Foreign exchange intervention and monetary policy design: a market microstructure analysis*

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Abstract

In this paper we extend a New Keynesian open economy model to include risk-averse FX dealers and FX intervention by the monetary authority. These ingredients generate deviations from the uncovered interest parity (UIP) condition. More precisely, in this setup dealers' portfolio decisions endogenously add a time-variant risk premium element to the traditional UIP that depends on FX intervention by the central bank and FX orders by foreign investors. We analyse the effectiveness of different strategies of FX intervention (eg unanticipated operations or via a preannounced rule) to affect the volatility of the exchange rate and the transmission mechanism of the interest rate. Additionally, we extend the model to include information heterogeneity in the FX market, in line with Bacchetta and van Wincoop (2006) and Vitale (2011). Also, we solve the model extending the methodology proposed by Townsend (1983) to solve dynamic stochastic general equilibrium (DSGE) models with heterogeneous expectations.

Our findings are as follows: (i) FX intervention has a strong interaction with monetary policy in general equilibrium; (ii) FX intervention rules can have stronger stabilisation power than discretion in response to shocks because they exploit the expectations channel; (iii) there are some trade-offs in the use of FX intervention: it can help isolate the economy from external financial shocks, but it prevents some necessary adjustments of the exchange rate in response to nominal and real external shocks; and (iv) in the context of the model with heterogeneous information, FX intervention can help restore the connection between the exchange rate and observed fundamentals.

Keywords: foreign exchange microstructure, exchange rate dynamics, exchange rate intervention, monetary policy.

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1 Introduction

IInterventions by central banks in FX markets have been common in many countries, and they have become even more frequent in the most recent past, in both emerging market economies and some advanced economies. These interventions have been particularly large during periods of capital inflows, when central banks bought foreign currency to prevent an appreciation of the domestic currency. Also, they have been recurrent during periods of financial stress and capital outflows, when central banks used their reserves to prevent sharp depreciations of their currencies. For instance, in Figure 1 we can see that during 2009–12 the amount of FX interventions as a percentage of FX reserves minus gold was between 30% and 100% in some Latin American countries, and considerably more than 100% in Switzerland. Also, these FX interventions were sterilised in most cases, thus enabling central banks to keep short-term interest rates in line with policy rates.

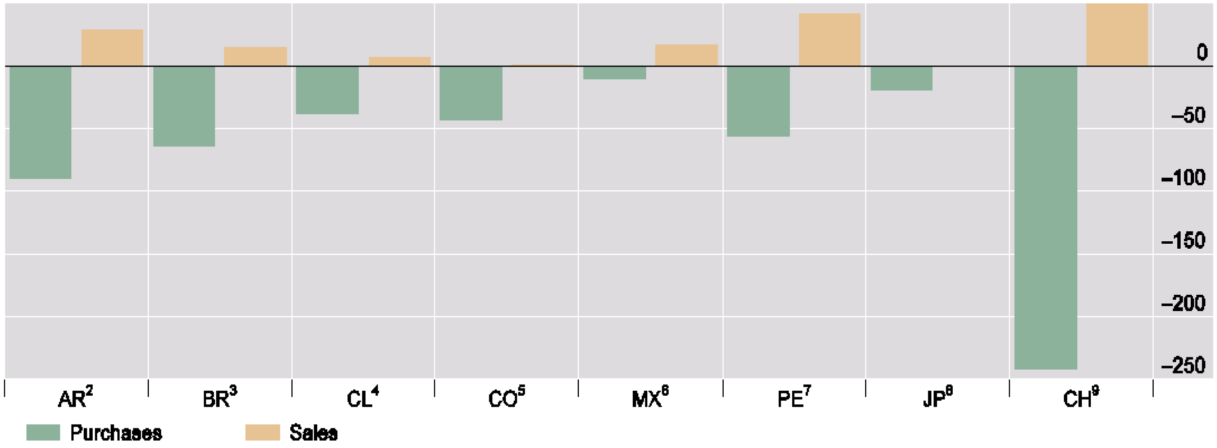
Given the scale of interventions in FX markets by some central banks, it should be important for them to include this factor in their policy analysis frameworks. A variety of questions need to be addressed, such as: How does sterilised intervention affect the transmission mechanism of monetary policy?, Which channels are at work? Are there benefits to intervention rules? What should be the optimal monetary policy design in the context of FX intervention? To analyse these questions we need an adequate framework of exchange rate determination in macroeconomic models.

There is substantial empirical evidence that traditional approaches of exchange rate determination (eg asset markets) fail to explain exchange rate movements in the short run (see eg Meese and Rogoff (1983) and Frankel and Rose (1995)). This empirical evidence shows that most exchange fluctuations at short- to medium-term horizons are related to order flows – the flow of transactions between market participants – as in the microstructure approach presented by Lyons (2001), and not to macroeconomic variables. However, in most of the models used for monetary policy analysis, the exchange rate is closely linked to macroeconomic fundamentals, as in the uncovered interest rate parity (UIP) condition. Such inconsistency between the model and real exchange rate determination in practice could lead in some cases to incorrect policy prescriptions, eg overestimation of the impact of fundamentals and corresponding underestimation of the impact of news and beliefs in the exchange rate.

Bacchetta and van Wincoop (2006) provide an alternative framework to analyse exchange rate determination. They introduce symmetric information dispersion about future macroeconomic fundamentals in a dynamic rational expectations model to explain some stylised facts of

Figure 1: Intervention in the foreign exchange market: 2009 - 2012[1]

(As a percentage of average foreign exchange reserves minus gold)



AR = Argentina; BR = Brazil; CH = Switzerland; CL = Chile; CO = Colombia; JP = Japan; MX = Mexico; PE = Peru.

[1] The bars correspond to the sum of amounts allocated/transacted in the period. Positive (negative) values indicate a sale (purchase) of US dollars by the central bank, unless otherwise indicated. [2] Spot sales. [3] Swap market auctions; spot market auctions; repo market auctions; trade finance loans and forward market auctions. [4] Reserve accumulation programme and swap auctions. [5] Options for reserve accumulation; options for exchange rate volatility control and direct purchase auctions. [6] USD sales mechanism; auctions with minimum price; auctions without a minimum price; Fed swaps; direct sales and option auctions. [7] Auctions; CDR certificates and CDLD certificates. [8] Interventions by the Japanese Ministry of Finance. [9] Inferred by changes in the foreign currency investments of the Swiss National Bank by currency; adjusted for exchange rate changes.

Sources: National data; BIS calculations.

exchange rates. When introducing information heterogeneity their model can account for the short-run disconnection between exchange rate fluctuations and observed fundamentals, while both variables become closely related over longer horizons. In their model, exchange rates are closely related to order flow, defined as the private information component of FX orders. In a related work, Vitale (2011) extends the Bacchetta and van Wincoop (2006) model to analyse the impact of FX intervention on FX markets. This model is useful to analyse how FX intervention influences exchange rates via both a portfolio balance and a signalling channel.

In order to provide a framework for analysis of FX intervention together with monetary policy, we extend a standard New Keynesian small open economy model including market microstructure of exchange rate determination in the spirit of Bacchetta and van Wincoop (2006). Unlike them, we present a fully dynamic stochastic general equilibrium model with nominal rigidities, where we can analyse the interaction with interest rate policy and FX intervention. In this setup, we introduce risk-averse FX dealers, which generates deviations from the UIP condition. More precisely, in this alternative setup, the portfolio decision of dealers adds a time-variant risk premium element to the traditional UIP that depends on both FX intervention by the central bank and FX orders from foreign investors.

In our model, central bank FX intervention affects exchange rate determination through two channels: the portfolio balance effect and the expectations/signalling effect. In the former, a sterilised intervention alters the value of the currency because it modifies the ratio between domestic and foreign assets held by the private sector; and according to the latter, operations in foreign exchange markets by the monetary authority may signal changes in future monetary policy, affecting market expectations and hence the exchange rate. We further extend the model by including information heterogeneity in the FX market, in line with Bacchetta and van Wincoop (2006) and Vitale (2010).

Our findings show that in general equilibrium, FX intervention can have important implications for monetary policy. In some cases it can mute the monetary transmission mechanism through exchange rates, reducing the impact on aggregate demand and prices, while in others it can amplify the impact. Also, FX intervention rules can have stronger stabilisation power in response to shocks because they exploit the expectations channel. We also show that there are some trade-offs in the use of FX intervention. On the one hand, it can help isolate the economy from external financial shocks, but on the other it prevents some necessary adjustments of the exchange rate in response to nominal and real external shocks. Moreover, we show in our extended model with heterogeneous information that FX intervention can magnify the response of exchange rates to unobservable capital flow shocks, but in the aggregate intervention can help restore the connection with observed fundamentals.

On the technical side, as the rational expectations equilibrium depends on portfolio decisions of FX dealers with heterogeneous expectations, which in turn depend on the conditional variance of the depreciation rate, the solution strategy follows an approach in line with Townsend (1983) and Bacchetta and van Wincoop (2006). That is, we solve a signal extraction problem of the investors to calculate the average expected depreciation rate in the modified uncovered interest parity condition with an endogenous risk premium, which feeds from the rational expectations solution of the model.

Related to our paper, other recent work also models FX intervention in the context of a New Keynesian model. For example, Benes et al (2013) analyse the performance of a wide range of hybrid inflation targeting and managed exchange rate regimes in the presence of different external shocks. They find that FX intervention can help insulate the economy against certain shocks, but it may also hinder some necessary exchange rate adjustments, for example in the presence of terms-of-trade shocks. In a related work, Vargas et al (2013) show that, given imperfect asset substitutability, sterilised FX intervention can have an effect on credit supply by changing the balance sheet composition of commercial banks.

In the next section we introduce the model, with a special focus on the FX market. In Section 3 we show results from the simulation of the model. In Section 4 we modify the model from Section 2, taking into account information heterogeneity in the dealer market, and show some additional results that complement Section 3. The last section concludes.

2 The model

The model starts from a small open economy with nominal rigidities, in line with the contributions from Obstfeld and Rogoff (1995), Chari et al (2002), Gali and Monacelli (2005), Christiano et al (2005) and Devereux et al (2006), among others. To maintain the concept of general equilibrium, we start from a two-country model taking the size of one of these economies as close to zero, such that the small (domestic) economy does not affect the large (foreign) economy.

In this setup, dealers in the small domestic economy operate the secondary bond market. They receive customer orders for the sale of domestic bonds from households and for the sale of foreign bonds from foreign investors and the central bank. Dealers invest each period in both domestic and foreign bonds, maximising their portfolio returns. This is a cashless economy. The monetary authority intervenes directly in the FX market, selling or purchasing foreign bonds in exchange for domestic bonds. The central bank issues the domestic bonds and sets the nominal interest rates paid by these assets. The central bank can control the interest rate regardless of the FX intervention, that is, we assume the central bank can always perform fully sterilised interventions. [1]

We assume the frequency of decisions is the same between dealers and other economic agents. Households consume final goods, supply labour to intermediate goods producers and save in domestic bonds. Firms produce intermediate and final goods. Additionally, we include monopolistic competition and nominal rigidities in the retail sector, price discrimination and price to market in the export sector, and incomplete pass-through from the exchange rate to imported good prices –

[1] However, in practice sterilised interventions have limits. For example, the sale of foreign currency by the central bank is limited by the level of foreign reserves. On the other hand, the sterilised purchase of foreign currency is limited by the availability of instruments to sterilise those purchases (eg given by the by the demand for central bank bonds or by the stock of treasury bills in the hands of the central bank). Also, limits to the financial losses generated by FX intervention can represent a constraint for intervention itself.

characteristics that are important to analyse the transmission mechanism of monetary policy in a small open economy. We also consider as exogenous processes foreign variables such as output, inflation, the interest rate and non-fundamental capital flows.

2.1 Dealers

In the domestic economy there is a continuum of dealers d in the interval $d \in [0,1]$. Each dealer d receives ϖ_t^d and $\varpi_t^{d,cb}$ in domestic bond sale and purchase orders from households and the central bank, and ϖ_t^{d*} and $\varpi_t^{d*,cb}$ in foreign bond sale orders from foreign investors and the central bank, respectively. These orders are exchanged among dealers, that is $\varpi_t^d - \varpi_t^{d,cb} + S_t \left(\varpi_t^{d*} + \varpi_t^{d*,cb}\right) = B_t^d + S_t B_t^{d*}$, where B_t^d and B_t^{d*} are the ex-post holdings of domestic and foreign bonds by dealer d, respectively. Each dealer receive the same amount of orders from households, foreign investors and the central bank. The exchange rate S_t is defined as the price of foreign currency in terms of domestic currency, such that a decrease (increase) of S_t corresponds to an appreciation (depreciation) of the domestic currency. At the end of the period, any profits -either positive or negative- are transferred to the households.

Dealers are risk averse and short-sighted. They select an optimal portfolio allocation in order to maximise the expected utility of their end-of-period returns, where their utility is given by a CARA utility function. The one-period dealer's horizon gives tractability and captures the feature that FX dealers tend to unwind their FX exposure at the end of any trading period, as explained by Vitale (2010). The problem of dealer d is

$$\max -E_t^d e^{-\gamma \Omega_{t+1}^d}$$

where E_t^d is the expectation operator for dealer d based on the information available at time t, γ is the coefficient of absolute risk aversion and Ω_{t+1}^d is total investment after returns, given by:

$$\begin{aligned}
\Omega_{t+1}^d &= (1+i_t)\, B_t^d + (1+i_t^*)\, S_{t+1} B_t^{d*} \\
&\approx (1+i_t)\left[\varpi_t^d - \varpi_t^{d,cb} + S_t\left(\varpi_t^{d*} + \varpi_t^{d*,cb}\right)\right] + \left(i_t^* - i_t + s_{t+1} - s_t\right) B_t^{d*}
\end{aligned}$$

where we have made use of the resource constraint of dealers, we have log-linearised the excess of return on investing in foreign bonds and $s_t = \ln S_t$. Since the only non-predetermined variable is s_{t+1}, assuming it is normal distributed with time-invariant variance, the first-order condition for the dealers is:[2]

$$0 = -\gamma\left(i_t^* - i_t + E_t^d s_{t+1} - s_t\right) + \gamma^2 B_t^{d*}\sigma^2$$

where $\sigma^2 = var_t\left(\Delta s_{t+1}\right)$ is the conditional variance of the depreciation rate. Then, the demand for foreign bonds by dealer d is given by the following portfolio condition:

$$B_t^{d*} = \frac{i_t^* - i_t + E_t^d s_{t+1} - s_t}{\gamma\sigma^2} \tag{2.1}$$

According to this expression, the demand for foreign bonds will be larger the higher its return, the lower the risk aversion or the lower the volatility of the exchange rate.

2.1.1 FX market equilibrium

Foreign bond equilibrium in the domestic market should sum FX market orders from foreign investors (capital inflows) and central bank FX intervention, that is:

$$\int_0^1 B_t^{d*} dd = \int_0^1 \left(\varpi_t^{d*} + \varpi_t^{d*,cb}\right) dd = \varpi_t^* + \varpi_t^{*,cb}.$$

Replacing the FX market equilibrium condition in the aggregate demand for foreign bonds yields the following arbitrage condition:

$$\overline{E}_t s_{t+1} - s_t = i_t - i_t^* + \gamma\sigma^2(\varpi_t^* + \varpi_t^{*,cb}) \tag{2.2}$$

where $\overline{E}_t s_{t+1}$ is the average rational expectation of the exchange rate next period across all dealers. Note that $\overline{E}_t s_{t+1} = E_t s_{t+1}$, given that all dealers have access to the same set of information, expected exchange rate depreciation would be the same as well (this assumption will be relaxed in Section 4). Condition (2.2) determines the exchange rate, and differs from the traditional UIP on both the expectation term and the endogenous risk premium component. According to (2.2), an increase (decrease) in capital inflows or sales (purchases) of foreign bonds by the central bank appreciates (depreciates) the exchange rate s_t, *ceteris paribus* the remaining variables. This effect is larger, the more risk-averse dealers are (larger γ) or the more volatile the expected depreciation rate is (larger σ^2).[3]

Equation (2.2) is useful to understand both mechanisms through which FX intervention can affect the exchange rate. The last term on the right hand side captures the portfolio-balance

[2] Conditions that are verified later as satisfied.

[3] Sterilised intervention implies that a sale (purchase) of foreign currency by the central bank is accompanied by purchases (sales) of domestic bonds by the monetary authority, such that the domestic interest rates are in line with the policy target rate.

channel. Given that dealers are risk-averse and hold domestic and foreign assets to diversify risk, FX intervention changes the composition of domestic and foreign asset held by the dealers. This will be possible only if there is a change in the expected relative rate of returns of these assets, which compensates for the change in the risk they bear. In other words, according to the portfolio balance channel, a sale (purchase) of foreign bonds by the central bank augments (reduces) the ratio between foreign and domestic assets held by dealers, inducing an appreciation (depreciation) of the domestic currency because dealers require a greater (smaller) risk premium to hold a larger (smaller) quantity of this currency.

The second mechanism at work is the expectations channel, also known as the signalling channel. When central banks intervene in the FX markets they also signal future changes in policy, which affect expectations as well. Therefore, changes in $\overline{E}_t s_{t+1}$ in the left hand side caused by FX intervention will also have an effect on the spot exchange rate.

In an alternative model, Benes et al (2013) show that, in addition to a portfolio allocation approach, a risk premium-adjusted UIP condition can be derived by including imperfect substitutability between domestic and foreign assets.

