

Joined at the hip: monetary and fiscal policy in a liquidity-dependent world

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Abstract: Because money is the unit of account, the price of money is the inverse of the price level. If prices are sticky, so is the price of money in terms of goods, and this is one important reason why money is liquid and attractive. By contrast, the price of government bonds is free to jump and often does, especially in response to news about changes in fiscal policy and the supply of bonds. Those movements in government bond prices affect available liquidity, and that matters for aggregate demand, inflation and output. In particular, we show that under these conditions, certain bond-financed fiscal expansions can be contractionary, causing deflation and a temporary recession. To avoid such effects, changes in fiscal policy and bond supply must be matched by changes in money supply and in the interest rate on money. We conclude that in a liquidity-dependent world, fiscal and monetary policies are *joined at the hip*.

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I. Introduction

In mid-April 2021, the Federal Reserve paid 0.10% interest on reserves held by commercial banks at the Fed. At around the same time, the interest rate on short-maturity Treasury bills oscillated around 0.02%.¹ If yield is (an admittedly imperfect) proxy for liquidity, with assets that provide larger liquidity services managing to pay lower interest rates, then in mid-April Treasury bills were more liquid than bank reserves. Or, in the standard parlance of macroeconomics, government bonds were more liquid than money.

From the point of view of the holder (the private sector), in an environment of low interest rates bonds become money-like. Krishnamurthy and Vissing-Jorgensen (2012) write:

Money is a medium of exchange for buying goods and services, has high liquidity, and has extremely high safety in the sense of offering absolute security of nominal repayment. Investors value these attributes of money and drive down the yield on money relative to other assets. We argue that a similar phenomenon affects the prices of Treasury bonds. The high liquidity and safety of Treasuries drive down the yield on Treasuries relative to assets that do not to the same extent share these attributes.

Krishnamurthy and Vissing-Jorgensen then go on to provide detailed econometric evidence to justify these claims.

But that is not the end of the story. From the point of view of the issuer (the government), in an environment of low interest rates money becomes bond-like: financing government operations by issuing bonds is very similar to financing government operations by issuing bank reserves (money). Here is what a recent IMF working paper has to say on the subject:

In modern times, money finance must be through bank reserves. The problem with relying on financing through massive excess reserves is that the financial system simply does not have any use for them... Bank reserves play an important role in the payments system but technological developments have enabled quadrillions of dollars, euros, and yen to be transferred every year without more than a small quantity of reserves being needed. Reserves in excess of this amount are simply deadweight on banks' balance sheets. Reserves suffer in comparison with treasury securities because they can only be held by banks, cannot be used as collateral efficiently like securities, and are subject to capital requirements. Treasury securities may be held by anyone residing anywhere, serve as collateral and support global capital markets, and are not subject to capital requirements when held by real money investors. Thus, if we compare the two instruments, beyond a minimum threshold required for payments, securities sell at a premium to reserves and are a more efficient and less costly way for the sovereign to finance itself. (Stella, Singh and Bhargava, 2021).

¹ Take, for instance, the 1-Month Treasury Constant Maturity Rate at <https://fred.stlouisfed.org/series/DGS1MO>.

So in a low interest rate environment, there are plenty of similarities between money and government bonds. But there remains one key difference: the way their prices are set.² Because money is the unit of account, the price of money is the inverse of the price level. If prices are sticky, so is the price of money in terms of goods —and this is an important part of what makes money so liquid and attractive. Keynes emphasized this point in the *General Theory*, putting forth what one of us (Calvo 2012) has labelled the “price theory of money”:

...the fact that contracts are fixed, and wages are somewhat stable in terms of money, *unquestionably* plays a large part in attracting to money so high a liquidity premium (Keynes, 1936).

By contrast, the price of bonds in terms of goods is free to jump all over the place —and often does, even in the case of bonds as safe and liquid as US Treasuries. Recall the infamous 2013 “taper tantrum” and the short-lived but intense repo-market tremors of September 2019 and March 2020, all of which involved spikes in the yields of US Treasuries or, what is the same, sharp drops in the price of those bonds. Again in March-April 2021, swings in bond prices have occurred as markets digest the implications of the most recent US stimulus package.³

This difference matters a great deal for a number of monetary and fiscal policy issues. Imagine a policy in which the government writes checks and mails them to households, as the US and many other governments have been doing during the pandemic. Whether families will actually spend that money or save it for a rainy day is of course at the center of current debates over the effects of fiscal policy. What is not always taken into account is that the answer to this question depends crucially on how the government finances those checks.

If it prints money to fund the resulting additional budget deficit, then households will find themselves with more money in their pockets.⁴ Given that goods prices are sticky in terms of money, at least in the short run, then the real stock of money will rise. If banks and households do not wish to hold this larger quantity of real money, the only way they can all get rid of it (in the aggregate, that is) is to spend more, so that goods prices will then rise. So a policy of money-financed checks can trigger an output boom and (eventually) higher inflation.

If the authorities are not comfortable with this outcome, then they can always increase the interest rate paid on money in order to induce commercial banks to hold larger reserves at the central bank. This is what happened, in a nutshell, during the Great Financial Crisis of 2007-09 and again during the Covid-19 Crisis. Remunerated bank reserves rose massively, as central banks created reserves to purchase all kinds of assets, both private and public.

² See Calvo (2021) for a detailed discussion of this point and its implications.

³ See “US Treasury bond wobble heightens concerns over health of \$21tn market”. *Financial Times*, 3 March 2021. <https://www.ft.com/content/1deec2b3-59d4-4f90-b752-fefd2a88b5b2>.

⁴ Budget deficits would actually be financed by using central bank reserves, which can only be held by commercial banks. These banks, in turn, would “create” the additional money that would end up in the accounts of the public. But this wrinkle is not important for the story here. That is why we assume away commercial banks, so that households have their own monetary accounts at the central bank, which they can use for transactions

The story is very different if the additional checks issued to households are financed via a bond issue. Now the private sector will find itself with a larger nominal (and, at least for an instant, real) stock of government bonds in its portfolio. If it is happy to hold those bonds, then nothing else need happen. But if the private sector gets a case of the shivers, in order to reduce the real stock of bonds outstanding it does not have to buy more goods so that the price level will rise. It is enough for the private sector to begin selling those bonds in exchange for other assets.

The immediate consequence is that the price of government bonds will fall or (what is the same) the yield on those bonds will rise. It could even happen that the price of bonds falls sufficiently so that the actual value of the total stock outstanding does not change. In that case, overall liquidity in the hands of the public would be the same as it was before the new policy, and the checks need not have any effects on consumption, output or inflation, so fiscal policy would be entirely neutral!

Again, it could well be that the central bank is not comfortable with that outcome. Perhaps it does not want the yield on government bonds to rise, or their price to fall, because it fears the impact that might have on the balance sheets of financial intermediaries and, indirectly, on financial stability. Or perhaps the central bank is in fact hoping that the checks will stimulate aggregate demand, because inflation has been running below target. If so, the central bank can issue central bank reserves, buy the newly-issued government bonds, and cause their price to rise and their yield to drop. But that, of course, would amount to money-financing of the checks!

That is the logic behind the title of this paper. In a world in which government bonds are held for liquidity purposes, fiscal policies can have unwanted effects —or no effects at all! To make sure they have the desired effects, certain kinds of fiscal policies (for instance, the issuance of checks to households) may have to be coupled with certain kinds of monetary policies (the issuance of bank reserves). In this sense, monetary and fiscal policies are *joined at the hip*.

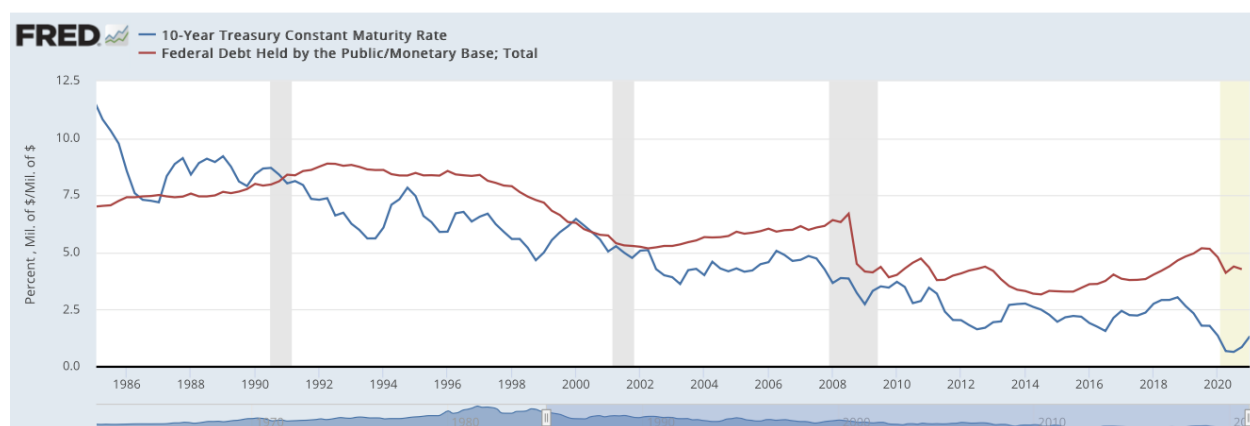


Figure 1

The core of our story relies on one relationship. As more government bonds are issued (or are expected to be issued), other things equal, the price of those bonds should tend to fall or, in what amounts to the same thing, their yield should rise. Figure 1 above suggests this is not just a theoretical construct. The figure plots the 10-year Treasury constant maturity rate and the Federal debt held by the public as a share of the monetary base. The two curves move together, and the correlation coefficient is 0.83. Note the red line shows debt outstanding as a share of monetary liabilities, suggesting that the way to keep in check the rise in bond yields is to allow the monetary base to expand.

