Equilibrium Bitcoin Pricing

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Standard money versus Cryptocurrency

Standard money: created by central and commercial banks

$\Rightarrow$ problem if institutions not trustworthy: inflation, expropriation (Zimbabwe, Venezuela, Turkey … China, financial crisis)

Cryptocurrency: created by internet network nodes, distributed

$\Rightarrow$ “allow online payments to be sent directly from one party to another without going through a financial institution” (Nakamoto, 2008) : Bitcoin

Can it work? Does bitcoin have value? Just fad, bubble about to burst?

Distinguished warnings

“Money, is an indispensable social convention backed by an accountable institution within the State that enjoys public trust… Private digital tokens posing as currencies, such as bitcoin and other crypto-assets that have mushroomed of late, must not endanger this trust in the fundamental value and nature of money”

Agustin Carstens, BIS

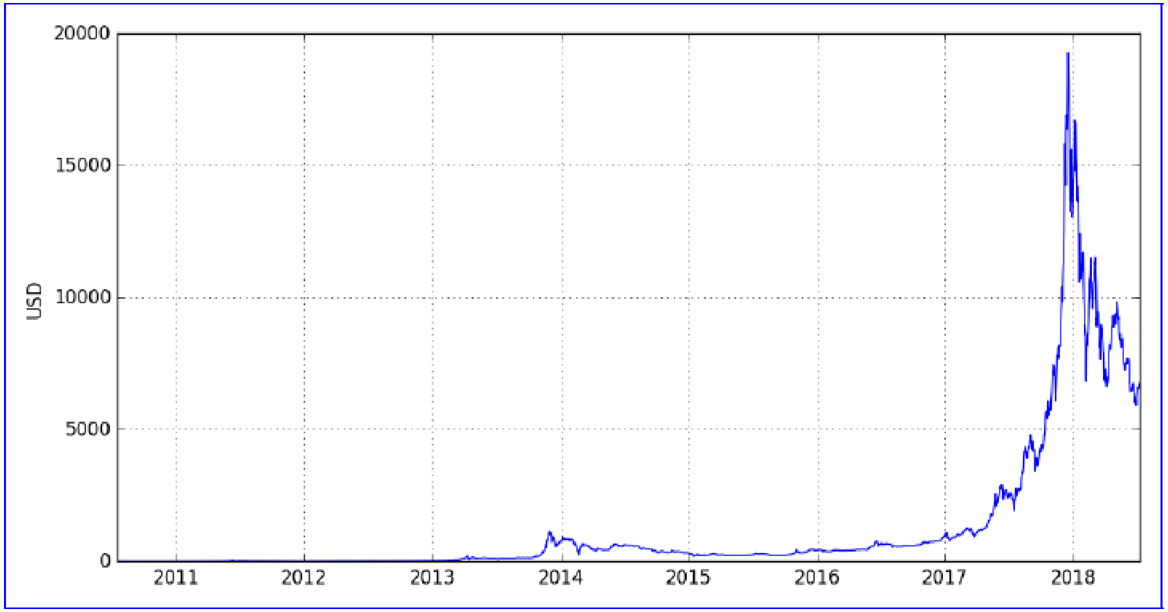
“bitcoin is a pure bubble, an asset without intrinsic value — its price will fall to zero if trust vanishes”

Jean Tirole, TSE

Yet

Market capitalisation of bitcoin today $143 billion

bitcoin < 10 cents in 2010 > 5,000 dollars in 2018



Is this a sign of irrationality or something reasonable consistent with basic economics ?

Our goal is to shed some light on this issue by offering a standard REE model of bitcoin, and confronting it to the data

What is money?