2.2 Monetary authority

The central bank in the domestic economy intervenes in the FX market by selling/buying foreign bonds to/from dealers in exchange for domestic bonds. Each period the central bank negotiates directly with dealers, such that every dealer receives the same amount of sales/purchases of foreign bonds from the central bank. Each period any dealer d receives a market order $\varpi_t^{d*,cb}$ from the central bank, where $\varpi_t^{d*,cb} > 0 \left(\varpi_t^{d*,cb} < 0\right)$ when the central bank sells (purchases) foreign bonds in exchange of domestic bonds. The total customer flow of foreign bonds received by dealer d equals $\varpi_t^{d*} + \varpi_t^{d*,cb}$. We assume the central bank can always perform fully sterilised FX interventions, therefore it maintains control over the interest rate regardless of the intervention. Moreover, we further assume the central bank does not have to distribute profits/losses to the households. That is, the monetary authority is not constrained by its balance sheet to perform interventions in the FX market.[4]

[4] The balance sheet of the central bank is the following: $S_t R_t^{cb} = B_t^{cb} + NW_t^{cb}$, where R_t^{cb}, B_t^{cb} and NW_t^{cb} are the central bank's reserves in foreign bonds, liabilities in domestic bonds and net worth, respectively. The first two components evolve according to: $R_t^{cb} = (1 + i_t^*)\, R_{t-1}^{cb} - \varpi_t^{*,cb}$ and $B_t^{cb} = (1 + i_t)\, B_{t-1}^{cb} - \varpi_t^{cb}$. Also, profits are given by: $P_t \Gamma_t^{cb} = \left[\frac{S_t\left(1+i_t^*\right)}{S_{t-1}} - 1\right] S_{t-1} R_{t-1}^{cb} - i_t B_{t-1}^{cb} - \left(S_t \omega_t^{*,cb} - \omega_t^{cb}\right)$

2.2.1 FX intervention

The central bank can have three different strategies of FX intervention. First, it can perform pure discretional intervention:

$$\varpi_t^{*cb} = \varepsilon_t^{cb1} \tag{2.3}$$

where the central bank intervenes via unanticipated or secret interventions. According to strategy (2.3), FX intervention by the central bank is not anticipated.

We also assume the monetary authority can intervene by a pre-announced rule. As a second case, the central bank can perform rule-based intervention taking into account the changes in the exchange rate:

$$\varpi_t^{*cb} = \phi_{\Delta s} \Delta s_t + \varepsilon_t^{cb2} \tag{2.4}$$

According to this rule, when there are depreciation (appreciation) pressures on the domestic currency, the central bank sells (purchases) foreign bonds to prevent the exchange rate from fluctuating. $\phi_{\Delta s}$ captures the intensity of the response of the FX intervention to pressures in the FX market.

Finally, the monetary authority can take into account misalignments of the real exchange rate as a benchmark for FX intervention:

$$\varpi_t^{*cb} = \phi_{rer} rer_t + \varepsilon_t^{cb3} \tag{2.5}$$

where rer_t captures deviations of the real exchange rate with respect to its steady state. In the same vein as rule 2, under this rule the central bank sells (purchases) foreign bonds when the exchange rate depreciates (appreciates) in real terms from its long-run value. However there are two important differences: rule 2 is expressed in nominal terms and takes into account only the current variation of the exchange rate, whilst rule 3 takes into account the deviations in the level of the exchange rate in real terms. That is, the rule in terms of real exchange rate misalignments is history-dependent, while the rule based on the change in the exchange rate is not. Intuitively, a key difference between rule 2 and 3 is that, under the former shocks to the exchange rate are accommodated, while under the latter they are reversed. This distinction is similar to that between inflation targeting and price level targeting for the case of shocks to the price level.

2.2.2 Monetary policy

The central bank implements monetary policy by setting the nominal interest rate according to a Taylor-type feedback rule that depends on CPI inflation. The generic form of the interest rate rule that the central bank uses is given by:

$$\frac{(1+i_t)}{(1+\bar{i})} = \left(\frac{\Pi_t}{\overline{\Pi}}\right)^{\varphi_\pi} \exp\left(\varepsilon_t^{MON}\right) \tag{2.6}$$

where $\varphi_\pi > 1$. $\overline{\Pi}$ and $\bar{i}$ are the levels in steady state of inflation and the nominal interest rate. The term ε_t^{MON} is a random monetary policy shock distributed according to $N \sim \left(0, \sigma_{MON}^2\right)$.

2.3 Households

2.3.1 Preferences

The world economy is populated by a continuum of households of mass 1, where a fraction n of them is allocated in the home economy, whereas the remaining $1-n$ is in the foreign economy. Each household j in the home economy enjoys utility from the consumption of a basket of final goods, C_t^j , and receives disutility from working, L_t^j. Households' preferences are represented by the following utility function:

$$\mathcal{U}_t = E_t\left[\sum_{s=0}^{\infty} \beta^{t+s} U\left(C_{t+s}^j, L_{t+s}^j,\right)\right], \tag{2.7}$$

where E_t is the conditional expectation on the information set at period t and β is the intertemporal discount factor, with $0 < \beta < 1$.

The consumption basket of final goods is a composite of domestic and foreign goods, aggregated using the following consumption index:

$$C_t \equiv \left[\left(\gamma^H\right)^{1/\varepsilon_H}\left(C_t^H\right)^{\frac{\varepsilon_H-1}{\varepsilon_H}} + \left(1-\gamma^H\right)^{1/\varepsilon_H}\left(C_t^M\right)^{\frac{\varepsilon_H-1}{\varepsilon_H}}\right]^{\frac{\varepsilon_H}{\varepsilon_H-1}}, \tag{2.8}$$

where ε_H is the elasticity of substitution between domestic (C_t^H) and foreign goods (C_t^M), and γ^H is the share of domestically produced goods in the consumption basket of the domestic economy. In turn, C_t^H and C_t^M are indices of consumption across the continuum of differentiated goods produced in the home country and those imported from abroad, respectively. These consumption indices are defined as follows:

$$C_t^H \equiv \left[\left(\frac{1}{n}\right)^{\frac{1}{\varepsilon}} \int_0^n C_t^H(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right]^{\frac{\varepsilon}{\varepsilon-1}}, C_t^M \equiv \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\varepsilon}} \int_n^1 C_t^M(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.9}$$

where $\varepsilon > 1$ is the elasticity of substitution across goods produced within the home economy, denoted by $C_t^H(z)$, and within the foreign economy, $C_t^M(z)$. Households' optimal demands for home and foreign consumption are given by:

$$C_t^H(z) = \frac{1}{n}\gamma^H \left(\frac{P_t^H(z)}{P_t^H}\right)^{-\varepsilon} \left(\frac{P_t^H}{P_t}\right)^{-\varepsilon_H} C_t, \tag{2.10}$$

$$C_t^M(z) = \frac{1}{1-n}\left(1-\gamma^H\right) \left(\frac{P_t^M(z)}{P_t^M}\right)^{-\varepsilon} \left(\frac{P_t^M}{P_t}\right)^{-\varepsilon_H} C_t \tag{2.11}$$

This set of demand functions is obtained by minimising the total expenditure on consumption P_tC_t, where P_t is the consumer price index. Notice that the consumption of each type of goods is increasing in the consumption level, and decreasing in their corresponding relative prices. Also, it is easy to show that the consumer price index, under these preference assumptions, is determined by the following condition:

$$P_t \equiv \left[\gamma^H \left(P_t^H\right)^{1-\varepsilon_H} + \left(1-\gamma^H\right)\left(P_t^M\right)^{1-\varepsilon_H}\right]^{\frac{1}{1-\varepsilon_H}} \tag{2.12}$$

where P_t^H and P_t^M denote the price level of the home-produced and imported goods, respectively. Each of these price indices is defined as follows:

$$P_t^H \equiv \left[\frac{1}{n}\int_0^n P_t^H(z)^{1-\varepsilon} dz\right]^{\frac{1}{1-\varepsilon}},\ P_t^M \equiv \left[\frac{1}{1-n}\int_n^1 P_t^M(z)^{1-\varepsilon} dz\right]^{\frac{1}{1-\varepsilon}} \tag{2.13}$$

where $P_t^H(z)$ and $P_t^M(z)$ represent the prices expressed in domestic currency of the variety z of home-produced and imported goods, respectively.

2.3.2 Households' budget constraint

For simplicity, we assume domestic households save only in domestic currency.[5] The budget constraint of the domestic household (j) in units of home currency is given by:

$$\varpi_t^j = \left(1+i_{t-1}\right)\varpi_{t-1}^j - \frac{\psi}{2}\left(\varpi_t^j - \overline{\varpi}\right)^2 + W_tL_t^j - P_tC_t^j + P_t\Gamma_t^j \tag{2.14}$$

[5] This way the only portfolio decision is made by dealers, which simplifies the analysis.

where ϖ_t^j is wealth in domestic assets, W_t is the nominal wage, i_t is the domestic nominal interest rate, and Γ_t^j are nominal profits distributed from firms and dealers in the home economy to the household j. Each household owns the same share of firms and dealer agencies in the home economy. Households also face portfolio adjustment costs, for adjusting wealth from its long-run level.[6] Households maximise (2.7) subject to (2.14).

2.3.3 Consumption decisions and the supply of labour

The conditions characterising the optimal allocation of domestic consumption are given by the following equation:

$$U_{C,t} = \beta E_t \left\{ U_{C,t+1} \left[\frac{1+i_t}{1+\psi\left(\varpi_t^j - \overline{\varpi}\right)} \right] \frac{P_t}{P_{t+1}} \right\} \tag{2.15}$$

where we have eliminated the index j for the assumption of representative agent. $U_{C,t}$ denotes the marginal utility for consumption. Equation (2.15) corresponds to the Euler equation that determines the optimal path of consumption for households in the home economy, by equalising the marginal benefits of saving to its corresponding marginal costs. The first-order conditions that determine the supply of labour are characterised by the following equation:

$$-\frac{U_{L,t}}{U_{C,t}} = \frac{W_t}{P_t} \tag{2.16}$$

where $\frac{W_t}{P_t}$ denotes real wages. In a competitive labour market, the marginal rate of substitution equals the real wage, as in equation (2.16).

2.4 Foreign economy

The consumption basket of the foreign economy is similar to that of the foreign economy, and is given by:

$$C_t^* \equiv \left[\left(\gamma^F\right)^{1/\varepsilon_F} \left(C_t^X\right)^{\frac{\varepsilon_F - 1}{\varepsilon_F}} + \left(1-\gamma^F\right)^{1/\varepsilon_F} \left(C_t^F\right)^{\frac{\varepsilon_F - 1}{\varepsilon_F}} \right]^{\frac{\varepsilon_F}{\varepsilon_F - 1}} \tag{2.17}$$

where ε_F is the elasticity of substitution between domestic (C_t^X) and foreign goods (C_t^F), respectively, and γ^F is the share of domestically produced goods in the consumption basket of the foreign economy. Also, C_t^X and C_t^F are indices of consumption across the continuum of differentiated goods produced similar to C_t^H and C_t^M defined in equations (2.9). The demand for each

[6] This assumption is necessary to provide stationarity in the asset position held by the households. See Schmitt-Grohe and Uribe (2004).

type of good is given by:

$$C_t^X(z) = \frac{1}{n}\gamma^F \left(\frac{P_t^X(z)}{P_t^X}\right)^{-\varepsilon} \left(\frac{P_t^X}{P_t^*}\right)^{-\varepsilon_H} C_t^* \tag{2.18}$$

$$C_t^F(z) = \frac{1}{1-n}\left(1-\gamma^F\right) \left(\frac{P_t^F(z)}{P_t^F}\right)^{-\varepsilon} \left(\frac{P_t^F}{P_t^*}\right)^{-\varepsilon_H} C_t^* \tag{2.19}$$

where P_t^X and P_t^F correspond to the price indices of exports and the goods produced abroad, respectively. P_t^* is the consumer price index of the foreign economy:

$$P_t^* \equiv \left[\gamma^F \left(P_t^X\right)^{1-\varepsilon_F} + \left(1-\gamma^F\right)\left(P_t^F\right)^{1-\varepsilon_F}\right]^{\frac{1}{1-\varepsilon_F}} \tag{2.20}$$

2.4.1 The small open economy assumption

Following Sutherland (2005), we parameterise the participation of foreign goods in the consumption basket of home households, $\left(1-\gamma^H\right)$, as follows: $\left(1-\gamma^H\right) = (1-n)(1-\gamma)$, where n represents the size of the home economy and $(1-\gamma)$ the degree of openness. In the same way, we assume the participation of home goods in the consumption basket of foreign households, as a function of the relative size of the home economy and the degree of openness of the world economy, that is $\gamma^F = n(1-\gamma^*)$.

This particular parameterisation implies that as the economy becomes more open, the fraction of imported goods in the consumption basket of domestic households increases, whereas as the economy becomes larger, this fraction falls. This parameterisation allows us to obtain the small open economy as the limiting case of a two-country economy model when the size of the domestic economy approaches zero, that is $n \to 0$. In this case, we have that $\gamma^H \to \gamma$ and $\gamma^F \to 0$. Therefore, in the limiting case, the use in the foreign economy of any home-produced intermediate goods is negligible, and the demand condition for domestic, imported and exported goods can be re-written as follows:

$$Y_t^H = \gamma \left(\frac{P_t^H}{P_t}\right)^{-\varepsilon_H} C_t \tag{2.21}$$

$$M_t = (1-\gamma)\left(\frac{P_t^M}{P_t}\right)^{-\varepsilon_H} C_t \tag{2.22}$$

$$X_t = (1-\gamma^*)\left(\frac{P_t^X}{P_t^*}\right)^{-\varepsilon_F} C_t^* \tag{2.23}$$

Thus, given the small open economy assumption, the consumer price index for the home and foreign economy can be expressed in the following way:

$$P_t \equiv \left[\gamma \left(P_t^H\right)^{1-\varepsilon_H} + (1-\gamma)\left(P_t^M\right)^{1-\varepsilon_H}\right]^{\frac{1}{1-\varepsilon_H}} \tag{2.24}$$

$$P_t^* = P_t^F \tag{2.25}$$

Given the small open economy assumption, the foreign economy variables that affect the dynamics of the domestic economy are foreign output, Y_t^* , the foreign interest rate, i^*, the external inflation rate, Π^*, and capital inflows, ϖ_t^*. To simplify the analysis, we assume these four variables follow an autoregressive process in logs.

2.5 Firms

2.5.1 Intermediate goods producers

A continuum of mass n of z intermediate firms exists. These firms operate in a perfectly competitive market and use the following linear technology:

$$Y_t^{int}(z) = A_t L_t(z) \tag{2.26}$$

$L_t(z)$ is the amount of labour demand from households, A_t is the level of technology.

These firms take as given the real wage, W_t/P_t, paid to households and choose their labour demand by minimising costs given the technology. The corresponding first-order condition of this problem is:

$$L_t(z) = \frac{MC_t(z)}{W_t/P_t} Y_t^{int}(z)$$

where $MC_t(z)$ represents the real marginal costs in terms of home prices. After replacing the labour demand in the production function, we can solve for the real marginal cost:

$$MC_t(z) = \frac{W_t/P_t}{A_t} \tag{2.27}$$

Given that all intermediate firms face the same constant returns to scale technology, the real marginal cost for each intermediate firm z is the same, that is $MC_t(z) = MC_t$. Also, given these firms operate in perfect competition, the price of each intermediate good is equal to the marginal cost. Therefore, the relative price $P_t(z)/P_t$ is equal to the real marginal costs in terms of consumption unit (MC_t).

2.5.2 Final goods producers

Goods sold domestically Final goods producers purchase intermediate goods and transform them into differentiated final consumption goods. Therefore, the marginal costs of these firms equal the price of intermediate goods. These firms operate in a monopolistic competitive market, where each firm faces a downward-sloping demand function, given below. Furthermore, we assume that each period t final goods producers face an exogenous probability of changing prices given by $(1-\theta^H)$. Following Calvo (1983), we assume that this probability is independent of the last time the firm set prices and the previous price level. Thus, given a price fixed from period t, the present discounted value of the profits of firm z is given by:

$$E_t\left\{\sum_{k=0}^{\infty}\left(\theta^H\right)^k\Lambda_{t+k}\left[\frac{P_t^{H,o}(z)}{P_{t+k}^H}-MC_{t+k}^H\right]Y_{t,t+k}^H(z)\right\} \tag{2.28}$$

where $\Lambda_{t+k}=\beta^k\frac{U_{C,t+k}}{U_{C,t}}$ is the stochastic discount factor, $MC_{t+k}^H=MC_{t+k}\frac{P_{t+k}}{P_{t+k}^H}$ is the real marginal cost expressed in units of goods produced domestically, and $Y_{t,t+k}^H(z)$ is the demand for good z in $t+k$ conditioned to a fixed price from period t, given by

$$Y_{t,t+k}^H(z)=\left[\frac{P_t^{H,o}(z)}{P_{t+k}^H}\right]^{-\varepsilon}Y_{t+k}^H$$

Each firm z chooses $P_t^{H,o}(z)$ to maximise (2.28). The first order condition of this problem is:

$$E_t\left\{\sum_{k=0}^{\infty}\left(\theta^H\right)^k\Lambda_{t+k}\left[\frac{P_t^{H,o}(z)}{P_t^H}F_{t,t+k}^H-\mu MC_{t+k}^H\right]\left(F_{t,t+k}^H\right)^{-\varepsilon}Y_{t+k}^H\right\}=0$$

where $\mu\equiv\frac{\varepsilon}{\varepsilon-1}$ and $F_{t,t+k}^H\equiv\frac{P_t^H}{P_{t+k}^H}$.

Following Benigno and Woodford (2005), the previous first order condition can be written recursively using two auxiliary variables, V_t^D and V_t^N, defined as follows:

$$\frac{P_t^{H,o}(z)}{P_t^H}=\frac{V_t^N}{V_t^D}$$

where

$$V_t^N=\mu U_{C,t}Y_t^H MC_t^H+\theta^H\beta E_t\left[V_{t+1}^N\left(\Pi_{t+1}^H\right)^{\varepsilon}\right] \tag{2.29}$$

$$V_t^D=U_{C,t}Y_t^H+\theta^H\beta E_t\left[V_{t+1}^D\left(\Pi_{t+1}^H\right)^{\varepsilon-1}\right] \tag{2.30}$$

Also, since in each period t only a fraction $\left(1-\theta^H\right)$ of these firms change prices, the gross rate of domestic inflation is determined by the following condition:

$$\theta^H \left(\Pi_t^H\right)^{\varepsilon-1} = 1 - \left(1-\theta^H\right)\left(\frac{V_t^N}{V_t^D}\right)^{1-\varepsilon} \tag{2.31}$$

The equations (2.29), (2.30) and (2.31) determine the supply (Phillips) curve of domestic production.

