Abstract

We set up and estimate a DSGE model of a small open economy that features two types of financial frictions: one in the relationship between depositors and banks (following Gertler and Karadi, 2011) and the other between banks and borrowers (along the lines of Bernanke et al., 1999). We use Chilean data to estimate the model, following a Bayesian approach. The model is used to understand the interaction between macroeconomic and financial factors. In particular, we try to answer the following questions: Is the presence of financial frictions useful in matching the dynamics of observed non-financial variables? Which of the financial frictions is more relevant in explaining the behavior of both financial and non-financial variables? How does the presence of financial frictions change the propagation of the main driving forces of the economy? Additionally, we perform an exercise to assess the benefits of “leaning against the wind” policies (i.e. using the monetary policy rate to smooth the credit cycle).
1 Introduction

The financial crisis of 2008 and the world recession that followed, as well as the turbulence in the Euro area, have renewed the interest for analyzing the interaction between macroeconomic and financial variables. The prevailing framework for monetary policy analysis before 2008 (the New Keynesian model) proved to be an incomplete tool, both because it did not include these interactions that ex-post were considered as relevant and also because it was not well suited to analyze the several “unconventional” policies that were implemented in response to the crisis. And while it seems that in many emerging countries the turbulence originated from the crisis have passed, the interest in analyzing the link between macroeconomic and financial variables remains. This is particularly so because of the need to evaluate different types of policies to complement the usual implementation of inflation targeting frameworks in a global context of significant fluctuations in capital flows to emerging countries.

The main goal of this paper is to characterize macroeconomic and financial interactions in Chile using a framework that can eventually be used for policy analysis. Assessing the empirical relevance of the several possible channels that should, at least a priori, play a role is most important before implementing any policy exercise. To that end, we extend in several dimensions a standard New Keynesian model of a small open economy to incorporate different aspects of the financial system. We estimate different versions of the model to assess how the several financial features contribute to explain both macroeconomic data (like, GDP, inflation, the exchange rate, etc.) and financial variables (such as spreads and credit).

To perform the analysis we develop a dynamic stochastic general equilibrium (DSGE) model of a small open economy featuring two types of domestic financial frictions. On one hand, there is a friction between depositors and banks that induces a spread between lending and deposit rates. We model this friction as a moral hazard problem following the work of Gertler and Karadi (2011) (GK). On the other hand, there is a spread between the lending rate and the return to capital (known as the external finance premium) that originates in a costly-state-verification problem, following Bernanke et al. (1999) (BGG). The model also features loans to finance working capital, although there are no informational asymmetries in this lending relationship. We estimate the model with quarterly Chilean data from 2001 to 2012, including both macro and financial variables, using Bayesian techniques.

We use the estimation of alternative versions of the model to assess several relevant issues. First, we explore whether a model with financial frictions can help to improve the goodness-of-fit of the model in terms of non-financial variables. We find that a model that includes frictions as in BGG can help to improve the fit of the model in some dimensions, performing at least as satisfactorily as a model without financial frictions. In contrast, model that feature GK frictions (alone or in tandem with a BGG setup) do not seem to be as useful in terms of replicating the observed macro variables.

Second, we investigate which of the financial frictions we considered (between depositors and banks, or between banks and firms) is most useful in accounting for the dynamics of financial variables. Our results show that the BGG friction appears to be more relevant in accounting for those dynamics. For instance, the versions of the model that include GK frictions tend to imply a much more volatile and less persistent spread than in the data. Overall, at least for the case of Chile, it seems that frictions between borrowers (firms) and banks appear to be

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1 This type of model-based analysis will provide a new useful tool for the analysis at the Central Bank of Chile. In particular, it can used to complement the forecast usually made with a larger-scale DSGE model (which does not consider financial frictions in an explicit way). Moreover, it can be used to analyze risk scenarios related to the potential shocks affecting financial intermediation. Finally, the model will also have the potential to analyze policies that complement the usual implementation of monetary policy.

2 The Gertler and Karadi framework has become quite popular in the recent macroeconomic literature, particularly for the analysis of unconventional monetary policies (see, for instance, Gertler and Kiyotaki, 2011; Gertler and Karadi, 2013; Dedola et al.; 2013; Kirchner and van Wijnbergen, 2012; Rannenberg, 2012).
more relevant than those between depositors and banks.

A third aspect that we analyze is related to the propagation of structural shocks. In particular, we describe how the presence of financial frictions changes the propagation of non-financial shocks. One of our findings is that the behavior of the real exchange rate and its interaction with financial frictions is key to understand how foreign shocks are propagated. For instance, when the economy is hit by a contractionary foreign shock, the real exchange rate tends to depreciate. In turn, because in our model the home good is fully tradable, the real depreciation improves, ceteris paribus, the financial position of these firms (relative to a model with no financial frictions). Thus, the negative effect on investment might be ameliorated in the presence of financial frictions.

Finally, we use the model with financial frictions that best fits the data (the BGG alternative) to assess if monetary policy can achieve better outcomes if it uses the policy rate to smooth the credit cycle; a strategy usually labeled as “leaning against the wind.” In particular, we expand the estimated benchmark policy rule (that reacts to inflation, output and lags of the policy rate) with the possibility of responding to three alternative measures of credit cycles: the growth rate of real loans, the ratio of loans to GDP, and the spread between lending and deposits rates. We find that reacting to these credit indicators can help to reduce the variance of inflation and output, particularly if the policy rate rule includes a reaction to the spread. However, we also show that the welfare gains obtained by reacting to these additional indicators are quite small.

Our study makes several contributions to the related literature. First, to the best of our knowledge, we are the first to set up a model combining banks as in Gertler and Karadi (2011) with entrepreneurs as in Bernanke et al. (1999) in a small open economy framework. In addition, we are the first to estimate a model featuring banks as in Gertler and Karadi (2011) for a small open economy. Finally, while several studies use estimated DSGE models to assess the role of financial frictions between domestic and foreign agents in propagating external shocks, we are among the few that assess the role of domestic financial frictions.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 describes the parametrization and estimation strategy. The goodness-of-fit analysis is presented in Section 4. Section 5 describes impulse responses obtained with the different versions of the model. Finally, Section 6 concludes and discusses some possible relevant extensions.

## 2 The Model

Our model shares many features with those in the literature of DSGE models for small open economies, particularly those used at central banks. The non-financial part of our framework is one of a small open economy with nominal and real rigidities. Domestic goods are produced with capital and labor, there is habit formation in consumption, there are adjustment costs in investment and capital utilization. Firms face a Calvo-pricing problem with partial indexation, and there is imperfect exchange rate pass-through into import prices in the short run due to local-currency price stickiness. In addition, households face a Calvo-type problem in setting wages, assuming also partial indexation to past inflation. We also assume that firms need to pay a fraction of their operating costs (working capital) in advance, which they finance with loans from banks. The economy also exports an exogenous

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3Rannenberg (2013) combines these two features but in a closed economy setup, using a calibrated model.
4Some examples of estimations in closed-economy frameworks with these types of banks are Villa (2013), Villa and Yang (2013), and Areosa and Coelho (2013).
5For instance, Tovar (2006) and Fernandez and Guhan (2012).
6Our base model (without financial frictions) is a simplified version of the model by Medina and Soto (2007), which is the DSGE model used for policy analysis and forecasting at the Central Bank of Chile. Given the simplifications that we make, the model is closer to that in Adolfson at al. (2007).
endowment of a commodity good; a feature that is introduced to account for the importance of commodity exports in Chile.

On top of that setup, we add two kinds of domestic financial frictions. On one hand, there are banks that intermediate credit from households to entrepreneurs (to finance capital accumulation) and to firms (for working capital), that are subject to a moral hazard problem along the lines of Gertler and Karadi (2011). On the other hand, capital accumulation by entrepreneurs is risky and subject to a costly-state-verification problem as in Bernanke et al. (1999), making the return on the loans obtained by banks state-contingent, as every period a fraction of the entrepreneurs will default on their loans.

The model features several exogenous sources of fluctuations: shocks to preferences, technology (neutral and investment-specific), commodity production, government expenditures, monetary policy, foreign demand, foreign inflation, foreign interest rates, the international price of the commodity good, and two financial shocks.

In the main part of the paper, we describe and set up the problems faced by each agent, leaving for the appendix the list of the relevant equilibrium conditions and the computation of the steady state.

2.1 Households

There is a continuum of infinitely lived households of mass one that have identical asset endowments and identical preferences that depend on consumption of a final good (C_t) and hours worked (h_t) in each period (t = 0, 1, 2, ...).7

Households save and borrow by purchasing domestic currency denominated government bonds (B_t) and by trading foreign currency bonds (B^{*}_t) with foreign agents, both being non-state-contingent assets. They can also deposit resources at banks (D_t). Expected discounted utility of a representative household is given by

$$E_t \sum_{s=0}^{\infty} \beta^s v_t^{t+s} \left[ \log (C_{t+s} - \varsigma C_{t+s-1} - \kappa h_{t+s}^{1+\phi} \right],$$

where v_t is an exogenous preference shock.

Following Schmitt-Grohe and Uribe (2006a, 2006b), labor decisions are made by a central authority, a union, which supplies labor monopsonistically to a continuum of labor markets indexed by i ∈ [0, 1]. Households are indifferent between working in any of these markets. In each market, the union faces a demand for labor given by

$$h_t(i) = \left[ W^n_t(i)/W^n_t \right]^{1-\epsilon} h^d_t,$$

where $W^n_t(i)$ denotes the nominal wage charged by the union in market i, $W^n_t$ is an aggregate hourly wage index that satisfies $W^n_t = \int_0^1 W^n_t(i)^{1-\epsilon} di$, and $h^d_t$ denotes aggregate labor demand by firms. The union takes $W^n_t$ and $h^d_t$ as given and, once wages are set, it satisfies all labor demand. Wage setting is subject to a Calvo-type problem, whereby each period the household (or union) can set its nominal wage optimally in a fraction $1 - \theta_W$ of randomly chosen labor markets, and in the remaining markets, the past wage rate is indexed to a weighted product of past and steady state CPI inflation with weights $\vartheta_W \in [0, 1]$ and $1 - \vartheta_W$, respectively.

Let $r_t$ and $r^*_t$ denote the gross real returns on $B_{t-1}$ and $B^{*}_{t-1}$, respectively. The real interest rate on deposits, by a non-arbitrage condition, will also equal $r_t$.

Further, let $W_t$ denote the real hourly wage rate, let $rer_t$ be the real exchange rate (i.e. the price of foreign consumption goods in terms of domestic consumption goods), let $T_t$ denote real lump-sum tax payments to the government and let $\Sigma_t$ collect real dividend income from the other hand, capital accumulation by entrepreneurs is risky and subject to a costly-state-verification problem as in Bernanke et al. (1999), making the return on the loans obtained by banks state-contingent, as every period a fraction of the entrepreneurs will default on their loans.

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7Throughout, uppercase letters denote variables containing a unit root in equilibrium (either due to technology or due to long-run inflation) while lowercase letters indicate variables with no unit root. Real variables are constructed using the domestic consumption good as the numeraire. In the appendix we describe how each variable is transformed to achieve stationarity in equilibrium. Variables without time subscript denote non-stochastic steady state values in the stationary model.
ownership of firms. The period-by-period budget constraint of the household is then given by

$$C_t + B_t + rer_t B^*_t + D_t + T_t = \int_0^1 W_t(i)h_t(i)di + r_tB_{t-1} + rer_tB^*_{t-1} + r_tD_{t-1} + \Sigma_t. \quad (2)$$

The household chooses $C_t$, $h_t$, $W^n_t(i)$, $B_t$, $B^*_t$ and $D_t$ to maximize (1) subject to (2) and labor demand by firms, taking prices, interest rates and aggregate variables as given. The nominal interest rates are implicitly defined as

$$r_t = R_{t-1}\pi_t^{-1}, \quad r^*_t = R^*_{t-1}\xi_t^{-1} (\pi^*_{t-1})^{-1},$$

where $\pi_t$ and $\pi^*_t$ denote the gross inflation rates of the domestic and foreign consumption-based price indices $P_t$ and $P^*_t$, respectively. The variable $\xi_t$ denotes a country premium given by

$$\xi_t = \xi \exp \left[ -\psi \frac{rer_t B^*_t/A_{t-1} - rer \times \tilde{b}^*}{rer \times \bar{b}^*} + \frac{\zeta_t - \zeta}{\zeta} \right],$$

where $\zeta_t$ is an exogenous shock to the country premium. The foreign nominal interest rate $R^*_t$ evolves exogenously, and the domestic central bank sets $R_t$.

### 2.2 Production and Pricing

The supply side of the economy is composed by a set of monopolistically competitive firms producing different varieties of a home good with labor and capital services as inputs, a set of monopolistically competitive importing firms, and three groups of perfectly competitive aggregators: one packing different varieties of the home good into a composite home good, one packing imported varieties into a composite foreign good, and a final group that bundles (with different combinations) the composite home and foreign goods to create a final goods that will be purchased by household consumption ($Y^C_t$), capital goods producers ($I_t$) and the government ($G_t$). All of these firms are owned by domestic households. In addition, there is a set of competitive firms producing a homogeneous commodity good that is exported abroad. A proportion of those commodity-exporting firms is owned by the government and the remaining proportion is owned by foreign agents. The total mass of firms in each sector is normalized to one. We denote productions/supply with the letter $y$ and inputs/demand with $x$.

#### 2.2.1 Final Goods

The final consumption good that generates utility for households, the final investment good that is used to increase the stock of capital, and expenditures by the government are produced with different technologies combining composite home and foreign goods. The three productions function are, respectively,

$$Y^C_t = \left[ (1 - \alpha_C) \frac{\partial}{\partial C} (X_t^{C,H}_t)^{\alpha_{C-1}} + \alpha_C \frac{\partial}{\partial C} (X_t^{C,F}_t)^{\alpha_{C-1}} \right]^{\frac{1}{\alpha_C}},$$

$$I_t = \left[ (1 - \alpha_I) \frac{\partial}{\partial I} (X_t^{I,H}_t)^{\alpha_{I-1}} + \alpha_I \frac{\partial}{\partial I} (X_t^{I,F}_t)^{\alpha_{I-1}} \right]^{\frac{1}{\alpha_I}},$$

$^8$See, for instance, Schmitt-Grohé and Uribe (2003) and Adolffson et al. (2007).

$^9$The variable $A_t$ (with $a_t \equiv A_t/A_{t-1}$) is a non-stationary technology disturbance, see below.
where $X_t^{C,H}$, $X_t^{I,H}$ and $X_t^{G,H}$ denote the demands of home composite goods by each representative firm, while $X_t^{C,F}$, $X_t^{I,F}$ and $X_t^{G,F}$ are the demands of foreign composite goods. Each representative firm is competitive and takes input prices ($p_t^H$ and $p_t^F$, measured in terms of the final consumption good) as well as selling prices (respectively, 1, $p_t^I$ and $p_t^G$ in terms of the final consumption good) as given.

### 2.2.2 Home Composite Goods

A representative home composite goods firm demands home goods of all varieties indexed by $j \in [0, 1]$ in amounts $X_t^H(j)$ and combines them according to the technology

$$Y_t^H = \left[ \int_0^1 X_t^H(j)^{\frac{1}{\sigma}-1} \, dj \right]^{\frac{\sigma}{\sigma-1}}.$$

Let $p_t^H(j)$ denote the price of the good of variety $j$ in terms of the home composite good. The profit maximization problem yields the following demand for the variety $j$:

$$X_t^H(j) = p_t^H(j)^{-\theta_H} Y_t^H.$$

### 2.2.3 Home Goods of Variety $j$

Each home variety $j$ is produced according to the technology

$$Y_t^H(j) = z_t K_t^h(j)^{\alpha} [A_t h_t^h(j)]^{1-\alpha},$$

where $z_t$ is an exogenous stationary technology shock, while $A_t$ (with $A_t \equiv A_t/A_{t-1}$) is a non-stationary technology disturbance, both common to all varieties. $K_t^h(j)$ denotes the demand for capital services by firm $j$ while $h_t^h(j)$ denotes this firm’s demand for labor. Additionally, we assume that a fraction $\alpha_t^{WC}$ of the operating costs needs to be financed with an intra-temporal loan (i.e. $L_t^{WC} = \alpha_t^{WC}[W_t h_t^h(j) + r_t^K K_t^h(j)]$), with a non-state contingent nominal rate of $R_t^{L,WC}$ (with $R_t^{L,WC} \equiv R_t^{L,WC}/\pi_t$). The firm producing variety $j$ has monopoly power but produces to satisfy the demand constraint given by (6). As the price setting decision is independent of the optimal choice of the factor inputs, the problem of firm $j$ can also be represented in two stages. In the first stage, the firm hires labor and rents capital to minimize production costs subject to the technology constraint (7). Thus, the firm’s real marginal costs in units of the final domestic good is given by

$$m^H_t(j) = \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \frac{(r_t^K)^{\alpha} W_t^{1-\alpha} [1 + \alpha_t^{WC}(R_t^{L,WC} - 1)]}{p_t^H z_t(A_t)^{1-\alpha}}.$$
to a weighted product of past inflation of home composite goods prices and steady state CPI inflation with weights \( \vartheta_H \in [0, 1] \) and \( 1 - \vartheta_H \).\(^{12}\)

### 2.2.4 Foreign Composite Goods

A representative foreign composite goods firm demands foreign goods of all varieties \( j \in [0, 1] \) in amounts \( X_t^F(j) \) and combines them according to the technology

\[
Y_t^F = \left[ \int_0^1 X_t^F(j) \frac{\sigma}{\sigma - 1} \, dj \right] ^{\sigma-1}. 
\]

Let \( p_t^F(j) \) denote the price of the good of variety \( j \) in terms of the foreign composite good. Thus, the input demand functions are

\[
X_t^F(j) = p_t^F(j)^{-\sigma} Y_t^F. \tag{9}
\]

### 2.2.5 Foreign Goods of Variety \( j \)

Importers buy an amount \( M_t \) of a homogenous foreign good at the price \( P_t^{F*} \) in the world market and convert this good into varieties \( Y_t^F(j) \) that are sold domestically, where \( M_t = \int_0^1 Y_t^F(j) \, dj \). The firm producing variety \( j \) has monopoly power but satisfies the demand constraint given by (9). As it takes one unit of the foreign good to produce one unit of variety \( j \), nominal marginal costs in terms of composite goods prices are

\[
P_t^{mc,F}(j) = P_t^F mc_t^F = S_t P_t^{F*}. \tag{10}
\]

Given marginal costs, the firm producing variety \( j \) chooses its price \( P_t^F(j) \) to maximize profits. In setting prices, the firm faces a Calvo-type problem, whereby each period the firm can change its price optimally with probability \( 1 - \theta_F \), and if it cannot change its price, it indexes its previous price according to a weighted product of past inflation of foreign composite goods prices and steady state CPI inflation with weights \( \vartheta_F \in [0, 1] \) and \( 1 - \vartheta_F \). In this way, the model features delayed pass-through from international to domestic prices.