Something I am willing to accept today, in exchange for goods and services, because I anticipate the others will accept it tomorow, when I want to buy goods and services

Simplest model to capture that idea: OLG

At t: young exchange part of endowment in goods for money

At t+1: now old, exchange money for part of endowment of t+1 young

OLG

Young consume endowment, save, buy dollar, and btc incurring transactions cost (φ_t)

$$c_t^y = e_t^y - s_t - q_t p_t - \hat{q}_t \hat{p}_t - \varphi_t(q_t) p_t.$$

p_t = price of bitcoin in consumption goods

Old consume endowment, savings, dollars and btc hoarded: fraction h_{t+1} of btc can be stolen, but btc brings transactional benefits (θ_{t+1})

$$c_{t+1}^o = e_{t+1}^o + s_t(1 + r_t) + (1 - h_{t+1})(1 + \theta_{t+1}) q_t p_{t+1} + \hat{q}_t \hat{p}_{t+1}$$

Similar to Kareken Wallace 1981 – Garratt Wallace 2018 except that we have hack risk (h_{t+1}), and transactions costs (φ_t) & benefits (θ_{t+1})

Transactions costs and benefits

Transactions benefits of using bitcoin: not expropriated/taxed/constrained by government, direct internet access, …

$$1 + \mathcal{T}_{t+1} = \frac{1 + \theta_{t+1}}{1 + \varphi_t'(X_t)}$$

Cost of buying bitcoin with dollars: transactions costs charged by exchanges, miners' fees, …

Equilibrium cryptocurrency price (Euler equation)

$$p_t = \frac{1}{1+r_t} E_t \left(\frac{u'(c^o_{t+1})}{E_t\left[u'(c^o_{t+1})\right]} (1 - h_{t+1}) \left(p_{t+1} + \mathcal{T}_{t+1} p_{t+1}\right) \right)$$

discount

risk neutral proba

hack risk

resale price

transactional net benefit

Similar to Tirole 1985 (OLG model of money) except that:

- we have random shocks
- we spell out hack risk, transactions costs & benefits

Comparison with stock price

$$p_t = \frac{1}{1+r_t} E_t \left(\frac{u'(c^o_{t+1})}{E_t\left[u'(c^o_{t+1})\right]} (1 - h_{t+1}) \left(p_{t+1} + \mathcal{T}_{t+1} p_{t+1}\right) \right)$$

discount | risk neutral proba | hack risk | resale price | transactional net benefit

stock not exposed to hack risk

dividend d_{t+1} instead of transactional benefit T_{t+1} p_{t+1}

dividends = fundamental of stock

transactional benefits = fundamental of currency (Tirole 1985)

Multiplicative structure

$$p_t = \frac{1}{1+r_t} E_t \left(\frac{u'(c^o_{t+1})}{E_t\left[u'(c^o_{t+1})\right]} (1-h_{t+1}) \left(p_{t+1} + \mathcal{T}_{t+1} p_{t+1}\right) \right)$$

Stocks: dividend d_{t+1} not multiplied by price

$\Rightarrow$ dividend anchors price (with something outside price)

Currency: transactional benefit $T_{t+1}\, p_{t+1}$ multiplied by price

$\Rightarrow$ no anchor (cannot anchor price with price…)

Consequences of multiplicative structure: equilibrium multiplicity

$$p_t = \frac{1}{1+r_t} E_t \left(\frac{u'(c^o_{t+1})}{E_t\left[u'(c^o_{t+1})\right]} (1 - h_{t+1}) \left(p_{t+1} + \mathcal{T}_{t+1} p_{t+1}\right) \right)$$

If future price expected high (resp. low), current price is high (resp. low)

$\Rightarrow$ multiple equilibria (Kareken Wallace 1981 ``indeterminacy result'')

Price = 0 is one of many possible equilibria

Consequences of multiplicative structure: Exogenous volatility

$$p_t = \frac{1}{1+r_t} E_t \left(\frac{u'(c^o_{t+1})}{E_t\left[u'(c^o_{t+1})\right]} (1-h_{t+1}) \left(p_{t+1} + \mathcal{T}_{t+1} p_{t+1}\right) \right)$$