Exported goods We assume that firms producing final goods can discriminate prices between domestic and external markets. Therefore, they can set the price of their exports in foreign currency. Also, when selling abroad they face an environment of monopolistic competition with nominal rigidities, with a probability $1-\theta^X$ of changing prices.

The problem of retailers selling abroad is very similar to that of firms that sell in the domestic market, which is summarised in the following three equations that determine the supply curve of exporters in foreign currency prices:

$$V_t^{N,X} = \mu\left(Y_t^X U_{C,t}\right) MC_t^X + \theta^X \beta E_t\left[V_{t+1}^{N,X}\left(\Pi_{t+1}^X\right)^{\varepsilon}\right] \tag{2.32}$$

$$V_t^{D,X} = \left(Y_t^X U_{C,t}\right) + \theta^X \beta E_t\left[V_{t+1}^{D,X}\left(\Pi_{t+1}^X\right)^{\varepsilon-1}\right] \tag{2.33}$$

$$\theta^X \left(\Pi_t^X\right)^{\varepsilon-1} = 1 - \left(1-\theta^X\right)\left(\frac{V_t^{N,X}}{V_t^{D,X}}\right)^{1-\varepsilon} \tag{2.34}$$

where the real marginal costs of the goods produced for export are given by:

$$\begin{aligned} MC_t^X &= \frac{P_t MC_t}{S_t P_t^X} \\ &= \frac{MC_t}{RER_t\left(\frac{P_t^X}{P_t^*}\right)} \end{aligned} \tag{2.35}$$

which depend inversely on the real exchange rate ($RER_t = \frac{S_t P_t^*}{P_t}$) and the relative price of exports to external prices $\left(\frac{P_t^X}{P_t^*}\right)$.

2.5.3 Retailers of imported goods

Those firms that sell imported goods buy a homogeneous good in the world market and differentiate it into a final imported good $Y_t^M(z)$. These firms also operate in an environment of monopolistic competition with nominal rigidities, with a probability $1-\theta^M$ of changing prices.

The problem for retailers is very similar to that of producers of final goods. The Phillips curve for importers is given by:

$$V_t^{N,M} = \mu \left(Y_t^M U_{C,t}\right) MC_t^M + \theta^M \beta E_t \left[V_{t+1}^{N,M} \left(\Pi_{t+1}^M\right)^{\varepsilon}\right] \tag{2.36}$$

$$V_t^{D,M} = \left(Y_t^M U_{C,t}\right) + \theta^M \beta E_t \left[V_{t+1}^{D,M} \left(\Pi_{t+1}^M\right)^{\varepsilon-1}\right] \tag{2.37}$$

$$\theta^M \left(\Pi_t^M\right)^{\varepsilon-1} = 1 - \left(1-\theta^M\right) \left(\frac{V_t^{N,M}}{V_t^{D,M}}\right)^{1-\varepsilon} \tag{2.38}$$

where the real marginal cost for the importers is given by the cost of purchasing the goods abroad ($S_t P_t^*$) relative to the price of imports (P_t^M):

$$MC_t^M = \frac{S_t P_t^*}{P_t^M} \tag{2.39}$$

where MC_t^M also measures the deviations from the law of one price.[7]

2.6 Market clearing

Total domestic production is given by:

$$P_t^{def} Y_t = P_t^H Y_t^H + S_t P_t^X Y_t^X \tag{2.40}$$

After using equations (2.21) and (2.22) and the definition of the consumer price index (2.24), the equation (2.40) can be decomposed into:

$$P_t^{def} Y_t = P_t C_t + S_t P_t^X Y_t^X - P_t^M Y_t^M \tag{2.41}$$

To identify the gross domestic product (GDP) of this economy, Y_t, it is necessary to define the GDP deflator, P_t^{def}, which is the weighted sum of the consumer, export and import price indices:

$$P_t^{def} = \phi_C P_t + \phi_X S_t P_t^X - \phi_M P_t^M \tag{2.42}$$

where ϕ_C, ϕ_X and ϕ_M are steady state values of the ratios of consumption, exports and imports to GDP, respectively. The demand for intermediate goods is obtained by aggregating the production

[7] See Monacelli (2005) for a similar formulation.

for home consumption and exports:

$$
\begin{aligned}
Y_t^{int}(z) &= Y_t^H(z) + Y_t^X(z) \\
&= \left(\frac{P_t^H(z)}{P_t^H}\right)^{-\varepsilon} Y_t^H + \left(\frac{P_t^X(z)}{P_t^X}\right)^{-\varepsilon} Y_t^X
\end{aligned} \tag{2.43}
$$

Aggregating (2.43) with respect to z, we obtain:

$$
Y_t^{int} = \frac{1}{n}\int_0^n Y_t^{int}(z)\,dz = \Delta_t^H Y_t^H + \Delta_t^X Y_t^X \tag{2.44}
$$

where $\Delta_t^H = \frac{1}{n}\int_0^n \left(\frac{P_t^H(z)}{P_t^H}\right)^{-\varepsilon} dz$ and $\Delta_t^X = \frac{1}{n}\int_0^n \left(\frac{P_t^X(z)}{P_t^X}\right)^{-\varepsilon} dz$ are measures of relative price dispersion, which have a null impact on the dynamic in a first-order approximation of the model. Similarly, the aggregate demand for labour is:

$$
L_t = \frac{MC_t}{W_t/P_t}\left(\Delta_t^H Y_t^H + \Delta_t^X Y_t^X\right) \tag{2.45}
$$

After aggregating households' budget constraints, firms' and dealers' profits, and including the equilibrium condition in the financial market that equates household wealth with the stock of domestic bonds, we obtain the aggregate resources constraint of the home economy:

$$
\begin{aligned}
\frac{B_t}{P_t} - \frac{B_{t-1}}{P_{t-1}} + \frac{\psi}{2}\left(\frac{B_t}{P_t} - \frac{\overline{B}}{P}\right)^2 &= \frac{P_t^{def}}{P_t} Y_t - C_t \\
&+ \left\{\frac{(1+i_{t-1})}{\Pi_t} - 1\right\}\frac{B_{t-1}}{P_{t-1}} + REST_t
\end{aligned} \tag{2.46}
$$

Equation (2.46) corresponds to the current account of the home economy. The left-hand side is the change in the net asset position in terms of consumption units. The right-hand side is the trade balance, the difference between GDP and consumption which is equal to net exports, and investment income. The last term, $REST_t \equiv \frac{P_t^M}{P_t} Y_t^M \left(1 - \Delta_t^M MC_t^M\right)$ is negligible and takes into account the monopolistic profits of retail firms.

A complete set of the log-linearised equations of the model can be found in Appendix B.

3 Results

3.1 Calibration

Instead of calibrating the parameters to a particular economy, we set the parameters to values that are standard in the new open economy literature, as shown in Table 1. The standard deviation of all exogenous processes was set to 0.01 and the autocorrelation coefficient to 0.5. The coefficient of absolute risk aversion for dealers was set to 500 as in Bacchetta and van Wincoop (2006). In the benchmark case, we calibrate the FX intervention reaction to exchange rate changes and real exchange rate misalignments to 0.5 for both cases, and analyse robustness to those parameters.

Table 1: **Baseline calibration**

Parameter	Value	Description
β	0.9975	Consumers time-preference parameter.
χ	0.5	Labour supply elasticity.
γ_c	1	Risk aversion parameter.
ε	0.75	Elasticity of substitution btw. home and foreign goods.
ε_X	0.75	Elasticity of substitution btw. exports and foreign goods.
ψ	0.6	Share of domestic tradables in domestic consumption.
θ_H	0.75	Domestic goods price rigidity.
θ_M	0.5	Imported goods price rigidity.
θ_X	0.5	Exported goods price rigidity.
ψ_b	0.1	Portfolio adjustment costs.
φ_π	1.5	Taylor rule reaction to inflation deviations.
γ	500	Absolute risk aversion parameter (dealers)
ϕ_ϖ	0.5	Net asset position over GDP ratio.
ϕ_C	0.68	Consumption over GDP ratio.
σ_x	0.01	S.D. of all shocks x
ρ_x	0.5	AR(1) coefficient for all exogenous processes

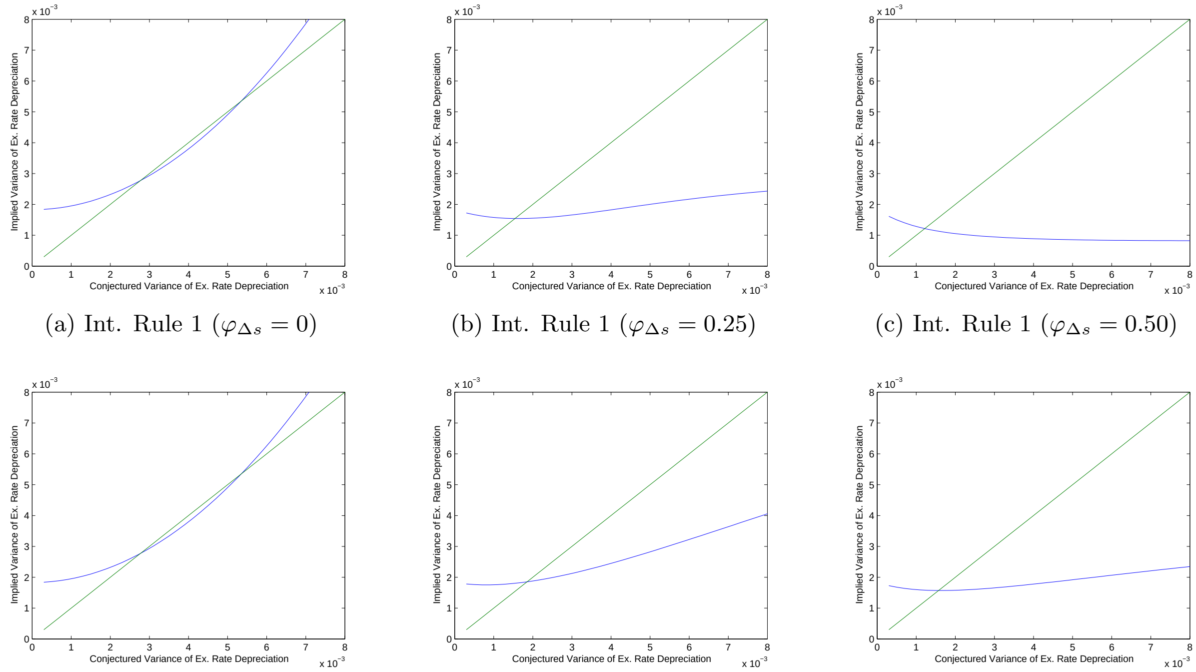
3.2 Model dynamics

3.2.1 Rational expectations (RE) equilibria

As shown in the previous section, the risk premium-adjusted uncovered interest parity condition (equation 2.2) depends, among other things, on the conditional variance of the depreciation rate. This, in turn, depends on the RE equilibrium of the model. Therefore, to solve for the linear RE equilibria entails solving for a fixed point problem on the conditional variance of the depreciation rate. In Figure 2 we plot the mappings of the conjectured and the implied conditional variance of the depreciation rate for different parameterisations of the FX intervention reaction function. Intersections with the 45-degree straight line correspond to fixed points for the conditional variance of the depreciation rate.

As shown in the left-hand panels, there are two RE equilibria in the case of no FX intervention, corresponding to a low-variance stable equilibrium and a high-variance unstable equilibrium.[8] This type of multiple equilibria is similar to the one found by Bacchetta and van Wincoop (2006) in a model without FX intervention. However, as shown in the centre and right-hand panels, FX intervention helps to rule out the second unstable equilibrium. Under both rules of FX intervention there is only a unique and stable equilibrium. Also, the intensity of FX intervention reduces the RE equilibrium variance of the exchange rate change.

The RE equilibrium variance of the exchange rate change also affects the direct impact of FX intervention and capital flows on the exchange rate, as shown in equation (2.2). Therefore, a more intensive FX intervention strategy also reduces its effectiveness as the reduction in variance, dampens the impact of interventions on the exchange rate.



(a) Int. Rule 1 ($\varphi_{\Delta s} = 0$) (b) Int. Rule 1 ($\varphi_{\Delta s} = 0.25$) (c) Int. Rule 1 ($\varphi_{\Delta s} = 0.50$)

(d) Int. Rule 2 ($\varphi_{rer} = 0$) (e) Int. Rule 2 ($\varphi_{rer} = 0.15$) (f) Int. Rule 2 ($\varphi_{rer} = 0.30$)

Figure 2: Existence of equilibria under FX intervention rules

[8] A slope lower (higher) than one of the mapping of the conjectured and the implied conditional variance of the depreciation rate, evaluated at the intersection with the 45-degree straight line, indicates a stable (unstable) equilibrium.

3.2.2 Intervention under rules and discretion

In Figure 3 we show the effects on impact of a 1% increase in sales of foreign bonds by the central bank. We compare the cases when the authority performs this policy under – entirely unanticipated – discretion and by deviations from an intervention rule. Since a shock to an FX intervention rule in the first period implies an endogenous response of intervention in the following periods, to make both scenarios comparable we have simulated in each case a series of discretional interventions that matches the series of endogenous rule-based interventions.

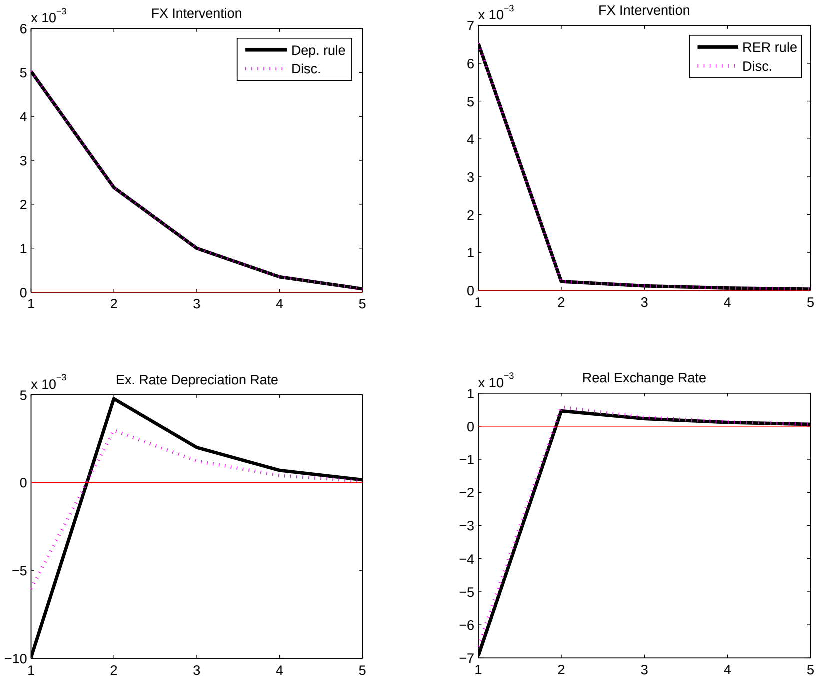
As shown, in both cases, the appreciation on impact of the exchange rate in response to the sale of foreign bonds by the central bank is larger under rules than under discretion. This effect is clearer in the case of the Δs_t rule. The reason is that under rule-based interventions, an intervention also announces a series of future interventions, consistent with the paths in the upper panels of Figure 3. Hence, the reaction on impact of the exchange rate is stronger. In the case of the second rule, which targets the misalignments of the real exchange rate, the differences will not be large, since interventions will not generate a strong expectation effect.[9]

Figure A.2 in the appendix shows the reaction of the economy to an FX intervention shock, where we do not normalise the size of interventions. In this case, we can compare the reaction of the economy to a pure surprise shock. Here we can observe the "surprise effect" as discretionary interventions have a larger effect than interventions under a rule. The reason is that under a rule-based intervention, FX sales by the central bank will be followed by contemporaneous purchases, as the former generate an appreciation of the currency. Hence, the market will not react on impact as much as in the case of full discretionary intervention. Nevertheless, once we net the surprise and expected effects, the central bank is able to stabilise the exchange rate through smaller interventions.

3.2.3 Impact of monetary policy

In Figure 4 we show the effects of a 1% increase in the policy rate under no intervention and both types of FX intervention rules. In this case we have not normalised by the amount of interventions since we want to understand how these rules affect the transmission mechanism of monetary policy. In all cases the exchange rate appreciates, but this is larger in the case of no FX intervention. This generates two opposite effects on inflation. In the first place, the lower

[9]It is easy to show this effect by substituting equations (2.4) and (2.5) into (2.2). An intervention based on changes of the exchange rate will generate a dynamic equation that amplifies the intervention through the expectation channel. In the case of the real exchange rate, this effect is determined by the size of the pass-through; a higher pass-through dampens this effect.



