Intermediary Leverage, Macroeconomic Dynamics, and Macroprudential Policy*

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Abstract

We develop a macroeconomic model in which financial intermediaries' optimally choose their leverage. The leverage choices of intermediaries both affect the financial accelerator and imply significant macroeconomic effects of changes in the risk facing intermediaries and the cost of their external funds. We use this model to evaluate several macroprudential policies. With regard to crisis policies, we find that capital injections conditioned upon voluntary recapitalization can be a more effective tool than direct lending/asset purchases. With regard to policies aimed at limiting the cyclical effects of financial disturbances, we demonstrate that policy strategies that lean against changes in aggregate credit, broad measures of asset prices, or leverage within the financial sector may significantly distort the economy's response to changes in fundamentals or have other unintended consequences. Within our model, policy strategies focused on mitigating shifts in the spread between borrowing rates and a risk-free interest rate appear to have better stabilization properties than other proposed macroprudential strategies.

1 Introduction

The recent financial crisis around the world has refocused the attention of economists to the role that financial intermediation plays in shaping cyclical dynamics. Moreover, policymakers have become more focused on the possible role that macroprudential policies may have in mitigating the adverse effects of large shocks to the financial system and on the potential for cyclically-focused macroprudential policy approaches to mitigate the business cycle.

In order to understand how shocks to financial conditions influence macroeconomic outcomes and the possible role of policies in mitigating such effects, we develop a macroeconomic model in which financial intermediaries optimally choose the mix of debt and equity in their liability base to finance their lending. The financial intermediaries in our model find debt financing attractive

*The analysis and conclusions set forth are those of the authors and do not indicate concurrence by other members of the research staff or the Board of Governors.

both because they face an external finance premium when forced to unexpectedly raise outside equity and because debt is assumed to receive preferential tax treatment. However, intermediary leverage is limited by the fact that increased leverage increases the probability of default, making investors unwilling to accept excessive debt levels.

Our model also embeds an inherent asymmetry between intermediaries assets (lending) and liabilities: Lending commitments cannot be adjusted quickly in response to changes in intermediaries' balance sheet condition. This friction, in conjunction with the premium on external funds, makes aggregate lending decisions sensitive to idiosyncratic risk within the intermediary sector or shocks to the external finance premium.

Overall, our framework yields a highly tractable quantitative framework with which we can assess implications of intermediaries' balance sheets for real economic activity. We first demonstrate that our framework creates a financial accelerator, amplifying somewhat the response of aggregate investment to technology or monetary shocks (although this amplification is modest, in line with other models of financial frictions summarized in Quadrini (2011); more significantly, shocks to risk within the intermediary sector or to the costs of external equity funds have substantial effects on macroeconomic activity, suggesting a possible role for macroprudential policies to mitigate these effects. Importantly, we demonstrate that our model implies a rich interaction between intermediary leverage, macroeconomic activity, and the source of financial disturbances to the intermediary sector. Specifically, an increase in risk within the financial sector leads to deleveraging and a contraction in real activity – as increased risk makes debt less attractive (because, absent changes in behavior, the likelihood of default rises) and leads to a pullback in lending. In contrast, an increase in the cost of external equity leads to an increase in leverage and a contraction in real activity – as intermediaries conserve internal funds to avoid raising equity and contract lending. Because financial shocks can lead to a decline in real activity and *either* an increase or decrease in leverage, we will see that macroprudential policies designed to lean against leverage can, in some cases, amplify the adverse effects of financial shocks.

Our analysis of macroprudential policies begins with a consideration of *crisis* policies, similar to that in Gertler and Karadi (2011) and Gertler et al. (2010). Specifically, we consider the relative merits of direct lending by the government and boosting the capital position at financial intermediaries to offset reductions in private credit supply. Our results indicate that the capital injection policy can be much more powerful than the direct lending/asset purchase policy in stabilizing output fluctuations. The key mechanism behind this difference is that an asset purchase policy suffers from a classic case of *crowding out*: while aggregate investment is lifted by the increase in government demand, higher government demand decreases private investment because it boosts asset prices, causing private investment to decline along its downward sloping demand curve. In contrast, the capital injection policy increases private demand for capital assets by improving the capital position directly, which boosts the risk appetite for risky assets.[1] This result is similar to that in our earlier work, which differed from that herein because intermediary leverage was assumed to be exogenously fixed in our earlier research (Kiley and Sim (2011b)). Overall, this finding expands on those in Gertler and Karadi (2011) and Gertler et al. (2010) by considering both direct lending and capital injections, thereby highlighting how a critical decision related to the Troubled Asset Relief Program (TARP) –that is, the decision to focus on capital injections rather than asset purchases–may have enhanced the effectiveness of that program.[2]

We then turn to a consideration of cyclical macroprudential policies. The Basel III process has developed a framework in which capital requirements may be adjusted in response to cyclical conditions, and a variety of researchers have considered the possibility of using, for example, a policy that leans against aggregate credit growth as a way to mitigate undesirable business cycle fluctuations (e.g., Drehmann et al. (2010), Repullo and Saurina (2011), and Edge and Meisenzahl (2011), which offer differing perspectives on this issue). We show that policies that lean against fluctuations in credit, leverage, or broad asset prices may significantly distort economic fluctuations. In particular, policies that lean against credit or broad measures of asset prices may significantly distort the economy's (desirable) adjustment to movements in technology. Policies that lean against leverage can also have unintended consequences: Specifically, a decline in perceived risk within the intermediation sector can lead to an increase in leverage and an unsustainable boom in activity–and leaning against leverage can mitigate this cycle; however, a decline in the cost of equity for financial intermediaries can lead to a decrease in leverage and unsustainable boom in activity, and leaning against leverage amplifies this cycle, rather than mitigating it. We show that,

[1] He and Krishnamurthy (2008) consider similar issues in a framework that cannot address the consequences for real activity.

[2] That said, we also do not focus on an important issue that is an area of focus in Gertler et al. (2010), namely how the presence of government interventions may alter private sector behavior

at least within our model, a policy that leans against credit spreads mitigates the financial cycle irrespective of whether the shock to the intermediation sector is focused on their costs of debt or equity.

This analysis of cyclical macroprudential policies expands upon previous work along several dimensions. As in Cúrdia and Woodford (2009), our model illustrates the potential benefits of policies that lean against movements in credit spreads, while significantly expanding their analysis through a model with debt and equity frictions and, hence, a much richer description of the interaction of intermediary leverage and macroeconomic fluctuations. As in Christensen et al. (2011), our framework provides an illustration of how policies that simply lean against credit movements may distort the economy's desirable adjustment to other shocks. However, our model, by making intermediary leverage an endogenous choice (rather than exogenously set by the regulatory policy) and considering financial disturbances affecting the desirability of both debt and equity, illustrates additional pitfalls potentially associated with policies that lean against leverage while also, showing how policies focused on credit spreads may, at least under certain conditions, be more robust. Indeed, our consideration of a range of indicators (credit, asset prices, leverage, and credit spreads) expands considerably the range of policies, and the potential pitfalls of such policies, that have been studied in dynamic macroeconomic models relative to the analyses in Christensen et al. (2011) and Angelini et al. (2011).

2 Model

The model economy consists of (i) a representative household, (ii) a representative firm producing intermediate goods, (iii) a continuum of monopolistically competitive retailers, (iv) a representative firm producing investment goods, and (v) a continuum of financial intermediaries. Time is discrete and the horizon is infinite.

The representative household lacks the skill necessary to directly manage financial investment projects. As a result, the household saves through financial intermediaries. In this sense, we assume that financial intermediation is a crucial part of our model economy. In addition to the assumed role of intermediation, we will adopt a framework in which raising equity from external funds is costly – a key financial friction in our model. As we discuss further below, a distinction between internal and external funds lies at the heart of much research in corporate finance (e.g., Myers and Majluf (1984)).

Finally, we assume a timing convention in intermediaries' financing decisions that is designed to highlight risks associated with intermediation. A key aspect of intermediation is that financial intermediaries make long-term commitments despite short-run funding risks. For example, a substantial portion of commercial and industrial lending by commercial banks are in the form of loan commitments; alternatively, banks have substantial mismatches between the maturities of their assets and liabilities. Rather than introducing long term assets, we adopt a simple framework which splits a time period into two. Lending and borrowing (e.g., asset and liability) decisions of intermediaries have to be made in the first half of the period t; idiosyncratic shocks to the returns of the projects made at time $t-1$ are realized in the second half of the period t, at which point lending and borrowing decisions cannot be reversed (until the subsequent period $t+1$). [3]

This set of assumptions has two advantages: First, the intra-period irreversibility in lending and borrowing decisions, in conjunction with costs of external equity financing, creates balance sheet risk and generates *precaution* in lending decisions; second, the timing convention helps us derive an analytical expression for the equity issuance and default triggers of intermediaries, allowing a sharp characterization of the equilibrium.

We now walk through the debt, equity, and payout decisions facing intermediaries. The model discussion of non-financial activities (of households and non-financial firms) is relatively brief, as those aspects of our model follow standard practice.

2.1 Financial Intermediary Sector

Financial intermediaries finance investment projects with debt and equity. Intermediaries wish to use debt – that is, to be leveraged – because a corporate income tax makes debt financing attractive and because raising equity from outside investors is costly.

[3] Another related approach would be the following. One can assume that a random fraction of households require early redemption of their debts/deposits at intermediaries in the second half of the period. In this case, the idiosyncratic redemption rate replaces the idiosyncratic shocks to the return on investment. Owing to the illiquidity of the investment project, the intermediary has to raise additional funds in interbank market or stock market to meet the "run". This will create a similar effect on the investment decision of the intermediary.

2.1.1 Intermediary Debt Contract

At the beginning of a period, a financial intermediary $i \in [0,1]$ and an investor enter a debt contract: the intermediary borrows $1 - m_t$ per dollar of investment, with this debt collateralized by the total investment project. If the intermediary does not default in the next period, it repays this debt (in amount $(1 + r^B_{t+1})(1 - m_t)$, where r^B_{t+1} is the interest rate on borrowing).

In the event of default, the investor receives the collateral asset, the market value of which will be a random variable Q_{t+1}. The investor liquidates the position immediately by selling the asset in the market (because investors are assumed to lack the skill to manage investment projects, thereby necessitating intermediation). We assume that the immediate liquidation by the creditor involves a distressed sales cost, a fraction $\eta \in (0,1)$ of the asset value. (CITATIONS TO MOTIVATE ASSUMPTION)

The intermediary's investment project delivers a random gross return, $1 + r^F_{t+1}$ after tax. The return on investment consists of an idiosyncratic component ϵ_{t+1} and an aggregate component $1 + r^A_{t+1}$ such that $1 + r^F_{t+1} = \epsilon_{t+1}(1 + r^A_{t+1})$ where the idiosyncratic component has a time-varying distributions $F_{t+1}(\cdot)$. In particular, we assume that the second moment of the distribution follows a Markov process (detailed further below), while the first moment is time-invariant (and normalized to equal one, $\mathbb{E}_t[\epsilon_{t+1}] = 1$). The time-variation in the second moment of the idiosyncratic return will have aggregate implications.

After the tax deduction on interest expenses, the debt burden of the intermediary is equal to $[1 + (1 - \tau_c) r^B_{t+1}](1 - m_t)$, where τ_c denotes the corporate income tax rate. A default occurs when the realized asset return $\epsilon_{t+1}(1 + r^A_{t+1})$ falls short of the value of the debt obligation $(1 + (1 - \tau_c) r^B_{t+1})(1 - m_t)$. This implies a default trigger ϵ^D_{t+1} – realizations of the idiosyncratic return below this value imply default

$$\epsilon_{t+1} \leq \epsilon^D_{t+1} \equiv (1 - m_t) \frac{1 + (1 - \tau_c) r^B_{t+1}}{1 + r^A_{t+1}}. \tag{1}$$

Intermediary Debt Pricing Households discount future cash flows with their stochastic discount factor, denoted by $M_{t,t+1}$. Given the default trigger (1) and the assumption regarding the bankruptcy costs, the no-arbitrage condition for the household should satisfy

$$1 - m_t = \mathbb{E}_t \left\{ M_{t,t+1} \left[(1-\eta) \int_0^{\epsilon^D_{t+1}} \epsilon_{t+1}(1 + r^A_{t+1}) dF_{t+1} + \int_{\epsilon^D_{t+1}}^{\infty} (1 - m_t)[1 + r^B_{t+1}] dF_{t+1} \right] \right\}. \quad (2)$$

The first term inside the parentheses on the right-hand side is the default recovery, where the recovery rate $1 - \eta$ owes to the costs of bankruptcy. The second term is the non-default income. The discounted value of this total return must equal the value of invested funds, $1 - m_t$.

Equation (2) works as the households' participation constraint in the intermediary's optimization problem for capital structure – that is, intermediaries must take into account the required returns to households on debt in deciding leverage. For later use, it is useful to replace r^B_{t+1} in the participation constraint with an expression including ϵ^D_{t+1}. Using the definition of the default trigger, we can express the borrowing rate as $r^B_{t+1} = \frac{1}{1-\tau_c}\left[\frac{\epsilon^D_{t+1}(1+r^A_{t+1})}{1-m_t} - 1\right]$. Substituting this in the participation constraint yields

$$1 - m_t = \mathbb{E}_t \left\{ M_{t,t+1} \left[\int_0^{\epsilon^D_{t+1}} (1-\eta)\epsilon_{t+1} dF_{t+1} + \int_{\epsilon^D_{t+1}}^{\infty} \left(\frac{\epsilon^D_{t+1}}{1-\tau_c} - \frac{\tau_c}{1-\tau_c}(1 - m_t) \right) dF_{t+1} \right] (1 + r^A_{t+1}) \right\}$$

2.1.2 Intermediary Equity Finance

We now turn to the problem of intermediary equity financing. We assume that the intermediaries have to sell new shares *at a discount*, which generates a dilution effect: issuing new equity with a notional value of dollar reduces the value of *existing* shares more than a dollar. As is standard in corporate finance literature, we assume that the managers of the intermediaries maximize the value of incumbent shareholders. Our approach, based on Bolton and Freixas (2000), is to assume a parametric form for the dilution cost: issuing new equity involves a constant per-unit issuance cost, $\varphi \in (0, 1)$.[4] We denote equity related cash flow by D_t. D_t is dividends paid when positive, and equity issuance when negative. With our assumption of costly equity issuance, actual cash inflow from the issuance $(-D_t)$ is $-(1 - \varphi)D_t$. Total equity related cash flow for the intermediary is $\bar{\varphi}(D_t) \equiv -D_t + \varphi \min\{0, D_t\}$.

[4] Our approach, based on Bolton and Freixas (2000), can be considered standard in corporate finance literature: See Gomes (2001), Cooley and Quadrini (2001), Hennessy and Whited (2007) and Bolton et al. (2009). Pursuing the micro-foundation for the existence of the dilution costs is beyond the scope of this paper. See Myers and Majluf (1984) and Myers (2000) for a more formal and micro-founded derivation.

Suppose that the intermediary invests in S_t units of asset whose market price is given by Q_t. The intermediary borrows $1 - m_t$ for each dollar of its investment. The cash inflow associated with this debt financing from households is given by $(1 - m_t)Q_tS_t < Q_tS_t$. To close the funding gap, the intermediary has three other sources: internal funds, N_t, equity issuance $\bar{\varphi}(D_t)$, and a (potential) lump-sum government transfer T_t such that

$$Q_tS_t = (1 - m_t)Q_tS_t + N_t + \bar{\varphi}(D_t) + T_t, \tag{3}$$

which is simply the flow of funds constraint facing the intermediary.

Without default, the internal funds of the intermediary are given by the difference between the total return from the asset minus the debt payment, i.e., $N_t = \epsilon_t(1 + r_t^A)Q_{t-1}S_{t-1} - (1 + (1 - \tau_c)r_t^B)(1 - m_{t-1})Q_{t-1}S_{t-1}$. However, owing to the limited liability of the intermediary, internal funds are truncated by zero. Hence the internal funds of the intermediary are given by

$$\begin{aligned} N_t &= [\max\{\epsilon_t, \epsilon_t^D\}(1 + r_t^A) - (1 - m_{t-1})(1 + (1 - \tau_c)r_t^B)]Q_{t-1}S_{t-1}. \\ &= [\max\{\epsilon_t, \epsilon_t^D\} - \epsilon_t^D(1 + r_t^A)]Q_{t-1}S_{t-1} \end{aligned} \tag{4}$$

Using (1), one can verify that the internal funds given by (4) is truncated by zero when a default occurs, i.e., $\epsilon_t < \epsilon_t^D$.