In a given equilibrium: exogenous random shocks (sunspots)

(take above equation, suppose it holds until t, it still holds if prices after t multiplied by exogenous iid random variable with mean 1)

$\Rightarrow$ volatility, unrelated to fundamentals

because no real anchor, only beliefs: Shiller 1981 critique does not apply (in REE currency can move lots more than fundamentals)

Similar equation for dollar

$$\hat{p}_t = \frac{1}{1 + r_t} E_t \left(\hat{p}_{t+1} \right)$$

Same as for btc (both are currencies), but no hack risk, no transactions costs, no transactional benefits

Equilibrium bitcoin dollar returns

Dividing btc price by dollar price $\rightarrow$ compute bitcoin dollar return ρ_{t+1}

Assuming risk neutrality, equilibrium required return such that

$$E_t\left[(1-h_{t+1})\frac{1+\theta_{t+1}}{1+\varphi_t'(X_t)}(1+\rho_{t+1})\right]-1=0.$$

$\Rightarrow$ Moment condition used to test model and estimate parameters = costs (φ) and benefits (θ) of holding btc

Interpreting equilibrium required return ρ_{t+1}

If hold 1 dollar today, still worth 1 dollar in 1 week

$$E_t\left[(1-h_{t+1})\frac{1+\theta_{t+1}}{1+\varphi_t'(X_t)}(1+\rho_{t+1})\right]-1=0.$$

If use dollar to buy btc:

fraction *h* of btc can be stolen (hacked)

I have to pay transaction costs φ to trade bitcoin (e.g., fees)

I can use my bitcoin to trade differently than with dollar $\rightarrow$ value θ

First order approximation

$$E_t\left[\rho_{t+1}\right] \approx \varphi_t'(X_t) + E_t(h_{t+1}) - E_t(\theta_{t+1}).$$

Equilibrium required expected return

costs

benefits

Goal of econometric analysis

Test rational expectations equilibrium pricing relation

using observed returns (ρ_{t+1}) and hacks (h_{t+1})

and observable variables to proxy for θ_{t+1} and φ_t

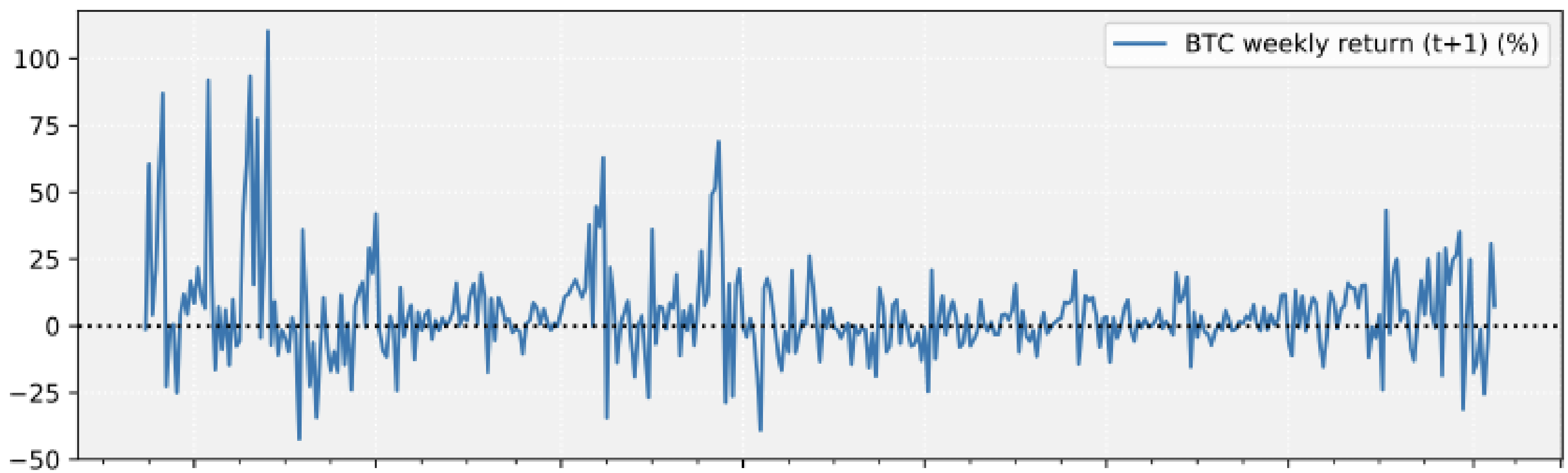
$\Rightarrow$ estimate fundamental value and costs of bitcoin