(a) Int. Rule 1 ($\varphi_{\Delta s} = 0.5$) (b) Int. Rule 2 ($\varphi_{RER} = 0.5$)

Figure 3: FX Intervention: rules vs. discretion

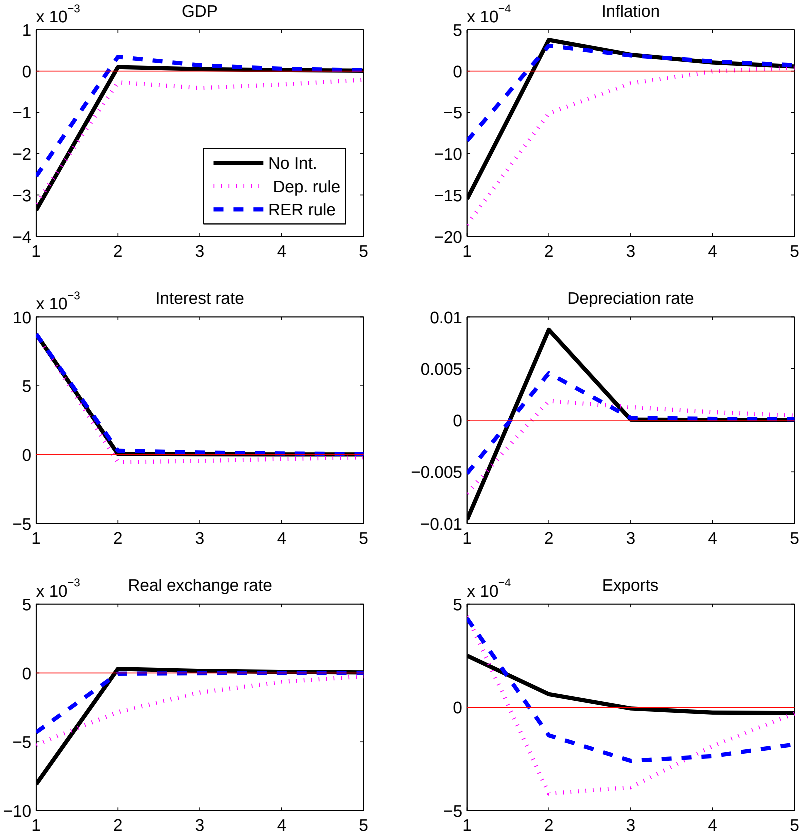
appreciation decreases inflation by less because of the pass-through from the exchange rate to inflation. On the other hand, FX intervention can lengthen the duration of real appreciation, which in turn dampens exports and aggregate demand, and consequently reduces inflation. As shown in Figure 4, under a nominal exchange rate smoothing rule, the real exchange rate remains deviated from its equilibrium value for a longer period, amplifying the impact of monetary policy on inflation. In contrast, under the real exchange rate stabilisation rule the former effect is more important, hence reducing the impact of monetary policy on inflation.

3.2.4 Transmission of external shocks

In Table 2 we present unconditional relative variances of some main macroeconomic variables assuming only one source of volatility at the time for different FX intervention regimes[10]. For comparison, relative variances are normalised with respect to the no intervention case.

[10]Exercises are simulated using the conditional variance of the depreciation rate in equilibrium in equation 2.2.

Figure 4: Reaction to a 1% monetary policy shock - rules vs. no intervention



As shown, not surprisingly, FX intervention reduces the volatility of the change of the exchange rate in all cases. However, this exercise highlights some trade-offs in the use of FX intervention. In particular, the effects of FX intervention on the volatility of other macroeconomic variables will depend on the source of the shock. FX intervention helps to isolate domestic macroeconomic variables from financial external shocks, but amplifies fluctuations in some domestic variables from nominal and real external shocks.

For instance, the volatility of consumption, exports, output and inflation generated by foreign interest rate and capital flow shocks is reduced under both types of FX intervention regimes. However, the use of FX interventions to smooth the nominal exchange rate amplifies the volatility of inflation and output generated by foreign inflation shocks. Similarly, the use of a real exchange rate misalignment rule increases the volatility of consumption, exports, output and inflation generated by foreign output shocks. In this case, FX intervention prevents the adjustment of the real exchange rate as a macroeconomic stabiliser.

In Figures A.3, A.4 and A.5 in the appendix we compare the dynamic effects of external shocks under discretion, rules and no intervention. Overall, the effectiveness of intervention rules is confirmed. In other words, given that it is known the central bank will enter the FX market to prevent large fluctuations in the exchange rate, the amount of intervention necessary to reduce fluctuations is smaller. This means that the FX sales and purchases by the central bank necessary to stabilise the exchange rate will be much higher under discretion because it does not influence expectations as in the case of an intervention rule.

In Figure A.3 we show the reaction to a portfolio or non-fundamental capital flow shock. Capital inflows generate an appreciation of the exchange rate that under no intervention affects the whole economy. In the case where the central bank intervenes through rules or discretion, the effects of these shocks are dampened, stabilising the economy. For the case of a foreign interest rate shock, in Figure A.4 we show how interventions can ease the pressure of capital outflows on the exchange rate. It is interesting to see how interventions have similar effects when reacting to non-fundamental (order flow) and fundamental (interest rate) shocks. Finally, in Figure A.5 we show the reaction to a foreign inflation shock. In this case, as in the previous ones, interventions provide a channel to counter the impact of external shocks on the economy. Foreign inflation will generate an exchange rate appreciation and a current account deficit. An active central bank is capable of reversing these effects through foreign exchange interventions, since the combination of a low nominal depreciation under the exchange rate smoothing rule with higher foreign inflation can generate a depreciation of the real exchange rate.

Table 2: **Macroeconomic volatility under FX intervention rules (Equilibrium $\sigma_{\Delta s}$)**

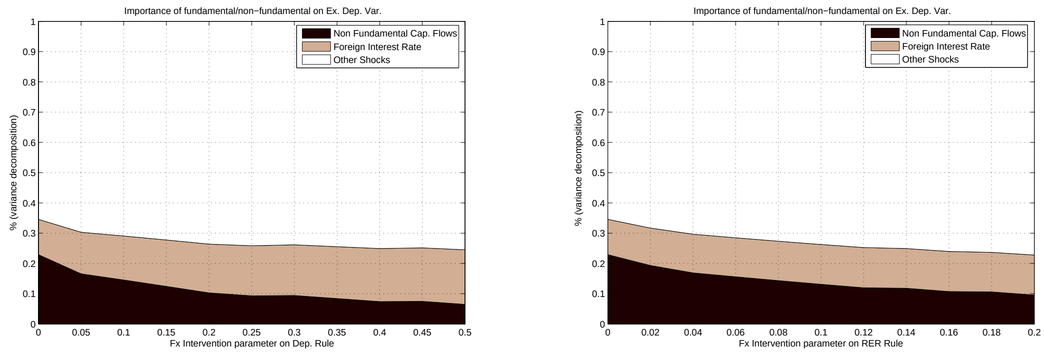
Endogenous variable	RER	Δ Ex. Rate	Cons.	Exp.	Int. Rate	Prod.	Infl.
Foreign interest rate shock (ε_{i^*})							
No intervention	1	1	1	1	1	1	1
$\varphi_{\Delta s} = 0.25$	0.31	0.69	0.48	0.92	0.44	0.12	0.41
$\varphi_{\Delta s} = 0.50$	0.10	0.56	0.26	0.89	0.21	0.03	0.17
$\varphi_{rer} = 0.15$	0.76	0.90	0.84	0.97	0.86	0.66	0.84
$\varphi_{rer} = 0.30$	0.64	0.85	0.75	0.96	0.78	0.49	0.74
Non-fundamental capital flow shock (ε^{ω^*})							
No intervention	1	1	1	1	1	1	1
$\varphi_{\Delta s} = 0.25$	0.25	0.23	0.31	0.28	0.34	0.32	0.31
$\varphi_{\Delta s} = 0.50$	0.15	0.13	0.21	0.17	0.24	0.21	0.21
$\varphi_{rer} = 0.15$	0.38	0.38	0.40	0.38	0.42	0.39	0.40
$\varphi_{rer} = 0.30$	0.24	0.23	0.25	0.24	0.27	0.25	0.26
Foreign inflation shock (ε^{π^*})							
No intervention	1	1	1	1	1	1	1
$\varphi_{\Delta s} = 0.25$	0.90	0.78	0.99	0.92	1.08	1.02	1.06
$\varphi_{\Delta s} = 0.50$	0.84	0.67	0.96	0.85	1.13	1.01	1.08
$\varphi_{rer} = 0.15$	0.76	0.74	0.77	0.75	0.81	0.78	0.80
$\varphi_{rer} = 0.30$	0.63	0.62	0.66	0.63	0.71	0.66	0.69
Foreign output shock (ε^{y^*})							
No intervention	1	1	1	1	1	1	1
$\varphi_{\Delta s} = 0.25$	1.28	0.93	0.89	0.99	0.72	0.92	0.63
$\varphi_{\Delta s} = 0.50$	1.42	0.85	0.85	0.98	0.59	0.89	0.46
$\varphi_{rer} = 0.15$	0.72	0.73	1.10	1.02	1.15	1.09	1.18
$\varphi_{rer} = 0.30$	0.59	0.62	1.15	1.02	1.24	1.14	1.30

Note: The table shows normalised unconditional relative variances of the model assuming the only source of volatility is the shock in the table heading. We have considered changes in variance produced by intervention rules themselves, and how these affect the overall volatility of the economy.

3.2.5 Contribution of shocks and FX intervention

Up to now we have proved the effectiveness of FX interventions by the central bank as a mechanism to cope with the effects of external shocks. To show this we have kept the variance of the exchange rate constant across regimes, as a way to make results comparable. However, as shown by Figure 2, intervention rules reduce the equilibrium value of the exchange rate volatility. This is key to understanding an additional effect of interventions. The impact of portfolio shocks on the exchange rate value is a function of the risk dealers bear for holding more foreign currency in their portfolio. Hence, a lower volatility will reduce the risk and consequently the premia they charge for these holdings. This makes interventions less effective when dealing with most external shocks, as shown by Table 2, while improving the resilience of the economy to portfolio or non-fundamental capital shocks.

Thus, an intervention rule that reduces the volatility of the exchange rate will affect as well the relative importance the shocks have in explaining this variance. In Figure 5 we show the variance decomposition of the exchange rate variation under different shocks. Our result is then a robust one, when the central bank intervenes in the FX market through rules, non-fundamental capital flows explain a smaller fraction of the fluctuations of the change of the exchange rate, while the effect of others, such as foreign interest rate shocks, increases.



(a) Δs rule (b) RER rule

Figure 5: Variance decomposition

4 Extension: dealers with heterogeneous information

The model presented in Section 2 is useful to analyse the interaction of different FX intervention strategies with conventional monetary policy. However, this model also shares the weakness of standard macroeconomic models in explaining the short-run fluctuations of the exchange rate. As documented by Meese and Rogoff (1983), Frankel and Rose (1995) and others, in the short run a random walk predicts exchange rates better than macroeconomic models. This disconnection between macroeconomic fundamentals and the exchange has been referred to by Lyons (2001) as "the exchange rate determination puzzle".

Bacchetta and van Wincoop (2011) suggested that certain assumptions about the information under which participants operate in the FX market are crucial to explain this puzzle. In particular, typically in macroeconomic models FX market participants are assumed to: i) have identical information; ii) perfectly know the model; iii) use the available information at all times. Assumptions that are quite inconsistent with how FX markets operate in reality. Bacchetta and van Wincoop (2011) show that relaxing these assumptions enables various exchange rate puzzles to be explained, such as the disconnection between exchange rates and fundamentals and the forward premium puzzle. In line with this, we extend the model by relaxing the first assumption, acknowledging that FX dealers can have access to different sources of information and can have different expectations about future macroeconomic variables. As shown by Bacchetta and van Wincoop (2006) in a more tractable model, these characteristics magnify the response of the exchange rate to unobserved variables and generate a disconnection in the short run between the exchange rate and observed fundamentals.

More precisely, we extend the model to include dealers that have access to heterogeneous information, as in Bacchetta and van Wincoop (2006). We introduce two sources of information heterogeneity among dealers: we assume they face idiosyncratic shocks in the amount of customer orders from foreign investors and also receive noisy signals about some future shocks. Therefore, as dealers have access to different sets of information, expected exchange rate depreciation in (2.2) would differ among them as well.

We assume the foreign investor exposure for each dealer is equal to the average plus an idiosyncratic term:

$$\varpi_t^{d*} = \varpi_t^* + \varepsilon_t^d \tag{4.1}$$

where ε_t^d has variance that approaches infinity, so that knowing one‘s own foreign investor exposure provides no information about the average exposure as in Bacchetta and van Wincoop (2006). ϖ_t^* is unobservable and follows an AR(1) process:

$$\varpi_t^* = \rho_{\varpi^*} \varpi_{t-1}^* + \varepsilon_t^{\varpi^*} \tag{4.2}$$

where $\varepsilon_t^{\varpi^*} \sim N\left(0, \sigma_{\varpi^*}^2\right)$. The assumed autoregressive process is known by all agents.

We assume that dealers observe past and current fundamental shocks, while they also receive private signals about some future shocks. More precisely, we assume dealers receive one signal each period about the foreign interest rate one period ahead.[11] That is, at time t dealer d receives

[11] This assumption can be extended to the case where dealers receive each period a vector of signals of a set of fundamental variables.

a signal

$$v_t^d = i_{t+1}^* + \varepsilon_t^{vd},\ \varepsilon_t^{vd} \sim N\left(0, \sigma_{vd}^2\right) \tag{4.3}$$

where ε_t^{vd} is independent from i_{t+1}^* and other agents' signals. This idiosyncratic signal can be reconciled with the fact that dealers have different models to forecast future fundamentals, so each can imperfectly observe future variables with an idiosyncratic noise. We also assume that the average signal received by investors is i_{t+1}^*, that is $\int_0^1 v_t^d dd = i_{t+1}^*$. The foreign interest rate follows an AR(1) process known by dealers:

$$i_t^* = \rho_{i^*} i_{t-1}^* + \varepsilon_t^{i^*} \tag{4.4}$$

where $\varepsilon_t^{i^*} \sim N\left(0, \sigma_{i^*}^2\right)$. Dealers solve a signal extraction problem for the unknown innovations $\left(\varepsilon_t^{\varpi^*}, \varepsilon_{t+1}^{i^*}\right)$, given the observed depreciation rate and signal $\left(\Delta s_t, v_t^d\right)$.[12]

4.1 Computational strategy

The computational strategy consists in dividing the system of log-linearised equations into two blocks. In the first block we take into account all the equations but the risk premium-adjusted UIP, which is included in the second block. Then, we solve for the rational expectations equilibrium of the first block taking the depreciation rate as an exogenous variable. This solution feeds into the second block to solve for the policy function of the depreciation rate. Note that with this computational strategy we also eliminate any informational spillovers between dealers and other economic agents, such as households and firms. However, the segmented information in the FX market seems a reasonable assumption, since it takes account of the fact that dealers have access to private information which is not known by other economic agents.

Accordingly, in the first block the depreciation rate only appears in the real exchange rate equation:

$$rer_t = rer_{t-1} + \Delta s_t + \pi_t^* - \pi_t \tag{4.5}$$

This system of equations can be written as:

$$A_0 \begin{bmatrix} X_t \\ E_t Y_{t+1} \end{bmatrix} = A_1 \begin{bmatrix} X_{t-1} \\ Y_t \end{bmatrix} + A_2 \Delta s_t + B_0 \epsilon_t \tag{4.6}$$

where $X_t = [rer_t, i_t, \pi_t^*, w_t^*, i_t^*, ...]'$ is a size n_1 vector of backward looking variables, $Y_t = [\pi_t, ...]'$ is

[12] As explained by Bacchetta and van Wincoop (2006), if the $B(L)$ polynomial in equation (4.10) is invertible, knowledge of the depreciation rate at times $t-1$ and earlier and of the interest rate shocks at time t and earlier, reveals the shocks ε^{ϖ^*} at times $t-1$ and earlier. That is, $\varepsilon_{t-s}^{\varpi^*}$ becomes observable at time t for $s \geqslant 1$.

a size n_2 vector of forward looking variables, such as $n_T = n_1 + n_2 + 1$ is the number of endogenous variables. ϵ_t is the vector of observable shocks in the model. $A_2 = [1, 0..0]'$ is a $(n_1 + n_2) \times 1$ matrix.[13]

The second block corresponds to the risk-premium adjusted UIP condition:

$$\overline{E}_t \Delta s_{t+1} = i_t - i_t^* + \gamma \sigma^2 \left(\varpi_t^* + \varpi_t^{*,cb} \right) \tag{4.7}$$

In the first stage we find the rational expectations solution of the system in (4.6) using the perturbation method, taking as exogenous Δs_t.[14] That is, we find the policy functions:

$$Y_t = M_1 X_{t-1} \tag{4.8}$$

$$X_t = M_2 X_{t-1} + M_3 \Delta s_t + M_4 \epsilon_t \tag{4.9}$$

In the second stage we use the previous solution to find the policy function of Δs_t using the Townsend (1983) method. More precisely, we conjecture a solution for Δs_t as a function of infinite lag polynomials of the shocks in the model:

$$\Delta s_t = \mathcal{A}(L) \varepsilon_{t+1}^{i^*} + \mathcal{B}(L) \varepsilon_t^{\varpi^*} + \mathcal{D}(L) \zeta_t \tag{4.10}$$

where $\varepsilon_{t+1}^{i^*}$ is an innovation to the future foreign interest rate (i_{t+1}^*), the fundamentals over which agents receive a signal, and $\varepsilon_t^{\varpi^*}$ is the shock to the unobservable capital flow (ϖ_t^*), which can be inferred with a lag. $\mathcal{A}(L)$ and $\mathcal{B}(L)$ are infinite lag polynomials, while $\mathcal{D}(L)$ is a vector of infinite lag polynomials operating ζ_t, the vector of remaining shocks. [15]

In the second stage we solve for the signal extraction problem of the dealers for the unobserved innovations $\left(\varepsilon_t^{\varpi^*}, \varepsilon_{t+1}^{i^*}\right)$, using both the depreciation rate and their signal $\left(\Delta s_t, v_t^d\right)$, which serves to calculate the average expectation of the future depreciation rate and its conditional variance in equation (4.7) as functions of shocks.[16]

What follows is equating the coefficients of (4.7) as a function of shocks with those of the conjectured solution (4.10), generating a system of non-linear equations on the unknown coefficients in $\mathcal{A}(L)$, $\mathcal{B}(L)$ and $\mathcal{D}(L)$. Following Bacchetta and van Wincoop (2006), we exploit the recursive

[13] Since information heterogeneity only enters the model through the exchange rate, the unobservable shocks are excluded from the first step.

[14] We use Dynare to solve for the rational expectations of the first block. For more information see: Villemont (2011) and Adjemian et al (2011).

[15] Note that ϵ_t and ζ_t are not exactly the same, since the latter can also include FX intervention shocks.

[16] In turn, given the solution of the first block, we can express the endogenous variables in (4.7) as function of shocks as well.

pattern of this infinite order set of equations and truncate the order of the polynomial after guaranteeing the stability of the solution. See appendix C for more details on the computational strategy.

