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Financial Intermediation, Risk Taking and Monetary Policy*

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Abstract

This paper explores the ability of interest rate policy to influence risk taking of financial intermediaries when the riskiness of their individual portfolios is unobserved and influenced by two opposing mechanisms. Low interest rates may lead to more risk taking through a portfolio channel, as intermediaries invest in fewer low-risk, low-return assets and more high-risk, high-return assets. However, low interest rates may also moderate risk taking. In light of new information regarding portfolio risk, high risk intermediaries with limited liability have incentives to adjust their initial investments by trading bonds in repo markets in exchange for additional resources. This borrowing is limited by the amount of safe bonds intermediaries can pledge as collateral, restricting the ability of high risk intermediaries to gamble resources. Hence, low policy rates, may not necessarily induce excessive risk-taking. We calibrate the model to U.S. data and find that, in a neighborhood of the optimal policy, lower interest rates lead to less risk taking as the collateral channel dominates. We extend the model to include private bonds issued by financial intermediaries and rating agencies that misrepresent the riskiness of these bonds. Private bonds weaken the collateral channel and, as the portfolio channel becomes dominant, low interest rates lead to more risk taking and sizable welfare losses.

Keywords: *Risk taking, financial intermediation, optimal monetary policy.*

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1 Introduction

The recent financial crisis has fostered interest in the relationship between monetary policy and the risk taking behavior of financial intermediaries. The conventional view is that low interest rate environments create incentives for intermediaries to take on more risk by shifting towards higher return, but riskier investments. We evaluate this view in a dynamic general equilibrium model in which financial intermediaries' investments into bonds and risky projects are directly influenced by monetary policy. In our baseline framework, low interest rates do not necessarily lead to more risk taking, which we measure as the resources devoted to high risk investments relative to the social optimum. However, augmenting the model to allow for market inefficiencies observed in the run-up to the recent financial crisis—such as the presence of credit rating agencies that facilitated a mispricing of assets—we find that policy rates and risk taking are negatively related. We conclude that the conventional view holds true in our framework with such market inefficiencies.

We consider two channels through which monetary policy can potentially affect risk taking. The *portfolio channel* emphasizes the direct link between interest rates and risk taking. Quantities of bonds purchased are negatively related to bond returns, leading intermediaries to invest more into risky projects at low interest rates. The *collateral channel* allows intermediaries to adjust their portfolios by borrowing against collateral in the repo market. These transactions are limited by the amount of bonds that intermediaries can pledge as collateral. Due to a small amount of bonds purchased in a low interest rate environment, there are fewer collateralized loans possible as further information regarding the riskiness of projects is revealed. This leads to less risk taking. In quantitative experiments, we find that the collateral channel dominates the portfolio channel and thus, the sign of the relationship between the level of interest rates and the risk taking is reversed relative to the conventional view. Namely, lower interest rates are associated with less risk taking by financial intermediaries.

In an extension of our benchmark economy, which allows for the mispricing of risky assets by rating agencies, we show that relaxing the collateral constraint faced by financial intermediaries, can reverse the relationship between the level of interest rates and risk taking. In this economy without binding collateral constraints, lower rates increase risk taking by financial intermediaries at a substantial cost to society.

Formally, we consider a model with incomplete markets in which financial intermediaries use deposits and equity received from households to invest in risky projects and risk free government bonds. Risk taking by intermediaries is influenced through the interest rate of government bonds. In this environment, we address two questions: (a) What is the optimal interest rate policy? (b) What are the consequences of deviating from the optimum? Important features of the environment are the presence of aggregate and idiosyncratic risk and the presence of a secondary bond market in which high and low risk financial intermediaries can privately trade repurchase agreements among themselves. The key imperfections in the model are government-backed deposit insurance, limited liability of financial intermediaries and market segmentation that make financial intermediation necessary to achieve the best outcome. We calibrate the model to match key characteristics regarding economic expansions and contractions, the interest rate policy and the financial sector in the U.S. We find that the first best social planner's problem solution is unattainable in a competitive equilibrium. This is mainly due to the financial sector frictions undermining the optimal portfolio choice of households.

Given that the first best is unattainable, we search for the second best interest rate policy and find that it is very close in welfare terms to the socially optimal resource allocation. Despite this, the second best policy induces too much risk taking by the financial sector. The intuition for the result is the following: to approximate the optimal outcome, the return on government bonds has to decrease sharply in a contraction period. In order to be able to achieve that and still remain close to the optimal portfolio shares, the return to government bonds in the good state needs to be higher than otherwise, thus leading to more risk taking in the good state, while keeping risk taking in the bad state very close to the social planner optimum.

Regarding our second question, we consider various deviations from the second best policy. As an extreme example, we consider the optimal solution in a model economy where there is no capital re-allocation via the repo market. Relative to the no repo market equilibrium, the second best policy leads to less risk taking, as it facilitates resources to flow to low risk intermediaries in contraction periods and constrains the risk taking of high risk intermediaries in expansion periods. Thus, the presence of the repo market may yield the benefit of greater stability in the economy. Next, we consider small symmetric deviations from the optimal interest rate policy. This analysis exposes that lower than optimal interest rates lead to less risk taking in a good economic state, but more risk taking in a bad economic state. The opposite is true for moderately higher than optimal interest rates. However, large upward deviations relative to the optimum, induce non-linear effects that can change the sign of the relationship between risk taking and the level of interest rates. Key for all of these results is that the optimal solution requires a situation, where financial intermediaries are constrained in their ability to trade bonds in the secondary market by the amount of collateral they have available. Thus, in a good state the optimal policy restricts the amount of risk taken by the high risk intermediary, and in the bad state it aims to facilitate a trade of resources from the high risk to the low risk intermediary. To do this the interest rate has to be very low, otherwise the high risk intermediaries will still be tempted to speculate. In welfare terms, we find that a lower than optimal interest rate is less costly, than an equivalently higher than optimal interest rate.

Up to now, we described the results for an economy in normal circumstances and established that optimal interest policy using the collateral channel can decrease risk taking even at low interest rate levels. To better understand the special circumstances of the recent crisis, we augment our model by introducing rating agencies who, in exchange for a fee, stamp bonds backed by high risk projects as viable safe collateral in the repo market. This market imperfection is essential for the reversal of our standard result. Beyond this feature, we allow for foreign demand for domestic bonds, both privately and government issued once, at the government set policy rate. Under these circumstances the previously binding collateral constraint becomes undone and lower interest rates make the problem of excessive risk taking by high risk intermediaries more sever. We find that the welfare consequences can be quite severe under these circumstances.

Our paper contributes to the growing literature studying the risk taking channel of monetary policy, as discussed for example in Borio and Zhu (2008) and Gambacorta (2009).[1] While initially this strand of the literature was motivated mainly by the recent financial crisis and the subsequent recession, now there have been attempts to empirically document the link between monetary policy, especially the prolonged periods of low interest rate, and the risk taking of financial intermediaries. The evidence is suggestive that such a link exists, though the strength of the channel is still under dispute.[2] Our paper contributes to this literature by measuring the importance of the risk taking

[1] A few other papers in this literature are: Acharya and Naqvi (2004), Diamond and Rajan (2009), Goodhart, Sunirand, and Tsomocos (2006)

[2] Ioannidou, Ongena, and Peydró (2009) show for Bolivia that low interest rates lead to more risky loans and lower interest rates for these loans. Jiménez, Ongena, Peydró, and Saurina (2009) consider the Spanish situation and find evidence that lower interest rates are associated with more risky loans in the short term and softer lending

channel for the United States in a quantitative general equilibrium model. We extend the idea put forth in Rajan (2006), namely that a lower interest rate decreases the resources invested in government bonds relative to risky project thus increasing the risk taking of the financial sector. One of our contributions in this regard is to show how interest rate policy may affect risk taking via its effects on collateralized trading in a secondary bond market which takes place as more information regarding the riskiness of the projects becomes available. We show that this *collateral channel* of monetary policy in fact dominates the *portfolio channel* in a neighborhood of the optimal interest rate policy.

Regarding the model side of our paper, Agur and Demertzis (2010) develop a dynamic model that also incorporates the portfolio channel present in our model. They show that a central bank that cares about financial stability sets higher average interest rates and reduces the rate sharply in a crisis, though keeping the periods of low interest rates short. Their paper is theoretical in nature, but finds qualitatively similar implications to what we find. Another paper complementary to our work is Giavazzi and Giovannini (2010).

The paper is organized as follows. Section 2 presents the model and some key results. Section 3 outlines the methods we use to pin down the parameters in our model. Given the parameters, we conduct various experiments in section 4 and derive the main results of the paper. Section 5 summarizes the main findings.

2 Model description

The economy is populated by a measure one of identical households, a measure π_m of identical nonfinancial firms, a measure $1-\pi_m$ of financial intermediaries and a government. Time is discrete and infinite. Each period, the economy is subject to an exogenous aggregate shock which affects the productivity of all firms, as outlined below. The aggregate state $s_t \in \{\overline{s}, \underline{s}\}$ follows a first-order Markov process.

Financial and nonfinancial firms differ in the way they are funded, in the types of investments they make and the productivity of these investments. Nonfinancial firms finance their operations

standards in the medium term. Delis and Kouretas (2010) and Altunbas, Gambacorta, and Marques-Ibane (2010) consider banks in the European Unions, the latter paper also considers the situation for the United States. Both find evidence for of the risk taking channel of monetary policy.

through household equity only.[3] All equity raised is invested into capital whose return depends on the productivity of the production technology in the nonfinancial sector, $q_m(s_t)$. Financial intermediaries finance their operations through household equity and deposits. The main difference between these two forms of funding is that equity returns are contingent on the realization of the aggregate state s_t, while returns to deposits are independent of s_t and guaranteed by deposit insurance. Intermediaries invest into safe government bonds and risky projects.

Key for our model's mechanism is the determination of risky investments by financial intermediaries. Initially, these intermediaries are identical and receive the same amount of equity and deposits from households and thus make the same investments into government bonds and risky projects. Investments are potentially subject to financial regulation which requires a minimum amount of equity for every unit of risky investment to provide some buffer for potential losses. After the initial investment decisions, intermediaries acquire more information about their risky projects. With probability π_h an intermediary has a high risk project with productivity $q_h(s_t)$ and with probability $\pi_l = 1 - \pi_h$ an intermediary has a low risk project with productivity $q_l(s_t)$. High risk intermediaries are more productive in an expansionary state of the economy, but are less productive in a contractionary state of the economy, compared to low risk intermediaries. Formally, the assumption we are making is that $q_h(\overline{s}) > q_l(\overline{s}) \geq q_l(\underline{s}) > q_h(\underline{s})$. Once $j \in \{h, l\}$ is known, but before the realization of s_t, intermediaries trade bonds in a secondary bond market in order to adjust the amount of resources invested into the risky projects. Transactions in the secondary market are observable only by intermediaries and can be interpreted as bilateral repurchasing agreements. As a result, financial intermediaries may violate the financial regulation constraint. If they do, this is only revealed in case of bankruptcy. More details on the timing of events in our model are presented in Section 2.2 and in Section A of the Appendix.

2.1 Households

There is a measure one of identical households. At the beginning of period t, the aggregate state s_t is revealed and households receive returns on their previous period investments, wage income and lump-sum taxes or transfers from the government. The resulting wealth, $w(s^t)$, is then split

[3] The important assumption here is that the nonfinancial sector is funded through state contingent claims. We use equity for simplicity, but we could also allow for state contingent corporate bonds.

between current consumption and investments that will pay returns in period $t+1$.

Investments take the form of deposits, nonfinancial sector equity and financial sector equity. Deposits, $D_h\left(s^{t-1}\right)$, earn a fixed return, $R^d\left(s^{t-1}\right)$, which is guaranteed by deposit insurance. Equity invested in financial intermediaries, $Z\left(s^{t-1}\right)$, is a risky investment which gives households a claim to the profits of the intermediaries. The return per unit of equity is $R^z\left(s^t\right)$. Similarly, the equity investment into the nonfinancial sector, $M\left(s^{t-1}\right)$, entitles the household to state contingent returns, $R^m\left(s^t\right)$.

Households supply labour inelastically. We assume that labour markets are segmented.[4] Fraction π_m of a household's time is spent working in the nonfinancial sector, and fraction $1-\pi_m$ is spent in the financial sector. Wage rates are conditional on the sector, the type of firm within the sector and the aggregate state of the economy. $W_m\left(s^t\right)$ is the wage rate paid by nonfinancial firms given history s^t, while $W_j\left(s^t\right)$ is the wage rate paid by financial intermediary of type $j\in\{h,l\}$. As a result of these assumptions, we can normalize labour supplied to each firm to one unit for any realization of the aggregate state.

The household's problem is given by:

$$\max\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\varphi\left(s^t\right)\log C\left(s^t\right)$$

$$\begin{aligned}
\text{subject to} &\quad : \\
w\left(s^t\right) &= R^m\left(s^t\right)M\left(s^{t-1}\right)+R^d\left(s^{t-1}\right)D_h\left(s^{t-1}\right)+R^z\left(s^t\right)Z\left(s^{t-1}\right) \\
&\quad +\pi_m W_m\left(s^t\right)+\left(1-\pi_m\right)\pi_l W_l\left(s^t\right)+\left(1-\pi_m\right)\pi_h W_h\left(s^t\right)+T\left(s^t\right) \\
w\left(s^t\right) &= C\left(s^t\right)+M\left(s^t\right)+D_h\left(s^t\right)+Z\left(s^t\right)
\end{aligned}$$

where $\varphi\left(s^t\right)$ is the probability of history s^t, $C\left(s^t\right)$ is consumption, $T\left(s^t\right)$ are lump-sum transfers if $T\left(s^t\right)\geq 0$ or lump-sum taxes otherwise, π_j with $j\in\{h,l\}$ is the probability of working for financial intermediary of type j where $\pi_h+\pi_l=1$ and $M\left(s^t\right), D_h\left(s^t\right)$ and $Z\left(s^t\right)$ are investments that pay off in period $t+1$.

