Financial Institution Dynamics and Capital Regulations*

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June 27, 2014

Preliminary

Abstract

This paper studies the implications of two frictions that banks likely face: (i) time inconsistency in dividend payout and equity issuance and (ii) moral hazard with respect to bank risk taking and default. We propose a model where there is a role for banking regulations due to these frictions. By characterizing the time-consistent Markov equilibrium, we highlight time-consistent outcome that features bank managers to pay high dividends and make capital accumulation difficult. In addition, an introduction of moral hazard makes banks take default risk due to limited liability. These two frictions also interact to compound the problem. While the time consistent outcome makes bank capital accumulation difficult, moral hazard leads to excessive leverage, together enhancing the default risk. Two types of regulations may be needed to reduce the effects of the two frictions. Minimum capital requirement would limit the extent of default-risk taking by reducing the excessive provision of loans. However, minimum requirement alone may not be sufficient to address the difficulty in capital accumulation due to time consistency. A regulation such as capital conservation buffer, directly restricting the distribution of earnings to achieve higher capital accumulation, may be necessary.

Keywords: Banks; Time inconsistency; Moral hazard; Capital requirement

JEL classification: G21; G28; G32; G35; E44

*The authors would like to thank the participants at the Bank of Canada-Bank of Japan Joint Workshop at the Bank of Japan in November 2013, the 5th BIS Consultative Council for the Americans Research Conference in May 2014, the Canadian Economic Association meetings in June 2014 and the Computing in Economics and Fiance in June 2014. The views expressed in this paper are those of the authors and not necessarily those of the Bank of Canada, the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

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1 Introduction

The recent financial crisis led to a creation of a new set of banking regulations, i.e., Basel III.[1] While these regulations aim to improve global and domestic financial stability, their full and broad impacts on an economy are still being assessed. An increasing number of studies analyze bank capital requirements, one main part of Basel III, however multiple layers of capital requirements make the analysis difficult. One challenge is the understanding of how each layer of the regulations interacts with different frictions that banks face. In this paper, we aim to highlight two such frictions: (i) bank manager's time-inconsistent incentive in bank equity issuance and dividend payout and (ii) moral hazard with respect to bank default and limited liability.[2]

In addition to minimum capital requirement, Basel III introduced *capital conservation buffer* as a layer of bank capital requirements. Capital conservation buffer requires banks to hold capital above their minimum capital requirements. It is designed so that banks build up capital buffers before the periods of stress which can be drawn down as losses materialize.[3] When banks draw down on the buffer, they are required to rebuild it by reducing discretionary distribution of earnings, including dividend and staff bonus payments. Empirical studies and observations on the recent financial crisis show that some troubled banks with depleting capital continued to pay dividends while bank executives kept receiving high compensations. While these observations motivate the need for such regulation, more analysis on what frictions can give rise to such bank behaviour is important.

The model with the two frictions that we study can qualitatively generate these behaviour, giving a role for such regulation. The main deviation from the macro-banking literature is the assumption that the bank manager and owners (or equity shareholders) have different objectives. While the manager has control over bank resources (including equity issuance and dividend payouts) and tries to maximize his consumption flows, the shareholders have incentive to receive higher dividends and to limit a dilution of their share value. This environment leads to the first friction, time-inconsistent incentives in equity issuance and dividend payouts from the perspectives of the manager. When issuing new equity (selling new shares)

[1] See Basel Committee on Banking Supervision (2011).

[2] We analyze these issues towards a longer-term objective of building a quantitative-analytic framework of financial institution dynamics and regulations.

[3] Besides the capital conservation buffer, countercyclical buffer (another layer of the capital requirements) also requires banks to maintain capital above the minimum threshold. Countercyclical buffer is designed to address a build-up of systemic risk in the economy when aggregate credit growth is "too high" whereas capital conservation buffer is designed for risk associated with individual banks.

today, the manager promises and the shareholders expect certain dividends in the future.[4] However, when the time comes to pay out dividends (or dilute away the share values), the manager has no incentive to follow through with the previous promise as the equity has already been raised. Time-consistent solution we propose is based on the generalized Euler (GEE) equation together with the application of the Markov equilibrium concept.[5] In this solution, the manager takes into account the dividend payout behaviour (as well as of equity issuance) of his future self such that past decisions are consistent with the current ones. The time-consistent solution can feature high dividend payout and low valuation of equity, making it more costly to accumulate bank capital. Another friction we study is moral hazard which can lead to excessive default-risk taking by the manager. Moral hazard, caused by limited liability, can give rise to excessive leverage with risky loans. Low bank capital together with high leverage in risky loans can lead to excessive risk taking.

Multiple regulations may be necessary to address these problems. While minimum capital requirements could reduce the excessive leverage issue, the difficulty associated with bank capital accumulation would require another regulation that hits the source of the friction, the time-inconsistency issue. Implementation of capital conservation buffer, which restricts dividend and manager compensation when capital is low, could more directly improve bank capital accumulation. We discuss these issues by characterizing the equilibrium of the model.

The literature on the analysis of time consistent policies goes back to Kydland and Prescott (1977). Building on this literature, Krusell, Kuruşçu, and Smith (2002) and Krusell and Smith (2003) introduced an analysis of generalized Euler equations (GEEs) and their numerical algorism to solve the model. They find that, although time consistency problems often lead to multiplicity of solutions and equilibria, the magnitude of the problem is reduced by focusing on GEEs and differentiable Markov equilibrium.

In corporate finance, our paper benefitted from Anagnostopoulos, Cárceles-Poveda, and Marcet (2010). They show time-inconsistency issues can arise when managers and owners have different objective functions. They analyze and characterize commitment solutions. In addition, the macroeconomic corporate finance literature has studied equity and debt issuance and their effects on business dynamics. For example, Cooley and Quadrini (2001) and Covas and den Haan (2011) analyze non-financial firm financing in both debt and equity.

[4] This is the reason why new shares can be sold at a positive price.

[5] See Krusell and Ríos-Rull (1999), Krusell, Quadrini, and Ríos-Rull (1996) and Klein, Krusell, and Ríos-Rull (2008).

Our paper also builds on a growing body of the macro-banking literature (see, for example, Gertler and Kiyotaki (2010), Meh and Moran (2010), Repullo and Saurina (2011), Cobae and D'erasmo (2012), He and Krishnamurthy (2012), Martinez-Miera and Suarez (2012), Cobae and D'erasmo (2013) and Brunnermeier and Sannikov (2013)). One main difference between our paper and theirs is the separation of bank managers and owners. In the literature, since bank equity is only internally generated (i.e., retained earnings), there is no external owner (i.e., outside equity holders). As a result, bank managers and owners are assumed to have the same objective function and hence no frictions exist between them.[6] Motivated by empirical observations and regulatory discussions during and since the crisis, our paper introduces a friction between managers and owners. In line with this model assumption (and with the capital conservation requirements under capital conservation buffer that restrict manager compensations), Eufinger and Gill (2013) propose capital requirements to be contingent on manager compensations and argue that these regulations can better control bank's risk-taking behaviour.

