MACRO POLICY RESPONSES TO NATURAL RESOURCE WINDFALLS
AND THE CRASH IN COMMODITY PRICES

Frederick van der Ploeg, University of Oxford*"**

Abstract

Policy prescriptions for managing natural resource windfalls are based on the permanent income hypothesis: none of the windfall is invested at home and saving in an intergenerational SWF is dictated by smoothing consumption across different generations. Furthermore, with Dutch disease effects the optimal response is to intertemporally smooth the real exchange rate, smooth public and private consumption, and limit sharp fluctuations in the intersectoral allocation of production factors. We show that these prescriptions need to be modified for the following reasons. First, to cope with volatile commodity prices precautionary buffers should be put in a stabilisation fund. Second, with imperfect access to capital markets the windfall must be used to curb capital scarcity, invest domestically and bring consumption forward. Third, with real wage rigidity consumption must also be brought forward to mitigate transient unemployment. Fourth, the real exchange rate has to temporarily appreciate to signal the need to invest in the domestic economy to gradually improve the ability to absorb the extra spending from the windfall. Fifth, with finite lives the timing of handing back the windfall to the private sector matters and consumption and the real exchange rate will be volatile. Finally, with nominal wage rigidity there is a short-run role for monetary and exchange rate policy to respond to a crash in oil prices.

Keywords: Dutch disease, permanent income, volatility, capital scarcity, domestic investment, Dutch disease, absorption constraints, overlapping generations, nominal wage rigidity

JEL codes: E60, F34, F35, F43, H21, H63, O11, Q33

August 2016

* Presented at the Closing Conference of the BIS CCA Research Network on “The Commodity Cycle: Macroeconomic and Financial Stability Implications”, Mexico City, 18-19 August 2016. Support from the BP funded OXCARRE is gratefully acknowledged. I have benefited from helpful comments by Assia Elgouacem, Chiara Ravetti and Sam Wills and the participants of the conference on an earlier draft.
** OXCARRE, Department of Economics, Manor Road Building, Oxford OX1 3UQ, United Kingdom. Phone: +44-1865-281285. E-mail: rick.vanderploeg@economics.ox.ac.uk. Also affiliated with the St. Petersburg State University, 7/9 Universitetskaya nab., St. Petersburg, 199034 Russia, and Vrije Universiteit Amsterdam.
1. Introduction

Countries with substantial parts of their government and export revenue consisting of the revenue from selling natural resources on world markets often have bad records for economic growth, especially in countries with poor institutions, inadequate rule of law, lack of financial development and not very open to international trade. These countries often experience sharp appreciations of the real exchange rate, which hurts non-resource export industries so that production factors are reallocated from the tradables to the non-traded sectors. As a result, private consumption and public spending suffer from excessive volatility which damages welfare. Many of these resource-rich countries are still on a developing path and seem unable to harness the once-in-a-lifetime opportunity of a temporary natural resource windfall to boost the process of economic development. In the light of this is it is important to ask how fiscal and monetary policy can contribute to a better use of natural resource windfalls.

The main policy prescriptions for managing natural resource windfalls are based on the permanent income hypothesis. It states that none of the windfall should be invested at home, so that all of the revenue is invested abroad preferably in an independently managed intergenerational sovereign wealth fund (SWF). It also states that saving and withdrawals of the fund are dictated by smoothing the consumption dividend across different generations, which implies borrowing ahead of the windfall, saving during the windfall and drawing a constant dividend from accumulated assets in the fund after the windfall. These prescriptions are akin to those that follow from the tax smoothing literature (e.g., Barro, 1979). We show that in a small open economy producing tradables and non-tradables this permanent-income approach to policy formulation is to smooth the time path of the real exchange rate to avoid big fluctuations in the sizes of the traded and non-traded sectors of the economy as well as smooth consumption. This provides a synthesis of the optimal management of windfalls based on the permanent income hypothesis and the literature on Dutch disease which focuses at the sectoral consequences of windfall revenues (e.g., Corden and Neary, 1982; Corden, 1984; Neary, 1988).

Although these permanent-income policy prescriptions serve as a useful benchmark, they do not speak to the specific challenges facing resource-rich developing countries (e.g., Venables,

---

1 The original empirical cross-country evidence was provided by Sachs and Warner (1977), which led to a voluminous literature surveyed by Frankel (2012) and van der Ploeg (2011). More recent micro evidence for the impact of natural resources based on more convincing econometric identification strategies is surveyed by Cust and Poelhekke (2015) and Cust and van der Ploeg (2016). Harding et al. (2016) offer empirical evidence that the real exchange rate appreciates following a giant oil discovery and Harding and Venables (2016) give evidence for the impact of non-resource exports. Kohlscheen and Sousa (2016) give empirical VAR evidence for Dutch disease effects.

2 This extends earlier analysis by Matsen and Torvik (2005) on the optimal Dutch disease to a more micro-based general equilibrium framework.
2016). Such countries have to cope with the notorious volatility of commodity prices, capital scarcity and the too low investment in the domestic economy. They are confronted with transient periods of unemployment and big swings in the real exchange rate and consequently in the production factors being reallocated between traded and non-traded sectors. They have to deal with lags and absorption constraints in getting the domestic economy ready to efficiently absorb the extra consumer and investment demands resulting from the new wealth associated with the windfall. They face non-neutrality of government debt so the timing of handing back dividends to the private sector matters.

We show in a succession of simple models how each of these features can be accommodated in the analysis of how to optimally manage natural resource windfalls. Starting from the simplest two-period framework for the permanent-income benchmark, we introduce stochastic volatility of commodity prices, and show that this necessitates saving of precautionary buffers, which can be put in a separate stabilisation fund alongside the intergenerational fund. We then show how with imperfect access to international capital markets the windfall must be used to curb capital scarcity, bring the cost of borrowing down, boost investment in the domestic economy and bring consumption forward from future relatively rich to current relatively poor generations.

We show that with real wage rigidity consumption must be brought forward too to mitigate transient unemployment. Allowing for absorption constraints we show that the real exchange rate must appreciate to boost investment in domestic home-grown structures (infrastructure, health, education, etc.) so that with time a more efficient spending of the windfall can take place (cf. van der Ploeg and Venables, 2013). We then move to an infinite-horizon framework with finite lives where Ricardian equivalence no longer holds (Blanchard, 1985). We thus have that the timing of handing out revenue from the windfall matters for private agents. We allow households to have a higher rate of time preference than the return on financial assets. We use this framework to show that even a permanent-income rule leads to volatility of the withdrawals from the fund. This will thus lead to overshooting of the real exchange rate and of real consumption, which is made possible by households running down assets and the current account eventually turning into surplus. If the whole of a temporary windfall is immediately spent, consumption and the relative price of non-tradables increase by more on impact but fall back to their original values after the windfall has ceased. Households accumulate assets during the windfall and decumulate afterwards. Combining absorption constraints into one model, we offer simulations to compare the benchmark permanent-income rule with more pragmatic rules such as the bird-in-hand rule used in Norway and the spend-all rule used in countries that do not save their windfall. Unemployment caused by real wage rigidity is highest under spend-all rule. We also comment on the short-run role for monetary and exchange rate policy for dealing with a
crash in commodity prices when nominal wage rigidity prevails and compare nominal exchange rate peg with Taylor rules that target inflation, unemployment and the real exchange rate.

Section 2 reviews the permanent-income analysis for managing windfalls. Section 3 allows for stochastic volatility of commodity prices. Section 4 allows for capital scarcity. Section 5 returns to the permanent-income benchmark to show the optimality of smoothing the real exchange rate. Section 6 allows for real wage rigidity and temporary unemployment. Section 7 analyses absorption constraints. Section 8 departs from Ricardian debt neutrality and shows how timing of resource dividend matters. Section 9 puts it all together and offers simulations for various policy scenarios. Section 10 allows for unemployment caused by real wage sluggishness and nominal wage sluggishness. Section 11 concludes.

2. Permanent-income benchmark: managing windfalls in small open economies

To illustrate the optimal response of a small open economy to temporary and permanent changes in the revenue from selling commodities on world markets, we first present a benchmark analysis with the assumptions of one homogenous good that is traded freely on international capital markets and perfect access to international capital markets. Section 5 extends this benchmark to a model with a traded and a non-traded sector. To keep the analysis simple, we adopt a two-period model where capital letters denote the ‘future’ (period 2) and small letters the ‘present’ (period 1). Prices are normalised such that the price of final goods in each period is unity. In period 1 the excess of production income, denoted by $y$, and natural resource revenue, $n$, over consumption, $c$, is used for investment in physical capital, $k$, and/or net foreign assets, $f$. In period 2 production income $Y = Y(k)$ depends on how much is invested in physical capital, $k$. Total income includes exogenous resource revenues, $N$, what is left of the capital stock, $(1 - \delta)k$ where the depreciation rate is denoted by $0 \leq \delta \leq 1$, and principal plus interest income on foreign assets, $(1 + r)f$ where $r$ denotes the exogenous global return on foreign assets. The period 1 and period 2 budget constraints of the economy are thus

\[ (1) \quad c + k + f = y + n \quad \text{and} \quad C = Y(k) + (1 - \delta)k + (1 + r)f. \]

Policy makers maximise the utilitarian social welfare function $\left( c^{1-\eta} + (1 + \rho)^{-1} C^{1-\eta} \right) / (1 - \eta)$, where $\rho \geq 0$ denotes society’s rate of pure time preference and $\eta > 0$ is society’s coefficient of relative intergenerational inequality aversion. Rawlsian social welfare corresponds to $\eta \to \infty$ and Benthamite or utilitarian social welfare to $\eta \to 0$. The optimal policies must satisfy

---

3 Throughout we use Roman letters to denote functions and italics to denote variables.
The first part of (2) states that the return on physical capital must equal the return on foreign assets. It gives our first key result: the optimal value of \( k \) does not depend on \( n \) or \( N \). A windfall should thus not be used for investing in the domestic economy. With access to perfect capital markets investment should already be at its optimal level. Many countries, even those that do not suffer capital scarcity such as the Netherlands, use their windfalls for all kinds of infrastructure projects even when the social benefits do not weigh up against the costs.