Since households own all firms in the economy, the household's problem determines the aggregate

[4] The assumption of a labour market segmentation is done for convenience. Relaxing this assumption to allow labour to move across firms and sectors, leads to a reenforcement of the risk taking channel present in our model as both capital and labour flow in the same direction.

state valuation system used to price risky returns by both the financial and the nonfinancial sector:

$$\lambda\left(s^t\right)=\frac{\varphi\left(s^t\right)}{C\left(s^t\right)}.$$

2.2 Financial sector

There is a measure $1-\pi_m$ of financial intermediaries. The defining feature of financial intermediaries is that they are funded through both deposits and equity. Deposits are claims to a return which is independent of the s_t realization and are backed by government deposit insurance. Equity promises state contingent returns. Equity holders are the residual claimants to an intermediary's profits, but due to limited liability of the intermediaries, equity returns are bounded below by zero.

The problem of an intermediary is to choose investments in safe bonds and risky projects that maximize the expected value of its equity. Initially, all financial intermediaries are identical: they all purchase an equal amount of bonds in the primary bond market and devote the remainder of their financial resources to risky projects. We refer to this as the first stage of an intermediary's problem.

Risky projects are of two different types $j\in\{h,l\}$, where h denotes high risk and l denotes low risk. In the second stage, intermediaries find out the type of their risky project. The probability of having a project of riskiness $j\in\{h,l\}$, π_j is time and state invariant and known. We refer to intermediaries as being high risk intermediaries or low risk intermediaries, based on the type j of their risky projects. The production technology for an intermediary of type j given the history $s^t=\left(s^{t-1},s_t\right)$ of the exogenous aggregate shock is given by $q_j\left(s_t\right)\left(k_j\left(s^{t-1}\right)\right)^{\theta}\left(l\left(s^{t-1}\right)\right)^{1-\theta-\alpha}$, where parameters θ and α satisfy $\alpha,\theta\in[0,1]$, $1-\alpha-\theta\geq 0$. Here, $q_j\left(s_t\right)$ is the productivity of an intermediary of type j given realization s_t of the aggregate state, $k_j\left(s^{t-1}\right)$ is the amount of resources invested in the risky project and $l\left(s^{t-1}\right)$ is the amount of labour employed. Note that capital and labor are both chosen prior to the revelation of the shock s_t. If $\alpha>0$ there is a fixed factor present in the production process. This factor's returns are payable to the equity holders in the absence of bankruptcy and to the depositors otherwise.

The important difference between the two intermediaries is their productivity given the different aggregate state $s_t\in\{\overline{s},\underline{s}\}$. We are focusing on the case where one intermediary is facing lower risk than the other intermediary regarding its productivity in the different states. The fundamental assumption we are making is: $q_h(\overline{s}) > q_l(\overline{s}) \geq q_l(\underline{s}) > q_h(\underline{s})$.

We now describe the two stages of a financial intermediary's problem that take place during period $t-1$. This shows how capital that is used for production during period t is determined.

First stage

After production in period $t-1$ has taken place, intermediaries receive resources from households and make investment decisions that will pay off in t. Financial intermediaries don't know their type and maximize the expected return to equity conditional on what they will do at the second stage, i.e. once they know their type. Throughout, they take the welfare weights of the equity holders, $\lambda\left(s^t\right)$, into account when maximizing their profits, $V_j\left(s^t\right)$.

Taking the amount of equity $z\left(s^{t-1}\right)$ issued by an intermediary as given, the first stage problem of an intermediary is to choose $k\left(s^{t-1}\right), b\left(s^{t-1}\right), d\left(s^{t-1}\right), l\left(s^{t-1}\right)$ that solve:

$$\max \sum_{j\in\{h,l\}} \pi_j \sum_{s^t|s^{t-1}} \lambda\left(s^t\right) V_j\left(s^t\right) \tag{P1}$$

subject to:

$$z\left(s^{t-1}\right) + d\left(s^{t-1}\right) = k\left(s^{t-1}\right) + p\left(s^{t-1}\right) b\left(s^{t-1}\right) \tag{1}$$

$$V_j\left(s^t\right) = \max \left\{ \begin{array}{l} q_j\left(s_t\right)\left[k\left(s^{t-1}\right) + \tilde{p}\left(s^{t-1}\right)\tilde{b}_j\left(s^{t-1}\right)\right]^{\theta}\left[l\left(s^{t-1}\right)\right]^{1-\theta-\alpha} \\ +q_j\left(s_t\right)(1-\delta)\left[k\left(s^{t-1}\right) + \tilde{p}\left(s^{t-1}\right)\tilde{b}_j\left(s^{t-1}\right)\right] \\ +\left[b\left(s^{t-1}\right) - \tilde{b}_j\left(s^{t-1}\right)\right] - R^d\left(s^{t-1}\right) d\left(s^{t-1}\right) - W_j\left(s^t\right) l\left(s^{t-1}\right), 0 \end{array} \right\} \tag{2}$$

$$z\left(s^{t-1}\right)/k\left(s^{t-1}\right) \geq \eta$$

The intermediary decides on the demand for deposits, $d\left(s^{t-1}\right)$, the split of its total resources, between safe investment in government bonds, $b\left(s^{t-1}\right)$, and risky investments, $k\left(s^{t-1}\right)$. Finally, the intermediary decides on the amount of labour it wishes to hire, $l\left(s^{t-1}\right)$. The compensation of labour is such that a type and state contingent contract is offered to the households, paying a wage $W_j\left(s^t\right) > 0$. The price of government bonds in the primary market is $p\left(s^{t-1}\right)$ and in the secondary market is $\tilde{p}\left(s^{t-1}\right)$. The amount of bonds traded in the secondary market is $\tilde{b}_j\left(s^{t-1}\right)$ and taken as given at the first stage of the problem. At this stage there exists a capital requirement constraint that states that the equity per unit of risky investment has to be larger than η. This constraint captures some of the requirements imposed by regulations stemming from the Basel II accord. The return to the equity holder is subject to a limited liability constraint. The order of payment is first to labour, $W_j\left(s^t\right) l\left(s^{t-1}\right)$, then to the depositors, $R^d\left(s^{t-1}\right) d\left(s^{t-1}\right)$, and finally to the equity holders, $V_j\left(s^t\right)$.

We abstract from labour redistribution across intermediaries after their risk-type is revealed and thus all of them have an equal amount of labor $l\left(s^{t-1}\right)$ which is normalized to 1. Notice that the undepreciated capital stock takes into account the productivity of the capital. In this regard, we consider the capital stock as being not only a means of production, but also a store of value. In contraction periods, the capital while not depreciating in a physical sense does so in an economic sense. This is especially true in the case of a bankruptcy and subsequent restructuring.[5]

Given the solution to this problem, the intermediaries enter the second stage.

Second stage

At this stage, intermediaries know their risk type, and are able to reallocate financial resources through a market for repurchasing agreements with 100% coverage of collateral. The repo market is shown to be an important margin of balance sheet adjustment by intermediaries, see, for example, Adrian and Shin (2010). The intermediaries' problem is to choose trades in the secondary bond market $\tilde{b}_j\left(s^{t-1}\right)$ that solve:

$$\max \sum_{s^t|s^{t-1}} \lambda\left(s^t\right) V_j\left(s^t\right) \tag{P2}$$

where $V_j\left(s^t\right)$ is given in equation (2) and $\tilde{b}_j\left(s^{t-1}\right) \in \left[-\frac{k\left(s^{t-1}\right)}{\tilde{p}\left(s^{t-1}\right)}, b\left(s^{t-1}\right)\right]$. The main choice is between increasing or decreasing the safe investment, $\tilde{b}_j\left(s^{t-1}\right)$, through trades in the secondary bond market at price $\tilde{p}\left(s^{t-1}\right)$.

There are two alternative interpretations of the trades taking place in the secondary market. The first one is that of a sale of bonds. The second one is that of a repurchasing agreement. Under the second interpretation, which we are following in this paper, some intermediaries, call them A^s, pledge their government bonds as collateral and receive extra resources from the other intermediaries A^b. These extra resources can be used for risky investment. There are two possible

[5] We aren't the first to introduce this feature into a stochastic general equilibrium model, see for example Gertler and Kiyotaki (2010) and Gertler, Kiyotaki, and Queralto (2010).

outcomes after the aggregate state is revealed. In case one, the intermediary A^s has insufficient profits to pay wages, the depositors and the other intermediaries A^b. In that case the intermediary declares bankruptcy and intermediaries A^b use the collateral to obtain returns. In the other case, intermediary A^s is able to satisfy all the obligations including those to the intermediaries A^b. Then the intermediary will cash in the bonds on his own.[6] We assume that the trading at this stage is not observable by the regulatory authority and thus only in the case of bankruptcy will the trade be revealed.

A key feature of the second stage problem is that intermediaries can only use as many bonds as they bought in the primary bond market, $b\left(s^{t-1}\right)$, as collateral.[7] This means that their ability to increase their risky investment is limited by their primary market activities. Put differently, higher primary bond market activities potentially lead to more secondary market risk taking despite the better visible balance sheets.[8] This also means that there are two possibilities: intermediaries can either collateralize a subset of their bonds, or they can use all their bonds as collateral which leaves them without any insurance against a potential negative productivity shock.

2.3 Nonfinancial sector

There are π_m identical nonfinancial firms. The nonfinancial sector is funded only through household equity which pays a state contingent return. Each nonfinancial firm enters period t with equity $M\left(s^{t-1}\right)/\pi_m$ from households which is invested into capital. Hence $M\left(s^{t-1}\right)/\pi_m = k_m\left(s^{t-1}\right)$. The problem of a nonfinancial intermediary is to choose capital and labor to produce output. The returns to capital and payments to labor are contingent on the realization of the aggregate state at time $t: s_t$.

$$\max\left\{y_m\left(s^t\right) + q_m\left(s_t\right)(1-\delta)\,k_m\left(s^{t-1}\right) - R^m\left(s^t\right)k_m\left(s^{t-1}\right) - W_m\left(s^t\right)l_m\left(s^{t-1}\right)\right\}$$

$$\text{subject to:} \qquad y_m\left(s^t\right) = q_m\left(s_t\right)\left(k_m\left(s^{t-1}\right)\right)^{\theta}\left(l_m\left(s^{t-1}\right)\right)^{1-\theta}$$

Notice that the return $R^m\left(s^t\right)$ is net of depreciation.

[6] In equilibrium, a financial intermediary that is not bankrupt will always be able to buy the bonds back.

[7] The assumption of 100% collateralization can be relaxed without qualitatively changing our results.

[8] A case study for example of Citigroup over the last decade reveils a significant exposure to off-balance sheet risk that was nearly invisible till after the the risk had been reveiled.

We introduce this sector mainly to allow for an alternative form of investment to equity in the financial sector. While the nonfinancial sector has significant quantitative implications and simplifies the calibration of the model, it is of minor importance for the qualitative properties of our model.

2.4 Government

The government issues bonds that the financial intermediaries can use either as an asset or as a medium of exchange on a secondary bond market. At the end of period $t-1$, the government sold bonds, $B\left(s^{t-1}\right)$, at price, $p\left(s^{t-1}\right)$. These bonds pay off during period t. Part of the proceeds from the bond sales was used to cover a proportional cost, τ, of issuing bonds, while the remainder was transformed into deposits.[9] Each financial intermediary received government deposits, $D_g\left(s^{t-1}\right)/\left(1-\pi_m\right)$, given by:

$$D_g\left(s^{t-1}\right) = \left(1-\tau\right) p\left(s^{t-1}\right) B\left(s^{t-1}\right)$$

During period t, to guarantee the fixed return on deposits the government provides deposit insurance at zero price which is financed through household taxation.[10] The government balances its budget after the production takes place at the beginning of period t :

$$T\left(s^{t}\right) + B\left(s^{t-1}\right) + \Delta\left(s^{t}\right) = R^d\left(s^{t-1}\right) D_g\left(s^{t-1}\right)$$

Here, $\Delta\left(s^{t}\right)$ is the amount of deposit insurance necessary to guarantee the fixed return on deposits, $R^d\left(s^{t-1}\right)$. Given the limited liability of intermediaries, if they are unable to pay $R^d\left(s^{t-1}\right)$ on deposits, they pay a smaller return on deposits which ensures they break-even. The rest is covered by deposit insurance.[11]

[9] Alternatively, the proceeds from the bond sales could be handed to the households via transfers. Our results would be unaffected by such a change.

[10] The assumption of a zero price of deposit insurance is not important for our purpose. What matters is that the insurance is not priced in a way that eliminates moral hazard. This means that, for example, deposit insurance can not be made contingent on the portfolio decisions of the intermediaries.

[11] Formally, the amount of deposit insurance is given by:

$$\Delta\left(s^{t}\right) = \left(1-\pi_m\right)\cdot\max\left(0, R^d\left(s^{t-1}\right) d\left(s^{t-1}\right) - \sum_{j\in\{l,h\}} \pi_j \tilde{R}_j^d\left(s^{t}\right) d\left(s^{t-1}\right)\right)$$

The main policy instrument is the price of government bonds on the primary market, $p\left(s^{t-1}\right)$. The government satisfies any demand for bonds given this price. The key decision from the government's perspective is to choose the bond price $p\left(s^{t-1}\right)$ that maximizes the welfare of the households in the decentralized economy.