Empirical observations relevant for our study comes in two sets that received attention during the crisis and in subsequent regulatory discussions. Acharya, Gujral, Kulkarni, and Shin (2011), studying large U.S. commercial and investment banks during the crisis, observed that troubled banks whose capital were depleting continued to pay dividends.[7] In parallel to dividend payments, executive compensations also received attention during the crisis. Names of executives at large U.S. investment banks and their high compensations despite sub-par performances of these banks made the news.[8] An international community of bank regulators placed importance on these observations such that Basel III explicitly restricts manager compensations and dividend payments when bank capital falls below the level required by the conservation buffer. Implementation of such capital requirements and restrictions in case of violation would introduce complex interactions between incentives of managers, owners and regulators, making an assessment of the effects of such restrictions on banking sector dynamics difficult. Hence, an analysis of a model incorporating these interactions would potentially shed light on the issue. In addition, Abreu and Gulamhussen (2013) also study dividend payouts of U.S. banks before and during the recent crisis. They find that agency

[6] Gertler and Kiyotaki (2010) discuss an extension of their baseline model, which includes outside equity. However, they assume that payoff rates are equal between inside and outside equity holders and hence no incentive issues arise.

[7] This occurred even though some of these banks received TARP funds to rebuild capital.

[8] See among others "Fury over Lehman's Executive Pay," Al Jazeera (October 7, 2008) and "CEO Pay Climbs Higher Despite Slow Economy," NBC News (June 15, 2008).

cost and signaling are important factors in bank dividend decisions. Regarding executive compensations, Bordeleau and Engert (2009) summarize their stylized facts for Canadian and U.S. banks. Non-banking studies, such as Chen, Steiner, and Whyte (2006), Chhaochharia and Grinstein (2009), analyze executive compensation structure and how they affect manager behavior.

Other literatures in corporate finance are also relevant for our study. Since our paper assumes a separation of bank manager and shareholders, understanding of corporate governance structures is important. Bertrand and Mullainathan (2000) and Bertrand and Mullainathan (2001) find that managers in a company with weak corporate governance can implicitly exhibit strong controls over their compensations. Lambrecht and Myers (2012) analyze a model where manager compensation and dividend payouts are tied and smooth over time. Finally, Eckbo (2008) present data on seasoned equity offerings and find that the number of offerings with anti-dilution protections has reduce over time.

The reminder of the paper is organized as follows. Section 2 discusses observations and regulatory discussions surrounding bank dividend payouts and executive compensations during the crisis. Section 3 briefly summarizes Basel III capital requirements. Section 4 presents and analyzes the model that highlights time-inconsistency issues. Section 5 introduces moral hazard issues to the model, together with those of time inconsistency. Section 6 concludes.

2 Bank Dividends and Executive Compensations During the Crisis and Regulatory Discussions

Banks were slow to cut payouts in the early stages of the financial crisis.[9] In a macroeconomic banking model where banks choose capital optimally with respect to their aggregate economic environment, banks tend to accumulate capital during bad times (e.g., negative technology shocks) and reduce it during good times (e.g., positive shocks).[10] Since dividends, if not paid, can be used to build up capital, incentives of bank managers to allocate earnings as dividends during bad times (i.e., low earnings periods) appear puzzling. There are potentially multiple reasons why banks continue to pay dividends while facing declining earnings. Abreu and Gulamhussen (2013) using the U.S bank holding company data find

[9]These observations were publicized by reports on individual bank cases of continuing dividend payouts. See, for example, an article by K. Cook, "Fed's Rosengren: Tougher Dividend Rules Needed," Reuters (October 10, 2010), and also by R. Blackden and P. Aldrick, "Barclays Lift Dividend Despite Profits Drop," The Telegraph (February 19, 2008).

[10]See, for example, Meh and Moran (2010).

dividend payouts as signalling of bank's future growth and also to reduce agency costs to be important.[11] Acharya, Gujral, Kulkarni, and Shin (2011) also suggest that managers trying to maximize shareholder values together with their aversion to dilute existing shareholder values may lead to continuous dividend payouts without raising common equity even if needed.

In a reaction to observed bank dividend payout behaviour, the Federal Reserve issued an updated supervisory guidance to bank holding companies in March 2009, reiterating the importance of conserving their capital especially when experiencing financial difficulties and/or receiving public funds. Specifically addressing under what conditions it would be appropriate for a TARP recipient to defer payment of dividends, it stated that "a board of directors' decision to defer a dividend on government investment - as well as all other instruments in the capital structure - should be based primarily on the extent to which deferral is necessary for the bank holding company to preserve capital to continue operating in a safe and sound manner and serving as a strength to its depository institution subsidiaries."[12]

Furthermore, in 2010, Boston Federal Reserve Bank's President stated that had a proactive approach to dividend retention been in place during the crisis, it could have helped 19 biggest U.S. banks to retain nearly $80 billion in capital and reduced the need for an emergency infusion of public funds. An aggressive policy has the potential to reduce potential taxpayer exposure to the banking sector, as well as serve to insulate the broader economy from the sorts of loan supply shocks, while prompting supervisors to look forward and proactively seek reductions in dividends in appropriate circumstances.[13]

Along with bank dividends, executive compensations received attention during the crisis. Several reports highlighted high executive compensations during the crisis period of low or negative bank earnings.[14] Bordeleau and Engert (2009) summarize some stylized facts on executive compensations at large banks in Canada and the U.S. They find that, during the period leading up to the recent financial crisis, U.S. investment banks relied more on annual cash bonuses to reward their executives rather than fixed pay, relative to their peers among U.S. commercial banks and Canadian banks. Since bank earnings can be distributed into dividends and/or compensations (e.g., bonuses), an international community of regulators

[11] Regarding agency costs, dividend may counterbalance the increased need for monitoring.

[12] See "TARP Recipients and Dividend Payments" (Speech), Federal Reserve Bank of Kansas City (September 30, 2010).

[13] See K. Cooke, "Red's Rosengren: Tougher Dividned Rules Needed," Reuters (October 10, 2010).

[14] See among others "Fury over Lehman's Executive Pay," Al Jazeera (October 7, 2008) and "CEO Pay Climbs Higher Despite Slow Economy," NBC News (June 15, 2008).

and supervisors reacted to these observations and came up with a set of principles for sound compensation practices by financial institutions.[15] Basel III capital requirements reflect these discussions on dividend payouts and staff bonuses.

3 Basel III Capital Requirements

The Basel Committee on Banking Supervision issued a document containing a framework of new post-crisis regulations, Basel III.[16] This section briefly discusses an overview of the capital requirements in Basel III.[17] Table 1 summarizes the four layers of capital requirements in Basel III.

[INSERT TABLE 1 HERE]

The definition of "capital" used in the Table 1 is *Common Equity Tier 1* capital which mainly consists of common shares and retained earnings.[18] Banks need to maintain the minimum capital requirement of 4.5% at all times.

A capital conservation buffer of 2.5% is added on top of the minimum requirement, making 7% the combined requirement. As the name suggests, it is a buffer below which capital can fall in a period of stress while banks should maintain capital above it during normal times. When buffers have been drawn down, banks should rebuild them by reducing discretionary distributions of earnings, such as dividend payouts and staff bonus payments. Table 2 shows how this restriction on discretionary distribution of earnings will be implemented.

[INSERT TABLE 2 HERE]

The table says that when the Common Equity Tier 1 capital is below 7.0% (i.e., minimum plus conservation buffer) of risk-weighted assets, the percentage of earnings specified in the right column must be conserved without paying them out as dividends or staff bonus payments. The percentage of earnings required to conserve increases as capital falls, guiding banks to rebuild capital back above 7%. The motivation for restricting banks from

[15] See Financial Stability Forum (2009).

[16] See Basel Committee on Banking Supervision (2011) for the details of the information presented in this section.

[17] Basel III consists of two broad categories of regulations: one on capital framework and another on liquidity standard.