We explore these issues in a bare-bones, conventional model. To generate liquidity services (or, more precisely, *transactional* liquidity) we assume a liquidity-in-advance constraint, so that households must have sufficient transactional liquidity (money and bonds) in their pockets in order to consume. This approach has limitations, but one great advantage: it is extremely simple and well understood. Since we are not making any claims about the origins of liquidity demand, but only about its implications, no great harm is done by relying on a standard model.⁵

Also to keep matters simple, we do away with commercial banks and equate money with central bank reserves. Readers uncomfortable with that idea can think of our money as a central bank-issued digital currency that households and firms hold in accounts at the central bank, and which can be used for all the standard transactions.⁶

This menu of assets gives rise to a menu of possible monetary policy frameworks. We focus on a policy regime that targets the quantity of money (central bank reserves) and the interest rate on those reserves, as the world's main central banks have been doing since the Great Financial Crisis. Given this, the interest rate on the "pure" bond adjusts to clear markets.

Our first set of results has to do with the effects of fiscal policies, defined as helicopter drops of zero-coupon government bonds (or, if you prefer, government transfers to households financed by debt issuance). The conventional wisdom is that such policies should be expansionary, contributing to raise aggregate demand and output. We show that there are some situations in which the conventional wisdom is borne out. But there are others in which helicopter drops of bonds are neutral, or even recessionary! It all depends on expectations and timing details.

An unanticipated and permanent bond issue is completely neutral. The price of bonds drops sufficiently so that total transactional liquidity, consumption and output are unchanged. The analogy is with an unanticipated and permanent step increase in the money supply in the context of perfectly flexible prices. No monetary economist is surprised to hear such a policy has zero real effects.

⁵ The main results of this paper go through if we instead assume that money and bonds enter the utility function, as long as the elasticity of substitution between consumption and transactional liquidity is sufficiently low (of course, a transactional-liquidity-in-advance constraint arises in the limit when this elasticity goes to zero). Recently there has been renewed interest in models in which money enters the utility function. See, for instance, Piazzesi, Rogers and Schneider (2021) and Diba and Loisel (2021).

⁶ As in Piazzesi, Rogers and Schneider (2021).

An unanticipated and transitory bond supply increase, on the other hand, has the expected expansionary effects—at least for a while. The intuition is that bond prices fall, but by less than the increase in nominal (and real, given predetermined bond prices) bond supply. So liquidity goes up, and so do aggregate demand and output.

This logic is reversed in the case of other fiscal policies. An anticipated and permanent future step increase in bond supply causes a drop in bond prices in the short run (when bond supply is still unchanged), and therefore a liquidity squeeze and a recession. The very realistic policy of having the fiscal authority issue bonds gradually for a finite period of time, returning at the end of that period to a policy of fixed nominal bond supply, also has a recessionary impact. Bond prices fall more quickly than bond supply increases, causing the value of government bonds outstanding to decline. The resulting drop in liquidity is, once again, recessionary.

So the moral of the story is simple: when bond prices are free to react to expected changes in fiscal policy, they can move in ways that curtail available liquidity, and therefore push down aggregate demand and output. That cannot happen in the case of conventional monetary policy, because the price of money in terms of goods is fixed whenever prices are sticky.

An important caveat is in order. We are not claiming that all fiscal policies can have non-standard effects. Changes in government consumption will typically have real effects, though they may be attenuated in our framework by the implications of bond prices that move over time. And money-financed fiscal expansion also have standard effects. It is bond-financed increases in government transfers to the private sector that can have non-standard effects or no effects at all. But of course, much of the recent fiscal expansion in response to the Covid-19 crisis consisted of sharp rises in government transfers, so our exercise is not without policy relevance.

What can monetary policy do in response? Casual observation of recent events suggests that liquidity provision is often targeted at stabilizing bond prices. That is what the Fed seems to have done, for instance, during recent episodes of volatility in the Treasury and Repo markets.

An instance of this liquidity provision occurred at the outset of Covid-19 crisis, when yields rose sharply and bid-offer spreads widened dramatically. Market stress was intensified by several technical features of the market, such as the limited collateral base of dealers and the weakness of centralized trading arrangements (Duffie, 2020). But it is unlikely that the episode of intense market stress would have occurred in the absence of huge expected fiscal deficits and the associated anticipated massive issue of Treasury bonds.

In response to market stress, the Federal Reserve purchased \$1 trillion of Treasuries in the three-week period from March 16, and then continued to buy at a high rate. Vissing-Jorgensen (2021) argues that “Fed purchases were causal for driving down yields” and that these “purchase effects” were due to acute liquidity needs on the part of sellers.

Given the apparent importance of monetary policy responses to bond market turmoil, our second set of results describes the monetary policies that must accompany expected bond helicopter

drops in order to ensure those bonds issues will not to have contractionary effects on liquidity, consumption and output. The model confirms the conjecture that to avoid that unwanted outcome, the central bank must cut the interest rate on money and also expand the money supply in specific ways that depend on the timing details of the bond helicopter drop.

For instance, following the announcement of a future and permanent increase in the supply of government bonds, the required monetary policy involves gradually increasing the nominal money supply— followed by a step decrease in nominal (and real) money just as bond supply takes a step increase.

So our story features a kind of fiscal dominance, but not the traditional kind. Monetary policy is not compelled to finance the budget deficit, as it often happens in emerging markets. Monetary policy is compelled to stabilize government bond prices, to prevent fluctuations in those prices from having unwanted effects on output and inflation. The motivation is different, but an outside and uninformed observer could easily conclude this is case of fiscal dominance.

The next section lays out the basic model, while section III explores the effects of fiscal shocks. Section IV considers complementary monetary policies while Section V concludes.

II. The basic model

Consider the simplest possible model. Time is continuous and the economy is closed. There is one good and two assets: money and a long-term government bond. We can describe the economy by a few equations (derivations are in the appendix).⁷ All variables are in log deviations from steady state levels, denoted by overbars. First, ℓ_t is liquidity, defined as

$$\ell_t - \bar{\ell} \equiv \alpha(m_t - \bar{m}) + (1 - \alpha)(q_t - \bar{q}) + (1 - \alpha)(z_t - \bar{z}) \quad (1)$$

where m_t is real money holdings, z_t real holdings of a long-term bond, q_t is the price of that bond in terms of money and $\alpha \in (0,1)$ is a parameter. For the sake of simplicity we assume bonds take the forms of consols (perpetuities) that yield liquidity services but pay no coupon.⁸

Next is a liquidity-in-advance constraint, which we assume always binds:

$$y_t - \bar{y} = c_t - \bar{c} \leq \ell_t - \bar{\ell} \quad (2)$$

where c_t is consumption and y_t is output, equal to consumption in a closed economy with no investment. For a given level of liquidity, agents have to decide how much money and how many bonds to hold. That decision is summarized by the portfolio balance equation

⁷ All variables except for interest and growth rates are in logs and overbars denote steady state values.

⁸ In the United States, 10-year Treasuries are often used as collateral in repo operations, which looks a lot like a liquidity-in-advance constraint.

$$\rho(m_t - \bar{m}) - \rho(q_t - \bar{q}) - \rho(z_t - \bar{z}) = i_t^m - \dot{q}_t \quad (3)$$

where i_t^m is the interest rate on money and ρ is the rate of time discounting. This is intuitive: the ratio of money to bonds (or the difference, in logs) depends on their relative returns.

The following identities complete the demand side of the model

$$\dot{m}_t = \mu_t - \pi_t \quad (4)$$

$$\dot{z}_t = \zeta_t - \pi_t \quad (5)$$

where μ_t is nominal money growth and ζ_t is nominal bond growth.