### 2.2.6 Commodities

A representative commodity producing firm produces a quantity of a commodity good \( Y_t^{Co} \) in each period. Commodity production evolves according to an exogenous process, and it is co-integrated with the non-stationary TFP process. The entire production is sold abroad at a given international price \( P_t^{Co*} \). The real foreign and domestic prices are denoted as \( p_t^{Co*} \) and \( p_t^{Co} \), respectively, where \( p_t^{Co*} \) is assumed to evolve exogenously. The real domestic currency income generated in the commodity sector is therefore equal to \( p_t^{Co} Y_t^{Co} \). The government receives a share \( \chi \in [0, 1] \) of this income and the remaining share goes to foreign agents.

### 2.3 Capital Accumulation

#### 2.3.1 Entrepreneurs

Entrepreneurs manage the economy’s stock of capital \( (K_t) \). Following Bernanke et al. (1999) (BGG for short), entrepreneurs have two distinctive features in this setup. On the one hand, they have a technology available to

\(^{12}\)This indexation scheme eliminates the distortion generated by price dispersion up to a first-order expansion.
transform new capital produced by capital-goods producers (described below) into productive capital that can be used by firms. In particular, if at $t$ they buy $K_t$ units of new capital, the amount of productive capital available to rent to firms in $t+1$ is $\omega_{t+1}^e K_t$. The variable $\omega_t^e > 0$ is the source of heterogeneity among entrepreneurs and it is distributed in the cross section with a c.d.f. $F(\omega_t^e; \sigma_{\omega,t-1})$, and p.d.f. $f(\omega_t^e; \sigma_{\omega,t-1})$, such that $E(\omega_t^e) = 1$. The variable $\sigma_{\omega,t}$ denotes the time-varying cross-sectional standard deviation of entrepreneurs’ productivity, which is known in advance,\textsuperscript{13} and it is assumed to follow an exogenous process, as in, for instance, Christiano et al. (2010, 2014). On the other hand, entrepreneurs have finite lifetimes (we describe this in more detail below) and when they exit the market they transfer all their remaining wealth to households.

In each period, after the idiosyncratic productivity shock is realized, entrepreneurs rent capital services (which for each individual entrepreneur equal $u_t \omega_t^e K_{t-1}$, where $u_t$ denotes capital utilization) to home goods producing firms, at a rental rate (in real terms) $r_t^K$.\textsuperscript{14} They face a utilization cost per unit of capital, which in real terms is given by

$$\phi(u_t) = \frac{r^K}{\phi_u} \{\exp[\phi_u(u_t - 1)] - 1\},$$

where $r^K$ is the steady state value of the rental rate of capital services, and $\phi_u$ governs the importance of these utilization costs.\textsuperscript{15} After non-depreciated capital is returned, they sell it to capital goods producers at a real price $q_t$. Afterwards, they buy new capital ($q_t K_t$).

We assume that purchases of new capital have to be financed by loans from intermediaries. However, due to an informational asymmetry (described below) entrepreneurs will not be able to obtain loans to cover for the whole operation. This will create the incentives for entrepreneurs to accumulate net worth ($N_t^e$) to finance part of the capital purchases. Thus, we have

$$q_t K_t = N_t^e + L^K_t,$$

where $L^K_t$ is the loan obtained from banks in real terms. We assume that the loan contract signed at $t$ is nominal and it specifies a non-contingent interest rate $R^{L,e}_t$ (with $R^{L,e}_t \equiv R^{L,e}_t \pi_t$). The fact that entrepreneurs have finite lifetimes prevents them from accumulating net worth beyond a point at which they can self-finance the operation.

The informational asymmetry takes the form of a costly-state-verification problem, as in BGG. In particular, we assume that $\omega_t^e$ is only revealed to the entrepreneur ex-post (i.e. after loan contracts have been signed) and can only be observed by a third party after paying a monitoring cost, equivalent to a fraction $\mu^e$ of the total revenues generated by the project. Thus, at the time entrepreneurs have to repay the loan they can choose to either pay it (plus the specified interest) or to default, in which case the intermediary will pay the monitoring cost and seize all entrepreneurial assets.

Following BGG, the optimal debt contract specifies a cut-off value $\bar{\omega}_{t+1}^e$ such that if $\omega_{t+1}^e \geq \bar{\omega}_{t+1}^e$ the borrower pays $\bar{\omega}_{t+1}^e [r^K_{t+1} u_{t+1} - \phi(u_{t+1}) + (1 - \delta)q_{t+1}] K_t$ units of final consumption goods to the lender and keeps $(\omega_{t+1}^e - \bar{\omega}_{t+1}^e) [r^K_{t+1} u_{t+1} - \phi(u_{t+1}) + (1 - \delta)q_{t+1}] K_t$, while if $\omega_{t+1}^e < \bar{\omega}_{t+1}^e$ the borrower receives nothing (defaults) and the lender obtains $(1 - \mu^e) \omega_{t+1}^e [r^K_{t+1} u_{t+1} - \phi(u_{t+1}) + (1 - \delta)q_{t+1}] K_t$. Therefore, under the assumption of a competitive

\textsuperscript{13}That is, at the time the financial contract is signed, everybody knows the distribution from which individual productivity will be drawn next period.

\textsuperscript{14}We are abusing the notation here, as $u_t$, $\omega_t^e$ and $K_{t-1}$ should have an index identifying the individual entrepreneur. However, as we assume that entrepreneurs are identical ex-ante, that technology is linear, and that $E(\omega_t^e) = 1$, in equilibrium aggregate capital services will be given by $u_t K_{t-1}$.

\textsuperscript{15}Note that the choice of $u_t$ is intra-periodic, so it does not depend on financing conditions.
the lending market, the mapping between the cut-off value and the interest rate on the loan $R_t^{L,e}$ satisfies

$$R_t^{L,e} = \tilde{\omega}_t^{e} \left[ \frac{K_t}{L_t^{e}} \pi_t^{e+1} \right] - \phi(u_{t+1}) + (1 - \delta) q_{t+1} \right],$$

where the right-hand side is the return obtained by the bank for each unit of money lent from an entrepreneur that pays back the loan. As we assume that entrepreneurs bear all the risk (as in BGG), this condition is assumed to hold state by state.

While $R_t^{L,e}$ denotes the interest rate of a loan signed at $t$, the ex-post return for the intermediary for each unit lent at $t$ (which we denote by $R_t^{L,K}$, with $r_t^{L,K} \equiv R_t^{L,K} / \pi_t$) is not equal to $R_t^{L,e}$ for two reasons: not all loans will be repaid and, from those entrepreneurs who default, the intermediary receives their assets net of monitoring costs. This in particular implies that, while the interest rate on the loan is known at the time the contract is signed, the return obtained by the intermediary is instead state-contingent, for it depends on the aggregate conditions that determine whether entrepreneurs default or not. Therefore, for the intermediary to be willing to lend it must be the case that

$$L_t^{K} r_t^{L,K} \leq g(\tilde{\omega}_t^{e}; \sigma_{\omega,t}) \left[ \frac{K_t}{L_t^{e}} \pi_t^{e+1} \right] - \phi(u_{t+1}) + (1 - \delta) q_{t+1} \right],$$

where the term in brackets on the right-hand side of (12) is the average (across entrepreneurs) revenue obtained at $t + 1$ if the amount of capital purchases at $t$ was $K_t$, and with

$$g(\tilde{\omega}_t^{e}; \sigma_{\omega,t-1}) \equiv \tilde{\omega}_t^{e} \left[ 1 - F(\tilde{\omega}_t^{e}; \sigma_{\omega,t-1}) \right] + (1 - \mu^e) \int_0^{\tilde{\omega}_t^{e}} \omega^e f(\omega^e; \sigma_{\omega,t-1})d\omega^e.$$

The first term on the right-hand side is the share of total revenues that the intermediary obtains from those who pay back the loan, while the second is the value of the assets seized from defaulting entrepreneurs, net of monitoring costs. As we will see below, the banks’ problem defines a non-arbitrage condition that relates the expected value of $R_t^{L,K}$ with other interest rates relevant for banks. Thus, (12) is the participation constraint for the banks to be willing to lend. As before, this condition holds state-by-state under the assumption that entrepreneurs bear all the risk.  

From the entrepreneurs’ viewpoint, the expected profits for the project of purchasing $K_t$ units of capital equals

$$E_t \left\{ \tilde{\omega}_t^{e} \left[ 1 - F(\tilde{\omega}_t^{e}; \sigma_{\omega,t}) \right] + (1 - \mu^e) \int_0^{\tilde{\omega}_t^{e}} \omega^e f(\omega^e; \sigma_{\omega,t})d\omega^e \right],$$

where

$$h(\tilde{\omega}_t^{e}; \sigma_{\omega,t-1}) \equiv \int_{\tilde{\omega}_t^{e}}^{\infty} \omega^e f(\omega^e; \sigma_{\omega,t-1})d\omega^e - \tilde{\omega}_t^{e} \left[ 1 - F(\tilde{\omega}_t^{e}; \sigma_{\omega,t-1}) \right].$$

The first term on the right-hand side of (14) is the expected share of average revenue that entrepreneurs obtain given their productivity. The second is the expected repayment. Both are conditional on not defaulting (i.e. $\tilde{\omega}_t^{e} \geq \omega_t^{e}$).

Defining $lev_t^{e} \equiv \frac{\tilde{\omega}_t^{e}}{N_t}$, and given the revelation principle, the optimal debt contract specifies a value for $lev_t^{e}$ and a state-contingent $\tilde{\omega}_t^{e+1}$ such that (13) is maximized subject to (12) being satisfied with equality for every possible aggregate state at $t + 1$. As shown in the appendix, the optimality condition for this contract can be
written as follows:

$$E_t \left\{ \frac{r^K_{t+1} u_{t+1} - \phi(u_{t+1}) + (1 - \delta)q_{t+1}}{q_t} \left[ h'(\omega_{1,t+1}^{e}; \sigma_{o,t}) g(h_{1,t+1}^{e}; \sigma_{o,t}) - h(\omega_{1,t}^{e}; \sigma_{o,t}) \right] \right\} =
E_t \left\{ r^{L,K}_{t+1} \frac{h'(\omega_{1,t+1}^{e}; \sigma_{o,t})}{g'(\omega_{1,t+1}^{e}; \sigma_{o,t})} \right\}, \quad (15)$$

The ratio $r_{p,t} \equiv E_t \left\{ \frac{r^K_{t+1} u_{t+1} - \phi(u_{t+1}) + (1 - \delta)q_{t+1}}{q_t} \right\}/E_t \left\{ r^{L,K}_{t+1} \right\}$ is known as the external finance premium which, as shown by BGG, is (up to first order) an increasing function of entrepreneurs’ leverage $\lambda(t)^e_t$.

Finally, average entrepreneurs’ net worth evolves over time as follows. The average return an entrepreneur gets after repaying its loan at $t$ is given by $[r^K_t u_t - \phi(u_t) + (1 - \delta)q_t]K_{t-1}h(\omega_{t}^{e}; \sigma_{o,t-1})$. We assume that only a fraction $v$ of entrepreneurs survives every period, and an equivalent fraction enters the market with an initial capital injection from households equal to $\bar{n}^eA_{t-1}$, with $\varepsilon > 0$ (i.e. a fraction $\bar{n}^e$ of balanced-growth-path net worth).\(^\text{17}^\) Thus, we have

$$N^e_t = v \left\{ [r^K_t + (1 - \delta)q_t]K_{t-1}h(\omega_{t}^{e}; \sigma_{o,t-1}) \right\} + \varepsilon n^e A_{t-1}.$$

### 2.3.2 Capital Goods

Capital goods producers operate the technology that allows to increase the economy-wide stock of capital. In each period, they purchase the stock of depreciated capital from entrepreneurs and combine it with investment goods (which they buy at a price $P^I_t$) to produce new productive capital. The newly produced capital is then sold back to the entrepreneurs and any profits are transferred to the households. A representative capital producer’s technology is given by

$$K_t = (1 - \delta)K_{t-1} + [1 - \Gamma (I_t/I_{t-1})] \bar{\gamma} I_t,$$

where $I_t$ denotes investment expenditures in terms of the final good as a materials input and

$$\Gamma \left( \frac{I_t}{I_{t-1}} \right) = \frac{\gamma}{2} \left( \frac{I_t}{I_{t-1}} - \bar{a} \right)^2$$

are convex investment adjustment costs. The variable $\bar{\gamma}$ is an investment shock that captures changes in the efficiency of the investment process (see, for instance, Justiniano et al., 2011).

### 2.4 Banks

We assume the presence of competitive financial intermediaries (banks) that take deposits from households and combine them with their own net worth to produce loans to both firms and to entrepreneurs. Following Gertler and Karadi (2011), the relationship between households and banks is characterized by a moral hazard problem that gives rise to a premium between the lending and deposit rates, even in the absence of BGG frictions.

The balance sheet of a representative financial intermediary at the end of period $t$ is given by

$$L^W_C + L^K_t = D_t + N_t,$$

where $D_t$ denote deposits by domestic households at this intermediary, $L^W_C$ and $L^K_t$ denote the intermediary’s

\(^{17}\) Entrepreneurs that leave that market transfer their remaining resources to households.
stock of loans to, respectively, home goods producing firms and entrepreneurs, and \( N_t \) denotes the intermediary’s net worth (all in real terms of domestic units). The latter evolves over time as the difference between earnings on assets and interest payments on liabilities:

\[
N_{t+1} = r_{t+1}^{L,WC} L_{t+1}^{WC} + r_{t+1}^{L,K} L_{t+1}^{K} - r_{t+1} D_t = (r_{t+1}^{L,WC} - r_{t+1}) L_t^{WC} + (r_{t+1}^{L,K} - r_{t+1}) L_t^{K} + r_{t+1} N_t
\]

where \( r_{t}^{L,WC} \) and \( r_{t}^{L,K} \) denote the real gross returns on both types of loans.\(^{18,19}\)

Financial intermediaries have finite lifetimes. At the beginning of period \( t+1 \), after financial payouts have been made, the intermediary continues operating with probability \( \omega \) and exits the intermediary sector with probability \( 1 - \omega \), in which case it transfers its retained capital to the household which owns that intermediary. Thus, the intermediary’s objective in period \( t \) is to maximize expected terminal wealth \( (V_t) \), which is given by

\[
V_t \equiv E_t \sum_{s=0}^{\infty} (1 - \omega)^s \beta^{s+1} \Xi_{t,t+s+1} N_{t+s+1},
\]

where \( \beta^{s} \Xi_{t,t+s} \) is the households’ stochastic discount factor for real payoffs.

Further, following Gertler and Karadi (2011), a costly enforcement problem constrains the ability of intermediaries to obtain funds from depositors. In particular, at the beginning of period \( t \), before financial payouts are made, the intermediary can divert an exogenous fraction \( \mu_t \) of total assets \( (L_t) \). The depositors can then force the intermediary into bankruptcy and recover the remaining assets, but it is too costly for the depositors to recover the funds that the intermediary diverted. Accordingly, for the depositors to be willing to supply funds to the intermediary, the incentive constraint

\[
V_t \geq \mu_t (L_t^{WC} + L_t^{K})
\]

must be satisfied. That is, the opportunity cost to the intermediary of diverting assets (i.e. to continue operating and obtaining the value \( V_t \)) cannot be smaller than the gain from diverting assets. As can be seen, shocks that increase \( \mu_t \) will make this constraint tighter, making the financial problem more severe.