[18] Numbers in the table are the required percentage of capital with respect to risk-weighted assets. A risk weight is assigned to each asset class by the Basel Committee and represents a degree of riskiness of the underlying asset. Hence, the higher it is the risk, the more capital is required. In our model, banks hold only one type of risky assets, hence, assuming the risk weight of one.

freely distributing earnings as dividends or bonuses appears to be based on the dividend and compensation observations during the crisis and the regulatory discussions that followed as presented in Section 2.

The third row of Table 1 specifies a countercyclical capital buffer of 0 to 2.5%. This buffer aims to address the risks of system-wide stress that varies with the macro-financial environment. The requirement is deployed by national jurisdictions when aggregate credit growth is deemed excessive. Finally, additional requirements are placed as extra loss absorbency for banks that are systemically important.[19] Banks are deemed systemically important based on indicators under several categories: cross-jurisdictional activity, size, interconnectedness, substitutability/financial institution infrastructure and complexity. Systemically important banks are charged with additional 1 to 3.5% of capital, depending on the values of the indicators.

In the next section, we present a model that aims to incorporate features addressing regulatory concerns of banks overly paying dividends and compensations at times of stress.

4 Time Inconsistency in a Model without Bank Default

The main deviation of this paper from the macro-banking literature is the separation of bank manager objectives from those of bank shareholders (i.e., owners). We assume that managers have control over bank's resource allocation. In each period, managers consume (i.e., manager compensation), give dividends to existing shareholders and extend loans. These activities are funded by the bank's beginning-of-period net worth, newly issued equity and deposits. Given the assumption of the separation of bank managers and bank shareholders, we analyze two potential sources of friction: (i) a time-inconsistency problem with respect to managers' incentive to pay dividends and issue new equity and (ii) bank manager's impatience to consume today over tomorrow relative to shareholders, i.e., bank manager's discount factor is lower than that of shareholders.

In this section, we focus on analyzing the time-inconsistency issue by presenting and comparing a commitment solution and time-consistent Markov equilibrium solution. For this purpose, we use a version of the model without bank default.

[19] These are banks that are deemed to potentially cause system-wide adverse impacts in case of their default. There are both global and domestic systemically important banks. See Basel Committee on Banking Supervision (2013) for discussions of global systemically important banks and Office of Superintendant of Financial Institutions (2013) for domestic systemically important banks in Canada.

4.1 Bank manager's problem

We assume that managers maximize their own value while facing a constraint placed by bank owners (e.g., the board of directors). The constraint partially restricts manager compensations so that, in order to increase the compensations, dividends also need to increase.[20] Below, we describe the environment and problem for bank managers.[21]

$$V\left(n;\Omega\right)=\max_{\{c,z,y,n',e,m\}}\left\{u\left(c\right)+\chi V\left(n';\Omega\right)\right\} \tag{1}$$

subject to

$$c+z+y=n+\alpha\,m; \tag{2}$$

$$n'=f\left(y\right); \tag{3}$$

$$m=\beta\,e\,\Omega\left(n'\right); \tag{4}$$

$$c\leq\psi\,z;\quad and \tag{5}$$

$$e\leq\frac{m}{m+\left(n-\gamma c-z\right)}; \tag{6}$$

where V is the value function for the bank manager, n the beginning-of-period bank net worth which is the state variable of the manager, Ω the value of bank's outstanding shares, u the utility function for the manager, and χ the subjective discount factor.[22] The prime indicates variables in the next period. Given the state variable, n, the manager maximizes $V(n;\Omega)$ by choosing c, z, y, n', e and m, or manager's own consumption, dividends, bank capital, bank net worth in the next period, new equity issuance (the fraction of the claims to the value of the bank, Ω) and the value of new equity raised, respectively.[23]

When making these decisions, the manager faces constraints 2 through 6. Constraint 2 is the budget constraint where $\alpha \in (0,1)$ specifies the proportional cost associated with new equity issuance. In constraint 3, f specifies the concave loan-return function with bank

[20]This constraint does not eliminate the time inconsistency problem.

[21]We follow a notational convention that a prime ($'$) indicates a next-period variable and also a variable with a subscript indicates a partial derivative of the variable with respect to the subscript.

[22]Given that u is concave, we assume risk-averse bank manager. Lambrecht and Myers (2012) analyze a model where dividend payout tends to be smooth if managers are risk averse and dividends are tied to manager compensation.

[23]Bertrand and Mullainathan (2000) and Bertrand and Mullainathan (2001) find that managers in a company with weak corporate governance can implicitly exhibit strong control over their own compensations, supporting our assumption of bank manager controlling the resource allocations of the bank.

capital, y, where $\frac{df}{dy} > 0$ and $\frac{d^2 f}{dy^2} < 0$ hold.[24] Constraint 4 specifies the relationship between the value of new equity raised, m, and the fraction of bank ownership (or of the claim to the total value of outstanding shares) held by new shareholders, e, where $e \cdot \Omega$ is the value of new shareholders in the next period and β the discount factor of shareholders.[25] The bank manager is assumed to be impatient, $\chi < \beta$. As seen in 4, the manager understands that the value of shares is a function of bank's net worth next period, $\Omega(n')$, and hence that how much equity, m, can be raised today depends also on manager's choice of n'. Constraint 5 restricts how much the manager can consume by the amount of dividends paid out to existing shareholders. We can think of this constraint to be a restriction on manager compensation placed by shareholders (e.g., the board of directors) to partially align manager's incentives to those of shareholders.[26] The last constraint, 6, specifies that e cannot exceed the fraction of the newly raised equity fund over total available resources (after paying out c and z), i.e., beginning-of-period post-expense net worth plus new equity.[27] γ dictates potential costs associated with manager compensation.[28]

The bank manager's problem above is indexed by a function $\Omega(n)$ which is exogenous to the manager. Ω captures the value of shareholders taking into account their expectation of manager's future actions. In equilibrium, this shareholder expectation is consistent with manager's actions on equity issuance, e, and dividend payouts, d. Since manager's decisions are a function of the state variable, n, shareholders' expectations also depend on n. In equilibrium, the value of existing shareholders is given by:

$$\Omega(n) = z(n) + \beta(1 - e(n))\,\Omega\left(n'(n)\right), \tag{7}$$

where $z(n)$, $e(n)$ and $n'(n)$ are manager's optimal decisions on dividend payouts, new equity issuance and next period net worth, respectively. Note that dividends are paid to only existing shareholders and that new equity issuance dilutes the existing shareholder value.

[24] For the model without bank default, we assume no deposits to finance loans.

[25] The equity issuance specification follows that by Covas and den Haan (2012).

[26] Without this constraint, equity shares will have no value in the time-consistent Markov equilibrium.

[27] Without constraint 6, 4 says that $e = 1$ is the optimal choice of the manager, implying that the value of existing shares is completely diluted, which is empirically implausible. Constraint 6 limits such behaviour by restricting e to be below the point where the relative claim of the existing shareholders $(1 - e)$ over the new shareholders (e) is inline with the funding they are entitled to. If the bank is liquidated, the existing shareholders are entitled to the liquidated value $n - \gamma c - z$ and the new shareholders m.

[28] The $\gamma < 1$ fraction is interpreted as a part of compensation costs to be accrued to existing shareholders as they are non-investment purpose and rather the rewards for past performance. In contrast, the $1 - \gamma$ fraction of compensation is for investment/productive purpose so that it accrues to future shareholders including both the existing and new.

4.2 Simplified problem

For tractability, we simplify the problem presented in Section 4.1. First, note that constraints 5 and 6 hold with equality. Since there is no direct benefit to the manger with respect to z, the manager can always increase c by decreasing z if 5 doesn't bind. Similarly, e does not directly benefit or cost the manager. If 6 doesn't bind, e can increase to raise more equity.