The second part of (2) is the Keynes-Ramsey rule or Euler equation, which states that growth in real consumption depends on the return on assets relative to the pure rate of time preference. If the pure rate of time preference is set to the return on assets, i.e., \( \rho = r \), this gives full smoothing of the time paths of consumption. Total differentiation of the budget constraints (1) and solving for the effects of the windfall on consumption and foreign asset holdings, we obtain

\[
(3) \quad dc = dC = \frac{1 + r}{2 + r} \left( dn + \frac{1}{1 + r} dN \right) \quad \text{and} \quad df = \frac{d(n - N)}{2 + r}.
\]

The first part of (3) establishes that the time path of consumption is fully smoothed. The change in consumption is a constant fraction of the change in resource wealth (the present value of current and future resource revenue) indicated by the terms in the round brackets. The second part of (3) shows that the country borrows ahead of an anticipated windfall \( (dn = 0, dN > 0) \) and saves during a temporary windfall \( (dn > 0, dN = 0) \). A permanent windfall has no effects on foreign asset accumulation \( (dn = dN > 0 \Rightarrow df = 0) \).

What does (3) imply for the recent crash in the oil price \( (dn < 0) \)? If policy makers believe that the crash is permanent, they should depress current and future consumption by the full whack of the windfall \( (df = 0) \). This means cutting private consumption (by cutting transfers) and cutting public consumption permanently. If policy makers believe the oil price will eventually recover, they should borrow and cut present and future consumption by a smaller amount \( (df < 0) \).

**Benchmark Result 1:** Optimally managing a resource windfall in a small open economy with only traded goods production and perfect access to international capital markets implies that the whole windfall is invested abroad and none of it is invested in the domestic economy. The time path of consumption is smoothed by saving foreign assets if the windfall is temporary and borrowing from abroad if it is anticipated. The optimal response to a temporary crash in commodity prices is to borrow from abroad and to a permanent crash is to cut spending.
The permanent-income rule serves as a useful benchmark and outlines the principles of optimally managing an intergenerational sovereign wealth fund (SWF). However, it is based on bold assumptions. To relax these, we show how our benchmark prescriptions must be modified to allow for volatility of natural resource windfalls (section 3) and capital scarcity (section 4). We then modify the benchmark permanent-income analysis to get the prescription of smoothing the real exchange rate in a model with Dutch disease effects (see Benchmark Result 2, section 5). We show how this needs to be modified in the presence of unemployment (section 6). We show how absorption constraints necessitate real exchange rate volatility and the need to use part of the windfall to invest in domestic structures (sections 7 and 9). We also show the importance of non-neutrality of government debt for the timing of spending the windfall (sections 8 and 9). Finally, we discuss unemployment caused by real wage rigidity and by nominal wage rigidity, and the potential role for monetary policy (section 10).

3. Volatile windfalls: the case for a stabilisation and an intergenerational SWF

The benchmark analysis of section 2 indicates the need for an intergenerational SWF to spread the benefits of a natural resource windfall evenly across generations. Here we advocate the additional need for a stabilisation SWF to cope with the notorious volatility of commodity prices and resource windfalls. Although in principle one could hedge such risks away using futures markets, such markets often do not exist or are too thin and costly. Furthermore, few ministers of finance are prepared to take the political risk of hedging as electorate may have little sympathy for spending lots of public mony on hedging when there are other pressing needs. A stabilisation fund may then allow the country to accumulate prudent buffers.

To illustrate this in the simple context of our benchmark analysis, we assume that the future windfall $\hat{N}$ is stochastic and distributed with mean $\bar{N}$ and variance $\sigma^2 > 0$. To make our point, we suppose for simplicity that the return on financial assets and future production are known with certainty. Policy makers now maximise expected social welfare, $\frac{c^{1-\eta}}{1-\eta} + \frac{1}{1+\rho} \frac{E[C]^{1-\eta}}{1-\eta}$, subject to the budget constraints (1). This gives rise to the modified optimality conditions

$$Y'(k) = r + \delta$$

and

$$E[C] = \left[1 + \eta(1 + \eta) \left(\frac{\sigma}{E[C]}\right)^{\frac{1}{\eta}}\right] c > c.$$

As we assume that $r$ is known, the first part of $(2')$ is unaffected so still none of the windfall is invested in the domestic economy. Taking a second-order Taylor-series expansion of the
stochastic Keynes-Ramsey rule, \( u'(c) = E[u'(C)] \), gives the second part of (2'). Despite choosing \( \rho = r \), the time path of expected consumption slopes upwards. The reason is that there is prudential saving in period 1, which increases with the standard deviation of the future windfall normalised by the expected level of future consumption, the coefficient of relative risk aversion, \( \eta \), and the coefficient of relative prudence, \( 1 + \eta \). This prudential saving provides a buffer against large negative shocks to the future windfall to be put in a stabilisation fund.

**Modification 1:** With uncertain future windfalls it is optimal to initially consume less and engage in prudential savings, especially if policy makers are relatively prudent and risk averse and the future windfall is large relative to the future size of the economy.

In an infinite-horizon context prudential saving is larger if the stochastic shocks to the windfall last longer. This approach has been used to calibrate and estimate the optimal sizes of intergenerational and stabilisation fund for Iraq, Ghana and Norway (van den Bremer and van der Ploeg, 2013). Since the optimal size of the intergenerational fund is larger the more temporary the windfall whilst the stabilisation fund is bigger the longer lasting the windfall and the more persistent oil price shocks, the intergenerational fund pales into insignificance with the stabilisation fund for Iraq with oil reserves potentially lasting for centuries and the opposite for Norway whose oil and gas revenues taper off quickly in the next decades. The prudent buffer approach can also explain why the current account of resource rich small open economies is more likely to be in surplus (Bems and De Carvalho Filho, 2011). Two extensions are possible. First, with endogenous natural resource extraction rates, the stochastic version of the Hotelling rule implies more rapacious depletion of the natural resource (van den Bremer et al., 2016; Elgouacem, 2016). Second, the derivation of prudent saving buffers can be integrated with a capital asset pricing model (CAPM) to allow for stochastic returns on a host of financial assets as well as stochastic return on natural resource assets in the ground (van den Bremer et al., 2016). The main lessons are that the portfolio allocation should hedge against commodity price volatility, the demand for risky assets should be leveraged up and gradually deleveraged as the natural resource is depleted, and there should be more prudential saving if the oil price cannot be fully diversified on international asset markets. Requiring the fund to go short in risky assets when natural resource reserves are still high may lead to political obstacles. A more pragmatic approach that avoids heavy borrowing to hedge the natural resource wealth in the ground and targets a general equity index (e.g., the FTSE index) must realise that portfolio allocation above
the ground accentuates the risk of resource wealth in the ground as returns on exhaustible resources such as oil have since 2008 become positively correlated with stock market returns. Since such a pragmatic approach avoids hedging with specific financial assets whose returns are negatively correlated with below-ground resource wealth, it has to hold less risky assets (equities) than suggested by the normal CAPM model and gradually increase holdings of risky assets to their normal levels as natural resource wealth is depleted.

Less prudential saving buffers and a smaller stabilisation fund are required if the country can hedge commodity price risks using financial derivatives such as forward contracts or put options. The welfare gains from using such instruments results from a reduction in export revenue volatility, the need for less precautionary buffers and the enhanced ability of the country to borrow against future income from commodity exports amounts to several percentage points of annual consumption (Borensztein et al., 2013; Lopez-Martin et al., 2016). This hedging is used in Mexico, where laws are in place to regulate how the derivatives should be used. Although there are some countries who hedge uncertainty about future commodity revenue, many other countries refrain from it for obvious political reasons, because futures markets are too thin, not long-term enough and too costly, and politicians of countries that are dominant on world commodity markets do not want to be seen to manipulate these markets.

4. Capital scarcity: using windfalls to boost domestic investment

The benchmark analysis of section 2 indicated that none of the windfall should be used for investments in the domestic economy if there is access to perfect international capital markets. Although this is reasonable for advanced economies, it is not empirically relevant for many developing economies that find it hard to borrow and thus struggle with sub-optimally low levels of private and public investment. For those countries natural resource windfalls may provide a once-in-a-lifetime opportunity to boost domestic investment and thus promote growth and development. To illustrate this, we assume that the country is indebted and faces a risk premium on borrowing (cf. van der Ploeg and Venables, 2011, 2012). Let the interest to be paid on borrowing from abroad thus be endogenous and given by

\[ r = \bar{\rho} + \pi(f) > \bar{\rho} \]

with \( \pi' < 0 \). and \( \bar{\rho} \) the world interest rate. Empirical evidence indeed suggests that interest spreads rise with foreign indebtedness. With the pure rate of time preference set to the world interest rate, \( \rho = \bar{\rho} \)

the optimality conditions (abstracting from uncertain windfalls) become

---

4 One explanation for this is that monetary policies have been constrained by the zero lower bound on the nominal interest rate (Datta et al., 2016).
(2’)
\[ Y'(k) = r + \pi(f) + \pi'(f)f + \delta \quad \text{and} \quad C = \left( \frac{1+\bar{r} + \pi(f) + \pi'(f)f}{1+r} \right)^{\frac{1}{\eta}}. \]

Using the budget constraints (1) and the second part of (2’), we get the equilibrium condition

\[ C = \left( \frac{1+\bar{r} + \pi(f) + \pi'(f)f}{1+r} \right)^{\frac{1}{\eta}} = \frac{Y(k) + (1-\delta)k + N + [1+\bar{r} + \pi(f)]f}{y + n - k - f} > 1. \]

It is simplest to solve the resulting optimal policies diagrammatically. The first part of (2’) gives the optimality condition for domestic investment, which implies that investment is too low compared with what it would be if there was access to perfect international capital markets and no risk premium has to be paid. Investment increases if debt reduction curbs the cost of capital. This relationship is displayed by the horizontal kk-schedule in panel (a) of figure 1 in case of perfect international capital markets and by the upward-sloping kk-schedule in panel (b) in case of capital scarcity. The right-hand side of equation (4) increases in \( k \) and typically increases in \( f \) whilst the left-hand side decreases in \( f \). Hence, equation (4) is in both panel (a) and (b) drawn as the downward-sloping cc-schedule, which shifts by changes in \( n \) or \( N \).

**Figure 1: SWF management and domestic investment in response to a windfall**

(a) **Perfect access to capital markets**

(b) **Capital scarcity**

**Key:** The solid lines indicate outcomes for when there is no windfall; the dashed lines for a temporary windfall; and the dotted lines for an anticipated windfall. A temporary windfall shifts the equilibrium from \( E \) to \( T \); an anticipated windfall shifts the equilibrium from \( E \) to \( A \).