2.5 Market clearing

There are eight market clearing conditions. The *labor market clearing conditions* state that labor demanded by financial intermediaries and nonfinancial firms equals labor supplied by households:

$$\begin{aligned}(1-\pi_m)\, l\left(s^{t-1}\right) &= 1-\pi_m \\ \pi_m l_m\left(s^{t-1}\right) &= \pi_m\end{aligned}$$

The *goods market clearing condition* equates total output produced with aggregate consumption and investment. Output produced by nonfinancial firms is $\pi_m q_m\left(s^t\right)\left(k_m\left(s^{t-1}\right)\right)^{\theta}$, while output produced by financial firms is $(1-\pi_m)\sum_{j\in\{l,h\}}\pi_j q_j\left(s^t\right)\left(k_j\left(s^{t-1}\right)\right)^{\theta}$, where $k_j\left(s^{t-1}\right)$ are resources allocated to the risky projects after secondary market trading.

$$\begin{aligned}C\left(s^t\right)+M\left(s^t\right)+D_h\left(s^t\right)+Z\left(s^t\right) &= \pi_m q_m\left(s_t\right)\left[\left(k_m\left(s^{t-1}\right)\right)^{\theta}+(1-\delta)\,k_m\left(s^{t-1}\right)\right] \\ &\quad +(1-\pi_m)\sum_{j\in\{l,h\}}\pi_j q_j\left(s_t\right)\left[\left(k_j\left(s^{t-1}\right)\right)^{\theta}+(1-\delta)\,k_j\left(s^{t-1}\right)\right]\end{aligned}$$

Financial markets clearing conditions ensure that the deposit markets, equity markets and bond markets clear. Deposits demanded by financial intermediaries equal deposits from the households and the government:

$$D_h\left(s^{t-1}\right)+D_g\left(s^{t-1}\right)=D\left(s^{t-1}\right)=(1-\pi_m)\,d\left(s^{t-1}\right)$$

In the primary bond market, total bond sales by the government equal the bond purchases by

where $\tilde{R}_j^d\left(s^t\right)$ is the gross return that intermediary $j\in\{h,l\}$ is able to pay once the returns from risky investments are realized. Here, $\tilde{R}_j^d\left(s^t\right)\equiv\min\left\{R^d\left(s^{t-1}\right),\ I_j\left(s^t\right)/d\left(s^{t-1}\right)\right\}$ and $I_j\left(s^t\right)=q_j\left(s_t\right)\left[\left(k\left(s^{t-1}\right)+\tilde{p}\left(s^{t-1}\right)\tilde{b}_j\left(s^{t-1}\right)\right)^{\theta}+(1-\delta)\left(k\left(s^{t-1}\right)+\tilde{p}\left(s^{t-1}\right)\tilde{b}_j\left(s^{t-1}\right)\right)\right]+\left(b\left(s^{t-1}\right)-\tilde{b}_j\left(s^{t-1}\right)\right)-W_j\left(s^t\right)l\left(s^{t-1}\right)$. Note that $I_j\left(s^t\right)\geq 0$.

financial intermediaries.

$$B\left(s^{t-1}\right)=\left(1-\pi_m\right)b\left(s^{t-1}\right)$$

In the secondary bond market, trades between the different types of intermediaries must balance.

$$\sum_{j\in\{l,h\}}\pi_j\tilde{b}_j\left(s^{t-1}\right)=0 \tag{3}$$

Total equity invested by households in the financial and nonfinancial sectors are distributed over the firms.

$$\begin{aligned} M\left(s^{t-1}\right) &= \pi_m k_m\left(s^{t-1}\right) \\ Z\left(s^{t-1}\right) &= \left(1-\pi_m\right)z\left(s^{t-1}\right) \end{aligned}$$

2.6 Social planner problem with costly reallocation

In order to have a reference, we now look at a social planner's problem without the distortions present in the decentralized competitive equilibrium. At the beginning of period t, production takes place using capital that the social planner has allocated to nonfinancial firms, $k_m\left(s^{t-1}\right)$, high risk financial intermediaries, $k_h\left(s^{t-1}\right)$, and low risk financial intermediaries, $\left(k_l\left(s^{t-1}\right)\right)$.[12] The wealth $w\left(s^t\right)$ is then split between consumption and resources allocated to risky investments. At the time of this decision the social planner does not know the type of financial intermediaries and allocates the same resources, $k\left(s^t\right)$, to all of them. Once their type is revealed the social planner reallocates resources between high risk and low risk intermediaries but incurs a cost for

[12] From the social planner's perspective there are no firms, only technologies. In order to relate to the competitive equilibrium, we stretch the language and for example refer to the technology used by the nonfinancial sector as nonfinancial firms, even in the social planner context.

this adjustment. The resulting capital is used for production in period $t+1$.

$$
\begin{aligned}
& \max E \sum_{t=0}^{\infty} \beta^t \log C\left(s^t\right) \\
\text{subject to} \quad & : \\
C\left(s^t\right)+\pi_m k_m\left(s^t\right)+\left(1-\pi_m\right) k\left(s^t\right) & = \pi_m q_m\left(s_t\right)\left[\left(k_m\left(s^{t-1}\right)\right)^\theta+(1-\delta) k_m\left(s^{t-1}\right)\right] \\
& +\left(1-\pi_m\right) \pi_l q_l\left(s_t\right)\left[\left(k_l\left(s^{t-1}\right)\right)^\theta+(1-\delta)\left(k_l\left(s^{t-1}\right)\right)\right] \\
& +\left(1-\pi_m\right) \pi_h q_h\left(s_t\right)\left[\left(k_h\left(s^{t-1}\right)\right)^\theta+(1-\delta) k_h\left(s^{t-1}\right)\right] \\
k_l\left(s^t\right) & = k\left(s^t\right)-\left(\frac{\pi_h}{\pi_l}+\iota_n\left(s^t\right) \tau\right) n\left(s^t\right) \\
k_h\left(s^t\right) & = k\left(s^t\right)+\left(1-\iota_n\left(s^t\right) \tau\right) n\left(s^t\right)
\end{aligned}
$$

where

$$
\iota_n\left(s^t\right)=\left\{\begin{array}{ccc}
1 & \text{if} & n \geq 0 \\
-1 & \text{if} & n<0
\end{array}\right\} .
$$

The variable n captures the amount of resources redistributed from one financial intermediary to another one. The costs of reallocation in this setup are τn, where τ is identical to the corresponding parameter in the competitive equilibrium. The reason for including ι_n is to allow for redistribution both from the high risk to the low risk intermediary's technology and vice versa without loosing the cost aspect. Without this indicator function a negative n would lead to windfalls instead of a cost.

2.7 Competitive Equilibrium Properties

In this section, we characterize the equilibrium of our model and present results on the relationship between equilibrium bond prices and the return to deposits. In addition, we propose a method for measuring the risk taking behavior of financial intermediaries and provide intuition for how interest rate changes affect risk.

Constrained and Unconstrained Equilibria

The model presented in Section 2 has several key features among which are the limited liability of financial intermediaries and the presence of the secondary bond market. These features allow for bankruptcy to occur in equilibrium, and for the riskiness of intermediaries' portfolios to be adjusted through repurchasing agreements.

Financial intermediaries maximize expected returns to equity, but benefit from limited liability. When a bad productivity shock occurs, intermediaries who are unable to pay the promised rate of return to depositors declare bankruptcy. Equity holders receive no return on their investments, while the returns to depositors are covered by deposit insurance. Limited liability introduces an asymmetry in that it allows the high risk intermediary to make investment decisions that bring high profits in good times, while being shielded from losses in bad times. In our numerical experiments, only the high risk intermediaries go bankrupt.

The redistribution of resources that takes place through the secondary bond market allows financial intermediaries to change their risk exposure in light of new information obtained about their investments. Intermediaries who use bonds as collateral in the secondary market increase the amount of resources allocated to risky investments. By the same token, intermediaries who give resources against the bond collateral decrease their risk exposure. From a social planner's perspective, it is optimal for resources to flow to high risk intermediaries during expansion periods and to low risk intermediaries during contractions. To induce these types of reallocation flows in the competitive equilibrium, bond prices need to be appropriately chosen. They should be relatively low in good times and high in bad times.

The extent of reallocation through the secondary market is limited by the bond holdings of each intermediary. Nonetheless, a financial intermediary who seeks to increase his exposure to risky investments can choose to pledge a fraction of his bonds as collateral, i.e $\tilde{b}_j\left(s^t\right) < b\left(s^t\right)$. We refer to these equilibria as having an *unconstrained secondary bond market*. Equilibria with a *constrained secondary bond market* are ones in which either the high risk or the low risk intermediaries pledge all their bond holdings as collateral.

For a given monetary policy, $p\left(s^t\right)$, multiple equilibria exist. A common situation is the co-existence of an equilibrium with positive government bond holdings and an equilibrium with zero bond holdings.[13] We focus our analysis on the former. In addition, we consider equilibria with a constrained secondary bond market. In these equilibria, intermediaries use their bonds as collateral

[13] This is not uncommon in the literature. For example in Overlapping Generations Models with money both trade equilibria and an autarky equilibrium coexist.

in the secondary market. As long as the cost of issuing bonds is positive, this is the only resource efficient way use of government bonds.

The limited liability of financial intermediaries and the constrained secondary bond market require non-linear techniques to solve for the equilibrium.

Bond Prices and the Return to Deposits

Proposition 1 *Consider an economy with positive government bond holdings. In the absence of capital regulation or if this regulation does not bind, the equilibrium bond prices and the return to deposits satisfy:* $p\left(s^{t-1}\right)=\tilde{p}\left(s^{t-1}\right)$ *and* $R^d\left(s^{t-1}\right)\geq\frac{1}{p(s^{t-1})}$. *The last inequality is strict in the case of a constraint secondary bond market. Moreover, in an equilibrium with binding capital regulation, bond prices and return to deposits are such that:* $p\left(s^{t-1}\right)>\tilde{p}\left(s^{t-1}\right)$ *and* $R^d\left(s^{t-1}\right)\geq\frac{1}{p(s^{t-1})}$.

Proof. These results follow from the first order conditions of the financial intermediaries' problems. Appendix B provides a sketch of the proof. ■

The intuition for these results are as follows. In the absence of financial regulation, there are no frictions in the model that would make primary and secondary bond prices different. When financial regulation binds and intermediaries are required to hold a minimum share of safe assets, they are only willing to purchase additional bonds at a price lower than in the primary market. In addition, returns to deposits are weakly greater than returns to bonds, since otherwise there would be a profit opportunity for the intermediary willing to pay a bit more to its depositors.

Proposition (1) is important for two reasons. First, it shows that as long as capital regulation does not constrain the choices financial intermediaries make, interest rate policy has a direct effect on the secondary bond market. Second, the return to depositors is bounded below by the implicit interest rate of government bonds. Thus, the interest rate policy not only affects the choices financial intermediaries make, but also affects the investment choices of households. In the quantitative experiments, we find the latter effect is weaker than the former.

2.7.1 Measuring Risk Taking Behavior

A natural question in our setup is: How does policy influence risk taking in the economy? To address this question, we first make the notion of risk taking precise.

Risk taking is defined as the percentage deviation of the amount of resources invested in the high risk project in a competitive equilibrium relative to the social planner's choice. Formally,

$$r = E\left[\frac{k_{h,t-1}^{CE} - k_{h,t-1}^{SP}}{k_{h,t-1}^{SP}}\right]$$

where we have used the same notational convention as before: $k_{j,t-1}^{i} \equiv k_{j}^{i}\left(s^{t-1}\right)$ for $j \in \{h, l\}$ and where the superscript $i \in \{CE, SP\}$ denotes whether the variable is the solution to the competitive equilibrium for a given interest rate policy or the social planners problem. Here, $k_{h,t-1}^{SP} = k_{f,t-1}^{SP} + \left(1 - \iota\left(n_{t-1}^{SP}\right)\tau\right) n_{t-1}^{SP}$ is the capital that the social planner invests in the high risk technology and $k_{h,t-1}^{CE} = k_{t-1}^{CE} + \tilde{p}_{t-1}^{CE}\tilde{b}_{t-1}^{CE}$ is the capital invested in the high risk intermediary in the competitive equilibrium.

We provide some intuition for how interest rate changes affect risk taking. When the economy is in an expansion, resources are optimally distributed from the low risk intermediary to the high risk intermediary. Figure 1 illustrates the impact of lowering the return to safe assets for risk taking. In the *primary market*, purchases of bonds are negatively related to bond returns, leading intermediaries to invest more capital into risky projects at low interest rates. In an *unconstrained secondary market* equilibrium, high risk intermediaries receive extra resources for risky investments and risk taking increases. However, in a *constrained secondary market* equilibrium, due to the smaller amount of bond purchases in the primary market, there are less bonds available as collateral and thus risk taking decreases for low bond interest rates.

In contrast, when the economy is in a contraction, resources are optimally distributed from the high risk intermediary to the low risk intermediary. As before, lower rates on safe assets push more capital into risky projects in the *primary market.* In the *secondary market*, in an *unconstrained equilibrium*, the low risk intermediaries receive extra resources and risk taking reduces. However, in a *constrained secondary market* equilibrium, due to fewer bond purchases in the primary market, there is limited re-trading and less resources are given from the low risk to the high risk intermediary, thus risk taking increases.