Furthermore, constraints 4 and 6 combine to eliminate e and become

$$\beta\Omega(n') = n + m - (1 + \psi\gamma)\, z. \tag{8}$$

Given 6 and 8, 7 in equilibrium becomes[29]

$$\Omega\,(n) = -\psi\gamma z\,(n) + n. \tag{9}$$

In addition, using 3 to eliminate n', 5 for c, 8 for m and using the expression 9 for $\Omega(n')$, the budget constraint, 2, becomes

$$(1 + \psi - \alpha\,(1 + \psi\gamma))\, z + y = (1 - \alpha)\, n - \alpha\beta\gamma\psi z\,(f\,(y)) + \alpha\beta f\,(y)\,, \tag{10}$$

which gives an explicit expression of z as a function of y:

$$z = \frac{1}{1 + \psi - \alpha\,(1 + \psi\gamma)}\,[(1 - \alpha)\, n - \alpha\beta\gamma\psi z\,(f\,(y)) + \alpha\beta f\,(y) - y]\,. \tag{11}$$

Taking these simplifications into account and defining $\widetilde{\psi} \equiv \psi/[1 + \psi - \alpha\,(1 + \psi\gamma)]$, bank manager's problem 1 becomes

$$V\,(n) = \max_{y} \left\{u\left(\widetilde{\psi}\,[(1 - \alpha)\, n - \alpha\beta\gamma\psi z\,(f\,(y)) + \alpha\beta f\,(y) - y]\right) + \chi V\,(f\,(y))\right\}. \tag{12}$$

The first-order condition of this problem is

$$\widetilde{\psi}\left(1 + \alpha\beta\gamma\psi z_n' f_y - \alpha\beta f_y\right) u_c = \chi V_n' f_y,$$

[29]This gives us the explicit expression of Ω in equilibrium such that we do not need to index the manager's problem with it any further.

and the envelop condition gives us

$$V_n = \widetilde{\psi}\,(1-\alpha)\,u_c,$$

such that the optimality condition is given by

$$u_c = \frac{\chi\,(1-\alpha)}{1-\alpha\beta f_y(1-\gamma\psi z_n')} f_y u_c'. \tag{13}$$

4.3 Time inconsistency problem

This paper focuses on time-consistent decisions by the bank manager without any commitment technology to force him to keep his promise. The solution presented in the previous section, equation 13, is called the Generalized Euler Equation (GEE).[30] This solution is time consistent and employs the concept of Markov equilibrium.[31]

The nature of time inconsistency in our environment is the following. Suppose the bank manager wants to raise new equity in some period by promising dividend payouts in the future. The new shareholders invest in bank equity, expecting the manager to pay dividends in the future. This expectation is captured by Ω which in turn limits how much new equity can be raised. However, once the equity is raised, the manager no longer has incentive to pay dividends as promised, i.e., time-inconsistent incentives exist.

In time-consistent Markov equilibrium, the manager in the current period explicitly takes into account the actions of himself in future dividend payouts (i.e., z') and their effects on $\Omega(n')$ through 9. As a result, equity issuance in this period and future dividend payouts become consistent and no incentive to change behaviour in the future.[32] This time consistency is observed in the GEE, 13, by the presence of the term z_n' which is the partial derivative of dividend next period with respect to bank net worth next period. The time-consistent manger understands that, when n' changes, dividend next period will also change. Both of

[30] A detailed discussion and derivation of the GEE in a model of a consumer with quasi-geometric discounting can be found in Krusell, Kuruşçu, and Smith (2002) and Krusell and Smith (2003).

[31] The equilibrium concept is the same as those used in Krusell and Ríos-Rull (1999), Krusell, Quadrini, and Ríos-Rull (1996) and Klein, Krusell, and Ríos-Rull (2008).

[32] A note on 5 and 6 is warranted. Without constraints 5 and 6, the time-consistent equilibrium entails $e = 1$ and $z = 0$. As a result, shareholders expecting this behaviour from the manager place zero value in bank shares, and hence $\Omega = 0\ \forall\ n$. Constraints 5 and 6 partially limit this time-inconsistency problem and thus support a time-consistent equilibrium with $\Omega > 0$ for some n but not entirely as the manager still has flexibility in adjusting future d away from what was promised.

these changes impact the value of bank equity next period and hence the value of new equity issuance today.

In order to concretely show the difference between the optimal solution with commitment and the one that is time consistent, we derive the solution with commitment in the next section.

4.4 Commitment

If commitment technology is available, the manager can credibly promise the path of future dividend payouts at time 0 and, in return, raise enough equity (given small n at time 0) to give out loans of the efficient amount. Knowing that the promise is always honored, the manager can sustain "high" Ω in equilibrium that is optimal for the manager. Specifically, we present a sequential version of problem 1 where the manager makes a sequence of choices, including equity issuance and dividends, at time 0:[33]

$$\max_{\{c_t,z_t,y_t,m_t,e_t,n_{t+1},\Omega_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \chi^t u\left(c_t\right)$$

subject to

$$\begin{aligned} c_t + z_t + y_t &= n_t + \alpha\, m_t; \\ m_t &= \beta e_t \Omega_{t+1}; \\ e_t &\leq \frac{m_t}{m_t + n_t - \gamma c_t - z_t}; \\ c_t &\leq \psi z_t; \\ \Omega_t &= z_t + \beta\left(1-e_t\right)\Omega_{t+1} \;\; \textit{and} \\ n_{t+1} &= f\left(y_t\right). \end{aligned}$$

If we again simplify this problem as we have done in Section 4.2, we have:

$$\max_{\{z_t,y_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \chi^t u\left(\psi z_t\right)$$

[33] The problem with commitment technology can also be recursively formulated. See Appendix A.

subject to

$$(1+\psi-\alpha\left(1+\psi\gamma\right))\,z_t+y_t=(1-\alpha)\,f(y_{t-1})-\alpha\beta\gamma\psi z_{t+1}+\alpha\beta f\left(y_t\right),\ \ \textit{and}$$
$$y_{-1}\ \ \textit{given}.$$

The first-order conditions with respect to z_t and y_t are given by

$$\psi u_{c_t}-\left(1+\psi-\alpha\left(1+\psi\gamma\right)\right)\mu_t-\alpha\beta\gamma\psi\chi^{-1}\mu_{t-1}=0\ \ \textit{and} \tag{14}$$
$$\left(1-\alpha\beta f_{y,t}\right)\mu_t-\chi\left(1-\alpha\right)f_{y,t}\mu_{t+1}=0, \tag{15}$$

respectively, where μ_t is a Lagrange multiplier on the budget constraint. Note that μ_{t-1} is the multiplier on the budget constraint at $t-1$. Without commitment technology, the manager would have an incentive to set $\mu_{t-1}=0$ in every period t. This highlights the source of time inconsistency and implies that the manager can have an incentive to set lower z_t than promised at time $t-1$ when he needs to raise capital.