The supply side is simply the Calvo-Phillips equation, as in Calvo (1983) or Galí (2015).

$$\dot{\pi}_t = \rho\pi_t - \kappa(y_t - \bar{y}) \quad (6)$$

Finally, we specify the policy regime: assume i_t^m , μ_t and ζ_t are (exogenous) policy instruments. The “pure” interest rate i_t is endogenous, as is q_t , the price of zero-coupon bonds in terms of money. This policy regime, of fixing the interest on money and also making asset purchase decisions (QE) that exogenously set the nominal stock of bonds and money, is not unlike what the Fed and other central banks are doing nowadays.

In steady state $i = \rho$, so that the steady state inflation rate is zero, as is i^m . In addition, in steady state the authorities are neither creating nor destroying nominal money or bonds, so $\mu = \zeta = 0$. Finally and intuitively, output is at its steady state level, so that

$$c = y = \bar{y} \quad (7)$$

and

$$\bar{q} + \bar{z} - \bar{m} = \log(1 - \alpha) - \log \alpha \quad (8)$$

With some algebra (see the Appendix) the economy can be reduced to:

$$\dot{s}_t = \rho(s_t - \bar{s}) + i_t^m + \zeta_t - \mu_t \quad (9)$$

$$\dot{\pi}_t = \rho\pi_t - \kappa(m_t - \bar{m}) - \kappa(1 - \alpha)(s_t - \bar{s}) \quad (10)$$

$$\dot{m}_t = \mu_t - \pi_t \quad (11)$$

where $(s_t - \bar{s})$ is the ratio (in levels) of bonds to money or the difference (in logs) between bonds and money:

$$(s_t - \bar{s}) \equiv (q_t - \bar{q}) + (z_t - \bar{z}) - (m_t - \bar{m}) \quad (12)$$

Differential equations (10) and (11) are a 2x2 system in m_t and π_t , with s_t as an exogenous variable. The appendix shows that, given that π_t is a “jumpy” variable and m_t is a state variable, the system is saddle-path stable. The associated phase diagram, with inflation on the vertical axis and real money balances on the horizontal axis, is shown below.

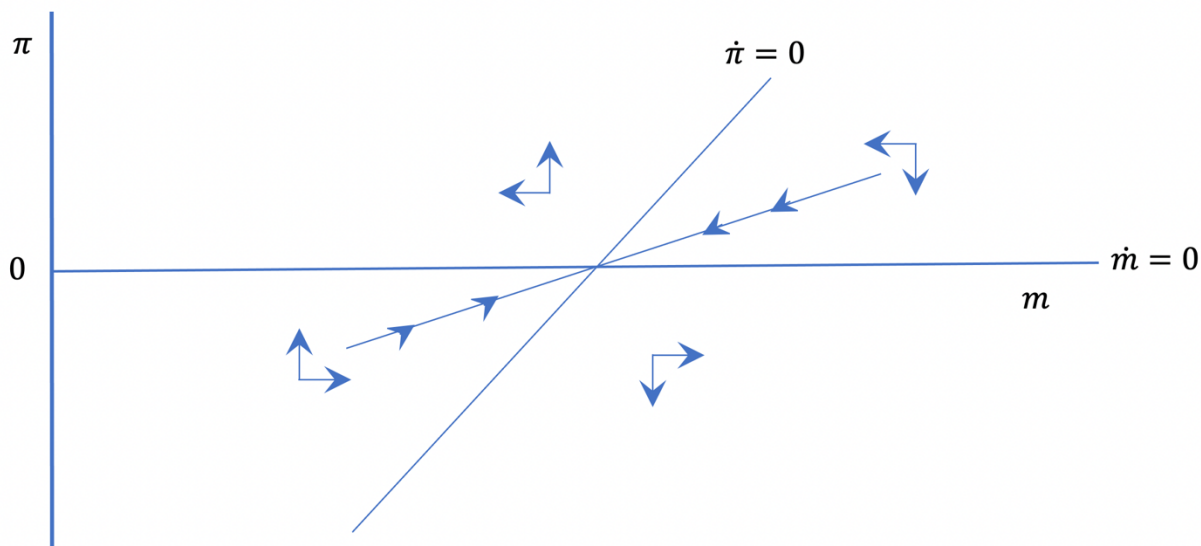


Figure 2

III. The effects of fiscal policy shocks

How does the economy react when the fiscal authority carries out transfers to citizens financed by issuing zero coupon bonds? Contrary to conventional wisdom, such transfers can be neutral or contractionary. But the devil is in the details. Let us see how.

Unanticipated and permanent helicopter drop of bonds

First, consider the effects of an unexpected and permanent increase in the supply of government bonds. Start from steady state and the expectation of $i_t^m = \mu_t = 0$ for all $t \geq 0$ and $\zeta_t = 0$ for all $t > 0$, and assume z rises from \bar{z} to $\bar{z}' > \bar{z}$. Note that what increases at time 0 is the nominal stock of bonds, but since the price level is predetermined, real bonds rise by the same amount. Now, steady state demand for bonds has not changed, so the higher supply prompts a lower price: \bar{q} falls to \bar{q}' , and $\bar{q} + \bar{z} = \bar{q}' + \bar{z}'$ remains the same.

In short: an unexpected and permanent helicopter drop of zero-coupon bonds causes a drop in their price, with no other effects.

Anticipated and permanent helicopter drop of bonds

Fiscal policies are typically announced before they are implemented. So in the interest of realism, consider next an anticipated and permanent helicopter drop of bonds.

Suppose at time 0 agents expect that nominal money balances will remain constant throughout, and nominal bonds outstanding will too, except for a step increase at time $T > 0$. Because the price level is predetermined, at time T the real stock of bonds outstanding will jump up as well. Assume finally that $i_t^m = \mu_t = 0$ always and that $\zeta_t = 0$ as well except for the discontinuity at T .

To sort out what happens, go back to the asset market equilibrium condition

$$\dot{q}_t = \rho(q_t - \bar{q}) + \rho(z_t - \bar{z}) - \rho(m_t - \bar{m}) \quad (13)$$

where we have set $i_t^m = 0$. In the time interval $[0, T)$ the difference $(z_t - \bar{z}) - (m_t - \bar{m}) = 0$ is unchanged, since that difference depends only on the nominal stocks of money and bonds, and those nominal stocks are fixed in that interval. We therefore have

$$\dot{q}_t = \rho(q_t - \bar{q}) \quad (14)$$

Because this differential equation is unstable, at time T the price of bonds must be at its steady state level —otherwise it would not converge. Moreover, that price cannot jump down at time T , because that would inflict a capital loss on bondholders, and arbitrage should prevent that. So it must be the case that $q_T + \bar{z}' = \bar{q}' + \bar{z}'$, where \bar{z}' is the post-shock stock of bonds outstanding and \bar{q}' is the new steady state price of bonds. It follows that q_t must jump down at time 0 and then fall gradually between 0 and T so that this last equation will hold exactly at time T . So it is a terminal condition —the requirement that q_t be at a lower level at T — that drives dynamics.

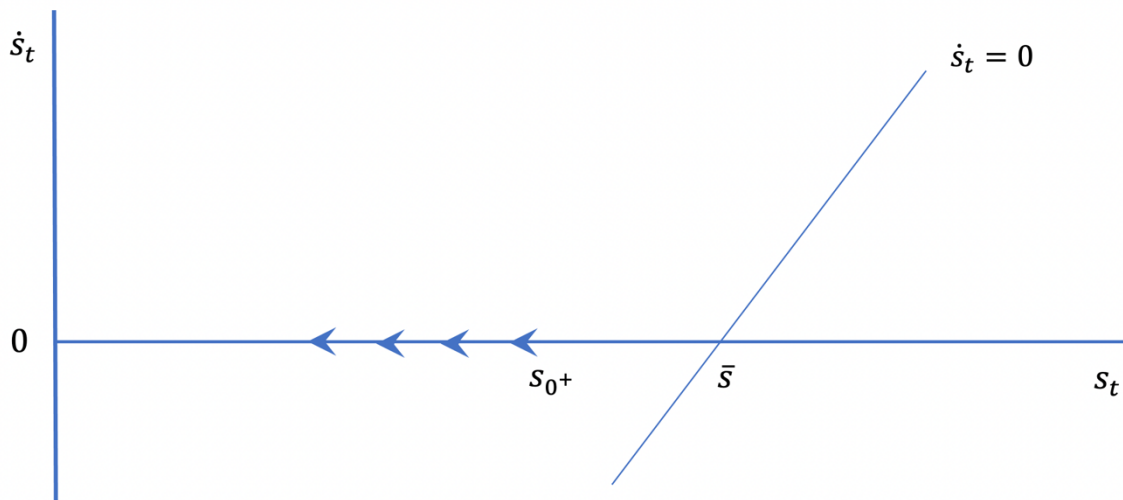


Figure 3

Recalling the definition of s_t , and given the trajectory of q_t , it follows that s_t must also drop on impact at time 0, and then continue to decline until it jumps back to its initial steady state at time T . Of course, the jump at T occurs not because of a movement in q_t , but because the nominal stock of bonds jumps at that time.