Competitive equilibria in a close neighborhood of the social planner's solution feature a constrained secondary bond market. Empirically, expansion periods are longer than contractions, which means that lowering interest rate will on average lead to less risk taking in our benchmark model.

3 Calibration

This section outlines our approach for determining the various parameters of the model and describes the various data that we used. We calibrate the following parameters: β, θ, τ, and the aggregate shock transition matrix Φ. We estimate π_m, δ, α, $q_m(\overline{s})$, $q_m(\underline{s})$, $q_l(\overline{s})$, $q_l(\underline{s})$, $q_h(\overline{s})$, $q_h(\underline{s})$ using a minimum distance estimation procedure. All parameter values are summarized in Tables 1 and 2.

The utility discount factor, β, is calibrated to ensure an annual real interest rate of 4%. In a quarterly model, this leads to a value of $\beta = 0.99$. The capital income share is determined using data from the U.S. National Income and Product Account (NIPA) provided by the Bureau of Economic Analysis (BEA) for the period 1947 to 2009. We find $\theta = 0.29$ for the business sector.[14] The cost of issuing government bonds, τ, is determined from existing literature. Stigum (1983, 1990) reports brokerage fees for U.S. Treasury bills and notes to be between 0.0013% and 0.008% of the amount issued. Green (2004) reports fees around 0.004%. A higher cost of issuing bonds has negative consequences for the mechanism in our paper, since it makes the use of bonds as a medium of exchange less desirable from a social perspective and it reduces welfare. Thus, to stress the robustness of our approach, we choose the highest cost estimate, namely $\tau = 0.008\%$.[15]

To calibrate the transition matrix for the aggregate state of the economy, we start by identifying turning points in the real value added of the U.S. business sector from 1947Q1 to 2010Q2. We use the approach outlined in Harding and Pagan (2002), which is based on the idea put forward in Burns and Mitchell (1946). The peaks and troughs identified through the turning point method allow us to identify expansion and contraction periods in business real value added.[16] We find 11 contractions between 1947 and 2010, with an average duration of 5 quarters. Based on these facts, the probability of switching from a bad realization of the aggregate shock at time $t-1$ to a good

[14] For the corporated business sector—where income is split into capital and labor by the BEA—we find $\theta = 0.29$. For noncorporate businesses which include proprietors, we need to split proprietor's income into capital and labor income in order to compute the capital income share. We attribute about 0.788 percent of proprietor's income to labor income and find a capital share for the noncorporate sector of 0.29. While 0.788 seems a bit high, it is not unreasonable.

[15] The costs have to be substantial (larger than 0.5%) to negate the usefulness of bonds as a medium of exchange.

[16] The business cycles we identify using data for the business sector mimic closely those determined by the NBER.

realization at time t is $\phi(s_t = \overline{s}|s_{t-1} = \underline{s}) = 0.20$. Moreover, the probability of switching from an expansion period to a contraction is $\phi(s_t = \underline{s}|s_{t-1} = \overline{s}) = 0.06$. The calibrated transition matrix, Φ, is given below.

$$\Phi = \begin{bmatrix} \phi(s_t = \underline{s}|s_{t-1} = \underline{s}) & \phi(s_t = \overline{s}|s_{t-1} = \underline{s}) \\ \phi(s_t = \underline{s}|s_{t-1} = \overline{s}) & \phi(s_t = \overline{s}|s_{t-1} = \overline{s}) \end{bmatrix} = \begin{bmatrix} 0.80 & 0.20 \\ 0.06 & 0.94 \end{bmatrix}$$

A parameter which is challenging to determined is the fraction of financial intermediaries who fund high risk projects, π_h. In our benchmark calibration, we set $\pi_h = 15\%$ and $\pi_l = 1 - \pi_h = 85\%$. We perform sensitivity analysis with respect to the value of π_h.

We estimate the following 9 parameters: the productivity parameters, $q_m(\overline{s})$, $q_m(\underline{s})$, $q_l(\overline{s})$, $q_l(\underline{s})$, $q_h(\overline{s})$, $q_h(\underline{s})$; the fixed factor parameter in the financial sector, α; the depreciation rate, δ; and the parameter, π_m, which guides the importance of the nonfinancial sector. We can normalize one parameter, since the absolute level of productivity is not important in our model. We set the productivity of the high risk intermediary in the good aggregate state, $q_h(\overline{s})$, to be 1. To determine the remaining eight parameters, we use eight moments from the data which are described below. Unless noted otherwise, we use data for 1987Q1 to 2010Q2. The reason for focusing on the post-Volcker time period is that inflation was low and stable. Moreover, till the recent financial crisis the consensus was that monetary policy had been successful in achieving its official goals.

The first moment we use in the estimation procedure is the output of the nonfinancial sector as a share of total output. This moment is strongly related to the parameter π_m in our model. We identify total output in the model with U.S. business sector value added published by the BEA. In addition, we identify the nonfinancial sector in our model with the corporate nonfinancial sector. We find that, from 1987Q1 to 2010Q2, the average value added share of the corporate nonfinancial sector was 66.9%. Note that we treat the remainder of the business sector, namely the corporate financial businesses and the noncorporate businesses, as the model's financial intermediation sector. Our interpretation is that the majority of noncorporate businesses are strongly dependent on the financial sector for funding. In our model, for simplicity, the financial intermediary is endowed with the technology of production of noncorporate businesses.

The parameter α influences the returns to equity in our model's financial sector, which in turn are dependant on the equity to total assets ratio of these intermediaries. We use the equity to asset ratio for corporate financial businesses as a second data moment to target in our estimation. Using data from the U.S. Flow of Funds from 1994$Q1$ to 2010$Q2$, we find this ratio to be on average 7.6%. In performing this calculation, we excluded mutual funds.[17] We choose the time period beginning in 1994, because the Basel capital regulation was in effect since then.

In our model, the depreciation rate is stochastic and is given by:

$$\frac{\pi_m q_{m,t} \delta k_{m,t} + (1 - \pi_m)(\pi_h q_{h,t} \delta k_{h,t} + \pi_l q_{l,t} \delta k_{l,t})}{\pi_m k_{m,t} + (1 - \pi_m)(\pi_h k_{h,t} + \pi_l k_{l,t})}$$

We determine the value of δ to ensure that the average depreciation rate in the model is very close to that found in the data, namely 2.5% per quarter.

We still need five data moments to pin down the five productivity parameters: $q_m(\overline{s})$, $q_m(\underline{s})$, $q_l(\overline{s})$, $q_l(\underline{s})$, $q_h(\underline{s})$.

1. We aim to match the average maximum decline in real output in the business sector during contraction periods. Here, contraction periods are identified by the Pagan and Harding turning points approach. The average decline over all contractions periods since 1947 is 6.48%, where we detrend output by a constant growth trend to make it stationary. We are using a longer period for this exercise, since the number of contractions is fairly small in our reference period, namely 3.

2. We next consider the coefficient of variation for business sector output. To do this we detrend the logarithm of output using a linear trend and measure the standard deviation of the residual to be 3.75%.

3. We measure the coefficient of variation of net worth of households in the economy. Here, we use U.S. Flow of Funds accounts data and detrend the logarithm of household net worth. The reason we focus on net-worth is that it is very close to the state variable $w(s^t)$ in our model. The trend we are using is a polynomial of order three. We find that the coefficient of variation of this detrended net-worth is 8.17%.

[17] The equity to asset ratio of depository institutions only—commercial banks, savings institutions and credit unions—is esentially identical to the ratio computed for the corporate financial sector excluding mutual funds.

4. The U.S. Flow of Funds accounts also provide us with information regarding the deposits held by households as a share of their total financial assets. Here, we find for our reference period a share of 17.2%.

5. The final moment we use is the recovery rate during bankruptcy. Here, we use an estimate for the whole economy provided by Acharya, Bharath, and Srinivasan (2003). They find that the average recovery rate on corporate bonds in the United States after a bankruptcy for the period from 1982 to 1999, was 42 cents on the dollar.

We determine all eight parameters jointly using a minimum distance estimator to match the moments above. Let Ω_i be a model moment and $\tilde{\Omega}_i$ be the corresponding data moment. Our procedure makes use of the problems given in (4) and (5) below. Notice that in (4) we impose restrictions on the ordering of productivity parameters across the different technology types.[18] For our benchmark calibration we are abstracting from capital adequacy requirement and set $\eta = 0$. The reason for this choice is that the introduction of this constraint leads to larger welfare losses relative to the social planner's solution. We will report results for $\eta = 0.08$, which is the current level in the U.S. as parts of our results section.

$$\begin{aligned} Q^* &= \arg \min_{\substack{Q=\{q_m(\overline{s}),q_m(\underline{s}),q_l(\overline{s}),\\ q_l(\underline{s}),q_h(\underline{s}),\delta,\alpha,\pi_m\}}} \sum_{i=1}^{8} \left(\frac{\Omega_i - \tilde{\Omega}_i}{\tilde{\Omega}_i} \right)^2 \\ \text{s.t.} &: \; q_h(\underline{s}) < q_m(\underline{s}) < q_l(\underline{s}) \leq q_l(\overline{s}) < q_m(\overline{s}) \leq q_h(\overline{s}) \text{ and} \\ & \quad \Omega_i \text{ is implied in a competitive equilibrium given policy } p^* \end{aligned} \tag{4}$$

$$\begin{aligned} p^* &= \arg\max_p E \sum_{t=0}^{\infty} \beta^t \log C\left(s^t\right) \\ \text{s.t.} &: \; \left\{C\left(s^t\right)\right\} \text{ is part of a competitive equilibrium given } Q^* \end{aligned} \tag{5}$$

We start out with a guess Q_1^* and solve the problem in (5) for an optimal policy p^*. Next, we take this optimal policy as given and choose parameters to minimize the distance between our model moments and the corresponding data moments, as shown in (4). This step yields Q_2^*.

[18] These restrictions can be relaxed without changes to our qualitive results. The only requirement is that the high risk technology is the most risky one in operation. Even when we relaxed this requirement in our estimation proceedure, it never was violated.

We continue the procedure till convergence is achieved. The reason for choosing this two-step procedure is because our model is highly nonlinear and the initial guess is very important in finding a competitive equilibrium solution. The guess we start with is the social planner's solution.

The estimated parameters are presented in Tables 2. Notice that despite the assumption that depreciation is stochastic, the model is able to perfectly match the average depreciation observed in the data. Table 3 shows that the model matches the targeted data moments well. Some moments—such as the capital depreciation rate, or the coefficient of variation of output—are matched very well, while others—the recovery rate after bankruptcy, or the deposits to asset ratio for households—are still a bit far from the data. Regarding the recovery rate in bankruptcy, one aspect to keep in mind is that the data target taken from Acharya, Bharath, and Srinivasan (2003) was for corporate bonds only, while the model considers recovery rates for small business bankruptcies. In addition, there is a tight relationship between the model's recovery rate, deposit and equity ratios. The reason for the low recovery rate is a low equity to asset ratio of financial intermediaries and a very strong decline of output during contractions. Given a low recovery rate in bankruptcy, households desire safe assets and choose to hold a high proportion of their wealth in deposits.

4 Results

This section first presents results for our benchmark model as outlined above. We also consider a modified version of our model that allows for the issuance of private bonds, evaluated by a rating agency and allowing for foreign demand for these private bonds. The extension of our model illustrates how the introduction of incorrect bond rating together with the presence of some external demand for bonds can undermine the key mechanism of the model and under very low interest rates lead to outcomes similar to what was seen in the run-up and the unfolding of the recent financial crisis. We are not claiming that our extended model is providing a thorough analysis of recent events. It mainly sheds light on key ingredients of the crisis in relationship to interest rate policy.

4.1 Benchmark model

The first main finding concerns the social planner solution. In a first attempt to find the optimal monetary policy we try to implement the first best allocation as derived from the social planner problem with costly reallocation as a competitive equilibrium. Our strategy here is to use the allocation and the implied behavior of both the financial sector and the non-financial sector to then derive all the decisions as well as the returns including the optimal monetary policy. For parameter constellations which have an active financial sector, we find that the first best allocation can not be implemented as a competitive equilibrium.[19] There are two areas where equilibrium conditions are violated. The first area concerns the portfolio choice problem of the household. Given the returns to deposits, financial sector equity and non-financial sector equity, we find that the household does not wish to hold any financial sector equity and deposits in the contractionary aggregate state. Thus the social planner allocation of resources across sectors cannot be supported directly in a competitive equilibrium. The second area, where a breakdown occurs is with regards to the optimal policy price of government bonds and the returns to deposits. Here the support of the optimal policy in the bad state requires that $R^d < 1/p$, thus violating a key arbitrage condition. The rational for this violation is the fact that the social planner allocation of resources between the intermediaries in the bad state of the economy would require at the same time that the high risk intermediaries are able to buy a large value of bonds in the secondary market and pay a very low return on deposits in order to avoid high risk intermediaries' bankruptcy in the bad state. The first requirements leads to a low price of bonds in the bad state, the second one to a low return on deposits with the combination violating the no-arbitrage condition.

Given that the first best solution is not implementable, the task for us is to find the second best solution by choosing $p\left(s^{t-1}\right)$ in order to maximize the welfare of the households:

$$\max_{p\left(s^{t-1}\right)} E\left[\sum_{t=0}^{\infty} \beta^t \log \tilde{C}\left(s^t\right)\right]$$

subject to:

$$\tilde{C}\left(s^t\right) \text{ is part of a competitive equilibrium given policy } p\left(s^{t-1}\right)$$

We solve this problem numerically, taking the function $p(\cdot)$ from the space of linear spline functions and searching for the defining parameters in order to maximize the unconditional welfare of the representative consumer. Here each policy function implies a particular competitive equilibrium

[19] While we derive this result for a specific parameter set, it appears to be general given the productivity ordering and the existence of expansion as well as contraction periods.

and the associated expected utility.