With $\mu_0=0$ and previously defined parameter $\widetilde{\psi}$, we can iteratively use 14 and 15 for $t=0$, 1 and 2 to obtain,

$$u_{c,0}=\frac{\chi\left(1-\alpha\right)f_{y,0}}{1-\alpha\beta f_{y,0}\left(1-\widetilde{\psi}\gamma(1-\alpha)\right)}u_{c,1}\ \ \textit{and}$$
$$u_{c,1}+\left(-\alpha\beta\chi^{-1}\widetilde{\psi}\gamma\right)u_{c,0}=\frac{\chi\left(1-\alpha\right)f_{y,1}}{1-\alpha\beta f_{y,1}\left(1-\widetilde{\psi}\gamma(1-\alpha)\right)}u_{c,2}.$$

By continuing further, we obtain

$$u_{c,t}+\sum_{j=1}^{t}\left(-\alpha\beta\chi^{-1}\widetilde{\psi}\gamma\right)^j u_{c,t-j}=\frac{\chi\left(1-\alpha\right)f_{y,t}}{1-\alpha\beta f_{y,t}\left(1-\widetilde{\psi}\gamma(1-\alpha)\right)}u_{c,t+1}. \tag{16}$$

Comparing 13 and 16, we note two differences. One is the second term on the left-hand side of 16: the summation of all past discounted marginal utilities. This indicates that, under commitment, all past promises need to be remembered and honored. Another difference is in the term in the parenthesis in the denominator of the right-hand side. They are $(1-\gamma\psi z_n')$ in 13 and $\left(1-\widetilde{\psi}\gamma(1-\alpha)\right)$ in 16 as well as the summation term on the left-hand side, implying that dividend payout decisions are different in two cases.

4.5 Steady state

Comparison of 13 and 16 at the steady state gives us an insight into the relationship between productive (or loan) efficiency and dividend payout policy. We have the time-consistent solution, 13, in the steady state as

$$f_y^{TC} = \frac{1}{\chi\left(1-\alpha\right)+\alpha\beta\left(1-\gamma\psi z_n'\right)}, \tag{17}$$

where TC indicates the time-consistent solution. Regarding the commitment solution, 16, in the steady state, we have

$$f_y^{CM} = \frac{1}{\left(1-\alpha\right)\chi+\alpha\beta}, \tag{18}$$

where CM indicates the commitment solution.[34]

In addition to the condition under commitment, which is without the friction arising from time inconsistency, it is informative to derive the result without another friction, i.e., the lower discount factor of bank manager relative to shareholders. From 18, we can simply assume $\chi = \beta$ and obtain

$$f_y^{FB} = \frac{1}{\beta}, \tag{19}$$

where FB indicates the first-best solution without any friction.

We can compare the level of y from each condition, given the assumption of concavity of f. Suppose $z_n' > 0$. Then, we obtain

$$y^{FB} > y^{CM} > y^{TC}. \tag{20}$$

That is, the first-best solution leads to the highest value of bank capital, y, followed by that with commitment and then that of time-consistent solution. Therefore, two frictions in the model incrementally leads to lower bank capital.

Since the discount factor of the manager is χ, the first-best level of y will not privately be chosen by the manager. Given that, the commitment solution is optimal for the manager as it leads to "high" Ω, giving the manager better terms in equity financing and hence an optimal amount of bank capital, y, for loan returns, $f(y)$.

[34] We derived the expression by assuming $u_{c,t} = u_{c,t+1}$ for all t and taking t to infinity.

4.6 Numerical analysis

In order to further analyze the steady state properties of the time-consistent solution obtained above, this section numerically characterize them. With GEE, a numerical analysis (even) at the steady state needs to consider not only the steady state values of the variables of interest (e.g., z^{ss} and y^{ss}) but also the decision rules (e.g., $z(n)$ and $y(n)$). Hence, the dynamics of all variables in the GEE need to be taken into account, albeit around the steady state. This is precisely because the GEE contains a derivative term, $z'_n = \frac{\partial z'}{\partial n'}$. We use a numerical algorithm described in Krusell, Kuruşçu, and Smith (2002) and Klein, Krusell, and Ríos-Rull (2008) to solve for the steady state and the derivatives of the decision rules.[35]

From 10 and 13, we have the budget constraint and the GEE. We can rewrite them as functional equations of $z(n)$ and $y(n)$ as follows

$$(1+\psi-\alpha(1+\psi\gamma))\,z(n)+y(n)=(1-\alpha)\,n-\alpha\beta\gamma\psi z\left(f\left(y(n)\right)\right)+\alpha\beta f\left(y(n)\right), \quad \textit{and}$$

$$u_c\left(\psi z(n)\right)=\frac{\chi\left(1-\alpha\right)f_y\Big(y(n)\Big)}{1-\alpha\beta f_y\Big(y(n)\Big)\Big(1-\gamma\psi z_n\Big(y(n)\Big)\Big)}u_c\left(\psi z\Big(y(n)\Big)\right).$$

Here, we have two unknown functions, $z(n)$ and $y(n)$, and the steady state value of n. The algorism approximates $z(n)$ and $y(n)$ by a polynomial of order k-1 and assumes $\partial^k z/\partial n^k = 0$. Then, we have $2k+2$ unknowns to pin down: k for each $z(n)$ and $y(n)$, $z(n^{ss})$ and $y(n^{ss})$, where n^{ss} is the steady state value of n.[36] Differentiating the two functional equations k times with respect to n give us $2k+2$ equations and unknowns. We start from $k=1$ and increase k until the solution converges. Our solution converged at $k=4$.

For the numerical analysis, we assume $u\left(c\right)=\log\left(c\right)$ and $f\left(y\right)=y^{\nu}$. Table 3 displays the parameter values used and Table 4 the results of steady state values in the time-consistent solution. The results under commitment are also presented for comparison. Three sets of observations are worth noting. First, we observe that $z'_n > 0$ in the steady state.

[INSERT TABLE 3 HERE]

[INSERT TABLE 4 HERE]

[35] The numerical approximation here is a variant of perturbation methods. See, for example, Judd (1998) for a discussion of perturbation methods.

[36] n^{ss} can be obtained by solving $n^{ss} = f\left(y(n^{ss})\right)$.

Second, bank net worth (n), capital (y) for loans and the value of outside equity Ω are smaller under time consistency relative to those under commitment. An interesting question is, why is y smaller under time consistency? Friction in time consistency shows up in the tradeoff between $\Omega(n')$ and z' in 9. The manager today understands that, when tomorrow comes, he will want to increase z' in order for him to consume more, c'.[37] But this increase in z' comes with a decrease in $\Omega(n')$ through 9. This occurs because, as z' increases, e' increases as well by 6. That is, higher z' gives more room for the manager tomorrow to dilute the value of the shareholders tomorrow. A decrease in $\Omega(n')$ directly affects equity financing for the manager today through 4. By understanding this mechanism, the manager today has less incentive to increase y and hence n' as this leads for the manager tomorrow to increase z'. As a result, (n), (y) and Ω are all lower under time consistency in the steady state.[38]

Third, there is excessive dividend payout (in dividend yield) under time consistency relative to commitment. Table 4 shows that z/Ω, our measure of dividend yield, is higher under time commitment. In line with the discussion in the second set of observations, the valuation of shares is lower under time consistency due to manager's dilution incentives. Although this leads to lower Ω, the manager still pays high dividend to maintain his consumption, resulting in excessive dividend yields.

5 Moral Hazard and Time Inconsistency in a Model with Bank Default

In the previous section, we presented a model that leads to under-capitalization due to frictions arising from manager's time inconsistent incentives in dividend payouts and equity issuance. We now introduce another friction to the model: moral hazard due to the possibility of bank default. Given deposit insurance, a social planner would care about externality brought by this friction. If the manager has incentive to keep paying dividends even when the bank with low capital faces a potential default, regulations such as Basel capital requirements can become important for two reasons. One is the excessive risk taking in leverage by the manager beyond what is socially optimal. Another is under-capitalization as shown in the previous section. Different capital requirements (e.g., the minimum requirement and the conservation buffer) will interact with both of these margins.

[37] Constraint 5 qualitatively aligns incentives solely with respect to dividend payouts between the manager and shareholders.