With capital scarcity, panel (b) indicates that a temporary increase in natural resource revenue shifts out the cc-schedule and thus boosts both \( f \) and \( k \). Part of a temporary windfall is thus saved and, different from before, part of it is used to invest in the domestic economy and promote growth and development. But with perfect access to capital markets, panel (a) indicates
that the temporary windfall does not affect investment in the domestic economy and merely leads to saving to smooth consumption across generations.

With capital scarcity, an anticipated future windfall shifts back the \( cc \)-schedule and thus \( f \) and \( k \) fall. The optimal response to an anticipated windfall is thus to borrow more on international markets to boost consumption ahead of the windfall. It is the extra borrowing that aggravates capital scarcity, pushes up the cost of borrowing for domestic investment projects, and thus curbs domestic investment. Without capital scarcity, the borrowing undertaken to smooth consumption does not adversely affect investment in the domestic economy – see panel (a).

Since from (4) \( C > c \), we establish that the outward shift of an increase in \( n \) exceeds the inward shift of an equal increase in \( N \). It follows that a permanent windfall of natural resource revenues \( (dn = dN > 0) \) shifts out the \( cc \)-locus (by less than a temporary windfall) and thus boosts \( f \) and \( k \) in case of capital scarcity. A permanent windfall is thus used to curb indebtedness and bring down the cost of domestic borrowing, which helps to bring domestic investment closer to that in economies without capital scarcity. Consumption increases both today and in the future. As a result of the debt reduction, the ratio of future to current consumption falls. This means that the windfall is frontloaded towards present consumption, because current generations are poorer than future generations and have a higher marginal utility of consumption.

Finally, the effects of a crash in oil prices and in natural resource revenue is to shift the \( cc \)-curve back in figure 1 and therefore induces an increase in debt, a rise in borrowing costs, and a drop in investment in the domestic economy.

**Modification 2:** If developing economies pay an interest premium that rises with indebtedness, it is optimal to use part of a natural resource windfall to cut foreign debt and alleviate capital scarcity, to cut the domestic cost of borrowing and to boost investment in the domestic economy. Another part is used to boost consumption, especially of current relatively poor generations. A crash in commodity price induces more indebtedness and a fall in domestic investment.

We have focused on a regime where the windfall so large that it does not alleviate all capital scarcity, but the windfall does the process of development a leg-up. With very large windfalls, capital scarcity may eventually be fully alleviated so that the domestic cost of borrowing is brought down to the world interest rate. In that case, the country eventually reverts to the regime discussed in section 2 and will accumulate part of the remaining windfall in a SWF to ensure a sustained boost to consumption out of interest on the SWF (van der Ploeg and Venables, 2011).
It is also important to allow for intertemporal absorption constraints as it is costly to increase spending quickly (Berg et al., 2013). An application of the implications of capital scarcity and intertemporal absorption constraints to managing oil revenue in Ghana shows how the windfall is used to bring consumption forward, to reduce capital scarcity, and to promote growth and development whilst not ramping up public investment too rapidly (van der Ploeg, 2012).

Although we have highlighted capital scarcity, there may be other factors why investment is sub-optimally low and why it is justified to spend part of the windfall on investment in the domestic economy. For example, if international capital markets count in political uncertainty and hold back investment in the economy, it might make sense to use part of the windfall to boost domestic investment. However, with political uncertainty there is the danger that the windfall is used to boost partisan investment which does not necessarily enhance growth.

Instead of modelling capital scarcity as an interest premium on foreign debt, one could say that there is maximum that can be borrowed by a country that depends on GNP. Following a commodity price crash, the real depreciation and fall in the price of non-tradables induces a drop in GNP and thus curbs the collateral and borrowing ability of the country. Dutch disease effects operating in reverse can thus aggravate borrowing and capital scarcity. These collateral constraints lead to sudden stops with dramatic effects on the current account and a slower extraction of the natural resource, especially when policy makers are learning about whether the drop in the international commodity price is permanent or not (Bejarano et al., 2016).

5. Revisiting the permanent-income benchmark: Dutch disease effects

Abstracting from uncertain windfalls and capital scarcity and also from investments in domestic capital, we now extend our permanent-income benchmark of section 2 to allow for structural change with a traded sector and a non-traded sector. We assume perfect competition on all factor and goods markets, full employment of all factors of production and free movement of these factors between the two sectors in each period. Each period is described by the model of Dutch disease put forward by Neary (1988). To give more flavour to the analysis, the public and the private sector are now disaggregated with the government having access to international capital markets but liquidity-constrained households do not. Prices are normalised such that the price of traded final goods in each period is unity. We use a unit expenditure function to describe homothetic preferences over consumption of tradables and non-tradables in each period. We assume that each sector has a production function that displays constant returns to

---

5 We abstract from learning by doing or agglomeration effects in the traded sector, so that a temporary windfall does not lead to a permanent loss in output as in van Wijnbergen (1984a).
scale and that all variable factors of production can be instantaneously adjusted. We use the concept of a GNP function, which is defined as the maximum revenue from selling tradables and non-tradables minus the costs of all variable factors of production.

\( a. \) The present

The equilibrium conditions for the present markets of non-tradables and tradables are

\[
(5) \quad e'(p)c = y_p(p,l) \quad \text{and} \quad e(p)c + f = y(p,l) + n,
\]

where \( p \) and \( c \) denote the relative price of non-tradables and real consumption (or utility) during period 1. The unit expenditure function for producing \( c \) is \( e(p) \) and the share of non-tradables in consumption is \( \theta = pe'(p)/c \). The GNP function is \( y(p, l) \), where the latter suppresses the supplies of fixed factors of production other than the aggregate supply of labour \( l \) and also suppresses the exogenous costs of variable factors of production. The supply of non-tradables and the wage in terms of tradables are \( y_p \) and \( y_l \), respectively. We suppose that tradables are intensive in labour, so \( y_{pl} \leq 0 \). The first part of (5) states that demand for non-tradables must equal the supply of non-tradables. The second part of (5) states that the excess of production and commodity income over consumer spending is saved.

Total differentiation of (5) yields the following comparative statics:

\[
(6) \quad c = d(n - f, l) \quad \text{with} \quad d_{n-f} = 1/e(p) > 0, \quad d_f = y_i / e(p) > 0, \quad \text{and}
\]

\[
p = p(n - f, l) \quad \text{with} \quad p_{n-f} = \frac{p}{e(p)c} \frac{1}{e'd + e'} > 0
\]

\[
(7) \quad p_f = \frac{1}{e(p)c} \frac{1}{e'd + e'} (y_i - \theta y_p) > 0,
\]

where \( e'd \equiv -pe'(p)/c'(p) > 0 \) is the price elasticity of the demand for non-tradables, \( e' \equiv py_{pp}/y_p > 0 \) is the price elasticity of the supply of non-tradables, and \( d_f > 0 \) is the real consumption wage. We conclude from (6) that a windfall boosts real consumption in as far as these revenues are not saved. Equation (7) indicates that as a consequence the real exchange rate appreciates (i.e., the relative price of non-tradables rises) to the extent that the windfall is not saved. The real appreciation is especially large if demand and supply of non-tradables do not respond much to changes in prices. Equilibrium after the windfall is restored by a contraction of the tradables sector and an expansion of the non-tradables sector, which results from factors moving from the tradables to the non-tradables sector. Still, consumption of tradables increases and is met by additional imports from abroad.
b. The future

The future is described by the market equilibrium conditions

\begin{align*}
E'(P)C = Y_P(P, L) & \quad \text{and} \quad Y(P, L) + (1 + r)f = E(P)C, \\
(5')
\end{align*}

The main difference between (5) and (5') is that the future has additional income (principal plus interest) from saving foreign assets. It thus follows from total differentiation of (5') that

\begin{align*}
C = D(N + (1 + r)f, L), & \quad D_p = 1/E(P) > 0, \quad D_L > 0, \quad \text{and} \\
(6')
P = P(N + (1 + r)f, L), & \quad P_{N + (1 + r)f} = P E(P) C (\varepsilon_d + \varepsilon_e) > 0, \quad P_L > 0.
\end{align*}

\begin{align*}
\mathcal{C} = (1 + r) \left( \frac{E(p)}{1 + \rho E(P)} \right)^{\frac{1}{\eta}} c & \quad \text{or} \quad \mathcal{C} = \left( \frac{E(p)}{E(P)} \right)^{\frac{1}{\eta}} c \quad \text{if} \quad \rho = r.
\end{align*}

\text{c. Optimal management of the SWF and the real exchange rate}

The permanent-income approach again underlies the optimal management of natural resource windfalls in this benchmark and is akin to the theory of tax smoothing and optimal government debt (Barro, 1979). The country maximises life-time utility,

\[ \frac{1}{1 - \eta} \left( c^{1-\eta} + \frac{1}{1 + \rho} C^{1-\eta} \right), \]

subject to the present-value budget constraint,

\[ e(p)c + \frac{1}{1 + r} E(P)C = y(p, l) + n + \frac{1}{1 + r} [Y(P, L) + N]. \]

This gives the Keynes-Ramsey rules for \textit{intertemporal} smoothing of real consumption:

\begin{align*}
\mathcal{C} = \left( \frac{1 + r}{1 + \rho E(P)} \right)^{\frac{1}{\eta}} c & \quad \text{or} \quad \mathcal{C} = \left( \frac{E(p)}{E(P)} \right)^{\frac{1}{\eta}} c \quad \text{if} \quad \rho = r.
\end{align*}

Without loss of generality, we let unit expenditure functions, \( \phi(\cdot) \) and \( \Phi(\cdot), \) and \( l \) and \( L \) be the same for the present and future. In that case, equation (9) implies full intertemporal smoothing.
of the real exchange rate, \( p = P \), and thus of real consumption, \( c = C \). Using equations (7) and (7'), we can solve for the optimal size of the SWF from

\[
p(n - f, l) = P(N + (1 + r)f, L).
\]

Total differentiation of (11) thus gives the optimal amount of saving for this economy as

\[
f = F(n, N), \quad F_n = \frac{p'(e)}{p'(e) + (1 + r)P'(E)} > 0, \quad F_N = -\frac{(1 + r)P'(E)}{p'(e) + (1 + r)P'(E)} < 0.
\]

From (10) and (11) we see that the permanent-income rule requires intertemporal smoothing of consumption and the real exchange rate. It is thus optimal to save a temporary windfall (\( dn > 0 \)) and to borrow ahead of an anticipated windfall of commodity revenue (\( dN > 0 \)). Hence, thanks to access to international capital markets, a boost to present or future windfall revenue boosts present and future consumption by equal amounts.