To see what happens to inflation and real balances, turn to the other phase diagram. If $s_t < \bar{s}$, the $\dot{\pi}_t = 0$ schedule shifts to the right. On impact, it goes from $\dot{\pi}_t = 0$ to $\dot{\pi}'_t = 0$. Thereafter it keeps shifting farther to the right, until it jumps back to its original position at time T . But that does not change the qualitative conclusion that between times 0 and the instant before T the system is guided by the dynamics emanating from a steady state such as point B .

So at time 0 the system drops from A to D . Thereafter it follows the arrows corresponding to the steady state at B , until at time T it reaches the saddle path that takes to the steady state at A .

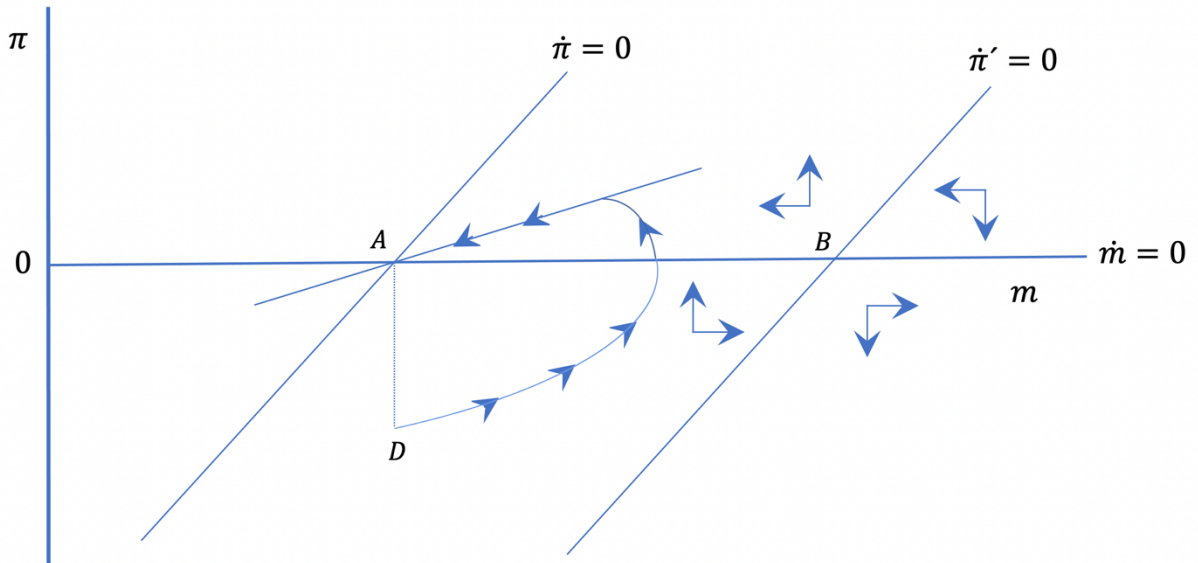


Figure 4

Between the time of the shock and the moment when we economy crosses the horizontal axis, inflation is negative but rising. The Calvo equation in its original form can be written as

$$\kappa(y_t - \bar{y}) = \rho\pi_t - \dot{\pi}_t \quad (15)$$

So if $\pi_t < 0$ and $\dot{\pi}_t > 0$, it follows that $y_t < \bar{y}$. Unambiguously, the economy is in a recession. Once the system crosses the horizontal axis, there follows a period in which $\pi_t > 0$ and $\dot{\pi}_t > 0$, so the output gap can be positive or negative. But after time T , as the economy travels along the saddle path, $\pi_t > 0$ and $\dot{\pi}_t < 0$, so that $y_t > \bar{y}$ without ambiguity. A naïve observer, noting that the economy enters a boom just as government drops begins to drop bonds on citizens, could conclude that the helicopter drop is expansionary. But of course that would neglect the fact that the expectation of the drop previously caused a recession!

Unanticipated and transitory increase in bond issues

Next, and also in the interest of realism, consider the effects of an unanticipated and transitory increase in ζ_t . Starting at time 0 the fiscal authority issues bonds at a constant speed for an interval of time of length T , and then stops:

$$\zeta_t = \begin{cases} \zeta > 0 & \text{for } 0 \leq t < T \\ 0 & \text{for } t \geq T \end{cases} \quad (16)$$

To figure out what happens, it helps to begin by asking what is the trajectory of s_t . If $i_t^m = \mu_t = 0$ always, which we assume, equation (9), which guides the evolution of s_t , can be written as

$$\dot{s}_t = \rho \left[s_t - \left(\bar{s} - \frac{\zeta}{\rho} \right) \right] \quad (17)$$

Between 0 and T the $\dot{s}_t = 0$ schedule shifts to the left, as depicted below. It follows that s_t drops on impact and then gradually rises, so that it can be back at its initial steady state at time T .

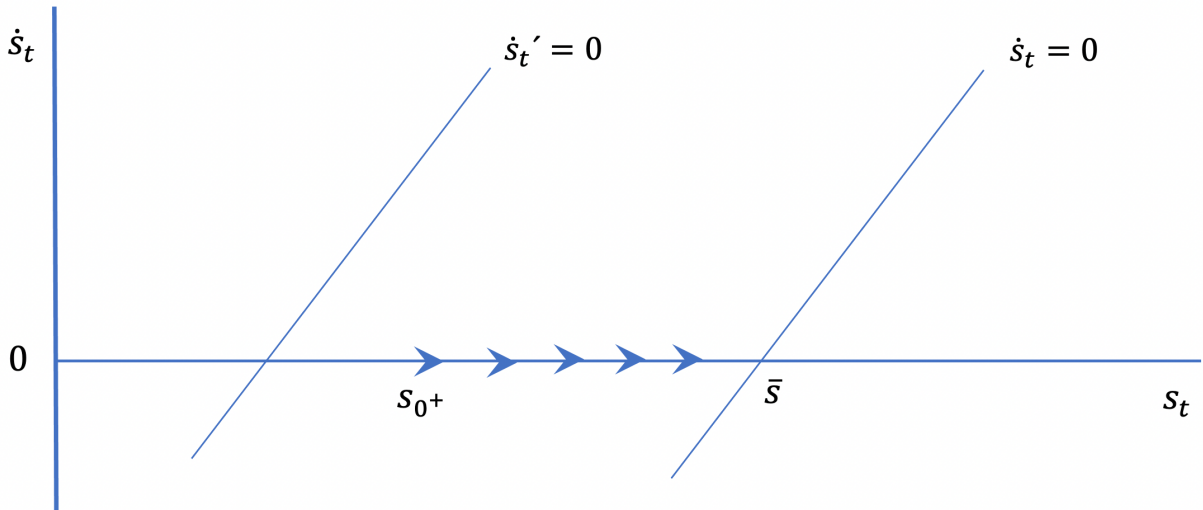


Figure 5

What happens to inflation, real money balances and output? Again we can infer this from the phase diagram with inflation and money balances. Between times 0 and T , $s_t < \bar{s}$. The shock again causes the $\dot{\pi}_t = 0$ schedule to shift to the right. On impact, it goes from $\dot{\pi}_t = 0$ to $\dot{\pi}_t' = 0$. The difference with the previous exercise is that now between times 0 and T it gradually shifts back to the left, until it is in its original position at time T . Still, between times 0 and the instant before T the system is guided by the dynamics emanating from a steady state such as that at B .