There are two metrics that are of particular use to us. The first one is risk taking which was defined in Section 2.7.1. We are interested in whether a particular policy implies too much or too little risk taking relative to what the social planner would do. Risk taking is measured in percent deviations from the social optimum: a positive number implies too much risk taking, while a negative number means too little risk taking. The second metric compares welfare across policies. We define the Lifetime Consumption Equivalents (LTCE) as the percentage decrease of the optimal consumption associated with the social planner problem with costly reallocation, required to make the consumer ex ante indifferent to the consumption pattern from a competitive equilibrium under a specific interest rate policy.

To improve our understanding of the model's implications, we conduct 4 experiments. Welfare and risk taking results for all experiments are summarized in Table 4**.**

As a first step, we try to assess the benefits of having a secondary bond market. A monetary policy that charges a very high price in the primary market for bonds induces a state of financial autarky, where no intermediary buys bonds in the primary market and thus risk does not get reallocated in the secondary market.

Exp. 1 Equilibrium without risk redistribution – financial autarky

For this experiment, we find that the welfare loss of shutting down the secondary market is quite substantial, namely 0.88% in terms of LTCE relative to the social planner with reallocation. Furthermore, we see from the risk taking measure that one of the reasons for this loss is an excessive risk taking under financial autarky. The social planner would want much more reallocation, than what takes place if there is no secondary market.

Given this result, it makes sense to search for a second best policy that tries to get as close to the first best allocation as possible.

Exp. 2 Second best policy, $p^*\left(s^{t-1}\right)$

From experiment 2, we find that the second best policy is very close to the first best in terms of welfare with 0.04% welfare loss. Yet, even for the best interest rate policy, the risk taking is elevated exceeding the one found from the social planner by 23.6%. Figure 2 presents simulation results from our baseline model. Here we can clearly see in the bottom two subplots, that the elevated risk taking under the second best is mostly due to too much risk taking in good states, when resources are reallocated from the low risk to the high risk intermediaries. In contrast, in the contraction state, where resources are allocated from the high risk to the low risk intermediaries the risk taking much lower, though still higher than optimal.

In order to better understand the influence that interest rate policy has on risk taking, we look at variations of the second best policy. In particular we look at:

Exp. 3 Level shifts in second best returns to bonds. In particular, we shift the second best policy uniformly by 0.1 percentage points at a quarterly rate: i.e. $\frac{1}{p^*(s^{t-1})} \pm 0.1$.

This experiment highlights the extend to which a too low interest rate can contribute to the risk taking of financial intermediaries. We learn from this that consistently with our previous intuition in Section 2.7.1, the second best places us into the constraint secondary market region such that an increase in return leads initially to more risk taking and a decrease in return leads to less risk taking. This result is local in the sense that a very strong increase in the return can lead to a decrease in risk taking as the secondary market becomes unconstraint. The latter point is made visible in the blue line in Figure 5, which plots risk taking for variations around the second best policy. The figure also illustrates the non linear aspect of our model. The kink in the benchmark risk taking marks the point at which the secondary bond market becomes unconstraint.

Finally, we consider the impact of introducing a capital adequacy requirement into our model. We choose $\eta = 8\%$, which is the currently implemented constraint in the United States, and reoptimize the interest rate policy.

Exp. 4 Second best policy given a capital adequacy ratio of $\eta = 0.08$, $p^*\left(s^{t-1};\eta\right)$.

The presence of a capital adequacy requirement, leads to a substantial decrease in risk taking, from 23.6% under the second best without regulation to -9.1% under a second best with regulation. Thus it actually is quite successful in reducing risk taking. A negative consequence of the regulation though is that it leads to a stronger welfare loss of 0.07%, relative to the unregulated second bests 0.04%.

4.2 How important is the share of high risk intermediaries?

Up to now, we reported experiments for $\pi_h = 0.15$. Next, we report some of the main implications of our model for a smaller and a larger fraction of high risk intermediaries, $\pi_h \in \{0.13, 0.17\}$. The results from this sensitivity analysis are reported in Table 5. While the quantitative results change, we find that the qualitative results remain intact.

A first interesting result is that while the potential welfare gains from having a secondary market are decreasing with the share of high risk intermediaries, the risk taking is going up. The welfare result is quite intuitive since the main gains from reallocation are due to the presence of high risk intermediaries and thus a need to adjust the resource distribution as the economic state varies. Considering the second best results, we see that the influence of interest rate policy decreases with the share of high risk intermediaries and thus the welfare gains decrease. Regarding variations of the interest rate around the second best, we find that lower rates lead to less risk taking regardless of the value of π_h and that indeed for the lowest considered value of π_h an upward deviation in the policy rate can become quite costly (-0.44% $LTCE$), relative to a downward deviation of similar magnitude (-0.0536% $LTCE$).

4.3 Allowing for a Rating Agency, Private Bonds and Foreign Investment

In this section, we consider an extension of our benchmark model, in which during an expansion phase, financial intermediaries can issue their own private bonds and sell them either to other financial intermediaries, or to foreign investors. We further assume the existence of rating agencies which can "stamp" those private bonds (at a proportional cost) as being safe bonds. Once stamped, private bonds appear to be as safe as government bonds and are traded at the same bond price $\tilde{p}$.[20] During an expansion the private bonds are fully repaid every period by their issuers. When a bad state happens, the high risk financial intermediaries default on their private bonds. In this case the government bails out domestic bond holders by fully guaranteeing their returns on stamped

[20] We could instead allow for some yield spread between the returns on private and government bonds. A fixed spread is not going to change our qualitative results. An endogenous risk spread is beyond the scope of this paper.

private bonds. This public guarantee justifies our assumption that domestic bond purchasers are indifferent between government and private bonds. In contrast, foreign investors are surprised to learn that their allegedly safe bonds return only 80 percent of what was due. Thus, we assume that the rating agencies mislead foreigners into believing that the returns on private bonds are fully guaranteed. Instead foreign investors are forced to take a 20 percent haircut on their bond values.

We introduce these new features into the benchmark model to allow for market inefficiencies which were believed to be at the root of the recent financial crisis. It has been often argued that credit rating agencies contributed to the propagation of asset mispricing by giving top ratings to derivative securities, which should have been assigned in a much riskier category. Also, the demand for top rated assets from domestic pension funds and foreign wealth funds was fueling the incentive to overlook risks and devise complex derivative securities, which would appear to be much safer than their underlying assets.

We introduced these new features in the benchmark model in a way which allows foreign demand for domestic bonds to relax the collateral constraints faced by financial intermediaries, while preserving as much as possible the original structure of the model.[21] Specifically, we assume that in any given period during an expansion phase, when financial intermediaries buy government bonds, they are not completely sure whether foreign investors will be willing to buy domestic private bonds. In the model, the existence or absence of foreign demand is revealed after domestic intermediaries trade government bonds among themselves in the domestic repo market. At this point, with probability π_F the foreigners are willing to buy domestic private bonds, in which case financial intermediaries who want to borrow more capital issue their own bonds, in the amount a_j for $j \in \{h, l\}$. Before they can sell these bonds, however, they must receive the approval stamp from the rating agencies. This stamp is given at the real cost of ξa_j ($\xi = 0.01$), which is due after the production takes place. Once the bonds are stamped, they can be sold at the same price as the government bonds, $\tilde{p}$.

Alternatively, with probability $(1 - \pi_F)$ the foreigners do not want to buy domestic bonds, in which case there are no private bonds issued and, at least some intermediaries, are constrained

[21] Using Flow of Funds data, it can be shown that the rest of the world holds a significan portion of US securities: Treasury and agency backed securities, commercial paper and corporate bonds. In our model, it is not essential that most of the private bonds end up with foreigners. What matters is the presence of a misperception of the actual risk of private bonds and the presence of some demand for these.

in their repo market bond sales. If the probability $(1 - \pi_F)$ is large enough, the intermediaries would still want to buy government bonds from the primary bond market, to facilitate their repo transactions. In the simulations we set π_F at 0.1 percent. Lastly, the government uses lump-sum taxes on the households to guarantee full returns to domestic private bond holders and partial returns to foreigners in case of a bad shock.

4.3.1 Implications of the extended model

In order to highlight the consequences of the changes we made to the model, we report simulation results for the very low likelihood event, that the foreign demand is positive in every period of an expansion phase. One can think of the simulation results as an upper bound on the possible welfare costs of loosing the collateral channel as a tool for influencing risk taking of the financial intermediaries. Alternatively, the experiment magnifies the welfare and risk taking consequences in these extreme situations and puts them in perspective relative to normal times.[22] The red curve on Figure 4 shows how welfare changes in response to uniform shifts away from the second best policy in the baseline model (i.e. the policy that gives the highest welfare in the original benchmark model). As we can see, with foreign demand, the welfare losses relative to a social planner are substantially worse. Even in the best scenario, i.e. the peak of the red line, the welfare is approximately 0.15 percentage points lower than at the second best. Also, the welfare peak in the red line is situated to the right of zero, which means that optimal policy rates in the extended model would need to be adjusted upwards relative to the benchmark model.

Figure 5 shows the risk taking implications of interest rate variations between the benchmark model and the extended model. The difference is remarkable. With foreign demand, lower policy rates unambiguously increase risk taking. This is precisely because the presence of foreign demand for private bonds, eliminates the collateral constraint channel of monetary policy. The high-risk intermediaries are not constrained in their risk taking, even if they buy very few government bonds rates in the primary market.

These results suggest that lower policy rate can indeed contribute to more risk taking, provided there are some other sources of inefficiency in the financial markets which leads participants to

[22] Obviously, the results under this extreme assumption are not representative for the average economy, but they serve well as a representation of extreme circumstances such as recently experienced in the United States.

misprice risk. In our extension the risk is being shifted partially to the government and partially to foreign investors. The market failures we highlight in our extended model may not be the only possible sources, but were certainly prominent features of the recent crisis.[23]

5 Conclusion

The recent financial crisis has stirred interest in the relationship between prolonged periods of low interest rates and risk taking behavior of private financial institutions. This paper analyzes this issue in the formal setup of a dynamic general equilibrium model that features limited liability of financial intermediaries, deposit insurance, as well as heterogeneity in the riskiness of financial intermediaries. There are two main channels through which policy influences risk taking. There is a direct channel which was previously highlighted by Rajan (2006), in which a lower policy rate reduces the returns to government bonds and thus leads to more resources being shifted to risky investments. Beyond this we have a second channel, in which a lower number of bond holdings by intermediaries reduce the amount of collateral for repurchasing activities in secondary markets. This constrains the ability of intermediaries to take on more risk as they receive further information regarding the riskiness of their projects. We show that for a calibrated version of our model, while the social planner solution is unattainable, the second best is very close in welfare terms, yet features too much risk taking. For variations around the second best solution, we find that both risk taking channels are relevant, but that the second channel dominates. This leads to the finding that if the interest rate policy is close to the optimum, lower interest rates lead to less risk taking.

We also extend our model to allow for rating agencies that misreport the riskiness of private bonds, as well as foreign demand for domestic bonds. For the extended model, we find too much risk taking for low interest rates and substantial welfare losses.

There are different potential extensions to our work. While we considered the implications of capital adequacy requirement in the context of our model, we do not try to characterize the features of an optimal capital regulation. Our results show that a state independent regulation is successful in reducing risk taking, but it also reduces welfare. This gives hope that a more flexible regulation can potentially lead to less risk taking and higher welfare than an optimal interest rate

[23] The presence of a binding capital adequacy regulation does not change the results, since the constraint mainly affects the balance sheet after the primary market.

policy alone. In the same context, a worthwhile exercise is to consider the coordination problem between regulation and interest rate policy. Another extension is concerned with the potential time inconsistency problem of interest rate policy. This paper focused on the case of a time consistent policy. It might be worthwhile to look at the consequences of a departure from this assumption.

References

Acharya, V. V., S. T. Bharath, and A. Srinivasan (2003): "Understanding the Recovery Rates on Defaulted Securities," *CEPR Discussion Papers 4098*.

Acharya, V. V., and H. Naqvi (2004): "The seeds of a crisis: A theory of bank liquidity and risk-taking over the business cycle," *NYU Stern School of Business Working Paper.*

Adrian, T., and H. S. Shin (2010): "Liquidity and Leverage," *Journal of Financial Intermediation*, 19, 418–437.

Agur, I., and M. Demertzis (2010): "Monetary policy and excessive bank risk taking," *European Banking Center Discussion Paper No. 2010-06S*.

Altunbas, Y., L. Gambacorta, and D. Marques-Ibane (2010): "Does Monetary Policy Affect Risk Taking?," *BIS Working Papers 298*.

Borio, C., and H. Zhu (2008): "Capital regulation, risk-taking and monetary policy: A missing link in the transmission mechanism?," *BIS Working paper no. 268*.

Burns, A. F., and W. C. Mitchell (1946): *Measuring business cycles*. National Bureau of Economic Research, New York, NBER.

Delis, M. D., and G. Kouretas (2010): "Interest rates and bank risk-taking," *Munich Personal RePEc Archive*.

Diamond, D. W., and R. G. Rajan (2009): "Illiquidity and Interest Rate Policy," *NBER Working Paper 15197*.

Gambacorta, L. (2009): "Monetary policy and the risk-taking channel," *BIS Quarterly Review*, December, 43–53.

Gertler, M., and N. Kiyotaki (2010): "Financial Intermediation and Credit Policy in Business Cycle Analysis," *Working Paper*.