**Benchmark Result 2:** Using an intergenerational SWF to smooth real consumption by borrowing ahead of a natural resource windfall and saving during a declining windfall also smooths the real exchange rate. This avoids volatile reallocations of labour and other production factors from the traded to the non-traded sector. The windfall leads instead to a small permanent contraction of the traded and expansion of the non-traded sector.

We use figure 2 to illustrate these optimal responses graphically and compare them with some non-optimal responses to a temporary crash in commodity prices and windfall revenue. Equations (7) and (7') are shown as the downward-sloping \( pp \)-schedule and the upward-sloping \( PP \)-schedule, respectively. The temporary crash shifts the \( pp \)-curve to the left, and as result under the permanent-income rule both the current and the future real exchange rate depreciate, the current account goes into deficit and the country borrows from abroad. This ensures that the real exchange rate and real consumption are smoothed over time. Today and in the future, the non-traded sector contracts and the traded sector expands.

If the country instead fails to smooth consumption and the real exchange rate by borrowing and not tightening spending at all, the new equilibrium is not PI but NT. As a result, current consumption, the current real exchange rate and sectoral allocation of production factors are unaffected, but future consumption falls by more and the future real exchange rate depreciates by more than under the permanent-income rule. Also, the future expansion of the traded sector and contraction of the non-traded sector are bigger than under the permanent-income rule.
Figure 2: Real exchange rate, foreign assets and a temporary commodity price crash

Key: The solid lines correspond to initial outcome and dashed lines to after a temporary crash in commodity prices. The equilibrium shifts from E to PI under the permanent-income rule, from E to NB under full tightening of spending if international borrowing is ruled out, and from E to NT if spending is not tightened at all and the country borrows from abroad instead.

The alternative of taking the full hit of the commodity price crash by cutting spending shifts the equilibrium from E to NB in which case future outcomes are unaffected, but the current real exchange rate has to depreciate by more and consumption has to fall by more than under the permanent-income rule. Both these latter policies introduce too much volatility in consumption, the real exchange rate and in sectoral allocation of production factors, and are thus sub-optimal.

6. Mitigating unemployment during a commodity price crash

A commodity price crash induces depreciation of the real exchange rate and, if the real consumption wage is rigid in the short run, can cause unemployment. To analyse how this affects Results 1 and 2, we split period 1 into an initial sub-period denoted by tildes with real wage rigidity and another one with full employment. Let the real consumption wage in this initial sub-period be fixed at its pre-crash value \( z \), so \( y_l(\tilde{p},\tilde{I}) / e(\tilde{p}) = z \). The real wage facing non-tradables producers, \( y_l / \tilde{p} = ze / \tilde{p} \), falls and that facing tradables producers, \( y_t = ze \), rises with the price of non-tradables, so employment falls in the traded sector but rises in the non-traded sector. If tradables production is intensive in labour, \( y_{pl} \leq 0 \), and \( y_{pl} < 0 \), this gives \( \tilde{I} = \tilde{I}(\tilde{p}, z) \) with \( \tilde{I}_l < 0 \) and \( \tilde{I}_p < 0 \). Matching demand and supply of non-tradables,
e'(\bar{p}) \hat{c} = y_p (\bar{p}, \bar{I}), \quad \text{gives} \quad \bar{p} = \hat{p}(\bar{c}, \bar{I}) \quad \text{with} \quad \hat{p}_c > 0 \quad \text{and} \quad \hat{p}_I > 0 \quad \text{so we solve for} \quad \bar{I} = \tilde{I}(\hat{c}, \hat{z}) \quad \text{with} \quad \tilde{I}_c > 0 \quad \text{and} \quad \tilde{I}_c < 0 \quad \text{where we assume} \quad \hat{p}_c \hat{p}_I < 0, \quad \text{and thus} \quad \bar{p} = \hat{p}(\hat{c}, \hat{z}) \quad \text{with} \quad \hat{p}_c > 0 \quad \text{and} \quad \hat{p}_I < 0. \quad \text{A drop in consumption is thus associated with real depreciation and a drop in employment. The country maximises welfare} \quad \left( \hat{c}^{-\eta} + \hat{c}^{1-\eta} + (1 + \rho)^{-\frac{1}{\eta}} \hat{I}^{1-\eta} \right) / (1 - \eta), \quad \text{subject to the present-value budget constraint} \quad e(\bar{p}) \bar{c} + e(\bar{p}) c + E(P) C / (1 + r) = y(\bar{p}, \bar{I}) + y(p, 1) + n + [Y(P, 1) + N] / (1 + r), \quad \text{where we let} \quad l = L = 1 \quad \text{and} \quad \rho = r. \quad \text{The direct effects of} \quad \bar{p}, P \quad \text{and} \quad P \quad \text{cancel out due to the equilibrium conditions for the markets for non-tradables but the effect via} \quad \tilde{I} \quad \text{has to be taken account of. Hence, full intertemporal smoothing takes place under full employment,} \quad p = P \quad \text{and} \quad c = C, \quad \text{but with a rigid real consumption wage we get} \quad \left( \frac{\hat{c}^{-\eta}}{c} \right) = \frac{e(\bar{p}) - y_p (\bar{p}, \bar{I}) \hat{c}^{-\eta} (\hat{c}, \hat{z})}{e(\bar{p})} < \frac{e(\bar{p})}{e(\bar{p})}. \quad \text{The Keynes-Ramsey rule is thus distorted as it follows that} \quad \hat{c} > c \quad \text{and} \quad \bar{p} > p. \quad \text{To soften the blow to unemployment of a commodity price crash, it is thus best to cut consumption by less when the real consumption wage is rigid and to limit the depreciation of the real exchange rate.}

**Modification 3:** *If the real consumption wage is rigid in the short run and the traded sector is intensive in labour, consumption should be brought forward after a commodity price crash to mitigate adverse effects of depreciation of the real exchange rate on short-run unemployment.*

In section 10 we introduce the adverse effect of the real depreciation on structures, intertemporal dynamics of the real consumption wage and nominal wage sluggishness.

### 7. Absorption constraints and the need to invest in home-grown structures

Many developing economies are confronted with bottlenecks in the productive capacity of the domestic economy. They thus find it tough to quickly and efficiently absorb the spending hike induced by a windfall and need the right amount of domestic infrastructure and capital, called ‘structures’, in place before the windfall can be spent. The permanent-income rule for smoothing the real exchange rate (result 1B) is then no longer valid. To show this, we allow for absorption constraints and structures (cf. van der Ploeg and Venables, 2013). We thus let production in each sector depend on home-grown structures, denoted by $s$. These consist of infrastructure, human capital, health, the quality of the civil service and the judiciary, etcetera, and must be produced by the non-traded sector. We assume that the non-traded sector is intensive in structures and the traded sector is intensive in labour, hence the short-run GNP.
function \( y(s, p, l) \) satisfies \( y_{pl}(s, p, l) - \delta > 0 \), where \( \delta > 0 \) is the depreciation rate of structures. If structures are produced entirely by the non-traded sector, the efficiency condition is \( y_s(s, p, l) = r + \delta - \frac{\dot{p}}{p} \). The user cost thus consists of the usual rental and depreciation charges minus the capital gains that result if firms anticipate prices of non-tradables to rise in the future. GNP may also depend on capital stock, \( k \), imported from abroad, but this can be w.l.o.g. subsumed in the production function. Real consumption is \( c = [e(p)\lambda]^{1/\eta} \), where \( \lambda \) is the social value of wealth which is smoothed over time, \( \dot{\lambda} = 0 \), as \( \rho = r \). With perfect capital markets the windfall revenue can be fully amortised. The windfall thus implies a drop in the social value of wealth and a permanent increase in consumer spending. We can trace the effects of a windfall-induced drop in \( \dot{\lambda} \) on structures and the real exchange rate from rewriting the condition for equilibrium in the market for non-tradables, \( y_p = e'(p)c + \ddot{s} + \delta s \), and rewriting the efficiency condition for structures, \( y_s(s, p, l) = r + \delta - \frac{\dot{p}}{p} \), as:

\[
\begin{align*}
\ddot{s} &= y_p(s, p, l) - e'(p)[e(p)\lambda]^{1/\eta} - \delta s, \quad s(0) = s_0, \\
\frac{\dot{p}}{p} &= r + \delta - y_s(s, p, l), \quad p(0) \text{ free},
\end{align*}
\]

where we have \( y_u = y_d = 0 \) if we assume that production of non-tradables only uses labour.

**Figure 3: Absorption constraints and Dutch disease**

- **Real exchange rate, \( p \)**
- **Structures, \( s \)**

**Key:** A permanent-income rule for managing a temporary windfall boosts consumer spending. This shifts the \( \ddot{s} = 0 \) locus to the right. The equilibrium shift up on impact from \( E \) to \( A \), and afterwards the equilibrium moves along the saddle-path (the bold solid arrow) to \( E' \).

Figure 3 gives the saddle-point diagram, where the \( \ddot{s} = 0 \) locus from (12) slopes downwards, the \( \frac{\dot{p}}{p} = 0 \) locus from (13) is horizontal, and the saddle-path (the fat solid arrow) slopes downwards.
Structures $s$ are predetermined and the real exchange rate $p$ is non-predicted. A windfall-induced permanent increase in consumption then leads to an immediate jump upwards of the relative price of non-tradables. This signals the need for expansion of the non-traded sector and contraction of the traded sector, which is made possible by a relocation of labour from tradables to non-tradables production. However, the additional spending on non-tradables is only fully made possible once the economy has accumulated enough structures to boost productive capacity of non-tradables enough. Over time investment in structures takes place and the relative price of non-tradables falls. Once absorption constraints are fully alleviated, the price of non-tradables has returned to its original equilibrium. In response to the windfall, we thus see that real consumption, $c = \left[ e(\rho) \lambda \right]^{1/\eta}$, undershoots its new higher long-run value.

**Modification 4:** If structures are needed to boost production of non-tradables relatively more than production of tradables, a windfall-induced permanent boost to consumer spending leads to temporary appreciations of the real exchange rate to boost investment in structures and over time improve the absorption capacity of the economy. This allows over time more labour and other production factors to be shifted from tradables and non-tradables production.

Since structures cannot be acquired on world markets, they must be home-grown. Since this takes time, structures slow down growth and development. This dynamic adjustment process captures the absorption constraints that hinder a quick harnessing of natural resource windfalls facing many developing countries. The mechanism behind these absorption constraints relies on the production of non-tradables being more intensive in structures than production of tradables ($y_{ps} - \delta > 0$). If this is not the case (i.e., $y_{ps} - \delta < 0$), the real exchange rate does not respond to a windfall at all and instead capital is slowly run down by wear and tear to allow gradual release of labour and other factors of production from the traded sector to allow the non-traded sector to expand. Such an adjustment process seems less applicable for developing countries.