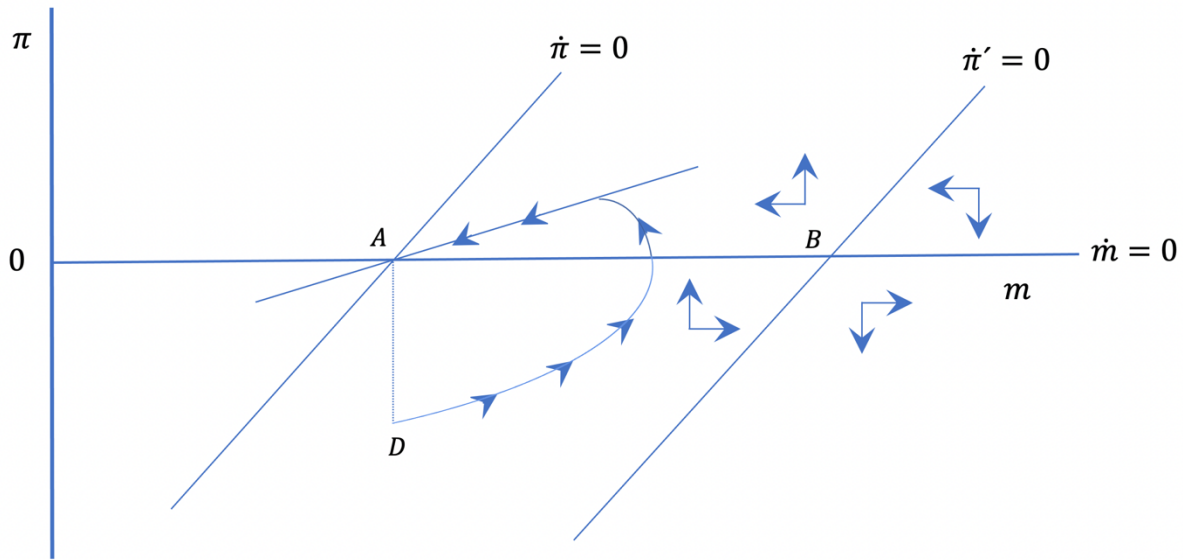


Figure 6

So at time 0 the system drops from A to point D. Thereafter it follows the arrows corresponding to the steady state at B, until at time T it reaches the saddle path that takes it back to point A. Again the economy goes through an initial recession. After the fiscal expansion has ended, the economy will (without ambiguity) be in a boom, with inflation positive (but falling) and output above its natural rate.

A naïve observer could once more be misled by this temporal sequence of events. For instance, an opponent of bond-financed fiscal expansion could argue that now that the helicopter drops have ended, “confidence is back” and “growth can resume”. That of course is the wrong conclusion. Our analysis shows that the boom in the closing part of the cycle is part-and-parcel of the earlier recession. There is no confidence regained, since fiscal sustainability is not the issue here. Rather, it is the drop in the price of bonds that is doing all the work here.

The intuition for these contractionary results follows from the behavior of q_t , the price of the zero-coupon bonds. Both the once-and-for-all anticipated helicopter drop and the temporary-but-sustained helicopter drop of bonds cause a fall in the money price of those bonds. As a result, the ratio of bond-liquidity to money-liquidity also falls (s_t goes down temporarily).

Total liquidity can be written as

$$\ell_t - \bar{\ell} = (m_t - \bar{m}) + (1 - \alpha)(s_t - \bar{s}) \quad (18)$$

We know $m_t - \bar{m}$ is fixed on impact, and $s_t - \bar{s}$ drops in both cases. It follows that of total liquidity drops, and because of the “liquidity-in-advance” constraint, so must aggregate demand and output. The resulting deflation is the only way the real value of liquidity can eventually be rebuilt. But it is a gradual and painful process, in that it involves a protracted recession.

Unanticipated and temporary helicopter drop of bonds

Lest readers conclude that helicopter drops of bonds are always contractionary, finally consider an unanticipated but temporary step increase in zero coupon bonds. At time 0 agents learn that nominal bond supply will rise immediately and remain at the new higher level until $T > 0$. At that time, nominal bond supply will return to its initial level.

Recalling the definition of s_t it is easy to see that, whatever happens to that variable on impact and in the short run, it must be back to its initial steady state level at T , since relative demands for money and bonds will not have changed. On impact m_t is unchanged and z_t jumps up (the nominal stock of bonds rises while the price level is predetermined).

What happens to q_t ? Intuition suggests that q_t should fall: the supply of bonds is going up, so their price should go down. The key observation is that the impact drop in q_t must be less than the impact upward jump up in z_t , for only in that way will s_t continue to rise after time 0 and find itself at a level such that it can jump back down to its initial steady state level at T —the time when the step reduction in nominal (and real) bond supply will take place.⁹

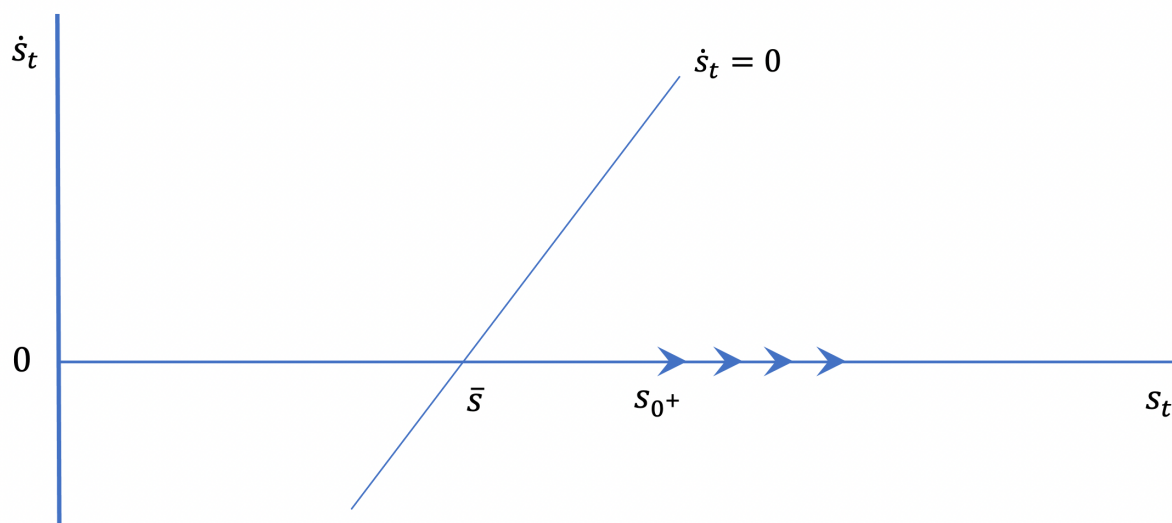


Figure 7

What happens to inflation, real money balances and output? We can describe the dynamics of the 2x2 system using the phase diagram below. If $s_t > \bar{s}$, the $\dot{\pi}_t = 0$ schedule shifts to the left. On impact, it goes from $\dot{\pi}_t = 0$ to $\dot{\pi}_t' = 0$. Then it keeps shifting farther to the left, until it jumps back to its original position at time T . Between times 0 and the instant before T , the system is guided by the dynamics emanating from a steady state such as point B .

⁹ Notice also that the price of bonds cannot jump at T , because that would amount to a huge capital gain on bonds that will naturally be arbitrated away.

So at time 0 the system jumps from A to D. Thereafter it follows the arrows corresponding to the steady state at B, until at T it reaches the saddle path that takes it back to the steady state at A.

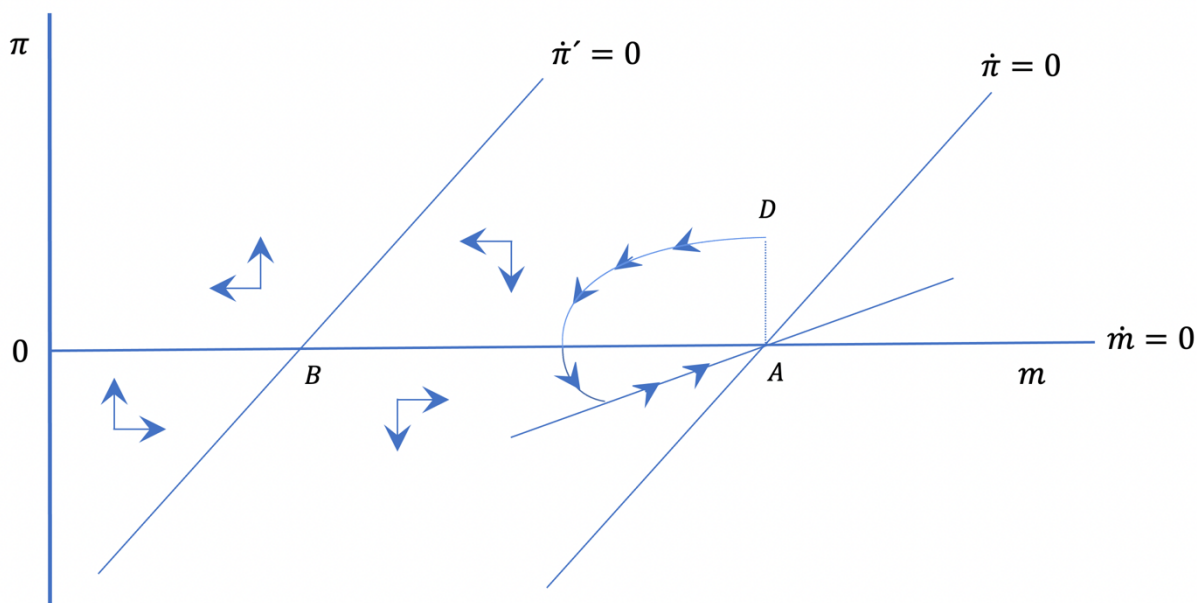


Figure 8

Between the time of the shock and the moment when the system crosses the horizontal axis, inflation is positive but falling. By arguments analogous to those used before, if $\pi_t > 0$ and $\dot{\pi}_t < 0$, it follows that $y_t > \bar{y}$. The economy is in a boom. Once the system crosses the horizontal axis, there follows a period in which $\pi_t < 0$ and $\dot{\pi}_t < 0$, so the output gap can have either sign. But after time T , as the economy travels along the saddle path, $\pi_t < 0$ and $\dot{\pi}_t > 0$, so that $y_t < \bar{y}$.

