $\theta$ and the Tradable Productivity Shock
Figure 7: Financial development: $\theta$ and the Non Tradable Productivity Shock
Figure 8: Financial development: $\theta$ and the Foreign Interest Rate Shock

- Agg. Output
- Agg. Cons.
- Current Acc.
- RER $p^{NT}/p^T$
- House prices
- Capital prices
- Output (T)
- Capital (T)
- Output (NT)
- Housing (NT)
- Dom. Int. Rate
- Labour
- Wages
- Real $q^h$
- Real $q^k$
- Real wages
Figure 9: Robustness: $\beta$ and the Tradable Productivity shock
Figure 10: Robustness: $\gamma$ and the Tradable Productivity shock

\[ \gamma = r \frac{W_{95}}{W} \]

\[ \gamma = r \frac{W_{98}}{W} \]

\[ \gamma = r \frac{W_{985}}{W} \]
Figure 11: Robustness: $\varepsilon$ and the Tradable Productivity shock
Figure 12: Robustness: $\gamma^T$ and the Tradable Productivity shock
Figure 13: Introducing nominal rigidities - Tradable Productivity shock
Figure 14: Nominal rigidities: $\omega$ and Tradable Productivity shock

- $\omega = 0.5$
- $\omega = 0.6$
- $\omega = 0.7$
Figure 15: Monetary Policy: $\phi_\pi$ and Tradable Productivity shock
Figure 16: Macroprudential Policies and Nominal Rigidity - Tradable Productivity Shock