8. **Managing windfalls with finite lives and Dutch disease**

So far, we have not disaggregated public and private sectors. With operational intergenerational bequest motives it does not really matter whether the government hands out the windfall directly or hands out the windfall via a permanent-income rule. To add realism, we depart from Ricardian debt neutrality so that the timing of natural resource handouts to private agents does
matter. We use an infinite-horizon, continuous-time, overlapping-generations model with finite lives (Blanchard, 1985). This also implies that an increase in government consumption does not lead to 100% crowding out of private consumption. This allows us to meaningfully discuss and compare the effects of permanent-income and spend-all rules for managing natural resource windfalls on the time paths of consumption, the current account, the real exchange rate, and asset holdings.\(^6\) We abstract from structures until section 9.

Each period households have a probability of death \( \bar{\gamma} > 0 \), so that \( 1/\bar{\gamma} \) is the expected lifetime. The rate of births is also \( \bar{\gamma} \), so the size at time \( t \) of a cohort born at time 0 is \( \bar{\gamma} \exp(-\bar{\gamma} t) \) and the aggregate population size remains constant at 1. Households have no operational bequests, but have access to life insurance companies with free entry and exit. The competitive premium is \( \bar{\gamma} \), so that households with wealth \( a \) pay a premium \( \bar{\gamma}a \) and bestow their assets \( a \) to the insurance companies upon death.

With logarithmic utility and a unit coefficient of intergenerational inequality aversion (\( \eta = 1 \)), the dynamics of private wealth accumulation and aggregate consumption are described by

\[
\dot{a} = ra + y(p,l) + h - e(p)c, \quad a(0) = a_0 \quad \text{and} \quad \dot{c} = \left( r - \rho - \frac{\bar{\gamma}}{e} \right) c - \gamma \frac{a}{e(p)},
\]

where \( h \) are lump-sum government handouts and \( y = \bar{\gamma}(\rho + \bar{\gamma}) > 0 \). Equilibrium on the market for non-tradables and the current account dynamics are now given by

\[
e(p)(c + g) = y_p(p, l) \quad \text{and} \quad \dot{f} = rf + y(p, l) + n - e(p)(c + g), \quad f(0) = f_0.
\]

Real government consumption is denoted by \( g \) and has the same unit-expenditure function as private consumption. Denoting assets in the intergenerational sovereign wealth fund by \( b \) and government debt by \( d \), the budget constraints of the fund and the government are

\[
b = rb + n - \tau, \quad b(0) = 0 \quad \text{and} \quad \dot{d} = rd + e(p)g + h - \tau, \quad d(0) = d_0.
\]

where \( \tau \) is the transfer from the fund to the government. Net foreign asset holdings are the sum of private and fund asset holdings minus government debt, so \( f = a + b - d \). The fund indicates above-ground financial wealth from accumulated saving of windfall revenue. Below-ground natural resource wealth, denoted by \( v \), is the present value of resource revenue. Permanent resource income is the return on resource wealth, so

\[
v(t) = \int_0^t s_n(s) e^{-r(s-t)} ds \quad \text{and} \quad n^r(t) = rv(t).
\]

\(^6\) In section 8 we use this framework to also discuss bird-in-hand rules.
The permanent-income rule states that the amount taken from the fund for general budget purposes must equal permanent resource income plus interest income on fund wealth, hence

\[ \tau(t) = r[n^P(t) + rb(t)] = n^P(t) + rb(t) = n^P(0), \quad \forall t \geq 0. \]

This transfer is constant over time, which implies that total above- and below-ground wealth is constant under the permanent-income rule too: \( v(t) + b(t) = n^P(0) / r \), \( \forall t \geq 0 \). Any running down of below-ground resource wealth must thus be exactly mirrored by accumulation of above-ground wealth in the fund. This echoes the Hartwick rule (Hartwick, 1977).

To focus at the operation of sovereign wealth fund, we abstract here from government debt and spending, so \( h = \tau \) and \( g = 0 \). Notice that the first part of (15) implies that \( \dot{c} / c = (e^d + e^c) \dot{p} / p \) and \( p = p(c) \) with \( p'(c) > 0 \), and notice that \( \dot{e} / e = \theta \dot{p} / p \). Hence, (14) can be rewritten as

\[ \dot{a} = ra + \tau - \Omega(c) \quad \text{and} \quad \dot{c} = \left( \frac{e^d + e^c}{e^d + e^c + \theta} \right) (r - p)c - \gamma \frac{a}{e(p(c))}, \]

where \( \Omega(c) \equiv e(p)c - y(p,l) \) with \( \Omega(c) = e(p) > 0 \). This is a saddle-point system of differential equations in the predetermined state, \( a \), and the non-predetermined state, \( c \). (The variables \( b \) and \( f \) are predetermined too.) The black lines in figure 4 indicate the effects of a permanent-income rule for the transfer from the fund to the government and to households.

**Figure 4: Permanent-income rule and Dutch disease with finite lives**

- **Key:** Under the permanent-income rule, the \( \dot{c} = 0 \) locus moves up. Under the permanent-income rule the equilibrium shifts on impact from E to A, and then moves along the saddle-path (fat solid black arrow) to PI. The effects of the spend-all rule of a temporary windfall are given by the dotted purple line during the windfall and by the solid purple arrow after the windfall. The permanent-income rule leads households to permanently decumulate assets, whilst the spend-all rule leads them to temporarily accumulate assets.
Figure 4 supposes that society’s pure rate of time preference exceeds the world return on financial assets \((\rho > r)\), so the \(\dot{c} = 0\) locus from the second part of (19) slopes downwards. The \(\dot{a} = 0\) locus from the asset accumulation equation in (19) slopes upwards. The saddle-path SS slopes downwards. The permanent increase in the transfer from the fund made possible by the temporary windfall moves up the \(\dot{c} = 0\) locus and shifts the equilibrium from E to PI. On impact, consumption and consequently the price of non-tradables jump up and overshoot their new steady-state values. This leads to a big expansion of the non-traded sector and contraction of the traded sector. Over time, households run down their financial assets. Due to the accumulation of assets in the fund, the country eventually has current-account surpluses and accumulates foreign assets. As a result, the price of non-tradables and consumption fall back a bit until the new equilibrium is reached. Hence, with finite lives the full smoothing of consumption and the real exchange rate discussed in section 5 no longer holds. Instead, there are volatile movements in these variables and as a result production factors move of the windfall from the traded to the non-traded sector on impact of the news, and subsequently this reallocation is reversed partially.

The purple lines in figure 4 indicate what happens under a spend-all rule where the transfer to the general budget equals resource revenue, which we assume to be constant for a finite period. Consumption jumps up again and by more than under the permanent-income rule, since households may not be alive to enjoy the benefits of future permanent transfers. With finite lives the present value of handouts is thus higher under the spend-all rule than under the permanent-income rule as Ricardian equivalence no longer holds. During the period of the windfall consumption falls back and households are accumulating assets to provide for after the windfall has ceased and to compensate for the government not saving via a permanent-income rule (see the dotted purple line). After the windfall consumption falls back to its pre-windfall level and households decumulated their assets back to what they were (see the solid purple saddle-path arrow). In this case, foreign assets are driven entirely by household assets.

The propensity to binge in this economy arises from the rate of time preference exceeding the return on assets \((\rho > r)\). If this is not the case \((\rho = r)\), we note from the second part of (19) that the \(\dot{c} = 0\) locus corresponds to the vertical axis in figure 4. In that case, managing a temporary windfall with a permanent-income rule implies that real consumption jumps up instantaneously on impact with no effect on household asset accumulation. Of course, the government will save under this rule and thus the country experiences current-account surpluses. With a spend-all rule, households engage in temporary saving to compensate for the government not saving any of the windfall revenue. The boost to real consumption is, as before, temporary.
Modification 5: With finite lives and a permanent-income rule, the real exchange rate appreciates and real consumption jumps up immediately with no effect on household asset accumulation upon news of a temporary windfall. However, if households are impatient ($p > r$), real consumption and the exchange rate overshoot their steady states. Over time, households decumulate assets but the country eventually accumulates foreign assets, consumption falls, and the real exchange rate depreciates, and the initial contraction of the non-traded sector and expansion of the traded sector are partially reversed. Under a spend-all rule consumption jumps up by a bigger amount and households accumulate assets during the windfall and decumulate them again afterwards, so the country has current-account surpluses during the windfall and deficits afterwards. The permanent-income allows a permanent smaller boost to consumption whilst the spend-all rule allows a temporary bigger boost to consumption.

It is thus essential to depart from Ricardian debt neutrality when analysing different rules for managing resource windfalls for otherwise they all have the same effect on the time path of consumption. Finite lives is one way of doing this, but one can also use infinite lives provided there is population growth and no intergenerational bequest motive (e.g., Weil, 1989). Another approach that can be used is to assume that the majority of households are liquidity constrained in which case the government has to step in to smooth consumption on their behalf. Ricardian debt neutrality also does not hold in the presence of distorting taxes.