2.1.3 Value Maximization

Symmetric Equilibrium In order to present a sharp characterization of the equilibrium, the timing convention mentioned earlier is important. Formally, we assume: (i) all aggregate information is known at the beginning of each period; (ii) based on aggregate information, intermediaries make lending/borrowing decisions, which are irreversible within a given period; (iii) idiosyncratic shocks are realized after the lending/borrowing decision; (iv) depending on the realization of idiosyncratic shocks, some intermediaries undergo the default/renegotiation process; (iv) finally, equity issuance/dividend payout decisions are made.

This timing convention, the risk neutrality of intermediaries, and the absence of persistence in the first moment of idiosyncratic shock imply a symmetric equilibrium in which all intermediaries choose the same lending/investment level and capital structure. The symmetric equilibrium also implies that all intermediaries face the same borrowing cost and default trigger at the borrowing/lending stage (e.g., the first half of period t).. The shadow value of the participation constraint (the no-arbitrage condition for a bond investor, (2)), denoted by θ_t, also has a degenerate distribution since the borrowing decision is made before the realization of the idiosyncratic shock.

However, the distribution of dividends and equity financing do depend on the realization of idiosyncratic shocks, and thus has a non-degenerate distribution. Since the flow of funds constraint depends on the realization of the idiosyncratic shock, the shadow value of the constraint, denoted by λ_t, also has a non-degenerate distribution.

To simplify the dynamic problem, we decompose the intermediary problem into two stages in a way that is consistent with the timing convention: in the first stage, the intermediary solves for the value maximizing strategies for lending and borrowing without knowing its realization of net-worth. In the second stage, the intermediary solves for the value maximizing dividend/issuance strategy based upon all information, including the realization of its net-worth.

Formally, we define two value functions, J_t and $V_t(N_t)$. J_t is the ex-ante value of the intermediary before the realization of idiosyncratic shock while $V_t(N_t)$ is the ex-post value of the intermediary after the realization of idiosyncratic shock. In our symmetric equilibrium, the ex-ante value function does not depend on the intermediary specific state variables and J_t is a function of aggregate state variables only. The ex-post value function, however, depends on the realized internal funds, N_t, which is a function of the realized idiosyncratic shock as shown by (4). To emphasize the dependence of the ex-post value function on realized internal funds, we use N_t as a function argument for $V_t(\cdot)$.

Since the first stage problem is based upon the conditional expectation of net-worth, not the realization, it is useful to define an expectation operator $\mathbb{E}_t^{\epsilon}(\cdot) \equiv \int \cdot dF_t(\epsilon)$, the conditioning set of which includes all information up to time t, except the realization of the idiosyncratic shock. Because of the assumed conditioning set, all aggregate state variables at time t can be taken out of the expectation operator. We can then formulate the intermediary problem as follows. All financial intermediaries are owned by the representative household, and hence discount future cash flows by the stochastic pricing kernel of the representative household, $M_{t,t+1} \equiv (\Lambda_{t+1}/P_{t+1})/(\Lambda_t/P_t)$.where Λ_t is the marginal utility of consumption and P_t is the price index of consumption basket. Before the realization of the idiosyncratic shock, the intermediary maximizes shareholder value by solving for the size of its lending, debt, and equity (through choices for the aggregate project size S_t, leverage m_t, and default trigger ϵ_t^D, subject to the households participation constraint equation 2),

$$\begin{aligned} J_t \; = \; & \min_{\theta_t} \max_{S_t, m_t, \epsilon_{t+1}^D} \Bigg\{ \mathbb{E}_t^\epsilon [D_t] + \mathbb{E}_t [M_{t,t+1} \cdot V_{t+1}(N_{t+1})] \\ & + \mathbb{E}_t^\epsilon \left[\lambda_t \left([\max\{\epsilon_t, \epsilon_t^D\} - \epsilon_t^D](1 + r_t^A) Q_{t-1} S_{t-1} + T_t + \bar{\varphi}(D_t) - m_t Q_t S_t \right) \right] \\ & + \theta_t Q_t S_t \mathbb{E}_t \left[M_{t,t+1} \left(\int_0^{\epsilon_{t+1}^D} (1-\eta) \epsilon_{t+1} dF_{t+1} + \int_{\epsilon_{t+1}^D}^\infty \frac{\epsilon_{t+1}^D}{1-\tau_c} dF_{t+1} \right) (1 + r_{t+1}^A) \right. \\ & \left. - (1 - m_t) \left(1 + M_{t,t+1} \int_{\epsilon_{t+1}^D}^\infty \frac{\tau_c}{1-\tau_c} dF_{t+1} \right) \right] \Bigg\} \end{aligned} \tag{5}$$

where, for the homogeneity of the problem, we scale up the Lagrangian multiplier for the participation constraint for the bond investors by the size of the balance sheet, $Q_t S_t$. We also replace $(1 - m_t)(1 + r_{t+1}^B)$ with $\epsilon_{t+1}^D (1 + r_{t+1}^A)$ using the default condition in the participation condition. After the realization of the idiosyncratic shock, the intermediary solves

$$\begin{aligned} V_t(N_t) \; = \; & \min_{\lambda_t} \max_{D_t} \Bigg\{ D_t + \mathbb{E}_t [M_{t,t+1} \cdot J_{t+1}] \\ & + \lambda_t \left[[\max\{\epsilon_t, \epsilon_t^D\} - \epsilon_t^D](1 + r_t^A) Q_{t-1} S_{t-1} + T_t + \bar{\varphi}(D_t) - m_t Q_t S_t) \right] \Bigg\} \end{aligned} \tag{6}$$

Problem (5) solves the optimal investment/borrowing problem only based upon $\mathbb{E}_t^\epsilon[N_t]$, $\mathbb{E}_t^\epsilon[D_t]$ and $\mathbb{E}_t^\epsilon[\lambda_t]$, which are aggregate information. At this stage, the intermediary does not know whether default, or issuance or distribution of dividends will occur under its optimal strategy. In contrast, problem (6) solves for the optimal level of distribution/issuance based on the knowledge of its realized net-worth. As a result, λ_t enters the program without the expectation operator. In the second stage problem, the truncated net worth N_t becomes a state variable of the decision problem.

The first-order conditions associated with problem (6) are given by the following (The appendix provides details of the derivation).

- FOC for D_t :

$$\lambda_t = \bar{\varphi}'(D_t) = \begin{cases} 1 & \text{if } D_t \geq 0 \\ 1/(1-\varphi) & \text{if } D_t < 0 \end{cases} \tag{7}$$

- FOC for S_t :

$$m_t \mathbb{E}_t^{\epsilon}[\lambda_t] = \mathbb{E}_t\{M_{t,t+1}\mathbb{E}_{t+1}^{\epsilon}[\lambda_{t+1}(\max\{\epsilon_{t+1}, \epsilon_{t+1}^D\} - \epsilon_{t+1}^D)](1 + r_{t+1}^A)\} \tag{8}$$

- FOC for m_t :

$$\mathbb{E}_t^{\epsilon}[\lambda_t] = \theta_t \left[1 + \frac{\tau_c}{1-\tau_c}\mathbb{E}_t\left(M_{t,t+1}[1 - F_{t+1}(\epsilon_{t+1}^D)]\right)\right] \tag{9}$$

- FOC for ϵ_{t+1}^D :

$$\begin{aligned} 0 \quad = \quad & \mathbb{E}_t\left\{M_{t,t+1}\mathbb{E}_{t+1}^{\epsilon}\left[\lambda_t\left(\frac{\partial \max\{\epsilon_{t+1}, \epsilon_{t+1}^D\}}{\partial \epsilon_{t+1}^D} - 1\right)\right]\right\} \\ & + \theta_t \mathbb{E}_t\left\{M_{t,t+1}\left[\left((1-\eta) - \frac{1}{1-\tau_c}\right)\epsilon_{t+1}^D f_{t+1}(\epsilon_{t+1}^D)\right](1 + r_{t+1}^A)\right\} \\ & + \theta_t \mathbb{E}_t\left\{M_{t,t+1}\left[\frac{1}{1-\tau_c}[1 - F_{t+1}(\epsilon_{t+1}^D)](1 + r_{t+1}^A) + (1-m_t)\frac{\tau_c}{1-\tau_c}f_{t+1}(\epsilon_{t+1}^D)\right]\right\} \end{aligned} \tag{10}$$

2.1.4 Discussion

Equation (7) states that the shadow value of the internal funds depends on the intermediary's realized equity regime: the marginal valuation of additional dollar is equal to one as long as it does not face any difficulty in closing the funding gap, and as a result, distributes a strictly positive amount of dividends; the shadow value can be strictly greater than 1 if it is facing short term funding problems and has to raise equity funds from the stock market. The gap between the two marginal valuations is determined by the size of the dilution cost per issuance φ.

To see the economic effects of balance sheet risk and time-variation in the value of intermediaries internal funds, first imposing the threshold level 0 for the dividend on the flow of funds constraint (3), and solving for ϵ yields the value for the realized idiosyncratic return below which outside equity must be raised – i.e., the "issuance trigger",

$$\epsilon_t^E \equiv (1 - m_{t-1})\frac{R_t^{B,\tau}}{R_t^F} + m_t\frac{Q_t S_t + T_t}{R_t^F Q_{t-1} S_{t-1}} = \epsilon_t^D + m_t\frac{Q_t S_t + T_t}{R_t^F Q_{t-1} S_{t-1}}. \tag{11}$$

If the realized idiosyncratic shock is greater than the trigger, the intermediary pays out a strictly positive amount of dividends. If the realized shock is less than the trigger value, the intermediary has to raise new equity by paying φ per-unit issuance cost.[5]

Since the shadow value takes one with probability $1 - F_t(\epsilon_t^E)$ and $1/(1-\varphi)$ with probability $F_t(\epsilon_t^E)$, the expected shadow value is given by

$$\mathbb{E}_t^{\epsilon}[\lambda_t] = 1 - F_t(\epsilon_t^E) + \frac{F_t(\epsilon_t^E)}{1-\varphi} = 1 + \mu F_t(\epsilon_t^E) \geq 1,\ \mu \equiv \frac{\varphi}{1-\varphi}. \tag{12}$$

The inequality is strict as long as $\varphi > 0$ and $\sigma_t > 0$. The fact that the expected shadow value of internal funds is always greater than 1 shows that the intra-period irreversibility of investment decision in the frictional capital market creates *caution* on the part of the risk neutral intermediaries. Though intermediaries know that they may be swamped with excess cash flow *ex-post*, they are led to conservative investment strategy due to the concern on potential balance sheet risks. Moreover, the degree of conservatism is endogenously time-varying as a function of macroeconomic developments (as captured in the aggregate state variables).

The caution, or behavior like risk aversion, caused by the frictional capital market plays an important role in the determination of asset prices. To see this point, consider the equation for S_t, (8), which is an asset pricing formula in our framework. The asset pricing formula is different from the textbook version for two reasons. First, it is a levered asset pricing formula in that the asset return is raised up by a factor $1/\tilde{m}_t$ where $\tilde{m}_t$ is the after-tax margin as defined earlier. If $\tilde{m}_t = 1$, in which case the intermediary is 100 equity funded, the formula collapses to $1 = \mathbb{E}_t[M_{t,t+1}^B \cdot (1 + \tilde{r}_{t+1}^A)/\Pi_{t+1}^p]$, yielding a more familiar expression.

Second, despite the ownership by the household, the intermediary pricing kernel deviates from that of the representative household. In general, a pricing kernel is a discounting factor reflecting the relative marginal valuation of cash flows for the asset holders, today vs tomorrow. In a frictionless neoclassical growth model, such a discounting factor can be constructed with the ratio of marginal utilities of the representative household since the marginal utility is the shadow value of the flow-of-funds constraint facing the household.

In our framework, asset valuation undergoes an important modification: owing to the fund-

[5] (11) shows that the support of the idiosyncratic shock is divided into three partitions: (i) $(0, \epsilon_t^D]$, (ii) $(\epsilon_t^D, \epsilon_t^E]$, and (iii) (ϵ_t^E, ∞). In the first interval, the intermediary defaults. In the second interval, the intermediary avoids default, but needs to raise new funds through stock market. In the third interval, the intermediary pays out strictly positive dividends to the shareholders. This characterization makes it clear that the intermediary defaults when it stops issuing equity to pay back its debts.

ing market frictions facing the financial intermediaries, the shadow value of the budget constraint of a risk neutral intermediary is not always equal to one, and as a result, the pricing kernel has an additional factor, the ratio of expected shadow values of internal funds today vs tomorrow, $\mathbb{E}^{\epsilon}_{t+1}[\lambda_{t+1}]/\mathbb{E}^{\epsilon}_{t}[\lambda_t]$, a ratio summarizing the intermediary's expectation about their dynamic balance sheet condition. In this sense, (8) and (14) can be thought of as an application of liquidity-based asset pricing model (LAPM, Holmström and Tirole (2001)) in a dynamic general equilibrium economy. [6]

Effects of Costly Equity Finance and Default Option To show how the financial market frictions in the paper affects the prices of assets and the financial investment decisions of the intermediaries, it is useful to transform the FOC for S_t into a more intuitive form in the following way. Dividing the FOC through by m_t and, substituting (12) for $\mathbb{E}^{\epsilon}_{t}[\lambda_t]$ and replacing $\epsilon^{D}_{t+1}(1+r^{A}_{t+1})$ with $(1-m_t)(1+(1-\tau_c)r^{B}_{t+1})$ yields

$$1 = \mathbb{E}_t \left\{ M^{B}_{t,t+1} \frac{1}{m_t} \left[(1+\tilde{r}^{A}_{t+1}) - (1-m_t)(1+(1-\tau_c)r^{B}_{t+1}) \right] \right\} \tag{13}$$

where

$$M^{B}_{t,t+1} \equiv M_{t,t+1} \frac{\mathbb{E}^{\epsilon}_{t+1}[\lambda_{t+1}]}{\mathbb{E}^{\epsilon}_{t}[\lambda_t]} = M_{t,t+1} \frac{1+\mu F_{t+1}(\epsilon^{E}_{t+1})}{1+\mu F_t(\epsilon^{E}_{t})} \tag{14}$$

and

$$\begin{aligned} 1+\tilde{r}^{A}_{t+1} &= \frac{\mathbb{E}^{\epsilon}_{t+1}[\lambda_{t+1}\max\{\epsilon_{t+1}, \epsilon^{D}_{t+1}\}]}{\mathbb{E}^{\epsilon}_{t+1}[\lambda_{t+1}]}(1+r^{A}_{t+1}) \\ &= \left\{ \frac{\mathbb{E}^{\epsilon}_{t+1}[\lambda_{t+1}\epsilon_{t+1}]}{\mathbb{E}^{\epsilon}_{t+1}[\lambda_{t+1}]} + \frac{\mathbb{E}^{\epsilon}_{t+1}[\lambda_{t+1}\max\{0, \epsilon^{D}_{t+1}-\epsilon_{t+1}\}]}{\mathbb{E}^{\epsilon}_{t+1}[\lambda_{t+1}]} \right\}(1+r^{A}_{t+1}). \end{aligned} \tag{15}$$

(13) can be considered an asset pricing formula. It is different from the text book version because, first, it is a levered asset pricing formula, and second, the pricing kernel of the financial intermediaries is a filtered version of the representative households. The wedge between the pricing kernels of the intermediaries and the representative household is determined by the dynamic liquidity condition measured by the ratio of expected shadow values of internal funds, today vs

[6] See He and Krishnamurthy (2008), who derive an intermediary specific pricing kernel by assuming risk aversion for the intermediary. Also see Jermann and Quadrini (2009), who derives a similar pricing kernel by assuming a quadratic dividend smoothing function.

tomorrow. This wedge effectively generates an external premium for the end users of credit, i.e., the productive firms because the wedge contains the price of risk component, where the risk is not originated from the intertemporal smoothing needs of the household sector, but from the funding frictions facing financial intermediaries. We define an implicit lending rate R^L_{t+1} as the return that satisfies $1 = \mathbb{E}_t[M^B_{t,t+1}R^L_{t+1}]$. It is a direct result of capital market friction that an external financing premium exists despite the assumption that the relationship between the lender and the firms are essentially frictionless.

