Systemic Liquidity Requirements*

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COMMENTS WELCOME

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Abstract

We examine the privately optimal liquidity and diversification choices of interlinked financial intermediaries and characterise their welfare properties. Costly liquidation generates strategic complementarity in the withdrawal decisions of creditors of a given intermediary, while diversification induces strategic complementarity across intermediaries. Individually beneficial diversification can result in *systemic fragility* - a run on one intermediary triggers a run on another. This externality renders privately optimal levels of liquidity as socially insufficient. A planner attains constrained efficiency by imposing a macro-prudential liquidity buffer. This buffer is high when liquidation costs are high and when expected investment returns are low. Hence, efficient liquidity regulation *leans against the wind* by imposing a high liquidity buffer during a boom.

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1 Introduction

There has been a secular decline in liquid assets as a share of bank balance sheets over the last three decades. Figure 1 depicts the liquidity ratio of the United States and the United Kingdom banking systems during this period. While the US banks' liquidity ratio was roughly constant at a level of 5% - 7% during the 1980s and early 1990s, it dropped to below 1% before the outbreak of the financial crisis in 2007. A similar picture arises for the UK, where the liquidity ratio was steady at a level of about 3% during the 1980s and early 1990s, dropping to a level of 1% and below in the 2000s.

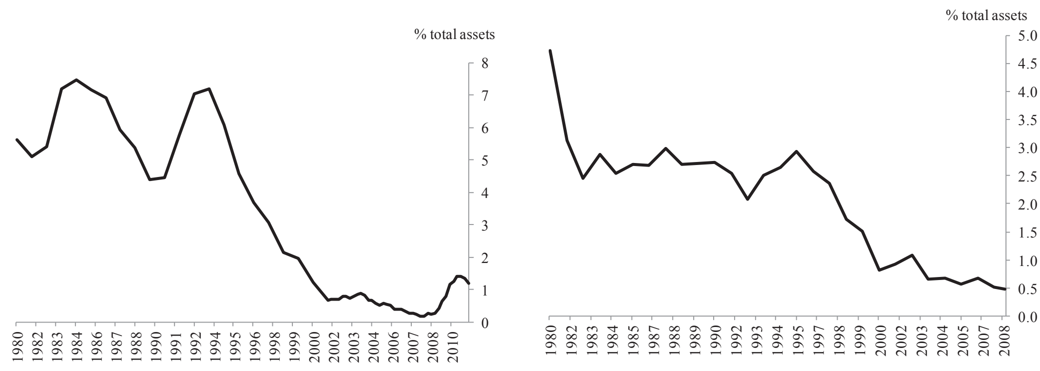

Figure 1: Liquid assets as a share of banks' balance sheet in percentage points for the US (left panel, 1980 − 2010) and the UK (right panel, 1980 − 2008). The chart for the US shows obligations of the US Treasury held by FDIC-insured commercial banks as a proportion of total FDIC-insured commercial bank assets. Source: www2.fdic.gov/hsob, Commercial Bank reports. The chart for the UK shows the 'broad' liquidity ratio of UK banks reported in Bank of England Financial Stability Report (October, 2008), which shows cash, central bank balances, money at call, eligible bills and UK gilts held by the UK banking sector as a proportion of total UK banking sector assets. Source: www.bankofengland.co.uk.

Liquidity regulation plays a major role in recent financial reform proposals. These proposals include the introduction of rules governing the composition of banks' balance sheets envisaged under the Basel Committee on Banking Supervision's proposed Liquidity Coverage Ratio (LCR) or Net Stable Funding Ratio (BCBS (2010)). Both regulatory tools seek to impose limits on the degree of "liquidity mismatch" on a bank's balance sheet by, for example, requiring a lower bound on the short-term and medium-term liquidity ratios. Liquidity regulation is also being considered for use as part of the macro-prudential toolkit (Bank of England (2011)).

We examine whether the privately optimal portfolios of interlinked financial intermediaries (banks, for brevity) are sufficiently liquid from a social perspective. In this context, this paper makes two contributions. The first is theoretical. We provide a theoretical model of financial intermediation with asset-side diversification, featuring a portfolio choice between liquidity and two risky assets in a bank-run context. Costly liquidation generates strategic complementarity between creditors of one bank, while diversification results in strategic complementarity between creditors of distinct but interconnected banks. The interaction of these forms of strategic complementarity in the withdrawal decisions of creditors is determined by banks that choose their portfolio in the best interest of its creditors because of free entry. Consequently, the incidence and the extent of the exposure to strategic complementarity between regions is *endogenous* in our model. Diversification trades off intra-regional strategic complementarities in creditors' withdrawing decisions with inter-regional strategic complementarities. While diversification is individually optimal, it exposes an interconnected financial system to systemic fragility in which a run on one bank triggers a run on another.

Second, we study constrained efficient regulation and examine its characteristics. We find that such regulation takes the form of a macro-prudential liquidity buffer in excess of the laissez-faire liquidity level, aimed at internalising the effects of costly liquidation on other regions (Proposition 3). We examine the comparative statics of this liquidity buffer and shed some light on its structural and cyclical determinants. In particular, there are two reasons for the liquidity buffer to be high when asset returns are low (as in the run-up to the recent financial crisis characterised by a "search for yield"). On the one hand, a systemic bank run is more likely when the expected returns on banks' investments are low. On the other hand, the opportunity cost of higher liquidity holdings given by the expected investment return is low. This establishes a *leaning-against-the-wind* property of macro-prudential liquidity regulation: the efficient liquidity buffer is high when expected investment returns are low (Proposition 4), which typically happens during a boom (e.g. Fama and French (1989)). We also show that the liquidity buffer is large when the liquidation cost is high.

We make these contributions in the following setting. Ours is a two-period economy consisting of two regions populated by agents with uncertain consumption needs. These agents occupy regionally segmented markets, placing their resources on call with a regional intermediary. In turn, intermediaries have access to regional investment projects and liquidity. Regional projects are long-dated, risky, and can be liquidated at a cost, while liquidity yields an inferior but safe return at a short horizon. As long as regional investment returns are not perfectly correlated, there is scope for diversification. One effect of diversification is the individually beneficial reduction of the volatility of a bank's portfolio, thereby reducing the ex-ante probability of a bank run.

Our model has a sequential structure. The portfolio choice of banks takes place at the initial date and is publicly observed. Each creditor receives a private signal about the profitability of his bank's investment project at the interim date and makes his withdrawal decision. We use global games techniques to solve for the unique equilibrium of the withdrawal subgame at the interim date. If the signal's noise is small enough, there exists a unique equilibrium characterised by a withdrawal threshold: a creditor withdraws ("runs") if and only if the investment return is believed to be sufficiently bad based on his private signal (Proposition 1). The equilibrium in the subgame is characterised by inefficient bank runs (Lemma 2). The fear of other creditors withdrawing prematurely, triggering costly liquidation, induces a creditor to withdraw in more states of the world than dictated by social efficiency.

At the initial date a bank chooses its level of liquidity and diversification in the best interest of its creditors because of free entry. Any *micro*-prudential regulatory motive is absent because of the full alignment between the incentives of a bank and its creditors. In our preferred specification, a bank that holds a larger amount of liquid assets discourages a run by increasing its available resources to meet the withdrawals of prematurely withdrawing consumers. Likewise, more diversification lowers the withdrawal threshold. By discouraging a run, holding liquidity means costly liquidation of risky projects occurs in fewer states of the world, increasing welfare.

Privately optimal portfolio choices are socially inefficient. Whereas each bank considers the beneficial effect of holding liquidity on its creditors, it does not internalise the positive effect on the creditors of the other bank, failing to internalise the systemic implications of its portfolio decision. Diversification implies that the creditors of one bank are also affected by the liquidation decision of the other bank, for example because of mark-to-market accounting. Consequently, individually beneficial diversification can generate *systemic fragility.* One bank's liquidity holding entails a positive externality that generates a role for regulatory intervention. Its form is naturally macro-prudential as it deals with inter-regional or economy-wide effects of liquidity.

This paper is organized as follows. We continue by reviewing the literature. Section (3) describes the economy and section (4) characterises its unique equilibrium. Section (5) contains the constrained planner's problem. Next, we establish the constrained efficiency of a macro-prudential liquidity buffer and study its comparative statics. Finally, section (7) concludes. Most derivations and proofs are delegated to the appendices.

2 Literature

Our paper is connected with two lines of literature. First, our paper builds on the literature on financial fragility and bank runs initiated by the seminal contribution of Diamond and Dybvig (1983). In a world of non-verifiable idiosyncratic liquidity shocks, banks offer demand-deposit contracts and improve upon the competitive equilibrium because of enhanced risk-sharing. However, strategic complementarity between creditors' withdrawal decisions arises in such a contract, giving rise to multiple equilibria. Apart from the efficient no-run equilibrium, the other pure-strategy equilibrium features an inefficient bank-run.

The theory of global games pioneered by Carlsson and van Damme (1993), developed in Morris and Shin (2001), and applied by Goldstein and Pauzner (2005) demonstrates that the introduction of a small amount of uncertainty about the long term investment return establishes a unique equilibrium.[1] Uncertainty about investment returns generates strategic uncertainty, preventing perfect co-ordination among creditors that is at the heart of the multiplicity result. The equilibrium is characterised by a unique withdrawal threshold. Co-ordination failure between creditors implies an inefficiently high withdrawal threshold since the fear of not being served at the final date induces creditors to withdraw for a socially inefficiently large range of posteriors.

Our paper follows the global games approach of establishing unique equilibria in a setup with global strategic complementarities as in Carlsson and van Damme (1993) and Morris and Shin (2001, 2003). In the spirit of Goldstein and Pauzner (2005), we analyse the effect of the ex-ante portfolio choice on the unique equilibrium of the continuation game. See also Eisenbach (2011) for a recent application to the optimal debt maturity structure. In contrast to these papers, we consider a multi-region economy with interregional diversification because of imperfectly correlated investment returns. Allen and Gale (2000) consider a multi-region economy with non-stochastic

[1] The strategic complementarity between the withdrawal decisions of creditors are global in Morris and Shin (2001) and one-sided in Goldstein and Pauzner (2005).

asset returns and negatively correlated liquidity shocks, while we consider stochastic but less than perfectly correlated investment returns and no interregional liquidity shocks. Our model also features a unique, endogenous, and positive bank run probability.

Our paper is also related to Dasgupta (2004) who extends the analysis of Goldstein and Pauzner (2005) to a two-region economy with negatively correlated liquidity shocks in the spirit of Allen and Gale (2000). Two separate forms of strategic complementarity occur *sequentially* in that framework, leading to contagious spillovers from a debtor bank's default to a creditor bank in equilibrium. There is no strategic complementarity across regions, however. By contrast, we consider the *simultaneous* interaction of intra- and interregional strategic complementarities in the withdrawal decision of creditors within banks and between different but related banks. Because of the correlation of regional investment returns, strategic complementarities between creditors across regions are present and the optimal extent to this exposure as well as the implications for liquidity regulation are explored in our paper.

From a conceptual point of view, our setup nests Goldstein (2005) who studies twin crises. In that model there is not only strategic complementarity within the groups of speculators and depositors, respectively, but also between these groups.This interaction gives rise to a vicious circle between banking crises and currency crises that leads to a destabilisation of the economy and a high correlation between these two crises. In our model, the incidence and the extent of the exposure to interbank strategic complementarity arises from the optimal behaviour of banks; these strategic complementarities are endogenous rather than exogenous as in Goldstein (2005). A bank chooses its portfolio in the best interest of its creditors at the initial date because of free entry. Thus, some strategic complementarity within a bank's creditors is optimally traded in for strategic complementarity between creditors of different banks. While the interaction between these forms of strategic complementarity is vicious at the interim date for a *given* portfolio choice as in Goldstein (2005), it is ex ante optimal from the creditors' point of view. In addition, we consider optimal regulation and formally derive the constrained efficient allocation.

Second, our paper is related to a growing literature on liquidity regulation and macroprudential policy. While a full survey of the literature on liquidity regulation is beyond the scope of this paper, we highlight some contributions. Cifuentes et al. (2005) is one of the first papers that discusses the usefulness of liquidity requirements for forestalling systemic crises. They are concerned with the unintended consequences of capital regulation in times of crisis, in which solvency requirements and marking-to-market can lead to fire sale spirals. Their simulations suggest that liquidity requirements are a useful tool for the mitigation of a contagious spread, noting that the privately chosen amount of liquidity may be suboptimal. We formalize the notion of a positive externality of liquidity provision, focus on solvency risk, and discuss a different transmission channel – diversification across banks. We also study the properties of the optimal liquidity regulation with respect to the extent of the market friction and the cyclical position.

Several papers use systemic liquidation costs as a point of departure.[2] Korinek (2011) studies an economy in which systemic externalities take the form of fire sales. In times of crisis, the distressed sale of assets depresses asset prices, imposing a pecuniary externality on other financial institutions that is not internalised by the seller. When markets are incomplete, a social planner can improve on the laissez faire outcome by imposing a tax on risk-taking, for instance in the form of mandatory capital injections which reduces the incidence of fire sales under distress. Wagner (2011) studies the diversification-diversity trade-off in a model of systemic liquidation risk. Here, due to fire sales, joint liquidation is costly, generating an incentive for investors to hold diversified portfolios. Wagner (2009) also stresses the role of endogenous liquidation costs and their effect on ex-ante portfolio choice. The more correlated a bank is with the average bank, the higher its cost in case of liquidation, which constitutes a cross-bank externality. The author studies the implication of this externality on the efficiency of the ex-ante chosen bank portfolio. While bank portfolios are generally inefficient, banks may be 'too correlated' (as in the standard case) or 'too diversified' under laissez faire, implying that regulatory treatment should be heterogeneous. While our paper also relates to the issues of liquidity and diversification, we endogenize the bank's liquidation decision by modelling creditor behaviour and privately optimal (yet inefficient) withdrawals. Another difference is that liquidation costs are not per se systemic in our paper, but interbank linkages that arise from a diversification motive spread the expected costs of liquidation throughout the system.