(relying on Generalised Method of Moments, Hansen 1982)

$\Rightarrow$ Is REE rejected ?

$\Rightarrow$ Is hypothesis that fundamental value significantly positive rejected?

Time series of weekly btc returns “variable to be explained”



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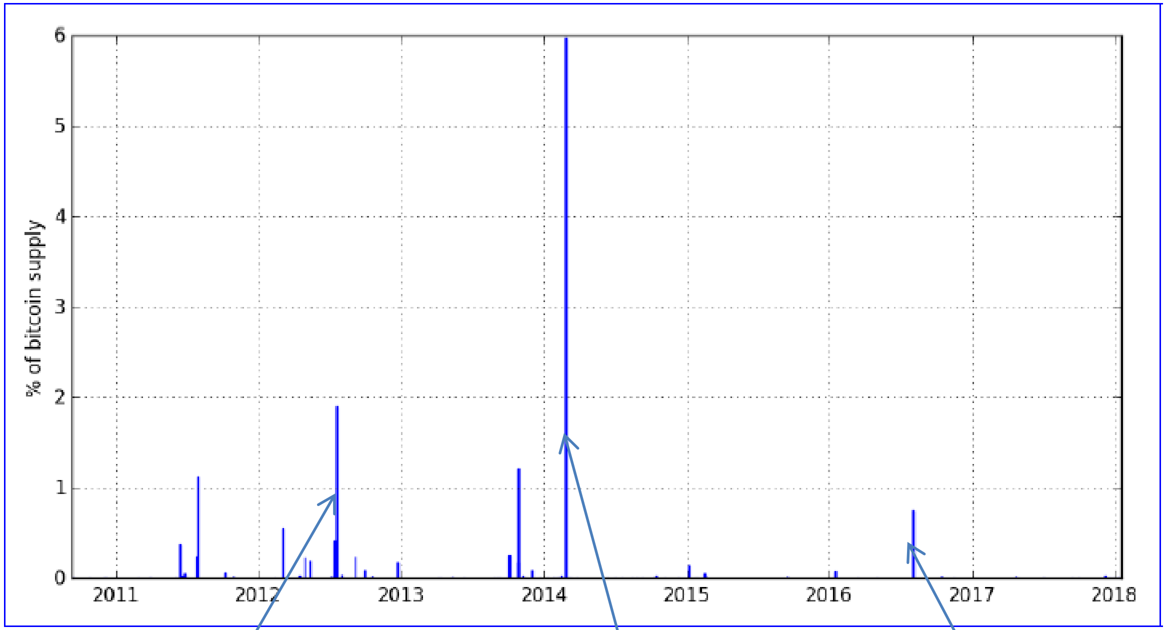
to be confronted to hacks, proxies for transactions costs and benefits

Browsing the net, construct time series of bitcoin thefts/hacks to serve as estimate of *h*

on average 0.04% of btc supply stolen per week
(so hack risk can explain only .04 percentage points of btc required returns)