The asset pricing formula (13) shows a rich relationship between the intermediaries stochastic discount factor and the returns to investment, equity issuance, and default decisions of intermediaries. Specifically, the return entering the asset valuation condition is composed of two terms reflecting the key financial market frictions in the model. The first component of the above is a direct result of costly equity financing friction as the factor collapses into 1 under a frictionless equity market ($\varphi = 0$, hence $\lambda_t = 1$).

As for the second part, consider the possibility that it is nil – i.e., no default. In this case and with costly equity financing ($\varphi > 0$), $\mathbb{E}^{\epsilon}_{t+1}[\lambda_{t+1}\epsilon_{t+1}]/\mathbb{E}^{\epsilon}_{t+1}[\lambda_{t+1}]$ is always less than one, as high realizations of the idiosyncratic return ϵ_{t+1} will be associated with lower values for internal funds (as high returns imply that there is no need to raise external funds, thereby lowering the *ex post* value of internal funds), and this negative covariance implies, via Jensen's inequality, that the numerator is less than the denominator of the valuation expression. [7] Under a diminishing marginal rate of return from capital, which is the case in this paper, this means that the asset return $1 + r^A_{t+1}$ must be higher than it would be in a frictionless market, implying that capital is under-accumulated in the model economy owing to the capital market frictions.

Now consider the second factor of (15). Clearly, this is the value of default option. Being an option value, the second factor is necessarily non-negative. In contrast to the equity market friction, the credit market friction under the limited liability encourages risk-taking, inducing the intermediaries to over-invest in capital assets.

An option is more valuable when the uncertainty of asset returns increases. This, however, does not imply that the financial intermediaries will increase their investments in risky assets

[7] In fact, this is the case in our earlier works (Kiley and Sim (2011a) and Kiley and Sim (2011b)), in which we consider the financial intermediary sector under a regulatory capital requirement.

at a time of heightened uncertainty: while a greater uncertainty boosts the risk appetite of the intermediaries through the default option, the same increase in uncertainty boosts the first term, thereby elevating the ex-ante weighted average cost of capital for the intermediaries, which then reduce investment in risky assets.

2.2 The Rest of the Model Economy

To close the model, we now turn the production, capital accumulation and the consumption/saving decisions of the representative household. Regarding the structure of production and capital accumulation, we take the approach of Gertler and Karadi (2011) and Gertler and Kiyotaki (2010), which has become a standard setting for studying the role of frictions in the financial intermediation. Namely, we assume that the production of consumption and investment goods are devoid of financial frictions. This assumption, while strong, helps us focus on the friction facing the financial intermediaries in their funding markets rather than the friction in their lending (investment) market.

2.2.1 Production and Investment

We assume that there is a competitive industry that produces intermediate goods using a constant returns to scale technology; without loss of generality, we assume the existence of a representative firm. The firm combines capital (K) and labor (H) to produce the intermediate goods using a Cobb-Douglas production function, $Y_t^M = a_t H_t^{\alpha} K_t^{1-\alpha}$, where the aggregate technology shock follows a Markov process, $\log a_t = \rho_a \log a_{t-1} + \sigma_v v_t$, $v_t \sim N(0,1)$.

As in Gertler and Karadi (2011) and Gertler and Kiyotaki (2010), the intermediate-goods producer issues state-contingent claims S_t to a financial intermediary, and use the proceeds to finance capital purchase, $Q_t K_{t+1}$. No-arbitrage condition implies that the price of the state-contingent claim must be equal to Q_t such that $Q_t S_t = Q_t K_{t+1}$. We assume that the financial intermediary has enough information to align the interests of the intermediary and the firm. After the production and sales of products, the firm sells its undepreciated capital at the market value, returns the profits and the proceeds of capital sales to the intermediary. The competitive industry structure implies that the firm's static profit per capital is determined by the capital share of the revenue, i.e., $r_t^K = (1-\alpha)P_t^M Y_t^M / K_t$, where P_t^M is the price level of the intermediate goods. Hence the after-tax return for the intermediary is given by

$$R_t^F = \frac{(1-\tau_c)(1-\alpha)P_t^M Y_t^M / K_t + [1-(1-\tau_c)\delta]Q_t}{Q_{t-1}}. \tag{16}$$

To endogeneize the value of capital assets, we introduce a friction in adjusting the level of investment at the aggregate level. More specifically, we assume that there is a competitive industry producing new capital goods combining existing capital stock and consumption goods using a quadratic adjustment cost of investment, $\chi/2(I_t/I_{t-1}-1)^2 I_{t-1}$.

2.2.2 Households

We specify a standard CRRA consumption utility for the representative household. More specifically, we specify $u(C_t, H_t) = \frac{1}{1-\gamma}[C_t^{1-\gamma}-1] - \frac{\zeta}{1+\nu}H^{1+\nu}$ for the momentary utility function. The representative household earns market wage by providing labor hours. The efficiency condition for labor hours is given by the FOC, $\frac{W_t}{P_t}C_t^{-\gamma} = \zeta H_t^{\nu}$, where P_t is the Dixit and Stiglitz (1977) type composite price level of the retail goods. As assumed earlier, households save through financial intermediaries, investing in either their debt or equity shares.

We denote the total outstanding debt of financial intermediaries by B_t. In equilibrium, $B_t = \int [1-m_{t-1}(i)]Q_{t-1}S_t(i)di = (1-m_{t-1})Q_{t-1}K_t$, where i is an index for an intermediary and the last equality is due to the symmetric equilibrium and the no-arbitrage condition aforementioned. We presented the no-arbitrage condition that prices intermediary debt (2) previously. Investment in the shares of the financial intermediary satisfies the equilibrium condition

$$1 = \mathbb{E}_t\left[M_{t,t+1}\frac{\mathbb{E}_{t+1}^{\epsilon}[\max\{D_{t+1},0\} + (1-\varphi)\min\{D_{t+1},0\}] + P_{t+1}^S}{P_t^S}\right] \tag{17}$$

where P_t^S is the ex-dividend price of an intermediary share. This is a standard dividend-price formula for the consumption CAPM, taking into account the effect of the equity issuance cost on dividend related cash flows to investors (as shown in the appendix)/ Note that in our symmetric equilibrium, $P_t^S(i) = P_t^S$ for all $i \in [0,1]$ because $P_t^S(i) = \mathbb{E}_t[M_{t,t+1} \cdot J_{t+1}]$ does not depend on intermediary specific variables. Finally, note that in general equilibrium, the existing shareholders and the investors in the new shares are the same entity, the representative household. Hence, the costly equity financing does not create a wealth effect for the household, but affects the aggregate allocation through the marginal efficiency conditions of the intermediaries.

2.2.3 Nominal Rigidity and Monetary Policy

We assume that a continuum of monopolistically competitive firms take the intermediate outputs as inputs and transform them into differentiated retail goods $Y_t(j)$, $j \in [0,1]$. To generate a nominal rigidity, we assume that the retailers face a quadratic cost in adjusting their prices $P_t(j)$ given by $\frac{\chi^p}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - \bar{\Pi}^p\right)^2 P_t Y_t$, where Y_t is the CES aggregate of the differentiated products with an elasticity of substitution ε, $\bar{\Pi}$ is the current inflation rate and steady state inflation rate, respectively, and ς is the indexation weight. We also assume a symmetric nominal rigidity in the determination of wage, where the adjustment cost of nominal wage is specified as $\frac{\chi^w}{2}\left(\frac{W_t(j)}{W_{t-1}(j)} - \bar{\Pi}^w\right)^2 W_t H_t$.

For the monetary policy, we specify a Taylor-type interest rule given by

$$R_t = R_{t-1}^{\rho^r}\left[\left(\frac{Y_t}{Y_t^*}\right)^{\kappa^y}\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\kappa\Delta p}\right]^{1-\rho^r}\epsilon_t^R \tag{18}$$

where Y_t^* is specified as the capacity level of output, given by $Y_t^* = K_t^{1-\alpha}(z_t H^*)^{1-\alpha}$, where z_t is the current level of aggregate technology and H^* is the steady state level of work hours. Thus, the output gap in this paper is defined as deviation from the capacity level of output.

2.2.4 Fiscal Policy

In our baseline model, the fiscal policy is simply dictated by the period-by-period balanced budget constraint, which is given by

$$T_t = \tau^c r_t^K K_t - \tau^c[\delta + (1 - m_{t-1})]Q_{t-1}K_t. \tag{19}$$

The first item is the proceeds of the corporate income tax and the second item is the sum of depreciation allowances and tax refund on debt holdings.

3 Model Properties: A Quantitative Evaluation

In order to illustrate more clearly the economic implications of our model framework, we explore its quantitative implications by illustrating the effect of economic disturbances – specifically, shifts in aggregate productivity/technology, monetary policy shocks, increases in the degree of risk within the financial system that lower the relative attractiveness of debt financing, and increases in the costs of external equity that lower the relative attractiveness of equity financing. The impulse response analyses we undertake will provide context for the degree to which our model of intermediation contains a mechanism that amplifies the effects of disturbances outside the financial sector – a *financial accelerator* – and the possible importance for developments within the financial sector for the macroeconomy more generally.

3.1 Calibration

Many of our parameters are set at standard values (see table 1. The discount factor *beta* is set to 0.985, implying a steady state real return to capital near 6 percent per year (given that we calibrate to a quarterly frequency). The households' risk aversion parameter γ is set to 2, a modest value; the Frisch labor-supply elasticity η is set to 1, a commonly used value in macroeconomic analyses. We set the labor share in production α to 0.60 and the depreciation rate δ to 0.025. With regard to adjustment costs for investment and prices/wages, we adopt a moderate value for investment adjustment costs (χ equal to 0.5) and a large value for prices and wages (χ^p and χ^w equal to 250); these values deliver reasonable responses of investment and price/wage inflation to shocks, are broadly consistent with empirical work suggesting very flat "Phillips curves", and, for reasonable variations, have little influence on our main results.

The parameters for the interest rate rule assume moderate persistence/inertia (ρ_r equal to 0.75), a response to the deviation of output from the level consistent with technology, the level of productive capital, and steady-state labor input equal to that of Taylor (1999) (κ_y equal to 0.25, or equal to 1 for interest rates expressed at an annual rate), and a response to inflation consistent with that of Taylor (1993) and Taylor (1999) ($\kappa_{\Delta p}$ equal to 1.5).

There are several aspects of our calibration that govern predictions for the macroeconomic effects of credit policies. We set the bankruptcy cost η equal to 0.05 – a low value; this is a relative

Table 1: Baseline Calibration

Description	Calibration
Preferences and production	
Time discounting factor	$\beta = 0.985$
Constant relative risk aversion	$\gamma = 2$
Elasticity of labor supply	$1/\nu = 1$
Value added share of labor	$\alpha = 0.6$
Depreciation rate	$\delta = 0.025$
Real/nominal rigidity and monetary policy	
Investment adjustment cost	$\chi = 5$
Price adjustment cost	$\chi^p = 250$
Wage adjustment cost	$\chi^w = 250$
Monetary policy inertia	$\rho^r = 0.75$
Taylor rule coefficient for output gap	$\kappa^y = 0.25$
Taylor rule coefficient for inflation gap	$\kappa^{\Delta p} = 1.5$
Financial Frictions	
Liquidation cost	$\eta = 0.05$
Dilution cost	$\varphi = 0.30$
Corporate tax	$\tau_c = 0.20$
Long run level of uncertainty	$\bar{\sigma} = 0.05$
Exogenous Stochastic Process	
Persistence of idiosyncratic uncertainty	$\rho^\sigma = 0.85$
Volatility of shock to uncertainty process	$\sigma^v = 0.10$
Persistence of technology shock	$\rho^z = 0.90$
Volatility of shock to technology process	$\sigma^z = 0.006$

modest degree of financial friction – as recovery rates on many assets are often estimated below 95 percent. The estimates/calibrations for the equity issuance cost varies a lot in the literature ranging from 0.08 in Gomes (2001) to 0.30 in Cooley and Quadrini (2001). We chose $\varphi = 0.30$, following Cooley and Quadrini (2001). While this choice is on the high side of the range, we made this choice to replicate the harsh financing environment seen during the recent financial turmoil.

The tax advantage of debt financing associated with the deductibility of interest in a corporate tax framework provides one of the rationales for leverage by financial intermediaries. The model does not include a complete specification of the tax system (indeed, by a wide margin). The corporate tax rate τ_c is set to 0.20 – far below the statutory or effective corporate rate in the United States, but a somewhat high value for the tax preference of debt relative to equity.[8]. The preferential tax treatment of debt will help ensure that intermediary leverage within our model is high

[8] for instance, Philippon (2009)chooses a tax shield of 0.12

(that is, near the observed value for the margin, as measured by total regulatory capital relative to risk-weighted assets at U.S. banks, of $12 - 1/2$ percent).

Finally, the distribution of idiosyncratic uncertainty, and particular its evolution of time, is a key factor. We assume the ϵ_t follows a log-normal distribution: The standard deviation of this distribution σ changes over time, while the mean also adjusts to ensure that the mean of ϵ always equals one. (That is, we only consider mean-preserving changes in the second moment of ϵ to focus on variation in risk). To model time-varying uncertainty, we assume the following process:

$$\log \sigma_t^v = (1 - \rho_\sigma) \log \bar{\sigma} + \rho_\sigma \log \sigma_{t-1}^v + u_t, \quad u_t \sim \text{iid } N(0, \Sigma^{v2}). \tag{20}$$

We set $\bar{\sigma_v}$ equal to 0.05, a relatively modest value for idiosyncratic risk; we set ρ_σ equal to 0.85, and Σ^v equal to $0.x$.

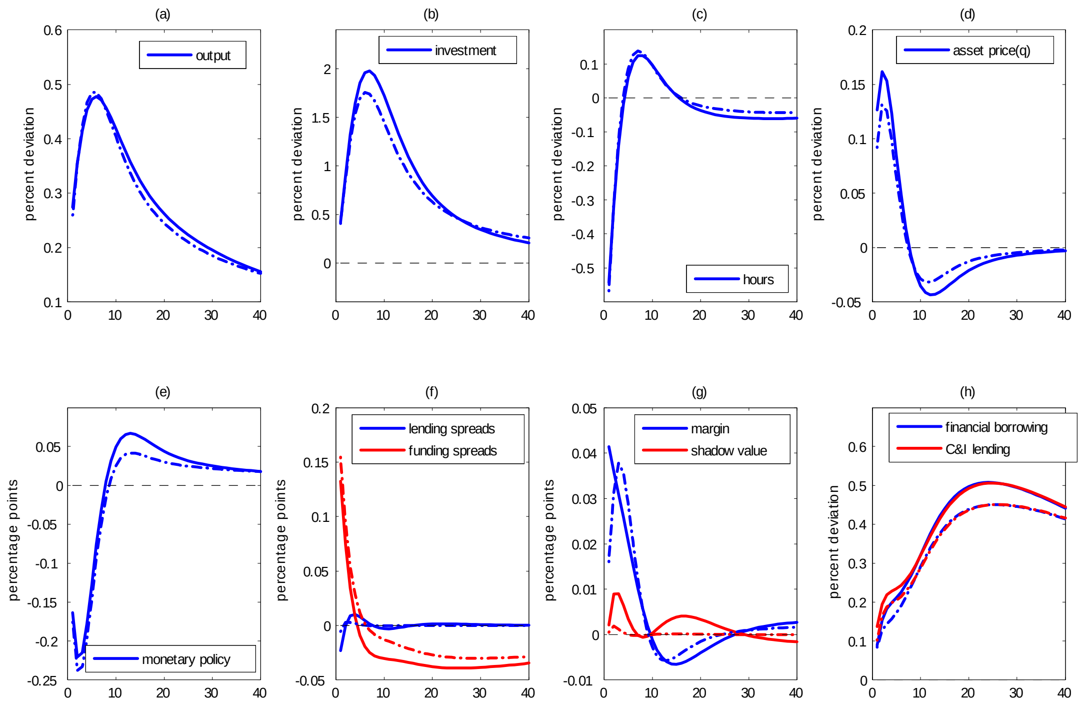
3.2 Technology and Monetary Shocks

We first present the response of the economy to a shift in aggregate productivity – a technology shock z_t. Figure 1 shows the response of output, investment, hours, asset prices, the monetary policy rate, the lending spread defined as $R^L - R$, the funding spread defined as $R^B - R$ (all at annual rates), the equity margin (i.e., m_t, the inverse of leverage) and value of intermediary internal funds $\mathbb{E}_t^\epsilon[\lambda_t]$, and intermediary borrowing and lending.