[2] While liquidity holdings typically save on liquidation costs, Calomiris et al. also consider other benefits of liquidity. In their model liquidity is observable and thus verifiable and its riskiness is invariant to the bank manager's unobservable investment in risk management that gives rise to moral hazard. Their focus is on the mix of capital and liquidity requirements as part of a macro-prudential toolkit and they abstract from the co-ordination issue of the depositors' withdrawal problem that is central to our analysis.

Uhlig (2010) also uses endogenous liquidation costs in a model with outside investors and a two-tiered banking sector. The arising system-wide externality generates strategic complementarities in the creditors' withdrawal decisions as in our model. While Uhlig focuses on a positive analysis of the previous financial crisis and discusses briefly some ex-post policy interventions, our focus is on optimal (liquidity) regulation from an ex-ante perspective. Studying ex-ante policy has the advantage of precluding the issue of moral hazard arising from an ex-post policy intervention, a theme also stressed by Farhi and Tirole (forthcoming). In line with the present paper, the authors also highlight that prudential policies are required to contain a crisis ex-ante, using a macro-prudential approach. The moral hazard aspect of an ex-post policy intervention, and the lender-of-last-resort (LOLR) policy in particular, has been stressed by Cao and Illing (2011a) and Cao and Illing (2011b). The authors show that ex-post liquidity provision by the central bank leads to excessive investment into the illiquid asset and optimal second-best policy requires both ex-post liquidity provision (to prevent inefficient liquidation) and an ex-ante minimum liquidity requirement. Liquidity will only be provided to banks that met the initial minimum requirements.

Castiglionesi et al. (2010) study the effect of international financial integration on financial stability. As interbank risk sharing becomes more widely available, banks may hold lower levels of liquidity, making the banking system more vulnerable to aggregate shocks. While we also study ex-ante liquidity holdings, our focus is on the prevention of costly liquidation and the associated inefficient bank runs arising from a demand-deposit contract absent in Castiglionesi et al. (2010).

Finally, Gai et al. (2011) present a network model that generates rich interactions between complexity and contagion. They show that higher liquidity requirements make the network of interbank exposures less prone to collapse. Using a completely exogenous setup, the authors study the effects of liquidity requirements calibrated to banks' systemic importance. They show that targeting the most systemically important institutions that are most instrumental in spreading contagion is more potent than a 'flat rate' liquidity rule. While our framework abstracts from such richness, we consider endogenous behaviour such as portfolio choice and a withdrawal decision. We also assess implications for the design of macro-prudential liquidity regulation of taking a welfare-maximising perspective.

3 The Model

The economy extends over three dates $t = 0, 1, 2$ and consists of two equally sized regions $k = A, B$. Our notion of a region is generic, comprising a particular sector, a country within an economic union, or a bank in a banking system. Each region contains a bank and a continuum of creditors operating in regionally segmented markets. Our notion of creditors is also broad: it is not limited to the traditional case of retail creditors and commercial banks but incorporates, for instance, money market funds (creditors) and investment banks (banks).[3] There is a single good used for consumption and investment.

Creditors Each region has ex ante identical creditors of mass $1 + \lambda$. There is idiosyncratic liquidity uncertainty (Diamond and Dybvig (1983)): a creditor is either impatient and wishes to consume at the interim date or patient and wishes to consume at the final date. The ex ante probability of being impatient is $\lambda/(1 + \lambda) \in (0, 1)$ and is identical across creditors. By the law of large numbers, this is also the share of early creditors in each region. Creditors are uncertain about their consumption needs at the initial date, but learn their preference privately at the beginning of the interim date. A creditor's utility function is:

$$U(c_1, c_2) = \begin{cases} c_1 & \text{w.p.} \ \frac{\lambda}{1+\lambda} \\ c_2 & \text{w.p.} \ \frac{1}{1+\lambda} \end{cases}, \tag{1}$$

where c_t is consumption at date t. Creditors are endowed with one unit at the initial date only, implying an aggregate endowment of $1 + \lambda$ at the initial date. A creditor either stores his endowment or deposits at the bank.

Investment opportunities Table 1 summarises the two types of investment opportunity available in each region at the initial date. First, there is storage that yields a unit safe return. Second, there is a long term regional investment project, such as loans, that matures after two periods and yields a stochastic return $\mathbf{r}_k$ with mean $\bar{r}$ and realisation r_k. Similar to Morris and Shin (2001), premature liquidation $l_k \in [0, 1]$ of the invested resources at par at the interim date results in the reduction of the final-date return to $r_k - (1+\chi)l_k$. The cost of premature liquidation $0 < \chi < \bar{r} - 1$ arises at the final date and is the source of strategic complementarities between

[3] This model's "creditors" and "banks" may also be re-interpreted as local and global banks in the spirit of Uhlig (2010). Then, a prematurely withdrawing creditor represents a run of one (local) bank on another (global) bank, an arguably reasonable feature of the recent financial crises.

Asset	$t = 0$	$t = 1$	$t = 2$
Storage $(0 \to 1)$	-1	1	
Storage$(1 \to 2)$		-1	1
Project $(0 \to 2)$	-1	l_k	$r_k - (1+\chi)l_k$

Table 1: Returns from investment opportunities

patient creditors within a given region. The upper bound on the liquidation cost prevents it from driving the result on its own. To avoid strict dominance of either investment opportunity, we assume $1 < \bar{r} < 1 + (1 + \chi)$. Our assumptions about the mean return and the liquidation cost can be summarized:

Assumption 1.

$$0 < \bar{r} - (1 + \chi) < 1 \tag{2}$$

Costly liquidation is a key but well-motivated feature of our model. As discussed in James (1991) and Mullins and Pyle (1994), in practice these costs comprise both direct liquidation expenses together with reductions in the 'going concern' value of bank assets under distress. The empirical literature typically finds these liquidation costs to be large: of the order of 30% of bank assets on average.[4] Another interpretation of liquidation costs relates to the concept of market liquidity and the haircut associated with it.

Banks At the initial date the bank in each region offers a contract to creditors that specifies possible withdrawal d_{1k} at the interim date and the portfolio composition of the bank (see figure 2). That is, part of the raised deposits is stored $y_k \in [0, (1 + \lambda)]$ and referred to as *liquidity* and the remainder invested $I_k \equiv 1 + \lambda - y_k$ in long term projects. Banks also *diversify* their portfolio by holding some of the long term investment in the other region's project $b_k \in [0, I_k]$, reflecting syndicated loans or outright purchases.[5] Despite the risk-neutrality of creditors, a diversification of the bank's portfolio is beneficial. As we will show, diversification implies a reduction in the volatility of the bank's portfolio without changing its expected return, thus lowering the probability of a costly bank run. A bank's portfolio choice is publicly observed.

[4] Mullins and Pyle (1994) and Brown and Epstein (1992) estimates of direct liquidation expenses of around 10%, varying between 17% for assets relating to owned real estate, down to 0% for liquid securities for assets in receivership at the FDIC. Adding to direct expenses losses associated with forced liquidation, James (1991) gives an average total cost of 30% of a failed bank's assets. Similar orders of magnitude are reported in Bennett and Unal (2011) whose sample runs for much longer, covering 1986-2007.

[5] See Sufi (2007) for an empirical work on syndicated loans.

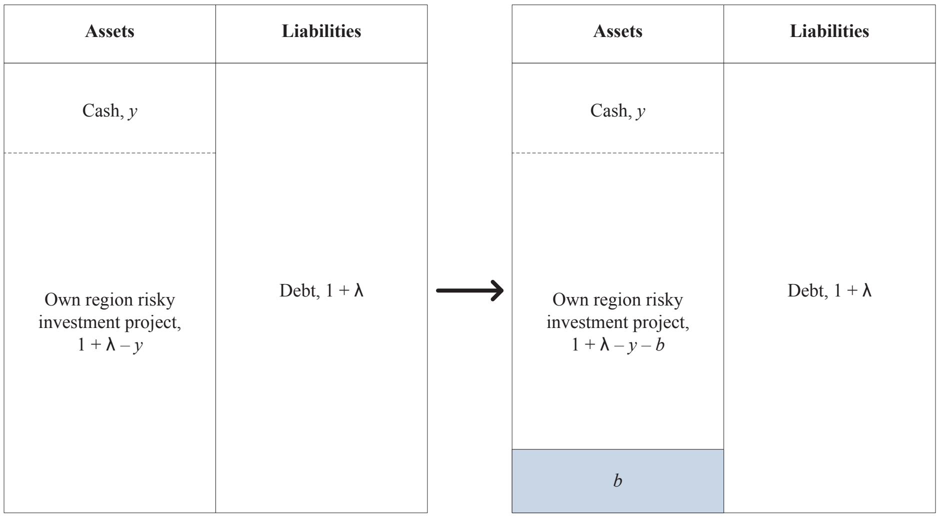

Figure 2: A bank's balance sheet at the initial date (before and after diversification)

Following Allen and Gale (2000), the bank has an optimal pecking order when serving withdrawals at the interim date, using liquidity first. Let $w_k \in [0, 1]$ denote the proportion of region k's patient creditors that withdraw at the interim date. If withdrawals from patient creditors are sufficiently high ($w_k > y_k - \lambda$), the bank partially liquidates its investment projects. It starts with its own region's project, which is more easily or faster liquidated.[6] The bank may hold excess liquidity ($y_k > \lambda$), that is more cash than needed to service withdrawals from impatient consumers. Holding excess liquidity drives a wedge between the proportion of patient creditors that withdraw w_k and the proportion of the investment project that is liquidated l_k, highlighting the usefulness of liquidity in preventing costly liquidation. At the final date the bank shares the proceeds of its remaining assets equally among the remaining creditors (mutual bank).

There is free entry to the banking sector such that the bank maximizes creditors' expected utility. If it did not, another bank would emerge and offer a better investment plan, attracting all deposits. Given the alignment of interest between the bank and its creditors as well as the bank's enhanced access to projects, all creditors deposit in full.[7]

[6]For syndicated loans, the lead bank may decide on liquidation such that the liquidation decision is forfeited. See also Sufi (2007).

[7]Note that only banks have access to investment projects. While Diamond and Dybvig (1983) demonstrate a role for the bank as provider of liquidity insurance in case of risk-averse creditors, banking arises in our model from the bank's enhanced access to investment projects. This common assumption can be motivated as an equilibrium outcome in a perturbed setting with moral hazard and monitoring. Delegation is optimal as long as banks are better at monitoring or can obtain monitoring technology more cheaply.

Throughout our analysis, we focus on $d_1 \equiv 1$ as in Dasgupta (2004) and a similar assumption in Goldstein (2005). Therefore, banks are viable at the interim date as the promised payment does not exceed the liquidation value. There might still be bank runs at the interim date, however. In the spirit of Morris and Shin (2001) and Morris and Shin (2003), we acknowledge that the given demand deposit contract need not be optimal. Rather, our focus is on studying the equilibrium outcome and the welfare characteristics of this contract in a setting with noisy signals, liquidation costs, and diversification. For other aspects of banking, see Goldstein and Pauzner (2005) for a one-regional model with bank insolvency at the interim date and, for instance, Ahnert and Georg (2012) for default on interbank obligations.

The following time line summarizes the model description:

Initial date $t = 0$

- Creditors receive endowment.
- Banks offer an investment plan to creditors. Other bank(s) may enter freely and make offers to creditors.
- Creditors decide whether and where to deposit.
- The bank in each region selects its portfolio by choosing an amount of liquidity. The remainder is invested in the long term projects of both regions (diversification). Portfolios are publicly observed.

Interim date $t = 1$

- Individual consumption preferences (impatient or patient) are drawn and privately observed by creditors.
- Each creditor receives a private signal about the return of his region's long term investment project. Creditors update their beliefs about the investment return and the proportion of prematurely withdrawing patient creditors in both regions.
- Creditors may withdraw.
- Banks service withdrawals. This may involve (partial) liquidation of the investment project, starting with its own region.

- Impatient creditors consume. Withdrawing patient creditors store their funds.

Final date $t = 2$

- The investment project matures.
- Remaining creditors receive an equal share of the bank's funds (mutual bank).
- Patient creditors consume.

3.1 Payoffs

We are now ready to determine the creditors' payoffs. An **impatient creditor** always withdraws at the interim date and receives unity. An impatient creditor's payoff is unaffected by other creditors' withdrawal decisions. A **patient creditor** who withdraws prematurely receives the same payoff as the bank does not observe a creditor's liquidity preference. If an impatient creditor keeps the funds at the bank, he receives an equal share of the bank's final-date assets A_{2k}:

$$d_{2k} = \frac{A_{2k}}{1 - w_k} \tag{3}$$

$$A_{2k} = \underbrace{\max\{0, y_k - \lambda - w_k\}}_{\text{excess liquidity}} + \underbrace{(I_k - b_k)[r_k - (1+\chi)l_k]}_{\text{investment proceeds}} + \underbrace{b_k[r_{-k} - (1+\chi) + l_{-k}]}_{\text{proceeds from diversification}} \tag{4}$$

$$l_k \equiv \max\{0, w_k + \lambda - y_k\} \tag{5}$$

where l_k is the total amount of liquidation by bank k. It decreases in the amount of liquidity y_k chosen by that region's bank at the initial date and increases in the proportion of prematurely withdrawing patient creditors w_k.