Gertler, M., N. Kiyotaki, and A. Queralto (2010): "Financial Crises, Bank Risk Exposure and Government Financial Policy," *Working Paper*.

Giavazzi, F., and A. Giovannini (2010): “Central banks and the financial system,” *CEPR Discussion Papers 7944*.

Goodhart, C. A., P. Sunirand, and D. P. Tsomocos (2006): “A model to analyze financial fragility,” *Economic Theory*, 27, 107–142.

Green, T. C. (2004): “Economic News and the Impact of Trading on Bond Prices,” *The Journal of Finance*, 59(3), 1201–1233.

Harding, D., and A. Pagan (2002): “Dissecting the Cycle: a Methodological Investigation,” *Journal of Monetary Economics*, 149, 365–381.

Ioannidou, V., S. Ongena, and J.-L. Peydró (2009): “Monetary policy, risk-taking and pricing: Evidence from a quasi-natural experiment,” *European Banking Centre Discussion Paper No. 2009-04S*.

Jiménez, G., S. Ongena, J. L. Peydró, and J. Saurina (2009): “Hazardous times for monetary policy: What do twenty-three million bank laons say about the effects of monetary policy on credit risk-taking?,” *Bank of Spain Working Paper No. 833*.

Rajan, R. G. (2006): “Has Finance Made the World Riskier,” *European Financial Management*, 12(4), 499–533.

Stigum, M. (1983): *The Money Market*. McGraw-Hill, Boston, New York, 2nd edn.

A Timing of Model Events

The timing in our model is as follows. At the beginning of period t, the aggregate shock s_t realizes and financial and nonfinancial firms find out their current productivity shock: $q_j(s_t)$ for intermediaries of type $j \in \{h, l\}$ and $q_m(s_t)$, respectively. All firms produce output using the capital that has been allocated to production at the end of period $t-1$. Nonfinancial firms pay state contingent returns to capital and labor and make zero profits. Financial intermediaries pay state contingent returns to labor and may declare bankruptcy, if they are unable to pay the fixed return on deposits. Equity holders in financial intermediaries receive returns only in the absence of bankruptcy. Households use current wealth to purchase consumption and make investments into deposits and equity that will pay off tomorrow. At the end of period t, financial intermediaries allocate the resources received to bonds and risky projects. They find out their type $j \in \{h, l\}$ and trade repurchasing agreements on government bonds in the secondary bond market. The resulting investments into the risky projects pay returns at the beginning of period $t+1$, after shock s_{t+1} realizes.

Let $s_t \in \{\bar{s}, \underline{s}\}$ be the aggregate shock at time t. Let $s^t = (s_1, s_2, ...s_t)$ be the history of the aggregate shock up to time period t. Note that $s^t = \left(s^{t-1}, s_t\right)$. The timing of the economy is as follows:

- Each nonfinancial firm enters period t with equity $M\left(s^{t-1}\right)/\pi_m$, capital $k_m\left(s^{t-1}\right)$ and labor $l_m\left(s^{t-1}\right)$.
- Each financial intermediary (FI) enters period t with $b_j\left(s^{t-1}\right)$ safe assets, $k_j\left(s^{t-1}\right)$ risky assets, $d\left(s^{t-1}\right)$ deposits, equity $z\left(s^{t-1}\right)$ and labor $l\left(s^{t-1}\right)$.
- Aggregate shock s_t realizes. The history s^t is now known.
- Nonfinancial firms: find out $q_m(s_t)$, produce using beginning of period capital and labor, pay wage income $W_m\left(s^t\right) l_m\left(s^{t-1}\right)$ and equity returns $R^m\left(s^t\right) k_m\left(s^{t-1}\right)$
- FI: find out $q_j(s_t)$, produce output, pay wage incomes $W_j\left(s^t\right) l\left(s^{t-1}\right)$, pay returns on deposits $R^d\left(s^{t-1}\right)$ and equity returns $R^z\left(s^t\right)$

- Bankrupt intermediaries are liquidated. Their equity holders receive no equity returns; government steps in to guarantee the fixed rate of return on deposits.

- Government uses lump-sum taxes or transfers $T\left(s^{t}\right)$ to cover expenses and balance budget

- Household wealth $w\left(s^{t}\right)$ is realized

- Households consume $C\left(s^{t}\right)$, make investment decisions that will pay off tomorrow: $M\left(s^{t}\right)$, $Z\left(s^{t}\right)$ and $D_{h}\left(s^{t}\right)$ and supply labor inelastically to financial intermediaries and nonfinancial firms.

- Each nonfinancial firms receives equity $M\left(s^{t}\right)/\left(1-\pi_{m}\right)$.

- FI receive deposits $d\left(s^{t}\right)$ and equity $z\left(s^{t}\right)$. They do not know the type of their future risky projects. FI purchase government bonds $b\left(s^{t}\right)$ at price $p\left(s^{t}\right)$. These bonds pay off tomorrow. Government covers bond issuance costs and deposits $D^{g}\left(s^{t}\right)/\left(1-\pi_{m}\right)$ with each intermediaries

- FI find out their type for period $t+1$ production: $j\in\{l,h\}$. Realization s_{t+1} is still unknown. Intermediaries trade bonds $\tilde{b}_{j}\left(s^{t}\right)$ on secondary mkt.

  - after trading, FI have bonds given by: $b\left(s^{t}\right)-\tilde{b}_{j}\left(s^{t}\right)$
  - after trading, FI have risky assets given by: $k_{j}\left(s^{t}\right)=k\left(s^{t}\right)+\tilde{p}\left(s^{t}\right)\tilde{b}_{j}\left(s^{t}\right)$

- Enter period $t+1$

B Sketch of Proof for Proposition 1

To simplify notation in our derivations, we use subscripts as a short hand notation for the entire history, s^{t-1}. For example, $\tilde{b}_{j,t-1} \equiv \tilde{b}_j\left(s^{t-1}\right)$ and $b_{t-1} \equiv b\left(s^{t-1}\right)$.

Deriving the relationship between bond prices and the return to deposits in our model involves studying three possible outcomes on the secondary bond market. Transactions of bonds either satisfy: (i) $\tilde{b}_{j,t-1} < b_{t-1}$ for both $j \in \{h,l\}$ or (ii) $\tilde{b}_{h,t-1} = b_{t-1}$ and $\tilde{b}_{l,t-1} < b_{t-1}$ or (iii) $\tilde{b}_{l,t-1} = b_{t-1}$ and $\tilde{b}_{h,t-1} < b_{t-1}$. Here, we sketch the proof of Proposition 1 for case (ii). The proof is obtained in an analogous fashion for cases (i) and (iii) and is omitted here for brevity.[24]

In case (ii), the high risk intermediary increases the amount of resources allocated to risky investments by selling all bond holdings in the secondary bond market.

B.1 Step 1: Some Key Relationships

In finding and characterizing the equilibrium, it is useful to define the share of resources a financial intermediary retains for risky investment in the primary market, call it x_{t-1}. Then,

$$k_{t-1} = x_{t-1}\left(z_{t-1} + d_{t-1}\right) \tag{6}$$
$$b_{t-1} = \frac{1 - x_{t-1}}{p_{t-1}}\left(z_{t-1} + d_{t-1}\right) \tag{7}$$

where the second equation was obtained from equation (1).

For the case presented here, high risk intermediaries use all their bonds as collateral in the secondary market, while low risk intermediaries give resources against this collateral. We have:

$$\tilde{b}_{h,t-1} = b_{t-1} = \frac{1 - x_{t-1}}{p_{t-1}}\left(z_{t-1} + d_{t-1}\right) \tag{8}$$
$$\tilde{b}_{l,t-1} = -\frac{\pi_h}{\pi_l} b_{t-1} = -\frac{\pi_h}{\pi_l}\frac{1 - x_{t-1}}{p_{t-1}}\left(z_{t-1} + d_{t-1}\right) \tag{9}$$

Lastly, using equations $(6) - (9)$, the resources allocated to risky investments by high and low risk intermediaries after the secondary market trades are given by (10) and (11).

$$k_{t-1} + \tilde{p}_{t-1}\tilde{b}_{h,t-1} = \left[x_{t-1} + \frac{\tilde{p}_{t-1}}{p_{t-1}}\left(1 - x_{t-1}\right)\right]\left(z_{t-1} + d_{t-1}\right) \tag{10}$$

[24] The full derivation is available upon request from the authors.

$$k_{t-1}+\tilde{p}_{t-1}\tilde{b}_{l,t-1}=\left[x_{t-1}-\frac{\pi_h}{\pi_l}\frac{\tilde{p}_{t-1}}{p_{t-1}}\left(1-x_{t-1}\right)\right]\left(z_{t-1}+d_{t-1}\right) \tag{11}$$

B.2 Step 2: Equilibrium Conditions for the Financial Sector

In what follows, we make use of the equilibrium result $l_{t-1}=1$.

We rewrite the secondary market problem given in $(P2)$ as below:

$$\max_{\tilde{b}_{j,t-1}}\sum_{s^t|s^{t-1}}1_{j,t}\lambda_t\left(\begin{array}{l} q_{j,t}\left[\left(k_{t-1}+\tilde{p}_{t-1}\tilde{b}_{j,t-1}\right)^{\theta}+(1-\delta)\left(k_{t-1}+\tilde{p}_{t-1}\tilde{b}_{j,t-1}\right)\right] \\ +\left(b_{t-1}-\tilde{b}_{j,t-1}\right)-R^d_{t-1}d_{t-1}-W_{j,t}\end{array}\right)$$

where $\tilde{b}_{j,t-1}\in\left[-\frac{k_{t-1}}{\tilde{p}_{t-1}},b_{t-1}\right]$ and $1_{j,t}$ is an indicator function given by $1_{j,t}\equiv\begin{cases}1 \text{ if } V_{j,t}>0\\ 0 \text{ otherwise}\end{cases}$.

The first order conditions with respect to bond trades, $\tilde{b}_{h,t-1}$ and $\tilde{b}_{l,t-1}$, are given by:[25]

$$\sum_{s^t|s^{t-1}}1_{j,t}\lambda_t\left\{q_{j,t}\tilde{p}_{t-1}\left[\theta\left(k_{t-1}+\tilde{p}_{t-1}\tilde{b}_{j,t-1}\right)^{\theta-1}+1-\delta\right]-1\right\}-\mu_{j,t-1}=0 \tag{12}$$

where $\mu_{j,t-1}$ for $j\in\{h,l\}$ are the Lagrange multipliers on the constraints $\tilde{b}_{j,t-1}\leq b_{t-1}$ and they satisfy the complimentary slackness conditions: $\mu_{j,t-1}\geq 0$, $\mu_{j,t-1}\left(b_{t-1}-\tilde{b}_{j,t-1}\right)=0$.

Notice that for the case we are analyzing here, $\mu_{l,t-1}=0$ and $\mu_{h,t-1}\geq 0$. Using this, along with the expressions in (10) and (11), we can rewrite equation (12) for $j\in\{h,l\}$ as (13) and (14) below:

$$\theta\left[\left(x_{t-1}-\frac{\pi_h}{\pi_l}\frac{\tilde{p}_{t-1}}{p_{t-1}}\left(1-x_{t-1}\right)\right)\left(z_{t-1}+d_{t-1}\right)\right]^{\theta-1}+1-\delta = \frac{\sum_{s^t|s^{t-1}}1_{l,t}\lambda_t}{\sum_{s^t|s^{t-1}}1_{l,t}\lambda_t q_{l,t}\tilde{p}_{t-1}} \tag{13}$$

$$\theta\left[\left(x_{t-1}+\frac{\tilde{p}_{t-1}}{p_{t-1}}\left(1-x_{t-1}\right)\right)\left(z_{t-1}+d_{t-1}\right)\right]^{\theta-1}+1-\delta \geq \frac{\sum_{s^t|s^{t-1}}1_{h,t}\lambda_t}{\tilde{p}_{t-1}\sum_{s^t|s^{t-1}}1_{h,t}\lambda_t q_{h,t}} \tag{14}$$

Notice that equation (13) can be equivalently written as:

$$\left[x_{t-1}-\frac{\pi_h}{\pi_l}\frac{\tilde{p}_{t-1}}{p_{t-1}}\left(1-x_{t-1}\right)\right]\left(z_{t-1}+d_{t-1}\right)=\left[\frac{1}{\theta}\left(\frac{\sum_{s^t|s^{t-1}}1_{l,t}\lambda_t}{\sum_{s^t|s^{t-1}}1_{l,t}\lambda_t q_{l,t}\tilde{p}_{t-1}}-1+\delta\right)\right]^{\frac{1}{\theta-1}} \tag{15}$$

[25] In equilibrium, the constraint $-\frac{k_{t-1}}{\tilde{p}_{t-1}}\leq\tilde{b}_{j,t-1}$ does not bind as returns to capital invested in risky projects would become infinite.