Two response for each variable are presented – the response for the baseline calibration and the response for a calibration in which the equity dilution cost ϕ is set to 0.01, rather then 0.30; the responses under the calibration with the lower value of the equity dilution cost illustrate the responses of an economy in which intermediary financial frictions are very modest (and hence an economy more like a prototypical dynamic general equilibrium macroeconomic model without financial frictions).

On balance, the responses to a technology shock are similar for the baseline calibration and that with low financial frictions. That said, there is a modest financial accelerator to investment in the case with significant financial frictions – as can be seen in the somewhat larger movement in investment in that case. It is also notable that aggregate lending moves significantly with the movements in investment, reflecting the increase in demand for credit associated with stronger in-

Figure 1: Effect of Technology Shock

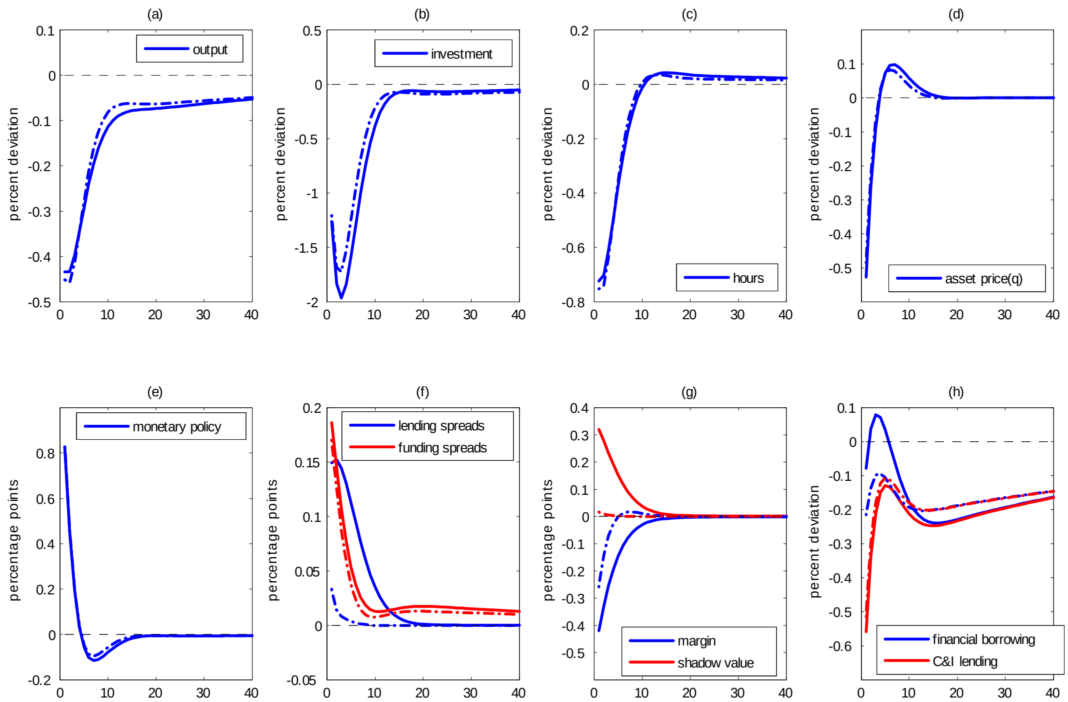


vestment; this movement will prove important in thinking about macroprudential policies below. In contrast, intermediary leverage moves little in this case: The technology shock has little effect on the relative attractiveness of debt and equity for intermediaries and hence the equity margin is relatively stable in response to technology shocks.

Figure 2 reports the impulse response following a 100 basis point (at an annual rate) shock to the monetary policy reaction function. Again, the model shows a modest financial accelerator. In other regards, the model responses to this shock are typical of those in the related literature.

Overall, our review of the quantitative impact of technology and monetary shocks yields three important insights we will use later. First, the model has a modest financial accelerator: As in the related literature summarized in Quadrini (2011) or Boivin et al. (2010), the financial accelerator, while present, is not especially large. Second, technology shocks, and indeed any shock with an important macroeconomic effect, leads to significant movements in aggregate credit as such lending finances movements in investment; indeed, because movements in overall investment financed by credit is not perfectly proportional to movements in real GDP, an indicator like the credit-to-GDP ratio will experience large movements in response to most macroeconomic dis-

Figure 2: Effect of Monetary Policy Shock



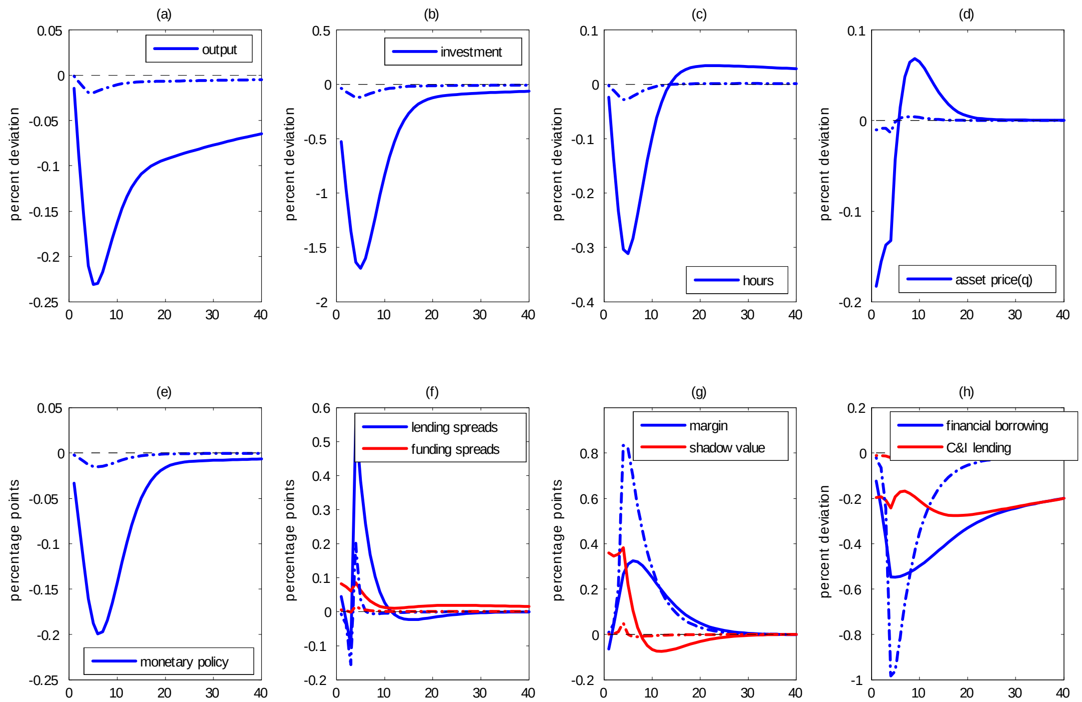
turbances. Finally, intermediary leverage does not necessarily move significantly in response to all macroeconomic disturbances–for example, the intermediaries' equity margin moves little in response to a shift in technology; rather, intermediary leverage is importantly influenced by the relative attractiveness of debt and equity, which may or may not change in response to certain economic disturbances.

3.3 Financial Disturbances

While the financial accelerator is only modest within our model (as in many others), our model creates a significant role for financial disturbances. We illustrate two such disturbances that are important in the presence of the financial frictions we have emphasized and that also have potentially different implications for macroprudential policies.

The first disturbance we consider is an increase in the idiosyncratic risk facing intermediaries investments. As we emphasized earlier, the inability of intermediaries to perfectly match the timing of information about the value of the assets and liabilities, and in particular the role of commitments in lending, imply that intermediaries adopt a precautionary approach to lending.

Figure 3: Effect of Anticipated Idiosyncratic Risk Shock



This precaution increases in response to an increase in idiosyncratic risk, causing intermediaries to pull back the supply of lending a hence lowering aggregate investment and output.

We illustrate this dynamic in figure 3, which shows the response of the economy to an anticipated (four-quarters in advance) increase in idiosyncratic risk. The increase in anticipated risk raises the value of internal funds at intermediaries, boosting the spread between the loan interest rate and the risk-free policy rate; as a result, lending and investment decline. The increase risk also leads financial intermediaries to, over time, increase their equity margin (i.e., deleveraging), as higher risk implies a higher probability of having to raise costly external funds and deleveraging mitigates intermediaries' exposure to this risk. This dynamic reflects the fact that higher risk makes debt less attractive – intermediaries with high debt and (now more likely due to increased risk) adverse realized returns must raise more costly equity; in contrast, equity is relatively more attractive with increased risk, reflecting limited liability. Finally, the adverse macroeconomic consequences are notable, with output declining significantly. Of course, these influences are only sizable in the presence of financial frictions: The low friction case shows little response of macroeconomic aggregates or overall lending to this disturbance.

Figure 4: Effects of Equity Cost Shock

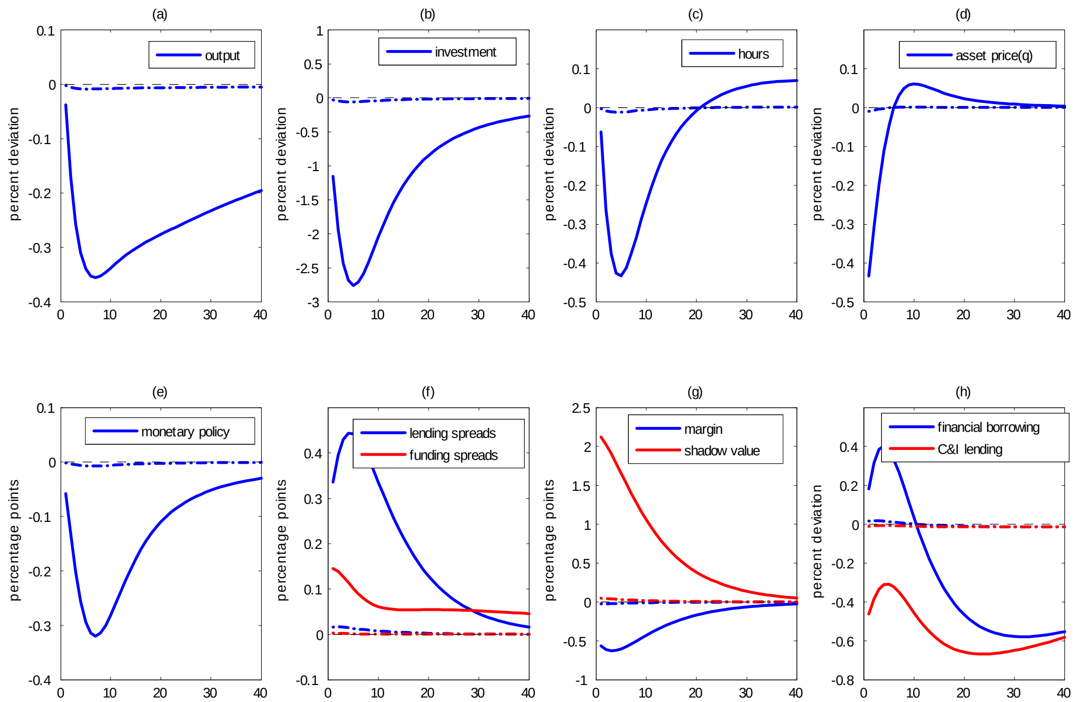
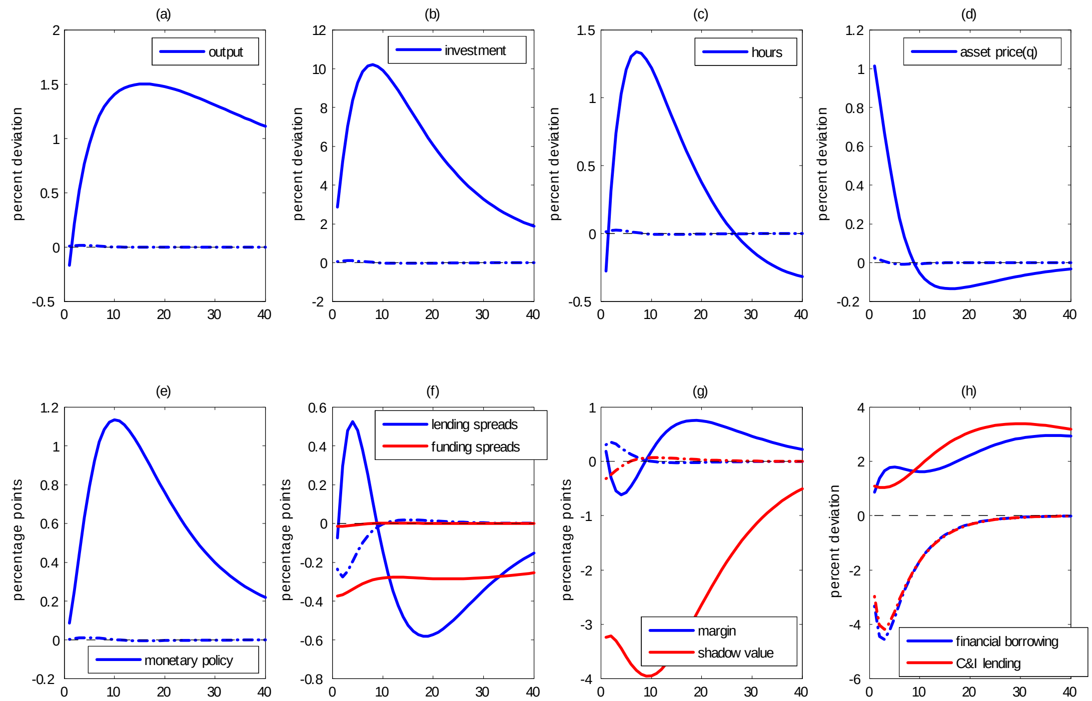


Figure 4 considers an alternative shock to intermediaries – an increase in the equity dilution cost. Such a shock to the expected costs of funds also leads intermediaries to decrease the supply of lending, which leads to a decline in investment and in aggregate output; as in the case of an increase in intermediary risk, this shock only has significant macroeconomic consequences in the presence of sizable financial frictions. However, an increase in the equity dilution cost, while similar to an increase in risk in that it leads intermediaries to contract their balance sheet, has one dynamic implication that is starkly different from that of increased risk: The contraction in the intermediary balance sheet (e.g., lending) is accompanied by a *lower* equity margin (i.e., increase in leverage), as equity has become the unattractive financing margin. This result illustrates how movements in intermediary leverage may not reflect shifts in intermediaries' willingness to supply credit.

In contrast to the divergent predictions for leverage associated with increased risk or an increase in the cost of equity, the lending spread rises following both shocks and provides a cleaner view of the shift in lending supply associated with each shift in financial conditions facing intermediaries.