There are two dimensions to the strategic behaviour of a patient creditor as depicted in figure (3). The first dimension is strategic interaction between the withdrawal decisions of patient creditors in a given region. More withdrawals by other patient creditors in your region have two effects. First, the bank draws down its excess liquidity and then liquidates a larger share of its investment in the long term projects. This effect is detrimental to a waiting patient creditor. Second, there are fewer patient creditors to share the remaining resources with at the final date. This effect is beneficial for a waiting patient creditor. Appendix (A.2) derives the partial derivative of the final-date payoff to a patient depositor with respect to the share of prematurely withdrawing patient depositors in his region, $\frac{\partial d_{2k}}{\partial w_k}$. Under the condition that the liquidation cost

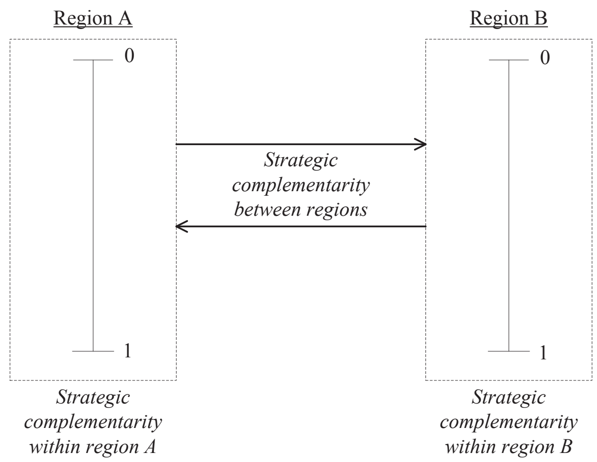

Figure 3: Interaction of two forms of strategic complementarities

is sufficiently high, there is strategic complementrarity between depositors of a given region.

The second dimension is a strategic complementarity between the withdrawal decisions of patient creditors across regions that arises from diversification ($b_k > 0$). The more patient creditors in region $-k$ withdraw, the more of region $-k$'s investment project is liquidated (once the excess liquidity is exhausted), reducing the final-date return to region $-k$'s investment project. Because of diversification, the bank in region k is also affected and its final-date asset value is reduced (for instance via mark-to-market accounting). This increases the incentive for a patient creditor in region k to withdraw as well. See Appendix (A.3) for a proof.

Note the presence of a system-wide effect, that is the **negative externality** of one bank's liquidation decision on the other bank's portfolio value. This externality motivates a policy intervention in our setup.

Definition 1. ***Macro-prudential*** *policy is a policy intervention that deals with economy-wide (i.e. across regions) externalities.*

3.2 Information structure

This part describes what is known about the stochastic asset returns by agents in the economy. We extend the notion of uncertainty about investment returns pioneered by Carlsson and van Damme (1993) and developed in Morris and Shin (2001) and Goldstein and Pauzner (2005) to a multi-regional setup. Bold variables denote random variables. All distributions are common knowledge. The stochastic economy-wide investment return $\boldsymbol{r}$, such as a key macroeconomic variable common to both regions, is distributed according to:

$$\boldsymbol{r} \sim \mathcal{N}\left(\bar{r}, \frac{1}{\alpha}\right) \tag{6}$$

where $\bar{r}$ is the mean of the economy-wide return, $\alpha \in (0, \infty)$ its precision, and r its realisation. The returns to the regional investment project consist of the economy-wide investment return as a common component and a regional disturbance $\boldsymbol{\eta}_k$:

$$\boldsymbol{r}_k \equiv \boldsymbol{r} + \boldsymbol{\eta}_k, \tag{7}$$

$$\boldsymbol{\eta}_k \sim \mathcal{N}\left(0, \frac{1}{\beta}\right) \tag{8}$$

where the regional shock $\boldsymbol{\eta}_k$ has precision $\beta \in (0, \infty)$, is identically and independently distributed across regions and independent of the economy-wide investment return. Regional noise adds a second layer of uncertainty relative to the setting of Morris and Shin (2001). Creditors use of the information structure to infer that regional returns are distributed according to:

$$\boldsymbol{r}_k \sim \mathcal{N}\left(\bar{r}, \frac{1}{\delta}\right) \tag{9}$$

where $\delta \equiv \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)^{-1}$ is the precision of the public signal of the regional return. The correlation between both investment project returns is $corr(\boldsymbol{r}_A, \boldsymbol{r}_B) = \frac{1}{1+\alpha/\beta} \in (0, 1)$.[8] Given the imperfect positive correlation, there is a diversification opportunity for banks. While diversification does not affect the expected return of the bank's portfolio, it reduces its variance, thereby lowering the probability of a bank run. At the interim date the investment project returns are realized but not publicly observed. Each creditor receives a private signal x_{ik} about his regional return:

$$\boldsymbol{x}_{ik} \equiv \boldsymbol{r}_k + \boldsymbol{\epsilon}_{ik}, \tag{10}$$

$$\boldsymbol{\epsilon}_{ik} \sim \mathcal{N}\left(0, \frac{1}{\gamma}\right), \tag{11}$$

where the idiosyncratic noise $\boldsymbol{\epsilon}_{ik}$ has precision $\gamma \in (0, \infty)$ and is independently and identically distributed across creditors and independent of the regional noise and the economy-wide return.

[8] See Allen et al. (forthcoming) for independent investment returns and Ahnert and Georg (2011) for a common shock, a non-diversifiable aggregate shock.

4 Equilibrium

The setup described is given by a two-stage game. We define and solve for its equilibrium in this section.

4.1 Equilibrium concept

At the initial date, there is a perfect-information game in which the **bank** in each region chooses its portfolio, taking the choice of the other bank as given (Nash equilibrium). Because of free entry, the bank will make zero profits and maximizes the expected utility of creditors in its region.[9] A bank's portfolio choice is publicly observed.

At the interim date, there is an imperfect-information subgame between **creditors**. Each creditors receives a private signal and updates his beliefs about regional investment returns r_k and r_{-k} as well as the proportion of prematurely withdrawing patient creditors w_k and w_{-k}. Capital letters denote posteriors distributions derived in Appendix (A.1):

$$\boldsymbol{R_{i,k}} \equiv r_k | x_{ik} \tag{12}$$

$$\boldsymbol{R_{i,-k}} \equiv r_{-k} | x_{ik} \tag{13}$$

$$\boldsymbol{W_{i,k}} \equiv w_k | x_{ik} \tag{14}$$

$$\boldsymbol{W_{i,-k}} \equiv w_{-k} | x_{ik} \tag{15}$$

Likewise, $L_k^* = W_k^* + \lambda - y_k$ denotes the posterior mean of the liquidation volume in region k. Given his information, each creditor decides whether or not to withdraw his funds from the bank. To ease notation, we assume that a creditor does not withdraw his funds from the bank if indifferent between both actions.

We determine the Bayesian Nash equilibrium of the subgame at the interim date. A *strategy* is a plan of action of patient creditor i in region k for each private signal x_{ik}. A *profile of strategies* is a collection of strategies of all creditors. A profile of strategies is a Bayesian Nash equilibrium in the subgame if the actions described by creditor i's strategy maximize his expected utility conditional on the information available and given the strategies followed by all

[9] This is similar to Gale (2010), in which the bank's optimal behaviour under free entry and subject to the creditors participation constraint can be expressed as the solution to a contracting problem in which the welfare of creditors is maximised subject to the zero-profit constraint of the bank.

other creditors. We will prove the uniqueness of this Bayesian Nash equilibrium and demonstrate that it is characterised by a *threshold strategy*. Patient creditor i in region k withdraws if and only if his signal falls short of a regional threshold ($x_{ik} < x_k^*$).

Solving backwards, a bank takes the withdrawal decision of its creditors into account when making its portfolio choice at the initial date. Thus, the bank chooses the Bayesian Nash equilibrium in the subgame that is best for its creditors. A bank takes into account the effect of its portfolio choice at the initial date on the withdrawal threshold at the interim date.

4.2 Equilibrium behaviour

We now study the equilibrium withdrawal behaviour of creditors in the subgame. Impatient creditors always withdraw. Strategic complementarity in withdrawal decisions, both within and between regions, arises from the strategic withdrawal decision of patient creditors. (A withdrawing patient creditor stores his funds for consumption at the final date.) The threshold posterior mean R_k^* equivalent to the threshold signal $x_{ik} = x_k^*$ is defined as the posterior mean in region k that makes a patient creditor indifferent between keeping and withdrawing. The **equilibrium condition** for region k is given by:

$$1 = d_{2k}(R_k^*, R_{-k}^*) = \frac{A_{2k}(R_k^*, R_{-k}^*)}{1 - W_k^*}, \tag{16}$$

where the left-hand side is the payoff from withdrawing and the right-hand side is the expected payoff from not withdrawing conditional on receiving the threshold signal.

We start with Lemma (1) that ensures positive equilibrium liquidation shares:

Lemma 1. *Let private signals be sufficiently precise. Then, there is always a positive amount of liquidation of the investment project $L_k^* > 0$ in equilibrium.*

A proof is given in Appendix (A.4). The intuition is as follows. Given the low level of liquidation, the final-date consumption level of a patient depositor always exceeds unity if the private signal is sufficiently precise. Thus, it is strictly dominant to wait, contradicting the supposition of equilibrium.

Equation (16) implicitly defines the best response function $R_k^*(R_{-k}^*)$, taking the other region's withdrawal threshold R^*_{-k} as given. Each region's withdrawal threshold depends on the other's, owing to the effect of expected withdrawals in the other region on final period payoffs. By virtue of Lemma 1, the best response function of creditors in region k reduces to:

$$\begin{aligned} R_k^* &= 1+\frac{(1+\chi)(I_k-b_k)-1}{I_k-(1-\kappa)b_k}[W_k(R_k^*)+\lambda-y_k] \\ &\quad -\frac{(1-\kappa)(\bar{r}-1)-(1+\chi)[W_{-k}(R_k^*,R_{-k}^*)+\lambda-y_k]}{I_k-(1-\kappa)b_k}b_k \end{aligned} \tag{17}$$

where κ is a constant defined in the Appendix that collects precision parameters. The equilibrium of the subgame is obtained as the intersection of the best-response functions and is summarised by the following proposition that states its existence, uniqueness, and symmetry:

Proposition 1. *Suppose that the idiosyncratic noise is sufficiently small and the regional noise is sufficiently large:*

$$\gamma > \underline{\gamma} < \infty \tag{18}$$

$$\delta < \overline{\delta} \in (0,\infty) \tag{19}$$

Then, there exists a unique and symmetric equilibrium threshold R^* in the subgame.

$$\begin{aligned} R^* &= 1+\frac{(1+\chi)(I_k-b_k)-1}{I_k-(1-\kappa)b_k}[W_k(R^*)+\lambda-y_k] \\ &\quad -\frac{(1-\kappa)(\bar{r}-1)-(1+\chi)[W_{-k}(R^*,R^*)+\lambda-y_k]}{I_k-(1-\kappa)b_k}b_k \end{aligned} \tag{20}$$

Proof. See Appendix (A.5). □

Note that sufficiently small idiosyncratic noise is a standard condition for a unique global game equilibrium (Morris and Shin (2001), Goldstein and Pauzner (2005)). The upper bound on the precision of the regional investment return $\overline{\delta}$ is in line with the global games literature in which the private signal needs to be sufficiently precise relative to the public signal for uniqueness of equilibrium. A low precision of the regional investment return can be attained by a low precision of the global investment return and the regional noise.

We now turn to the efficiency properties of the equilibrium threshold R^* in the subgame. The threshold can be grouped in three terms. First, the efficiency level is unity ($R^{FB} = 1$). Second, the factor on L^* is positive if and only if there is strategic complementarity within a region's creditors (see appendix A.2). Third, there is a term associated with diversification (also part of the second term). Thus, there are efficient bank runs in the absence of diversification and strategic interaction ($b_k = 0, \chi = \overline{\chi}$). Absent any diversification ($b_k = 0$), strategic complementarities in the withdrawal decision of late creditors ($\chi > \overline{\chi}$) result in co-ordination failure between creditors and a withdrawal threshold above the efficient level. A patient creditor to withdraw even though the posterior mean exceeds unity (inefficient bank runs similar to Morris and Shin (2001)).

Finally, we establish an upper bound and lower bound on the equilibrium withdrawal threshold:

Lemma 2. *The equilibrium withdrawal threshold always exceeds unity but never exceeds the expected investment return* ($1 < R^* < \bar{r}$).

Each inequality of this lemma can be proven in two steps. First, the defining equation of R^*, equation (20), implies that neither $R^* = \bar{r}$ nor $R^* = 1$ are equilibria due to the sufficiency conditions. The left-hand side is too high and too low, respectively, in these cases. Second, the sufficiency conditions for equilibrium uniqueness imply that the slope of the left-hand side exceeds the slope of the right-hand side, implying that the equilibrium level R^* must lie above uity and below $\bar{r}$. ■

This establishes the presence of **inefficient equilibrium bank runs**.

4.3 Changes to threshold R^*

We consider parameter changes like the expected asset returns $\bar{r}$ and liquidation costs χ. As a bank takes into account the effect of its portfolio choice on the withdrawal threshold, we also state the change of the withdrawal threshold with respect to the levels of liquidity (both y_k and y_{-k}) and the amount of diversification b_k.

Proposition 2. *The withdrawal threshold* R^* *varies according to*

(a) $\frac{\partial R^*}{\partial \chi} > 0$*: the withdrawal threshold is high when liquidation costs are high.*

(b) $\frac{\partial R^*}{\partial \bar{r}} < 0$*: the withdrawal threshold is low when the expected investment return is high.*

(c) $\frac{\partial R^*}{\partial y_k}$ *has ambiguous sign.*

(d) $\frac{\partial R^*}{\partial y_{-k}} < 0$*: the withdrawal threshold is low when the other bank's liquidity holding is large.*

(e) $\frac{\partial R^*}{\partial b_k}$ *has ambiguous sign.*

Proof. See Appendix (A.6). □

The intuition for these results is as follows. A high liquidation cost implies a low value of a patient creditor's funds at the final date in case of a premature withdrawals of patient depositors. Thus, a patient creditor finds it optimal to withdraw his funds for a larger range of posterior means. A high liquidation cost is thus associated with an intensified co-ordination failure problem between patient creditors. A high expected return on the investment project makes it worthwhile to abandon the investment project only if the private signal is sufficiently negative. Hence, premature withdrawal happens for a smaller range of posterior means.

A high level of liquidity held has two implications. First, more own region liquidity makes liquidation of the risky project less likely, tending to make bank runs less likely. Second, with higher liquidity, high returns are foregone as otherwise high yielding assets are overlooked in favour of low yielding liquidity (opportunity cost is R^* in equilibrium). At low levels of liquidity, we would expect the first effect to dominate. At some point however, more liquidity reduces the availble investment in risky assets, lowering returns and increasing run probability. For any symmetric portfolio choice, a large degree diversification reduces the variance of an individual bank's portfolio and thus the individual probability of a bank run. Premature withdrawals then occurs for a smaller range of posterior means.