Using equations $(6)-(11)$ we rewrite the primary market problem given in $(P1)$ as below:

$$\max_{\substack{x_{t-1}\in[0,1]\\ d_{t-1}\geq 0}} \sum_{j\in\{h,l\}} \pi_j \sum_{s^t|s^{t-1}} \lambda_t V_{j,t}$$

subject to :

$$V_{l,t} = \max\left\{\begin{array}{l} q_{l,t}\left[\left(x_{t-1}-\frac{\pi_h}{\pi_l}\frac{\tilde{p}_{t-1}}{p_{t-1}}\left(1-x_{t-1}\right)\right)\left(z_{t-1}+d_{t-1}\right)\right]^{\theta} \\ +q_{l,t}\left(1-\delta\right)\left(x_{t-1}-\frac{\pi_h}{\pi_l}\frac{\tilde{p}_{t-1}}{p_{t-1}}\left(1-x_{t-1}\right)\right)\left(z_{t-1}+d_{t-1}\right) \\ +\frac{1}{\pi_l}\frac{\left(1-x_{t-1}\right)}{p_{t-1}}\left(z_{t-1}+d_{t-1}\right)-R^d_{t-1}d_{t-1}-W_{l,t},\ 0 \end{array}\right\}$$

$$V_{h,t} = \max\left\{\begin{array}{l} q_{h,t}\left[\left(x_{t-1}+\frac{\tilde{p}_{t-1}}{p_{t-1}}\left(1-x_{t-1}\right)\right)\left(z_{t-1}+d_{t-1}\right)\right]^{\theta} \\ +q_{h,t}\left(1-\delta\right)\left(x_{t-1}+\frac{\tilde{p}_{t-1}}{p_{t-1}}\left(1-x_{t-1}\right)\right)\left(z_{t-1}+d_{t-1}\right)-R^d_{t-1}d_{t-1}-W_{h,t},\ 0 \end{array}\right\}$$

$$z_{t-1}-\eta x_{t-1}\left(z_{t-1}+d_{t-1}\right)\geq 0$$

Let ζ_{t-1} be the Lagrange multiplier on the capital regulation constraint. The first order conditions with respect to x_{t-1} and d_{t-1} are given by (16) and (17), respectively.[26]

$$\begin{aligned} & \frac{1}{p_{t-1}}\sum_{s^t|s^{t-1}}\lambda_t 1_{l,t} \qquad (16)\\ = & \left\{\theta\left[\left(x_{t-1}-\frac{\pi_h}{\pi_l}\frac{\tilde{p}_{t-1}}{p_{t-1}}\left(1-x_{t-1}\right)\right)\left(z_{t-1}+d_{t-1}\right)\right]^{\theta-1}+1-\delta\right\}\left(1+\frac{\pi_h}{\pi_l}\frac{\tilde{p}_{t-1}}{p_{t-1}}\right)\pi_l\sum_{s^t|s^{t-1}}1_{l,t}\lambda_t q_{l,t} \\ & +\left\{\theta\left[\left(x_{t-1}+\frac{\tilde{p}_{t-1}}{p_{t-1}}\left(1-x_{t-1}\right)\right)\left(z_{t-1}+d_{t-1}\right)\right]^{\theta-1}+1-\delta\right\}\left(1-\frac{\tilde{p}_{t-1}}{p_{t-1}}\right)\pi_h\sum_{s^t|s^{t-1}}1_{h,t}\lambda_t q_{h,t} \\ & -\zeta_{t-1}\eta \end{aligned}$$

$$\begin{aligned} & R^d_{t-1}\sum_{j\in\{h,l\}}\pi_j\sum_{s^t|s^{t-1}}1_{j,t}\lambda_t \qquad (17)\\ = & \left\{\theta\left[\left(x_{t-1}-\frac{\pi_h}{\pi_l}\frac{\tilde{p}_{t-1}}{p_{t-1}}\left(1-x_{t-1}\right)\right)\left(z_{t-1}+d_{t-1}\right)\right]^{\theta-1}+1-\delta\right\}\pi_l\sum_{s^t|s^{t-1}}1_{l,t}\lambda_t q_{l,t} \\ & +\left\{\theta\left[\left(x_{t-1}+\frac{\tilde{p}_{t-1}}{p_{t-1}}\left(1-x_{t-1}\right)\right)\left(z_{t-1}+d_{t-1}\right)\right]^{\theta-1}+1-\delta\right\}\pi_h\sum_{s^t|s^{t-1}}1_{h,t}\lambda_t q_{h,t} \\ & -\zeta_{t-1}\eta \end{aligned}$$

[26] In order to obtain equation (17), we derive the first order condition with respect to deposits and simplify it by using the expression in (16).

B.3 Step 3: Bond Prices

Using (15), we rewrite the equilibrium condition for the choice of x_{t-1}, equation (16), as below:

$$
\begin{aligned}
&\left(\frac{1}{p_{t-1}} - \frac{\pi_l}{\tilde{p}_{t-1}}\left(1+\frac{\pi_h}{\pi_l}\frac{\tilde{p}_{t-1}}{p_{t-1}}\right)\right)\sum_{s^t|s^{t-1}} 1_{l,t}\lambda_t \\
= \quad &\left\{\theta\left[\left(x_{t-1}+\frac{\tilde{p}_{t-1}}{p_{t-1}}(1-x_{t-1})\right)(z_{t-1}+d_{t-1})\right]^{\theta-1}+1-\delta\right\}\left(1-\frac{\tilde{p}_{t-1}}{p_{t-1}}\right)\pi_h\sum_{s^t|s^{t-1}} 1_{h,t}\lambda_t q_{h,t} \\
&-\zeta_{t-1}\eta
\end{aligned}
$$

Using $\pi_l + \pi_h = 1$, we can simplify the left hand side of the above equation and write it equivalently as:

$$
\left(1-\frac{\tilde{p}_{t-1}}{p_{t-1}}\right)\cdot \Xi - \zeta_{t-1}\eta = 0 \tag{18}
$$

$\Xi \equiv \left\{\theta\left[\left(x_{t-1}+\frac{\tilde{p}_{t-1}}{p_{t-1}}(1-x_{t-1})\right)(z_{t-1}+d_{t-1})\right]^{\theta-1}+1-\delta\right\}\pi_h \sum_{s^t|s^{t-1}} 1_{h,t}\lambda_t q_{h,t} + \frac{\pi_l \sum_{s^t|s^{t-1}} 1_{l,t}\lambda_t}{\tilde{p}_{t-1}}$. Notice that $\Xi > 0$, unless all financial intermediaries go broke. Then, equation (18) implies that, in the absence of capital regulation or if the capital regulation does not bind (i.e. $\eta = 0$ or $\zeta_{t-1} = 0$), the primary and secondary market bond prices are equated, $\tilde{p}_{t-1} = p_{t-1}$. However, if $\eta > 0$ and capital regulation binds $\zeta_{t-1} > 0$, then equation (18) implies that $\tilde{p}_{t-1} < p_{t-1}$.

B.4 Step 4: Primary Market Bond Price and Return to Deposits

We combine equations (16) and (17) to eliminate the term $\zeta_{t-1}\eta$. We find:

$$
\begin{aligned}
&\frac{1}{p_{t-1}}\sum_{s^t|s^{t-1}} 1_{l,t}\lambda_t - R^d_{t-1}\sum_{j\in\{h,l\}}\pi_j\sum_{s^t|s^{t-1}} 1_{j,t}\lambda_t \\
= \quad &\left\{\theta\left[\left(x_{t-1}-\frac{\pi_h}{\pi_l}\frac{\tilde{p}_{t-1}}{p_{t-1}}(1-x_{t-1})\right)(z_{t-1}+d_{t-1})\right]^{\theta-1}+1-\delta\right\}\pi_h\frac{\tilde{p}_{t-1}}{p_{t-1}}\sum_{s^t|s^{t-1}} 1_{l,t}\lambda_t q_{l,t} \\
&-\left\{\theta\left[\left(x_{t-1}+\frac{\tilde{p}_{t-1}}{p_{t-1}}(1-x_{t-1})\right)(z_{t-1}+d_{t-1})\right]^{\theta-1}+1-\delta\right\}\frac{\tilde{p}_{t-1}}{p_{t-1}}\pi_h\sum_{s^t|s^{t-1}} 1_{h,t}\lambda_t q_{h,t}
\end{aligned} \tag{19}
$$

Using (13) and (14), equation (19) becomes $R^d_{t-1} \geq \frac{1}{p_{t-1}}$. This completes the proof of Proposition 1 for the case in which the high risk intermediary sells all bonds in the secondary bond market. The other cases are derived analogously, but are omitted here to keep the exposition short.

C Model Extension: Private Bonds

In the model extension discussed in Section 4.3, the financial intermediaries' first stage problem changes to:

$$\max \sum_{j\in\{h,l\}} \pi_j \sum_{s^t|s^{t-1}} \lambda\left(s^t\right) V_j\left(s^t\right)$$

subject to:

$$z\left(s^{t-1}\right) + d\left(s^{t-1}\right) = k\left(s^{t-1}\right) + p\left(s^{t-1}\right) b\left(s^{t-1}\right)$$

$$V_j\left(s^t\right) = \left(1-\pi_F\right)\max\left\{\begin{array}{l} q_j\left(s_t\right)\left[k\left(s^{t-1}\right)+\tilde{p}\left(s^{t-1}\right)\tilde{b}_j\left(s^{t-1}\right)\right]^{\theta}\left[l\left(s^{t-1}\right)\right]^{1-\theta-\alpha} \\ +q_j\left(s_t\right)\left(1-\delta\right)\left[k\left(s^{t-1}\right)+\tilde{p}\left(s^{t-1}\right)\tilde{b}_j\left(s^{t-1}\right)\right] \\ +\left[b\left(s^{t-1}\right)-\tilde{b}_j\left(s^{t-1}\right)\right]-R^d\left(s^{t-1}\right)d\left(s^{t-1}\right)-W_j\left(s^t\right)l\left(s^{t-1}\right),0 \end{array}\right\}$$

$$+\pi_F\max\left\{\begin{array}{l} q_j\left(s_t\right)\left[k\left(s^{t-1}\right)+\tilde{p}\left(s^{t-1}\right)\tilde{b}_j\left(s^{t-1}\right)+\tilde{p}\left(s^{t-1}\right)a_j\left(s^{t-1}\right)\right]^{\theta}\left[l\left(s^{t-1}\right)\right]^{1-\theta-\alpha} \\ +q_j\left(s_t\right)\left(1-\delta\right)\left[k\left(s^{t-1}\right)+\tilde{p}\left(s^{t-1}\right)\tilde{b}_j\left(s^{t-1}\right)+\tilde{p}\left(s^{t-1}\right)a_j\left(s^{t-1}\right)\right] \\ +\left[b\left(s^{t-1}\right)-\tilde{b}_j\left(s^{t-1}\right)\right]-\left(1+\xi_j\left(s^{t-1}\right)\right)a_j\left(s^{t-1}\right) \\ -R^d\left(s^{t-1}\right)d\left(s^{t-1}\right)-W_j\left(s^t\right)l\left(s^{t-1}\right),0 \end{array}\right\}$$

$$z\left(s^{t-1}\right)/k\left(s^{t-1}\right) \geq \eta$$

where the real cost of stamping bonds $\xi_j\left(s^{t-1}\right)$ is defined as:

$$\xi_j\left(s^{t-1}\right) = \left\{\begin{array}{ccc} 0.01 & \text{if} & a_j\left(s^{t-1}\right) > 0 \\ 0 & \text{if} & a_j\left(s^{t-1}\right) \leq 0 \end{array}\right\}$$

Since these are assumed to be the real costs, we need to subtract $\left(1-\pi_m\right)\sum_{j=l,h}\pi_j\xi_j\left(s^{t-1}\right)a_j\left(s^{t-1}\right)$ from the total output. Also, in order for the private bond market to clear we must have:

$$\sum_{j=l,h}\pi_j a_j\left(s^{t-1}\right) + a_F\left(s^{t-1}\right) = 0,$$

where $a_F\left(s^{t-1}\right) < 0$ is the quantity of private bonds purchased by foreign investors. In exchange, foreign investors give $\tilde{p}\left(s^{t-1}\right) a_F\left(s^{t-1}\right)$ in real capital to domestic financial intermediaries. After the production takes place, foreigners receive $a_F\left(s^{t-1}\right)$ in return, if the aggregate state is good, and only $0.8 \times a_F\left(s^{t-1}\right)$, if the aggregate state is bad.[27]

[27] The values of π_F, ξ and the recovery rate of 80 percent are arbitrary and were chosen for illustration purposes.

D Tables

Table 1: Calibrated Parameters

Parameter/Value	Moment
$\beta = \left(\frac{1}{1.04}\right)^{1/4}$	Real interest rate of 4 percent
$\theta = 0.29$	Capital income share[1]
$\tau = 0.008\%$	Brokerage fees for the issuance of U.S. T-bills[2]
$\Phi = \begin{bmatrix} 0.9447 & 0.0553 \\ 0.20 & 0.80 \end{bmatrix}$	Average length of expansions/contractions of business sector
$\pi_l = 0.85$, $\pi_h = 1 - \pi_l = 0.15$	Sensitivity analysis

[1]This is the average share for the corporate non-financial sector from 1948 to 2009. [2]Stigum (1983, 1990) reports values between 0.0013% and 0.008%. We use the upper bound.

Table 2: ESTIMATED PARAMETERS

PARAMETER	VALUE
The following parameters are determined jointly to match the moments in Table 3	
Mean value added share of corporate nonfinancial sector[1]	$\pi_m = 0.6949$
Depreciation rate	$\delta = 0.0264$
Fixed factor income share	$\alpha = 0.00070317$
Productivity parameters	
nonfinancial firms	$q_m(\overline{s}) = 0.9617$
	$q_m(\underline{s}) = 0.9281$
low risk financial firms	$q_l(\overline{s}) = 0.9381$
	$q_l(\underline{s}) = 0.9344$
high risk financial firms	$q_h(\overline{s}) = 1$ (normalization)
	$q_h(\underline{s}) = 0.6785$

[1]The value added share of the corporate non-financial sector is computed relative to the business sector.