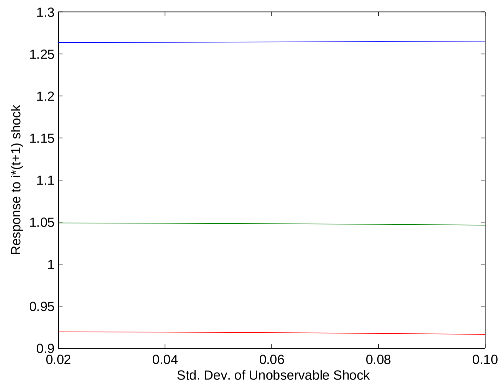
4.2 The effects of heterogeneous information

Bacchetta and van Wincoop (2006) proved that by adding heterogeneous information in an exchange rate determination model it is possible to account for the short-run disconnection between the exchange rate and observed fundamentals. This way, the exchange rate becomes closely associated with order flow, which is associated with the private information component of total market orders. The mechanism at work is a magnification effect of unobserved fundamentals, such as capital flows in our model, on the exchange rate. They showed that, under heterogeneous information, there is rational confusion when the exchange rate changes: dealers do not know whether this is driven by unobserved fundamentals (eg capital flows) or by information of other dealers about future macroeconomic fundamentals (eg foreign interest rates).

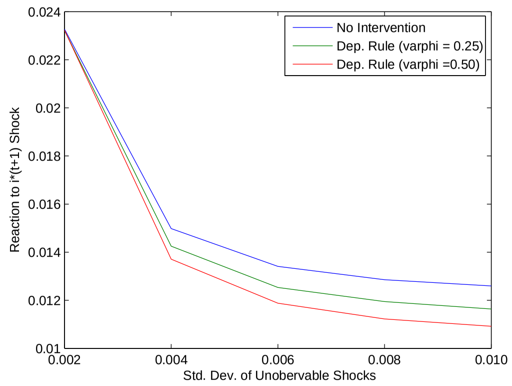
This rational confusion magnifies the impact of the unobserved capital flows on the exchange rate, since dealers confuse them with changes in average private signals about future foreign interest rates/fundamentals. This magnification effect depends on the precision of the public signal (the exchange rate) relative to that of the private signal (v_t^d). It will be larger when the public signal is more precise relative to the private signal, for instance when the exchange rate is less volatile.

In the context of heterogeneous information, FX intervention can affect the magnification effect and the connection of the exchange rate with observed fundamentals. We show in figure 6 the response on impact of the exchange rate change to future foreign interest rate shocks (i_{t+1}^*) and unobserved capital flow shocks (ϖ_t^*) , that is coefficients a_1 and b_1 respectively. We show in the first column the responses in a model with common knowledge, defined as one in which all dealers have access to the same information, and in the second column the responses in a model with heterogeneous information.[17] In the last column we present the differences between the responses in the heterogeneous information and common knowledge models. These responses are plotted for different values of the standard deviation of unobserved capital flow shocks $\left(\sigma_{\varpi_t^*}\right)$, for three degrees of FX intervention intensity under rule 1 (no intervention, $\phi_{\Delta s} = 0.25$, and $\phi_{\Delta s} = 0.25$).

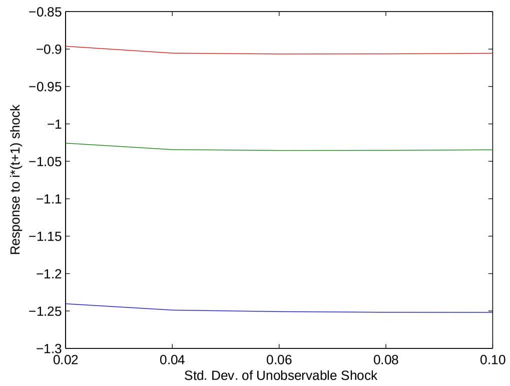
[17]Therefore, in a common knowledge model capital flows become an observable variable and all dealers observe signal shock (ε_t^{vd})



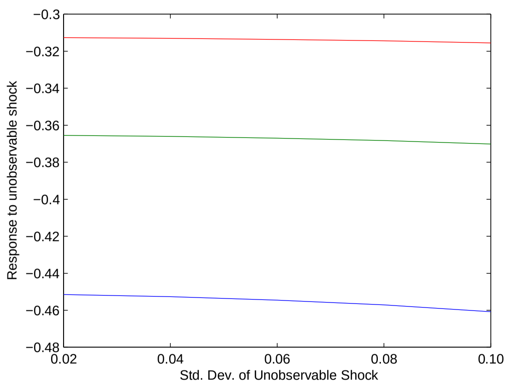
(a) Reaction to a i^*_{t+1} - CK



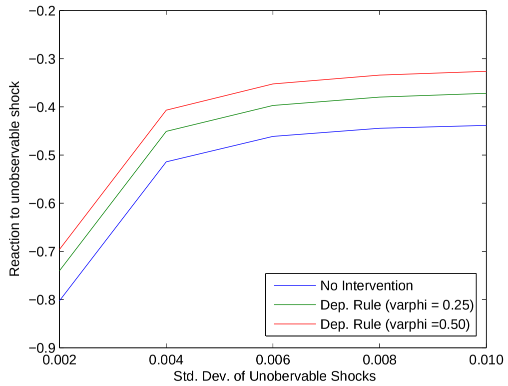
(b) Reaction to a i^*_{t+1} - HI



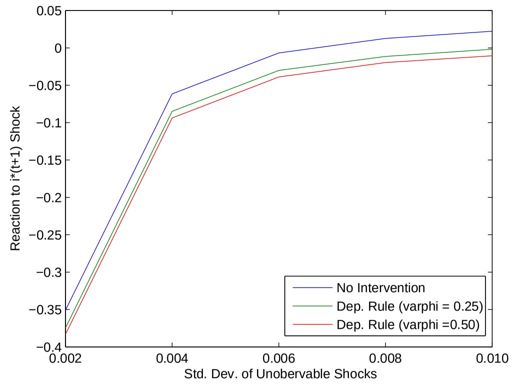
(c) Difference (HI - CK)



(d) Reaction to a ω^*_t - CK

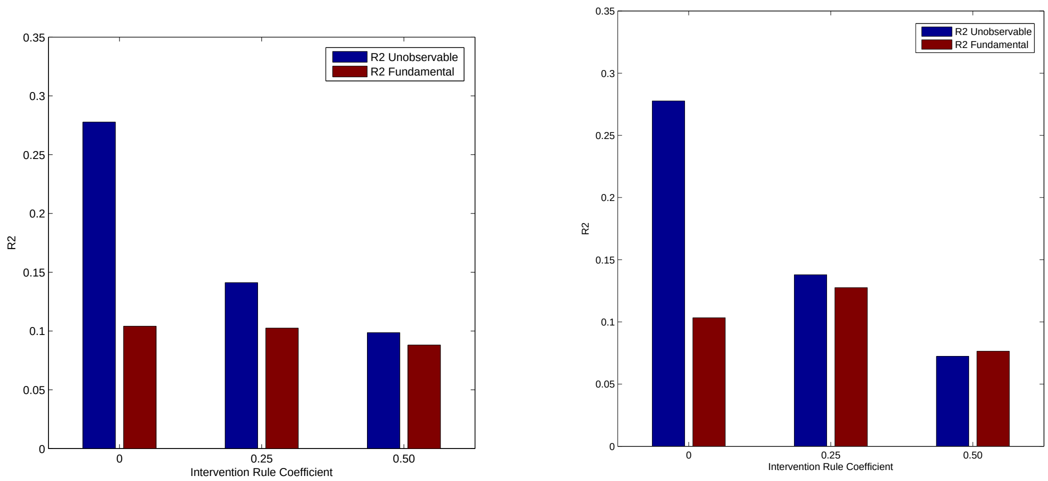


(e) Reaction to a ω^*_t - HI



(f) Magnification Effect (HI - CK)

Figure 6: Reaction to unobservable and fundamental shocks under heterogeneous information and common knowledge



(a) Δs rule

(b) RER rule

Figure 7: Regression of Δs_t on unobservable and fundamental shocks.

The following things are important to note: i) Under common knowledge and heterogeneous information, FX intervention dampens the impact of both unobserved capital flow shocks and future foreign interest rate shocks. ii) The standard deviation of unobserved capital flow shocks $\left(\sigma_{\varpi_t^*}\right)$ affects the responses under heterogeneous information, but not under common knowledge. This is because the response of the exchange rate depends on the precision of the signals only in the former model. iii) There is evidence of a magnification effect. That is, the response to unobservable capital shocks is much stronger in the heterogeneous information than in the common knowledge model. The opposite is true for the response to future foreign interest rate shocks. iv) The magnification effect is larger, the stronger the intensity of FX intervention. The main mechanism for this result is that, when FX intervention reduces the exchange rate volatility, it also increases the precision of the public signal, which amplifies the magnification effect[18][19].

These results illustrate an additional effect that intervention can have in the FX market, that is, the magnified response of the exchange rate to unobservable shocks, such as capital flows. However, the magnification effect is not strong enough to increase the disconnection between the exchange rate and observed fundamentals. Figure 7 reports the R^2 of a regression of Δs_t on

[18]On the other hand, as shown in figure 6, the magnification effect is larger when the unobservable capital flows are more volatile, because that increases the exchange rate volatility.

[19]However, this result could change if intervention can bring additional information about future fundamentals to the FX market, as analysed by Vitale (2011).

unobserved capital flow shocks (ϖ_t^*) and future interest rates (i_{t+1}^*). As shown, FX intervention reduces the contribution of unobserved capital flow shocks to exchange rate changes, and as a counterpart increases the connection between observed fundamentals and the exchange rate.

5 Conclusions

In this paper we present a model to analyse the interaction between monetary policy and FX intervention by central banks which also includes microstructure fundamentals in the determination of the exchange rate. We introduce a portfolio decision of risk-averse dealers, which adds an endogenous risk premium to the traditional uncovered interest rate condition. In this model, FX intervention affects the exchange rate through both a portfolio balance and an expectations/signalling channel. Also, an extension of the model with heterogeneous information in the FX market generates the disconnection observed in the data between exchange rates and macroeconomic fundamentals. On the technical side, we also propose an extension of the Townsend (1983) method to solve DSGE models with heterogeneous information.

Our results illustrate that FX intervention has strong interactions with monetary policy. Intervening to smooth real exchange rate misalignments can mute the monetary transmission mechanism through exchange rates, reducing the impact on aggregate demand and prices, while intervening to smooth nominal exchange rate fluctuations can amplify the impact. Also, FX intervention rules can be more powerful in stabilising the economy as they exploit the expectations channel. When we analyse the response to foreign shocks, we show that FX intervention rules have some advantages as a stabilisation tool, because they anchor expectations about future exchange rates. Therefore, the amount of FX intervention needed to stabilise the exchange rate under rules is much smaller than under discretion. We also show that there are some trade-offs in the use of FX intervention. On the one hand, it can help isolate the economy from external financial shocks, but on the other it prevents some necessary adjustments of the exchange rate in response to nominal and real external shocks. Moreover, in our extension of the model with heterogeneous information, we also show that FX intervention can magnify the response of exchange rates to unobservable capital flow shocks, but in the aggregate intervention can help restore the connection with observed fundamentals.

References

[1] Adjemian, S, H Bastani, M Juillard, F Karamé, F Mihoubi, G Perendia, J Pfeifer, M Ratto and S Villemot (2011): "Dynare: Reference Manual, Version 4," *Dynare Working Papers* 1, CEPREMAP.

[2] Bacchetta, P and E van Wincoop (2006): "Can information heterogeneity explain the exchange rate determination puzzle?", *American Economic Review*, 96, pp 552–76.

[3] ——— (2012): "Modeling exchange rates with incomplete information", in J James, I W Marsh and L Sarno (eds), *Handbook of Exchange Rates*, John Wiley & Sons, Inc., Hoboken, NJ, USA. doi: 10.1002/9781118445785.ch13.

[4] Benes, J, A Berg, R Portillo and D Vabra (2013): "Modeling sterilized interventions and balance sheet effects of monetary policy in a new-Keynesian framework", *IMF Working Paper* WP/13/11.

[5] Benigno, P and M Woodford (2005): "Inflation stabilization and welfare: the case of a distorted steady state", *Journal of the European Economic Association* 3(6), 1185–236.

[6] Chari, V V, P Kehoe and E McGrattan (2002): "Can sticky price models generate volatile and persistent real exchange rates?", *Review of Economic Studies* 69(3), 533–63.

[7] Christiano, L, M Eichenbaum and C Evans (2005): "Nominal rigidities and the dynamic effects of a shock to monetary policy", *Journal of Political Economy* 113(1), 1–45.

[8] Devereux, M, P Lane and J Xu (2006): "Exchange rates and monetary policy in emerging market economies", *Economic Journal* 116(511), 478–506.

[9] Frankel, J and A Kose (1995): "Empirical research on nominal exchange rates, in G M Grossman and K Rogoff (eds), *Handbook of International Economics*, volume III, pp 1689–1729.

[10] Gali, J and T Monacelli (2005): "Monetary policy and exchange rate volatility in a small open economy", *Review of Economic Studies*, 72(3), 707–34.

[11] Lyons, R (2001): *The microstructure approach to exchange rates*, Cambridge, MA: MIT Press.

[12] Meese, R and K Rogoff (1983): “Empirical exchange rate models of the seventies: do they fit out of sample?”, *Journal of International Economics* 14, pp 345–73.

[13] Obstfeld, M and K Rogoff (1995): “Exchange rate dynamics redux”, *Journal of Political Economy* 103(3),624–60.

[14] Schmitt-Grohe,S and M Uribe (2004: ”Closing small open economies”, *Journal of International Economics* 61(1), pp 163–185.

[15] Sutherland, A (2005): “Incomplete pass-through and the welfare effects of exchange rate variability”, *Journal of International Economics* 62(5), 375–99.

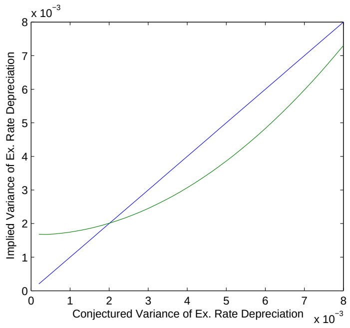
[16] Townsend, R (1983): “Forecasting the forecasts of others”, *Journal of Political Economy* 91(4), pp 546–88.

[17] Vargas, H, A Gonzalez and D Rodriguez (2013): “Foreign exchange rate intervention in Colombia”, *Banco de la Republica Working Paper* no 757.

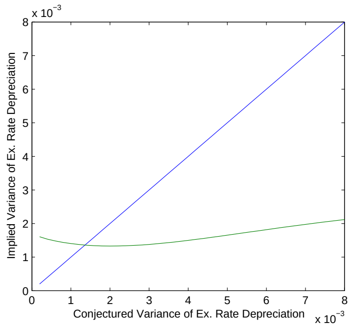
[18] Villemot, S, (2011): ”Solving rational expectations models at first order: what Dynare does,” *Dynare Working Papers* 2, CEPREMAP.

[19] Vitale, P (2011): “The impact of FX intervention on FX markets: a market microstructure analysis”, *International Journal of Finance & Economics*, vol 16 (1), pages 41–62.

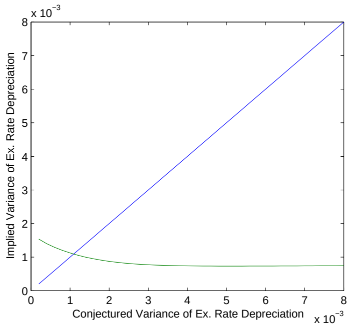
A Figures and tables



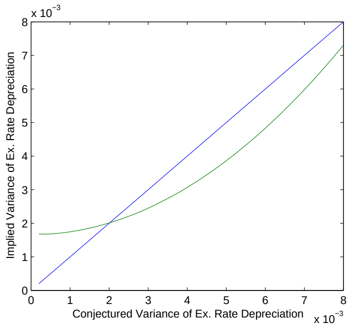
(a) Int. Rule 1 ($\varphi_{\Delta s} = 0$)



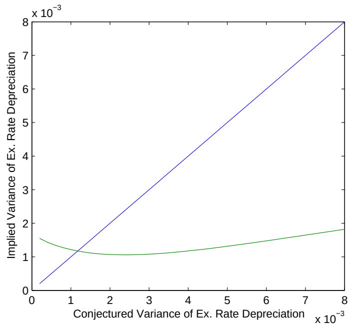
(b) Int. Rule 1 ($\varphi_{\Delta s} = 0.25$)



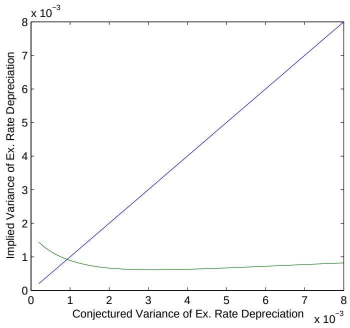
(c) Int. Rule 1 ($\varphi_{\Delta s} = 0.50$)



(d) Int. Rule 2 ($\varphi_{rer} = 0$)



(e) Int. Rule 2 ($\varphi_{rer} = 0.15$)