Finally, a high level of liquidity held by the other bank prevents costly liquidation in that region for a larger range of posterior means. This benefits not only creditors in the other region, but also creditors in this region who hold a stake in the other region's investment project via their diversified portfolio. This constitutes a positive externality from holding liquidity: more liquidity holdings in one region reduce the withdrawal threshold in the other region, motivating macro-prudential policy. The externality is the stronger, the larger the amount of diversification and the larger the liquidation costs.

4.4 Optimal portfolio choice

We complete the characterisation of the equilibrium by studying the bank's optimal behaviour. A bank's objective function is the expected utility of its creditors derived in Appendix (A.7) and given by:

$$\mathrm{E}[U_k] = (1+\lambda)\bar{r} - (\bar{r}-1)y_k - b_k(1+\chi)\int_{-\infty}^{\infty}\Phi(\cdot)f(r)dr - (I_k - b_k)\int_{-\infty}^{\infty}\Phi(\cdot)\left[1+\frac{\phi(\cdot)}{\sqrt{\beta}\Phi(\cdot)} - r\right]f(r)dr \tag{21}$$

where $f(r)$ the probability density of the economy-wide investment return and $\Phi(\cdot)$ is the probability of a bank run in either region for a given investment return r. Note that the probability of a bank run decreases in the investment return r ($\frac{\partial\Phi(\cdot)}{\partial r} < 0$).

The expected utility consists of four terms. The first term is the expected return if all resources are invested in the long asset, which serves as benchmark. The second term is the reduction in expected returns because of holding liquidity, which has a lower expected return (but reduces the withdrawal threshold R^*, as shown above). The third term reflects the negative externality arising from a bank run in the *other* region, which happens with probabilty $\Phi(\cdot)$. The fourth term measures the loss from a bank run in the creditor's region. The measure of exposed assets is $I_k - b_k$, a run occurs with probability $\Phi(\cdot)$, and the unit loss given default is $[1+\frac{\phi(\cdot)}{\sqrt{\beta}\Phi(\cdot)} - r]$ for any given investment return r.

Note that a higher withdrawal threshold implies a larger area of inefficient runs and thus lowers the expected utility ($\frac{\partial \mathrm{E}[U_k]}{\partial R^*} < 0$) as shown in Appendix (A.7). In addition, the direct effect of holding liquidity is negative as the opportunity cost of liquidity, the expected investment project return, exceeds the return to liquidity ($\frac{\partial \mathrm{E}[U_k]}{\partial y_k} < 0$). The positive externality from holding liquidity, or the equivalent negative externality from liquidation, is again present ($\frac{d\mathrm{E}[U_k]}{dy^*_{-k}} = \frac{\partial \mathrm{E}[U_k]}{\partial R^*}\frac{\partial R^*}{\partial y_{-k}} > 0$). Again, this externality is the stronger, the larger the liquidation cost and the more diversified a bank is.

We are now equipped to determine the optimal portfolio choice of a bank. To focus on non-trivial portfolio choices, we restrict attention to parameters that ensure $\tilde{D}\frac{\partial R^*}{\partial b_k} < 0$ and $\tilde{D}\frac{\partial R^*}{\partial y_k} < 0$. To obtain intuition, we first consider exogenous amounts of diversification($0 < b_k < I_k$). Section (6) considers the case of endogenous diversification and confirms our results.

Definition 2. *The* ***privately optimal*** *or equilibrium level of liquidity y^*_k solves the bank's portfolio choice problem at the initial date, taking the creditor's response at the interim date into* *account and taking the other bank's level of liquidity y_{-k} as given:*

$$y_k^* \equiv \arg\max_{y_k} \mathrm{E}[U_k] \;\; s.t. \;\; R^* = R^*(y_k, y_{-k}) \tag{22}$$

The competitive bank's first-order condition yields:

$$0 = \frac{\partial \mathrm{E}[U_k]}{\partial y_k} + \frac{\partial \mathrm{E}[U_k]}{\partial R^*}\frac{\partial R^*}{\partial y_k} \tag{23}$$

The first-order condition balances the (direct) private cost of holding liquidity in terms of foregone investment return ($\frac{\partial \mathrm{E}[U_k]}{\partial y_k} < 0$) with the (indirect) private benefits from holding liquidity in terms of a lower withdrawal threshold ($\frac{\partial R^*}{\partial y_k} < 0$). The equilibrium liquidity holding is symmetric ($y_k^* = y^*$) and it is the unique global maximizer of the bank's objective function. The latter part arises from the global concavity of the objective function, which stems purely from the effect of liquidity holdings on the withdrawal threshold, and is established in Appendix (A.8).

5 Welfare

This section compares the optimal portfolio choice of the competitive bank with that of a social planner. We adopt the notion of constrained efficiency also studied by Lorenzoni (2008). The withdrawal decision of depositors at the interim date and the demand-deposit contract are taken as given by the constrained planner[10]. In contrast to a bank, the planner internalizes the beneficial effects of holding liquidity on the creditors of *other* banks.

Definition 3. *The* ***socially efficient*** *levels of liquidity (y_k^{SP}, y_{-k}^{SP}) solve the planner's portfolio choice problem at the initial date, taking the creditor's response at the interim date into account.*

$$(y_k^{SP}, y_{-k}^{SP}) \equiv \arg\max_{y_k; y_{-k}} \mathrm{E}[U_k] + \mathrm{E}[U_{-k}] \;\; s.t. \;\; R^* = R^*(y_k, y_{-k})$$

The first-order condition for the social planner's problem is (by symmetry):

$$0 = \underbrace{\frac{\partial \mathrm{E}[U_k]}{\partial y_k}}_{-} + \underbrace{\frac{\partial \mathrm{E}[U_k]}{\partial R^*}}_{-}\underbrace{\left[\frac{\partial R^*}{\partial y_k} + \frac{\partial R^*}{\partial y_{-k}}\right]}_{-} \tag{24}$$

The planner balances the (direct) social cost of holding liquidity in terms of foregone investment

[10]Direct choice of the threshold R^* would achieve the first-best allocation $R^{FB} = 1$.

return ($\frac{\partial \mathrm{E}[U_k]}{\partial y_k} < 0$) with the (indirect) social benefits from holding liquidity in terms of a lower withdrawal threshold. The private and social costs of holding liquidity coincide. There are two components to the social benefits from holding liquidity. Apart from the beneficial effect from holding liquidity on the regional creditor ($\frac{\partial R^*}{\partial y_k} < 0$) identical to the private benefit from holding liquidity, the beneficial effect of holding liquidity on the other region's creditors ($\frac{\partial R^*}{\partial y_{-k}} < 0$) is also present. Thus, the social benefit from holding liquidity exceeds the private benefit.

The global concavity of the social welfare function $\mathrm{E}[U_k] + \mathrm{E}[U_{-k}]$ is established in Appendix (A.8), establishing that the planner's allocation is the unique global maximiser of the social welfare function. Symmetry is again obtained ($y_k^{SP} = y^{SP}$). We proceed by comparing the optimal and the efficient levels of liquidity:

Proposition 3. *The social planner chooses a higher liquidity holding than the competitive bank. Thus, the planner imposes a macro-prudential liquidity buffer* $y^{SP} - y^* > 0$.

Proof. Observe that the right-hand side of (24) has an additional positive term, the positive externality of holding liquidity on the other region. Thus, the social benefits from holding liquidity exceed the social cost of liquidity when evaluated at the competitive level y^*. In other words,

$$\left.\frac{d\mathrm{E}[U_k] + \mathrm{E}[U_{-k}]}{dy_k}\right|_{y_k = y_k^*} > 0$$

Given the strict global concavity of the objective function, we obtain $y^{SP} > y^*$. □

This is the first central result of our analysis. A social planner imposes a macro-prudential liquidity requirement in order to internalise the social costs of liquidation that arise in an inter-linked financial system.

5.1 Comparative statics

This sections studies how the equilibrium allocation and the planner's allocation vary with the exogenous parameters of the model.

Proposition 4. *The privately optimal and socially efficient levels of liquidity vary according to*

(a) $\partial y^{SP}/\partial b_k < \partial y^*/\partial b_k < 0$, *such that more diversification by banks requires fewer liquidity holdings as chosen privately and socially.*

(b) $\partial y^{SP}/\partial \chi > \partial y^*/\partial \chi > 0$*, such that higher liquidation costs, which generate more costly distress, raise the liquidity levels as chosen privately and socially.*

(c) $\partial y^{SP}/\partial \bar{r} < \partial y^*/\partial \bar{r} < 0$*, such that higher investment returns (low asset prices – such as in a recession) require lower private and social levels of liquidity.*

Proof. See Appendix (A.9). □

Lemma 1 follows directly:

Corollary 1. *The macro-prudential liquidity buffer* $y^{SP} - y^*$ *raises as*

(a) there is a lower amount of diversification b_k*;*

(b) there is a higher liquidation cost χ*;*

(c) there is a lower expected return of the investment project $\bar{r}$*.*

The mechanisms underlying these results are as follows. First, when banks portfolios are more diversified, a run in a given region is less likely (Proposition 2). This diversification provides a natural hedge against region-specific return shocks, reducing the need of liquidity buffers. This highlights the substitutability between liquidity (self-insurance) and diversification (co-insurance). Second, the strength of the externality is captured by the liquidation cost parameter χ. Thus, when the costs of financial distress are high, the social planner imposes a high macro-prudential liquidity buffers since individual and systemic runs are likely (Proposition 2).

Third, when asset returns are low (or asset prices high), the social planner requires a high liquidity buffer. This result is the consequence of two forces going in the same direction. On the one hand, low asset returns mean a low opportunity cost of holding liquidity from an ex-ante perspective, thus making holding liquidity relatively inexpensive. On the other hand, a bank run is more probable from an ex-ante perspective as the economy-wide investment return is low (Proposition 2). In sum, both effects imply that the macro-prudential buffer is small when investment return is high.

A vast literature following the seminal paper by Fama and French (1989) links the cyclical position to expected investment returns. This literature's consensus appears to be that expected returns are high during recessions and low during booms, documenting a countercyclicality in asset returns (see also Campbell and Diebold (2009)). In light of this finding, our result suggests a high macro-prudential liquidity buffer during booms when expected returns are low. We close by applying our result to the period prioir to the recent financial crisis. This period was characterised by low expected returns, a *search for yield*, sugessting that the liquidity buffer should have been high.

6 Endogenous diversification

Banks now choose the amounts of liquidity y_k and diversification b_k at the initial date. Our results confirm the optimality of a macro-prudential liquidity buffer. What is more, a prudential regulator would optimally limit the amount of diversification.

There is now an additional first-order condition for diversification that complements the one for liquidity. The first-order condition for diversification of the bank is identical to the one of the planner since there are no externalities associated with diversification. The first-order condition is given by:

$$0 = \frac{\partial \mathrm{E}[U_k]}{\partial b_k} + \frac{\partial \mathrm{E}[U_k]}{\partial R^*}\frac{\partial R^*}{\partial b_k} \tag{25}$$

which again yields a symmetric solution. To gain intuition, the following figure depicts the first-order conditions of both the bank and the planner in diversification - liquidity space (b,y).[11] Let y refer to the first-order condition for liquidity and b for diversification. Likewise, we label the competitive equilibrium and the social planner allocations by $*$ and SP, respectively. Consider the bank's portfolio choice first. The first-order condition for diversification is steeper than the first-order condition for liquidity (proven in Appendix (A.8)):

$$\left(\frac{dy_k}{db_k}\right)_*^b < \left(\frac{dy_k}{db_k}\right)_*^y < 0 \tag{26}$$

Next, we determine how the planner's allocation will differ from the competitive equilibrium. Solving for the slope of the planner's first-order condition for liquidity in linkages - liquidity space (see Appendix (A.8)), we obtain that the planner's first-order condition for liquidity is flatter than the bank's one:

$$\left(\frac{dy_k}{db_k}\right)_{SP}^y < \left(\frac{dy_k}{db_k}\right)_*^y \tag{27}$$

This result is intuitive: the planner is less willing to trade off liquidity for diversification as it

[11] Appendix (A.8) verifies the validity of the second-order conditions, which now involve a Hessian, and demonstrates the uniqueness of the allocation.

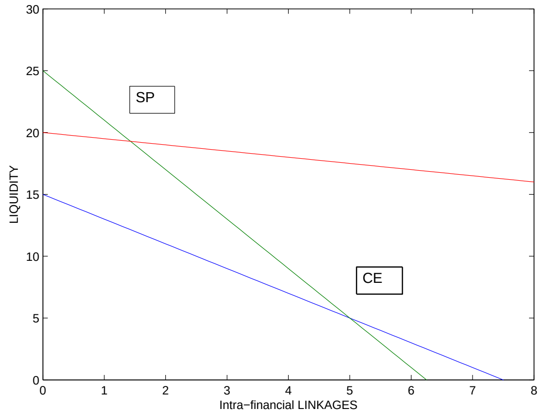

Figure 4: Allocations with endogenous diversification and liquidity. Qualitative illustration only. Liquidity is on the vertical axis and intra-financial linkages or diversification on the horizontal axis. The social planner allocation (SP) is at the top left, while the private competitive equilibrium (CE) is at the bottom right, implying that the planner chooses a higher level of liquidity and a lower level of diversification.

considers the full social cost of liquidation, internalizes the negative externality of liquidation on the other bank's creditors. Also note that the planner's first-order condition for liquidity lies strictly above the one for the competitive bank.[12] This is a consequence of our first key result, stated in graphical form: the planner imposes a macro-prudential liquidity buffer because the social benefits from holding liquidity exceed the private benefits for any level of diversification. Finally, recall that the the first-order conditions for diversification are the same by symmetry.