[38] The net result of the tradeoff (in constraint 4) between higher e' (increasing m') and lower $\Omega(n')$ (decreasing m) on m is that equity issuance is slightly lower under time commitment, which in turn suppresses dividend and manager compensation today.

Given this motivation, we present an extension of the model where individual banks face idiosyncratic shocks to loan returns and the manager has an outside option to leave the bank when defaulted. These two elements generate a possibility of bank default and incentives to hold bank capital to absorb adverse shocks in loan returns. Thus, we now explicitly model loans as $\ell = d + y$, where ℓ is the loan amount, d the deposits and y the bank capital. Loans given in time t are subject to bank-specific capital-quality shock, η', that affects their returns in time $t+1$. We assume that banks face a bankruptcy-threshold level of capital, $\underline{n}$, under which they default. Given the limited liability assumption, the manager of defaulting banks has an outside option, $V(\underline{n})$. This assumption works as an insurance for the manager against risk of $n' < \underline{n}$, giving incentives to take more risk, i.e., morel hazard.[39] We augment model 12 along these dimensions and present it as follows:

$$V(n) = \max_{z,y,\ell} \left\{ u(\psi z) + \underbrace{\chi \int_{\eta'_*(\ell,y)} V\left(f\left(\ell, y, \eta'\right)\right) dG\left(\eta'\right)}_{\text{continuation value of survival}} + \underbrace{\chi V(\underline{n}) G\left(\eta'_*(\ell, y)\right)}_{\text{outside option}} \right\} \tag{21}$$

subject to

$$\begin{aligned}
&(1 + \psi - \alpha(1 + \psi\gamma)) z + y = \\
&\qquad (1-\alpha) n + \alpha\beta \int_{\eta'_*(\ell,y)} \left[f(\ell, y, \eta') - \psi\gamma z'\Big(f(\ell, y, \eta')\Big) \right] dG\left(\eta'\right); \qquad (22) \\
&f\left(\ell, y, \eta'\right) = \max\left\{ \left[\eta' R \ell^{-\gamma} - R_d - h(\ell - y)\right] \ell + \left[R_d + h(\ell - y)\right] y,\, \underline{n} \right\}; \ \ \textit{and} \qquad (23) \\
&\eta'_*(\ell, y) = \max\left\{ \min\left\{ \frac{\underline{n} - (R_d + h(\ell - y))\, y}{R} \ell^{\gamma-1} + \frac{R_d + h(\ell - y)}{R} \ell^{\gamma}, 1 \right\}, 0 \right\} \qquad (24)
\end{aligned}$$

where η'_* defines the threshold value of the shock, η', below which the bank defaults.[40] The loan return function f now is a function of ℓ, y and η'. G is the cumulative distribution function of η'. We consider the shock, η', to be a credit-risk shock such that we restrict the support of G to be $\eta' \in [0, 1]$. The manager chooses dividends, bank capital and loans subject to constraints 22 through 24. Constraint 22 is the budget constraint similar to 10 after incorporating loan-return shocks and default in the present value of bank shareholders,

[39]We implicitly assume deposit insurance: a government exists to cover the loss when a defaulted bank does not have enough bank capital to pay back for depositors. This procedure can be costly, involving some dead-weight loss and hence externality from bank default. Therefore, the government (i.e., a social planner) has more incentive to avoid default than the manager who is insured by his outside option.

[40]η'_* is an endogenous function of ℓ and y, intuitively, as these variables together define bank's capital buffer against negative shocks.

which affects how much equity can be raised today. Constraint 23 describe the loan return function exhibiting decreasing-returns-to-scale (DSR) technology in ℓ with $\gamma \in (0,1)$.[41] R is a parameter normalizing the rate of returns on loans, R_d the risk-free deposit rate and $h(\ell - y)$ the bank's internal cost in raising deposit.[42] We assume that deposit rate is constant at R_d but the internal cost increasing in d, i.e., $dh/dd > 0$.[43] Constraint 24 is obtained by finding η' that equates the two terms in the max function of 23. Note that, now, Ω is a known function of n, exactly as derived in 9.

5.1 Excessive leverage and moral hazard

Before analyzing the full model, let us first look at the "second stage" problem of the manager: a decision of d (or ℓ) given y. Let the value for the banker $V(n)$. Then, in the lending stage the bank manager solves

$$\max_{d} \left\{ \int_{\eta'_*(d+y,y)} V\left[f\left(d+y,y,\eta'\right)\right]\ dG\left(\eta'\right) + \underbrace{G\left(\eta'_*\left(d+y,y\right)\right)\ V\left(\underline{n}\right)}_{\text{prob times value of bkrptcy}} \right\}.$$

Notice that dR_d is fully paid only with probability $1 - G[\eta'_*(y,d)]$ due to limited liability, the source of moral hazard. The solution is a function $d(y)$ (or implied $\ell(y)$) defined by

$$\int_{\eta'_*(d+y,y)} V'_n\ f_d\ dG\left(\eta'\right) = 0.$$

We show the difference between the implied rate of return in d for any socially optimal arrangement (assuming for now the value of V'_n is constant) given by $f_d = 0$ and that implied by the first-order condition of the manager who only internalizes the payment for deposits with probability $1 - G(\eta'_*(d+y,y))$.[44]

In order to show that manager's decisions in d leads to "excessive" leverage, let us compare

[41] The DRS function in 23 is partly derived from the assumption that the borrower/producer has a DRS production technology of the form, $\tilde{f}(k,l,\eta') = \eta' k^{\theta} l^{\sigma}$, and maximizes profits, $\tilde{f}(k,l,\eta') - w\,l$, where k and l are physical capital and labour, respectively, and $\theta + \sigma < 1$. The borrower finances physical capital by bank loans and pay the bank all profits in return. From the producer's perspective, η' can be considered as an asset-quality shock.

[42] The internal deposit cost function, $h(d)$, captures costs associated with non-rate competition for deposit. For example, when opening a new checking account, banks sometimes have a promotional gift.

[43] This assumption is motivated by the observation that retail deposits are "sticky" or more difficult to raise.

[44] Given $\underline{n} = f[d+y, y, \eta'_*(d+y,y)]$ and f being decreasing-returns to scale in ℓ, we can state that $\partial\eta'_*/\partial y = -f_y/f_{\eta'} < 0$ so that an increase in y will reduce the bankruptcy threshold of η' as more bank capital provides a buffer against adverse shocks. This is an intuitive result since y is a buffer against adverse shocks in loan returns so that, more y the bank has, lower probability of bad enough shock to deplete all capital.

it to the choice, d^{SP}, of a social planner. Since the planner always pays $d\,R_d$ to depositors, the planner cares only about productive efficiency and choose d accordingly:

$$\max_{d}\int_0^1 f\left(d+y,d,\eta'\right)dG\left(\eta'\right).$$

The solution implied by FOC is

$$\begin{aligned}
&f_d=(1-\gamma)\eta' R\Big(d^{SP}+y\Big)^{-\gamma}-\left(R_d+h+h_d\cdot d^{SP}\right)=0\ \ or\\
&(1-\gamma)R\Big(d^{SP}+y\Big)^{-\gamma}\bar{\eta}=R_d+h+h_d\,d^{SP},
\end{aligned}$$

where $\overline{\eta}=\int_0^1\eta' dG\left(\eta'\right)$. Evaluating the Banker's marginal value of deposit at d^{SP},

$$\int_{\eta'_*(y,d^{SP})}^{1} V'_n\cdot\left(R_d+h+h_d\,d^{SP}\right)\left(\frac{\eta'}{\bar{\eta}}-1\right)dG\left(\eta'\right)\propto E\left[\eta'\mid\eta'>\eta'_*\right]-E\left[\eta'\right]>0$$

if the preference is linear. Hence, the banker has incentive to set d above d^{SP} if he is not too risk-averse. Note that the expectation of η' conditional on non-default is higher than its unconditional expectation. This term highlights the fact that the banker does not fully repay depositors (i.e., when $\eta'<\eta'_*$) whereas the planner does.