Using the method of undetermined coefficients, \( V_t \) can be expressed as follows (see the appendix):

\[
V_t = \varrho_t^{L,WC} L_t^{WC} + \varrho_t^{L,K} L_t^{K} + \varrho_t^N N_t,
\]

where

\[
\begin{align*}
\varrho_t^{L,WC} &= \beta E_t \left\{ \Xi_{t,t+1} \left[ (1 - \omega)(r_{t+1}^{L,WC} - r_{t+1}) + \omega \frac{L_{t+1}^{WC}}{L_t} \beta_{t+1}^{L,WC} \right] \right\}, \\
\varrho_t^{L,K} &= \beta E_t \left\{ \Xi_{t,t+1} \left[ (1 - \omega)(r_{t+1}^{L,K} - r_{t+1}) + \omega \frac{L_{t+1}^{K}}{L_t} \beta_{t+1}^{L,K} \right] \right\}, \\
\varrho_t^N &= \beta E_t \left\{ \Xi_{t,t+1} \left[ (1 - \omega)r_{t+1} + \omega \frac{N_{t+1}}{N_t} \beta_{t+1}^N \right] \right\}
\end{align*}
\]

**Footnotes:**

\(^{18}\) These real rates relate to their nominal counterparts in a similar way as the real domestic deposit rate \( r_t \) defined above.

\(^{19}\) We assume that, while loans to working capital are intra-periodic, firms repay loans after banks’ choices in period \( t \) have been made, which is the same as assuming that the return from this loan is received in the next period as in (16). This is in line with the assumption of working capital loans in the related literature without banks (e.g. Christiano et al. 2014).
The intermediary maximizes (18) subject to (17) taking \( N_t \) as given. The first-order conditions to this problem are as follows:

\[
L_t^{WC} : (1 + \kappa_t)\xi_t^{LWC} - \mu_t \kappa_t = 0, \\
L_t^K : (1 + \kappa_t)\xi_t^{LK} - \mu_t \kappa_t = 0, \\
\kappa_t : \xi_t^{LWC}L_t^{WC} + \xi_t^{LK}L_t^K + \xi_t^N N_t - \mu_t(L_t^{WC} + L_t^K) \geq 0,
\]

where \( \kappa_t \geq 0 \) is the multiplier associated with the incentive constraint. The second condition holds with equality if \( \kappa_t > 0 \), otherwise it holds with strict inequality. Notice that the optimality conditions for each type of loans imply that \( \xi_t^{LWC} = \xi_t^{LK} \equiv \xi_t^L \). In other words, as the incentive constraint is symmetric for both types of loans, banks need to be indifferent ex-ante between lending one unit to firms or to entrepreneurs. However, the arbitrage condition is not simply that the expected return of both loans are ex-ante identical (not even up to first order), because the marginal value for each type of loan depends on the growth rate of each of these loans. In addition, either of the conditions for the choice of loans imply that

\[
\kappa_t = \frac{\xi_t^L}{\mu_t - \xi_t^L},
\]

such that the constraint is strictly positive if \( \mu_t > \xi_t^L \). That is, the incentive constraint holds with equality if the marginal gain to the financial intermediary from diverting assets and going bankrupt (\( \mu_t \)) is larger than the marginal gain from expanding assets by one unit of deposits (i.e. holding net worth constant) and continuing to operate (\( \xi_t^L \)). We assume that this is the case in a local neighborhood of the non-stochastic steady state. The condition for \( \kappa_t \) holding with equality implies that

\[
L_t \equiv L_t^{WC} + L_t^K = lev_t N_t,
\]

where

\[
lev_t \equiv \frac{\xi_t^N}{\mu_t - \xi_t^L}
\]

denotes the intermediary’s leverage ratio. As indicated by (19), higher marginal gains from increasing assets \( \xi_t^L \) support a higher leverage ratio in the optimum, the same is true for the higher marginal gains of net worth \( \xi_t^N \), while a larger fraction of divertable funds \( \mu_t \) lowers the leverage ratio.

The aggregate evolution of net worth follows from the assumption that a fraction \( 1 - \omega \) of intermediaries exits the sector in every period and an equal number enters. Each intermediary exiting the sector at the end of period \( t - 1 \) transfers their remaining net worth \((\bar{N}_{e,t} \equiv (r_t^{LWC} - r_t) L_t^{WC} + (r_t^{LK} - r_t)L_{t-1}^K + r_t N_{t-1})\) to households. At the same time, households transfer starting capital equal to \( \bar{N}_{n,t} \equiv \frac{\rho}{1-\omega} \bar{N}_{e,t} \) to each new intermediary; with \( \rho > 0 \) (i.e. the transfer equals a fraction \( \frac{\rho}{1-\omega} \) of balanced-growth-path net worth). Aggregate net worth then evolves as follows:

\[
N_t = \omega \bar{N}_{e,t} + (1 - \omega) \bar{N}_{n,t} = \omega \left[ (r_t^{LWC} - r_t)L_t^{WC} + (r_t^{LK} - r_t)L_{t-1}^K + r_t N_{t-1} \right] + \rho N_{t-1}.
\]

We define the average lending-deposit spread as

\[
sp_{rt} = \left( \frac{R_t^{LWC}L_t^{WC} + R_t^{LK}L_t^K}{L_t} \right) 1 \frac{1}{R_t}.
\]
i.e. the average of both contractual loan rates, weighted by the size of each loan on total loans, relative to the deposit rate. This measure would be the data counterpart of the spread we will use for the estimation.

Finally, it is relevant to highlight that our treatment of working capital loans in the bank’s problem is different than in the related literature. In the original paper by Gertler and Karadi, this type of loans were not considered. Other papers that do consider these loans in models featuring GK banks (e.g. Rannenberg, 2013; Villa, 2013; Villa and Yang, 2013; and Areosa and Coelho, 2013) assume that working capital loans are not subject to the moral hazard problem assumed by GK. However, it is not clear why this is an appropriate assumption, for it implies that somehow the ability of diverting funds by banks is different depending on the type of loans, or that depositors can easily distinguish between the several uses banks give to their funds. We believe that our setup, in which there is an indifference condition for both types of loans that is affected by the frictions faced by banks, is more realistic. Moreover, as we will latter describe, this assumption generates an additional propagation channel in the model.

2.5 Fiscal and Monetary Policy

The government consumes an exogenous stream of final goods \( G_t \), levies lump-sum taxes, issues one-period bonds and receives a share \( \chi \) of the income generated in the commodity sector. We assume for simplicity that the public asset position is completely denominated in domestic currency. Hence, the government satisfies the following period-by-period constraint:

\[
p_t G_t + r_t B_t = T_t + B_t + \chi p_t C o Y_t C o.
\]

Monetary policy is carried out according to a Taylor rule of the form

\[
R_t = \left( \frac{R_{t-1}}{R} \right)^{\rho R} \left[ \left( \frac{\pi_t}{\bar{\pi}} \right)^{\alpha \pi} \left( \frac{Y_t}{Y_t-1} \right)^{\alpha y} \right]^{1-\rho R} \exp(\varepsilon_t^R),
\]

where \( \bar{\pi} \) is target inflation, \( Y_t \) is real GDP (defined below), \( \bar{a} \) is the long-run growth rate of TFP, and \( \varepsilon_t^R \) is an i.i.d. Gaussian shock that captures deviations from the rule.

2.6 The Rest of the World

Foreign agents demand home composite goods and buy the domestic commodity production. There are no transaction costs or other barriers to trade. The structure of the foreign economy is identical to the domestic economy, but the domestic economy is assumed to be small relative to the foreign economy. The latter implies that the foreign producer price level \( P_t^F \) is identical to the foreign consumption-based price index \( P_t^C o \). Further, let \( P_t^H \) denote the price of home composite goods expressed in foreign currency. Given full tradability and competitive export pricing, the law of one price holds separately for home composite goods and the commodity good, i.e. \( P_t^H = S_t P_t^H \) and \( P_t^C o = S_t P_t^C o \). That is, domestic and foreign prices of both goods are identical when expressed in the same currency. Due to local currency pricing, a weak form of the law of one price holds for foreign composite goods according to (10). Therefore, the real exchange rate \( rer_t \) satisfies

\[
rer_t = \frac{S_t P_t^*}{P_t} = \frac{S_t P_t^F}{P_t} = \frac{P_t^F mc_t^F}{P_t} = p_t^F mc_t^F,
\]
and the commodity price in terms of domestic consumption goods is given by

\[ p_{t}^{Co} = \frac{P_{t}^{Co}}{P_{t}} = \frac{S_{t} P_{t}^{Co*}}{P_{t}} = \frac{S_{t} P_{t}^{*}}{P_{t}} = p_{t}^{F} P_{t}^{Co*}. \]

We also have the relation \( rer_{t}/rer_{t-1} = \pi_{t}^{S}/\pi_{t} \), where \( \pi_{t}^{S} = S_{t}/S_{t-1} \). Further, foreign demand for the home composite good \( X_{t}^{H*} \) is given by the schedule

\[ X_{t}^{H*} = o^{*} \left( \frac{P_{t}^{H*}}{P_{t}} \right)^{-\eta^{*}} Y_{t}^{*}, \]

where \( Y_{t}^{*} \) denotes foreign aggregate demand. Both \( Y_{t}^{*} \) and \( \pi_{t}^{*} \) evolve exogenously.

### 2.7 Aggregation and Market Clearing

Taking into account the market clearing conditions for all the different markets, we can define the trade balance in units of final goods as

\[ TB_{t} = p_{t}^{H} X_{t}^{H*} + rer_{t} p_{t}^{Co*} Y_{t}^{Co} - rer_{t} M_{t}. \quad (22) \]

Further, we define real GDP as follows:

\[ Y_{t} \equiv C_{t} + I_{t} + G_{t} + X_{t}^{H*} + Y_{t}^{Co} - M_{t}. \]

Then, the GDP deflator \( (p_{t}^{Y}, \text{expressed as a relative price in terms of the final consumption good}) \) is implicitly defined as

\[ p_{t}^{Y} Y_{t} = C_{t} + p_{t}^{I} I_{t} + p_{t}^{G} G_{t} + TB_{t}. \]

Finally, we can show that the net foreign asset position evolves according to

\[ rer_{t} B_{t}^{*} = rer_{t+1}^{*} B_{t-1}^{*} + TB_{t} - (1 - \chi) rer_{t} p_{t}^{Co*} Y_{t}^{Co}. \]

### 2.8 Driving Forces

The exogenous processes in the model are \( v_{t}, \omega_{t}, z_{t}, a_{t}, \zeta_{t}, R_{t}^{*}, \pi_{t}^{*}, p_{t}^{Co}, y_{t}^{Co}, y_{t}^{*}, g_{t}, \mu_{t} \) and \( \sigma_{\omega, t} \). For each of them, we assume a process of the form

\[ \log \left( \frac{x_{t}}{\bar{x}} \right) = \rho_{x} \log \left( \frac{x_{t-1}}{\bar{x}} \right) + \varepsilon_{t}^{x}, \quad \rho_{x} \in [0, 1), \quad \bar{x} > 0, \]

for \( x = \{ v, \omega, z, a, \zeta, R^{*}, \pi^{*}, p^{Co}, y^{Co}, y^{*}, g, \mu, \sigma_{\omega} \} \), where the \( \varepsilon_{t}^{x} \) are i.i.d. Gaussian shocks.

### 2.9 Alternative Versions of the Model

In addition to this complete model, for comparison purposes we will also consider three alternative versions. The Base model is one in which there are no financial frictions (i.e. with no banks and where entrepreneurs do not face idiosyncratic shocks), and where households lend directly to both firms and entrepreneurs. The GK model features banks following Gertler and Karadi (2011) as we have described, but entrepreneurs face no financial frictions. In this version of the model, we force the share of total capital purchases financed by loans to be the
same as in the model with BGG entrepreneurs. Moreover, following Gertler and Karadi (2011), we assume in this version of the model that loans to entrepreneurs are state-contingent, with the interest rate equal to the return on capital. In other words, in the GK model the average spread is

\[
\frac{(R^L_{it} - WC_{it}) + E_t\{R^L_{t+1} - WC_{t+1}\}}{L_t} \quad \frac{1}{R^L_{it}},
\]

where,

\[
\tilde{R}^L_{it} \equiv \left[ \frac{\mu^K(u_t) + \rho_t(1 - \delta)}{\alpha_L \rho_{t-1}} - \frac{(1 - \alpha_L^K) \rho_t K_t}{\alpha_L \rho_{t-1} K_{t-1}} \right] \pi_{t+1},
\]

with \( \alpha_L^K \) being the share of total capital purchases financed by loans. This rate is the one that generates zero-profits for entrepreneurs in such a model.

In addition, the BGG model is one with entrepreneurs facing the costly-state-verification problem we have detailed above, but where they obtain funds directly from households. In this version, working capital loans are made at the risk-free rate, thus the spread is simply

\[
\frac{R^L_{it}}{R^L_{it}}.
\]

Finally, the full model we have described will be labeled as GK+BGG.

3 Parametrization Strategy

Our empirical strategy combines both calibrated and estimated parameters. The calibrated parameters and targeted steady state values are presented in Table 1. For most of the parameters not related with financial frictions we draw from related studies using Chilean data, as indicated in the table, while others are endogenously determined in steady state to target some first moments (\( \beta, \pi^*, \kappa, \sigma^*, \bar{g}, \bar{b} \) and \( \bar{y}_C \)). The parameters that deserve additional explanation are those related with financial frictions: \( \bar{\mu} \) (the steady state value of the fraction of divertible funds), \( \omega \) (the fraction of surviving banks), \( \iota \) (the capital injection for new banks), \( \mu_e \) (bankruptcy costs), \( \upsilon \) (the fraction of surviving entrepreneurs), \( \iota^e \) (the capital injection for new entrepreneurs), and \( \sigma^\omega \) (the steady state value of entrepreneurs’ dispersion).

We target the following averages for financial variables. We set the spread between the interest rate on entrepreneurs (\( R^L_e \)) and the deposit rate (\( R \)) to 380 basis points, which corresponds to the average spread between 90-days loans and the monetary policy rate.\(^{20,21}\) We further set the bank leverage ratio to 9. This statistic is not easy to calibrate, for banks’ balance sheets are more complicated in the data than in the model. Consolidated data from the banking system in Chile implies an average leverage ratio of around 13 between 2001 and 2012, but on the assets side of the balance sheet there are other types of assets that are not loans. To pick the value that we use, we compute an average ratio of the stock of loans to total consolidated assets of the banking system of 66% and adjusted the observed average leverage of the banking system by this percentage (i.e. 9 \( \approx \frac{13}{0.66} \)). For the entrepreneurs’ problem, we choose a steady state leverage of 2.05, which corresponds to

\(^{20}\) All the rates and spread figures are presented here in annualized terms, although in the model they are included on a quarterly basis.