The temporary helicopter drop of government bonds does produce an output boom —but at the cost of an eventual recession. The recession arrives around the time the bond supply expansion is to be undone, which could trigger calls for the continuation of the “expansionary” fiscal policy.

Of course, this policy exercise is not very realistic, because governments that issue a large stock of bonds one year do not typically retire every last penny worth of those bonds T periods later. On the contrary, bond supply expansions tend to be, if not permanent, at least very long-lived.

The intuition as to why this kind of helicopter drop induces an initial boom, while the others caused initial recessions, has to do with the availability of liquidity. In the first two cases, the increase in bond supply is either eventual (at time T) or gradual. Since the pricing of bonds is forward-looking, the price jumps down right away. It follows that total available bond liquidity (quantity times price) goes down —and so do total liquidity, aggregate demand and output. In the contractionary case, this logic is reversed. Supply of bonds rises today. But because that increase is expected to be undone at time T , the drop in bond prices is less than proportional. So total available bond liquidity goes up, and so do aggregate demand and output.

IV. What is to be done?

Monetary and fiscal authorities do have the tools at their disposal to avoid unwanted effects on inflation and output from the different kinds of helicopter drops of bonds. But what needs to be done depends on the details of the specific policy shock. For simplicity and brevity, we focus only on those fiscal policies that cause an initial recession.

One possible monetary policy

Start from the anticipated future step increase in bond-holdings. Can the recessionary effects of that fiscal expansion be avoided? A monetary policy that achieves such an aim must satisfy three requirements.

First, given the liquidity-in-advance constraint, for output and inflation to be at their steady state levels between times 0 and T , liquidity must be constant throughout.

Second, during that time interval the price q_t has to decline, so that it will be at its new steady state level at time T .

Third, since q_t will fall, something else must rise to ensure the constancy of liquidity. By the very nature of this exercise, bond-holdings z cannot move. That leaves money balances as the only candidate. And since inflation needs to be zero, real balances can change only if nominal balances are changing.

These three requirements immediately rule out a standard Taylor rule, in which the interest responds to deviations of inflation and output from their steady state levels, as an effective tool in achieving the desired goal. We will see below that manipulating the interest rate on money changes the speed with which the price q_t falls. But fall it must, and that means ceteris paribus liquidity will be contracting. Unless nominal and real balances are increased, of course, but that increase cannot be achieved via a Taylor rule alone. Additional QE-like policies, which target the quantity of money (or central bank reserves, if you prefer) are needed.

Recall the asset market equilibrium condition that governs the evolution of q_t :

$$\dot{q}_t = i_t^m + \rho(q_t - \bar{q}) + \rho(z_t - \bar{z}) - \rho(m_t - \bar{m}) \quad (19)$$

If in addition we require that liquidity be constant at its steady state level, and given that $(z_t - \bar{z}) = 0$ throughout, this becomes

$$\dot{q}_t = i_t^m + \frac{\rho}{\alpha}(q_t - \bar{q}) \quad (20)$$

So the evolution of bond prices depends on the interest rate on money and on bond prices themselves. It is easy to check that the higher the interest rate on money, the larger the initial drop in q_t . If the central bank wanted to prevent a crash in bond prices at time 0, it could move the interest rate to prevent that. But notice that interest rate would have to be negative! Otherwise q_t would not begin falling at time 0, when $q_0 = \bar{q}$.

Countries like Sweden, Switzerland and Denmark have paid negative rates on excess bank reserves, so this is not out of the question. But there are limits on how negative that rate can be. To avoid wading into that controversial terrain, here we simply adopt the criterion of minimizing the initial drop subject to the zero lower bound on the nominal interest rate. So, for simplicity, we assume $i_t^m = 0$ throughout. Given this assumption and the terminal condition $q_T = \bar{q}'$, there is only one initial q_t —call it q_{0+} —that satisfies the solution to differential equation (20).

Given that the price of bonds will fall on impact, to keep total liquidity constant the central bank must engineer a step increase in nominal and real balances, equal to

$$m_{0+} - \bar{m} = -\left(\frac{1-\alpha}{\alpha}\right)(q_{0+} - \bar{q}) \quad (21)$$

In addition, and given q_t will also fall gradually after time 0, constancy of liquidity is ensured if

$$\dot{m}_t = \mu_t = -\left(\frac{1-\alpha}{\alpha}\right)\dot{q}_t \quad (22)$$

So nominal balances have to keep rising to keep liquidity constant. Combining equation (20) (with $i_t^m = 0$) and (22) to eliminate \dot{q}_t yields

$$\mu_t = -\left(\frac{1-\alpha}{\alpha}\right)\frac{\rho}{\alpha}(q_t - \bar{q}) > 0 \quad (23)$$

which is the policy rule. It has a clear interpretation: given that $q_t < \bar{q}$ in $t \in (0, T)$, $\mu_t > 0$ during that time period. Again, this behavior of money follows from the need to keep liquidity constant.

Now, the instant before time T real money balances will be above their steady state level. Since the price level is predetermined and cannot jump, the only way to mop up this excess money is for the central bank to buy it through an open market operation involving the “pure” bond, since the path of zero-coupon bonds is pinned down by the initial policy announcement. At time T the step increase in bond supply will also take place, so the ratio of bonds to money will rise

In short: there is a monetary policy that can avoid unwanted fluctuations in aggregate demand, output and inflation following the announcement of a future helicopter drop in government bonds. That policy involves keeping the interest rate on money at zero during the transition and gradually increasing the nominal money supply during that time interval— followed by a step decrease in nominal money just as zero-coupon bond supply takes a step increase at T .

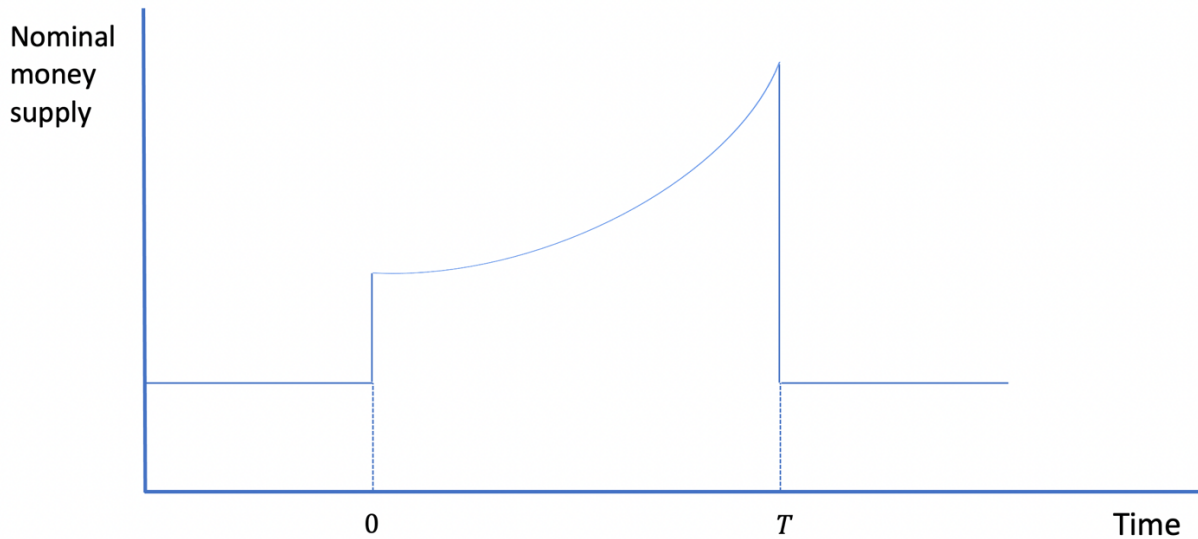


Figure 9

This monetary policy bears more than a passing resemblance to what we have seen in practice, and across many central banks, during the Great Financial Crisis and more recently during the Covid-19 crisis: ultra-low interest rates accompanied by QE operations that cause the money supply to expand, followed by an eventual “normalization” that undoes the previous monetary expansion. In spite of what looks like a very aggressive monetary policy, inflation is zero!