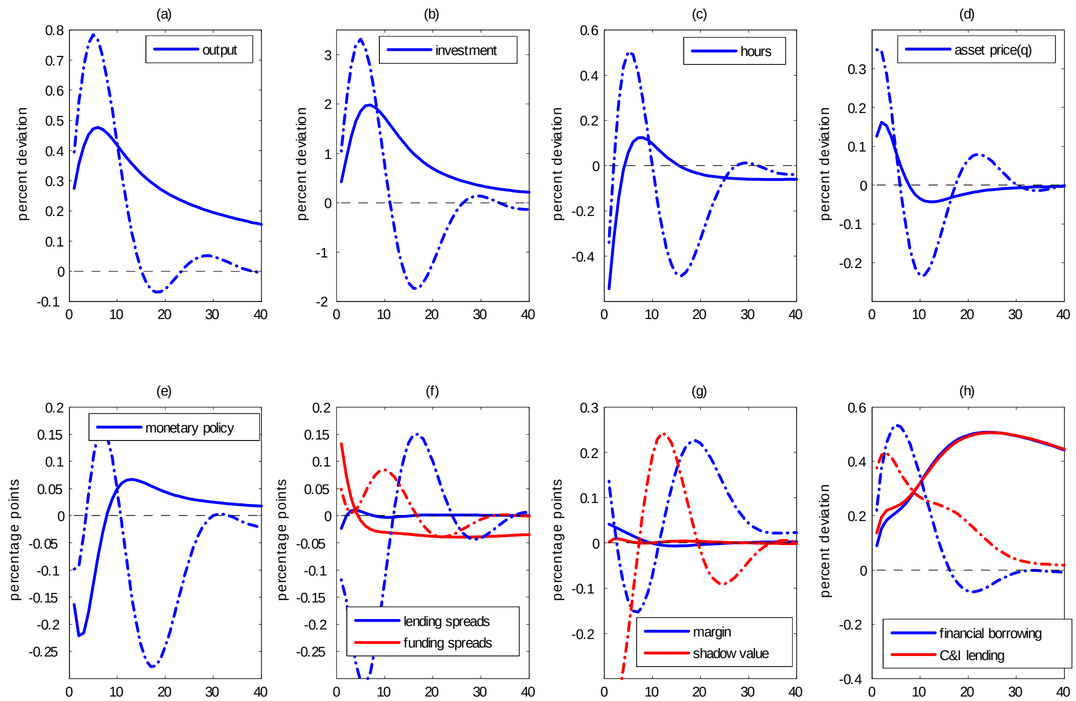
Figure 5: Effects of Asset Purchase and Capital Injection Policies



4 Macroprudential Policies

The integration of intermediary balance sheet management, lending decisions, and macroeconomic fluctuations makes our model framework ideal for analysis of macroprudential policies. We first consider what crisis policies of the sort analyzed in Gertler and Karadi (2011), Kiley and Sim (2011b), and Gertler et al. (2010); the first two of these studies examined similar issues in model's without an endogenous choice between intermediary debt and equity. We then expand the focus from crisis policies to stabilization policies, thereby moving beyond Gertler et al. (2010) to the set of issues laid out in Drehmann et al. (2010); while there has been some work in dynamic macroeconomic models exploring this issue (e.g., Christensen et al. (2011)), the focus of much of this literature has been on macroprudential rules that lean against credit growth, and our consideration of debt and equity choices at intermediaries will highlight a range of challenges that may encounter macroprudential stabilization policies and have not previously been analyzed.

Figure 6: Effect of Technology Shock Under Credit-to-GDP Macroprudential Rule



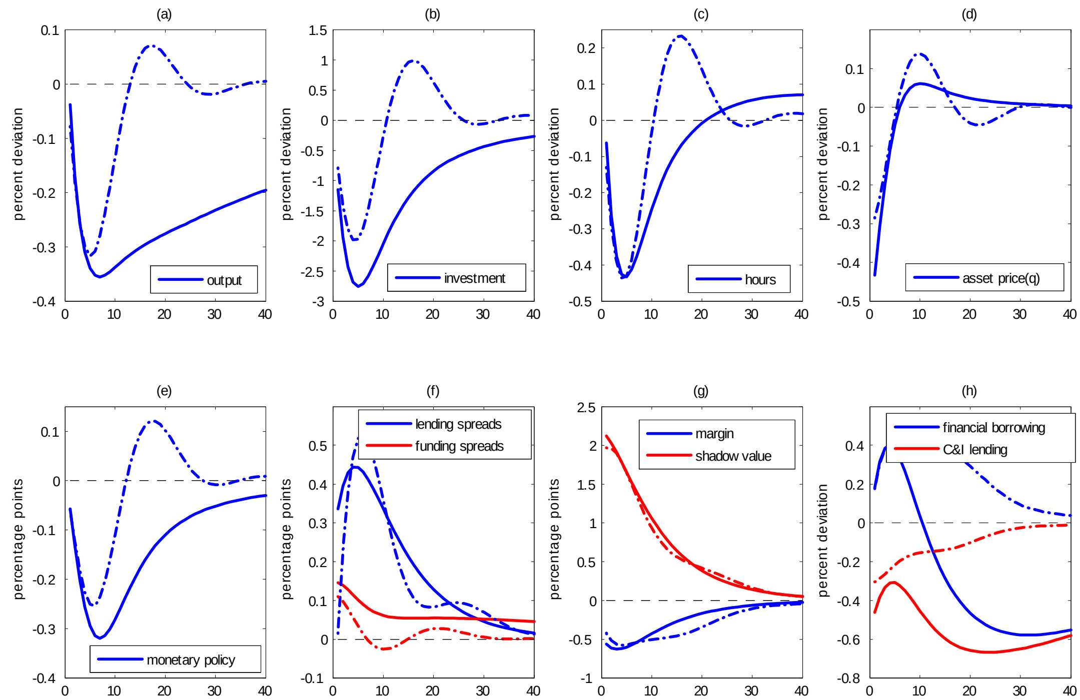
4.1 Crisis Policies

In response to a sharp decline in lending supply, Gertler and Karadi (2011) and Gertler et al. (2010) considered policies under which the government directly finances private (non-financial) investment; their analyses indicate that such programs, properly designed, can substantially mitigate a decline in private lending supply (while also potentially creating undesirable incentives when intermediaries incorporate the possibility of such policies into their private decisions).

We consider similar policies, and expand the set of crisis policies to include government injections of equity to intermediaries, as in Kiley and Sim (2011b); such injections are reminiscent of the use of government funds under the TARP program, which considered direct purchase of financial products backing non-financial lending but, in the end, entailed bank capital injections. To compare government-financed investment and capital injections, we assume (essentially) equivalent government outlays: We set the size of initial shocks so that the outlays of each type equal about 2 percent of output, roughly matching the 13/4 percent share of GDP that was devoted to bank recapitalization under TARP.

The direct lending policy can be understood in the following way. Without the government

Figure 7: Effect of Equity Cost Shock Under Credit-to-GDP Macroprudential Rule



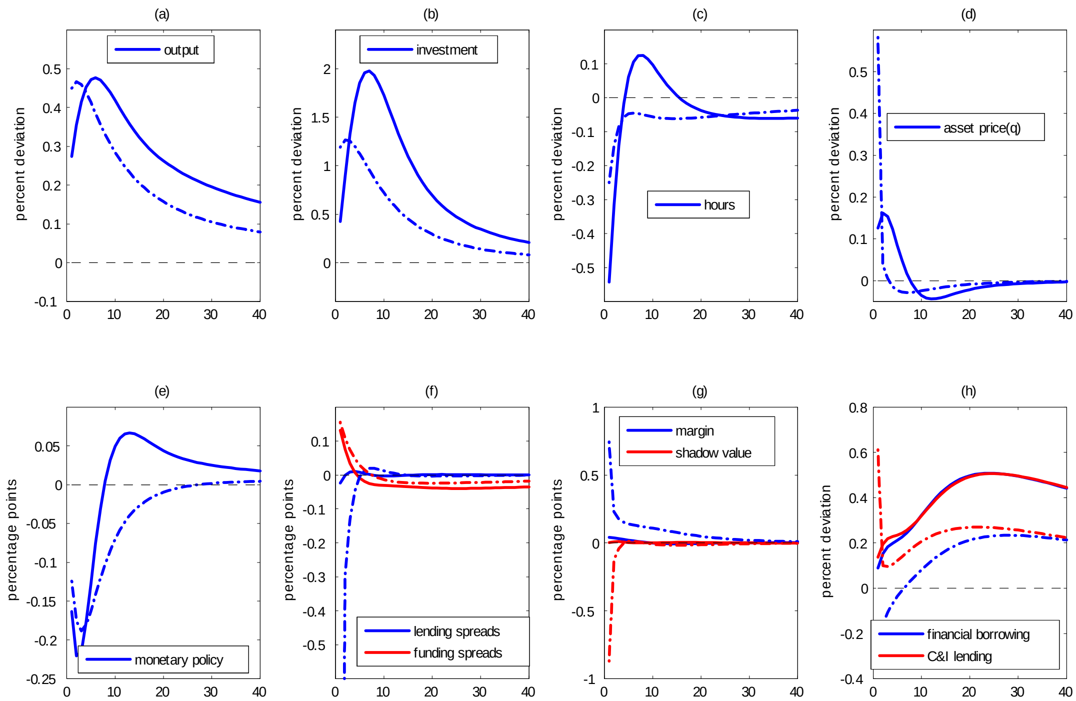
policy, the market clearing condition for capital assets is given by $S_t = K_{t+1}$. Let $s_t^{G,K}$ and $s_t^{B,K}$ denote the shares of capital assets owned by the government and by the intermediaries, respectively. Using these notations, the market clearing condition for capital assets can be expressed as $1 = s_t^{G,K} + s_t^{B,K}$. We assume that the government does not consider a short-sale policy – e.g., government investment is always greater than or equal to zero, restricting the space of $s_t^{G,K}$ to $[0, 1]$. We also assume that the government maintains a balanced budget and imposes a lump sum tax to meet the balanced budget constraint,

$$T_t^H = s_{t+1}^{G,K} Q_t K_{t+1} - s_t^{G,K} R_t^A Q_{t-1} K_t. \tag{21}$$

Note that we use R_t^A for asset returns, not R_t^F. This is because we assume that the government purchases pro rata shares of all assets and, as a result, idiosyncratic returns are washed out.

As for the capital injection policy, we envision a situation where the government purchases the new shares issued by the intermediaries in capital markets. We denoted the government share by $s_{t+1}^{G,S}(i)$ of a particular intermediary $i \in [0, 1]$. In so doing, the government refunds the cost of equity issuance to the issuing intermediaries. Technically, this is equivalent to paying higher prices

Figure 8: Effect of Technology Shock Under Q Macroprudential Rule



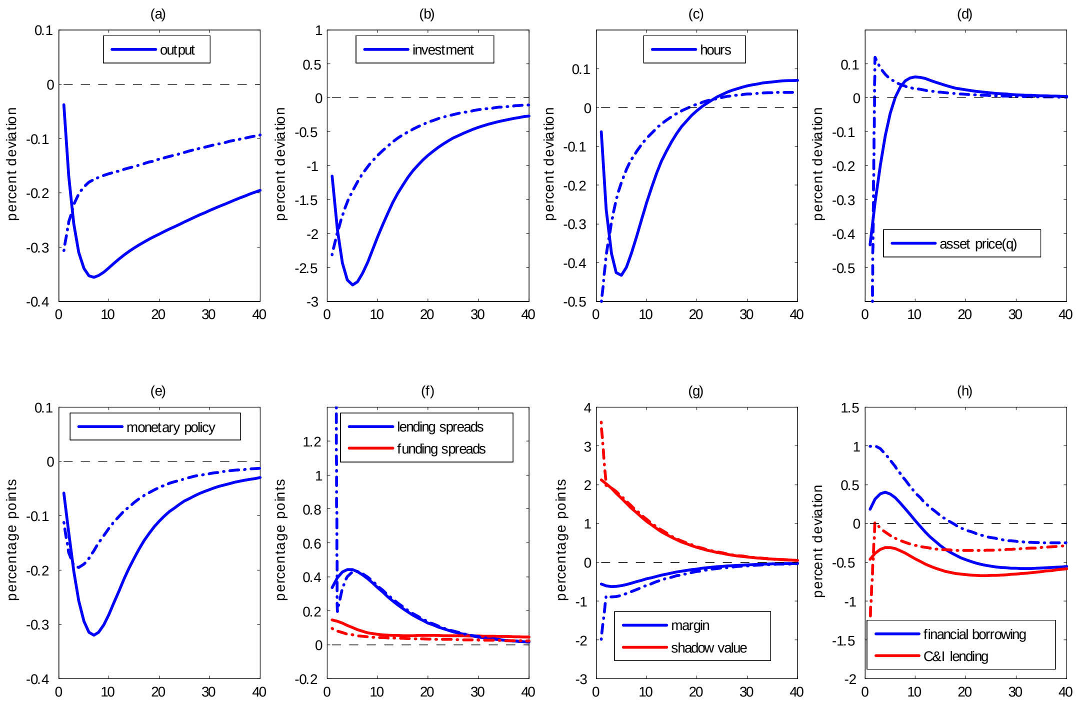
for the newly issued shares, higher than the prices available in capital markets (capital injection). Under this policy, the total cost of issuing equity is reduced to $\varphi(1 - s^{G,S}_{t+1}(i)) = \varphi(1 - s^{G,S}_{t+1})$ where the equality is due to the assumption that the government purchases *pro rata* shares of all issuing intermediaries. In the appendix, we show that the total funding needs to implement this policy evolves over time.

$$T^H_t = F_t(\epsilon^E_t)P^S_t s^G_{t+1} - F_{t-1}(\epsilon^E_{t-1})s^G_t \int_0^1 \left[\max\{D_t(i), 0\}di + P^S_{t-1,t}(i)\right] di.$$

where $P^S_{t-1,t}(i)$ is the current value of outstanding shares at the beginning of period t. Again, we assume that the government can impose a lump sum taxation on the household.

Figure 5 presents the economic effects of the two policies. The blue solid line shows the case of capital injection policy and black solid-dotted line the case of direct lending/asset purchase policy. The direct lending/asset purchase policy generates useful impact as supposed, as can be seen in the drop in the funding pressure measured by the shadow value of internal funds, which then lowers the lending spreads. However the magnitude of such desirable effect is too small to be economically important. The impact on the real variables is negligible. The reason is simple (and

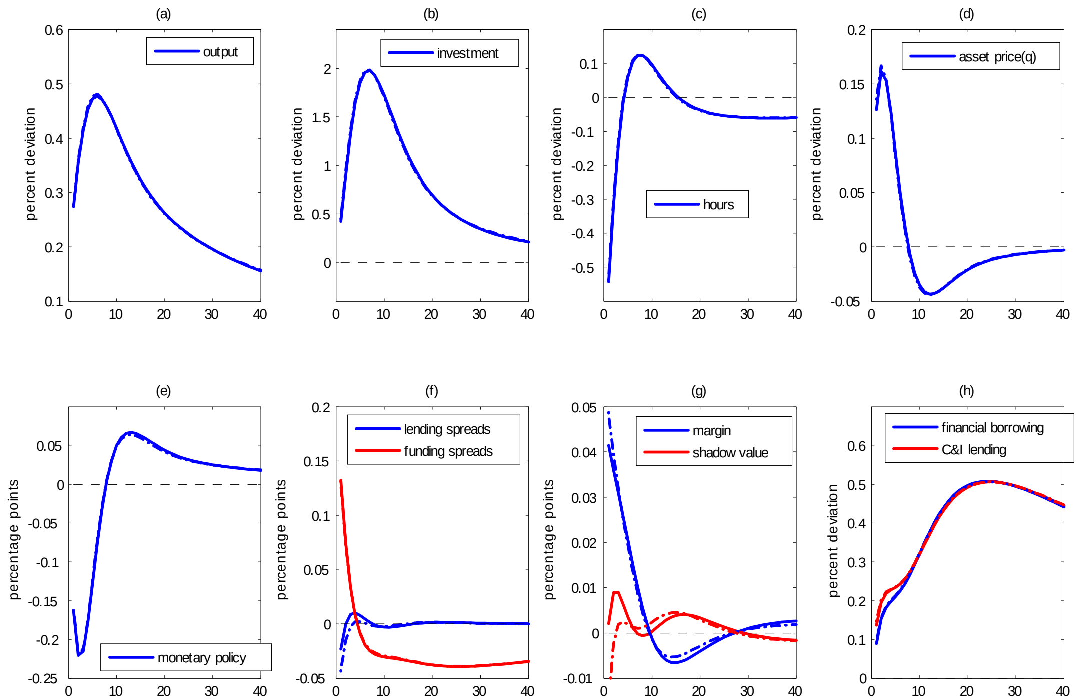
Figure 9: Effect of Equity Cost Shock Under Q Macroprudential Rule



similar to that in Kiley and Sim (2011b)): Government purchases of non-financial assets (direct investment) *crowds out* private lending and investment, despite the modest improvement in intermediaries balance sheets. In panel (h), one can see that the borrowing and lending in the private sector undergoes a persistent decline, which is offset by the expansion of public sector lending.