\(^{21}\) We match this spread instead of the one defined in (20) because the computation of the steady state simplifies significantly with this choice. At the posterior mode, the difference between these two spreads is less than 3 annualized basis points.
# Table 1: Calibrated Parameters.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Description</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>Risk aversion</td>
<td>1</td>
<td>Medina and Soto (2007)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Frisch elasticity</td>
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<td>Adolfson et al. (2008)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital share in production</td>
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<td>Medina and Soto (2007)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Capital depreciation</td>
<td>0.06/4</td>
<td>Medina and Soto (2007)</td>
</tr>
<tr>
<td>$\epsilon_H$</td>
<td>E.O.S. domestic aggregate</td>
<td>11</td>
<td>Medina and Soto (2007)</td>
</tr>
<tr>
<td>$\epsilon_F$</td>
<td>E.O.S. imported aggregate</td>
<td>11</td>
<td>Medina and Soto (2007)</td>
</tr>
<tr>
<td>$\sigma_C$</td>
<td>Share of $F$ in $Y^C$</td>
<td>0.26</td>
<td>Input-output matrix (2008-2012)</td>
</tr>
<tr>
<td>$\sigma_I$</td>
<td>Share of $F$ in $I$</td>
<td>0.36</td>
<td>Input-output matrix (2008-2012)</td>
</tr>
<tr>
<td>$\sigma_G$</td>
<td>Share of $F$ in $G$</td>
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<td>Normalization</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Government share in commodity sector</td>
<td>0.61</td>
<td>Average (1987-2012)</td>
</tr>
<tr>
<td>$s^b_H$</td>
<td>Trade balance to GDP in SS</td>
<td>4%</td>
<td>Average (1987-2012)</td>
</tr>
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<td>$s^g$</td>
<td>Gov. exp. to GDP in SS</td>
<td>11%</td>
<td>Average (1987-2012)</td>
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<td>$s^{Co}$</td>
<td>Commodity prod. to GDP in SS</td>
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<td>Average (1987-2012)</td>
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<td>$\bar{\pi}$</td>
<td>Inflation in SS</td>
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<td>Inflation Target in Chile</td>
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<td>Normalization</td>
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<td>Hours in SS</td>
<td>0.3</td>
<td>Normalization</td>
</tr>
<tr>
<td>$\bar{a}$</td>
<td>Long-run growth</td>
<td>2.50%</td>
<td>4.5% GDP - 2% labor force grth. (avg. 01-12)</td>
</tr>
<tr>
<td>$R$</td>
<td>MPR in SS.</td>
<td>5.80%</td>
<td>Fuentes and Gredig (2008)</td>
</tr>
<tr>
<td>$R^*$</td>
<td>Foreign rate in SS</td>
<td>4.50%</td>
<td>Fuentes and Gredig (2008)</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Country premium in SS</td>
<td>140bp</td>
<td>EMBI Chile (avg. 01-12)</td>
</tr>
<tr>
<td>$lev$</td>
<td>Leverage financial sector</td>
<td>9</td>
<td>Own calculation (see text)</td>
</tr>
<tr>
<td>spread</td>
<td>90 days lending-borrowing spread</td>
<td>380bp</td>
<td>Loan rate vs. MP rate (avg. 01-12)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Injection for new bankers</td>
<td>0.002</td>
<td>Gertler and Karadi (2011)</td>
</tr>
<tr>
<td>$\mu^e$</td>
<td>Bankruptcy cost</td>
<td>0.12</td>
<td>Christiano et al. (2010)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Survival rate of entrepreneurs</td>
<td>0.97</td>
<td>Bernanke et al. (1999)</td>
</tr>
<tr>
<td>$\tau p$</td>
<td>Entrepreneurs’ external finance premium</td>
<td>120bp</td>
<td>Spread A vs. AAA, corp. bonds (avg. 01-12)</td>
</tr>
<tr>
<td>$lev^e$</td>
<td>Entrepreneurs’ leverage</td>
<td>2.05</td>
<td>For the non-financial corp. sector (avg. 01-12)</td>
</tr>
<tr>
<td>$\rho_{yc^o}$</td>
<td>Auto corr. $y^{Co}$</td>
<td>0.4794</td>
<td>Own estimation</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>Auto corr. $g$</td>
<td>0.6973</td>
<td>Own estimation</td>
</tr>
<tr>
<td>$\rho_{R^*}$</td>
<td>Auto corr. $R^*$</td>
<td>0.9614</td>
<td>Own estimation</td>
</tr>
<tr>
<td>$\rho_{y^*}$</td>
<td>Auto corr. $y^*$</td>
<td>0.8665</td>
<td>Own estimation</td>
</tr>
<tr>
<td>$\rho_{\pi^*}$</td>
<td>Auto corr. $\pi^*$</td>
<td>0.3643</td>
<td>Own estimation</td>
</tr>
<tr>
<td>$\rho_{p^{Co}}$</td>
<td>Auto corr. $p^{Co}$</td>
<td>0.962</td>
<td>Own estimation</td>
</tr>
<tr>
<td>$\sigma_{yc^o}$</td>
<td>St. dev. shock to $y^{Co}$</td>
<td>0.0293</td>
<td>Own estimation</td>
</tr>
<tr>
<td>$\sigma_g$</td>
<td>St. dev. shock to $g$</td>
<td>0.0145</td>
<td>Own estimation</td>
</tr>
<tr>
<td>$\sigma_{R^*}$</td>
<td>St. dev. shock to $R^*$</td>
<td>0.0011</td>
<td>Own estimation</td>
</tr>
<tr>
<td>$\sigma_{y^*}$</td>
<td>St. dev. shock to $y^*$</td>
<td>0.0062</td>
<td>Own estimation</td>
</tr>
<tr>
<td>$\sigma_{\pi^*}$</td>
<td>St. dev. shock to $\pi^*$</td>
<td>0.0273</td>
<td>Own estimation</td>
</tr>
<tr>
<td>$\sigma_{p^{Co}}$</td>
<td>St. dev. shock to $p^{Co}$</td>
<td>0.1413</td>
<td>Own estimation</td>
</tr>
</tbody>
</table>

Note: All rates and spreads are annualized figures.
the average leverage between 2001 and 2012 for the largest Chilean firms.\textsuperscript{22} In addition, we also calibrate the external finance premium in steady state ($rp$), for which we choose a value of 120 basis points, which corresponds to the average between the A vs. AAA corporate-bond spread and the BBA vs. AAA spread, for the sample from 2001 to 2012.\textsuperscript{23} Finally, as the steady state for both financial problems impose less restriction than parameters, we normalize $\iota = 0.002$ (as in Gertler and Karadi, 2011), $\nu = 0.97$ (the value used by BGG) and $\mu^e = 0.12$ (in the range used by Christiano et al., 2010, for the US and the EU). Thus, the parameters $\bar{\mu}$, $\bar{\omega}$, $\bar{\iota}^e$ and $\sigma^\omega$ are endogenously set in steady state to match these targets.

We also calibrate the parameters characterizing those exogenous processes for which we have a data counterpart. In particular, for $g$ we use linearly-detrended real government expenditures, for $y^{Co}$ we use linearly-detrended real mining production, for $R^*$ we use the LIBOR rate, for $y^*$ we use linearly-detrended real GDP of commercial partners, for $\pi^*$ we use CPI inflation (in dollars) for commercial partners, and for $p^{Co*}$ we use international copper price deflated by the same price index used to construct $\pi^*$.\textsuperscript{24}

The other parameters of the model were estimated using Bayesian techniques, solving the model with a log-linear approximation around the non-stochastic steady state. The list of these parameters and the priors are described in columns one to four of Table 4.\textsuperscript{25} Because we estimate different version of the models, we used different datasets for each model. In all cases, the following variables were used (all from 2001Q3 to 2012Q4): the growth rates of real GDP, private consumption and investment, the CPI inflation rate, the monetary policy rate, the multilateral real exchange rate, the growth rate of real wages, and the EMBI Chile (a proxy for $\xi_t$). We also include as observables the variables used to estimate the exogenous processes previously described.\textsuperscript{26} For future references, we will call this the Macro dataset.

In addition, for those models including financial frictions, we also use as observables the spread between the 90-days loans rate and the monetary policy rate (as a counterpart of $spr_t$), and the growth rate of total loans in the banking system.\textsuperscript{27} Finally, for the estimation of the GK+BGG we also use the spread between the A vs. AAA corporate-bond as a proxy for $rp_t$.\textsuperscript{28}

Overall, the GK+BGG model is estimated with 17 variables. Our estimation strategy also includes i.i.d. measurement errors for all the observables. For all the variables except for the real exchange rate, the variance of this measurement errors was set to 10% of the variance of the corresponding observables. For the real exchange rate this variance was estimated.\textsuperscript{29}

\textsuperscript{22}This average is computed by consolidating balance sheet data compiled by the SVS (the stock market authority in Chile). On average, this includes the largest 300 firms in the country.
\textsuperscript{23}Here we follow Christiano et al. (2010) who use the spread on corporate bonds of different credit ratings as a proxy for the premium paid by riskier firms.
\textsuperscript{24}The data source for all Chilean-related data is the Central Bank of Chile, while the other variables are obtained from Bloomberg.
\textsuperscript{25}The prior means were set to represent (when available) the estimates of related papers for the Chilean economy (e.g. Medina and Soto, 2007).
\textsuperscript{26}While the parameters of these exogenous processes were calibrated, including these variables in the data set is informative for the inference of the innovations associated with these exogenous processes.
\textsuperscript{27}Results do not significantly change if the growth rate of commercial loans only is used instead.
\textsuperscript{28}We have also tried estimating the GK+BGG model using only loans and the spread as financial variables, although we found that one of the financial shocks was not properly identified, in the sense that the likelihood was not significantly changing the prior for the standard deviations of the innovations of these shocks. When we add the risk premium as an observable, both shocks were adequately identified.
\textsuperscript{29}Similar to other papers estimating this type of models (e.g. Adolfson et al., 2007), the model cannot adequately match the variance of the real exchange rate, which motivates estimating its measurement-error variance.
4 Inference and Goodness of Fit

In this section, we first compare the models in terms of goodness-of-fit, in order to understand if and how the presence of financial frictions helps to improve the ability of the model to account for the dynamics observed in the data. We then compare the inferred parameters and variance decomposition for each of the estimated models, with a special focus on the versions of the model that provide better match to the data.

To have an overall measure of goodness of fit, we compute the marginal data density for different sets of variables implied by the estimated posterior mode for each model. It is important to notice that, because different models were estimated with different data sets, the marginal data densities for the estimating samples are not comparable. Thus, what we have done is to compute, after obtaining the posterior mode for each model, the marginal data density of a different set of variables evaluated at the obtained posterior mode; which are indeed comparable across models. In particular, in Table 2 we compute this statistic for the Macro data set (i.e all non-financial variables) and for a data set that also includes the growth rate of loans and the borrowing-lending spread.

Table 2: Log Marginal Data Density for Different Sets of Variables

<table>
<thead>
<tr>
<th>Model</th>
<th>Macro</th>
<th>Loans + Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>-957.5</td>
<td></td>
</tr>
<tr>
<td>GK</td>
<td>-1006.3</td>
<td>-1144.0</td>
</tr>
<tr>
<td>BGG</td>
<td>-993.0</td>
<td>-1134.2</td>
</tr>
<tr>
<td>GK+BGG</td>
<td>-1020.9</td>
<td>-1201.8</td>
</tr>
</tbody>
</table>

Note: These were computed using a Laplace approximation at the posterior mode.

In terms of the Macro variables, the Base model does the best job. In principle, this result might be expected given that the Base model was estimated to match only these variables and the other models need to match other series as well. But what we see is that the difference is quite large between the Base model and the others. Thus, at least from this overall measure of goodness of fit, it seems that including financial frictions and financial variables does not improve the ability of the model to fit Macro series. Still, we should notice that of the three alternatives with financial frictions, the BGG model is the one providing the closest fit to the Base model. And in terms of the ability of the models with financial frictions to match both the Macro and the financial variables, we can see that the BGG alternative is the one showing the best performance.

To have a more detailed idea of which aspects of the data are better matched by each model, Table 3 reports the standard deviation and first-order autocorrelation of a number of observed variables, both in the data and at the posterior mode for each model. We will first focus on non-financial variables. In terms of standard deviations (Panel A) we can see that, although overall the Base model does a fairly good job, it misses some of the characteristics of the data. For instance, the Base model cannot replicate the variance of consumption growth being larger than that of GDP growth. Moreover, it exacerbates somehow the volatility of investment growth,

---

30These are approximated using the Laplace method
31Another exercise that we have performed but that we do not report is to estimate all models using only the Macro dataset, shutting down the financial shock (because they were not properly identified without the inclusion of financial variables). In that comparison, the performance of the model with financial friction was similar to the Base model, and none of the models with financial frictions generated a significant improvement over (nor they were significantly outperformed by) the Base model.
while it underestimate that of the trade-balance-to-output ratio. Additionally, the model implies a real exchange rate that is much more volatile than in the data, a result that is also consistent with the fact that the model generates a more persistent behavior of this variable. At the same time, the Base model does a fairly good job in accounting for the variability of wages, the nominal interest rate and inflation. And in term of auto-correlations (Panel B), the Base model generally implies higher persistence for many of the observables, particularly for real variables.

Table 3: Selected Second Moments

<table>
<thead>
<tr>
<th>Variable</th>
<th>Data</th>
<th>Base</th>
<th>GK</th>
<th>BGG</th>
<th>GK+BGG</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Standard Deviation (%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ΔGDP</td>
<td>1.02</td>
<td>0.99</td>
<td>1.0</td>
<td>0.9</td>
<td>0.96</td>
</tr>
<tr>
<td>ΔC</td>
<td>1.10</td>
<td>0.95</td>
<td>1.07</td>
<td>1.10</td>
<td>1.14</td>
</tr>
<tr>
<td>ΔI</td>
<td>3.75</td>
<td>4.37</td>
<td>6.06</td>
<td>3.18</td>
<td>5.55</td>
</tr>
<tr>
<td>TB/GDP</td>
<td>5.32</td>
<td>3.65</td>
<td>4.30</td>
<td>3.59</td>
<td>3.72</td>
</tr>
<tr>
<td>ΔW</td>
<td>0.62</td>
<td>0.60</td>
<td>0.78</td>
<td>0.63</td>
<td>0.84</td>
</tr>
<tr>
<td>R</td>
<td>0.46</td>
<td>0.51</td>
<td>1.30</td>
<td>0.68</td>
<td>1.16</td>
</tr>
<tr>
<td>π</td>
<td>0.74</td>
<td>0.62</td>
<td>1.13</td>
<td>0.68</td>
<td>0.99</td>
</tr>
<tr>
<td>rer</td>
<td>5.41</td>
<td>10.55</td>
<td>20.28</td>
<td>12.40</td>
<td>15.49</td>
</tr>
<tr>
<td>ΔL</td>
<td>1.41</td>
<td>1.52</td>
<td>1.31</td>
<td>1.07</td>
<td></td>
</tr>
<tr>
<td>spr</td>
<td>0.26</td>
<td>1.04</td>
<td>0.49</td>
<td>0.92</td>
<td></td>
</tr>
</tbody>
</table>

B. Autocorrelation of Order 1

<table>
<thead>
<tr>
<th>Variable</th>
<th>Data</th>
<th>Base</th>
<th>GK</th>
<th>BGG</th>
<th>GK+BGG</th>
</tr>
</thead>
<tbody>
<tr>
<td>ΔGDP</td>
<td>0.25</td>
<td>0.43</td>
<td>0.57</td>
<td>0.31</td>
<td>0.50</td>
</tr>
<tr>
<td>ΔC</td>
<td>0.63</td>
<td>0.60</td>
<td>0.70</td>
<td>0.66</td>
<td>0.71</td>
</tr>
<tr>
<td>ΔI</td>
<td>0.40</td>
<td>0.70</td>
<td>0.88</td>
<td>0.38</td>
<td>0.79</td>
</tr>
<tr>
<td>TB/GDP</td>
<td>0.73</td>
<td>0.92</td>
<td>0.94</td>
<td>0.92</td>
<td>0.92</td>
</tr>
<tr>
<td>ΔW</td>
<td>0.40</td>
<td>0.48</td>
<td>0.32</td>
<td>0.51</td>
<td>0.50</td>
</tr>
<tr>
<td>R</td>
<td>0.88</td>
<td>0.92</td>
<td>0.97</td>
<td>0.94</td>
<td>0.97</td>
</tr>
<tr>
<td>π</td>
<td>0.63</td>
<td>0.66</td>
<td>0.85</td>
<td>0.73</td>
<td>0.85</td>
</tr>
<tr>
<td>rer</td>
<td>0.73</td>
<td>0.93</td>
<td>0.98</td>
<td>0.95</td>
<td>0.96</td>
</tr>
<tr>
<td>ΔL</td>
<td>0.56</td>
<td>0.16</td>
<td>0.43</td>
<td>0.60</td>
<td></td>
</tr>
<tr>
<td>spr</td>
<td>0.68</td>
<td>0.20</td>
<td>0.89</td>
<td>0.21</td>
<td></td>
</tr>
</tbody>
</table>

Table 3 also shows that models with financial frictions can help to improve the goodness of fit of some these second moments, particularly the BGG model. In terms of the relative variance of consumption and GDP, all models with financial frictions can generate a more volatile consumption. Relative to the Base model, the BGG alternative implies a smaller variance of investment (although it undershoots that in the data) while the other two exacerbate this volatility (particularly the GK model). The variance of the trade-balance-to-output ratio is somewhat larger and closer to the data in the GK and in the GK+BGG, and the BGG model displays a figure similar to that in the Base setup.

Looking at the standard deviation of wages, the monetary policy rate and inflation, the BGG model is overall as good as the Base model, while both the GK and the GK+BGG models over-estimate these variances. And in terms of the real exchange rates, the presence of financial friction seems to exacerbate even further the implied volatility; if anything, the BGG model is the one that induces a variance for this variable that is closer to the Base model. Finally, focusing on the auto-correlation figures, we can see that the BGG model either improves the performance relative to the Base or it has similar implications. The other two models with financial frictions

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32 As we already mentioned, this shortcoming is many times found in estimations of DSGE models for small open economies.
33 The only exception is the auto-correlation of inflation, that in the Base model is clearly closer to the data than with the BGG
generally do a worst job than the either the Base or the BGG model.

In terms of both financial variables, we can see that the BGG setup outperforms the other two models with financial frictions in terms of matching the volatility and autocorrelation of both loans growth and the spread. The difference is most notorious in terms of the latter: both the GK and the GK+BGG models generate a much more volatile and less persistent spread than in the data.

Overall, this goodness-of-fit analysis yields two main conclusions. First, in terms of matching the dynamics of macro (non-financial) variables the BGG model can help to improve the ability of the model to match the data in some dimensions, particularly in terms of the moments we have analyzed. In contrast, both the GK and the GK+BGG alternative generally perform less satisfactorily, particularly the former. The second conclusion is that, in terms of matching the dynamics of the observed financial variables, the BGG model also outperforms the other two setups with financial frictions.

In the rest of this section we describe the posterior mode for the different models, with a special focus on the BGG alternative. Columns 6 to 13 in Table 4 displays the posterior mode and standard deviation of the estimated parameters for the four variants of the model. We will comment on those parameters whose inference is different between models to see how the presence of financial frictions may affect their posterior. One parameter that changes significantly across models is the one governing investment adjustment costs ($\gamma$). In the Base model the posterior mode is around 2.6, in the GK model is somewhat lower (close to 1.6) while in models that include BGG frictions this parameter is larger than in the Base model (around 10 in the BGG alternative and 4.3 in the GK-BGG). It is expected that the presence of financial frictions changes the inference about this parameter.