An alternative monetary policy

Consider, finally, the temporary increase in ζ_t , the rate of nominal bond issuance. Recall that this policy also induces an initial recession, followed by a boom. Can monetary policy prevent these fluctuations in output and inflation?

Again there is a change in the steady state level of the bond price q_t . Between times 0 and T , the nominal stock of zero coupon bonds is growing (by assumption). If inflation is zero, as required to keep output constant, the real stock of bonds must be going up as well. Since the price of bonds cannot jump at any time after time 0, it follows that at T the price of bonds must be such that the value of that stock of bonds is exactly equal to that in the original steady state. And that price is $q_T = \bar{q}' < \bar{q}$, meaning the price of bonds must be falling between 0 and T .

Keeping liquidity constant during that time period requires

$$\dot{m}_t = \mu_t = -\left(\frac{1-\alpha}{\alpha}\right)\zeta - \left(\frac{1-\alpha}{\alpha}\right)\dot{q}_t \quad (24)$$

So in principle real balances could be going up or down. Which will it be?

Consider a constant setting for i^m so that between 0 and T , the evolution of q_t is given by

$$\dot{q}_t = i^m + \frac{\rho}{\alpha}(q_t - \bar{q}) + \frac{\rho}{\alpha}(z_t - \bar{z}) \quad (25)$$

Again, the higher the interest rate on money, the larger the initial drop in q_t . Here too we adopt the criterion of minimizing the initial drop subject to the zero lower bound on the interest rate. Once more this implies keeping $i^m = 0$ between 0 and T . It follows that the price of bonds will take a step jump down at time 0, requiring a compensating step up in money balances:

$$m_{0^+} - \bar{m} = -\left(\frac{1-\alpha}{\alpha}\right)(q_{0^+} - \bar{q}) \quad (26)$$

But notice: now there is no step change in the stock of bonds at time T , so there cannot be a step change in money holdings at that time either. If nominal and real balances go up at time 0 to compensate for the drop in the price of bonds, then the stock of real money must decline in the transition, so that it will be back at its steady state level at time T . Combining (24) and (25) with $i^m = 0$ to eliminate \dot{q}_t yields the policy rule for the central bank:

$$\mu_t = -\left(\frac{1-\alpha}{\alpha}\right)\left[\zeta + \frac{\rho}{\alpha}(q_t - \bar{q}) + \frac{\rho}{\alpha}(z_t - \bar{z})\right] \quad (27)$$

It must be the case that $\mu_t < 0$ (money must be falling in the transition, after the initial increase), which in turn requires the total value of bonds to be rising. Now, the (real) stock of bonds will be going up (since ζ is positive and inflation is zero) but the price of bonds will be falling. So the requirement is that the price should not fall too quickly, which in turn depends on several parameters, among them the length T of the shock.

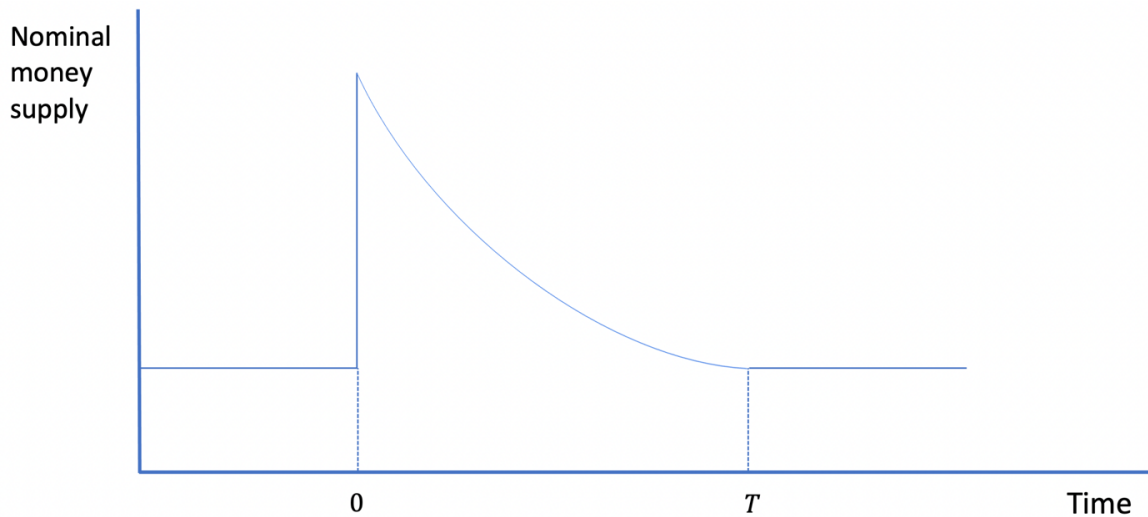


Figure 10

Again the joined-at-the-hip logic is at work. The monetary policy that prevents fluctuations in inflation and output associated with sustained helicopter drops of government bonds involves an initial step increase in money, keeping interest rates on money as low as possible (at zero), and gradually mopping up that newly created real balances, so as to keep liquidity constant.

V. Concluding remarks

Because money is the unit of account, the price of money is the inverse of the price level. If prices are sticky, so is the price of money in terms of goods. And, as Keynes emphasized in what Calvo (2012) has labelled the “price theory of money”, this is an important reason why money is attractive. By contrast, the price of government bonds is free to jump and often does, especially in response to news about current and future changes in fiscal policy and the supply of bonds.

Those movements in government bond prices affect the liquidity available to agents, and can affect (in our simple model, *does* affect) aggregate demand, inflation and output. To avoid such impacts, expected changes in bond supply must be matched by expected changes in money supply and in the interest rate on money. We conclude that in a liquidity-dependent world, fiscal and monetary policies are *joined at the hip*.

One special feature of our analysis is that the only fiscal policies we have considered consist of transfers to households, whether bond- or money-financed. Of course, not all fiscal policies consist just of transfers. Government consumption and investment directly increase aggregate demand. If those expenditures are financed via bonds, the larger bond supply would still affect bond prices, and potentially affect overall liquidity and consumption as well.

So the effects we have emphasized in this paper would still be present, but they would push in the opposite direction than would direct government expenditure and its conventional effects. Which would dominate and what would happen to aggregate demand would then be an empirical question, dependent on parameter values and the composition of public expenditure.

The analysis presented in this paper has relevance for current debates about the effectiveness of fiscal stimulus and, whether the Biden administration’s \$1.9 trillion fiscal stimulus, coming on the heels of other large stimulus packages carried out by the Trump White House, is too large and will cause overheating and the return of inflation.

Of course the model we have developed here is far too stylized for detailed policy analysis, but a theme that emerges is indeed relevant: what matters is the joint stance of fiscal and monetary policy — in particular not only the “low for long” announced path for interest rates, but also the perceived commitment (whether illusory or real) on the part of the Federal Reserve to stabilize asset prices in general, and the price of Treasuries in particular. Our model suggests that without that perceived commitment, a policy of fiscal transfers to households need not be expansionary, and in some circumstances could even be contractionary.

We have illustrated the implications of “jumpy” bond prices (in contrast to “sticky” money prices) by focusing on current fiscal and monetary policy debates. But the point is more general. Keynesian macroeconomics is built upon the very realistic premise of nominal wage and price stickiness, which in turn is what endows monetary policies with real effects. In a simple world in which money and liquidity are synonymous (so that there is only one source of transactional liquidity), and in which goods prices are sticky in terms of money, by controlling the money supply (or the cost of holding money), the authorities enjoy a great deal of control over the economy.

But in a more complex and realistic world in which other assets like government bonds also provide transactional liquidity services and, crucially, in which goods prices are not sticky in terms of the prices of those other assets, financial and macro dynamics can be quite different to what conventional, off-the-shelf models suggest.

Calvo (2021) shows that, without further assumptions, barter and other (dis)equilibrium solutions cannot be ruled out. Barter, for instance, can be shown to be an equilibrium solution unless someone (e.g., the central bank) guarantees a positive price for the zero-coupon bond. Moreover, depending on their resilience, government bonds could help to generate full employment while money and government bond markets exhibit excess demand or supply – giving rise to multiple disequilibrium solutions, in addition to the neo-Keynesian configuration in this paper. These observations suggest that the existence of more than one transactional liquid asset opens up a rich set of still largely unexplored avenues for future research.