In a stark contrast, aggregate output (panel (a)) rises much more under the capital injection policy, reflecting the larger increase in investment. The capital injection improves the liquidity/balance sheet condition of intermediaries, raising private investment; indeed, this increase is much stronger than direct investment by the government because intermediaries *leverage* the equity capital to boost the supply of lending significantly (with the increase in lending supply leading to a sharp decline in the lending spread, panel (f)). The strength of this effect owes partly to the tying of the cash injection to the amount of private equity financing raised by the intermediary: Specifically, this tying implies that only intermediaries that would otherwise wish to reduce lending to a greater extent reveal their balance sheet condition and allows the public resources to be directed to the right place – directly at the potential reduction in credit supply. In contrast, the direct asset purchase policy strengthens the balance sheets condition of all intermediaries, not

Figure 10: Effect of Technology Shock Under Leverage (m) Macroprudential Rule



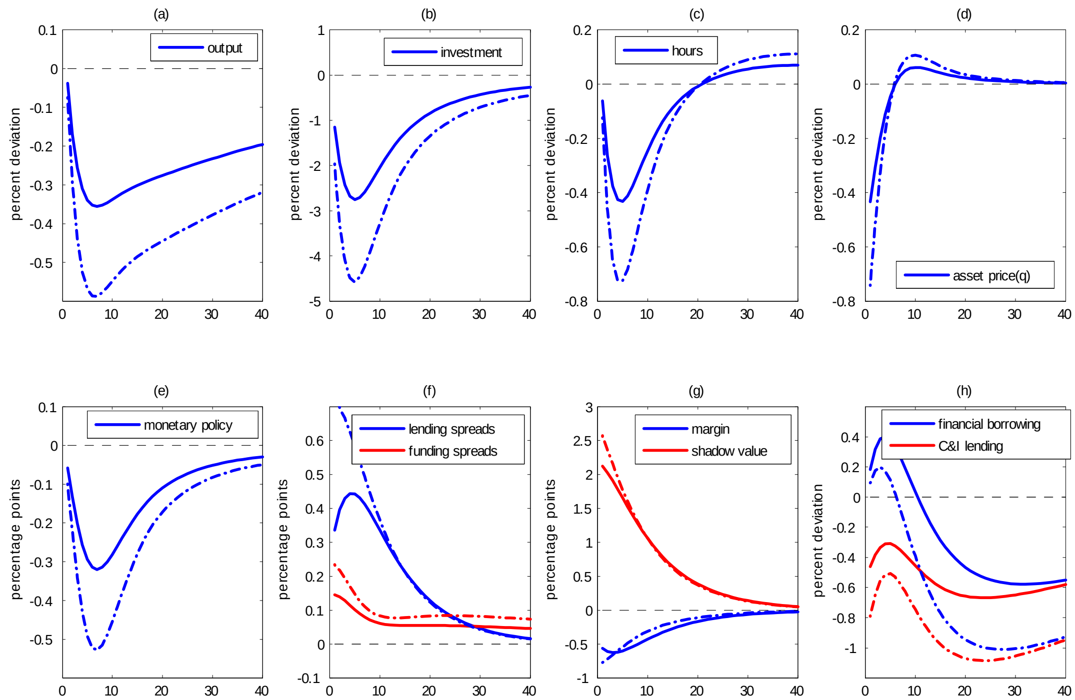
only the cash strapped institutions, and cannot prevent the ones with large amount of surplus cash flow from paying out the extra profits as dividends.

4.2 Stabilization Policies

Our analysis now moves beyond the focus on crisis policies in Gertler and Karadi (2011), Kiley and Sim (2011b), and Gertler et al. (2010) to a consideration of stabilization policies designed to curb fluctuations due to financial shocks. In particular, we consider the possibility of macroprudential rules to lean against financial imbalances in a manner that mitigates the influence of shocks to the financial sector (which absent financial frictions would not influence macroeconomic outcomes) on the macroeconomy.

The focus herein will be on macroprudential policies that influence intermediaries' choices regarding their level of capitalization – that is, that influence their equity margin or leverage choice. This focus is consistent with the framework being developed as part of the Basel III process, which has included consideration of a cyclical buffer for intermediary capital levels that could be adjusted in response to some cyclical indicator; specific focus in recent research has focused on poli-

Figure 11: Effect of Equity Cost Shock Under Leverage (m) Macroprudential Rule



cies to raise bank capital in response to an increase in aggregate credit (lending) relative to output (GDP) (e.g., Drehmann et al. (2010), Christensen et al. (2011), and Edge and Meisenzahl (2011)).

To implement such a cyclical tool within our framework, we assume that the government imposes a tax on leverage/equity margin that encourages or discourages equity accumulation in response to a certain indicator. This tax rate, which we call the margin tax, is equal to zero in the steady state, but can become positive (which encourages intermediaries to raise equity) or negative (which encourages intermediaries to lower equity) in response to a rule linked to an indicator of potential financial imbalances.

We focus on macroprudential rules tied to four indicators. The first indicator is the lending-to-output ratio, which is the approach suggested in Drehmann et al. (2010) and considered in some other research, such Christensen et al. (2011). We also consider adjusting the macroprudential instrument in response to asset prices (q), as leaning against asset prices is an approach often discussed, at least informally. Our analysis also includes two approaches more directly focused on financial sector developments: A macroprudential rule linked directly to leverage (which aims to keep leverage stable directly) and one linked to the lending rate spread over the risk-free rate.

Figure 12: Effect of Anticipated Risk Shock Under Leverage (m) Macroprudential Rule

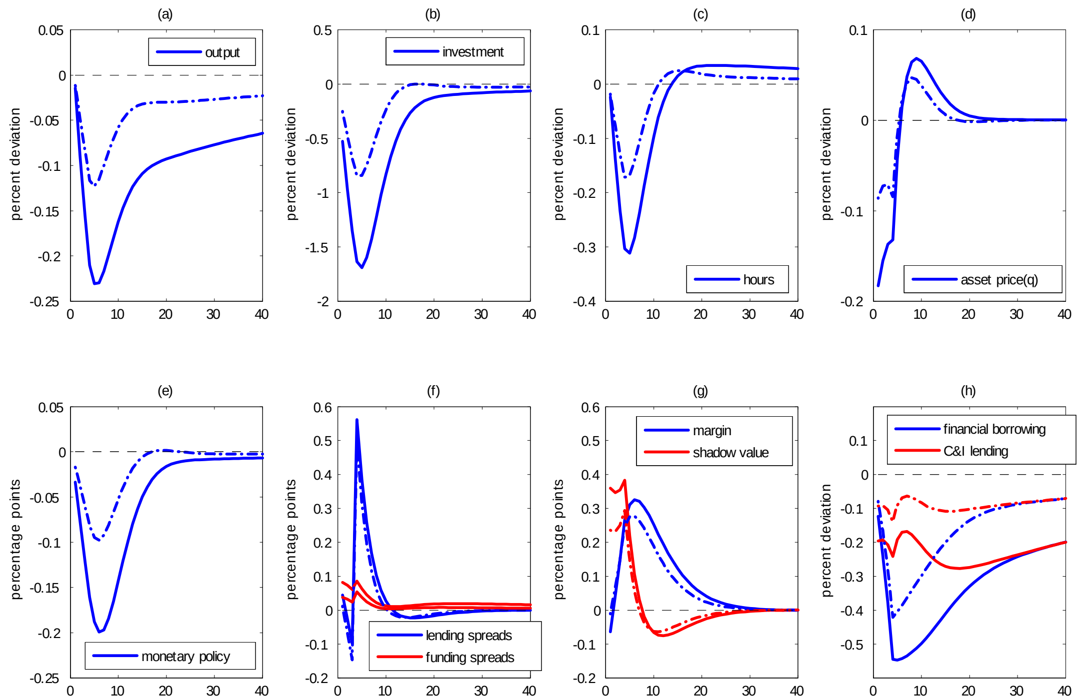
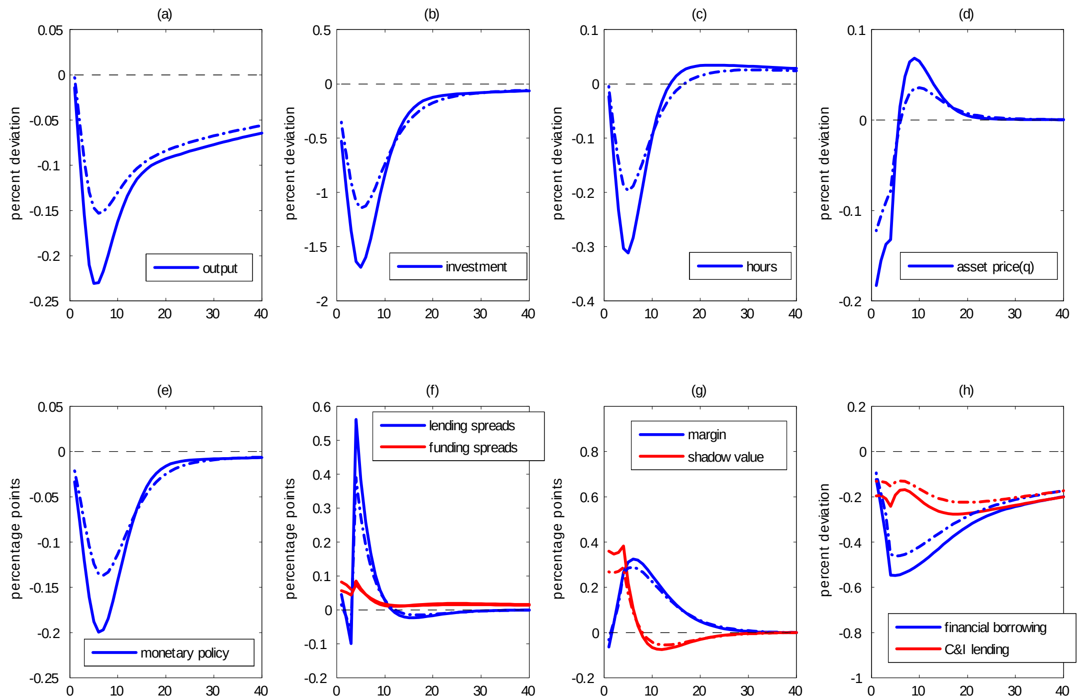


Figure 6 presents the responses of the economy to a technology shock under the credit rule, along with the responses in which their is no cyclical macroprudential instrument (i.e., our earlier baseline calibration). As is readily apparent, leaning against the credit-to-GDP ratio significantly distorts the economy's adjustment to a technology shock–and in an undesirable manner. Specifically, aggregate lending (panel (h)) fluctuates in a manner quite different from that of aggregate output following a technology shock, and trying to mitigate fluctuations in credit-to-GDP induces additional volatility in investment and output. This result is not especially surprising, as credit is responsive to any shock, not just changes in financial distortions.

While the rule linked to credit acts in an undesirable manner following a technology shock, it does mitigate the implications of a financial shock for output and investment. This can be seen in figure 7, which presents the response following an increase in the cost of outside equity for intermediaries under the baseline calibration (without a macroprudential instrument) and the credit-based rule: By leaning against credit, the policy rule does lower the impact of the financial shock. On balance, these results highlight how adjusting a macroprudential instrument in response to fluctuations in credit may have desirable effects in response to some fluctuations within

Figure 13: Effect of Anticipated Risk Shock Under Lending Spread Macroprudential Rule



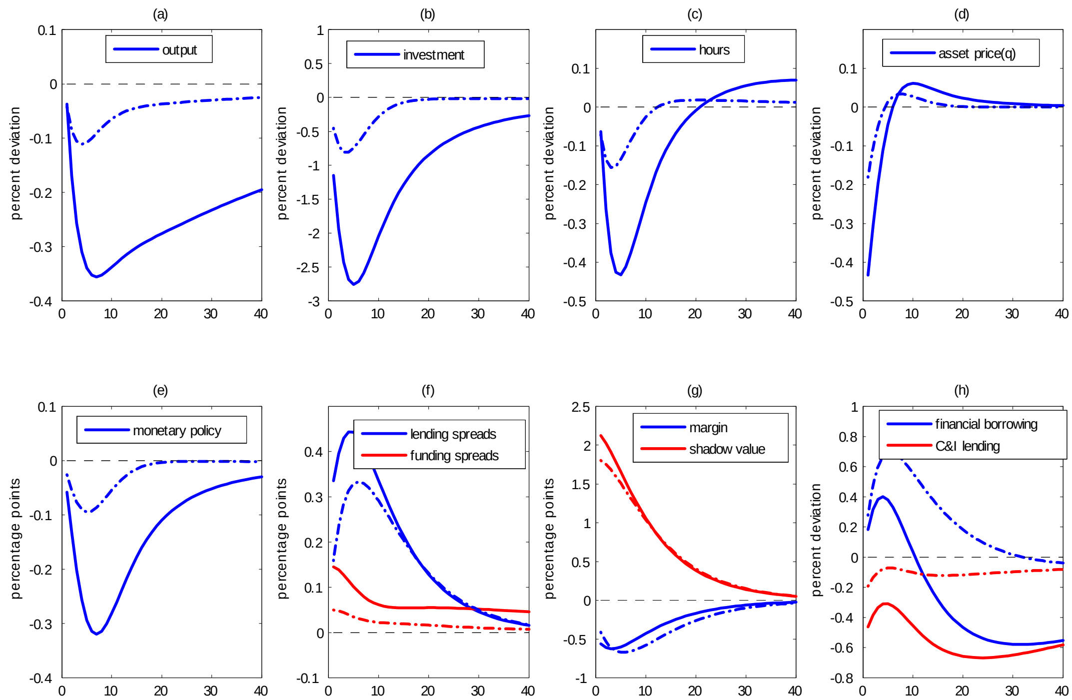
the financial sector, but at the cost of exacerbating fluctuations in response to other changes in macroeconomic conditions.

Figure 8 and figure 9 present the responses of the economy following a technology and equity cost shock under the asset-price-based (q) macroprudential rule: While the distorting effect of the policy rule following a technology shock is more modest here, reflecting the modest response of the asset price under the baseline calibration, the q-based rule is still somewhat distortionary following the technology shock and only partially effective following the financial shock.

The remaining two rules focus on rules that adjust the macroprudential instrument in response to developments within the financial sector. The first rule leans against leverage within the financial sector. Following a technology shock (as illustrated in figure 10, this approach induces relatively little distortion; as emphasized above, a technology shock has little influence on intermediaries preference for debt or equity and hence does not influence leverage significantly (as shown in panel (g)).

However, a rule that leans against leverage can amplify the effects of financial shocks in some cases, as can be seen in figures 11 and 12, which report the responses under this macroprudential

Figure 14: Effect of Equity Cost Shock Under Lending Spread Macroprudential Rule



rule following an increase in the cost of equity and an increase in anticipated idiosyncratic risk. Following an increase in the cost of equity (figure 11), intermediaries both lower equity/increase leverage and reduce lending supply; encourage more equity exacerbates the financial distortion associated with the higher cost of equity, amplifying the decline in lending supply, investment, and output. However, following an increase in anticipated risk, intermediaries deleverage and reduce lending supply, and leaning against the deleveraging mitigates the decline lending, investment, and output.

Overall, a policy that leans against leverage has induces little change in the economy's response to a technology shock, but can amplify or mitigate the effect of a financial disturbance depending on whether the disturbance increases or decreases the attractiveness of leverage. This sensitivity to the type of financial disturbance suggests that a macroprudential policy that leans against intermediary leverage may not have robust stabilization properties.

However, the macroprudential policy that leans against the lending spread performs well in response to all the shocks we have analyzed. As with the leverage-based policy, it does not distort the response to a technology shock, as spreads are little affected by such a disturbance in our model. (As a result, we do not report this impulse response, which is essentially identical to those in figure 10.) Because both shock to anticipated risk and the cost of equity boost the lending spread (reflecting the decline in lending supply associated with the higher value intermediaries place on internal funds under a riskier environment or an environment with a higher cost of outside equity), a policy that leans against the spread stabilizes lending, investment, and output under both financial disturbances (figures 14 and 13 – suggesting this approach is more robust.

5 Conclusions

We have developed a macroeconomic model in which financial intermediaries optimally choose their leverage – that is, the mix of debt and equity that finance their balance sheet. The leverage choices of intermediaries both affect the financial accelerator and imply significant macroeconomic effects of changes in the risk facing intermediaries and the cost of their external funds. We used this model to evaluate several macroprudential policies.