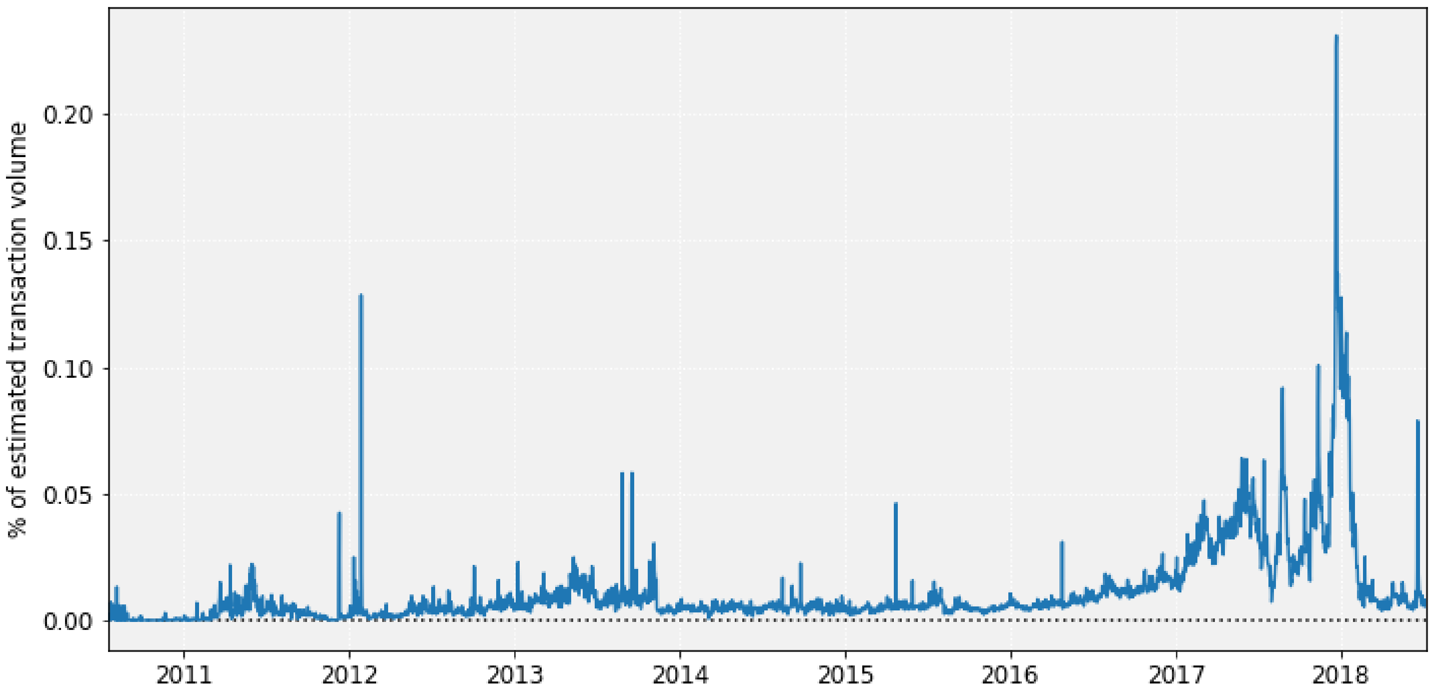


BST Ponzi scheme

MtGox collapse

Bitfinex hack

Download blockchain, construct time series of transaction fees, requested by miners for including transactions in blocks, to proxy for φ