Taking these three observations together, we obtain the key result in the case of the endogenous diversification:

Proposition 5. *Let both liquidity and diversification be endogenous. Then, the social planner chooses a higher amount of liquidity and a lower amount of diversification than in the equilibrium,* $y^{SP} > y^*$ *and* $b_k^{SP} < b_k^*$.

7 Conclusion

We examine whether the privately optimal portfolios of interlinked financial intermediaries are sufficiently liquid from a social perspective. We explore an economy in which liquidation costs and diversification lead to strategic complementarities between creditors of the same bank as

[12]To see this, fix diversification. Then, note that the additional term in the planner's first-order condition for liquidity, relative to the bank's first-order condition, is positive. Hence, liquidity needs to change to restore equality, which is brought by a rise in liquidity levels as $\frac{d^2 \mathrm{E}[U_k]}{dy_k^2} < 0$.

well as across banks. A novel feature of our model is the optimal choice of the exposure to strategic complementarity between creditors of different financial institutions. Diversification is individually beneficial but generates *systemic fragility*: one bank is exposed to runs on other banks through interbank exposures. Interbank linkages from diversification generate a positive liquidity externality. Subsequently, the equilibrium of this economy is constrained inefficient despite the perfect alignment between the bank and its creditors.

As a result, a (constrained) planner imposes a macro-prudential liquidity requirement, forcing banks to internalise the costs of liquidation imposed on the creditors of banks in the other region. That is, intrafinancial spillovers generate a role for macro-prudential policy. The size of the macroprudential liquidity buffer varies with the parameters of the economy. In particular, we find that the macro-prudential liquidity buffer is larger when the cost of liquidation is high and the return on banks' investment is low, such as during 'search for yield' episodes (booms).

Our framework provides a natural laboratory for studying macro-prudential policies in a general micro-founded setting. While preserving the co-ordination failure aspect of bank creditors as in Morris and Shin (2001) and Goldstein and Pauzner (2005), the presence of spillovers between regions puts the macro-prudential aspect of regulation at the center of our analysis in an intuitive way. Consequently, we have abstracted from micro-prudential regulation motives, such as moral hazard. We also abstracted from other regulatory tools like capital requirements or taxes on interbank exposures in our current analysis, but plan to approach these in subsequent research.

A Appendix

A.1 Posterior distributions and their means

Figure 5 summarizes the updating. Additional noise arises from another creditor's private signal (γ) and from regional investment return shocks (2β).

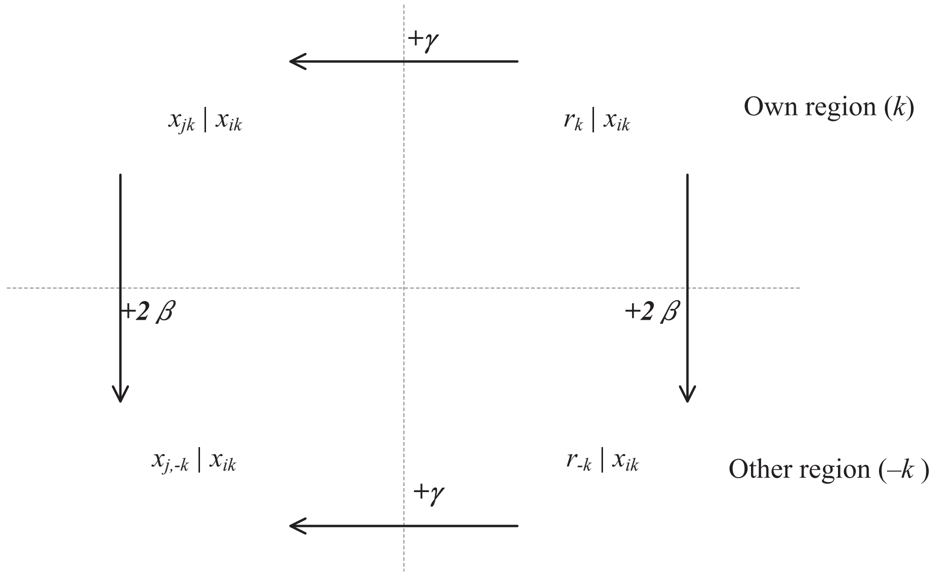

Figure 5: Updating: additional noise

Investment return in region *k* The mean of the posterior distribution is a weighted average of the mean of the prior distribution and the private signal, in which the relative weights are given by the respective precisions. The precision of the posterior distribution is the sum of the prior's and signal's precisions. Normality is preserved, such that:

$$\boldsymbol{R}_{ik} - \bar{r} \sim \mathcal{N}\left(R_{i,k} - \bar{r}, \frac{1}{\gamma + \delta}\right) \tag{28}$$

$$R_{i,k} - \bar{r} \equiv \frac{x_{ik} - \bar{r}}{1 + \frac{\delta}{\gamma}} \tag{29}$$

where $\delta \equiv \frac{\alpha\beta}{\alpha+\beta}$ is the precision of the public signal about the regional return. Three remarks are in order. First, the ratio of the prior's precision to the private signal, $\frac{\delta}{\gamma}$, determines the extent to which the posterior relies on the private signal. The more precise the private signal relative to the prior, the more the posterior is determined by the private signal. In the limit ($\frac{\delta}{\gamma} \to 0$), the posterior mean converges to the private signal. Second, the one-region result of Morris and Shin (2001) is obtained for vanishing regional noise ($\beta \to \infty$). Third, it is convenient to rewrite the threshold strategy in terms of a threshold of the posterior mean R_k^*:

Remark 1. *(Threshold strategy) A patient creditor i in region k withdraws prematurley if and only if $R_{i,k} < R_k^*$.*

Investment return in region $-k$ Because of the common component of regional returns, a creditor's private signal is also informative about the other region. The other region's posterior return is distributed according to:

$$\boldsymbol{r}_{-k} \sim \mathcal{N}\left(x_{ik}, \frac{1}{\mu}\right) \tag{30}$$

where $\mu \equiv \left(\frac{1}{\gamma} + \frac{1}{\beta} + \frac{1}{\beta}\right)^{-1} < \gamma$ is the precision of the private signal of a creditor i in k about region $-k$. The fact that $\mu < \gamma$ implies that the private signal is more precise about region k (the creditor's own region) than about region $-k$ (the other region). The precision of regional noise β determines the usefulness of the private signal for predicting the other region's return. If regional noise is large, knowledge about the common component is less valuable, and the private signal becomes less useful for predicting the other region's return. The private signal ceases to be included into the updating as regional noise becomes large ($\beta \to 0$) — even if idiosyncratic noise vanishes ($\gamma \to \infty$).

The posterior return of creditor i in region k over region $-k$'s return is:

$$\boldsymbol{R}_{i,-k} - \bar{r} \sim \mathcal{N}\left(R_{i,-k} - \bar{r}, \frac{1}{\mu + \delta}\right) \tag{31}$$

$$R_{i,-k} - \bar{r} \equiv = \kappa[R_{i,k} - \bar{r}] \tag{32}$$

$$\kappa \equiv \frac{1 + \frac{\delta}{\gamma}}{1 + \frac{\delta}{\mu}} \in (0, 1) \tag{33}$$

where $\kappa < 1$ as $\mu < \gamma$. Relative to the posterior mean of his own region's investment return, creditor i in k relies less on his private signal and more on the prior ($\kappa < 1$) in forming his posterior mean over region $-k$'s return due to the additional regional noise. Thus, $R_{i,-k} = \kappa R_{ik} + (1 - \kappa)\bar{r} > R_{i,k}$ for any signal below its mean, $x_{ik} < \bar{r}$. The difference between the posterior means vanishes ($\kappa \to 1$) as regional noise becomes small ($\beta \to \infty$).[13]

Figure 6 illustrates the posterior distributions of regional returns. It also contains the posterior distribution for creditor i over another creditor j's signal. creditor i uses this to form the posterior of the proportion of prematurely withdrawing patient creditors. When inferring another creditor's possible signal $x_{jk}|x_{ik}$, additional idiosyncratic noise γ leads to an unchanged

[13] As idiosyncratic noise vanishes ($\gamma \to \infty$), the precision parameter converges to $\kappa \to \frac{\alpha+\beta}{3\alpha+\beta} \in (0, 1)$.

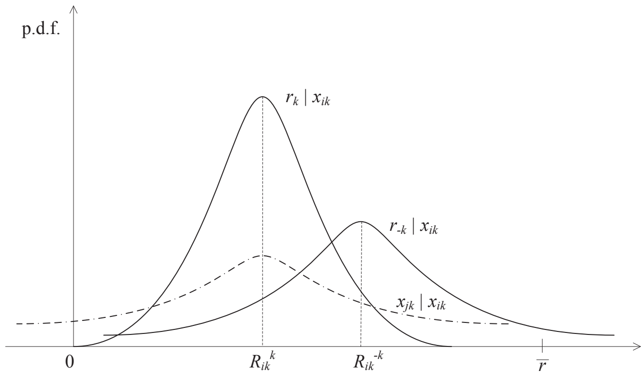

Figure 6: Different posteriors (for a given $x_{ik} < \bar{r}$)

posterior mean but a more dispersed posterior distribution.

Prematurely withdrawing patient creditors in region $\boldsymbol{k}$ This posterior proportion $\boldsymbol{W}_{i,k}$ can be written as the indicator of the event in which an arbitrary patient creditor withdraws prematurely. The proof relies on the independence of the individual noise across creditors in a given region and the knowledge that everyone plays the threshold strategy. The posterior distribution of the proportion of prematurely withdrawing patient creditors is:

$$\begin{aligned}
\boldsymbol{W}_{i,k} &= \boldsymbol{w}_k | x_{ik} && (34)\\
&= \int_{j\in[0,1]} \mathbf{1}\{\text{agent j in region k withdraws}|x_{ik}\}dj && (35)\\
&= \int_{j\in[0,1]} \mathbf{1}\{\boldsymbol{R}_{jk}^k \leq R_k^*|x_{ik}\}dj && (36)\\
&= \int_{j\in[0,1]} \mathbf{1}\{\boldsymbol{x}_{jk} \leq \frac{\delta}{\gamma}(R_k^* - \bar{r}) + R_k^*|x_{ik}\}dj && (37)\\
&= \int_{j\in[0,1]} \mathbf{1}\{\boldsymbol{r}_k + \boldsymbol{\epsilon}_{jk} \leq \frac{\delta}{\gamma}(R_k^* - \bar{r}) + R_k^*|x_{ik}\}dj && (38)\\
&= \int_{j\in[0,1]} \mathbf{1}\{\boldsymbol{\epsilon}_{jk} \leq \frac{\delta}{\gamma}(R_k^* - \bar{r}) + (R_k^* - r_k)|x_{ik}\}dj && (39)\\
&= 1 * \mathbf{1}\{\boldsymbol{\epsilon}_{jk} \leq \frac{\delta}{\gamma}(R_k^* - \bar{r}) + (R_k^* - r_k)|x_{ik}\} && (40)\\
&= \mathbf{1}\{\boldsymbol{x}_{jk} \leq x_k^*|x_{ik}\} = \mathbf{1}\{\boldsymbol{x}_{jk} - \bar{r} \leq x_k^*\bar{r}|x_{ik}\} && (41)
\end{aligned}$$

where the second line uses the definition of the proportion $\boldsymbol{w}_k$, the third line the the definition of threshold strategy, the fourth line the posterior distribution about the investment return by patient household $j \neq i$, the fifth line j's private signal, and the sixth line the independence of idiosyncratic noise from both the economy-wide investment return and the return to region's investment project return.

The posterior mean of the proportion is then the probability of that event:

$$W_{i,k} = \mathrm{E}[\boldsymbol{W}_{i,k}] \;=\; \Phi\left(\sqrt{\delta_1}\,[R_k^* - \bar{r}] + \sqrt{\delta_1}\gamma\delta\,[R_k^* - R_{i,k}]\right) \tag{42}$$

$$\delta_1 \;\equiv\; \frac{\frac{\delta^2}{\gamma^2}}{\frac{1}{\delta+\gamma} + \frac{1}{\gamma}} = \frac{\delta^2}{\gamma}\frac{\delta+\gamma}{\delta+2\gamma} \tag{43}$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution and δ_1 summarizes precision parameters. A patient creditor that receives the threshold signal x_k^* thus forms the following posterior mean:[14]

$$W_k^* \equiv W_{i,k}|_{R_{i,k}=R_k^*} = \Phi\left(\sqrt{\delta_1}\,[R_k^* - \bar{r}]\right) \tag{44}$$

Note that $\delta_1 \to 0$ as the private signal becomes very precise ($\gamma \to \infty$) such that $W_k^* \to \frac{1}{2}$.

Prematurely withdrawing patient creditors in region $-k$ The posterior proportion $\boldsymbol{W}_{i,-k}$ is again the indicator of the event that an arbitrary patient creditor in the other region withdraws prematurely:

$$\boldsymbol{W}_{i,-k} \;=\; \mathbf{1}\{\boldsymbol{x}_{j,-k} \leq x_{-k}^* | x_{ik}\} \tag{45}$$

$$W_{i,-k} \;\equiv\; \mathrm{E}[\boldsymbol{W}_{i,-k}] = \Pr\{\boldsymbol{x}_{j,-k} \leq x_{-k}^* | x_{ik}\}\ , \tag{46}$$

where the conditional probability takes the same form as before but differs in the distributions $\boldsymbol{x}_{j,-k}|x_{ik}$:

$$(x_{j,-k} - \bar{r})|x_{ik} \;=\; (\boldsymbol{r}_{-k} - \bar{r})|x_{ik} + \boldsymbol{\epsilon}_{j,-k} \tag{47}$$

$$= \mathcal{N}\left(R_{i,-k} - \bar{r}, \left[\frac{\gamma\,[\delta + \gamma\mu]}{\delta + 2\gamma\mu}\right]^{-1}\right) \tag{48}$$

$$W_{i,-k} \;=\; \Phi\left(\sqrt{\delta_2}\,\left[R_{-k}^* - \bar{r}\right] + \sqrt{\frac{\gamma(\delta+\mu)}{\delta+\gamma+\mu}}\,\left[R_{-k}^* - R_{i,-k}\right]\right) \tag{49}$$

[14]As regional noise vanishes ($\beta \to \infty$), the multi-region model collapses to the one-region model of Morris and Shin (2001), with the coefficient being equivalent to equation (2.6) in Morris and Shin (2001).