With regard to crisis policies, we found that capital injections conditioned upon voluntary recapitalization can be a more effective tool than direct lending/asset purchases. With regard to policies aimed at limiting the cyclical effects of financial disturbances, we demonstrated that policy strategies that lean against changes in aggregate credit, broad measures of asset prices, or leverage within the financial sector may significantly distort the economy's response to changes in fundamentals or have other unintended consequences. Within our model, policy strategies focused on mitigating shifts in the spread between borrowing rates and a risk-free interest rate appear to have better stabilization properties than other proposed macroprudential strategies.

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Appendix. Model Expressions for Intermediary Leverage, Macroeconomic Dynamics and Macroprudential Policy (Not for Publication)

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March 6, 2012

Abstract

This appendix derives the model expressions that are used for the computation of the paper, *Intermediary Leverage, Macroeconomic Dynamics and Macroprudential Policy.*

1 Model

1.1 Bond Pricing Equation

The default condition that defines the trigger level of idiosyncratic shock ϵ^D_{t+1} is obtained by equating the (gross) investment return and the after-tax debt burden

$$\epsilon^D_{t+1}(1+r^A_{t+1}) \equiv (1-m_t)[1+(1-\tau_c)r^B_{t+1}]. \tag{A1}$$

Solving for the borrowing rate yields $r^B_{t+1} = \frac{1}{1-\tau_c}\left[\frac{\epsilon^D_{t+1}(1+r^A_{t+1})}{1-m_t} - 1\right]$. For simplicity, we assume that the bond investor is not subject to interest rate income tax. Using the expression above, the debt payment to the investor can be expressed as

$$\begin{aligned}(1-m_t)(1+r^B_{t+1}) &= (1-m_t)\left\{1+\frac{1}{1-\tau_c}\left[\frac{\epsilon^D_{t+1}(1+r^A_{t+1})}{1-m_t} - 1\right]\right\} \\ &= (1-m_t)\left(1-\frac{1}{1-\tau_c}\right) + \frac{1}{1-\tau_c}\epsilon^D_{t+1}(1+r^A_{t+1}).\end{aligned}$$

Substituting the above in the bond pricing equation and rearranging the terms yields

$$\begin{aligned}&(1-m_t)\left[1-\left(1-\frac{1}{1-\tau_c}\right)\mathbb{E}_t\left(M_{t,t+1}\int_{\epsilon^D_{t+1}}^{\infty} dF_{t+1}\right)\right] \\ =\ & \mathbb{E}_t\left\{M_{t,t+1}\left[\int_0^{\epsilon^D_{t+1}}(1-\eta)\epsilon_{t+1}dF_{t+1} + \int_{\epsilon^D_{t+1}}^{\infty}\frac{1}{1-\tau_c}\epsilon^D_{t+1}dF_{t+1}\right](1+r^A_{t+1})\right\}.\end{aligned}$$

We define the standardized default trigger as

$$s^D_{t+1} \equiv \sigma^{-1}_{t+1}[\log\epsilon^D_{t+1} + 0.5\sigma^2_{t+1}]. \tag{A2}$$

Using this and the property of truncated lognormal distribution, $\int_0^{\epsilon_{t+1}^D} \epsilon_{t+1} dF_{t+1} = \Phi(s_{t+1}^D - \sigma_{t+1})$, one can rewrite the bond pricing equation as

$$\begin{aligned}
& (1-m_t)\left[1+\frac{\tau_c}{1-\tau_c}\mathbb{E}_t\left(M_{t,t+1}[1-\Phi(s_{t+1}^D)]\right)\right] \qquad \text{(A3)} \\
= \; & \mathbb{E}_t\left\{M_{t,t+1}\left[(1-\eta)\Phi(s_{t+1}^D-\sigma_{t+1})+\frac{1}{1-\tau_c}\epsilon_{t+1}^D(1-\Phi(s_{t+1}^D))\right](1+r_{t+1}^A)\right\}
\end{aligned}$$

1.2 Intermediary Optimization Conditions

The net-worth of an intermediary is given by $N_t = \max\{0, \epsilon_t(1+r_t^A)Q_{t-1}S_{t-1} - (1-m_{t-1})[1+(1-\tau_c)r_t^B]Q_{t-1}S_{t-1}\}$, where the max operator is due to the limited liability condition. Using (A1), we simplify the net-worth equation as $N_t = [\max\{\epsilon_t, \epsilon_t^D\} - \epsilon_t^D](1+r_t^A)Q_{t-1}S_{t-1}$. With that in mind, we express the intermediary optimization problem as

$$\begin{aligned}
J_t \; = \; & \min_{\theta_t}\max_{S_t, m_t, \epsilon_{t+1}^D}\Bigg\{\mathbb{E}_t^\epsilon[D_t]+\mathbb{E}_t\left[M_{t,t+1}\cdot V_{t+1}(N_{t+1})\right] \\
& +\mathbb{E}_t^\epsilon\left[\lambda_t\left(N_t+T_t+\bar{\varphi}(D_t)-[m_t+\tau_t^m(1-m_t)]Q_tS_t\right)\right] \\
& +\theta_tQ_tS_t\mathbb{E}_t\Bigg[M_{t,t+1}\left((1-\eta)\Phi(s_{t+1}^D-\sigma_{t+1})+\frac{1}{1-\tau_c}\epsilon_{t+1}^D(1-\Phi(s_{t+1}^D))\right)(1+r_{t+1}^A) \\
& -(1-m_t)\left(1+\frac{\tau_c}{1-\tau_c}\mathbb{E}_t\left(M_{t,t+1}[1-\Phi(s_{t+1}^D)]\right)\right)\Bigg]\Bigg\}
\end{aligned}$$

and

$$V_t(N_t) = \min_{\lambda_t}\max_{D_t}\left\{D_t+\mathbb{E}_t\left[M_{t,t+1}\cdot J_{t+1}\right]+\lambda_t\left(N_t+T_t+\bar{\varphi}(D_t)-[m_t+\tau_t^m(1-m_t)]Q_tS_t\right)\right\}$$

where we modify the flow of funds constraint to include the macroprudential policy τ_t^m. The optimization conditions are given by the following four conditions:

- FOC for D_t: $\lambda_t = \bar{\varphi}(D_t)\begin{cases} 1 & \text{if } D_t \geq 0 \\ 1/(1-\varphi) & \text{if } D_t < 0\end{cases}$

- FOC for S_t: $[m_t+\tau_t^m(1-m_t)]Q_t\mathbb{E}_t^\epsilon[\lambda_t] = \mathbb{E}_t\left[M_{t,t+1}\frac{\partial N_{t+1}}{\partial S_t}V_{t+1}'(N_{t+1})\right]$

- FOC for m_t: $(1-\tau_t^m)\mathbb{E}_t^\epsilon[\lambda_t] = \theta_t\left[1+\frac{\tau_c}{1-\tau_c}\mathbb{E}_t\left(M_{t,t+1}[1-\Phi(s_{t+1}^D)]\right)\right]$

- FOC for ϵ_{t+1}^D:
$$\begin{aligned}
0 = \; & \theta_tQ_tS_t\mathbb{E}_t\Bigg\{M_{t,t+1}\Bigg[(1-\eta)\frac{\phi(s_{t+1}^D-\sigma_{t+1})}{\sigma_{t+1}\epsilon_{t+1}^D}(1+r_{t+1}^A) \\
& +\frac{1}{1-\tau_c}\left((1-\Phi(s_{t+1}^D))-\frac{\phi(s_{t+1}^D)}{\sigma_{t+1}}\right)(1+r_{t+1}^A) \\
& -\;(1-m_t)\left(1+\frac{\tau_c}{1-\tau_c}[1-\Phi(s_{t+1}^D)]\right)\Bigg]\Bigg\}+\mathbb{E}_t\left[M_{t,t+1}\cdot\frac{\partial N_{t+1}}{\epsilon_{t+1}^D}V_{t+1}'(N_{t+1})\right]
\end{aligned}$$

1.2.1 Expected Shadow Value of Internal Funds

We define the equity issuance trigger ϵ_t^E as the value of idiosyncratic shock that exactly satisfies the flow of funds constraint when $D_t = 0$, i.e., $0 = \max\{0, \epsilon_t^E - \epsilon_t^D\}(1 + r_t^A)Q_{t-1}S_{t-1} + T_t - [m_t + \tau_t^m(1 - m_t)]Q_t S_t$, or equivalently

$$\epsilon_t^E \equiv (1 - m_{t-1})\frac{1 + r_t^B}{1 + r_t^A} + [m_t + \tau_t^m(1 - m_t)]\frac{Q_t S_t + T_t}{(1 + r_t^A)Q_{t-1}S_{t-1}}. \tag{A4}$$

When $\epsilon_t \geq \epsilon_t^E$, $D_t \geq 0$ and $\lambda_t = 1$ while $\epsilon_t < \epsilon_t^E$, $D_t < 0$ and $\lambda_t = 1/(1 - \varphi)$. We denote the standardized issuance trigger by

$$s_t^E \equiv \sigma_t^{-1}[\log \epsilon_t^E + 0.5\sigma_t^2]. \tag{A5}$$

We can then compute the expected shadow value of internal funds as a weighted average,

$$\mathbb{E}_t^\epsilon[\lambda_t] = 1 - F_t(\epsilon_t^E) + \frac{1}{1 - \varphi}F_t(\epsilon_t^E) = 1 - \Phi(s_t^E) + \frac{1}{1 - \varphi}\Phi(s_t^E) = 1 + \mu\Phi(s_t^E), \;\; \mu \equiv \frac{\varphi}{1 - \varphi}. \tag{A6}$$

1.2.2 FOC for Investment

Directly differentiating the net-worth equation yields

$$\frac{\partial N_{t+1}}{\partial S_t} = [\max\{\epsilon_{t+1}, \epsilon_{t+1}^D\} - \epsilon_{t+1}^D](1 + r_{t+1}^A)Q_t.$$

Applying Benveniste-Scheinkman's formula, $V_t'(N_t) = \lambda_t$, updating one period and combining it with the marginal effect of investment on the net-worth shows that the FOC for S_t is equivalent to

$$\begin{aligned}[m_t + \tau_t^m(1 - m_t)]\mathbb{E}_t^\epsilon[\lambda_t] &= \mathbb{E}_t\{M_{t,t+1}\lambda_{t+1}[\max\{\epsilon_{t+1}, \epsilon_{t+1}^D\} - \epsilon_{t+1}^D](1 + r_{t+1}^A)\} \\ &= \mathbb{E}_t\{M_{t,t+1}\mathbb{E}_{t+1}^\epsilon[\lambda_{t+1}(\max\{\epsilon_{t+1}, \epsilon_{t+1}^D\} - \epsilon_{t+1}^D)](1 + r_{t+1}^A)\}\end{aligned}$$

where the law of iterated expectation is used in the second line. Dividing through by $\mathbb{E}_t^\epsilon[\lambda_t]$ yields

$$[m_t + \tau_t^m(1 - m_t)]\mathbb{E}_t^\epsilon[\lambda_t] = \mathbb{E}_t\left[M_{t,t+1}\frac{\mathbb{E}_{t+1}^\epsilon[\lambda_{t+1}]}{\mathbb{E}_t^\epsilon[\lambda_t]}\left(\frac{\mathbb{E}_{t+1}^\epsilon[\lambda_{t+1}\max\{\epsilon_{t+1}, \epsilon_{t+1}^D\}]}{\mathbb{E}_{t+1}^\epsilon[\lambda_{t+1}]} - \epsilon_{t+1}^D\right)(1 + r_{t+1}^A)\right]$$

Dividing through by $\tilde{m}_t \equiv m_t + \tau_t^m(1 - m_t)$ and , substituting (A6) for $\mathbb{E}_t^\epsilon[\lambda_t]$ and replacing $\epsilon_{t+1}^D(1 + r_{t+1}^A)$ with $(1 - m_t)(1 + (1 - \tau_c)r_{t+1}^B)$ yields

$$1 = \mathbb{E}_t\left\{M_{t,t+1}^B\frac{1}{\tilde{m}_t}\left[1 + \tilde{r}_{t+1}^A - (1 - m_t)[1 + (1 - \tau_c)r_{t+1}^B]\right]\right\} \tag{A7}$$

where

$$M_{t,t+1}^B \equiv M_{t,t+1}\frac{\mathbb{E}_{t+1}^\epsilon[\lambda_{t+1}]}{\mathbb{E}_t^\epsilon[\lambda_t]} = M_{t,t+1}\frac{1 + \mu\Phi(s_{t+1}^E)}{1 + \mu\Phi(s_t^E)} \tag{A8}$$

and

$$1 + \tilde{r}_{t+1}^A \equiv \frac{\mathbb{E}_{t+1}^\epsilon[\lambda_{t+1}\max\{\epsilon_{t+1}, \epsilon_{t+1}^D\}]}{\mathbb{E}_{t+1}^\epsilon[\lambda_{t+1}]}(1 + r_{t+1}^A).$$

To derive an analytical expression for the modified return $1 + \tilde{r}_{t+1}^A$, we first rewrite it as

$$1 + \tilde{r}_{t+1}^A = \left\{\frac{\mathbb{E}_{t+1}^\epsilon[\lambda_{t+1}\epsilon_{t+1}]}{\mathbb{E}_{t+1}^\epsilon[\lambda_{t+1}]} + \frac{\mathbb{E}_{t+1}^\epsilon[\lambda_{t+1}\max\{0, \epsilon_{t+1}^D - \epsilon_{t+1}\}]}{\mathbb{E}_{t+1}^\epsilon[\lambda_{t+1}]}\right\}(1 + r_{t+1}^A).$$

The first term inside the curly bracket can be evaluated as

$$\begin{aligned}\mathbb{E}^{\epsilon}_{t+1}[\lambda_{t+1}\epsilon_{t+1}] &= \int_0^{\epsilon^E_{t+1}} \frac{\epsilon_{t+1}}{1-\varphi} dF_{t+1} + \int_{\epsilon^E_{t+1}}^{\infty} \epsilon_{t+1} dF_{t+1} \\ &= \frac{1}{1-\varphi}\Phi(s^E_{t+1}-\sigma_{t+1}) + 1 - \Phi(s^E_{t+1}-\sigma_{t+1}) = 1 + \mu\Phi(s^E_{t+1}-\sigma_{t+1}).\end{aligned}$$

Similarly, we can derive the analytical expression for the second term as

$$\begin{aligned}\mathbb{E}^{\epsilon}_{t+1}[\lambda_{t+1}\max\{0, \epsilon^D_{t+1}-\epsilon_{t+1}\}] &= \int_0^{\epsilon^D_{t+1}} \frac{\epsilon^D_{t+1}-\epsilon_{t+1}}{1-\varphi} dF_{t+1} \\ &= \frac{1}{1-\varphi}[\epsilon^D_{t+1}\Phi(s^D_{t+1}) - \Phi(s^D_{t+1}-\sigma_t)]\end{aligned}$$

where we use the fact that $\lambda_{t+1} = 1/(1-\varphi)$ when $\epsilon_{t+1} \le \epsilon^D_{t+1} < \epsilon^E_{t+1}$. Combining the two expressions above with (A6) yields

$$1 + \tilde{r}^A_{t+1} \equiv \left\{ \frac{1+\mu\Phi(s^E_{t+1}-\sigma_{t+1})}{1+\mu\Phi(s^E_{t+1})} + \frac{\epsilon^D_{t+1}\Phi(s^D_{t+1}) - \Phi(s^D_{t+1}-\sigma_t)}{(1-\varphi)[1+\mu\Phi(s^E_{t+1})]} \right\} (1+r^A_{t+1}) \tag{A9}$$

1.2.3 FOC for margin

Simply substituting in (A6) yields

$$(1-\tau^m_t)[1+\mu\Phi(s^E_t)] = \theta_t \left[1 + \frac{\tau_c}{1-\tau_c}\mathbb{E}_t\left(M_{t,t+1}[1-\Phi(s^D_{t+1})]\right)\right] \tag{A10}$$