In fact, in one of the earliest contributions to the financial accelerator literature, Carlstrom and Fuerst (1997) present a financial accelerator similar to BGG as an alternative to capital adjustment costs in order to account for the hump-shaped dynamics of investment. From that perspective, one might expect the presence of financial frictions to diminish the need for capital adjustment costs to fit the data (i.e. a lower value for $\gamma$). However, it is also true that when both capital adjustment costs and financial frictions are considered, one can also expect an interaction between both channels. In particular, it might be the case that with a lower $\gamma$ the price of capital (Tobin’s $q$) is too volatile (particularly after investment shocks), generating an excessive volatility in spreads that will get amplified by the presence of financial frictions.

Another parameter that shows different values between models is the share of working capital that needs to be financed by loans ($\alpha_{W,C}^{W,C}$). In particular, it seems that the versions that include BGG frictions require a smaller value for this parameter relative to the Base model, while when only the GK friction is present the value is much larger. The main difference in terms of the working capital channel between models is that, in setups featuring GK banks, the interest rate on working-capital loans is directly affected by the friction between banks and depositors, while that is not the case in the BGG model (where that rate just equals the monetary policy rate). Thus, given that we have described how the BGG model generally outperforms the other alternatives in terms of goodness of fit, one might think that modifying the models featuring GK banks in a way that working capital loans are not affected by the bank spread (e.g. if households were to extend these loans directly to firms), or even to eliminate completely these loans from the model, could help to improve the fit of these models. We have implemented these alternatives (although not reported) but in general they deliver worst outcomes in terms of goodness of fit than the models we have presented here. Thus, it seems that the working capital channel is an alternative.

34Evidently, given that the marginal-data-density comparison implies a large difference between the Base and the BGG model in terms of matching the Macro data set, it has to be the case that the Base model does significantly better than the BGG model in other dimensions (other than variances and first-order autocorrelations) that we have not explored. We focus on these statistics as they are the common ground of comparison in the related literature.
<table>
<thead>
<tr>
<th>Prior</th>
<th>Posterior Mode</th>
<th>BGG</th>
<th>GK+BGG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>Mode</td>
<td>Mean</td>
<td>St.Dev.</td>
</tr>
<tr>
<td>(\zeta) Habits</td>
<td>0.7</td>
<td>0.1</td>
<td>0.74</td>
</tr>
<tr>
<td>(\psi) Count. Prem. Elast.</td>
<td>0.01</td>
<td>(\Gamma^{-1})</td>
<td>(\infty)</td>
</tr>
<tr>
<td>(\eta^C) E.O.S.</td>
<td>1.4</td>
<td>0.4</td>
<td>1.38</td>
</tr>
<tr>
<td>(\eta^I) E.O.S.</td>
<td>1.4</td>
<td>0.4</td>
<td>1.30</td>
</tr>
<tr>
<td>(\eta^*) Elast. exports</td>
<td>0.3</td>
<td>0.2</td>
<td>0.40</td>
</tr>
<tr>
<td>(\gamma) Inv. Adj. Cost</td>
<td>(N)</td>
<td>4</td>
<td>1.5</td>
</tr>
<tr>
<td>(\theta_W) Calvo prob. (W)</td>
<td>0.75</td>
<td>0.1</td>
<td>0.94</td>
</tr>
<tr>
<td>(\theta) Index. past infl. (W)</td>
<td>0.5</td>
<td>0.2</td>
<td>0.33</td>
</tr>
<tr>
<td>(\theta_H) Calvo prob. (H)</td>
<td>0.75</td>
<td>0.1</td>
<td>0.48</td>
</tr>
<tr>
<td>(\theta_F) Index. past infl. (H)</td>
<td>0.5</td>
<td>0.2</td>
<td>0.48</td>
</tr>
<tr>
<td>(\theta_F) Index. past infl. (F)</td>
<td>0.75</td>
<td>0.1</td>
<td>0.79</td>
</tr>
<tr>
<td>(\rho_R) MPR Rule (R_{t-1})</td>
<td>0.75</td>
<td>0.1</td>
<td>0.83</td>
</tr>
<tr>
<td>(\alpha_p) MPR Rule (\pi_t)</td>
<td>(N)</td>
<td>1.5</td>
<td>1.54</td>
</tr>
<tr>
<td>(\alpha_p) MPR Rule growth</td>
<td>(N)</td>
<td>0.13</td>
<td>0.1</td>
</tr>
<tr>
<td>(\phi_u) Utilization Cost</td>
<td>(N)</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>(\alpha_L^W) Share work. cap.</td>
<td>(N)</td>
<td>0.7</td>
<td>0.3</td>
</tr>
<tr>
<td>Auto Correl. Shocks</td>
<td>(\rho_{\sigma}) Pref.</td>
<td>(\beta)</td>
<td>0.75</td>
</tr>
<tr>
<td>(\rho_{\omega}) Inv.</td>
<td>(\beta)</td>
<td>0.75</td>
<td>0.1</td>
</tr>
<tr>
<td>(\rho_{\sigma}) Temp. TFP</td>
<td>(\beta)</td>
<td>0.75</td>
<td>0.1</td>
</tr>
<tr>
<td>(\rho_{\sigma}) Perm. TFP</td>
<td>(\beta)</td>
<td>0.38</td>
<td>0.1</td>
</tr>
<tr>
<td>(\rho_{\beta}) Country prem.</td>
<td>(\beta)</td>
<td>0.75</td>
<td>0.1</td>
</tr>
<tr>
<td>(\rho_{\mu}) (\mu_t)</td>
<td>(\beta)</td>
<td>0.75</td>
<td>0.1</td>
</tr>
<tr>
<td>(\rho_{\omega}) (\sigma_{\omega_1})</td>
<td>(\beta)</td>
<td>0.75</td>
<td>0.1</td>
</tr>
<tr>
<td>St.Dev. Shocks</td>
<td>(\sigma_{\tau}) Pref.</td>
<td>(\Gamma^{-1})</td>
<td>0.01</td>
</tr>
<tr>
<td>(\sigma_{\tau}) Inv.</td>
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relevant one for each model individually, and that the ranking of models in terms of the ability to fit the data is not due to this particular model feature.

In terms of nominal rigidities, the main difference across models seems to be the price rigidities in Home goods. The Base model and the BGG alternative have similar values for both $\theta_H$ and $\vartheta_H$, while in both the GK and the GK-BGG model the estimated parameters indicate a higher degree of price-stickiness but with a lower importance for the indexation to past inflation. However, as we described before, both the Base and the BGG models seem to better match the dynamics of inflation, thus we regard this estimation with a lower degree of price stickiness but a more important degree of indexation as more likely.

The parameter that governs the elasticity of exports to the real exchange rate ($\eta^*$) is also affected in the presence of financial frictions. In particular, relative to the Base model, when financial frictions are included this elasticity is estimated to be smaller. As we will see, this parameter plays an important role in explaining the propagation of several shocks when financial frictions are included, for it governs how much home producers (and entrepreneurs) benefit from a real depreciation through the exports channel.

Finally, the parameters describing the exogenous driving forces also change between models. However, instead of comparing these parameters directly, it is more instructive to see how these different values affect the way each shock explains aggregate fluctuations. To that end, Table 5 displays the variance decomposition obtained for each version of the model for the relevant domestic observables, computed at their respective posterior modes. In the Base model, the shocks to the marginal efficiency of investment ($\varpi$) and to the stationary TFP process ($z$) are estimated to be the main driving forces of real GDP growth, while preference shocks and foreign driving forces play a more limited but non-trivial role as well. The preference shock is quite relevant to explain consumption fluctuations, and the investment shock helps to explain a large part of investment fluctuations. Foreign shocks also have an important role in explaining the components of aggregate demand, particularly for the trade balance, and they are also quite relevant for the dynamics of the real exchange rate. In terms of nominal variable, stationary TFP shocks play the most relevant role in explaining the evolution of wages and inflation.

When financial frictions are considered the relative contribution of each shock is quite different. In terms of GDP, the investment shock plays a relatively smaller role in all models with financial frictions, while preference shocks become more important. Focusing in the BGG setup the major differences are that commodity price shocks play a larger role for consumption and the trade balance, and that the importance of the shock to the marginal efficiency of investment is even larger for investment dynamics. Finally, we can see that financial shocks, in any of the models with financial frictions, have a very limited role in explaining the variance of non-financial variables, while they are most relevant in explaining fluctuations in the spread. Moreover, in the BGG model the investment shock is also an important determinant of financial variables.

5 Impulse Responses

The goal of this section is to understand how the presence of financial frictions changes the propagation of the main driving forces in the economy, which we will assess through the use of impulse-response analysis. While we will mainly focus on the comparison between the Base and the BGG models (given the results of our goodness-of-fit exercises) we are also interested in understanding the dynamics under the other two alternatives to see if we can understand what type of dynamics are rejected by the data. To keep the comparison as clean as possible, we omit some of the shocks because they do not have a significant impact to explain the variance of these variables. The only exception is the shock to the endowment of commodities that explains between 15 and 20% of the variance of real GDP growth but it has a negligible impact on other variables.
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we will fix the estimated parameters at the posterior mode obtained with the BGG model, and then compute the responses for the other alternatives by shutting down or activating the relevant channels.

We begin by analyzing the effects generated by a monetary policy shock, displayed in Figure 1. As it is usual in New-Keynesian models of small and open economies, in the Base model a positive shock to the Taylor rule leads to a fall in investment, consumption and GDP, while inflation decreases and the real exchange appreciates. In the BGG setup, the effect of the shock in activity gets amplified. As the shock generates a contraction in aggregate demand, financial conditions for entrepreneurs will, ceteris paribus, worsen; leading to a rise in the external finance premium and in the spread. This will further contract economic activity, activating the financial accelerator. As a result, in the BGG model investment falls much more than in the Base model. Consumption also drops slightly more which, together with the effect in investment, leads to a larger contraction in GDP. The dynamics of inflation, the real exchange rate and the policy rate seem to be similar in both the Base and the BGG models.

An interesting (and a priori counter-intuitive) result is that in the BGG model the contractionary policy shock raises loans to finance entrepreneurs, while a simple intuition may lead us to think that loans should fall. However, the key variable for the determination of the external finance premium is not just the loans obtained but entrepreneurs’ leverage (which equals the value of capital over net worth), that indeed drops after the contractionary policy shock. In principle, for a given level of net worth, one would expect that the fall in the value of capital generated by the policy shock should lead to a drop in loans received by entrepreneurs. However, net worth does not remain fixed; it decreases after the shock. Thus, the result that we obtain is generated because, for the chosen parametrization, net worth fall by more than the contraction in the value of capital, and therefore loans to entrepreneurs will actually rise. In other words, the observed average valued for leverage and spreads generate a net worth that is more elastic to changes in financial conditions than the value of assets. We have experimented with alternative calibrations and there are some values for parameters describing the BGG friction that can generate the opposite response for loans; however these alternative parameterizations do not seem to be favored by the data.

The other two models with financial frictions also feature an amplified response of investment. The main qualitative difference is the behavior of inflation, that rises after a contractionary policy shock. A possible explanation for these dynamics is the working capital channel. In both the GK and the GK+BGG setups, the interest rate relevant for working capital loans is not only affected by the policy rate (as in the Base or the BGG models) but also by the friction at the bank. As a consequence actual and expected marginal costs for firms will, ceteris paribus, rise by much more in models featuring GK banks. Thus, while neither in the Base nor in the BGG model the rise in the policy rate generated an increase in inflation trough the working capital channel, in models featuring GK banks this channel seems more relevant.

Figure 2 displays the responses to an increase in the world interest rate. In the Base model this shock is contractionary for consumption (through both a negative-wealth and an intertemporal-substitution effect) and for investment (as the shock increases the real interest rate). This drop in aggregate absorption leads to a real depreciation, which in turn raises aggregate inflation due to the increase in the domestic price of foreign goods. As a consequence, the policy rate rises and, trough the working capital channel, output falls despite the rise in the trade balance generated by the real depreciation. This shocks then shares the contractionary effects generated by

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36 Total loans still fall as working-capital loans drop by more than the rise in loans to entrepreneurs.
37 Actually, we have experimented with the BGG setup in many alternative versions of the model (e.g., closed economies, real models, calibrations closer to more developed countries, among others) and realized that this rise in loans arises in many other cases as well.

23
Figure 1: Impulse Responses to a Monetary Policy Shock.

Note: The blue solid line is the baseline model, the dashed red line corresponds to the GK model, the dash-dotted black line BGG, and the crossed-solid green line is the GK-BGG model. The variables are GDP, consumption, investment, the trade-balance-to-output ratio, inflation, the monetary policy rate, the real exchange rate, the spread, the external finance premium, loans to working capital, loans to entrepreneurs, and the variable being shocked. All variables are measured in percentage deviations with respect to the steady state, and all rates and spreads are expressed in a quarterly basis.
Figure 2: Impulse Responses to a Foreign Interest Rate Shock.

Note: See figure 1.
an increase in the domestic rate, but the crucial difference is that the real exchange rate depreciates here while with the domestic shock the effect was the opposite.

As can be seen in the figure, in the presence of financial frictions the contraction in investment is milder than in the Base model. While the price of capital (not shown) tends to fall after the shock (which tends to worsen financial conditions), there is an offsetting effect. Namely, the persistent real depreciation induces an increase in the marginal product of capital (equal to the rental rate) because home firms produce tradable goods. In equilibrium, and given the estimated parameters, the increase in the external finance premium is only mild and it even falls after some quarters. Thus, relative to the Base model, the cost of financing investment rises by little but the return increases due the interaction between the real depreciation and financial frictions, leading to a milder contraction in investment. Consumption, in contrast, drops by more in the presence of financial frictions; an effect that might be due to the larger increase in the policy rate path.

This lack of amplification of foreign interest rate shocks may seems at odds with a large body of literature that precisely emphasize the opposite: that financial frictions amplify the propagation of this type of shock.\textsuperscript{38} The key difference is that in those related studies financial frictions arise between domestic borrowers and foreign lenders, while in our case both are domestic agents. Thus, in the usual setup in that related literature, it is emphasized that the real depreciation generated by the shock leads to a negative effect in the balance sheet of borrowers, for it increases the burden of liabilities in domestic units. In contrast, this channel is not present in our model.\textsuperscript{39} One of the reason that lead us to omit this channel is that the usual interpretation of frictions between domestic and foreign agents is the presence of debt denominated in foreign currency (e.g. due to liability dollarization). But it is less clear whether this would be a relevant channel for Chile, as liability dollarization is not as a widespread phenomenon as in other emerging countries.\textsuperscript{40}

Another foreign shock that was identified as an important driver in the previous section is the commodity price shock. Figure 3 shows the responses to this price. Qualitatively, this shock generates a positive wealth effect (which is quite large given the estimated persistence for this shock) that raises consumption. In turn, by increasing the demand for domestic goods, the rise in desired consumption raises the marginal product of capital, expanding also investment. This increase in absorption leads to a real appreciation. In the Base model with no financial frictions, inflation experiences a fall, led by a reduction in the domestic price of imported goods due to the real appreciation. Consequently, the policy rate drops.

In models with financial frictions, the rise in investment is relatively milder than in the Base model, particularly for the GK model. Here we also have the two opposite effects that we described before but in the other direction: while the rise in aggregate demand tends to make investment more attractive, the real appreciation reduces the marginal product of capital for tradable firms. Thus, the spread and the external finance premium drop only slightly; a change that is not enough to generate an amplification on investment. In fact, in equilibrium the real exchange rate appreciation seems to be more persistent in models with financial frictions, which further emphasize

\textsuperscript{38}See, for instance, early contributions by Neumeyer and Perri (2005) and Uribe and Yue (2008) in simple, one-sector models, or Cespedes et al. (2004), Devereux et al. (2006), and Gertler et al. (2007) in contexts with multiple sectors and nominal rigidities.

\textsuperscript{39} Actually, in a reduced-form way it could be present, for the real exchange rate is a determinant of the country premium. However, the estimated elasticity for the premium is quite low; so this particular channel does not seems to be quantitatively relevant (in fact, we have played around with priors that would imply a higher elasticity a priori, but the posterior always assigns low values for this parameters).

\textsuperscript{40} A more subtle interpretation of frictions between domestic and foreign lenders would be that, although borrowers may have contracts in domestic currency, banks are the ones that obtain part of their funding from abroad. Thus, given that our setup also considers frictions at the bank level, this might amplify rather than dampen the effect of shocks to the world interest rate. However, preliminary exercises that we have performed estimating models that allow banks to receive financing from abroad (in addition to domestic deposits) tend to have a less satisfactorily fit to the data than the models we consider here. In a sense, this is not surprising given that we have shown that the GK frictions at the bank level that we consider do not seems to help much in fitting the model to the data. We discuss some other possibilities in the concluding section.