Table 3: Comparison of Data and Model Moments

Moment	Data in %	Model in %
Coefficient of variation of output[1]	3.75	3.94
Coefficient of variation of household net worth[2]	8.17	9.11
Average maximum decline in output during contractions[3]	6.48	6.98
Average deposits over total household financial assets[2]	17.2	26.0
Recovery rate in case of bankruptcy[4]	42.0	28.4
Mean output share of nonfinancial sector[5]	66.9	71.3
Average capital depreciation rate in economy	2.5	2.5
Equity to asset ratio of the financial sector[2,6]	7.6	5.2

[1]Output is measured as the value added for the business sector from 1987Q1 to 2010Q2. This is also the reference period for the other moments, unless otherwise stated. [2]From the U.S. Flow of Funds accounts. [3]This is the absolute decline taking the growth trend into account. [4]As reported in Acharya, Bharath, and Srinivasan (2003). [5]We identify the nonfinancial sector with the corporate non-financial sector. [6]When calculating the equity to asset ratio, we exclude mutual funds.

Table 4: Welfare and Risk Taking Results Relative to Social Planner with Resource Reallocation

Experiment	LTCE in %	Risk taking in %
Equilibrium without secondary market activity	−0.8754	33.1
2nd best	−0.0431	23.6
2nd best: −0.1 percentage points	−0.0433	21.1
2nd best: +0.1percentage points	−0.0436	26.2
2nd best with capital regulation	−0.0716	−9.1

Table 5: Sensitivity Analysis for Fraction of High Risk Intermediaries

	LTCE in %			Risk taking in %		
π_h value	0.13	**0.15**	0.17	0.13	**0.15**	0.17
Equilibrium without secondary market activity	−0.7814	**−0.8754**	−0.9624	37.4	**33.1**	29.1
2nd best: −0.1 percentage points	−0.0536	**−0.0433**	−0.0474	9.4	**21.1**	13.3
2nd best	−0.0439	**−0.0431**	−0.0397	20.4	**23.6**	21.6
2nd best: +0.1percentage points	−0.4403	**−0.0436**	−0.0428	89.4	**26.2**	30.6

E Figures

Figure 1: Risk Allocation in the Primary and the Secondary Market.

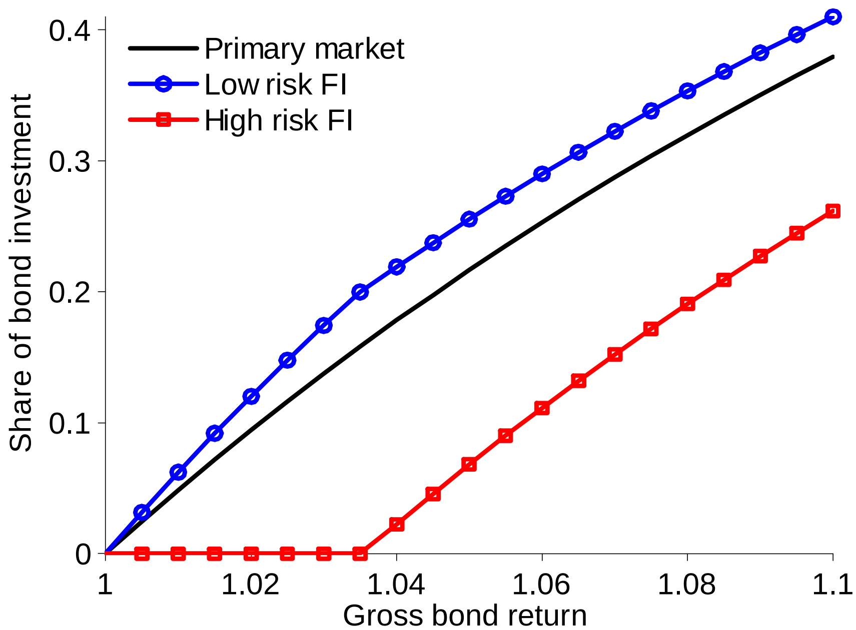

Figure 2: Simulation Results for Baseline Model

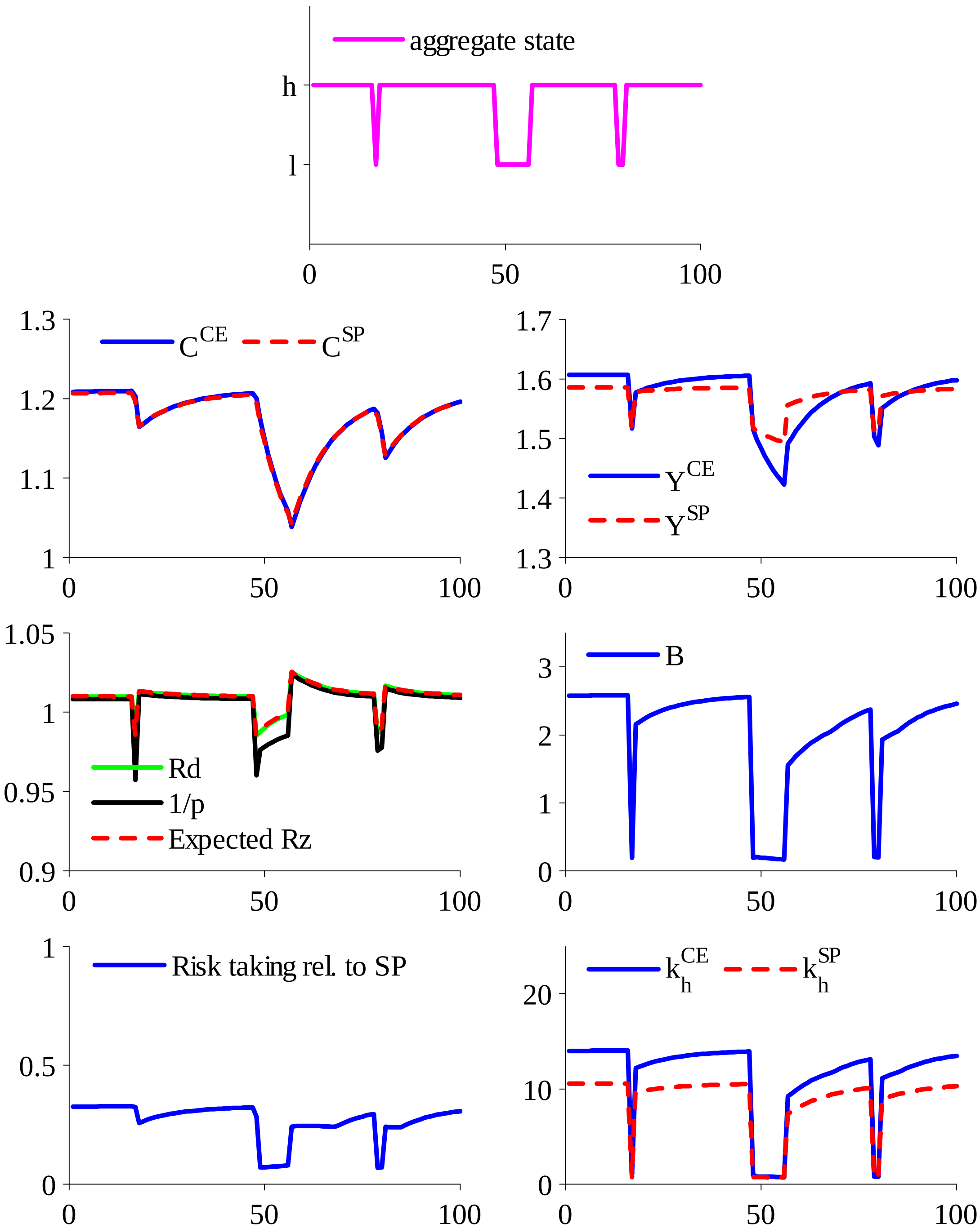

Figure 3: Simulation Results for Model Extension

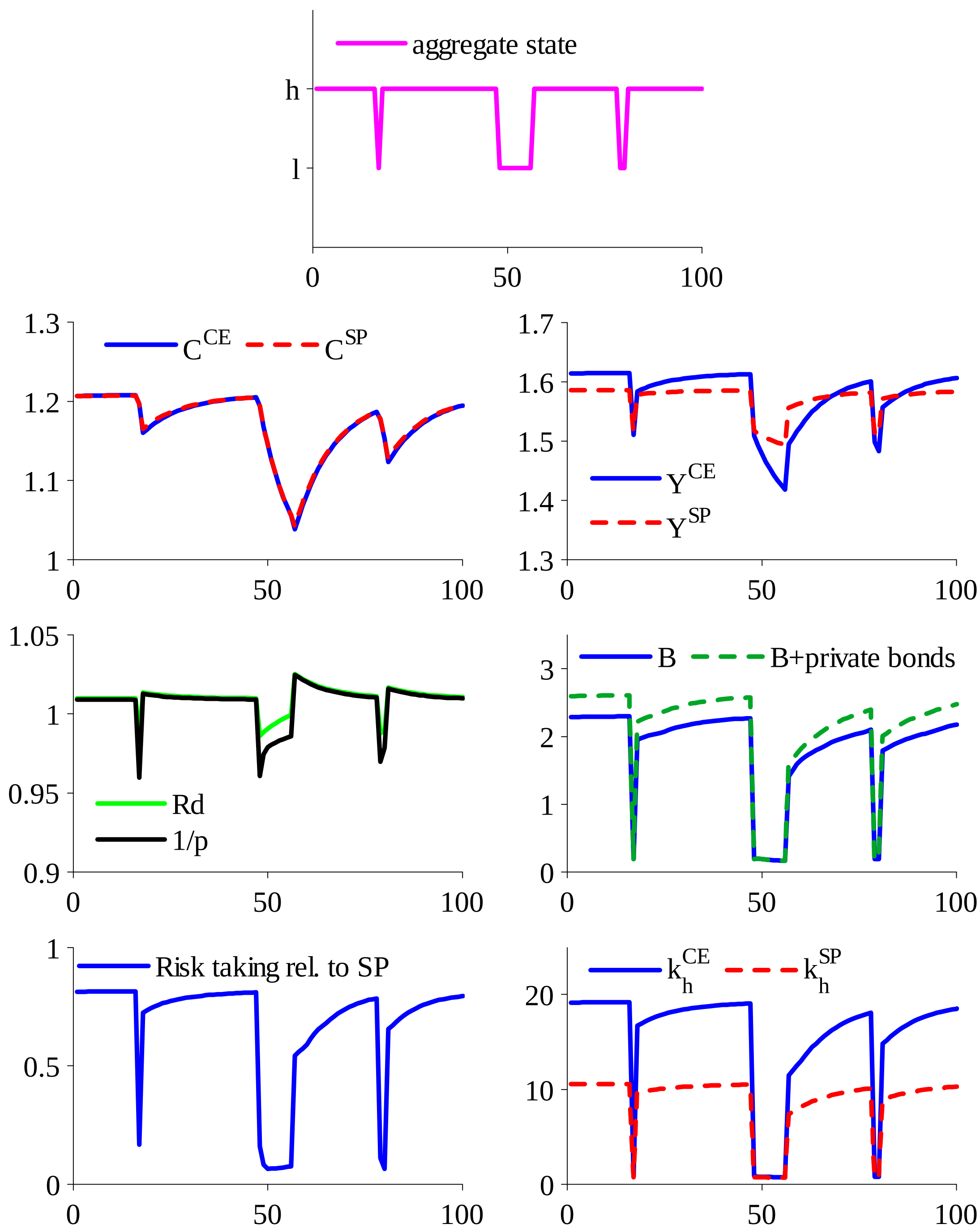

Figure 4: Welfare Loss Relative to the Social Planner, in LTCE (in Percent)

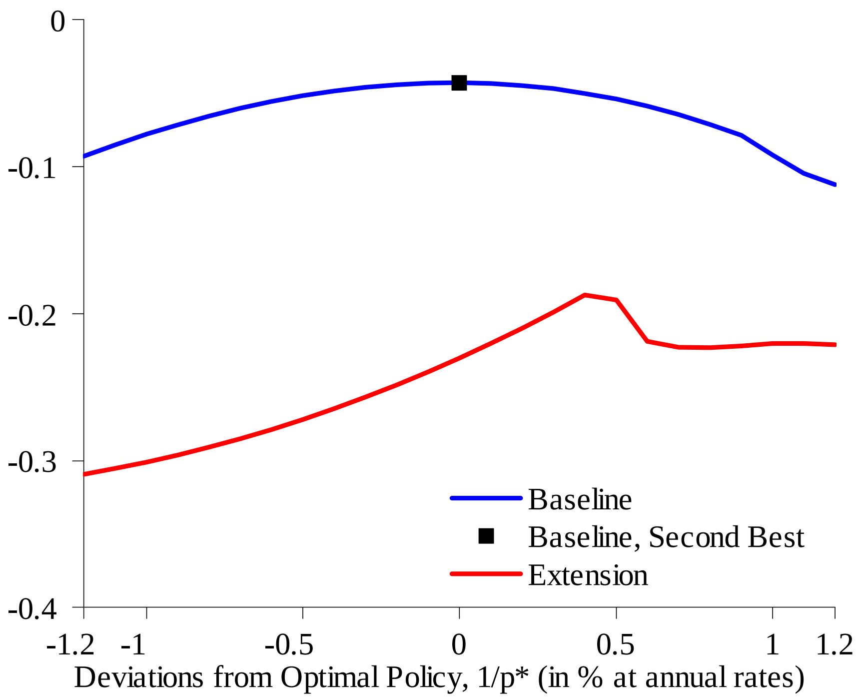

Figure 5: Risk Taking Relative to the Social Planner (in Percent)