The posterior mean is again the probability of the event:

$$W_{i,-k} \equiv \mathrm{E}\left[\boldsymbol{W}_{i,-k}\right] = \Phi\left((1+\frac{\delta}{\gamma})\sqrt{\delta_2}\left[R^*_{-k}-\bar{r}\right] - \kappa\sqrt{\delta_2}\left[R_{ik}-\bar{r}\right]\right) \tag{50}$$

$$\delta_2 \equiv \frac{\gamma[\beta+(2+\frac{\beta}{\gamma})\delta]}{2(\beta+\gamma)+(2+\frac{\beta}{\gamma})\delta} \tag{51}$$

A patient creditor who receives the threshold signal x^*_k will form the following posterior mean of the proportion of prematurely withdrawing patient creditors in its neighbouring region:

$$W^*_{-k} \equiv W_{i,-k}|_{R_{i,k}=R^*_k} = \Phi\left(\sqrt{\rho_1}\left(R^*_{-k}-\bar{r}\right) - \sqrt{\rho_2}\left(R^*_k-\bar{r}\right)\right) \tag{52}$$

$$\rho_1 \equiv \left(\frac{\delta+\gamma}{\gamma}\right)^2 \frac{1}{\frac{1}{\delta+\mu}+\frac{1}{\gamma}} \tag{53}$$

$$\rho_2 \equiv \kappa^2 \frac{1}{\frac{1}{\delta+\mu}+\frac{1}{\gamma}} \tag{54}$$

where $\rho_1 > \rho_2$.

A.2 Proof: Strategic complementarities within region

The relevant case around equilibrium is a positive liquidation by the regional bank only (see A.4). Thus, the final-date payoff of a patient creditor who keeps his funds at the bank are

$$d_{2k} = \frac{(I_k-b_k)[r_k-(1+\chi)(w_k+\lambda-y_k)]+b_k[r_{-k}-(1+\chi)l_{-k}]}{1-w_k} \tag{55}$$

Partially differentiating with respect to the porportion of prematurely withdrawing patient creditors:

$$\frac{\partial d_{2k}}{\partial w_k} = \frac{(I_k-b_k)[r_k-I_k(1+\chi)]+b_k[r_{-k}-(1+\chi)l_{-k}]}{(1-w_k)^2} \tag{56}$$

Locally around the equilibrium threshold R^*_k, we have:

$$\frac{\partial d_{2k}}{\partial w_k} = \frac{1-(1+\chi)(I_k-b_k)}{(1-w_k)} \tag{57}$$

such that $\chi > \underline{\chi} \equiv \frac{1+b_k-I_k}{I_k-b_k}$ ensures the presence of *strategic complementarities* within a region.

A.3 Proof: Strategic complementarities between region

We show that the partial derivative of the final-date payoff to a patient creditor with respect to the share of prematurely withdrawing patient creditors in the *other* region is (strictly) negative if and only if the amount of interbank diversification is (strictly) positive and the excess liquidity exhausted:

$$\frac{\partial d_{2k}}{\partial w_{-k}} = -\frac{(1+\chi)b_k}{(1-w_k)} < 0 \tag{58}$$

A.4 Proof: positive expected liquidation share in equilibrium

This proof is by contradiction: suppose that creditors expect no liquidation in equilibrium, $L_k^* = 0$. The idea of the proof is to show that this implies $d_{2k} \neq 1$ for a sufficiently large precision of the private signal. For the following steps, we employ the posterior distributions derived in Appendix (A.1).

Step 1: Fix any amount of liquidity $\lambda \leq y_k < \frac{1}{2} + \lambda$. We focus on parameters that make it never optimal to hold a larger amount of liquidity in equilibrium, that is a sufficiently high mean return $\bar{r}$. Then, we have:

$$W_k^* \leq y_k - \lambda \tag{59}$$

Step 2: Using the posterior mean of the proportion of prematurely withdrawing patient creditors in region k, we find the implied bound on the equilibrium mean investment return R_k^*:

$$R_k^* \leq \bar{r} + \frac{1}{\sqrt{\delta_1}} \left[\Phi^{-1}(y_k - \lambda)\right] \tag{60}$$

where $\Phi^{-1}(\cdot)$ denotes the inverse of cumulative distribution function of a standard normal random variable. Note that $R_k^* \to -\infty$ as $\gamma \to \infty$.

Step 3: The implied final-date payoff to patient despitors is:

$$d_{2k}^* = \frac{I_k R_k^* + (1-\kappa)b_k(\bar{r} - R_k^*) - (1+\chi)b_k(W_{-k}^* + \lambda - y_{-k})}{1 - W_k^*} \tag{61}$$

where $L_{-k}^* = W_{-k}^* + \lambda - y_{-k}$ is the equilibrium liquidation share in the other region.

Step 4: Given $y_k < \lambda + \frac{1}{2}$, there exists a boundary γ_0 such that $d_{2k}^* \neq 1$ for any $\gamma > \gamma_0$. This contradicts the supposition of R_k^* being an equilibrium in the withdrawal game. (If $y_k = \lambda + \frac{1}{2}$, then there is exists only one b_k that satisfies the equation $d_{2k}^* = 1$. However, the withdrawal subgame equilibrium threshold R_k^* ought to be determined for a range of portfolio choices. The portfolio choices in turn are determined at the initial stage, implying that this is not an equilibrium either.) ■

In sum, we have shown there is always liquidation in equilibrium for a sufficiently precise private signal and $y_k < \frac{1}{2} + \lambda$.

A.5 Proof: uniqueness of threshold equilibrium

The existence and uniqueness proof of the threshold equilibrium has two steps. First, we show that the best response function is bounded and that its lies strictly within zero and one, $\frac{dR_k^*}{dR_{-k}^*} \in (0, 1)$. This establishes that there exists an threshold equilibrium and that it is unique. Second, an individual creditor must not find it profitable to deviate from the threshold R_k^* given that all other creditors in both regions play a threshold strategy with threshold R_k^*. Using the iterated deletion of strictly dominated strategies, this establishes that *any* equilibrium strategy is a threshold strategy with threshold R^*. Hence, there is a unique Bayesian Nash equilibrium in the subgame that entails a threshold strategy. We discuss both steps in turn.

A.5.1 Slope of best-response function

Consider the bounds of the best-response functions first. If the threshold in the other region becomes smaller ($R_{-k} \to -\infty$), creditors in the other region never withdraw ($W_{-k} \to 0$). Let $\underline{R_k}$ denote the best response of a creditor in region k to that strategy. It is given by the solution to (20). We have $-\infty < \underline{R_k} < \bar{r}$. Likewise, we determine $\overline{R_k}$ for $R_{-k} \to +\infty$, then $W_{-k} \to 1$, that is all patient creditors always withdraw in the other region. We have $\underline{R_k} < \overline{R_k} < \infty$.

Existence and uniqueness of equilibrium requires the slope of the best-response function to lie strictly within zero and one, $\frac{dR_k^*}{dR_{-k}^*} \in (0, 1)$. The partial derivative is given by:

$$\frac{dR_k^*}{dR_{-k}^*} = \frac{(1+\chi)\sqrt{\rho_1}\phi(z_2)b_k}{D} \tag{62}$$

$$D \equiv I_k - (1-\kappa)b_k + (1+\chi)\sqrt{\rho_2}\phi(z_2)b_k - \sqrt{\delta_1}\phi(z_1)\left[(1+\chi)(I_k - b_k) - 1\right] \tag{63}$$

$$z_1 \equiv \sqrt{\delta_1}\left[R_k^* - \bar{r}\right] \tag{64}$$

$$z_2 \equiv \sqrt{\rho_1}\left[R_{-k}^* - \bar{r}\right] - \sqrt{\rho_2}\left[R_k^* - \bar{r}\right] \tag{65}$$

Both conditions are ensured when $D > (1+\chi)\sqrt{\rho_1}\phi(z_2)b_k > 0$. The second inequality never binds. Letting idiosyncratic noise vanish ($\gamma \to \infty$) and with the upper bound on the pdf

($\phi(z) \leq \frac{1}{\sqrt{2\pi}}$), the first inequality yields:

$$\frac{b_k}{I_k}\left[(1+\chi)\sqrt{\frac{\beta}{4\pi}}\left(\frac{2\frac{\alpha}{\beta}}{\sqrt{3(\frac{\alpha}{\beta})^2+4\frac{\alpha}{\beta}+1}}\right)\frac{2\frac{\alpha}{\beta}}{1+3\frac{\alpha}{\beta}}\right] < 1 \tag{66}$$

As $b_k \leq I_k$, a sufficiently low precision of the regional noise for a given correlation between the investment projects ρ suffices to ensure this condition. In other words, for any portfolio choice (b_k, y_k) there exists ($\underline{\gamma}$, $\overline{\delta}$) such that for any ($\beta < \overline{\delta}$, $\gamma > \underline{\gamma}$) that ensures a the existence of a unique (and symmetric) equilibrium.

A.5.2 No profitable deviation

We are left to demonstrate that there is no profitable deviation for an individual creditor if all other creditors play the threshold stategy with threshold R_k^*. We employ a standard argument of iterated deletion of strictly dominated strategies (adapted from Morris and Shin (2001)) to show that it is indeed optimal for patient creditor i from region k to follow a threshold strategy with threshold R_k^*.

First, we determine the payoff $v(\cdot)$ of the deviating creditor. If all patient players in region k and $-k$ play a threshold strategy characterised by R_k and R_{-k}, patient creditor i's uses its private signal to update his beliefs about regional returns and the shares of prematurely withdrawing patient creditors. His payoff from keeping its funds in the bank is given by:

$$v\left(R_{ik}, R_k, R_{-k}\right) \equiv \frac{(I_k-b_k)[R_{ik}-(1+\chi)(W_{i,k}+\lambda-y_k)]+b_k[\kappa R_{ik}+(1-\kappa)\bar{r}(1+\chi)]W_{i,-k}+\lambda-y_{-k}}{1-W_{ik}} \tag{67}$$

$$W_{i,k} = \Phi(z_{1i}) \tag{68}$$

$$W_{i,-k} = \Phi(z_{2i}) \tag{69}$$

$$z_{1i} \equiv (\sqrt{\delta_1}[R_k^*-\bar{r}]+\delta\gamma\sqrt{\delta_1})[R_k^*-R_{ik}] \tag{70}$$

$$z_{2i} \equiv (\sqrt{\rho_1}[R_{-k}^*-\bar{r}]-\sqrt{\rho_2}[R_{i,k}-\bar{r}] \tag{71}$$

(72)

It is useful for the following argument to define the partial derivatives of the payoff function:

$$v_1 \equiv \frac{\partial v}{\partial R_{ik}} = (1 - W_{ik})^{-1} \left[[I_k - (1-\kappa) b_k + \gamma\delta\sqrt{\delta_1}\phi(z_1 i)[(1+\chi)(I - k - b_k) - d_{2ik}] \right] > 0 \tag{73}$$

$$v_2 \equiv \frac{\partial v}{\partial R_k^*} = -\frac{\sqrt{\delta_1}(1+\delta\gamma)\phi(z_{1i})}{1 - W_{ik}} \left[A_{2ik} - (1+\chi)(I_k - b_k) \right] < 0 \tag{74}$$

$$v_3 \equiv \frac{\partial v}{\partial R_{-k}^*} = -\frac{(1+\chi)\sqrt{\rho_1}\phi(z_{2i})}{1 - W_{ik}} b_k < 0 \tag{75}$$

Note that the strong monotonicity of the payoff function in the first argument arises for the sufficiency conditions of a unique equilibrium and implies the optimality of a threshold strategy. Likewise, the strong monotonicity of the second partial derivative is due to the sufficiency conditions. Take the first step of the iteration argument.

Step 1: If the realised regional return r_k is sufficiently bad, agent i's private signal and posterior mean R_{ik} will be so bad that withdrawing is a dominant strategy, i.e. other players' actions do not matter. Formally, there exists a $\underline{R}^{(1)}$ such that

$$v(R_{ik}, R_k, R_{-k}) < 1 \;\; \forall (R_k, R_{-k}) \tag{76}$$

for *any* $R_{ik} < \underline{R}^{(1)}$. Note that this lower bound applies to any creditor in both regions by symmetry. The lower bound is implicity defined by

$$\lim_{R_k \to -\infty} \lim_{R_{-k} \to -\infty} \left(v(\underline{R}^{(1)}, R_k, R_{-k}) \right) = 1 \tag{77}$$

Hence, $R_{ik} \geq \underline{R}^{(1)}$. The same analysis is done by other patient creditors in region k: $R_k \geq \underline{R}^{(1)}$. By symmetry, this lower bound holds for the other region's threshold as well.

Step 2: Given that other patient creditors ruled out all thresholds below the lower bound $\underline{R}^{(1)}$, there is a second-stage lower bound $\underline{R}^{(2)}$ for patient creditor i defined by:

$$1 = v\left(\underline{R}^{(2)}, \underline{R}^{(1)}, \underline{R}^{(1)} \right) \tag{78}$$

Given the partial derivatives of the payoff function v, we have $\underline{R}^{(2)} > \underline{R}^{(1)}$, where any strategy that keeps funds in the bank for a posterior lower than $\underline{R}^{(2)}$ is strictly dominated in the second round.