26
Figure 3: Impulse Responses to a Foreign Commodity Price Shock.

\[ p^{\text{Co}} \Rightarrow GDP \]

\[ p^{\text{Co}} \Rightarrow C \]

\[ p^{\text{Co}} \Rightarrow I \]

\[ p^{\text{Co}} \Rightarrow TB/GDP \]

\[ p^{\text{Co}} \Rightarrow \pi \]

\[ p^{\text{Co}} \Rightarrow R \]

\[ p^{\text{Co}} \Rightarrow rer \]

\[ p^{\text{Co}} \Rightarrow \text{spr} \]

\[ p^{\text{Co}} \Rightarrow rp \]

\[ p^{\text{Co}} \Rightarrow LWC \]

\[ p^{\text{Co}} \Rightarrow LK \]

\[ p^{\text{Co}} \Rightarrow p^{\text{Co}} \]

Note: See figure 1.
the interaction between financial conditions and the real exchange rate.

Finally, we analyze the two shocks that were identified in the BGG model as the most relevant to explain financial variables. The first is the investment shock, that is displayed in Figure 4. In the Base model this shock increases the return on investment, leading to a rise in investment and the stock of capital. This shift in the supply of capital lowers its price. Inflation increases (and so does the monetary policy rate) due to pressure on aggregate demand for final goods from higher investment, which offsets the supply-side effect that the shocks generates.\footnote{By this we mean that the shock should, ceteris paribus, decrease the expected rental rate of capital as the stock of capital gets accumulated. Thus, ceteris paribus, the shock generates a reduction in expected marginal costs.}

The real exchange rate depreciation can be tracked to the fact that, as investment goods required a large share of imported goods, the current account worsen and foreign debt rises. This, in turn, rises the country premium persistently, and more than the domestic interest rate, which explains the depreciation. Output increases by the rise in the stock of capital and also because the demand for labor rises as well (as its marginal product increases with a higher capital stock). Finally, consumption rises due to the positive wealth effect generated.

In the BGG model, we can see that the expansionary effect on investment is reduced. This happens because, at the same time entrepreneurs would like to accumulate more capital to take advantage of the increased marginal return on investment, the price of capital drops. Thus, they need to rely more on external finance (loans to entrepreneurs increase) but at the same time the value of their assets is negatively affected by the drop in the price of capital. Thus, leverage is reduced and the premium rises. As a result of the milder expansion in aggregate investment, the upward pressure on inflation is milder, and the supply side effect dominates, generating a minor fall on inflation (and on the policy rate). Overall, the expansion in GDP is smaller, the trade-balance reduction is milder, and the real depreciation is not as large. Finally, the expansion in consumption is larger, as the expected path of the policy rate generates further incentives to consume.

In the GK model, the need for external financing is faced by banks, as they would like to lend more to (frictionless) entrepreneurs but they face a financing constraint. If that would be the only difference, the propagation of the shock would not be so different between the GK and the BGG model. However, in our GK setup, the working capital channel adds other dynamics. In particular, the additional need for external financing will rise not only the rate charged to entrepreneurs but also that for working capital loans. As a results, expected marginal cost for firms increase, increasing inflation even further. Moreover, this rise in prices leads the central bank to increase interest rates, further increasing the cost for working capital. As a results, in the GK model the shock is highly inflationary and, at the same time, contractionary for consumption and GDP.

In the GK+BGG model this additional working capital channel is present, but the effect is quantitatively smaller. The reason is that entrepreneurs finance part of the desired increase in the stock of capital with their own resources, therefore they require less loans from banks than in the GK model. As a results, the rise in the spread is milder (closer to the BGG case) and then the effect on inflation through the expected rise in working-capital costs is not as large.

Finally, the other driving force that was particularly important to explain the evolution of the spread in the BGG model is the shock to the expected dispersion of entrepreneurs. Figure 5 shows the impulse responses. The shock acts as an exogenous increases in the external finance premium, generating a drop in investment, and, through a negative wealth effect, a reduction in consumption as well. Inflation is affected by two channels that generate opposing effects, similar to what we describe for the investment shock. On one hand, the contraction on aggregate demand will tend to reduce inflation but, on the other, the rental rate of capital will increase, putting an upward pressure on inflation through the expected rise in marginal costs. In equilibrium, the second seems to
Figure 4: Impulse Responses to an Investment Shock.

Note: See figure 1.
Figure 5: Impulse Responses to a Shock to the Expected Dispersion of Entrepreneurs.

Note: See figure 1. The response of $\sigma_\omega$ shows an increase in period one and not at zero to emphasize that a shock to this variable in a given period changes the variance of the outcomes for entrepreneurs in the following period.
dominated in the first periods, while the demand effect appears to dominated after a few quarters. However, as can be seen, the predicted change in most variables is only mild, which explains why this shock does not appear as a relevant driving force for variables other than the spread.

6 Leaning Against the Wind

The goal of this section is to use the model with financial frictions that best fits the data (the BGG alternative) to analyze if allowing the policy rate to react to financial variables may help to attain better macroeconomic outcomes. In particular, we extend the monetary policy rule to be,

\[ \frac{R_t}{R_{t-1}} = \left( \frac{R_t}{R_{t-1}} \right)^{\rho_R} \left[ \left( \frac{\pi_t}{\bar{\pi}} \right)^{\alpha_{\pi}} \left( \frac{Y_t}{Y_{t-1}} \right)^{\alpha_y} \left( \frac{x_t}{\bar{x}} \right)^{\alpha_x} \right]^{1-\rho_R} \exp(\epsilon^R_t), \]

and we consider there alternative variables that the rule can react to \((x_t)\): the growth rate of real loans, the ratio of loans to GDP, and the spread. The exercise we perform is to fix all other parameters at the posterior mode of the BGG model and to choose the value of \(\alpha_x\) to attain some alternative goals.

We consider several alternative goals. A first set of them has to do with minimizing the variance of either inflation or GDP growth. The results of these exercises are displayed in Table 6, in which Panel A includes the standard deviation of some selected variables evaluated at the posterior mode of the BGG model (these come from Table 3). Panel B shows the results obtained when the rule responds to loans growth. When the goal is to minimize the variance of inflation (first line in Panel B), the best alternative is to set the reaction parameter at a small value (0.07), which allows to obtain minor reduction in the volatility of inflation of near 0.3 percentage points (pp). Such a value for the reaction parameter also produces an almost negligible reduction in the volatility of GDP growth. Finally, the volatility of the spread experiences a negligible increment and the variance of loans growth shows a minor drop.

When the reaction to loans growth is used to reduce output volatility, the optimal rule requires a more aggressive response. In particular, if the growth rate of loans was to increase by 1% in a quarter, the monetary policy rate should increase by 0.75 pp (almost 3 pp in annualized terms). With this reaction, the variance of output growth is reduced by almost 5%. However, there is a trade-off with the volatility of inflation, for in this case it would increase by near 6% relative to the benchmark. Moreover, this optimized rule, while reducing the variance of loan growth, it increases the standard deviation of the spread.

In policy discussions, many times some ratio of credit to GDP is used as the metric to assess the stance of the financial cycle. Thus, we explore the possibility of allowing the policy rate to respond to this ratio. It is interesting to notice that, no matter whether the goal is to minimize the variance of inflation or that of output growth, the optimal value of the reaction parameter is negative. This might be surprising if one has the prior that increases in the ratio of loans to GDP are good indicators of the credit booms and vice-versa; and form that perspective one would think that the optimal reaction parameter should be positive. The problem with such a reasoning is that, while it is true that increases in the policy rate reduce credit, they also tend to contract GDP.

Regardless of this, we can see that if the goal is to minimize the variance of output growth, responding to this ratio yield a higher variance than what can be obtained if the rule responds.

\footnote{This point has lead to recent debate in some central banks that have moved the policy rate in response debt to income ratios. See, for instance, the discussion for Sweden by Svensson (2013, 2014).}
Table 6: Policy Rules that Respond to Financial Variables to Reduce Volatility

<table>
<thead>
<tr>
<th>Variance to minimize</th>
<th>Response parameter</th>
<th>( \pi )</th>
<th>( \Delta GDP )</th>
<th>( spr )</th>
<th>( \Delta L )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Benchmark Rule</td>
<td></td>
<td>0</td>
<td>0.676</td>
<td>0.804</td>
<td>0.494</td>
</tr>
<tr>
<td>B. Rule responds to ( \Delta L_t )</td>
<td>( \pi )</td>
<td>0.071</td>
<td>0.674</td>
<td>0.797</td>
<td>0.500</td>
</tr>
<tr>
<td>C. Rule responds to ( L_t/(p_t Y_t) )</td>
<td>( \pi )</td>
<td>-0.010</td>
<td>0.667</td>
<td>0.795</td>
<td>0.482</td>
</tr>
<tr>
<td>D. Rule responds to ( spr_t )</td>
<td>( \pi )</td>
<td>-0.516</td>
<td>0.656</td>
<td>0.804</td>
<td>0.418</td>
</tr>
<tr>
<td></td>
<td>( \Delta GDP )</td>
<td>-0.044</td>
<td>0.735</td>
<td>0.785</td>
<td>0.453</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.516</td>
<td>0.656</td>
<td>0.804</td>
<td>0.418</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.224</td>
<td>0.661</td>
<td>0.801</td>
<td>0.458</td>
</tr>
</tbody>
</table>

Note: Panel A reports the standard deviations obtained in the BGG model. The first line in Panel B, C and D show the results of choosing the parameter \( (\alpha_x) \) that indicates how the policy rate should react to, respectively, the growth rate of real loans, the ratio of loans to GDP, and the spread, in order to minimize the variance of inflation in the BGG model. The second line in each of these panels is analogous but the goal is to minimize the variance of real GDP growth. All other parameters were fixed to the mode obtained in the BGG model.

Another indicator of the credit cycle is the spread (it tends to fall in credit booms), so we allow the policy rate to react to this variable. In this case, the required response is larger when the goal is to minimize the variance of inflation than when it seeks to reduce the volatility of output. In particular, in the former the policy rate should increase by around 0.5% when the spread drops by 1%, while in the later this elasticity is almost a half. Moreover, it seems that if the policy rule reacts to this spread there is no trade-off: the variance of inflation is reduced even in the case in which the goal is to minimize the variance of output.

Finally, we study the optimal reaction to these alternative policy variables when the metric of desirability is welfare, measured as the unconditional expectation of expected discounted utility. In particular, we compute the value of \( \alpha_x \) that minimizes

\[
E \sum_{t=0}^{\infty} \beta^t v_t \left[ \log \left( C(\alpha_x)_t - \varsigma C(\alpha_x)_{t-1} \right) - \kappa \frac{h(\alpha_x)^{1+\phi}}{1 + \phi} \right],
\]

were, in an abuse of notation, we highlight that allocations will depend on the chosen value for \( \alpha_x \). We approximate the value of this expected utility using a second-order Taylor approximation around the non-stochastic steady state, following Schmitt-Grohe and Uribe (2007a,b). We also compute the consumption equivalent that would make the household indifferent between the equilibrium with the optimal reaction \( (\alpha_x^{opt}) \) and that obtained with the benchmark rule \( (\alpha_x = 0) \). In other words, we define \( \lambda \) such that

\[
E \sum_{t=0}^{\infty} \beta^t v_t \left[ \log \left( C(\alpha_x^{opt})_t - \varsigma C(\alpha_x^{opt})_{t-1} \right) - \kappa \frac{h(\alpha_x^{opt})^{1+\phi}}{1 + \phi} \right] = E \sum_{t=0}^{\infty} \beta^t v_t \left[ \log \left[ (1 - \lambda) \left( (C(0)_t - \varsigma C(0)_{t-1}) \right) - \kappa \frac{h(0)^{1+\phi}}{1 + \phi} \right] \right],
\]

We compute a second order approximation to \( \lambda \) around the non-stochastic steady state.
Table 7: Policy Rules that Respond to Financial Variables to Maximize Welfare

<table>
<thead>
<tr>
<th>Response parameter</th>
<th>$100 \times \lambda$</th>
<th>$\pi$</th>
<th>$\Delta GDP$</th>
<th>$spr$</th>
<th>$\Delta L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Benchmark Rule</td>
<td>0</td>
<td>0.676</td>
<td>0.804</td>
<td>0.494</td>
<td>1.314</td>
</tr>
<tr>
<td>B. Rule responds to $\gamma L_t$</td>
<td>-0.370</td>
<td>0.698</td>
<td>0.848</td>
<td>0.471</td>
<td>1.355</td>
</tr>
<tr>
<td>C. Rule responds to $L_t/(p^Y_t Y_t)$</td>
<td>0.000</td>
<td>0.674</td>
<td>0.803</td>
<td>0.494</td>
<td>1.313</td>
</tr>
<tr>
<td>D. Rule responds to $spr_t$</td>
<td>-0.841</td>
<td>0.660</td>
<td>0.813</td>
<td>0.382</td>
<td>1.278</td>
</tr>
</tbody>
</table>

Results are displayed in Table 7. The reaction parameters in each case are quite different than those obtained when the goal was to minimize some volatility. However, the general result that emerges from the exercise is that the welfare gains of expanding the benchmark rule to consider financial variables are quite small (less than a hundred of a percentage point of consumption). Another way to read this result, compared to those in Table ??, is that the minimization of either the variance of inflation or GDP growth may not be a good approximation to maximizing welfare.

7 Conclusions

In this paper we have set up and estimated a DSGE model of a small open economy that includes two types of domestic financial frictions: one between domestic depositors and banks, and another between banks and domestic borrowers. We have estimated several versions of the model using Chilean data from 2001 to 2012. Our main goal was to understand how the presence of these financial frictions change the prediction of a standard DSGE model that has no financial sector.

We have reached several relevant lessons. First, we have found that the presence of financial frictions can help to improve the fit of the model in several relevant dimensions of the data. In particular, the friction between banks and borrowers (the BGG setup) seems to be more relevant than the friction embedded in the GK framework. However, it is not clear that such a model can generally outperform the goodness of fit of the Base model. Second, we highlighted that the presence of financial frictions significantly alter the propagation of structural shocks, particularly for foreign driving forces. Finally, the policy exercises we have performed indicated that, while “leaning against the wind” strategies may help to reduce the variance of inflation and output, the gains in terms of welfare are quite limited. If anything, it seems that moving the interest rate in response to the spread can lead to the best outcomes.

To conclude, there are several aspect of our framework that deserve to be discussed, for they can point to future improvements in the analysis. First, in our model financial frictions are always binding. In contrast, part of the literature has emphasized financial frictions that are only occasionally binding, particularly in the lending relationship between domestic and foreign agents. Assuming that frictions are always binding is convenient from a computational point of view (for it allows to solve the model using perturbation methods), but of course

43Some examples are Mendoza (2010), Benigno et al. (2013), and Bianchi (2011).

44Linearization not only allows to estimate the model with a likelihood approach more easily, but it also allows to consider many other potentially relevant features of the economy in the model. Global solution methods, required to solve models with occasionally binding constraints, generally require to limit significantly the size of the model (for instance, it would be quite costly to compute
we can be missing important dynamics. For instance, while the EMBI spread for Chile has been relatively small, it still experienced a spike during the 2008 world financial crisis. A similar sudden increase can also be observed in domestic spreads in the same period. This might reflect that financial conditions became suddenly more restrictive than in normal times. Thus it might be of interest to extend our analysis by considering a model in which financial frictions bind occasionally. Nonetheless, while of course this might be relevant from a quantitative point of view, qualitatively the analysis in this paper is still useful to understand the relevant channels that might be part of the propagation of foreign shocks.

In addition, given the highlighted relevance of the real exchange rate, it would be of interest to consider a multi-sector model, with tradables and non-tradables. Arguably, some of the channels that we have emphasized arise because all goods are tradable and in that way, for instance, a real depreciation may improve the financial position of these firms. But if firms in the non-traded sector are also subject to financial constraints, a real depreciation will deteriorate their financial conditions, making less clear what the final effect would be.

Another relevant issue that we did not tackle in this paper is the relative importance of domestic vis-a-vis foreign financial frictions in propagating external shocks. To perform such a comparison, one would need to set up a model with both types of frictions at the same time. For instance, one could consider that banks obtain funds also from abroad, subject to the same type of frictions that we assume between banks and domestic depositors. In such a setup, movements in the real exchange rate will also alter the banks’ balance sheet, leading to an additional amplification channel.

Alternatively, one could consider that firms (at least part of them) obtain funds not only in the domestic market but also abroad (for instance through corporate debt or equity markets). In particular, this can lead to additional relevant dynamics; as emphasized, for instance, by Caballero (2002) to explain how the Asian crisis propagated to the Chilean economy. If large firms can obtain funds both domestically and abroad while smaller firms (particularly in the non-traded sector) have only access to domestic financing, a sudden stop in capital inflows will lead these large firms to turn to the domestic market for financing, crowding-out the credit available for smaller firms. Thus, in such a setup both domestic and foreign financial frictions would be relevant for the propagation of external shocks.

Finally, the policy exercises we performed took as given the reaction parameters for inflation, output and past values of the interest rate in the policy rule. It would be of interest to also optimize over these parameters, in particular to assess if the presence of financial frictions change the preferred reaction function for the policy rate. We left these extensions for future research.

8 References


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Holland, Amsterdam.