9. Medium- and long-run macro effects of managing windfalls

Combining the models of absorption constraints of section 7 and that of finite lives and Dutch disease of section 8 and assuming that the transfer from the fund is allocated according to a ratio of government spending to lump-sum handouts of $\psi$ and that none of the transfer is used for debt reduction, we specify the following model for analysing the management of windfalls:

\begin{align*}
(20a) \quad \dot{s} &= y_p(s, p, l) - e(p)(c + g) - \delta s, \quad s(0) = s_0, \\
(20b) \quad \dot{a} &= ra + y(s, p, l) + h - e(p)c, \quad a(0) = a_0, \\
(20c) \quad \dot{p}/p &= r + \delta - y_s(s, p, l),
\end{align*}

\footnote{With overlapping generations it is possible that households save more than is socially optimal if population growth exceeds the steady-state marginal return on capital. Remedies may then be needed to overcome such dynamic inefficiencies. For example, Pareto improvements are possible if current generations eat part of the capital stock and consumption of future generations is held constant.}
(20d) \( \dot{c} = \left( r - \rho - \theta \left[ r + \delta - y(s, p, l) \right] \right) c - \gamma \frac{\alpha}{e(p)}, \quad c(0) \) free,

where \( h = \frac{1}{1 + \psi} \tau, \quad e(p)g = \frac{\psi}{1 + \psi} \tau, \quad f = a + b - d(0). \) In section 8.2 we present simulations of the model (20) for the three policy rules for the transfer of the fund towards the general government budget, \( \tau \), and the size of the fund, \( b \), discussed in section 8.1. It is possible to allow for capital scarcity (section 3) or volatile commodity prices too (section 4), but we have kept these features out to focus on the novel features introduced in this paper. We do extend this model in section 10 to allow for nominal and real wage rigidity to analyse the crash in commodity prices.\(^8\)

9.1. Permanent-income, bird-in-hand and spend-all rules

The transfer to the general government budget is given by either a permanent rule, \( \tau(t) = n^P(0) \), a spend-all rule, \( \tau(t) = n(t) \), or a bird-in-hand rule, \( \tau(t) = 0.04 b(t), \forall t \geq 0 \), which simply takes a fraction of accumulated fund assets towards the general government budget. We suppose a temporary exponentially declining windfall of the form \( n(t) = \bar{n} e^{-\mu t}, t \geq 0 \), where \( \mu > 0 \) denotes the rate of decline of windfall revenue. We then have a permanent-income rule with

\[ \tau(t) = \left( \frac{r}{r + \mu} \right) \bar{n} \]

and below-ground resource and fund wealth given by \( v(t) = n(t) / (r + \mu) \) and

\[ b(t) = \frac{\bar{n}}{r + \mu} (1 - e^{-\mu t}) \]

respectively. Under the bird-in-hand rule fund wealth is given by

\[ b(t) = \frac{\bar{n}}{0.04 - r - \mu} \left( e^{-\mu t} - e^{-(0.04 - r) t} \right). \]

The bird-in-hand rule is very conservative as it does not allow borrowing against future windfalls; instead, the government takes a fixed fraction, say 4%, from the fund for the general government budget.\(^9\) As a result, the government’s withdrawal is gradually ramped up during and gradually winded down as fund wealth vanishes after the windfall. The transfers from the fund under the three policy rules are thus

\[
\tau(t) = \begin{cases} 
\bar{n} e^{-\mu} & \text{under spend-all policy} \\
0.04 \left( \frac{\bar{n} e^{-\mu} - e^{-(0.04 - r) t}}{r + \mu} \right) & \text{under bird-in-hand policy} \\
\frac{r \bar{n}}{r + \mu} & \text{under permanent-income policy.}
\end{cases}
\]

\(^8\) A numerical two-country DSGE model of Norway and the rest of the world with wage and price rigidities, monopolistic competition, adjustment costs for investment, habit formation and incomplete exchange rate pass through has been used to analyse the effects of fiscal and monetary policy with and without a sovereign wealth fund and a bird-in-hand rule (Bergholt and Lansen, 2016).

\(^9\) Piaschacon (2012) uses a real one-sector framework DSGE model to study how a more pragmatic rules such as the bird-in-hand rule (used in Norway) fares in Mexico.
Figure 5 shows for a windfall defined by $\bar{n} = 1.125$ and $\mu = 0.025$ with $r = 0.02$ the withdrawals from the fund and fund wealth under these policy rules (in black). It also gives natural resource wealth (in red). If the windfall is immediately spent (solid lines denoted by SA), the government receives a declining amount. If the government pursues a permanent-income rule (dashed lines denoted by PI), the contribution to the government budget is constant, less in the beginning but more after a while than the spend-all rule. The permanent-income rule achieves this by accumulating fund wealth as resource wealth is run down.

**Figure 5: Three different policy rules for managing natural resource windfalls**

**Key:** Outcomes under the spend-all (SA), permanent-income (PI) rule and bird-in-hand (BIH) are denoted by black solid, dashed and dotted lines, respectively.

The bird-in-hand rule (dotted lines denoted by BIH) starts very conservative with very low withdrawals from the fund, but eventually they are much higher than under the permanent-
income rule. Eventually the contribution to the budget under the bird-in-hand rule vanishes, but this takes a very long time as fund wealth under this rule is very high for a long intermediate time. Total below- and above-ground wealth under this rule is also high for a long time.

9.2. Dynamic policy simulations

For purposes of simulation (20) we assume the following functional forms. First, we assume a Cobb-Douglas expenditure function, \( e(p) = p^\theta \), where \( 0 < \theta < 1 \) is the share of non-tradables in the consumption basket. Second, output of tradables is a linear function of labour use only, and that output of the non-tradables is a Cobb-Douglas function of structures and labour use. This implies the short-run GNP function

\[
y(s, p, l) = A^T l + \alpha p A^N \left[ (1 - \alpha) p A^N / A^T \right]^{(1 - \alpha)/\alpha} s, \quad \text{where } \alpha
\]

is the share of structures in the production of non-tradables, \( A^N \) is total factor productivity in production of non-tradables, and \( A^T \) is that of tradables. The parameter values used for these functional forms and for the remaining parts of (20) are given in appendix 1, which also gives the no-windfall steady state.\(^{10}\) Figure 6 contrasts the effects of the three different policy rules given in (21) on structures, household assets, the real exchange rate, GNP, real consumption, and foreign asset accumulation.

The most striking feature is that households temporarily save and accumulate if government hands out the windfall immediately to them (indicated by solid lines under the spend-all rule) but borrow permanently if the government employs a permanent-income rule (dashed lines). To an extent, private agents thus engage in saving themselves when the government fails to smooth withdrawals from the fund, but not entirely due to finite lives. Under the bird-in-hand rule (dotted lines) private agents borrow even more to offset the very prudent saving policy implemented by the government but over time households start saving again under this rule as fund assets and thus the government withdrawal start declining. Interestingly, under all three policy rules there is a sharp temporary appreciation of the real exchange rate which signals the importance of investing in structures to expand the sector producing non-tradables.

The stock of structures rises most under the spend-all rule, because under that rule the real exchange rate appreciates most as under finite lives private agents cannot fully offset the lack of saving by the government under this rule. The rise in the stock of structures, however, is temporary will eventually vanish under the spend-all rule as it does also under the bird-in-hand rule, but the rise is permanent under the permanent-income rule. As a result, production of non-

\(^{10}\) Appendix 2 explains the spectral decomposition algorithm that has been used to solve the transient paths of the saddle-path model (20)-(21) with \( k \) and \( a \) predetermined, and \( p \) and \( c \) non-predetermined variables.
Figure 6: Dynamic policy simulations of the three different rules for managing the fund

Key: Solid lines – spend-all rule; dashed lines = PI rule; and dotted lines = BIH rule
Figure 6: Dynamic policy simulations – continued

Key: Solid lines – spend-all rule; dashed lines = PI rule; and dotted lines = BIH rule
tradables and GNP rise higher in the short run under the spend-all rule and return to pre-windfall levels under the spend-all rule (as it does under the and bird-in-hand rule), but stay permanently higher with the permanent-income rule. The pattern is repeated for real consumption: the increases are concentrated up front for the spend-all rule and for a prolonged time in the medium run for the bird-in-hand rule, but effects vanish in the long run for these two rules. The permanent-income rule manages to smooth the time path of consumption most, albeit that consumption rises a little more in the short than in the long run. The permanent-income rule also manages to avoid volatility in the allocation of labour to the two sectors more than the other two rules. Finally, as private liabilities rise in the long run under the permanent-income rule and not under the other two rules, net foreign assets end up lower.

10. Short-run policy responses to a crash in commodity prices

Most windfalls of natural resources last for decades or even centuries, hence it seems natural to focus on the medium and long run as we have done mostly so far. However, news about a pending windfall or a crash in commodity prices can also have large short-run effects on macroeconomic outcomes when labour markets respond sluggishly. Here we briefly discuss the short-run effects and potential policy responses under both real and nominal wage rigidity.

10.1. Real wage rigidity and temporary unemployment

Here we allow for real wage sluggishness and thus extend the short-run analysis of section 6. The GNP function \( y(s, p, l) \) gives the maximum GNP that can be produced given an aggregate stock of structures \( k \) and employment \( l \) when the price of non-tradables is \( p \). For purposes of medium and long-run analysis, aggregate employment \( l \) is pinned down by exogenous labour supply \( \bar{T} \). However, empirical evidence suggests the presence of real wage rigidity so the real consumption wage, \( z \equiv y_l(s, p, l) / e(p) \), responds sluggishly to deviations from full employment (cf., Bean, 1988): \( \dot{z} / z = \zeta(l - \bar{T}) \), where \( \zeta > 0 \) is the speed of real wage adjustment. With a linear technology for tradables (as in the simulations of section 8), the nominal wage is constant and thus (using (20e) and \( y_{ss} = y_{sl} = 0 \) from (A2)) employment is

\[
(22) \quad l = \bar{T} - (\theta / \zeta) \dot{p} / p = \bar{T} - (\theta / \zeta) \left[ r + \delta - y_s(s, p, l) \right] \Rightarrow l = l(p), \quad l'(p) = (\theta / \zeta) y_{sp} > 0.
\]

Hence, aggregate employment decreases in the rate of appreciation of the real exchange rate or equivalently increases in the marginal product of structures. Given that non-tradables are intensive in structures, \( y_{sp} > 0 \), employment increases in the price of non-tradables, \( p \).
Hence, a resource discovery that causes on impact a temporary appreciation of the real exchange rate and boost to the real consumption wage leads to temporary shortages of labour (i.e., $l > \overline{T}$). Over time, as the real exchange appreciation is reversed and the real consumption wage falls back, employment falls back to its full-employment level.

Conversely, a temporary crash in commodity prices gives on impact a sharp depreciation of the real exchange rate which erodes the real consumption wage and causes unemployment ($l < \overline{T}$). As the real exchange rate and the real consumption wage recover, employment recovers until all unemployment is eliminated. Figure 7 illustrates this by showing the temporary effects on the unemployment rate of a permanent drop in commodity prices by a half when the average lag of real wage adjustment is five years, $\zeta = 0.2$. Real wage rigidity thus leads to higher transient unemployment under a spend-all rule than under the bird-in-hand rule and even more so than under the permanent-income rule, because the former rule is associated with sharper temporary depreciations of the real exchange rate than the other two rules.