Therefore, this implies that, given y, $d^{Banker}>d^{SP}$ and thus $\ell^{Banker}>\ell^{SP}$. That is, manager's optimal choice of loans leads to excessive leverage and banks defaulting too much, $\eta'_*(d^{Banker}+y,y)>\eta'_*(d^{SP}+y,y)$. This is excessive risk taking emerging from moral hazard with respect to manager's outside option as insurance.

5.2 Moral hazard and time inconsistency

Now we go back to the full problem and analyze low y and high leverage together through simultaneous choice of y and ℓ. Let us further simplify 21 by substituting out z using the budget constraint, $z=Q\left(\ell,y,n\right)$. Then, we have the following problem with two choice variables, ℓ and y:

$$V\left(n\right)=\max_{y,\ell}\left\{u\left(\psi Q\left(\ell,y,n\right)\right)+\chi\int_{\eta'_*(\ell,y)}V\left[f\left(\ell,y,\eta'\right)\right]\,dG\left(\eta'\right)+\underbrace{\chi\;\;G\left(\eta'_*\left(\ell,y\right)\right)\;V\left(\underline{n}\right)}_{\text{prob times value bkrptcy}}\right\}$$

subject to

$$Q(\ell, y, n) = \frac{1}{1+\psi-\alpha(1+\psi\gamma)}\left[(1-\alpha)n + \alpha\beta\int_{\eta'_*(\ell,y)}\left[f(\ell,y,\eta') - \psi\gamma z'\left(f(\ell,y,\eta')\right)\right]dG(\eta') - y\right];$$

$$f(\ell,y,\eta') = \max\left\{\left(R\ell^{-\gamma}\eta' - (R_d + h(\ell-y))\right)\ell + (R_d + h(\ell-y))y, \underline{n}\right\};\ \ \textit{and}$$

$$\eta'_*(\ell,y) = \max\left\{\min\left\{\frac{\underline{n}-(R_d+h(\ell-y))y}{R}\ell^{\gamma-1} + \frac{R_d+h(\ell-y)}{R}\ell^{\gamma}, 1\right\}, 0\right\}.$$

The first-order condition for ℓ and y, respectively, are:[45]

$$u_c\alpha\beta\left\{\int_{\eta'_*}\left(1-\psi\gamma z'_n\right)\left[(1-\gamma)R\ell^{-\gamma}\eta' - R_d - h - h_d(\ell-y)\right]dG(\eta') - \underline{\Omega}\frac{\partial\eta'_*}{\partial\ell}g(\eta'_*)\right\}$$
$$+\chi(1-\alpha)\int_{\eta'_*}u'_c\left[(1-\gamma)R\ell^{-\gamma}\eta' - R_d - h - h_d(\ell-y)\right]dG(\eta') = 0$$

$$u_c\left\{-1+\alpha\beta\int_{\eta'_*}\left(1-\psi\gamma z'_n\right)(R_d+h+h_d)(\ell-y)\,dG(\eta') - \alpha\beta\underline{\Omega}\frac{\partial\eta'_*}{\partial y}g(\eta'_*)\right\}$$
$$+\chi(1-\alpha)\left[\int_{\eta'_*}u'_c(R_d+h+h_d)(\ell-y)\,dG(\eta')\right] = 0$$

As we similarly discussed in Section 4.3, this is the GEE in this version of the model. In these first-order conditions, the presence of the term $(1-\psi\gamma z'_n)$ makes the solution time consistent. The bank manger today takes into account the reaction of himself tomorrow to his actions today.

5.3 An example: $\eta' \in \{0, 1\}$

To understand the model dynamics more clearly, we present a simple example of the model, where η' takes only two values, $\eta' = 1$ with probability p_1 or 0 with probability $1-p_1$. The implication of this particular assumption is that all banks survive when $\eta' = 1$ and all default when $\eta' = 0$. The manager's first-order condition for ℓ is

$$\left[\alpha\beta\left(1-\psi\gamma z'_{n,1}\right)u_{c,1} + \chi(1-\alpha)u'_{c,1}\right]f_{\ell,1}p_1 = 0.$$

[45]Note that $\underline{\Omega} = \lim_{\varepsilon\to0^+}\left[z'(\underline{n}+\varepsilon) + \underline{n} + \varepsilon\right]$. Also, we have $\partial\eta'_*/\partial\ell > 0$ unless $\underline{n} > R_d y$ and ℓ is very small. $\partial\eta'_*/\partial y < 0$.

Thus,

$$f_{\ell,1} = (1-\gamma) R\ell^{-\gamma} - R_d - h - h_d (\ell - y) = 0.$$

We can compare these results from the manager's problem to those of the planner's. The planner chooses ℓ by taking into account the expected returns over both the good state ($\eta' = 1$) and the bad ($\eta' = 0$) as the planner does not consider limited liability. Then, we have for the planner:

$$f_{\ell,SP} = p_1 \ (1-\gamma) R\ell^{-\gamma} - R_d - h - h_d (\ell - y) = 0.$$

We observe that the first term of the planner's condition is weighted down by p_1 relative to the manager's, implying that the manager chooses a larger ℓ for given y due to moral hazard.

We can also show, using the manager's condition, that ℓ is increasing in y:

$$\frac{d\ell}{dy} = \frac{h_{dd} (\ell - y) + 2h_d}{\gamma (1-\gamma) R\ell^{-\gamma-1} + h_{dd}(\ell - y) + 2h_d} \in (0, 1).$$

From these, we can conclude that the manager takes on excessive leverage, given y.

Now, turning to the choice of y, we have the manager's first-order condition to be:

$$u_c \left[1 - \alpha\beta \left(1 - \psi\gamma z_n'\right) f_y p_1\right] = \chi (1-\alpha) u_c' f_y p_1,$$

where

$$f_y = R_d + h + h_d (\ell - y) > 0 \ \ and$$
$$f_{yy} = (2h_d + h_{dd} (\ell - y)) \left(\frac{\partial \ell}{\partial y} - 1\right) < 0.$$

Thus, f is increasing and concave in y. Given this result, in steady state, we have a familiar condition:

$$f_y = \frac{1}{p_1 \left[\chi (1-\alpha) + \alpha\beta (1 - \psi\gamma z_n')\right]}.$$

This equation is similar to 17 except that we have an extra term, p_1. Given this condition, we can similarly state that, if $z_n' > 0$, we have low bank capital, y.

5.4 Bank capital regulations: a discussion

Our analysis so far implies that, under time consistency, banks with small n may be less able to increase capital, y, due to (i) high dividend payments and (ii) low Ω. At the same time, we saw that banks' privately optimal solution leads to excessive leverage with risky loans due to moral hazard. Against these two frictions, two types of regulations would likely be necessary. Minimum capital requirements are designed to put a cap on banks' risk-weighted leverage. This would help curve down risk taking from excessive leverage. However, the difficulty with capital accumulation due to the existence of time-inconsistency incentives may be better amended by a regulation that directly encourage capital accumulation. In this line of argument, capital conservation buffer in Basel III would complement the minimum requirements by restricting dividend payouts and manager compensation of banks with low capital so that these banks can better rebuild their capital when needed. As a result, Basel III will likely impact (more directly than Basel II does) with the dividend payout policy, $z(n)$, and hence address issues arising from both time inconsistency and moral hazard.