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Appendix

The representative individual maximizes the objective function

$$U = \int_0^{\infty} \log C_t e^{-\rho t} dt \quad (\text{A1})$$

where C_t is consumption and ρ the subjective rate of discount, subject to the budget constraint

$$\dot{B}_t + Q_t \dot{Z}_t + \dot{M}_t = Y_t + T_t + (i_t - \pi_t)B_t + i_t^m M_t - C_t, \quad (\text{A2})$$

where Z_t is the real stock of zero-coupon bonds, Q_t is the money price of those bonds, M_t is the real stock of money held, i_t is the nominal rate of interest on a “pure” government bond, i_t^m is the interest rate paid on money, π_t is the rate of inflation, Y_t is output (all of which accrues to the household) and T_t is real transfers from the government.

If we define total real asset-holdings as $A_t = B_t + Q_t Z_t + M_t$, (A2) can be written as

$$\dot{A}_t = Y_t + T_t + (i_t - \pi_t)A_t - \left(i_t - \frac{\dot{Q}_t}{Q_t}\right)Q_t Z_t - (i_t - i_t^m)M_t - C_t, \quad (\text{A3})$$

The government budget constraint is

$$\dot{B}_t + Q_t \dot{Z}_t + \dot{M}_t = T_t + (i_t - \pi_t)B_t - \pi_t Q_t Z_t + (i_t^m - \pi_t)M_t \quad (\text{A4})$$

Consolidating the representative household’s budget constraint and the government’s budget constraint we obtain $C_t = Y_t$, as should be in a closed economy with no investment.

Maximization by the representative individual is also subject to the liquidity constraint

$$C_t \leq L_t \quad (\text{A5})$$

where liquidity is defined as

$$L_t = \frac{M_t^\alpha (Q_t Z_t)^{1-\alpha}}{\alpha^\alpha (1-\alpha)^{(1-\alpha)}} \quad (\text{A6})$$

The Hamiltonian for the household’s problem is

$$\mathcal{H} = \log C_t + \lambda_t \left[Y_t + T_t + (i_t - \pi_t)A_t - \left(i_t - \frac{\dot{Q}_t}{Q_t}\right)Q_t Z_t - (i_t - i_t^m)M_t - C_t \right] + \lambda_t \gamma_t (L_t - C_t) \quad (\text{A7})$$

where λ_t is the co-state associated with state variable λ_t and γ_t is the multiplier associated with the liquidity-in-advance constraint.

First-order conditions are

$$\frac{1}{C_t} = \lambda_t(1 + \gamma_t) \quad (\text{A8})$$

$$\left(i_t - \frac{\dot{Q}_t}{Q_t}\right) Q_t Z_t = \gamma_t(1 - \alpha)L_t \quad (\text{A9})$$

$$(i_t - i_t^m)M_t = \gamma_t \alpha L_t \quad (\text{A10})$$

$$\dot{\lambda}_t = -\lambda_t(i_t - \pi_t - \rho) \quad (\text{A11})$$

Taking logs of (A8) yields

$$-\log C_t = \log \lambda_t + \log(1 + \gamma_t) \cong \log \lambda_t + \gamma_t \quad (\text{A12})$$

Taking time derivatives (small case variables denote logs) and combining with the last first-order condition we have the Euler condition for this problem:

$$\dot{c}_t = (i_t - \pi_t - \rho) - \dot{\gamma}_t \quad (\text{A3})$$

The last term on the RHS is non-standard, and it comes from the liquidity-in-advance constraint (see Reis, 2017). Given the existence of a (binding) liquidity constraint and a policy regime that targets the interest rate paid on money and the quantity of money (or of central bank reserves, if you prefer), the Euler equation does not play any role in the determination of output, inflation or asset prices. Once these variables have been determined, the Euler equation pins down the path of the interest rate on the “pure” bond that does not yield liquidity services.

Next, combine (A9) and (A10) to obtain

$$\alpha \left(i_t - \frac{\dot{Q}_t}{Q_t}\right) Q_t Z_t = (1 - \alpha)(i_t - i_t^m)M_t \quad (\text{A14})$$

Log-linearizing around the steady state (overbars denote steady state levels) we obtain

$$\rho(m_t - \bar{m}) - \rho(q_t - \bar{q}) - \rho(z_t - \bar{z}) = i_t^m - \dot{q}_t \quad (\text{A15})$$

which is the expression that appears in the text as (3).

Next, take (A9) and (A10) and use them in the definition of liquidity

$$L_t = \frac{1}{\alpha^\alpha (1-\alpha)^{(1-\alpha)}} \frac{\gamma_t^\alpha \alpha^\alpha L_t^\alpha}{(i_t - i_t^m)^\alpha} \frac{\gamma_t^{1-\alpha} (1-\alpha)^{1-\alpha} L_t^{1-\alpha}}{\left(i_t - \frac{\dot{Q}_t}{Q_t}\right)^{1-\alpha}} \quad (\text{A6})$$

This yields

$$\gamma_t = (i_t - i_t^m)^\alpha \left(i_t - \frac{\dot{Q}_t}{Q_t}\right)^{1-\alpha} \quad (\text{A17})$$

so that

$$\bar{\gamma} = \rho^\alpha \rho^{1-\alpha} = \rho \quad (\text{A18})$$

Log-linearizing around the steady state yields

$$(\gamma_t - \bar{\gamma}) = \alpha(i_t - i_t^m - \rho) + (1-\alpha)(i_t - \dot{q}_t - \rho) \quad (\text{A19})$$

which also appears in the text, this time as (4).

In logs, the equations of motion for money and bonds are

$$\dot{m}_t = \mu_t - \pi_t \quad (\text{A20})$$

$$\dot{z}_t = \zeta_t - \pi_t \quad (\text{A21})$$

which appear in the text as (4) and (5), and where μ_t is nominal money growth and ζ_t is nominal bond growth. Adding these two to asset market equilibrium condition (A15) yields

$$\dot{q}_t + \dot{z}_t - \dot{m}_t = i_t^m + \zeta_t - \mu_t + \rho(q_t - \bar{q}) + \rho(z_t - \bar{z}) - \rho(m_t - \bar{m}) \quad (\text{A22})$$

Define

$$(s_t - \bar{s}) \equiv (q_t - \bar{q}) + (z_t - \bar{z}) - (m_t - \bar{m}) \quad (\text{A23})$$

Given this definition, differential equation (A22) becomes

$$\dot{s}_t = \rho(s_t - \bar{s}) + i_t^m + \zeta_t - \mu_t \quad (\text{A3})$$

which appears as (9) in the text.

In log deviations from steady state, liquidity ℓ_t is

$$\ell_t - \bar{\ell} \equiv \alpha(m_t - \bar{m}) + (1-\alpha)(q_t - \bar{q}) + (1-\alpha)(z_t - \bar{z}) \quad (\text{A4})$$

which is equation (1) in the text.

Given that in a closed economy with no investment and no government consumption market clearing requires $y_t - \bar{y} = c_t - \bar{c}$ always, the liquidity-in-advance constraint is

$$y_t - \bar{y} \leq \ell_t - \bar{\ell} \quad (\text{A5})$$

which always binds. Bringing these two together, assuming the constraint binds and using the definition of $(s_t - \bar{s})$ we have

$$y_t - \bar{y} = (m_t - \bar{m}) + (1 - \alpha)(s_t - \bar{s}) \quad (\text{A27})$$

Next recall the Calvo-Phillips equation (in the text it is equation (6)):

$$\dot{\pi}_t = \rho\pi_t - \kappa(y_t - \bar{y}) \quad (\text{A28})$$

Combining (A27) and (A28) yields

$$\dot{\pi}_t = \rho\pi_t - \kappa(m_t - \bar{m}) - \kappa(1 - \alpha)(s_t - \bar{s}) \quad (\text{A29})$$

which appears as (10) in the text.

Equations (A20) and (A29) are a 2x2 system of differential equations which in matrix form is

$$\begin{bmatrix} \dot{\pi}_t \\ \dot{m}_t \end{bmatrix} = \Omega \begin{bmatrix} \pi_t \\ m_t \end{bmatrix} + \begin{bmatrix} \kappa\bar{m} - \kappa(1 - \alpha)(s_t - \bar{s}) \\ \mu_t \end{bmatrix} \quad (\text{A30})$$

where

$$\Omega = \begin{bmatrix} \rho & -\kappa \\ -1 & 0 \end{bmatrix} \quad (\text{A31})$$

Therefore $\text{Det}(\Omega) = -\kappa < 0$, and $\text{Tr}(\Omega) = \rho > 0$. It follows that one of the eigenvalues of Ω is positive and the other is negative. Since π_t is a “jumpy” variable and m_t is a state variable, we conclude that the 2x2 system is saddle-path stable, as asserted in the text.