**Figure 7: Effects of a permanent commodity price crash on the unemployment rate**

Key: Solid lines – spend-all rule; dashed lines = PI rule; and dotted lines = BIH rule

**10.2. Nominal wage rigidity and short-run monetary response to windfall**

Real wage rigidity may be pervasive in labour markets, but nominal wage rigidity is important too especially in the short run. The 1980’s style models of real exchange rate overshooting feature nominal wage rigidity in combination with forward-looking exchange markets. They have been used to analyse the adverse effect on competitiveness and Dutch disease and the appropriate monetary response (e.g., Buiter and Miller, 1981; Buiter and Purvis, 1983). They
have also been used to show that, upon news of an oil discovery, markets anticipate a future windfall-induced increase in spending which leads before the revenue has started pouring in to an appreciation of the real exchange rate, a drop in net non-resource exports and temporary unemployment (Eastwood and Venables, 1982).\(^{11}\) The recent strand of new Keynesian micro-founded macro models has been used to show that the wealth effects of an anticipated resource bonanza imply that the forward-looking component of inflation suppresses output below its natural level, leading to a recession; if the central bank follows a simple Taylor rule that tightens aggressively against inflation it will exacerbate this recession (Wills, 2015). Since the welfare-relevant output gap in such economies depends on oil technology, the weight of on the output gap in the Taylor rule increases in the size of the oil sector (Ferrero and Seneca, 2015). With substantial spill-overs of oil to the rest of the economy, a crash in oil price then warrants a cut in the nominal interest rate. This contrasts with the situation where the central bank’s mandate is to stabilise CPI inflation in which case the nominal interest rate must be raised to curb inflationary impacts of exchange rate depreciation.

Our two-sector framework can be adopted to allow for sluggish nominal wage growth and adaptive expectations.\(^{12}\) Growth in real wages is thus \(\dot{z}/z = \zeta(l - \bar{l}) + \pi' - \pi\) with \(\pi' = \sigma(\pi - \pi'), \sigma > 0\), where \(\pi'\) and \(\pi\) are expected and actual inflation rates, respectively.

The Fisherian hypothesis implies that the nominal interest rate is the sum of the long-run real interest rate and expected inflation. The Taylor rule lets the nominal interest rate react positively to the inflation gap (in such a way that the real interest rate increases to cool the economy) and negatively to unemployment (Taylor, 1993). Following the suggestion of Frankel (2012), one can also allow for the possibility that the central bank loosens monetary policy when the real exchange rate appreciates.\(^{13}\) Unemployment then increases if there is a depreciation of the real exchange rate (i.e., fall in \(y\) and \(p\) as before), expected inflation rises above target, and the real exchange rate is low necessitating a tightening of monetary policy. It is possible to show that such a Taylor rule performs better following a crash in commodity prices than a nominal exchange rate peg even though three quarters of such countries has a nominal exchange rate peg.

---

\(^{11}\) Typically, there is a lag between discovering oil and beginning production. Empirical findings using giant oil discoveries show that the short-run effect of a natural resource discovery on income is negative (despite a boom in exploration investment) until the natural resource production commences; private agents respond directly to an oil discovery causing the current account to depreciate (Arezki, et al., 2016).

\(^{12}\) Scandinavian two-sector models have been used before to analyse how monetary policy might respond to short-run disequilibrium effects of resource windfalls (e.g., Wijnbergen, 1984b). An alternative to adaptive expectations is staggered price setting (e.g., Calvo, 1982).

\(^{13}\) Alternatively, central banks could target some basket of commodity prices. Note that many new Keynesian models imply ‘divine coincidence’, so that there is no need for the nominal interest rate to respond to the real exchange rate or commodity prices as this is already taken care of by targeting inflation and the output gap (e.g., Monacelli, 2013).
A peg thus severely constrains monetary policy’s ability to respond to demand shocks including global shocks to commodity prices.\textsuperscript{14}

A crash in commodity prices implies that investors in the mining and exploration sectors scale back investment and production activity. Furthermore, households anticipate a drop in future income and thus cut spending, firms in non-tradable sectors hold off investment in structures, and there the government needs to cut spending. With nominal wage rigidity these negative demand shocks cause temporary falls in output and unemployment. Offsetting these shocks requires the price of non-tradables to fall relative to tradables which takes time with sticky prices and therefore unemployment is inevitable. A looser monetary policy can avoid this by inducing households to bring spending forward and the nominal exchange rate to depreciate, but many resource-exporting countries face constraints on monetary policy (e.g., due to badly functioning financial markets or a peg) so the fall in demand cannot perfectly be offset.

The assumption that home and foreign assets are perfect substitutes is strong. A portfolio framework that allows for imperfect substitution between home and foreign assets and disaggregates the fund, the treasury and the central bank is then more appropriate.\textsuperscript{15} Central banks trying to prevent worsening competitiveness and an appreciating currency in the wake of a resource windfall will sell their own currency and buy up foreign reserves, which can lead to substantial accumulation of foreign reserves at the central bank – akin to a de-facto SWF (Wills and van der Ploeg, 2014). Central banks faced with a crash in commodity prices and a government that does not use fiscal policy to smooth consumption and the real exchange rate might step in to prevent the currency from depreciating and inflation rising too fast. This policy will, however, as we have seen in recent years, lead to a rapid depletion of foreign reserves.

\textbf{11. Concluding remarks}

The benchmark prescription for managing natural resource windfalls is the permanent-income rule. This states that all of the revenue should be invested abroad and preferably managed by an independent intergenerational SWF and that judicious use of borrowing and saving policies should smooth the time paths of private and public consumption. Furthermore, these policy prescriptions imply that the time path of the real exchange rate must be smoothed too in order to avoid big swings in the relocation of factors of productions from the sectors producing non-tradables and tradables. A temporary windfall, whether it is anticipated or not, when managed

\textsuperscript{14} If the East African Monetary Union goes ahead, this will constrain the newly resource-rich members (Kenya, Tanzania and Uganda) to respond to commodity price shocks (Wills and van der Ploeg, 2014).

\textsuperscript{15} For a good discussion of how this might be done in a Scandinavian model, see Rødseth (2000).
by a permanent-income rule will thus lead to a modest permanent appreciation of the real exchange rate, contraction of the traded sector, expansion of the non-traded sector and increase in private and public consumption. Although the permanent-income policy prescriptions make sense for developed countries, they make little sense for resource-rich developing countries.

First, developing countries have less access to future markets and financial derivatives and must make more use of a stabilisation fund and precautionary saving to cope with the stochastic volatility of commodity prices and their windfalls. Consumption thus cannot be as high in the early stages as in the later stages of development to make room for precautionary saving. A longer-lasting windfall requires a bigger stabilisation fund, but smaller intergenerational fund.

Second, developing countries have worse access to international markets and suffer from high borrowing costs and too low levels of investment. A windfall should then be used to cut the cost of borrowing, alleviate capital scarcity, boost investment in the domestic economy, and promote economic development, but also frontload the increase in consumption towards current, relatively poor generations. Investing all windfall revenue in foreign assets is thus a bad idea.

Third, short-run unemployment caused by a commodity price crash when the real wage is rigid also necessitates bringing consumption forward to mitigate unemployment.

Fourth, developing countries need structures to boost production of non-tradables. As structures are mostly produced at home, such countries must invest to more efficiently absorb the extra consumption and investment spending associated with a windfall. Even a permanent-income rule for handing the revenue of the windfall to households requires a sharp temporary appreciation of the real exchange rate to signal gradual expansion of non-tradables production by investing in structures. Over time, more and more labour and other production factors are drawn out of tradables into non-tradables production. As this happens, the price of non-tradables falls back to its original level.

Fifth, Ricardian debt equivalence is not very relevant for many households, especially those in developing countries. The timing of the natural resource dividends thus matters. With finite lives and impatient households, using a permanent-income rule to manage a temporary windfall leads on impact to a sharp appreciation of the price of non-tradables and upward jump in real consumption, overshooting their higher steady-state values. As households decumulate assets and consumption falls, the price of non-tradables falls and current-account deficits switch into surpluses. Over time, the depreciation of the real exchange rate partially reverses the contraction of non-tradables production and expansion of tradables production. If the temporary windfall is immediately handed out to citizens, consumption jumps up by a bigger amount and households
accumulate assets during the windfall and decumulate them again afterwards as the government does not engage in saving on their behalf.

These insights are borne out by numerical policy simulations of permanent-income, spend-all and bird-in-hand rules in a model of Dutch disease, absorption constraints and finite lives. Households temporarily save under the spend-all rule but borrow permanently under the permanent-income rule. Households thus save if the government fails to smooth withdrawals from the fund. Households initially borrow even more under the bird-in-hand rule to offset the prudent saving by the government, but over time start saving again as transfers start declining. Under all three policy rules there is a sharp temporary appreciation of the real exchange rate and a consequent boost in structures, production of non-tradables and GNP, especially so under the spend-all rule as then the real exchange rate appreciates most as with finite lives households cannot fully offset the lack of government saving. The rise in structures and GNP will, however, be temporary under the spend-all and the permanent-income rules, but permanent under the permanent-income rule. Increases in real consumption are upfront for the spend-all rule and in the medium run for the bird-in-hand rule, and vanish in the long run, but the permanent-income rule smooths the time path of consumption and avoids sharp reallocations of labour best.

Finally, the short-run impact of natural resource windfalls and of commodity price crashes can be considerable. If real wages respond sluggishly to unemployment and non-tradables production is intensive in structures, a crash in commodity prices causes transient periods of unemployment and more so under the spend-all rule than under the permanent-income or bird-in-hand rule due to the sharper depreciation of the real exchange rate associated with not saving the windfall. If nominal wages are rigid in the short run, a monetary policy response is required. If the central bank steps in to prevent sharp appreciations of the currency when the government fails to smooth consumption, it accumulates foreign reserves, akin to a de-facto SWF. Conversely, if the central bank steps in during a crash in commodity prices to prevent rapid nominal depreciation of the currency and inflation, foreign reserves will be rapidly depleted and may lead to a speculative attack on the currency. Also, during a crash in commodity prices governments in many developing economies find it tough to cut spending or raise non-resource taxes to make up for the drop in resource revenue, even though this is needed if the crash is expected to last a long time. Fund wealth will then be rapidly depleted and government debt will escalate until the market is no longer willing to buy more debt. The subsequent switch to monetary finance boosts inflation, and if this switch to money finance is anticipated higher inflation can occur already before this switch (Sargent and Wallace, 1981). This myriad of short-run macro misery caused by the crash in commodity prices highlights the importance of
sound medium- and long-run management of resource wealth to cope with inevitable volatility in both resource production and commodity prices.