A Model Appendix

A.1 Intermediary Objective

This section shows that the objective of financial intermediaries, given by

\[ V_t = E_t \sum_{s=0}^{\infty} (1 - \omega) \omega^s \beta^{s+1} \xi_{t,t+s+1} \left[ (r_{t+1+s}^{L,WC} - r_{t+1+s}) L_{t+s}^{WC} + (r_{t+1+s}^{L,K} - r_{t+1+s}) L_{t+s}^{K} + r_{t+1+s} N_{t+s} \right], \]

can be expressed as

\[ V_t = \varphi_t^{L,WC} L_t^{WC} + \varphi_t^{L,K} L_t^{K} + \varphi_t^{N} N_t. \]

First, notice that

\[ V_t = E_t \sum_{s=0}^{\infty} (1 - \omega) \omega^s \beta^{s+1} \xi_{t,t+s+1} \left[ (r_{t+1+s}^{L,WC} - r_{t+1+s}) L_{t+s}^{WC} + (r_{t+1+s}^{L,K} - r_{t+1+s}) L_{t+s}^{K} + r_{t+1+s} N_{t+s} \right]. \]

Thus,

\[ \varphi_t^{L,WC} \equiv E_t \sum_{s=0}^{\infty} (1 - \omega) \omega^s \beta^{s+1} \xi_{t,t+s+1} (r_{t+1+s}^{L,WC} - r_{t+1+s}) \frac{L_{t+s}^{WC}}{L_t}, \]

\[ \varphi_t^{L,K} \equiv E_t \sum_{s=0}^{\infty} (1 - \omega) \omega^s \beta^{s+1} \xi_{t,t+s+1} (r_{t+1+s}^{L,K} - r_{t+1+s}) \frac{L_{t+s}^{K}}{L_t}, \]

and

\[ \varphi_t^{N} \equiv E_t \sum_{s=0}^{\infty} (1 - \omega) \omega^s \beta^{s+1} \xi_{t,t+s+1+r_{t+1+s}} \frac{N_{t+s}}{N_t}. \]

In terms of \( \varphi_t^{N} \),

\[ \varphi_t^{N} = E_t \left\{ (1 - \omega) \beta \xi_{t,t+1} r_{t+1} + \sum_{s=1}^{\infty} (1 - \omega) \omega^s \beta^{s+1} \xi_{t,t+s+1+r_{t+1+s}} \frac{N_{t+s}}{N_t} \right\}, \]

or,

\[ \varphi_t^{N} = E_t \left\{ (1 - \omega) \beta \xi_{t,t+1} r_{t+1} + \beta \omega \xi_{t,t+1} \frac{N_{t+1}}{N_t} \sum_{s=1}^{\infty} (1 - \omega) \omega^s \beta^{s+1} \xi_{t+1,t+s+1+r_{t+1+s}} \frac{N_{t+s}}{N_{t+1}} \right\}. \]

Finally, changing the index in the sum by \( j = s - 1 \), we obtain

\[ \varphi_t^{N} = E_t \left\{ (1 - \omega) \beta \xi_{t,t+1} r_{t+1} + \beta \omega \xi_{t,t+1} \frac{N_{t+1}}{N_t} \sum_{j=0}^{\infty} (1 - \omega) \omega^j \beta^{j+1} \xi_{t+1,t+1+j+1+r_{t+1+j+1}} \frac{N_{t+j+1}}{N_{t+1}} \right\}, \]

or, using the definition of \( \varphi_t^{N} \) evaluated at \( t+1 \),

\[ \varphi_t^{N} = E_t \left\{ (1 - \omega) \beta \xi_{t+1,t+1} r_{t+1} + \beta \omega \xi_{t+1,t+1} \frac{N_{t+1}}{N_t} \varphi_t^{N} \right\} = \beta E_t \left\{ \xi_{t+1,t+1} \left[ (1 - \omega) r_{t+1} + \omega \frac{N_{t+1}}{N_t} \varphi_t^{N} \right] \right\}. \]

With an analogous procedure we can obtain the expression for \( \varphi_t^{L,WC} \) and \( \varphi_t^{L,K} \).
A.2 Entrepreneurs’ Optimization Problem

Using the definition for \( lev^e_t \) and (13), the Lagrangian for the optimal-contract problem can be written as,

\[
E_t \left\{ \frac{r_{t+1}^K}{q_t} \left[ \omega_{t+1}^e + (1 - \delta) q_{t+1} \right] h(\omega_{t+1}; \sigma_{t+1}) + \eta_{t+1} \left[ g(\omega_{t+1}^e; \sigma_{t+1}) \frac{r_{t+1}^K}{q_t} \left[ \omega_{t+1}^e + (1 - \delta) q_{t+1} \right] - \left( lev^e_t - (lev^e_t - 1) r_{t+1}^L \right) \right] \right\},
\]

where \( \eta_{t+1} \) is the Lagrange multiplier. The choice variables are \( lev^e_t \) and a state-contingent \( \omega_{t+1}^e \). The first order conditions are the constraint holding with equality and

\[
E_t \left\{ \frac{r_{t+1}^K}{q_t} \left[ \omega_{t+1}^e + (1 - \delta) q_{t+1} \right] h(\omega_{t+1}; \sigma_{t+1}) + \eta_{t+1} \left[ g(\omega_{t+1}^e; \sigma_{t+1}) \frac{r_{t+1}^K}{q_t} \left[ \omega_{t+1}^e + (1 - \delta) q_{t+1} \right] - r_{t+1}^L \right] \right\} = 0,
\]

\[h'(\omega_{t+1}^e) + \eta_{t+1} g'(\omega_{t+1}^e) = 0.\]

Combining these to eliminate \( \eta_{t+1} \) and rearranging we obtain (15) in the text.

Finally, we need a functional form for \( F(\omega^e_t; \sigma_{t+1}) \). We follow BGG and assume that \( \ln(\omega^e_t) \sim N(-0.5 \sigma_{t+1}^2, \sigma_{t+1}^2) \) (so that \( E(\omega^e_t) = 1 \)). Under this assumption, we can define

\[ aux^1_t \equiv \ln(\omega^e_t) + 0.5 \sigma_{t+1}^2, \]

and, letting \( \Phi(\cdot) \) be the standard normal c.d.f. and \( \phi(\cdot) \) its p.d.f., we can write,

\[
g(\omega^e_t; \sigma_{t+1}) = \bar{\omega}_t [1 - \Phi(\omega^e_t)] + (1 - \mu^e) \phi(\omega^e_t) \frac{1}{\sigma_{t+1}} - \omega^e_t, \\
g'(\omega^e_t; \sigma_{t+1}) = [1 - \Phi(\omega^e_t)] - \omega^e_t \phi(\omega^e_t) \frac{1}{\sigma_{t+1}} - (1 - \mu^e) \phi(\omega^e_t) \frac{1}{\sigma_{t+1}}, \\
h(\omega^e_t; \sigma_{t+1}) = 1 - \Phi(\omega^e_t) - \omega^e_t \phi(\omega^e_t) \frac{1}{\sigma_{t+1}}, \\
h'(\omega^e_t; \sigma_{t+1}) = -\phi(\omega^e_t) \frac{1}{\sigma_{t+1}} - \omega^e_t \phi(\omega^e_t) \frac{1}{\sigma_{t+1}} - [1 - \Phi(\omega^e_t)] + \omega^e_t \phi(\omega^e_t) \frac{1}{\sigma_{t+1}}.
\]

Finally, a technical note is in order. As we have stated the model, it turns out that whether the participation constraint (11 in the text) holds state-by-state or in expectations (as in, for instance, Rannenberg, 2013) is (up to first order) irrelevant for the characterization of the optimal contract (in equilibrium it will hold without expectations anyway, as in Rannenberg, 2013). What is key to allow to merge the BGG model within the Gertler and Karadi framework is the assumption that the loan rate \( r^L_e \) is not contingent on the aggregate state, and if this is not the case the equilibrium is indeterminate. The intuition for this result is as follows. In the original BGG model, if the participation constraint for the lender holds state-by-state, the nature of \( r^L_e \) is irrelevant. This is so because, as the required return \( r_{t+1}^{L,K} \) is determined elsewhere, the participation constraint pins down the current value of \( \omega_{t+1}^e \) and then the other optimality condition of the optimal contract ((15) in the text) pins down the external finance premium (in fact, given that such a setup is the usual way the BGG model is implemented, an equation like (11) is generally omitted as an equilibrium condition). However, if in the original BGG model the participation constraint for the lender holds in expectations, we do require \( r_{t+1}^{L,e} \) to be non-contingent. In such

\[45\]See, for instance, the appendix of Devereux et al. (2006).
A case, it is precisely equation (11) that pins down $\omega_{t+1}$, while the participation constraint alone just determines (up to first order) $E_t\{\omega_{t+1}'\}$. 

In our setup the reason why we need $r_{t+1}^{Lc}$ to be non-contingent is because $r_{t+1}^{LK}$ is not determined by any other equilibrium condition (the intermediaries’ problem just pins down $E_t\{r_{t+1}^{LK}\}$). Thus, in our framework, equation (11) pins down $\omega_{t+1}$ and, given that value, (12) determines $r_{t+1}^{LK}$. Under the other alternative the equilibrium is indeterminate because only equation (12) displays both $r_{t+1}^{LK}$ and $\omega_{t+1}$, and there is no other equation that determines one of these.

### A.3 Equilibrium Conditions

The variables in uppercase that are not prices contain a unit root in equilibrium due to the presence of the non-stationary productivity shock $A_t$. We need to transform these variables to have a stationary version of the model. To do this, with the exceptions we enumerate below, lowercase variables denote the uppercase variable divided by $A_{t-1}$ (e.g. $c_t \equiv \frac{c_t}{A_{t-1}}$). The only exception is the Lagrange multiplier $\Lambda_t$ that is multiplied by $A_{t-1}$ (i.e. $\lambda_t \equiv \Lambda_t A_{t-1}$), for it decreases along the balanced growth path.

The rational expectations equilibrium of the stationary version of the model model is the set of sequences

$$\{\lambda_t, c_t, h_t, h^d, w_t, \bar{w}_t, mc_t^W, f_t^W, \Delta_t^W, i_t, k_t, r_t^K, q_t, y_t, \pi_t^C, \pi_t^F, \pi_t^H, \pi_t^C.H, \pi_t^L.H, \pi_t^I.H, \pi_t^S, \pi_t^A, R_t, \}
$$

$$\{\xi_t, \pi_t, \pi_t^S, \pi_t, r_t^K, r_t^{L.K}, r_t^{L.WC}, R_t^{L.K}, R_t^{L.WC}, \psi_t, \omega_t, \sigma_t, \rho_t, \epsilon_t\}_{t=0}^\infty,$$

(64 variables) such that for given initial values and exogenous sequences

$$\{v_t, w_t, z_t, a_t, \epsilon_t, \epsilon_t^R, R_t^*, \pi_t^S, \pi_t^C, \pi_t^C, \pi_t, \mu_t, \sigma_{\omega,t}\}_{t=0}^\infty,$$

the following conditions are satisfied. Households:

$$\lambda_t = \left(c_t - \frac{c_t-1}{\bar{a}_{t-1}}\right)^{-1} - \beta E_t\left\{\frac{v_{t+1}}{v_t} (c_{t+1} a_t - c_t)^{-1}\right\}, \quad \text{(E.1)}$$

$$w_t mc_t^W = \frac{h^d_t}{\lambda_t}, \quad \text{(E.2)}$$

$$\lambda_t = \frac{\beta}{a_t} R_t^* E_t\left\{\frac{v_{t+1}}{v_t} (\pi_{t+1}^S)^{\lambda_{t+1}}\right\}, \quad \text{(E.3)}$$

$$\lambda_t = \frac{\beta}{a_t} R_t^* E_t\left\{\frac{v_{t+1}}{v_t} (\pi_{t+1}^S)^{\lambda_{t+1}}\right\}, \quad \text{(E.4)}$$

$$f_t^W = mc_t^W \bar{w}_t^{-\epsilon_W} h_t^{\epsilon_W} + \beta \theta W E_t\left\{\frac{v_{t+1}}{v_t} (\pi_t^S)^{-\epsilon_W} \left(\frac{c_t^{\omega W} \pi_t^{1-\omega W}}{\pi_t^{1-\omega W}}\right) \left(\frac{w_t^{\omega W}}{w_t^{1-\omega W}}\right) \left(\frac{w_t^{1-\omega W}}{w_t^{1-\omega W}}\right) f_t^{W+1}\right\}, \quad \text{(E.5)}$$

$$f_t^W = \bar{w}_t^{1-\epsilon_W} h_t^{(\epsilon_W - 1)} + \beta \theta W E_t\left\{\frac{v_{t+1}}{v_t} (\pi_t^S)^{1-\epsilon_W} \left(\frac{c_t^{\omega W} \pi_t^{1-\omega W}}{\pi_t^{1-\omega W}}\right) \left(\frac{w_t^{\omega W}}{w_t^{1-\omega W}}\right) \left(\frac{w_t^{1-\omega W}}{w_t^{1-\omega W}}\right) f_t^{W+1}\right\}, \quad \text{(E.6)}$$

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\[ 1 = (1 - \theta_W)w_t^{1-\epsilon_W} + \theta_W \left( \frac{w_{t-1} \pi_W^{1-\alpha} \pi_t^{1-\alpha}}{w_t} \right)^{1-\epsilon_W}, \] 
(E.7)

\[ \Delta_t^W = (1 - \theta_W)\bar{w}_t^{-\epsilon_W} + \theta_W \left( \frac{w_{t-1} \pi_W^{1-\alpha} \pi_t^{1-\alpha}}{w_t} \right)^{-\epsilon_W} \Delta_{t-1}^W, \] 
(E.8)

\[ h_t = h_t^d \Delta_t^W. \] 
(E.9)

Composite final goods:

\[ y_t^C = [1 - \alpha_G] \frac{1}{\eta_C} \left(x_t^{C,H}\right)^{1-\alpha_C} + \alpha_G \left(x_t^{C,F}\right)^{1-\alpha_C}, \] 
(E.10)

\[ i_t = [1 - \alpha_I] \frac{1}{\eta_I} \left(x_t^{I,H}\right)^{1-\alpha_I} + \alpha_I \left(x_t^{I,F}\right)^{1-\alpha_I}, \] 
(E.11)

\[ g_t = [1 - \alpha_G] \frac{1}{\eta_G} \left(x_t^{G,H}\right)^{1-\alpha_G} + \alpha_G \left(x_t^{G,F}\right)^{1-\alpha_G}. \] 
(E.12)

\[ x_t^{C,H} = (1 - \alpha_C) \left(p_t^H\right)^{-\eta_C} y_t^C, \] 
(E.13)

\[ x_t^{C,F} = \alpha_C \left(p_t^F\right)^{-\eta_C} y_t^C, \] 
(E.14)

\[ x_t^{I,H} = (1 - \alpha_I) \left(p_t^H\right)^{-\eta_I} i_t, \] 
(E.15)

\[ x_t^{I,F} = \alpha_I \left(p_t^F\right)^{-\eta_I} i_t, \] 
(E.16)

\[ x_t^{G,H} = (1 - \alpha_G) \left(p_t^H\right)^{-\eta_G} g_t, \] 
(E.17)

\[ x_t^{G,F} = \alpha_G \left(p_t^F\right)^{-\eta_G} g_t. \] 
(E.18)

Home goods:

\[ m_{e_t}^H = \frac{1}{\alpha^\alpha (1 - \alpha)^{1-\alpha}} \left( r_t^K \right)^{-\alpha} u_{t-1}^{1-\alpha} \left[ 1 + \alpha^\alpha \left( p_t^L WC - 1 \right) \right], \] 
(E.19)

\[ \mu_{k_t-1}^R = \frac{u_{t-1} k_{t-1}}{b_t^d} = \frac{u_{t-1}}{\pi_t} w_t \left( 1 - \alpha_t^R \right), \] 
(E.20)

\[ y_t^{WC} = \alpha_L \left( u_t h_t^d + r_t^K u_t \frac{k_{t-1}}{\alpha_t-1} \right), \] 
(E.21)

\[ f_t^H = (p_t^H)^{-\epsilon_H} y_t^H m_{e_t}^H + \beta H E_t \left( \frac{u_{t+1} \lambda_{t+1}}{v_t} \right) \left( \frac{w_{t+1} \pi_{t+1}^{1-\alpha} \pi_{t+1}}{w_t} \right)^{-\epsilon_H} \left( \frac{p_t^H}{p_{t-1}^H} \right)^{-\epsilon_H} (1 - \epsilon_H) f_{t+1}^H, \] 
(E.22)

\[ f_t^H = (p_t^H)^{-\epsilon_H} y_t^H \left( \frac{\epsilon_H - 1}{\epsilon_H} \right) + \beta H E_t \left( \frac{u_{t+1} \lambda_{t+1}}{v_t} \right) \left( \frac{w_{t+1} \pi_{t+1}^{1-\alpha} \pi_{t+1}}{w_t} \right)^{-\epsilon_H} \left( \frac{p_t^H}{p_{t-1}^H} \right)^{-\epsilon_H} (1 - \epsilon_H) f_{t+1}^H, \] 
(E.23)

\[ y_t^H \Delta_t^H = z_t \left( \frac{u_{t+1} k_{t-1}}{a_t-1} \right)^{1-\alpha}, \] 
(E.24)
1 = \theta_H \left( p_{l+1}^H \pi_{l+1}^{1-\delta_H} \right) \frac{1-\varepsilon_H}{\pi_t} + (1 - \theta_H) (\tilde{p}_t^H)^{1-\varepsilon_H}, \quad (E.25)