1.2.4 FOC for default trigger

To transform the FOC for ϵ^D_{t+1} into a form that is more convenient for computation, we need to evaluate the following differentiation

$$\begin{aligned}& \mathbb{E}_t\left[M_{t,t+1} \cdot \frac{\partial N_{t+1}}{\epsilon^D_{t+1}} V'_{t+1}(N_{t+1})\right] \\ =\ & \mathbb{E}_t\left[M_{t,t+1} \frac{\partial \max\{\epsilon_{t+1}, \epsilon^D_{t+1}\} - \epsilon^D_{t+1}}{\partial \epsilon^D_{t+1}} (1+r^A_{t+1}) Q_t S_t V'_{t+1}(N_{t+1})\right] \\ =\ & \mathbb{E}_t\left\{M_{t,t+1}\mathbb{E}^{\epsilon}_{t+1}\left[\lambda_{t+1}\left(\frac{\partial \max\{\epsilon_{t+1}, \epsilon^D_{t+1}\}}{\partial \epsilon^D_{t+1}} - 1\right)\right](1+r^A_{t+1})Q_t S_t\right\}\end{aligned}$$

where we used the envelope condition $V'_{t+1}(N_{t+1}) = \lambda_{t+1}$ and the law of iterated expectation in the third line. To that end, first, we think of $\max\{\epsilon_{t+1}, \epsilon^D_{t+1}\}$ as a function of a 'variable' ϵ^D_{t+1} for a given 'parameter' ϵ_{t+1} and take a differentiation of $\max\{\epsilon_{t+1}, \epsilon^D_{t+1}\}$ with respect to ϵ^D_{t+1} as follows

$$\frac{\partial \max\{\epsilon_{t+1}, \epsilon^D_{t+1}\}}{\partial \epsilon^D_{t+1}} = \begin{cases} 0 & \text{if } \epsilon^D_{t+1} \le \epsilon_{t+1} \\ 1 & \text{if } \epsilon^D_{t+1} > \epsilon_{t+1} \end{cases}.$$

Second, we now think of the above as a function a 'variable' ϵ_{t+1} for a given 'parameter' ϵ^D_{t+1} since we now need to integrate this expression over the support of ϵ_{t+1}. Reminding that the shadow value is equal to $1/(1-\varphi)$ when $\epsilon_{t+1} \leq \epsilon^D_{t+1} < \epsilon^E_{t+1}$, one can see immediately that

$$\mathbb{E}^{\epsilon}_{t+1}\left[\lambda_{t+1}\frac{\partial \max\{\epsilon_{t+1},\epsilon^D_{t+1}\}}{\partial \epsilon^D_{t+1}}\right] = \int_0^{\epsilon^D_{t+1}} 1 \cdot \frac{dF_{t+1}}{1-\varphi} = \frac{\Phi(s^D_{t+1})}{1-\varphi}.$$

Using this expression, we can rewrite the FOC for ϵ^D_{t+1} as

$$\begin{aligned} 0 \;=\; & \theta_t \mathbb{E}_t \left\{ M_{t,t+1}\left[(1-\eta)\frac{\phi(s^D_{t+1}-\sigma_{t+1})}{\sigma_{t+1}\epsilon^D_{t+1}} + \frac{1}{1-\tau_c}\left([1-\Phi(s^D_{t+1})] - \frac{\phi(s^D_{t+1})}{\sigma_{t+1}}\right)\right](1+r^A_{t+1}) \right. \qquad \text{(A11)} \\ & \left. - (1-m_t)\left(1+\frac{\tau_c}{1-\tau_c}[1-\Phi(s^D_{t+1})]\right)\right]\Bigg\} + \mathbb{E}_t\left\{M_{t,t+1}\left[\frac{\Phi(s^D_{t+1})}{1-\varphi} - [1+\mu\Phi(s^E_{t+1})]\right](1+r^A_{t+1})\right\} \end{aligned}$$

1.3 Household Optimization Conditions

We denote the total outstanding of intermediary debts by B_t. In equilibrium, $B_t = \int[1-m_{t-1}(i)]Q_{t-1} S_t(i)di = (1-m_{t-1})Q_{t-1}K_t$, where $i \in [0,1]$ is an index for intermediary. The last equality is due to the symmetric equilibrium and the no-arbitrage condition mentioned in the main text. The realized aggregate return on intermediary debts, denoted by $1+\tilde{r}^B_t$, is given by

$$1+\tilde{r}^B_t \equiv \left[\int_0^{\epsilon^D_t}(1-\eta)\epsilon_t dF_t + \int_{\epsilon^D_t}^{\infty}(1-m_t)(1+r^B_t)dF_t\right]\frac{1+r^A_t}{1-m_{t-1}}.$$

Using $1+\tilde{r}^B_t$, we can express the household's budget constraint as

$$0 = W_tH_t + (1+\tilde{r}^B_t)B_t - B_{t+1} - P_tC_t - \int_0^1 P^S_t(i)S^F_{t+1}(i)di + \int_0^1[\max\{D_t(i),0\} + P^S_{t-1,t}(i)]S^F_t(i)di$$

where W_t is a nominal wage rate, H_t is labor hours, and $S^F_t(i)$ is the number of shares outstanding at time t. $P^S_{t-1,t}(i)$ is the time t value of shares outstanding at time $t-1$.[1] $P^S_t(i)$ is the ex-dividend value of equity at time t. The two values are related by the following accounting identity, $P^S_t(i) = P^S_{t-1,t}(i) + X_t(i)$ where $X_t(i)$ is the value of new shares issued at time t. The costly equity finance assumption adopted for the financial intermediary implies that $X_t(i) = -(1-\varphi)\min\{D_t(i),0\}$. Using the last two expressions, one can see that the budget constraint is equivalent to

$$\begin{aligned} 0 \;=\; & W_tH_t + (1+\tilde{r}^B_t)B_t - B_{t+1} - P_tC_t - \int_0^1 P^S_t(i)S^F_{t+1}(i)di \\ & + \int_0^1[\max\{D_t(i),0\} + (1-\varphi)\min\{D_t(i),0\} + P^S_t(i)]S^F_t(i)di. \end{aligned}$$

The household's FOCs for asset holdings are summarized by two conditions,

- FOC for B_{t+1} : $1 = \mathbb{E}_t\left[M_{t,t+1}(1+\tilde{r}^B_{t+1})\right]$
- FOC for $S^F_{t+1}(i)$: $1 = \mathbb{E}_t\left[M_{t,t+1}\frac{\mathbb{E}^{\epsilon}_{t+1}[\max\{D_{t+1},0\}] + (1-\varphi)\mathbb{E}^{\epsilon}_{t+1}[\min\{D_{t+1},0\}] + P^S_{t+1}}{P^S_t}\right]$.

[1] In our actual computation, we assume that the bankruptcy cost $\eta\Phi(s^D_t - \sigma_t)(1+r^A_t)Q_{t-1}K_t$ is transferred back to the household. This is to focus on the implications of the debt market friction through the FOCs of the intermediaries. Our main conclusion in this paper is not affected by this assumption.

where $\mathbb{E}^{\epsilon}_{t+1}[\max\{D_{t+1},0\}] = \int_0^1 \max\{D_t(i),0\}di$ and $\mathbb{E}^{\epsilon}_{t+1}[\min\{D_{t+1},0\}] = \int_0^1 \min\{D_t(i),0\}di$.

It is straightforward to verify that the FOC for intermediary debts is equivalent to the participation constraint of the household in the intermediary debt contract. In our actual computation, we use the following analytical expressions to compute the return on equity.

$$1 = \mathbb{E}_t\left[M_{t,t+1}\frac{D^+_{t+1} - (1-\varphi)D^-_{t+1} + P^S_{t+1}}{P^S_t}\right] \tag{A12}$$

where using the flow of funds constraint for intermediaries, one can show that

$$\begin{aligned} D^+_{t+1} &\equiv \mathbb{E}^{\epsilon}_t[\max\{D_t,0\}] = \{1-\Phi(s^E_t-\sigma_t) - \epsilon^D_t[1-\Phi(s^E_t)\}R^A_t Q_{t-1}S_{t-1} \\ &\quad -[1-\Phi(s^E_t)]\{[m_t + \tau^m_t(1-m_t)]Q_tS_t - T_t\} && \text{(A13)} \\ D^-_{t+1} &\equiv -\mathbb{E}^{\epsilon}_t[\max\{D_t,0\}] = -1/(1-\varphi)[\Phi(s^E_t-\sigma_t) - \Phi(s^D_t-\sigma_t) - \epsilon^D_t\Phi(s^E_t)]R^A_t Q_{t-1}S_{t-1} \\ &\quad +\Phi(s^E_t)/(1-\varphi)\{[m_t+\tau^m_t(1-m_t)]Q_tS_t - T_t\} && \text{(A14)} \end{aligned}$$

1.4 Lump-sum Taxation on Household for Capital Injection Policy

Under the capital injection policy considered in the main text, With this policy, $1 = s^{G,S}_t(i) + s^{H,S}_t(i)$ replaces the market clearing condition $1 = S^F_t(i)$ where $s^G_t(i)$ is the share owned by the government. The government purchases the shares of the financial intermediaries at market prices and refunds the cost of equity issuance only to the institutions that are raising equities and are owned by the government. At an aggregate level, the cost of raising outside equity is given by $\Xi_t = -\int_0^1 \varphi\min\{D_t(i),0\}di$. With this policy, the cost of raising outside equity is modified into

$$\begin{aligned} \Xi_t &= -\int_0^1 \varphi\min\{D_t(i),0\}di + \int_0^1 \varphi\min\{D_t(i),0\}s^{G,S}_{t+1}(i)di \\ &= -\int_0^1 \varphi\min\{D_t(i)[1-s^{G,S}_{t+1}(i)],0\}di \\ &= -(1-s^{G,S}_{t+1})\int_0^1 \bar{\varphi}\min\{D_t(i),0\}di = \bar{\varphi}(1-S^G_{t+1})D^-_t \end{aligned}$$

where the last line is due to the assumption that the government purchases *pro rata* shares, i.e., $s^{G,S}_{t+1}(i) = s^{G,S}_{t+1}$. The government funding required to implement this policy is given by

$$T^H_t = \int_0^1 \mathbf{1}(D_t(i)\le 0)P^S_t(i)s^{G,S}_{t+1}di - \int_0^1 \mathbf{1}(D_{t-1}(i)\le 0)[\max\{D_t(i),0\} + P^S_{t-1,t}(i)]s^{G,S}_t di.$$

Under the symmetric equilibrium, the ex-dividend value of equity is the same for all intermediaries, i.e., $P^S_t(i) = P^S_t$ for all $i \in [0,1]$. The assumption of no persistency in the first moment of the idiosyncratic shock and the law of large number imply

$$\begin{aligned} &\int_0^1 \mathbf{1}(D_{t-1}(i)\le 0)[\max\{D_t(i),0\} + P^S_{t-1,t}(i)]di \\ = &\left[\int_0^1 \mathbf{1}(D_{t-1}(i)\le 0)di\right]\int_0^1[\max\{D_t(i),0\} + P^S_{t-1,t}(i)]di \\ = &\left[\int_0^1 \mathbf{1}(D_{t-1}(i)\le 0)di\right]\int_0^1[\max\{D_t(i),0\} + P^S_t + (1-\varphi)\min\{D_t(i),0\}]di \end{aligned}$$

where we use $P^S_{t-1,t}(i) = P^S_t(i) - X_t(i) = P^S_t + (1-\varphi)\min\{D_t(i),0\}$. Hence, the funding needs can be written as

$$T_t = \Phi(s_t)P^S_t s^{G,S}_{t+1} - \Phi(s_{t-1})(D^+_t - (1-\varphi)D^-_t + P^S_t)s^{G,S}_t. \tag{A15}$$

2 Derivation of Steady State

2.1 Equilibrium Rates of Return

Using a numerical root finder, one can jointly solve for ϵ^D, r^B, r^K, ϵ^E, s^E, s^D, and m that satisfy the followings:

$$
\begin{aligned}
\epsilon^D &= (1-m)\frac{1+(1-\tau_c)r^B}{(1-\tau_c)r^K+1-(1-\tau_c)\delta} \\
\epsilon^E &= \epsilon^D + \frac{[m+\tau^m(1-m)](1+t)}{(1-\tau_c)r^K+1-(1-\tau_c)\delta} \\
s^E &= \frac{1}{\sigma}(\log\epsilon^E+\sigma^2) \\
s^D &= \frac{1}{\sigma}(\log\epsilon^D+\sigma^2) \\
1-m &= \left\{(1-\eta)\Phi(s^D-\sigma)+\frac{\tau_c}{1-\tau_c}\epsilon^D[1-\Phi(s^D)]\right\}\frac{(1-\tau_c)r^K+1-(1-\tau_c)\delta}{1-\beta\left(1-\tau_c/(1-\tau_c)\right)[1-\Phi(s^D)]} \\
1 &= \frac{\beta}{m+\tau^m(1-m)}\left\{\left[\frac{1+\mu\Phi(s^E-\sigma)}{1+\mu\Phi(s^E)}+\frac{\epsilon^D\Phi(s^D)-\Phi(s^D-\sigma)}{1-\varphi+\varphi\Phi(s^E)}\right]\right. \\
&\quad \left.\times[(1-\tau_c)r^K+1-(1-\tau_c)\delta]-(1-m)[1+(1-\tau_c)r^B]\right\} \\
0 &= \theta\beta\left[(1-\eta)\frac{\phi(s^D-\sigma)}{\sigma\epsilon^D}+\frac{1}{1-\tau_c}\left([1-\Phi(s^D)]-\frac{\phi(s^D)}{\sigma}\right)\right] \\
&\quad \times[(1-\tau_c)r^K+1-(1-\tau_c)\delta]-\beta(1-m)\left(1+\frac{\tau_c}{1-\tau_c}[1-\Phi(s^D)]\right) \\
&\quad +\beta\left[\frac{\Phi(s^D)}{1-\varphi}-[1+\mu\Phi(s^E)]\right][(1-\tau_c)r^K+1-(1-\tau_c)\delta]
\end{aligned}
$$

where the last three equations are the steady state versions of (A3), (A7) and (A11).

2.2 Levels

Once the equilibrium returns are obtained, we can analytically solve for endogenous quantities. From the price Phillips curve, $p^M = (\vartheta^p-1)/\vartheta^p$. From the FOC for capital rental decision, we have $\frac{y}{k} = \frac{r^K}{(1-\alpha)p^M} = \frac{\varepsilon r^K}{(1-\alpha)(\varepsilon-1)} \equiv \rho_{y/k}$. Substituting this in the resource constraint of the steady state, we can compute the consumption/capital ratio as $\frac{c}{k} = \frac{Y}{K} - \frac{i}{k} = \rho_{y/k} - \delta$. By dividing the production function by k, we have $\frac{y}{k} = \left(\frac{h}{k}\right)^\alpha$ or equivalently, $\frac{h}{k} = \left(\frac{y}{k}\right)^{1/\alpha} = \rho_{y/k}^{1/\alpha}$. Hence,

$$
\frac{h}{c} = \frac{\rho_{y/k}^{1/\alpha}}{\rho_{y/k}-\delta}, \text{ or equivalently, } h = \left(\frac{\rho_{y/k}^{1/\alpha}}{\rho_{y/k}-\delta}\right)c.
$$

The wage Phillips curve in the steady state is given by $\frac{\vartheta^w}{\vartheta^w-1}\zeta h^\nu = w(1-a\beta)(1-a)^{-\gamma}c^{-\gamma}$. From the FOC for the labor hours, we have $w = \alpha p^M\frac{y}{h} = \alpha p^M\frac{y/k}{h/k} = \alpha p^M\rho_{y/k}^{\frac{\alpha-1}{\alpha}}$. Substituting this in the wage Phillips curve yields

$$
\frac{\vartheta^w}{\vartheta^w-1}\zeta h^\nu = \alpha p^M\rho_{y/k}^{\frac{\alpha-1}{\alpha}}(1-a\beta)(1-a)^{-\gamma}c^{-\gamma}
$$

Substituting $h = \left(\frac{\rho_{y/k}^{1/\alpha}}{\rho_{y/k}-\delta}\right) c$ in the above yields c,

$$c = \left[\frac{\vartheta^w - 1}{\zeta\vartheta^w}\alpha p^M (1 - a\beta)(1 - a)^{-\gamma}\rho_{y/k}^{\frac{\alpha-1-\nu}{\alpha}}\left(\rho_{y/k} - \delta\right)^{\nu}\right]^{1/(\gamma+\nu)}$$

The levels of the other variables can be computed straightforwardly from this.