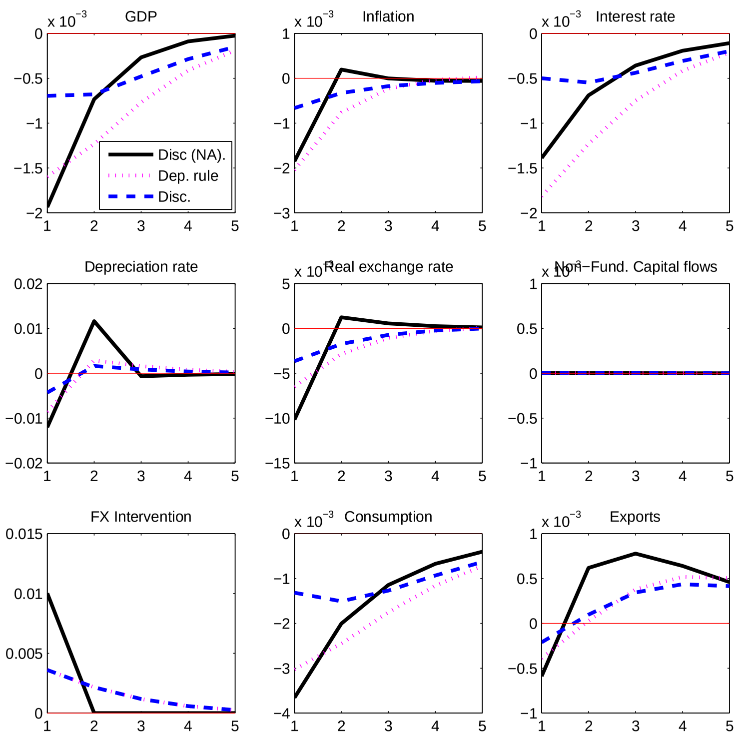


(f) Int. Rule 2 ($\varphi_{rer} = 0.30$)

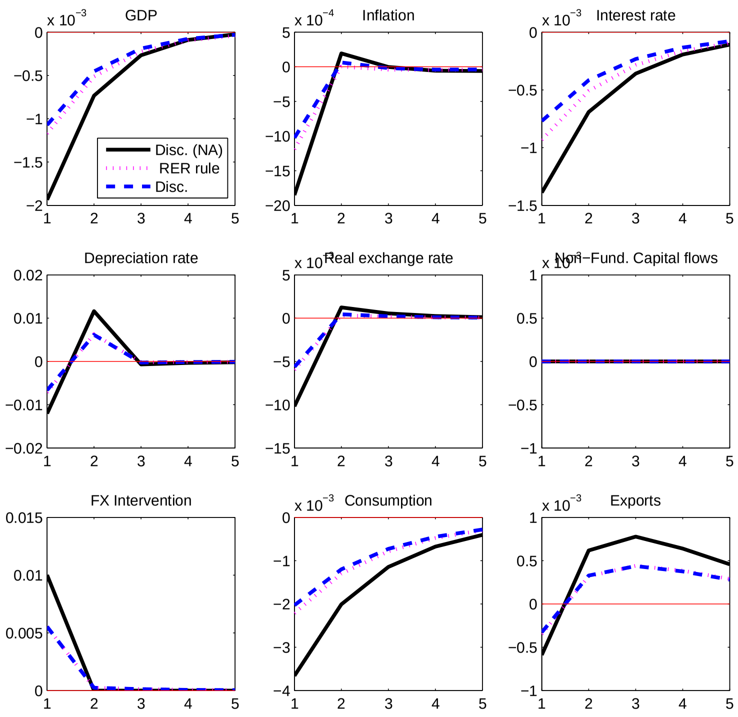
Figure A.1: Existence of equilibria under FX intervention Rules and heterogeneous information (HI)

Figure A.2: Reaction to a 1% FX Intervention Shock

(a) Int. Rule 1 ($\varphi_{\Delta s} = 0.5$)



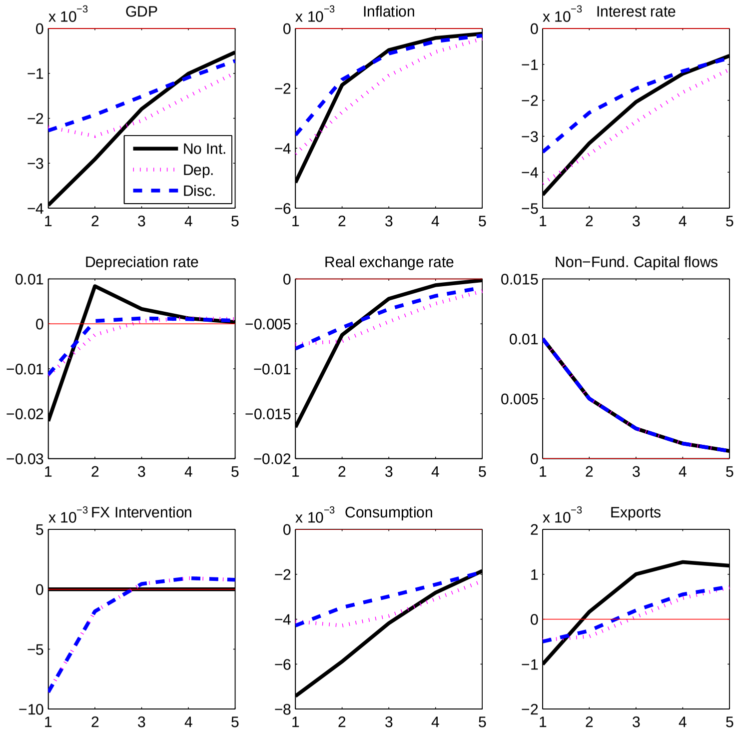
(b) Int. Rule 2 ($\varphi_{RER} = 0.5$)



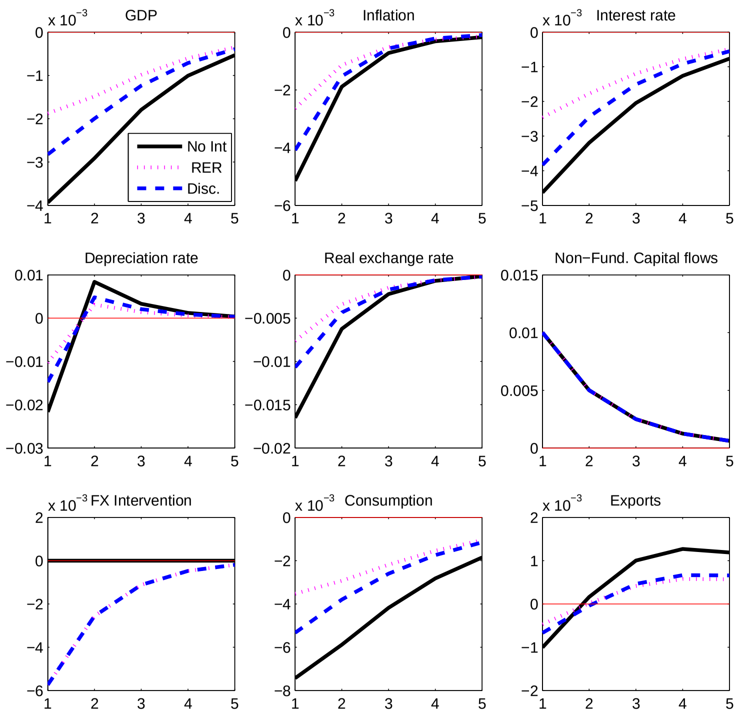
Note: Intervention under discretion normalised to the implied intervention path under rules.

Figure A.3: Reaction to a 1% Portfolio Balance Shock

(a) Int. Rule 1 ($\varphi_{\Delta s} = 0.5$)



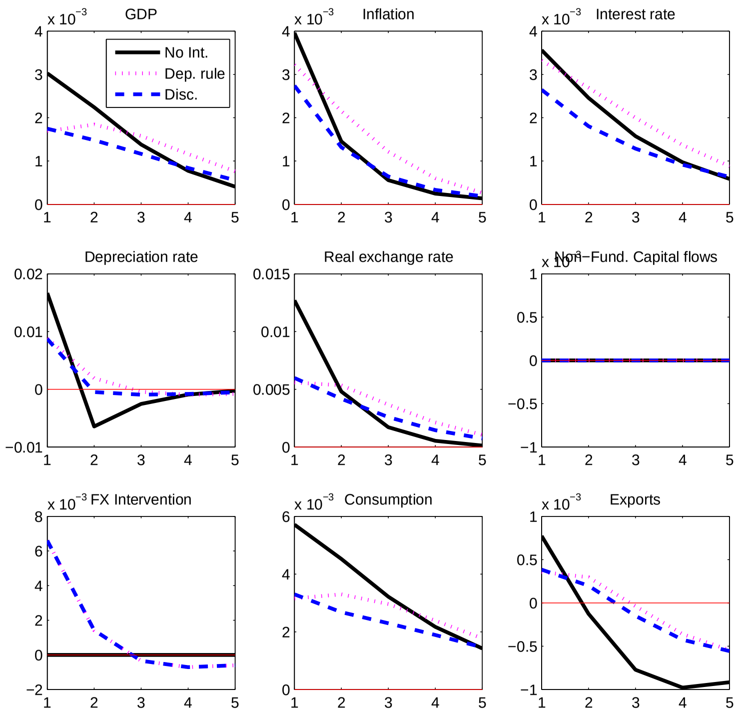
(b) Int. Rule 2 ($\varphi_{RER} = 0.5$)



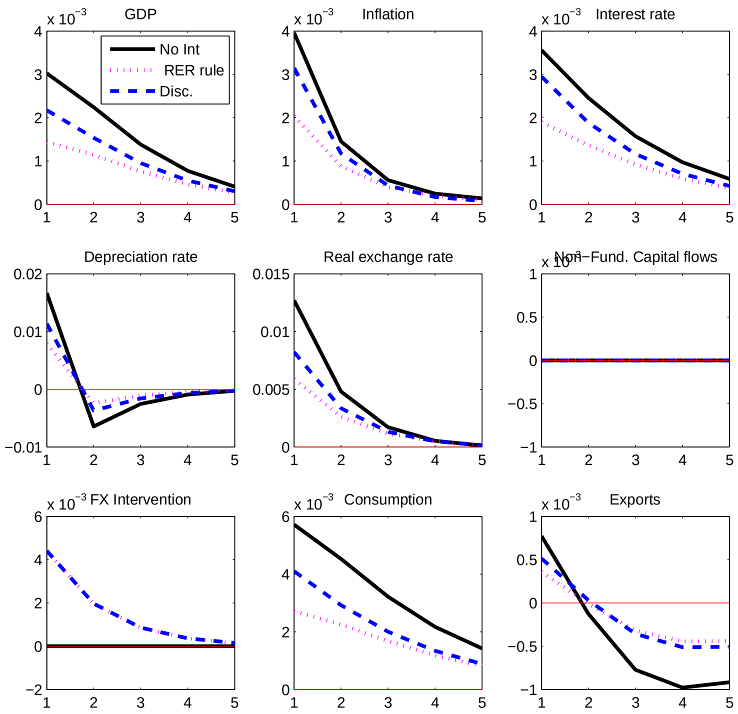
Note: Intervention under discretion normalised to the implied intervention path under rules.

Figure A.4: Reaction to a 1% Foreign Interest Rate Shock

(a) Int. Rule 1 ($\varphi_{\Delta s} = 0.5$)



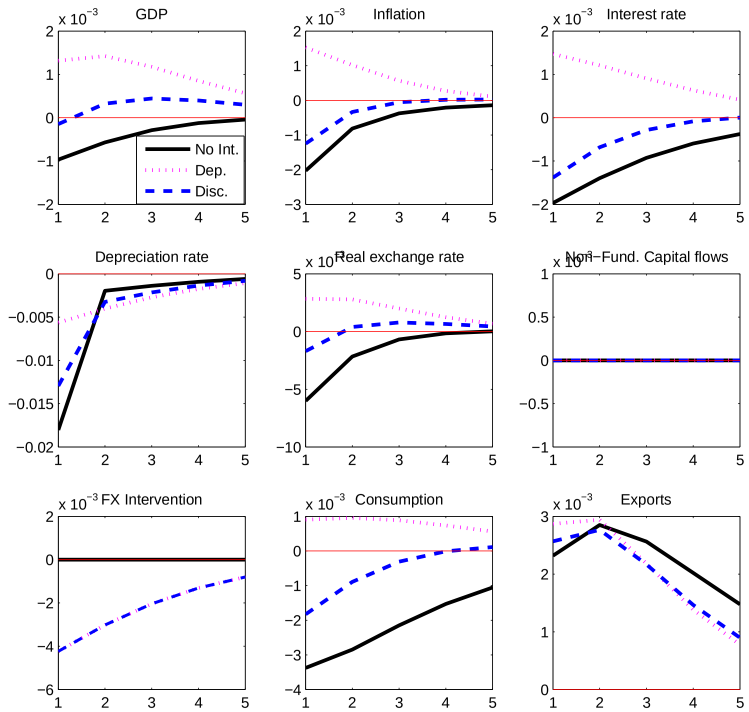
(b) Int. Rule 2 ($\varphi_{RER} = 0.5$)



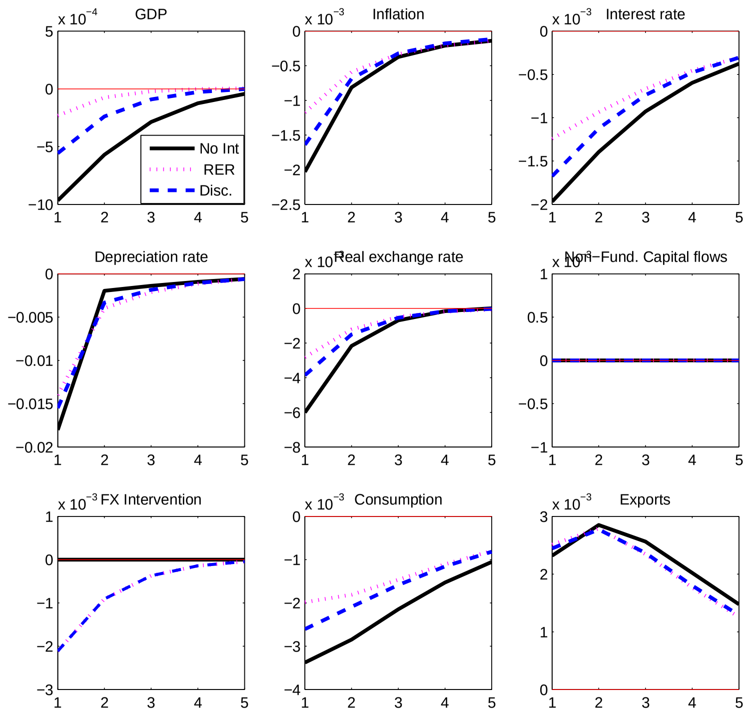
Note: Intervention under discretion normalised to the implied intervention path under rules.

Figure A.5: Reaction to a 1% Foreign Inflation Rate Shock

(a) Int. Rule 1 ($\varphi_{\Delta s} = 0.5$)



(b) Int. Rule 2 ($\varphi_{RER} = 0.5$)



Note: Intervention under discretion 0 to the implied intervention path under rules.

B The log-linear version of the model

B.1 Aggregate demand

Aggregate demand (y_t)

$$y_t = \phi_C(c_t) + \phi_X(x_t) - \phi_M(m_t) + g_t \tag{B.1}$$

GDP deflator $\left(t_t^{def}\right)$

$$t_t^{def} = \phi_X(rer_t + t_t^X) - \phi_M t_t^M \tag{B.2}$$

Real exchange rate (rer_t)

$$rer_t = rer_{t-1} + \Delta s_t + \pi_t^* - \pi_t \tag{B.3}$$

Euler equation (λ_t)

$$\lambda_t = \hat{\imath}_t + E_t(\lambda_{t+1} - \pi_{t+1}) - \psi_b b_t \tag{B.4}$$

Marginal utility (λ_t)

$$\lambda_t = -\gamma_u c_t \tag{B.5}$$

Exports (x_t)

$$x_t = -\varepsilon^x(t_t^X) + y_t^*; \tag{B.6}$$

Relative price of exports $\left(t_t^X\right)$

$$t_t^X = t_{t-1}^X + \pi_t^X - \pi_t^*; \tag{B.7}$$

Imports (m_t)

$$m_t = -\varepsilon(t_t^M) + c_t; \tag{B.8}$$

Relative price of imports $\left(t_t^M\right)$

$$t_t^M = t_{t-1}^M + \pi_t^M - \pi_t; \tag{B.9}$$

Home-produced goods demand $\left(y_t^H\right)$

$$y_t^H = -\varepsilon(t_t^H) + c_t; \tag{B.10}$$

Relative price of home-produced goods $\left(t_t^H\right)$

$$t_t^H = -\left(\frac{1-\psi}{\psi}\right) t_t^M \tag{B.11}$$

B.2 Aggregate supply

Total CPI (π_t):

$$\pi_t = \psi \pi_t^H + (1 - \psi)\, \pi_t^M + \mu_t \tag{B.12}$$

Phillips curve for home-produced goods (π_t^H):

$$\pi_t^H = \kappa_H \left(mc_t - t_t^H \right) + \beta E_t \pi_{t+1}^H \tag{B.13}$$

Real marginal costs (mc_t)

$$mc_t = wp_t - a_t; \tag{B.14}$$

Phillips curve for imported goods (π_t^M):

$$\pi_t^M = \kappa_M mc_t^M + \beta E_t \pi_{t+1}^M \tag{B.15}$$

Marginal costs for imports $\left(mc_t^M\right)$

$$mc_t^M = rer_t - t_t^M \tag{B.16}$$

Phillips curve for exports (π_t^X)

$$\pi_t^X = \kappa_X mc_t^X + \beta E_t \pi_{t+1}^X \tag{B.17}$$

Marginal costs for exports $\left(mc_t^X\right)$

$$mc_t^X = mc_t - rer_t - t_t^X \tag{B.18}$$

B.3 Labour market

Labour demand (l_t)

$$l_t = y_t - a_t; \tag{B.19}$$

Labour supply (wp_t)

$$wp_t = \gamma_u c_t + \chi l_t \tag{B.20}$$

B.4 FX markets and current account

Risk-premium adjusted UIP (Δs_t)

$$\overline{E}_t \Delta s_{t+1} = \hat{\imath}_t - \hat{\imath}_t^* + \gamma \sigma^2 \left(\varpi_t^* + \varpi_t^{*,cb} \right) \tag{B.21}$$

Current account (b_t)

$$\phi_{\varpi}\left(b_t - \beta^{-1}b_{t-1}\right) = t_t^{def} + y_t - \phi_C c_t + \frac{\phi_{\varpi}}{\beta}\left(i_{t-1} - \pi_t\right) \tag{B.22}$$

B.5 Monetary policy

Interest rate ($\hat{\imath}_t$)

$$\hat{\imath}_t = \varphi_\pi(\pi_t) + \varepsilon_t^{int} \tag{B.23}$$

FX intervention $\left(\varpi_t^{*,cb}\right)$

$$\varpi_t^{*,cb} = \varphi_{\Delta s}\Delta s_t + \varphi_{rer} rer_t + \varepsilon_t^{cb} \tag{B.24}$$

B.6 Foreign economy

Foreign output (y_t^*):

$$y_t^* = \rho_{y^*} y_{t-1}^* + \varepsilon_t^{y^*} \tag{B.25}$$

Foreign inflation (π_t^*):

$$\pi_t^* = \rho_{\pi^*} \pi_{t-1}^* + \varepsilon_t^{\pi^*} \tag{B.26}$$

Foreign interest rates (i_t^*):

$$i_t^* = \rho_{i^*} i_{t-1}^* + \varepsilon_t^{i^*} \tag{B.27}$$

Capital inflows-order flows (ϖ_t^*)

$$\varpi_t^* = \rho_{\varpi^*} \varpi_{t-1}^* + \varepsilon_t^{\varpi^*} \tag{B.28}$$

B.7 Domestic shocks

Productivity shocks (a_t):

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a \tag{B.29}$$

Demand shocks (g_t):

$$g_t = \rho_g g_{t-1} + \varepsilon_t^g \tag{B.30}$$

Mark up shocks (μ_t):

$$\mu_t = \rho_\mu \mu_{t-1} + \varepsilon_t^\mu \tag{B.31}$$

Thus we have in total 31 equations, 24 from the original model and seven auxiliary equations. We have included two exogenous shock processes - demand (g_t) and mark-up/inflation (μ_t) shocks - to perform additional analysis. The variables in the model are: a_t, y_t, c_t, x_t, m_t, y_t^*, y_t^H, l_t, λ_t,

ϖ_t^*, mc_t, mc_t^X, mc_t^M, t_t^{def}, t_t^X, t_t^M, t_t^H, π_t, π_t^H, π_t^X, π_t^M, π_t^*, rer_t, Δs_t, i_t, i_t^*, wp_t, b_t, $\varpi_t^{*,cb}$, g_t, μ_t. The minimum state variable (MSV) set is composed of 12 variables: a_t, y_t^*, b_t, ϖ_t^*, t_t^X, t_t^M, π_t^*, rer_t, i_t, i_t^*, g_t, μ_t.