\Delta_t^H = (1 - \theta_H) (\tilde{p}_t^H)^{-\varepsilon_H} + \theta_H \left( \frac{p_{l+1}^H \pi_{l+1}^{1-\delta_H}}{\tilde{p}_t^H} \right)^{-\varepsilon_H} \Delta_{t-1}^H. \quad (E.26)

Capital accumulation:

\begin{align*}
k_t &= (1 - \delta) \frac{k_{t-1}}{\alpha_{t-1}} + \left[ 1 - \frac{\gamma}{\varepsilon} \left( \frac{a_{t-1}}{\alpha_{t-1}} - \bar{a} \right)^2 \right] \tau_t, \\
p_t &= \left[ 1 - \frac{\gamma}{\varepsilon} \left( \frac{a_{t-1}}{\alpha_{t-1}} - \bar{a} \right)^2 - \frac{\gamma}{\varepsilon} \left( \frac{a_{t-1}}{\alpha_{t-1}} - \bar{a} \right) \frac{\alpha_{t-1}}{a_{t-1}} \right] \tau_t \\
&+ \beta \gamma E_t \left\{ \frac{v_{t+1} \lambda_{t+1} q_{t+1}}{v_t \lambda_t q_t} \left( \frac{a_{t+1}}{\alpha_{t+1}} - \bar{a} \right) \left( \frac{\alpha_{t+1}}{\alpha_{t+1}} \right)^2 \tau_{t+1} \right\}. \quad (E.27)
\end{align*}

Imported goods:

\begin{align*}
m_t &= v_t^F \Delta_t^F, \\
f_t^F &= (\tilde{p}_t^F)^{-\varepsilon_F} v_t^F + \beta \theta_F E_t \left\{ \frac{v_{t+1} \lambda_{t+1} q_{t+1}}{v_t \lambda_t q_t} \left( \frac{\tilde{p}_{t+1}}{\tilde{p}_t} \right)^{-\varepsilon_F} \left( \frac{p_t}{\tilde{p}_{t+1}} \right)^{-\varepsilon_F} \right\}. \quad (E.29, E.30)
\end{align*}

Entrepreneurs:

\begin{align*}
r_t^K &= r_t^K \exp[\phi(u_t - 1)], \\
l_t^K &= g(\omega_t; \sigma_{\omega,t-1}) \cdot (r_t^K) - \phi(u_t) + (1 - \delta) q_t k_{t-1}. \quad (E.35, E.36)
\end{align*}

\begin{align*}
E_t \left\{ \frac{r_{t+1}^K u_{t+1} - \phi(u_{t+1}) + (1 - \delta) q_{t+1}}{q_t} \right\} = & \left[ \frac{h'(\omega_{t+1}; \sigma_{\omega,t}) g(\omega_{t+1}; \sigma_{\omega,t})}{g'(\omega_{t+1}; \sigma_{\omega,t})} - h(\omega_{t+1}; \sigma_{\omega,t}) \right] = \\
E_t \left\{ \frac{\pi_{\omega,t}^K h'(\omega_{t+1}; \sigma_{\omega,t})}{g'(\omega_{t+1}; \sigma_{\omega,t})} \right\}. \quad (E.37)
\end{align*}

\begin{align*}
r_{t-1}^L &= \omega_t r_t^K u_t - \phi(u_t) + (1 - \delta) q_t k_{t-1} \frac{k_{t-1}}{r_t^K}, \quad (E.38)
\end{align*}

\begin{align*}
n_t^e &= \frac{v}{\alpha_{t-1}} \left[ \{ r_t^K u_t - \phi(u_t) + (1 - \delta) q_t k_{t-1} h(\omega_{t}; \sigma_{\omega,t-1}) \} + \varepsilon n^e, \quad (E.39)
\end{align*}
Market clearing and definitions:

\[ r_{pt} = \frac{E_t \left\{ \frac{L^K_{t+1}u_{t+1} - \phi(u_{t+1}) + (1-\delta)q_{t+1}}{\omega} \right\}}{E_t \left\{ L^K_t \right\}}, \]  
(E.40)

\[ lev_t^n = \frac{q_Lk_t}{n_t^\pi}, \]  
(E.42)

\[ r_{t-1}^{L,c} = \frac{R_{t-1}^{L,c}}{\pi_t}. \]  
(E.43)

Banks:

\[ g_t^L = \frac{\beta}{\alpha_t} E_t \left\{ \frac{v_{t+1} \lambda_{t+1}}{\lambda_t} \left[ (1 - \omega)(r_{t+1}^{L,WC} - r_{t+1}) + \omega \frac{l^{WC}_t}{l^*_t} \right] \right\}, \]  
(E.44)

\[ g_t^L = \frac{\beta}{\alpha_t} E_t \left\{ \frac{v_{t+1} \lambda_{t+1}}{\lambda_t} \left[ (1 - \omega)(r_{t+1}^{L,K} - r_{t+1}) + \omega \frac{l^{K}_t}{l^*_t} \right] \right\}, \]  
(E.45)

\[ g_t^N = \frac{\beta}{\alpha_t} E_t \left\{ \frac{v_{t+1} \lambda_{t+1}}{\lambda_t} \left[ (1 - \omega)r_{t+1} + \omega \frac{n_{t+1}}{n_t} \right] g_t^n \right\}, \]  
(E.46)

\[ lev_t = \frac{g_t^N}{\mu_t - g_t^n}, \]  
(E.47)

\[ l_t = lev_t n_t, \]  
(E.48)

\[ l_t = l_t + l^{WC}_t, \]  
(E.49)

\[ d_t = l_t - n_t, \]  
(E.50)

\[ n_t = \frac{\omega}{\alpha_{t-1}} \left[ (r_{t+1}^{L,WC} - r_{t+1})n_{t+1} + (r_{t+1}^{L,K} - r_{t+1})n_{t+1} + r_t n_{t+1} \right] + m, \]  
(E.51)

\[ spr_t = \frac{(R_{t}^{L,WC}_t)^{WC} + R_{t}^{L,K}_t n_{t+1}}{l_t} \]  
(E.52)

\[ r_t^{L,WC} = \frac{R_{t}^{L,WC}_t}{\pi_t}, \]  
(E.53)

Rest of the world:

\[ x_t^{H} = \sigma_t \left( \frac{p_t^H}{\tilde{r}_{t}} \right)^{-\eta_t} y_t^*, \]  
(E.54)

\[ \xi_t = \tilde{\xi} \exp \left[ -\psi \frac{\tilde{r}_{t} b_t^* - \tilde{r}_{t} \times \tilde{b}_t^*}{\tilde{r}_{t} \times \tilde{b}_t^*} + \zeta_t - \zeta_t^* \right]. \]  
(E.55)

Monetary policy:

\[ R_t = \frac{R_{t-1}}{R_t} \left( \frac{\pi_t}{\pi_x} \right)^{\alpha_p} \left[ \frac{y_t a_{t-1}}{y_{t-1} \tilde{a}} \right]^{\alpha_p} 1 - \rho_R \exp(\varepsilon_t R). \]  
(E.56)

Market clearing and definitions:

\[ y_t^H = x_t^{C,H} + x_t^{G,H} + x_t^{G,H} + x_t^{H*}, \]  
(E.57)

\[ \frac{\tilde{r}_{t}}{\tilde{r}_{t-1}} = \frac{\pi_t^S}{\pi_t}, \]  
(E.58)

\[ y_t = e_t + i_t + g_t + x_t^{H*} + y_t^{Co} - m_t, \]  
(E.59)
Also, from (E.36), (E.37), (E.41) and (E.42),

\[ rp \left[ h'(\bar{x}; \sigma_\omega)g(\bar{x}; \sigma_\omega) - h(\bar{x}; \sigma_\omega)g'(\bar{x}; \sigma_\omega) \right] = h'(\bar{x}; \sigma_\omega), \]
\[
\frac{\nu^e - 1}{\nu^e} = g(\tilde{\omega}^e; \sigma_\omega) \nu^e.
\]

These two equations can be solved numerically to obtain \(\tilde{\omega}^e\) and \(\sigma_\omega\). Then, from the definition of \(\Gamma\),

\[
R^{L,e} = \Gamma R,
\]

and combining (E.36) and (E.38),

\[
R^{L,K} = \frac{R^{L,e} \nu^e - 1}{\nu^e}, \quad r^{L,K} = R^{L,K}/\pi.
\]

Thus, from (E.44) and (E.45)

\[
R^{L,WC} = R^{L,K}, \quad r^{L,WC} = r^{L,K}.
\]

From (E.10)-(E.18),

\[
p^F = \frac{1 - \alpha C (p^H)^{1 - \eta_C}}{\alpha C}, \quad p^I = [(1 - \alpha I) (p^H)^{1 - \eta_I} + \alpha I (p^F)^{1 - \eta_I}]^{1/\eta_I}, \quad p^G = [(1 - \alpha G) (p^H)^{1 - \eta_G} + \alpha G (p^F)^{1 - \eta_G}]^{1/\eta_G}.
\]

From (E.25), (E.32) and (E.7),

\[
\tilde{p}^H = 1, \quad \tilde{p}^F = 1, \quad \bar{w} = 1.
\]

From (E.26), (E.34) and (E.8),

\[
\Delta^H = (\tilde{p}^H)^{-\epsilon_H}, \quad \Delta^F = (\tilde{p}^F)^{-\epsilon_F}, \quad \Delta^W = \bar{w}^{-\epsilon_W}.
\]

From (E.22)-(E.23), (E.30)-(E.31) and (E.5)-(E.6),

\[
mc^H = \frac{\epsilon_H - 1}{\epsilon_H} \tilde{p}^H, \quad mc^F = \frac{\epsilon_F - 1}{\epsilon_F} \tilde{p}^F, \quad mc^W = \left(\frac{\epsilon_W - 1}{\epsilon_W}\right) \bar{w}.
\]

From (E.9),

\[
h^d = h/\Delta^W.
\]

From (E.28),

\[
q = \frac{p^I}{\bar{w}}.
\]

From (E.41),

\[
r^K = q \left[r_p \times \rho^{L,K} - (1 - \delta)q\right].
\]

From (E.19),

\[
w = \left\{\frac{\alpha^\alpha (1 - \alpha)^{1 - \alpha} \rho^H m^H \frac{1}{c}}{(r^K)^{1 + \alpha} (R^{L,WC} - 1)}\right\}^{1/\alpha}.
\]

From (E.5),

\[
f^W = \bar{w}^{-\epsilon_W} h^d m^W / (1 - \beta \theta_W).
\]
From (E.20),
\[ k = \frac{\alpha ah d}{(1 - \alpha) r^K}. \]
From (E.24),
\[ y^H = z (k/a)^\alpha (ahd)^{1-\alpha} / \Delta^H. \]
From (E.22),
\[ f^H = mc^H (\tilde{p}^H)^{-\epsilon^H} y^H / (1 - \beta\theta_H). \]
From (E.27),
\[ i = k \left( \frac{1 - (1 - \delta)/a}{\omega} \right). \]
From (E.29),
\[ rer = mc^F p^F. \]
From (E.15)-(E.16),
\[ x^{I,H} = (1 - o_I) \left( \frac{p^H}{p^I} \right)^{-\eta_I} i, \]
\[ x^{I,F} = o_I \left( \frac{p^F}{p^I} \right)^{-\eta_I} i. \]

Let \( mon = \mu^c [r^K + (1 - \delta)q_{z_\eta}^\frac{\lambda}{r} \Phi(aux^1 - \sigma_x) \) be the monitoring costs paid in steady state. From GDP equal to value added, equivalent to (E.62), and (E.33),
\[ p^Y y = p^H y^H + p^Y ys + p^F (1 - mc^F \Delta^F) y^F - mon. \]
Using (E.63) and (E.14),
\[ p^Y y = p^H y^H + p^Y ys + p^F (1 - mc^F \Delta^F) \left[ o_C (p^F)^{-\eta_C} y_C + x^{IF} + o_G \left( \frac{p^F}{p^G} \right)^{-\eta_G} g \right] - mon. \]
Using (E.64),
\[ p^Y y = p^H y^H + p^Y ys + p^F (1 - mc^F \Delta^F) \left[ o_C (p^F)^{-\eta_C} (c + mon) + x^{IF} + o_G \left( \frac{p^F}{p^G} \right)^{-\eta_G} g \right] - mon. \]
Using (E.62),
\[ p^Y y = p^H y^H + p^Y ys + p^F (1 - mc^F \Delta^F) \left[ o_C (p^F)^{-\eta_C} (p^Y y(1 - s^H - s^G) - p^I i + mon) + x^{IF} + o_G \left( \frac{p^F}{p^G} \right)^{-\eta_G} p^Y ys^g \right] - mon. \]
Thus,
\[ p^Y_y = \frac{p^H_y + p^F(1 - mc^F\Delta^F)[-o_C(p^F)^{-\eta_C} (p^F i - \text{mon}) + x^F] - \text{mon}}{1 - s^{Co} - p^F(1 - mc^F\Delta^F)[o_C(p^F)^{-\eta_C} (1 - s^{tb} - s^g) + o_G \left( \frac{p^F}{p^G} \right)^{-\eta_G} \frac{s^g}{s^c}]} \]

From \( s^{tb} = \frac{tb}{p^Y_y} \), \( s^g = \frac{p^G g}{(p^Y_y)g} \), \( s^{Co} = rer \times p^{Co^*}/p^Y_{Co} \) and the exogenous process for \( g_t \),
\[ tb = s^{tb}p^Y_y, \quad g = \bar{g} = \frac{s^g p^Y_y}{p^G}, \quad y^{Co} = \bar{g}^{Co} = s^{Co}p^Y_y/(rer \times p^{Co^*}). \]

From (E.62),
\[ e = p^Y_y - p^I_i - p^G g - tb. \]

From (E.64),
\[ y^C = c + \text{mon}. \]

From (E.13)-(E.14) and (E.17)-(E.18),
\[ x^{C,H} = (1 - o_C)(p^H)^{-\eta_C} y^C, \]
\[ x^{C,F} = o_C(p^F)^{-\eta_C} y^C, \]
\[ x^{G,H} = (1 - o_G) \left( \frac{p^H}{p^G} \right)^{-\eta_G} g, \]
\[ x^{G,F} = o_G \left( \frac{p^F}{p^G} \right)^{-\eta_G} g. \]

From (E.57),
\[ x^{H*} = y^H - x^{C,H} - x^{I,H} - x^{G,H}. \]

From (E.63),
\[ y^F = x^{C,F} + x^{I,F} + x^{G,F}. \]

From (E.30),
\[ f^F = mc^F(p^F)^{-\epsilon_F} y^F/(1 - \beta \theta_F). \]

From (E.33),
\[ m = y^F \Delta^F. \]

From (E.59),
\[ g = c + i + g + x^{H*} + y^{Co} - m. \]

From (E.62),
\[ p^Y = (c + p^I_i + p^G g + tb)/y. \]

From (E.1),
\[ \lambda = \left( \frac{c - \zeta c}{\Delta} \right)^{-1} - \beta \zeta \left\{ (ca - \zeta c)^{-1} \right\}. \]

From (E.2),
\[ \kappa = mw^W \lambda w/h^\phi. \]
From (E.54),
\[ o^* = (x^H/y^*)(p^H/\text{rer})^y. \]
From (E.61),
\[ b^* = \bar{b}^* = \frac{tb - (1 - \chi)\text{rer} \times p^{Co\times y^{Co}}}{\text{rer} [1 - (R^* + \xi)/(\pi^*a)]}. \]
From (E.38),
\[ r^{Le} = \bar{\omega}^e \frac{[r^K + (1 - \delta)q]k}{l}. \]
From (E.42),
\[ n^e = \frac{qk}{\text{lev}^e}. \]
From (E.40),
\[ l^K = qk - n^e. \]
From (E.39),
\[ \epsilon^e = \{n^e - \omega^e \left( [r^K + (1 - \delta)q]kh(\bar{\omega}^e, \sigma_{\omega}) \right) \} / n^e. \]
From (E.51),
\[ \omega = a \left( 1 - i \right) / \left( [r^{L,K} - r] \text{lev} + r \right). \]
From (E.46),
\[ g^N = \frac{\beta}{a} \frac{1 - \omega}{1 - \beta \omega} r. \]
From (E.44),
\[ g^L = \beta \frac{1 - \omega}{a} \frac{1}{1 - \beta \omega} (r^{L,K} - r). \]
From (E.47),
\[ \mu = g^L + \frac{g^N}{\text{lev}}. \]
From (E.21),
\[ l^{WC} = \alpha_{L}^{WC} \left( wh + r^K k \frac{a}{a} \right). \]
From (E.49),
\[ l = l^{WC} + l^K. \]
From (E.50),
\[ n = \frac{l}{\text{lev}}. \]
From (E.50),
\[ d = l - n. \]
From (E.52),
\[ \text{spr} = \frac{(R_{L}^{L,W C}l^{WC} + R_{L}^{L,K})}{l} \frac{1}{R}. \]