Step 3: Iterating on these steps, we obtain a strictly increasing sequence $\underline{R^{(1)}} < \underline{R^{(2)}} < \cdots < \underline{R^{(l)}} < ...$, where the initial value $\underline{R^{(1)}}$ is given above and $\underline{R^{(l+1)}}$ is recursively defined by

$$1 = v\left(\underline{R^{(l+1)}}, \underline{R^{(l)}}, \underline{R^{(l)}}\right) \quad l = 1, 2, ... \tag{79}$$

Any posterior mean $R < \underline{R^{(m)}}$ does not survive the m^{th} round of deletion of strictly dominated stratgies. Let $\underline{R}$ be the limit of this sequence defined by

$$1 = v\left(\underline{R}, \underline{R}, \underline{R}\right) \tag{80}$$

An identical argument holds for a strictly decreasing sequence $\overline{R^{(1)}} > \overline{R^{(2)}} > \cdots > \underline{R^{(l)}} > ...$ that converges to $\overline{R} : 1 = v\left(\overline{R}, \overline{R}, \overline{R}\right)$.

Step 4: We proved that there is a unique R^* that solves $1 = v\left(R^*, R^*, R^*\right)$. Thus, both sequences converge to $\underline{R} = R^* = \overline{R}$. This establishes that there is a unique Bayesian Nash equilibrium in the subgame at the interim date. It prescribes that every creditor in either region will use a threshold strategy with threshold R^*, withdrawing his funds if and only if his posterior mean falls short of this threshold. This completes the proof for the existence and uniqueness of the equilibrium in threshold strategies. ■

A.6 Proofs: changes of threshold R^*

In analogy to the previous short hands (z_1,z_2), we define

$$z_1' \equiv \sqrt{\delta_1}\left[R^* - \bar{r}\right] \tag{81}$$

$$z_2' \equiv \left(\sqrt{\rho_1} - \sqrt{\rho_2}\right)\left[R^* - \bar{r}\right] \tag{82}$$

All partial derivatives share a common denominator $\tilde{D} = D - (1+\chi)\sqrt{\rho_1}\phi(z_2')b_k > 0$. The difference to the previous denominator arises as a regional bank takes into account the effect of its portfolio choice decision on both best response functions, that is on R_{-k} as well, while a creditor takes R^*_{-k} as given. (The bank still takes the other bank's portfolio choice as given.) Mathematically, this yields the above an additional negative term. The positive sign is assured by the sufficiency conditions of the unique equilibrium. Recall $\rho_1 > \rho_2$ and that $\delta_1 \to 0$ as $\gamma \to \infty$, such that $\sqrt{\delta_1} < (\sqrt{\rho_1} - \sqrt{\rho_2})$ as private noise vanishes. Consequently, $W_k(R^*) > W_{-k}(R^*, R^*)$ as $R^* < \bar{r}$. The smaller precision about the other regions makes a depositor belief that the equilibrium proportion of withdrawals is higher in his region than in the other region.

The partial derivatives with respect to parameters are:

$$\tilde{D}\frac{\partial R^*}{\partial \chi} = [I_k - b_k](W_k^* + \lambda - y_k) + b_k(W_{-k}^* + \lambda - y_{-k}) > 0 \tag{83}$$

$$\tilde{D}\frac{\partial R^*}{\partial \bar{r}} = -(1-\kappa)b_k - [(1+\chi)(I_k - b_k) - 1]\sqrt{\delta_1}\phi(z_1') - (1+\chi)b_k(\sqrt{\rho_1} - \sqrt{\rho_1})\phi(z_2') < 0 \tag{84}$$

The positive sign of the first bracket is ensured by the sufficient condition for strategic complementarity within a region ($\chi > \underline{\chi}$). Moving to the partial derivatives with respect to a bank's portfolio choice variables:

$$\tilde{D}\frac{\partial R^*}{\partial y_{-k}} = -(1+\chi)b_k < 0 \tag{85}$$

$$\tilde{D}\frac{\partial R^*}{\partial y_k} = R^* - (1+\chi)[1 - W_k^* - b_k] \tag{86}$$

$$\tilde{D}\frac{\partial R^*}{\partial b_k} = -[(1-\kappa)(\bar{r} - R^*) + (1+\chi)(W_k^* - y_k - W_{-k}^* + y_{-k}) \tag{87}$$

The first partial derivative illustrates the key channel for a beneficial policy intervention in our model. This beneficial effect of reducing the withdrawal threshold is only internalised by the planner. The partial derivatives for the portfolio choice variables of a bank b_k and y_k are ambiguous for an arbitrary portfolio choice. However, $\tilde{D}\frac{\partial R^*}{\partial b_k} < 0$ in any symmetric equilibrium $y_k^* = y^*$.

To focus on non-trivial portfolio choices, we restrict attention to parameters that ensure $\tilde{D}\frac{\partial R^*}{\partial b_k} < 0$ and $\tilde{D}\frac{\partial R^*}{\partial y_k} < 0$.

Second-order derivatives To varify the second-order conditions and to compute comparative statics on the privately optimal and socially efficient portfolio allocation, we also obtain the second-order effects on the withdrawal threshold. Note that the partial derivatives of the constant

$\tilde{D}$ are:

$$\frac{\partial \tilde{D}}{\partial y_k} = -1 + (1+\chi)\phi(z_1')\sqrt{\delta_1} < 0 \tag{88}$$

$$\frac{\partial \tilde{D}}{\partial b_k} = -(1-\kappa) + \sqrt{\delta_1}(1+\chi)\phi(z_1') - (1+\chi)\phi(z_2')(\sqrt{\rho_1} - \sqrt{\rho_2}) < 0 \tag{89}$$

where the signs arise as γ becomes large. This implies the following second-order partial derivatives:

$$\frac{\partial^2 R^*}{\partial y_k^2} = \underbrace{-\frac{\partial \tilde{D}}{\partial y_k}}_{+} \frac{1}{\tilde{D}} \frac{\partial R^*}{\partial y_{-k}} \tag{90}$$

$$\frac{\partial^2 R^*}{\partial b_k^2} = \underbrace{-\frac{\partial \tilde{D}}{\partial b_k}}_{+} \frac{1}{\tilde{D}} \frac{\partial R^*}{\partial b_{-k}} \tag{91}$$

$$\frac{\partial^2 R^*}{\partial y_k \partial b_k} = \frac{1}{\tilde{D}} \left[(1+\chi) - \frac{\partial \tilde{D}}{\partial b_k} \frac{\partial R^*}{\partial y_{-k}} \right] \tag{92}$$

$$\frac{\partial^2 R^*}{\partial b_k \partial y_k} = \frac{1}{\tilde{D}} \left[(1+\chi) - \frac{\partial \tilde{D}}{\partial y_k} \frac{\partial R^*}{\partial b_{-k}} \right] \tag{93}$$

Thus $\frac{\partial^2 R^*}{\partial y_k^2}$ inherits the sign of $\frac{\partial R^*}{\partial y_{-k}}$ and $\frac{\partial^2 R^*}{\partial b_k^2}$ inherits the sign of $\frac{\partial R^*}{\partial b_{-k}}$. The sign of the cross second partial derivatives is ambiguous.

A.7 Derivation of expected utility $\mathrm{E}[U_k]$

To determine the expected utility in region k, $\mathrm{E}[U_k]$, we first obtain the payoffs to creditors in four cases. As regional noise vanishes, there are two possible cases: full runs (partial bank runs exist only with non-vanmishing noise) or no run at all. Combining these cases for each region $k = \{A, B\}$, we have four cases. A run takes place if the realization of the regional return is below the threshold R^*. Note that the private signal converges to the regional return as the individual noise vanishes. In the limit, $x_{ik} \to r_k$ as $\gamma \to \infty$. Then,

$$R_{ik} < R^* \Leftrightarrow r_k < R^*$$

First, case (a) occurs when investment returns in both regions are sufficiently good to prevent runs, so $r_k \geq R^*$ and $r_{-k} \geq R^*$. In this case no liquidation takes place in either region. Impatient creditors of mass λ receive their promised payment and patient creditors of unit mass receive the remainder. Let π_k^a be the total payoff in region k in case (a), then:

$$\pi_k^a = [1 \times \lambda] + [(y_k - \lambda) + (I_k - b_k)r_k + b_k r_{-k}] \times 1 \quad (94)$$

$$= y_k + (I_k - b_k)r_k + b_k r_{-k} \quad (95)$$

Second, case (b) occurs when investment returns in k are sufficient to prevent a run in k, but bad investment returns in $-k$ generate a run there, so $r_{-k} < R^* \leq r_k$. Then, no liquidation takes place region k but full liquidation takes place in $-k$. This imposes liquidation costs on the bank in region k through the erosion of the value of the swap. Impatient creditors of mass λ receive their promised payment and patient creditors of unit mass receive the remainder, giving:

$$\pi_k^b = [1 \times \lambda] + [(y_k - \lambda) + (I_k - b_k)r_k + b_k (r_{-k} - (1 + \chi))] \times 1 \quad (96)$$

$$= \pi_k^a - b_k(1 + \chi). \quad (97)$$

Next, case (c) occurs when bad investment returns in region k trigger a run there, but good investment returns in the other region prevent contagion, so $r_k < R^* \leq r_{-k}$. In this case liquidation takes place in region k only. The bank in region k liquidates all of its assets and all agents receive an equal share of the funds (pro rata).[15] Again, liquidation costs are imposed on the other bank in the other region $-k$:

$$\pi_k^c = [1 \times \lambda] + [(y_k - \lambda) + (I_k - b_k) + b_k r_{-k}] \times 1 \quad (98)$$

$$= 1 + \lambda - b_k + b_k r_{-k}. \quad (99)$$

Finally, case (d) occurs when bad investment returns in both regions mean $r_k < R^*$ and $r_{-k} < R^*$. Liquidation takes place in both regions. Banks liquidate their assets and mutually impose liquidation costs on each other[16]. All agents receive an equal share of the remaining

[15]Part of the liquidation proceeds of the bank is a share of the interbank claim. Ex-post trade ensures that these claims are held by patient creditors without any welfare loss. Thus, the competitive bank and the planner are concerned with the total value of the assets only.

[16]Note that the amount creditors obtain from withdrawing at the interim date falls short of unity in this case. This is consistent with the withdrawal threshold derived earlier as withdrawing is a strictly dominant strategy in this case

funds (pro rata), giving

$$\begin{aligned} \pi_k^d &= [1 \times \lambda] + [(y_k - \lambda) + (I_k - b_k) + b_k\,(r_{-k} - (1+\chi))] \times 1 & (100) \\ &= \pi_k^c - b_k(1+\chi) & (101) \end{aligned}$$

We can identify the benefits of a asset diversification. In case (c), diversification is beneficial. Also, the swap substitutes some strategic complementarity between creditors within a given region with that between regions. This arises in the form of a negative externality imposed by the liquidation decision of one bank the other. This is reflected in the payoffs: $\pi_k^a > \pi_k^b$ and $\pi_k^c > \pi_k^d$.

Also note that the payoff π_k is weakly increasing in the investment return r_k with a discrete jump at R^* reflecting liquidation costs. Thus, a marginal decrease in the threshold R^* – as implied by larger liquidity holdings in the other region – strictly increases welfare.

We are now ready to determine the expected utilty by integrating over the four cases. Let $f(r) = \phi(\sqrt{\alpha}(r - \bar{r}))$ and $g(\eta_k) = \phi(\sqrt{\beta}\eta_k)$ denote the probability distribution functions of $\boldsymbol{r}$ and $\boldsymbol{\eta}_k$, respectively, where $\phi(\cdot)$ is the probability distribution function of the standard normal distribution. Note that the event $r_k < R^*$ can be rewritten as $\eta_k < \bar{\eta}(r) \equiv R^* - r$ for a given economy-wide investment return r. Using these distribution functions and the payoffs derived above, total welfare in region k is given by

$$\begin{aligned} \mathrm{E}[U_k] \equiv \int_{-\infty}^{\infty} \int_{-\infty}^{\bar{\eta}(r)} \left[\int_{-\infty}^{\bar{\eta}(r)} \pi_k^d g(\eta_{-k}) d\eta_{-k} + \int_{\bar{\eta}(r)}^{\infty} \pi_k^c g(\eta_{-k}) d\eta_{-k} \right] g(\eta_k) d\eta_k + \\ + \int_{\bar{\eta}(r)}^{\infty} \left[\int_{-\infty}^{\bar{\eta}(r)} \pi_k^b g(\eta_{-k}) d\eta_{-k} + \int_{\bar{\eta}(r)}^{\infty} \pi_k^a g(\eta_{-k}) d\eta_{-k} \right] g(\eta_k) d\eta_k f(r) dr. \end{aligned}$$

Integrating over η_{-k}, we have:

$$
\begin{aligned}
\int_{\bar\eta(r)}^{\infty} \pi_k^c g(\eta_{-k}) d\eta_{-k} &= [1+\lambda+b_k(r-1)][1-\Phi(\sqrt{\beta}\bar\eta)] \\
&\quad + b_k \mathbb{E}\left[\eta_{-k}|\eta_{-k} \geq \bar\eta\right] \\
\int_{-\infty}^{\bar\eta(r)} \pi_k^d g(\eta_{-k}) d\eta_{-k} &= \Phi(\sqrt{\beta}\bar\eta)[1+\lambda+b_k(r-1)-b_k(1+\chi)] \\
&\quad + b_k \mathbb{E}\left[\eta_{-k}|\eta_{-k} \leq \bar\eta\right] \\
\int_{\bar\eta(r)}^{\infty} \pi_k^a g(\eta_{-k}) d\eta_{-k} &= [y_k+(1+\lambda-y_k)r+(1+\lambda-b_k-y_k)\eta_k][1-\Phi(\sqrt{\beta}\bar\eta)] \\
&\quad + b_k \mathbb{E}\left[\eta_{-k}|\eta_{-k} \geq \bar\eta\right] \\
\int_{-\infty}^{\bar\eta(r)} \pi_k^b g(\eta_{-k}) d\eta_{-k} &= [y_k+(1+\lambda-y_k)r+(1+\lambda-b_k-y_k)\eta_k-b_k(1+\chi)]\Phi(\sqrt{\beta}\bar\eta) \\
&\quad + b_k \mathbb{E}\left[\eta_{-k}|\eta_{-k} \leq \bar\eta\right]
\end{aligned}
$$

The conditional expectations on η_{-k} vanish when adding up. By contrast, we find a closed-form expression for the conditional expectation on η_k by substitution.