The nine shocks comprise four foreign economy shocks $(\varepsilon_t^{y^*}, \varepsilon_t^{\pi^*}, \varepsilon_t^{i^*}, \varepsilon_t^{\varpi^*})$, three domestic economy shocks $(\varepsilon_t^a, \varepsilon_t^g, \varepsilon_t^\mu)$ and two policy shocks $\left(\varepsilon_t^{int}, \varepsilon_t^{cb}\right)$.

C Details of the computational strategy

The log-linearised system of equations of the model can be written as:

$$A_0 \begin{bmatrix} X_t \\ E_t Y_{t+1} \end{bmatrix} = A_1 \begin{bmatrix} X_{t-1} \\ Y_t \end{bmatrix} + A_2 \Delta s_t + B_0 \epsilon_t \tag{C.1}$$

and

$$\overline{E}_t \Delta s_{t+1} = i_t - i_t^* + \gamma \sigma^2 \left(\varpi_t^* + \varpi_t^{*,cb} \right) \tag{C.2}$$

where $A_2 = [1, 0..0]'$ is a $(n_1 + n_2) \times 1$ matrix and the definitions of the other matrices and vectors are as in Section 3. This is the state space form of the model.

C.1 Solving the first block

As an illustration, we will solve the system in (C.1) under some simplifying assumptions. For a more general solution, see Villemot (2011). The system in (C.1) can be written as:[20]

$$\begin{bmatrix} X_t \\ E_t Y_{t+1} \end{bmatrix} = A_0^{-1} A_1 \begin{bmatrix} X_{t-1} \\ Y_t \end{bmatrix} + A_0^{-1} A_2 \Delta s_t + A_0^{-1} B_0 \epsilon_t$$

or

$$\begin{bmatrix} X_t \\ E_t Y_{t+1} \end{bmatrix} = A \begin{bmatrix} X_{t-1} \\ Y_t \end{bmatrix} + \begin{bmatrix} a_{11} \Delta s_t \\ 0_{(n_1+n_2-1)\times 1} \end{bmatrix} + B \epsilon_t$$

after making $A = A_0^{-1} A_1$, $B = A_0^{-1} B_0$ and a_{11} the $(1,1)$ element of A_0^{-1}. Using the Jordan decomposition of $A = P \Lambda P^{-1}$, it becomes:

$$P^{-1} \begin{bmatrix} X_t \\ E_t Y_{t+1} \end{bmatrix} = \Lambda P^{-1} A \begin{bmatrix} X_{t-1} \\ Y_t \end{bmatrix} + \begin{bmatrix} p_{11} a_{11} \Delta s_t \\ 0_{(n_1+n_2-1)\times 1} \end{bmatrix} P^{-1} B \Delta s_t + P^{-1} C \epsilon_t$$

Making $R = P^{-1}B, \Lambda = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix}$, $P^{-1} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$, $R = \begin{bmatrix} R_1 \\ R_2 \end{bmatrix}$ and p_{11} the $(1,1)$

[20] Assuming A_0 is invertible, otherwise we can generalise this for the case of a non-invertible matrix.

element of P^{-1}. Λ_1 (Λ_2) is the diagonal matrix of stable (unstable) eigenvalues of size n_1 (n_2). The system of equations become:

$$\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} X_t \\ E_t Y_{t+1} \end{bmatrix}$$
$$= \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} X_{t-1} \\ Y_t \end{bmatrix} + \begin{bmatrix} p_{11} a_{11} \Delta s_t \\ 0_{(n_1+n_2-1)\times 1} \end{bmatrix} + \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} \epsilon_t.$$

Making $\widetilde{X}_{t-1} = P_{11}X_{t-1} + P_{12}Y_t$, $\widetilde{Y}_t = P_{21}X_{t-1} + P_{22}Y_t$, the system becomes:

$$\begin{bmatrix} \widetilde{X}_t \\ E_t \widetilde{Y}_{t+1} \end{bmatrix} = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix} \begin{bmatrix} \widetilde{X}_{t-1} \\ \widetilde{Y}_t \end{bmatrix} + \begin{bmatrix} p_{11} a_{11} \Delta s_t \\ 0_{(n_1+n_2-1)\times 1} \end{bmatrix} + \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} \epsilon_t$$

According to Blanchard & Kahn, given that Λ_2 is the diagonal of unstable eigenvalues, the only stable solution is given by: $\widetilde{Y}_t = 0 = P_{21}X_{t-1} + P_{22}Y_t$.

Then, the solution for the forward looking variables is:

$$Y_t = (P_{22})^{-1} P_{21} X_{t-1}. \tag{C.3}$$

The solution for the system of stable (backward looking) equations is:

$$\widetilde{X}_t = \Lambda_1 \widetilde{X}_{t-1} + \begin{bmatrix} p_{11} a_{11} \Delta s_t \\ 0_{(n_1-1)\times 1} \end{bmatrix} + R_1 \epsilon_t \tag{C.4}$$

C.2 Solving the second block

C.2.1 The $MA(\infty)$ representation of the first block

Now we change the classification of endogenous variables in block 1 to focus on those which are part of the minimum state variables (MSV) set. We call these variables Z_t, while the remaining endogenous variables are referred to as Z_t^-. In our case the Z_t is formed by *12* variables as defined in appendix B.

The transition and policy functions can be written as:

$$\begin{bmatrix} Z_t \\ Z_t^- \end{bmatrix} = \begin{bmatrix} W \\ W^- \end{bmatrix} Z_{t-1} + \begin{bmatrix} V \\ V^- \end{bmatrix} \epsilon_t^* \tag{C.5}$$

where $\epsilon_t^{*\prime} = [\epsilon_t', \Delta s_t]$ appends the depreciation rate in the vector of shocks. Evaluating the transition function in $t-1$ and replacing it in (C.5), we have:

$$\begin{bmatrix} Z_t \\ Z_t^- \end{bmatrix} = \begin{bmatrix} W \\ W^- \end{bmatrix} \left(W Z_{t-2} + V\epsilon_{t-1}^*\right) + \begin{bmatrix} V \\ V^- \end{bmatrix} \epsilon_t^*$$

Repeating this process many times, we get:

$$\begin{bmatrix} Z_t \\ Z_t^- \end{bmatrix} = \begin{bmatrix} W \\ W^- \end{bmatrix} \left[(W)^n Z_{t-n-1} + (W)^{n-1} V\epsilon_{t-n}^* + ... + WV\epsilon_{t-2}^* + V\epsilon_{t-1}^*\right] + \begin{bmatrix} V \\ V^- \end{bmatrix} \epsilon_t^*$$

which allows us to write the solution as a $MA(\infty)$:

$$\begin{bmatrix} Z_t \\ Z_t^- \end{bmatrix} = \begin{bmatrix} W \\ W^- \end{bmatrix} \sum_{i=1}^{\infty} (W)^{i-1} V\epsilon_{t-i}^* + \begin{bmatrix} V \\ V^- \end{bmatrix} \epsilon_t^* \tag{C.6}$$

Given the form of matrix W, the impact of shocks diminishes over time, allowing us to approximate the solution using a fixed number of lags. We focus on the solution for i_t in this step and replace it back into (C.2) In our setup i_t^* follows an exogenous process which is easy to express as a function of shocks. Finally, the last term $\gamma\sigma^2\left(\varpi_t^* + \varpi_t^{*,cb}\right)$ is a combination of the conditional volatility term σ^2, the first order autoregressive process of ϖ_t^* and other endogenous variables in the policy rule for $\varpi_t^{*,cb}$, which can also be expressed as a function of shocks.

C.2.2 Conditional moments and solution method

In order to calculate the conditional volatility of the depreciation rate, we need to make use of the strategy proposed by Bacchetta and van Wincoop (2006), based on the Townsend method.

First we conjecture a solution for the depreciation of exchange rate of the form:

$$\Delta s_t = \mathcal{A}(L)\varepsilon_{t+1}^{i^*} + \mathcal{B}(L)\varepsilon_t^{\varpi^*} + \mathcal{D}(L)'\zeta_t \tag{C.7}$$

where $\mathcal{A}(L)$ and $\mathcal{B}(L)$ are infinite order lag polynomials, while $\mathcal{D}(L)$ is an infinite order lag polynomials vector operating ζ_t, the vector of remaining shocks. Writing $\mathcal{A}(L) = a_1 + a_2L + a_3L^2 + ...$ (and a similar definition for $\mathcal{B}(L)$ and $\mathcal{D}(L)$), we evaluate forward the conjecture (C.7) to obtain the value in $t+1$.

$$\Delta s_{t+1} = a_1\varepsilon_{t+2}^{i^*} + b_1\varepsilon_{t+1}^{\varpi^*} + d_1'\zeta_{t+1} + \vartheta'\xi_t + \mathcal{A}^*(L)\varepsilon_t^{i^*} + \mathcal{B}^*(L)\varepsilon_{t-1}^{\varpi^*} + \mathcal{D}^*(L)'\zeta_t$$

where $\xi_t = \left(\varepsilon_{t+1}^{i^*}, \varepsilon_t^{\varpi^*}\right)'$ contains the unobservable innovations, $\vartheta' = (a_2, b_2)$ stands for the parameters associated with these shocks, $\mathcal{A}^*(L) = a_3 + a_4L + ...$ (similar definition for $\mathcal{B}^*(L)$) and $\mathcal{D}^*(L) = d_2 + d_3L + ...$. $\mathcal{A}^*(L)\varepsilon_t^{i^*} + B^*(L)\varepsilon_{t-1}^{\omega^*} + \mathcal{D}^*(L)'\zeta_t$ represents the term of all observable and past known shocks. Taking expectations for dealer d over the previous equations yields:

$$E_t^d(\Delta s_{t+1}) = \vartheta' E_t^d(\xi_t) + \mathcal{A}^*(L)\varepsilon_t^{i^*} + \mathcal{B}^*(L)\varepsilon_{t-1}^{\varpi^*} + \mathcal{D}^*(L)\zeta_t \tag{C.8}$$

while the conditional variance as a function of unobservable innovations is:

$$var_t(\Delta s_{t+1}) = a_1^2 var_t(\varepsilon_{t+2}^{i^*}) + b_1^2 var_t(\varepsilon_{t+1}^{\varpi^*}) + (d_1)' var_t(\zeta_{t+1}) d_1 + \vartheta' var_t(\xi_t)\vartheta. \tag{C.9}$$

Here $\sigma^2 \equiv var_t(\Delta s_{t+1})$ is constant given that $var_t(\xi_t)$ is also constant. In order to obtain the conditional moments we need to obtain the conditional expectation and variance of the unobservable component ξ_t.The computation of the conditional moments is then obtained following Townsend (1983).

FX traders extract information from the observed depreciation exchange rate Δs_t and the signal v_t^d. To focus on the informational content of observable variables, we subtract the known components from these observables and define these new variables as Δs_t^* and v_t^{d*}. We follow the authors' notation, hence in this case the observation equation of this part of the problem is given by:

$$Y_t^d = H'\xi_t + w_t^d \tag{C.10}$$

where $Y_t^d = \left(\Delta s_t^*, v_t^{d*}\right)'$, $w_t^d = (0, \varepsilon_t^{vd})'$, and

$$H' = \begin{bmatrix} a_1 & b_1 \\ 1 & 0 \end{bmatrix}$$

The unconditional means of ξ_t and w_t^d are zero, while we define their unconditional variances as $\widetilde{P}$ and R respectively. Hence we can write:

$$E_t^d(\xi_t) = MY_t^d \tag{C.11}$$

where:

$$M = \widetilde{P}H\left(H'\widetilde{P}H + R\right)^{-1}.$$

For the conditional variance of the unobservable component we have $P \equiv var_t(\xi_t)$, where

$$P = \widetilde{P} - MH'\widetilde{P}. \tag{C.12}$$

Substituting (C.10) and (C.11) in (C.8) and averaging over dealers gives the average conditional expectation of the depreciation rate in terms of the shocks:

$$\overline{E}_t \Delta s_{t+1} = \vartheta' M H' \xi_t + \mathcal{A}^*(L)\varepsilon_t^{i^*} + \mathcal{B}^*(L)\varepsilon_{t-1}^{\varpi^*} + \mathcal{D}^*(L)\zeta_t. \tag{C.13}$$

Moreover, replacing in equation (C.2) the FX intervention policy rule (B.24), the $MA(\infty)$ representation of the endogenous variables (C.6) and the definition of σ^2 from (C.9), we obtain:

$$\overline{E}_t \Delta s_{t+1} = \hat{\imath}_t - \hat{\imath}_t^* + \gamma\sigma^2 \left(\varpi_t^* + \varphi_{\Delta s}\Delta s_t + \varphi_{rer} rer_t + \varepsilon_t^{cb}\right) \tag{C.14}$$

$$\begin{aligned} \overline{E}_t \Delta s_{t+1} = & \mathcal{F}_i(L)\epsilon_t^* - \mathcal{G}(L)\varepsilon_t^{i^*} + ... \\ & + \gamma \left[(a_1^2 var_t(\varepsilon_t^{i^*}) + b_1^2 var_t(\varepsilon_t^{\varpi^*}) + (d_1)' var_t(\zeta_t) d_1 + \vartheta' P \vartheta\right] \\ & \times \left[\mathcal{J}(L)\varepsilon_t^{\omega^*} + \varphi_{\Delta s}\Delta s_t + \varphi_{rer}\mathcal{F}_{rer}(L)\epsilon_t^* + \varepsilon_t^{cb}\right] \end{aligned} \tag{C.15}$$

where $\mathcal{F}_z(L)\epsilon_t^*$ stands for $z_t = \{i_t, rer_t\}$, $\mathcal{G}(L)\varepsilon_t^{i^*}$ for i_t^*, and $\mathcal{J}(L)\varepsilon_t^{\omega^*}$ for ω_t^*. This is the "fundamental equation" $MA(\infty)$ representation.

To solve for the parameters of $\mathcal{A}(L)$, $\mathcal{B}(L)$ and $\mathcal{D}(L)$ we need to match the coefficients from equations (C.13) and (C.15).

C.2.3 Solution of parameters

Now we go through the algebra. Define $z_j^{y,x} \equiv \frac{dy_t}{dx_{t-j+1}}$ as the linear impulse response in the first step of the endogenous variable y_t with respect to the exogenous variable x_{t-j+1}. With this auxiliary variable we identify the parameters multiplying each shock. For this, we use the method of undetermined coefficients comparing equations (C.13) and (C.15).

Solution without rule-based FX intervention For simplicity, we solve first for the parameters assuming first there is no rule-based FX intervention, that is: $\varphi_{\Delta s} = \varphi_{rer} = 0$.