6 Conclusion

This paper explores how two frictions that banks likely face interact with each other and discusses their implications for capital regulations. We highlighted time-consistent solutions which can lead for bank managers to pay high dividends and make capital accumulation difficult in the time-consistent Markov equilibrium. In addition, an introduction of moral hazard made banks to take default risk due to limited liability. These two frictions also interact to compound the problem. While the time-consistent outcome makes bank capital accumulation difficult, moral hazard leads to excessive leverage with risky loans. The two enhance the default risk of banks.

Two types of regulations would help reduce the adverse effects of the two frictions. Minimum capital requirement would limit the extent of default-risk taking by reducing the excessive leverage. However, minimum requirement alone may not be sufficient to address the difficulty in capital accumulation due to time-consistent outcomes. Thus, a regulation such as the capital conservation buffer which directly restricts distribution of bank earning away from capital accumulation may be necessary.

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Tables

Table 1: Four layers of capital requirements.

	Requirement (% of risk-weighted assets)
Minimum	4.5
Conservation Buffer	2.5
Counter Cyclical Buffer	0-2.5
Systemically Important Banks	1-3.5

Table 2: Conservation buffers and restrictions on distributions of earnings

Common Equity Tier 1 Capital (% of risk-weighted assets)	Capital Conservation Ratio (% of earnings)
4.5 - 5.125	100
5.125 - 5.75	80
5.75 - 6.375	60
6.375 - 7.0	40
7.0 -	0

Table 3: Parameter Values

α	β	γ	χ	ψ	ν
0.98	0.99	0.5	0.9	1.0	0.9

Table 4: Commitment versus Time-Consistent in Steady State

	z'_n	n	y	e	z	m	Ω	y/n	z/Ω	m/Ω
Commitment	–	0.35	0.31	0.09	0.035	0.03127	0.33	0.89	0.10	0.09
Time Consistent	0.036	0.30	0.26	0.11	0.034	0.03125	0.28	0.87	0.12	0.11

Appendix

A Recursive formulation of the bank manager's problem with commitment

The sequential problem of the bank manager with commitment in Section 4.4 can also be formulated recursively. Because the commitment solution is specific to the starting period, we need to separate the first period problem (with value function W_0) and the rest (with value function W). Under commitment, the manager can choose and credibly promise the optimal path of Ω's. Other choice variables, $\{c, z, y, e\}$, can be expressed as a function of n, Ω and Ω'.

The first period problem is special in that the manager has flexibility in choosing both Ω and Ω' and given by:

$$W_0(n) = \max_{\Omega,\Omega'} \left\{ u\left(c(n,\Omega)\right) + \chi W\left(f(y(n,\Omega,\Omega')),\Omega'\right) \right\}$$

subject to

$$\begin{aligned}
c(n,\Omega) + z(n,\Omega) + y(n,\Omega,\Omega') &= n + \alpha\, m(n,\Omega,\Omega'); \quad \textit{where} \\
c(n,\Omega) &= \frac{n-\Omega}{\gamma\psi}; \\
z(n,\Omega) &= \frac{n-\Omega}{\gamma}; \\
m(n,\Omega,\Omega') &= \beta\Omega' - n + (1+\gamma\psi)z(n,\Omega); \quad \textit{and} \\
y(n,\Omega,\Omega') &= (1-\alpha)n + \alpha\beta\Omega' - (1+\psi-\alpha-\alpha(1+\gamma\psi))\frac{n-\Omega}{\gamma\psi}.
\end{aligned}$$

From the second period onward, the state variables include both n and Ω. Ω is chosen in the previous period and only Ω' is the decision in this period. The value function also becomes consistent over time:

$$W\left(n,\Omega\right) = \max_{\Omega'} \Big\{ u\Big(c(n,\Omega)\Big) + \chi W\Big(f\Big(y(n,\Omega,\Omega'),\Omega'\Big)\Big) \Big\}$$

subject to

$$\begin{aligned}
c(n,\Omega)+z(n,\Omega)+y(n,\Omega,\Omega') &= n+\alpha\, m(n,\Omega,\Omega'); \;\; \textit{where} \\
c(n,\Omega) &= \frac{n-\Omega}{\gamma\psi}; \\
z(n,\Omega) &= \frac{n-\Omega}{\gamma}; \\
m(n,\Omega,\Omega') &= \beta\Omega' - n + (1+\gamma\psi)z(n,\Omega); \;\; \textit{and} \\
y(n,\Omega,\Omega') &= (1-\alpha)n + \alpha\beta\Omega' - (1+\psi-\alpha-\alpha(1+\gamma\psi))\frac{n-\Omega}{\gamma\psi}.
\end{aligned}$$

In characterizing the solution to this problem, the first-order condition and the two envelop conditions with respect to n and Ω are, respectively:

$$\begin{aligned}
&\alpha\beta W_n' f_y + W_\Omega' = 0; \\
&W_n = \frac{1}{\gamma}u_c + \chi\left(1-\alpha-\frac{1+\psi-\alpha\,(1+\psi\gamma)}{\psi\gamma}\right) W_n' f_y; \;\; \textit{and} \\
&W_\Omega = -\frac{1}{\gamma}u_c + \chi\frac{1+\psi-\alpha\,(1+\psi\gamma)}{\psi\gamma} W_n' f_y.
\end{aligned}$$

By adding the two envelop conditions, we obtain

$$W_n + W_\Omega = \chi\,(1-\alpha)\,W_n' f_y. \tag{A1}$$

Also from the first envelop condition and using the definition of $\widetilde{\psi}$ from Section 4.2, we have

$$\chi W_n' f_y = \frac{W_n - (1/\gamma)\,u_c}{1-\alpha-(\gamma\widetilde{\psi})^{-1}}.$$

Using this with the combined envelop conditions, A1, we have

$$W_\Omega = \frac{(\gamma\widetilde{\psi})^{-1}}{1-a-(\gamma\widetilde{\psi})^{-1}} W_n - \frac{1-\alpha}{\gamma\left(1-\alpha-(\gamma\widetilde{\psi})^{-1}\right)} u_c.$$

Substituting this expression into the first-order condition, we obtain

$$-\left[1-\left(1-\gamma\widetilde{\psi}\,(1-\alpha)\right)\alpha\beta f_y\right]\left(-\chi f_y W_n'\right) = \widetilde{\psi}\chi\,(1-\alpha)\,f_y u_c'.$$

Using the second envelope condition with the first-order condition lagged by one period gives

$$\begin{aligned}-\chi f_y W_n' &= -\left[\frac{1}{\gamma}u_c + W_\Omega\right]\gamma\widetilde{\psi} \\ &= -\widetilde{\psi}u_c - \alpha\beta\gamma\widetilde{\psi}\chi^{-1}\left(-\chi f_{y,-1}W_n\right)\end{aligned}$$

This implies

$$-\chi f_{y,t}W_{t+1} = -\widetilde{\psi}\sum_{j=0}^{t}\left(-\alpha\beta\overline{\psi}\chi^{-1}\right)^j u_{c,t-j}.$$

Then, the Euler equation under commitment is given by

$$\left[1-\left(1-\gamma\widetilde{\psi}\left(1-\alpha\right)\right)\alpha\beta f_{y,t}\right]\sum_{j=0}^{t}\left(-\alpha\beta\gamma\widetilde{\psi}\chi^{-1}\right)^j u_{c,t-j} = \chi\left(1-\alpha\right)f_{y,t}u_c'.$$

Hence, we obtain the identical Euler equation from the recursive formulation of the problem as obtained from the sequential formulation found in 16.