$$
\mathbb{E}\left[\eta_k|\eta_k \geq \bar\eta\right] = \frac{\exp(-\frac{1}{2}\beta\bar\eta^2)}{\sqrt{\beta}\sqrt{2\pi}} = \frac{\phi(\sqrt{\beta}\bar\eta)}{\sqrt{\beta}}
$$

$$
\begin{aligned}
\mathrm{E}[U_k] \equiv & \int_{-\infty}^{\infty} (\Phi(\cdot)[1+\lambda+b_k(r-1)-b_k(1+\chi)\Phi(\cdot)]+) + (I_k-b_k)\mathbb{E}\left[\eta_k|\eta_k \geq \bar\eta\right] \\
& +(y_k+I_k r)(1-\Phi(\cdot)) - b_k(1+\chi)\Phi(\cdot)(1-\Phi(\cdot))f(r)dr \\
= & \int_{-\infty}^{\infty} -b_k(1+\chi)\Phi(\sqrt{\beta}\bar\eta) + (1-\Phi(\cdot))(y_k+I_k r) + \Phi(\cdot)[1+\lambda+b_k(r-1)] \\
& +(I_k-b_k)\mathbb{E}\left[\eta_k|\eta_k \geq \bar\eta\right] f(r)dr \\
\mathrm{E}[U_k] = & \int_{-\infty}^{\infty} (y_k+I_k r) f(r)dr + \frac{1}{\sqrt{\beta}}(I_k-b_k)\int_{-\infty}^{\infty} \phi(\sqrt{\beta}\bar\eta) f(r)dr + \\
& \int_{-\infty}^{\infty} \Phi(\sqrt{\beta}\bar\eta)[I_k-b_k-b_k(1+\chi)-r(I_k-b_k)]f(r)dr
\end{aligned}
$$

The expected utility reduces to

$$
\begin{aligned}
\mathrm{E}[U_k] = y_k + I_k\bar r + \frac{1}{\sqrt{\beta}}(I_k-b_k)\int_{-\infty}^{\infty} \phi(\sqrt{\beta}\bar\eta) f(r)dr & \\
+ \int_{-\infty}^{\infty} \Phi(\sqrt{\beta}\bar\eta)[I_k-b_k-b_k(1+\chi)-r(I_k-b_k)]f(r)dr. &
\end{aligned}
$$

Note that the level of your region's liquidity y_k enters directly via payoffs and indirectly via the withdrawal threshold R^*, while the level of the other region's lquidity y_{-k} enters indirectly throug the withdrawal threshold only. We consider the partial derivatives with respect to both amounts of liquidity and the withdrawal threshold in turn. First, we have that

$$\frac{\partial \mathrm{E}[U_k]}{\partial R^*} = -\sqrt{\beta}[(R^*-1)I_k + b_k(2+\chi-R^*)] \int_{-\infty}^{\infty} \phi(\sqrt{\beta}[R^*-r])f(r)dr < 0$$

Similarly

$$\frac{\partial \mathrm{E}[U_k]}{\partial y_k} = \int_{-\infty}^{\infty} \left[(1-r)[1-\Phi(\sqrt{\beta}\bar{\eta})] - \frac{\phi(\sqrt{\beta}\bar{\eta})}{\sqrt{\beta}}\right] f(r)dr < 0 \quad (102)$$

$$\frac{\partial \mathrm{E}[U_k]}{\partial b_k} = \int_{-\infty}^{\infty} \Phi(\sqrt{\beta}\bar{\eta})[-\frac{\phi(\sqrt{\beta}\bar{\eta})}{\sqrt{\beta}\Phi(\sqrt{\beta}\bar{\eta})} + r - 1 - (1+\chi)]f(r)dr < 0 \quad (103)$$

such that $\frac{d\mathrm{E}[U_k]}{dy_k}$ and $\frac{d\mathrm{E}[U_k]}{db_k}$ have ambiguous sign in general. Also note that

$$\frac{\partial^2 \mathrm{E}[U_k]}{\partial y_k^2} = 0 = \frac{\partial^2 \mathrm{E}[U_k]}{\partial b_k^2} = \frac{\partial^2 \mathrm{E}[U_k]}{\partial y_k \partial b_k} \quad (104)$$

$$\frac{\partial^2 \mathrm{E}[U_k]}{\partial y_k \partial R^*} = \sqrt{\beta}(R^*-1)\int_{-\infty}^{+\infty} \phi(\cdot)f(r)dr > 0 \quad (105)$$

$$\frac{\partial^2 \mathrm{E}[U_k]}{\partial (R^*)^2} = -\underbrace{\sqrt{\beta}(I_k-b_k)\int_{-\infty}^{+\infty} \phi(\cdot)f(r)dr}_{+} + \cdots \quad (106)$$

$$\cdots + \sqrt{\beta^3}\underbrace{\int_{-\infty}^{+\infty}(R^*-r)\phi(\cdot)f(r)dr}_{-}\underbrace{[(R^*-1)I_k + b_k(2+\chi-R^*)]}_{+} < 0 \ , \quad (107)$$

where the first line states that the concavity of the objective function $\mathrm{E}[U_k]$ comes from the indirect effect on the withdrawal threshold R^* only, while the second and third line state second-order partial derivatives with respect to a choice variable and the withdrawal threshold, which will be used later. Some remarks for the fourth line are in order. Note that the second integral is negative. This term contains two probability distributions. First, $\phi(\cdot)$ is centered around R^* and would on its own imply a zero term. However, the pdf $f(r)$ that is centered around $\bar{r} > R^*$ pushes the joint distribution up and yields an overall negative expression.

A.8 Second-order conditions

First, consider the case of the competitive bank. We find the second-derivative of the objective function with respect to liquidity:

$$\frac{d^2\mathrm{E}[U_k]}{dy_k^2} = \frac{\partial^2\mathrm{E}[U_k]}{\partial y_k^2} + \frac{\partial^2\mathrm{E}[U_k]}{\partial y_k \partial R^*}\frac{\partial R^*}{\partial y_k} + \left[\frac{\partial^2\mathrm{E}[U_k]}{\partial y_k \partial R^*} + \frac{\partial^2\mathrm{E}[U_k]}{\partial (R^*)^2}\frac{\partial R^*}{\partial y_k}\right]\frac{\partial R^*}{\partial y_k} + \frac{\partial \mathrm{E}[U_k]}{\partial R^*}\frac{\partial^2 R^*}{\partial y_k^2} \tag{108}$$

$$= \frac{\partial^2\mathrm{E}[U_k]}{\partial (R^*)^2}\left(\frac{\partial R^*}{\partial y_k}\right)^2 + \frac{\partial^2\mathrm{E}[U_k]}{\partial y_k \partial R^*}\frac{\partial R^*}{\partial y_k} + \frac{\partial}{\partial y_k}\underbrace{\left[-\frac{\partial \mathrm{E}[U_k]}{\partial y_k} + \frac{\partial \mathrm{E}[U_k]}{\partial R^*}\frac{\partial R^*}{\partial y_k}\right]}_{=0} \tag{109}$$

$$= \underbrace{\frac{\partial R^*}{\partial y_k}}_{-}\left[\underbrace{\frac{\partial^2\mathrm{E}[U_k]}{\partial y_k \partial R^*}}_{+} + \underbrace{\frac{\partial^2\mathrm{E}[U_k]}{\partial (R^*)^2}}_{-}\frac{\partial R^*}{\partial y_k}\right] < 0 \tag{110}$$

The signs of the second derivatives of the objective function are derived in Appendix (A.7).

Second, an identical argument applies for the social planner's objective function:

$$\frac{d^2(\mathrm{E}[U_k] + \mathrm{E}[U_{-k})}{dy_k^2} = \frac{\partial^2\mathrm{E}[U_k]}{\partial y_k^2} + \frac{\partial^2\mathrm{E}[U_k]}{\partial y_k \partial R^*}\left(\frac{\partial R^*}{\partial y_k} + \frac{\partial R^*}{\partial y_{-k}}\right) + \left[\frac{\partial^2\mathrm{E}[U_k]}{\partial y_k \partial R^*} + \frac{\partial^2\mathrm{E}[U_k]}{\partial (R^*)^2}\frac{\partial R^*}{\partial y_k}\right]\frac{\partial R^*}{\partial y_k} \tag{111}$$

$$\cdots + \frac{\partial \mathrm{E}[U_k]}{\partial R^*}\left(\frac{\partial^2 R^*}{\partial y_k^2} + \frac{\partial^2 R^*}{\partial y_{-k}\partial y_k}\right)$$

$$= \frac{\partial^2\mathrm{E}[U_k]}{\partial (R^*)^2}\left(\frac{\partial R^*}{\partial y_k}\right)\left(\frac{\partial R^*}{\partial y_k} + \frac{\partial R^*}{\partial y_{-k}}\right) + \frac{\partial^2\mathrm{E}[U_k]}{\partial y_k \partial R^*}\left(\frac{\partial R^*}{\partial y_k} + \frac{\partial R^*}{\partial y_{-k}}\right) + \cdots \tag{112}$$

$$\cdots + \frac{\partial}{\partial y_k}\underbrace{\left[-\frac{\partial \mathrm{E}[U_k]}{\partial y_k} + \frac{\partial \mathrm{E}[U_k]}{\partial R^*}\left(\frac{\partial R^*}{\partial y_k} + \frac{\partial R^*}{\partial y_{-k}}\right)\right]}_{=0} < 0$$

Third, consider the case of endogenous diversification. We have:

$$\frac{d^2\mathrm{E}[U_k]}{db_k^2} = \frac{\partial R^*}{\partial b_k}\left[\frac{\partial^2\mathrm{E}[U_k]}{\partial b_k \partial R^*} + \frac{\partial^2\mathrm{E}[U_k]}{\partial (R^*)^2}\frac{\partial R^*}{\partial b_k}\right] < 0 \tag{113}$$

$$\frac{d^2\mathrm{E}[U_k]}{db_k dy_k} = \frac{\partial R^*}{\partial b_k}\left[\frac{\partial^2\mathrm{E}[U_k]}{\partial y_k \partial R^*} + \frac{\partial^2\mathrm{E}[U_k]}{\partial (R^*)^2}\frac{\partial R^*}{\partial y_k}\right] < 0 \tag{114}$$

$$\Rightarrow \frac{d^2\mathrm{E}[U_k]}{dy_k^2}\frac{d^2\mathrm{E}[U_k]}{db_k^2} - \left(\frac{d^2\mathrm{E}[U_k]}{db_k dy_k}\right)^2 > 0 \tag{115}$$

Thus,

$$\left(\frac{dy_k}{db_k}\right)^y_{CE} = -\frac{\frac{d^2\mathrm{E}[U_k]}{dy_k db_k}}{\frac{d^2\mathrm{E}[U_k]}{dy_k^2}} < 0 \tag{116}$$

$$\left(\frac{dy_k}{db_k}\right)^{b_k}_{CE} = -\frac{\frac{d^2\mathrm{E}[U_k]}{db_k^2}}{\frac{d^2\mathrm{E}[U_k]}{db_k dy_k}} < \left(\frac{dy_k}{db_k}\right)^y_{CE} \tag{117}$$

and

$$\frac{d^2(\mathrm{E}[U_k]+\mathrm{E}[U_{-k}])}{dy_k^2} = \left[\frac{\partial R^*}{\partial y_k}+\frac{\partial R^*}{\partial y_{-k}}\right]\left(\frac{\partial^2\mathrm{E}[U_k]}{\partial(R^*)^2}\left[\frac{\partial R^*}{\partial y_k}+\frac{\partial R^*}{\partial y_{-k}}\right]+\frac{\partial^2\mathrm{E}[U_k]}{\partial y_k\partial R^*}\right) < 0 \quad (118)$$

$$\frac{d^2(\mathrm{E}[U_k]+\mathrm{E}[U_{-k}])}{dy_k db_k} = \frac{\partial R^*}{\partial b_k}\left(\frac{\partial^2\mathrm{E}[U_k]}{\partial(R^*)^2}\left[\frac{\partial R^*}{\partial y_k}+\frac{\partial R^*}{\partial y_{-k}}\right]+\frac{\partial^2\mathrm{E}[U_k]}{\partial y_k\partial R^*}\right) < 0 \quad (119)$$

A.9 Comparative Statics

This section examines the dependence of the competitve equilibrium allocation and the (constrained) social planner's allocation on the exogenous parameters of the model. We proceed by totally differentiating the equilibrium conditions (23) and (24). Using an envelope-theorem argument, the partial derivative of the liquidity level with respect to any variable in $a \in A = \{\chi, b_k, \bar{r}\}$, can be written as:

$$\frac{\partial y^{CE}}{\partial a} = \Gamma^{CE}\frac{\partial R^*}{\partial a} \quad (120)$$

$$\frac{\partial y^{CE}}{\partial a} = \Gamma^{SP}\frac{\partial R^*}{\partial a}\ , \quad (121)$$

where Γ^{CE} and Γ^{SP} are positive parameters with $\Gamma^{SP} > \Gamma^{CE} > 0$ given by:

$$\Gamma^{CE} = -\frac{\frac{\partial^2 S_k}{\partial y_k\partial R^*}+\frac{\partial^2 S_k}{\partial(R^*)^2}\frac{\partial R^*}{\partial y_k}}{\frac{\partial^2 S_k}{\partial(R^*)^2}(\frac{\partial R^*}{\partial y_k})^2}$$

$$\Gamma^{SP} = -\frac{\frac{\partial^2 S_k}{\partial y_k\partial R^*}+\frac{\partial^2 S_k}{\partial(R^*)^2}[\frac{\partial R^*}{\partial y_k}+\frac{\partial R^*}{\partial y_{-k}}]}{\frac{\partial^2 S_k}{\partial(R^*)^2}(\frac{\partial R^*}{\partial y_k}+\frac{\partial R^*}{\partial y_{-k}})^2}$$

The ranking $\Gamma^{CE} < \Gamma^{SP}$ arises from direct differentiation with respect to the new term $\frac{\partial R^*}{\partial y_{-k}}$.

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