References


Appendix 1: Specification of expenditure and GNP function and no-windfall steady state

We assume a Cobb-Douglas expenditure function, so $e(p) = p^\theta$ and set the share of non-traded goods in the consumption basket to $\theta = 0.6$. We set the rate of time preference to 3% per year and the world rate of return on financial assets to 2% per year, so $\rho = 0.03$ and $r = 0.02$. We set the expected lifetime of households to fifty years, so that $\bar{\gamma} = 0.02$ and $\gamma = \bar{\gamma}(\rho + \bar{\gamma}) = 0.001$. The share of government spending in fund withdrawals is 30% and initial real public spending is zero, so $\psi = 0.25$, $e(p)g = 0.2h$ and $\tau = 0.8h$ during the windfall and $g = \tau = 0$ before.

To derive the GNP function, we suppose that output of non-tradables is given by $A^N_s(l^N)^{1-\alpha}$ and output of tradables by $A^T_t$, where $A^N$, $A^T$, $l^N$ and $l^T$ denote total factor productivity in production and employment in production of non-tradables and tradables, respectively. Production of non-tradables is thus intensive in structures and, for simplicity, we abstract from other production factors such as capital which can be subsumed in the total factor productivity terms if their costs are pinned down on the world market. Firms choose the allocation of labour and structures, $k$, to maximise profits, $pA^N_s(l^N)^{1-\alpha} + A^T(l-l^N)-(r+\delta-p\psi)ps$. This requires that the marginal productivity of labour in each sector equals the wage, so that $(1-\alpha)pA^N_s(l^N)^{\alpha} = A^T$. This gives conditional labour demands $l^N = \left((1-\alpha)pA^N/A^T\right)^{1/\alpha} s$ and $l^T = l - \left((1-\alpha)pA^N/A^T\right)^{1/\alpha} s$, hence a higher level of structures or a higher price of non-tradables demands a relocation of labour from the traded sector to the structures-intensive non-traded sector. The short-run GNP function is given by
The demand for structures follows from

\[ y_s(s, p, l) = \alpha p A^N (l^N / s)^{1-\alpha} = \alpha p A^N \left[ (1-\alpha) p A^N / A^T \right]^{(1-\alpha)/\alpha} \]

Note \( y_{ss} = 0 \). We set the production share of structures to 20% and the depreciation rate of structures to 2% per year, so that \( \alpha = 0.2 \) and \( \delta = 0.02 \). We set total labour supply to \( l = 1 \), and set \( A^T = 2 \) and \( A^N = 1.50854 \), so the steady-state price non-tradables

\[ p^* = \left( \frac{r + \delta}{\alpha} \right) \left( \frac{A^T}{1-\alpha} \right)^{1-\alpha} \frac{1}{A^N}, \]

and that of the consumer price index (denoted by asterisk) are normalised to unity, \( p^* = e^* = 1 \).

We suppose that government spending in the no-windfall steady state is zero. The no-windfall steady state follows from the stationarity conditions of (20), which give

\[ \gamma a^* = (r - \rho) e(p^*) c^* \]

and thus \( e(p^*) c^* = \frac{\gamma [y^* - e(p^*) g^*]}{\gamma + r(\rho - r)} \). From \( ra^* + y^* = e(p^*) (c^* + g^*) \), since we consider the “borrowing” case with \( \rho > r, \ c^* > 0 \) and \( a^* < 0 \). Steady-state output of non-tradables can be written as \( y_p^* = (r + \delta) s^* / (\alpha p^*) \), so from \( y_p - \delta s^* = e'(p^*) (c^* + g^*) \) we get

\[ s^* = \frac{\gamma [A^T l - e(p^*) g^*] \theta / p}{(r + \delta)(\alpha p)^{1-\delta} [\gamma + r(\rho - r)] - \gamma (r + \delta) \theta / p}. \]

Hence, \( y^* = A^T l + (r + \delta) s^* \). With this calibration we get \( s^* = 6.25, c^* = 1.875, a^* = -18.75 \) and \( y^* = 2.25 \). No-windfall, steady-state levels of output of non-tradables and tradables are then 1.25 and 1, respectively, and half of labour supply is then devoted to non-tradables production and the other half to tradables production.

Finally, we set the size of the temporary windfall to 50% of no-windfall steady-state GNP, so \( \tilde{n} = 1.125 \). We assume that the expected duration of the windfall is 40 years, so \( \mu = 0.025 \). This implies a permanent value of the windfall of \( r\tilde{n} / (r + \mu) = 0.5 \) of which \( e(p) g = 0.1 \) is spent by the government and \( \tau = 0.4 \) is handed out as transfers to households.

**Appendix 2: Algorithm for simulating the saddle-point system (22)**

We linearise the system (20) around its steady state, denoted by an asterisk, to get the system:

\[ \dot{\hat{x}} = A\hat{x} + B\tau \quad \text{with} \quad \hat{x} \equiv \begin{pmatrix} \hat{x}^p \\ \hat{x}^N \end{pmatrix}, \quad \hat{x}^p \equiv \begin{pmatrix} s - s^* \\ a - a^* \end{pmatrix}, \quad \hat{x}^N \equiv \begin{pmatrix} p - p^* \\ c - c^* \end{pmatrix}. \]

where the matrices

\[ (A1) \quad pA^N s^*(l^N)^{1-\alpha} + A^T l^T = A^T l + \alpha p A^N \left[ (1-\alpha) p A^N / A^T \right]^{(1-\alpha)/\alpha} s \equiv y(s, p, l). \]

The demand for structures follows from

\[ (A2) \quad y_s(s, p, l) = \alpha p A^N (l^N / s)^{1-\alpha} = \alpha p A^N \left[ (1-\alpha) p A^N / A^T \right]^{(1-\alpha)/\alpha} = (r + \delta - \rho / p) p. \]
\[
A = \begin{pmatrix}
y_y - \delta & 0 & y_p - e''(p)(c + g) & -e'(p) \\
r + \delta & r & 0 & -e \\
0 & 0 & -y_y & 0 \\
0 & -\gamma/e & \theta y_y c + \gamma \alpha(p e)^{-1} & r - \rho
\end{pmatrix}
\quad \text{and} \quad
B = \begin{pmatrix}
(\theta/p)y(1+\psi)^{-1} \\
(1+\psi)^{-1} \\
0 \\
0
\end{pmatrix}
\]

are evaluated at steady state with \(y_y = (r + \delta)/\alpha p^*, \ y_p = (1-\alpha)(r+\delta)s^*/(\alpha p^*)^2 > 0, \ y_{ss} = 0 \) and \(\tau^* = 0\). The solution to this system consists of the sum of the solution to the homogeneous system and the particular integral, denoted by superscripts \(H\) and \(NH\), respectively. We use the spectral decomposition \(A = M \Lambda M^{-1} = N^{-1} \Lambda N\), where the matrix \(\Lambda\) contains the eigenvectors in decreasing order of magnitude with two with positive real parts and two with negative real parts in order for the saddle-path property to be satisfied (i.e., 0.1808, 0.0375, -0.0283 and -0.2 for our calibration). Defining \(\tilde{y} = N\tilde{\xi}\), we get \(\tilde{y}_y = \Lambda H \tilde{y}^H\) for the homogenous system of which the non-exploding solution is \(y_y = \sum \alpha_i e^{\lambda_i t}\) and \(\tilde{y}_y = \sum \alpha_i e^{\lambda_i t}\). Hence, \(\tilde{x}_y = M_y y_y = M_y e^{N t} y_y(0) = M_y e^{N t} (M_y)^{-1} \tilde{x}_y(0)\) and \(\tilde{x}_{NH} = M_{NH} (M_{NH})^{-1} \tilde{x}_y(0), \forall t \geq 0\).

The particular integral for the permanent-income rule is \(\tilde{x}_N(t) = \sum \alpha_i e^{\lambda_i t}\) with \(\tilde{b}_{PI} = -A^{-1}B\). Given that \(\tilde{x}_y(0) = \tilde{B}, \ \tilde{x}_{PI}(0) = -n^P(0)\tilde{B}_{PI}\) and the solution under this rule is

(A6) \[
\tilde{x}_{PI}(t) = \tilde{x}_y(t) + \tilde{x}_N(t) = \left[\tilde{b}_{PI} - \left(\frac{M_{PP}}{M_{NP}}\right) e^{N t} (M_{NP})^{-1} \tilde{b}_{PI}\right] n^P(0), \forall t \geq 0.
\]

On impact non-predetermined variables jump to \(\tilde{x}_{PI}(0) = \left[\tilde{b}_{PI} - \frac{M_{NP}}{M_{PP}} (M_{PP})^{-1} \tilde{b}_{PI}\right] n^P(0)\).

The particular integral for the spend-all rule, \(\tau(t) = \bar{\tau}e^{-\mu t}\), is \(\tilde{x}_N(t) = \bar{\tau}e^{-\mu t}\tilde{b}_{SA}, \forall t \geq 0\), with \(\tilde{b}_{SA} = -(A + \mu I)^{-1}B\). The full solution under the spend-all rule is thus analogously given by

(A7) \[
\tilde{x}_{SA}(t) = \left[\tilde{b}_{SA} e^{-\mu t} - \left(\frac{M_{PP}}{M_{NP}}\right) e^{N t} (M_{NP})^{-1} \tilde{b}_{SA}\right] \bar{\tau}, \forall t \geq 0.
\]

The particular integral for the bird-in-hand rule, \(\tau(t) = \frac{0.04}{0.04 - r - \mu} \bar{\tau} e^{-\mu (0.04 - r)t}\) from

(21) \[
\tilde{x}_N(t) = \left(e^{-\mu} \tilde{b}_{SA} e^{-\mu (0.04 - r)t} \tilde{b}_{BH}\right) \frac{0.04}{0.04 - r - \mu} \bar{\tau}, \forall t \geq 0, \text{ with} \tilde{b}_{BH} = -(A + (0.04 - r)I)^{-1} B. \]

The full solution under the bird-in-hand rule is thus
\[ x^{SA}(t) = \left[ b^{SA} e^{-\mu t} - \left( \frac{M^{PP}}{M^{NP}} \right) e^{\lambda t} \left( \frac{0.04}{0.04 - r - \mu} \right) n + \right] \]

\[ - \left[ b^{BH} e^{-0.04 r t} - \left( \frac{M^{PP}}{M^{NP}} \right) e^{\lambda t} \left( \frac{0.04}{0.04 - r - \mu} \right) n, \forall t \geq 0. \]

Note \( \lim_{t \to \infty} x^{SA}(t) = \lim_{t \to \infty} x^{BH}(t) = \bar{0} \) but \( \lim_{t \to \infty} \tilde{x}^{PI}(t) = \bar{b}^{PI} n^{P}(0). \)