We start taking derivatives to the right-hand side of equations (C.13) and (C.15) with respect to $\varepsilon_t^{i^*}, \varepsilon_{t-1}^{i^*}, ..., \varepsilon_{t-s+3}^{i^*}$, respectively:

$$a_3 = \frac{di_t}{d\varepsilon_t^{i^*}} + \left(\frac{di_t}{d\Delta s_t}\frac{d\Delta s_t}{d\varepsilon_t^{i^*}} + \frac{di_t}{d\Delta s_{t-1}}\frac{d\Delta s_{t-1}}{d\varepsilon_t^{i^*}}\right) - \frac{di_t^*}{d\varepsilon_t^{i^*}}$$
$$a_4 = \frac{di_t}{d\varepsilon_{t-1}^{i^*}} + \left(\frac{di_t}{d\Delta s_t}\frac{d\Delta s_t}{d\varepsilon_{t-1}^{i^*}} + \frac{di_t}{d\Delta s_{t-1}}\frac{d\Delta s_{t-1}}{d\varepsilon_{t-1}^{i^*}} + \frac{di_t}{d\Delta s_{t-2}}\frac{d\Delta s_{t-2}}{d\varepsilon_{t-1}^{i^*}}\right) - \frac{di_t^*}{d\varepsilon_{t-1}^{i^*}}$$
$$\vdots$$
$$a_s = \frac{di_t}{d\varepsilon_{t-s+3}^{i^*}} + \sum_{j=1}^{s-1}\left(\frac{di_t}{d\Delta s_{t+1-j}}\frac{d\Delta s_{t+1-j}}{d\varepsilon_{t-s+3}^{i^*}}\right) - \frac{di_t}{d\varepsilon_{t-s}^{i^*}}$$

In this case the direct effect is zero, because i_t^* only appears in the risk premium-adjusted UIP condition, that is $\frac{di_t}{d\varepsilon_{t-s+3}^{i^*}} = 0$. Then the solution for a_3, a_4, ...is given by:

$$a_s = \sum_{j=1}^{s-1} z_{s-j}^{i,\Delta s} a_j - \rho_{i^*}^{s-3} \qquad \text{for } s \geqslant 3 \tag{C.16}$$

Similarly, taking derivatives with respect to $\varepsilon_{t-1}^{\varpi^*}, \varepsilon_{t-2}^{\varpi^*}, ..., \varepsilon_{t-s+2}^{\varpi^*}$, respectively **:**

$$b_3 = \frac{di_t}{d\varepsilon_{t-1}^{\varpi^*}} + \left(\frac{di_t}{d\Delta s_t}\frac{d\Delta s_t}{d\varepsilon_{t-1}^{\varpi^*}} + \frac{di_t}{d\Delta s_{t-1}}\frac{d\Delta s_{t-1}}{d\varepsilon_{t-1}^{\varpi^*}}\right) + \gamma\sigma^2\frac{d\varpi_t^*}{d\varepsilon_{t-1}^{\varpi^*}}$$
$$\vdots$$
$$b_s = \frac{di_t}{d\varepsilon_{t-s+2}^{\omega^*}} + \sum_{j=1}^{s-1}\left(\frac{di_t}{d\Delta s_{t+1-j}}\frac{d\Delta s_{t+1-j}}{d\varepsilon_{t-s+2}^{i^*}}\right) + \gamma\sigma^2\frac{d\varpi_t^*}{d\varepsilon_{t-s+2}^{i^*}}$$

Similarly to the previous case, the direct effect is zero here, that is $\frac{di_t}{d\varepsilon_{t-s+2}^{\varpi^*}} = 0$. Then the solution for the b_3, b_4, ... is given by:

$$b_s = \sum_{j=1}^{s-1} z_{s-j}^{i,\Delta s} b_j + \gamma\sigma^2\rho_{\omega^*}^{s-2} \qquad \text{for } s \geqslant 3 \tag{C.17}$$

Using the same approach, we take derivatives with respect to $\varepsilon_t, \varepsilon_{t-1}, ..., \varepsilon_{t-s}$ for $\varepsilon \in \zeta$:

$$
\begin{aligned}
d_2^{\varepsilon} &= \frac{di_t}{d\varepsilon_t} + \left(\frac{di_t}{d\Delta s_t}\frac{d\Delta s_t}{d\varepsilon_t}\right) + \gamma\sigma^2\left(\mathcal{I}_{\varepsilon=\varepsilon^{\varpi^*,cb}}\right)\\
d_3^{\varepsilon} &= \frac{di_t}{d\varepsilon_{t-1}} + \left(\frac{di_t}{d\Delta s_t}\frac{d\Delta s_t}{d\varepsilon_{t-1}} + \frac{di_t}{d\Delta s_{t-1}}\frac{d\Delta s_{t-1}}{d\varepsilon_{t-1}}\right) + \gamma\sigma^2\left(\rho_{\varepsilon}\mathcal{I}_{\varepsilon=\varepsilon^{\varpi^*,cb}}\right)\\
&\vdots\\
d_s^{\varepsilon} &= \frac{di_t}{d\varepsilon_{t-s+2}} + \sum_{j=1}^{s-1}\left(\frac{di_t}{d\Delta s_{t+1-j}}\frac{d\Delta s_{t+1-j}}{d\varepsilon_{t-s+2}}\right) + \gamma\sigma^2\left(\rho_{\varepsilon}^{s-2}\mathcal{I}_{\varepsilon=\varepsilon^{\varpi^*,cb}}\right)
\end{aligned}
$$

where $\mathcal{I}_{\varepsilon=\varepsilon^{\varpi^*,cb}}$ is an indicator value of 1 when the shock ε equals $\varepsilon^{\omega^*,cb}$. This system is summarised by:

$$
d_s^{\varepsilon} = z_{s-1}^{i,\varepsilon} + \sum_{j=1}^{s-1} z_{s-j}^{i,\Delta s} d_j + \gamma\sigma^2\left(\rho_{\varepsilon}^{s-2}\mathcal{I}_{\varepsilon=\varepsilon^{\varpi^*,cb}}\right) \tag{C.18}
$$

which is valid for $s \geqslant 2$. Note also that $\frac{di_t}{d\varepsilon_{t-s+2}} = 0$ when $\varepsilon = \varepsilon^{\omega^*,cb}$.

This set of equations (C.16), (C.17) and (C.18) allows us to express the whole system as a function of parameters a_1, a_2, b_1, b_2 and the vector of parameters d_1.

For the two unobservable $\left\{\varepsilon_{t+1}^{i^*}, \varepsilon_t^{\varpi^*}\right\}$ shocks we get:

$$
(\vartheta' M H')_1 = z_1^{i,\Delta s} a_1 \tag{C.19}
$$

$$
(\vartheta' M H')_2 = z_1^{i,\Delta s} b_1 + \gamma\sigma^2 \tag{C.20}
$$

substituting back the values for the matrices, we obtain a non-linear system of equations on the unknowns:

$$
[a_2\ b_2]\, M \begin{bmatrix} a_1 \\ 1 \end{bmatrix} = z_1^{i,\Delta s} a_1 \tag{C.21}
$$

$$
[a_2\ b_2]\, M \begin{bmatrix} b_1 \\ 0 \end{bmatrix} = z_1^{i,\Delta s} b_1 + \gamma\sigma^2 \tag{C.22}
$$

Note that considering (C.21) and (C.22) we have two equations and four unknowns, which hinder us from solving for the system. Bacchetta & van Wincoop (2006), solve the system of difference equations and impose a non-explosive solution, obtaining the two additional restrictions. In our case, it is not possible to obtain analytically a non-explosive solution. Hence we decide to follow a numerical approach by limiting the number of lags and analysing the stability of the solution. The numerical strategy relies then on the convergence of the values of $\mathcal{A}(L)$, $\mathcal{B}(L)$ and $\mathcal{D}(L)$. We set up the non-linear system of equations on the first elements of both infinite lag polynomials and search for a numerical solution using the trust-region-dogleg method implemented by MATLAB. The extra restrictions in our case are given by selecting a limit to the lags and setting the parameters associated with this lag in zero. Since these are functions of the first parameters (the unknowns), we can solve the system and obtain the solution. We change this limit sequentially and obtain solutions in each step. The algorithm stops when a fixed point is achieved, revealing that the inclusion of additional lags has a negligible effect on the result. We set the convergence criteria on **1e-003** . The result will be sensitive to the initial values, since non-linear problems will yield more than one solution. For this reason we start each step using the previous step result as initial point.

The system of equations: We can represent the system of equations using some auxiliary matrices.

The A system

The equations (C.21) and (C.16) can be written as:

$$\begin{bmatrix} a_3 \\ a_4 \\ \vdots \\ a_{n+1} \\ a_{n+2} \end{bmatrix} = \begin{bmatrix} z_1^{i,\Delta s} & 0 & \dots & 0 & 0 \\ z_2^{i,\Delta s} & z_1^{i,\Delta s} & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ z_{n-1}^{i,\Delta s} & z_{n-2}^{i,\Delta s} & \dots & z_1^{i,\Delta s} & 0 \\ z_n^{i,\Delta s} & z_{n-1}^{i,\Delta s} & \dots & z_2^{i,\Delta s} & z_1^{i,\Delta s} \end{bmatrix} \begin{bmatrix} a_2 \\ a_3 \\ \vdots \\ a_n \\ a_{n+1} \end{bmatrix} - \begin{bmatrix} 1 \\ \rho_{i^*} \\ \vdots \\ (\rho_{i^*})^n \\ (\rho_{i^*})^{n-1} \end{bmatrix} + a_1 \begin{bmatrix} z_2^{i,\Delta s} \\ z_3^{i,\Delta s} \\ \vdots \\ z_n^{i,\Delta s} \\ z_{n+1}^{i,\Delta s} \end{bmatrix}$$

These equations can be written in the matrix form, after assuming that the value of $a_{n+2} \longrightarrow 0$:

$$Z_1 A = Z_2^i A - X_{i^*} + a_1 Z_3^{i,\Delta s} \tag{C.23}$$

where $Z_1 = \begin{bmatrix} 0_{(n-1)\times 1} & I_{n-1} \\ 0 & 0_{1\times(n-1)} \end{bmatrix}$, $A = [a_2, ..., a_{n+1}]'$ is a $n \times 1$ vector, Z_2^i is the lower triangular matrix that pre-multiplies A, $X_{i^*} = \left[1, \rho_{i^*}, ..., (\rho_{i^*})^{n-1}\right]'$, and $Z_3^{i,\Delta s} = \left[z_2^{i,\Delta s}, z_3^{i,\Delta s}, ..., z_{n+1}^{i,\Delta s}\right]'$

The B system:

Similarly, equations (C.17) can be written as:

$$Z_1 B = Z_2^i B + \gamma\sigma^2 \rho_{\varpi^*} X_{\varpi^*} + b_1 Z_3^{i,\Delta s} \tag{C.24}$$

where $B = [b_2, b_2, .., b_{n+1}]'$ and $X_{\varpi^*} = \left[1, \rho_{\varpi^*}, ..., (\rho_{\varpi^*})^{n-1}\right]'$.

The D system

In the same vein, the system for $D^{\varepsilon} = [d_1^{\varepsilon}, d_2^{\varepsilon}, .., d_n^{\varepsilon}]'$ is the following

$$\begin{aligned} Z_1 D^{\varepsilon} &= Z_2^i D^{\varepsilon} + Z_3^{\varepsilon} \qquad \text{when } \varepsilon \neq \varepsilon^{\varpi^*,cb} \\ Z_1 D^{\varpi^*,cb} &= Z_2^i D^{\varpi^*,cb} + \gamma\sigma^2 X_{\varpi^*,cb} \quad \text{otherwise} \end{aligned}$$

where $Z_3^{i,\varepsilon} = \left[z_1^{i,\varepsilon}, z_2^{i,\varepsilon}, ..., z_n^{i,\varepsilon}\right]'$ and $X_{\varpi^*,cb} = \left[1, \rho_{\varpi^*,cb}, ..., (\rho_{\varpi^*,cb})^{n-1}\right]'$.

The complete system of equations.

Then, after making use of $Z = Z_1 - Z_2^i$, the total system of non-linear equations become:

$$\begin{aligned} [a_2 \ b_2]\, M \begin{bmatrix} a_1 \\ 1 \end{bmatrix} &= z_1^{i,\Delta s} a_1 \\ [a_2 \ b_2]\, M \begin{bmatrix} b_1 \\ 0 \end{bmatrix} &= z_1^{i,\Delta s} b_1 + \gamma\sigma^2 \\ A &= -Z^{-1}\left(X_{i^*} - a_1 Z_3\right) \\ B &= Z^{-1}\left(\gamma\sigma^2 \rho_{\varpi^*} X_{\varpi^*} + b_1 Z_3\right) \\ D^{\varepsilon} &= Z^{-1} Z_3^{\varepsilon} \\ D^{\varpi^*,cb} &= \left(\gamma\sigma^2\right) Z^{-1} X_{\varpi^*,cb} \\ \sigma^2 &= a_1^2 var_t(\varepsilon_{t+2}^{i^*}) + b_1^2 var_t(\varepsilon_{t+1}^{\varpi^*}) + (d_1)' var_t(\zeta_{t+1}) d_1 + \vartheta' var_t(\xi_t)\vartheta \end{aligned} \tag{C.25}$$

Note the system has $n \times \#$ of shocks+3 equations and unknowns, which only $n \times 2 + 3$ are non-linear equations (those corresponding to the B and $D^{\varpi^*,cb}$ system and the equations for a_1, b_1 and σ^2).

Solution with FX intervention rules When we allow for FX intervention, the equations (C.16), (C.17), (C.18), (C.21) and (C.22) are replaced by:

$$a_s = \sum_{j=1}^{s-1} z_{s-j}^{i,\Delta s} a_j - \rho_{i^*}^{s-3} + \gamma\sigma^2 \left[\varphi_{\Delta s} a_{s-1} + \varphi_{rer} \sum_{j=1}^{s-1} z_{s-j}^{rer,\Delta s} a_j \right] \tag{C.26a}$$

$$b_s = \sum_{j=1}^{s-1} z_{s-j}^{i,\Delta s} b_j + \gamma\sigma^2 \left[\rho_{\omega^*}^{s-2} + \varphi_{\Delta s} b_{s-1} + \varphi_{rer} \sum_{j=1}^{s-1} z_{s-j}^{rer,\Delta s} b_j \right] \tag{C.26b}$$

$$d_s^{\varepsilon} = z_{s-1}^{i,\varepsilon} + \sum_{j=1}^{s-1} z_{s-j}^{i,\Delta s} d_j + \gamma\sigma^2 \left[\begin{array}{c} \rho_{\varpi^*,cb}^{s-2} \mathcal{I}_{\varepsilon=\varepsilon^{\varpi^*,cb}} + \\ \varphi_{\Delta s} d_{s-1} + \varphi_{rer} \sum_{j=1}^{s-1} z_{s-j}^{rer,\Delta s} d_j^{\varepsilon} \end{array} \right] \tag{C.26c}$$

$$[a_2 \; b_2]\, M \begin{bmatrix} a_1 \\ 1 \end{bmatrix} = z_1^{i,\Delta s} a_1 + \gamma\sigma^2 \left(\varphi_{\Delta s} a_1 + \varphi_{rer} z_1^{rer,\Delta s} a_1 \right) \tag{C.26d}$$

$$[a_2 \; b_2]\, M \begin{bmatrix} b_1 \\ 0 \end{bmatrix} = z_1^{i,\Delta s} b_1 + \gamma\sigma^2 \left(1 + \varphi_{\Delta s} b_1 + \varphi_{rer} z_1^{rer,\Delta s} b_1 \right) \tag{C.26e}$$

We can also express this with linear algebra. For example, the A system can be written as:

$$Z_1 A = Z_2^i A + \gamma\sigma^2 \left(\varphi_{\Delta s} I_n + \varphi_{rer} Z_2^{rer} \right) A - X_{i^*} + a_1 \left(Z_3^i + \gamma\sigma^2 \varphi_{rer} Z_3^{rer} \right)$$

Then, after making use of $Z^{FX} = Z_1 - Z_2^i - \gamma\sigma^2 \left(\varphi_{\Delta s} I_n + \varphi_{rer} Z_2^{rer} \right)$, the total system of non-linear equations becomes:

$$\begin{aligned}
[a_2 \; b_2]\, M \begin{bmatrix} a_1 \\ 1 \end{bmatrix} &= z_1^{i,\Delta s} a_1 + \gamma\sigma^2 \left(\varphi_{\Delta s} + \varphi_{rer} z_1^{rer,\Delta s} \right) a_1 \\
[a_2 \; b_2]\, M \begin{bmatrix} b_1 \\ 0 \end{bmatrix} &= z_1^{i,\Delta s} b_1 + \gamma\sigma^2 \left(1 + \varphi_{\Delta s} b_1 + \varphi_{rer} z_1^{rer,\Delta s} b_1 \right) \\
A &= -\left(Z^{FX} \right)^{-1} \left[X_{i^*} - a_1 \left(Z_3^i + \gamma\sigma^2 \varphi_{rer} Z_3^{rer} \right) \right] \\
B &= \left(Z^{FX} \right)^{-1} \left(\gamma\sigma^2 \rho_{\varpi^*} X_{\varpi^*} + b_1 \left(Z_3^i + \gamma\sigma^2 \varphi_{rer} Z_3^{rer} \right) \right) \\
D^{\varepsilon} &= \left(Z^{FX} \right)^{-1} Z_3^{\varepsilon} \\
D^{\varpi^*,cb} &= \left(\gamma\sigma^2 \right) \left(Z^{FX} \right)^{-1} X_{\varpi^*,cb} \\
\sigma^2 &= a_1^2 var_t(\varepsilon_{t+2}^{i^*}) + b_1^2 var_t(\varepsilon_{t+1}^{\varpi^*}) + (d_1)' var_t(\zeta_{t+1}) d_1 + \vartheta' var_t(\xi_t) \vartheta
\end{aligned} \tag{C.27}$$

where $M = \frac{1}{(a_1)^2 \sigma_{i^*}^2 \sigma_v^2 + (b_1)^2 \sigma_{\omega^*}^2 \left(\sigma_{i^*}^2 + \sigma_v^2 \right)} \begin{bmatrix} a_1 \sigma_{i^*}^2 \sigma_v^2 & (b_1)^2 \sigma_{\omega^*}^2 \sigma_{i^*}^2 \\ b_1 \sigma_{\omega^*}^2 \left(\sigma_{i^*}^2 + \sigma_v^2 \right) & -a_1 b_1 \sigma_{i^*}^2 \sigma_{\omega^*}^2 \end{bmatrix}$

and $P = var_t(\xi_t) = \frac{\sigma_{i^*}^2 \sigma_v^2 \sigma_{\omega^*}^2}{(a_1)^2 \sigma_{i^*}^2 \sigma_v^2 + (b_1)^2 \sigma_{\omega^*}^2 \left(\sigma_{i^*}^2 + \sigma_v^2\right)} \begin{bmatrix} (b_1)^2 & -a_1 b_1 \\ -a_1 b_1 & (a_1)^2 \end{bmatrix}$