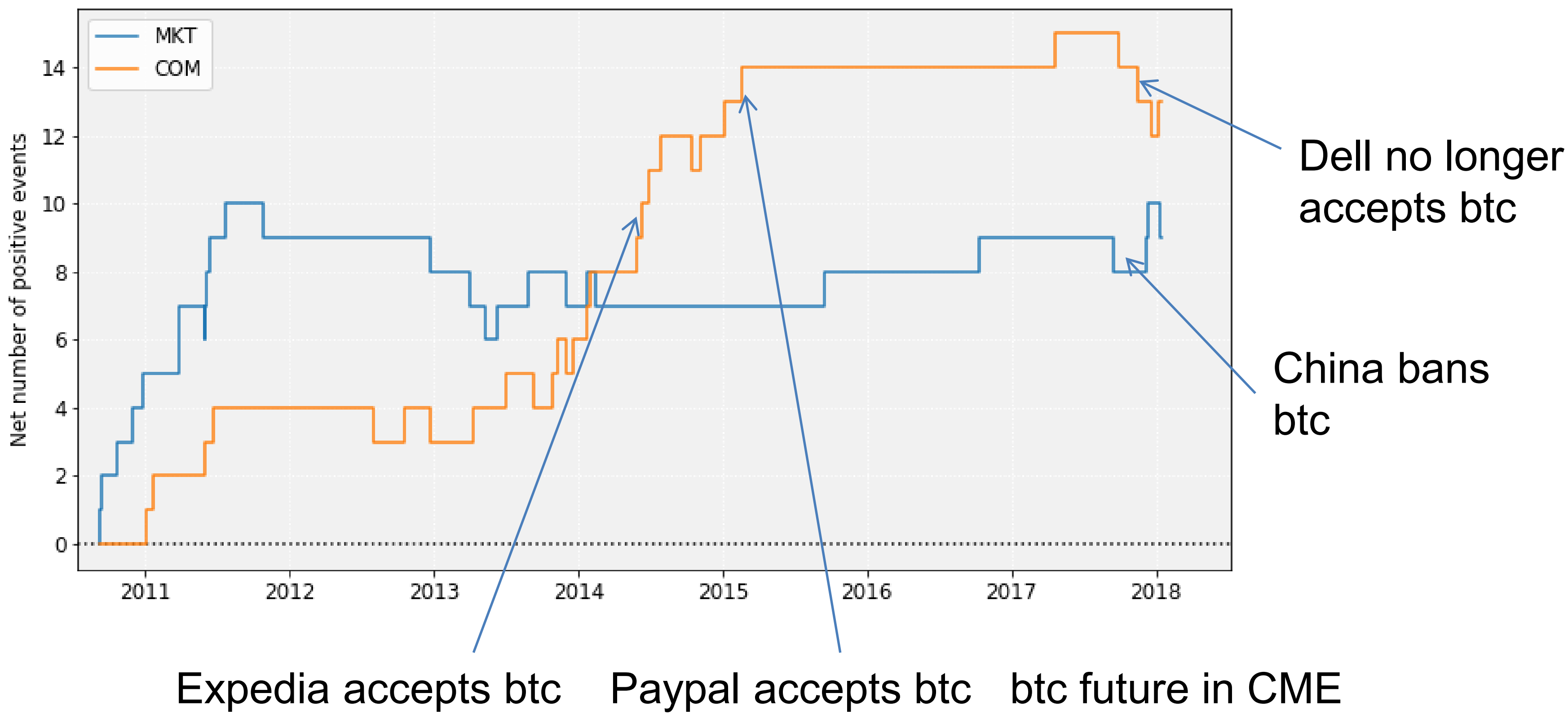
Large transactions fees towards end of 2017 (rise in btc attracts many investors, raise transaction volume, congest network)
Otherwise fees rather low (.0109% of transaction volume, on average)

Browsing net, find events affecting:

- ability to trade bitcoin for dollars, to also help proxy for φ
- ease to trade bitcoin for goods and services, to proxy for θ

(similar to Auer and Claessens, 2018)

Coding each positive as +1 and negative as -1, index of ease to trade btc



Ability to exchange btc for $ improved early (MKT), ease to exchange btc for goods/services improved later (COM)

GMM

Impose moment condition

$$E_t\left[(1-h_{t+1})\frac{1+\theta_{t+1}}{1+\varphi_t'(X_t)}(1+\rho_{t+1})\right]-1=0.$$

with θ affine in COM and φ affine in MKT and transaction fees

Instruments: year dummies, COM, MKT, transaction fees, lagged btc return, …

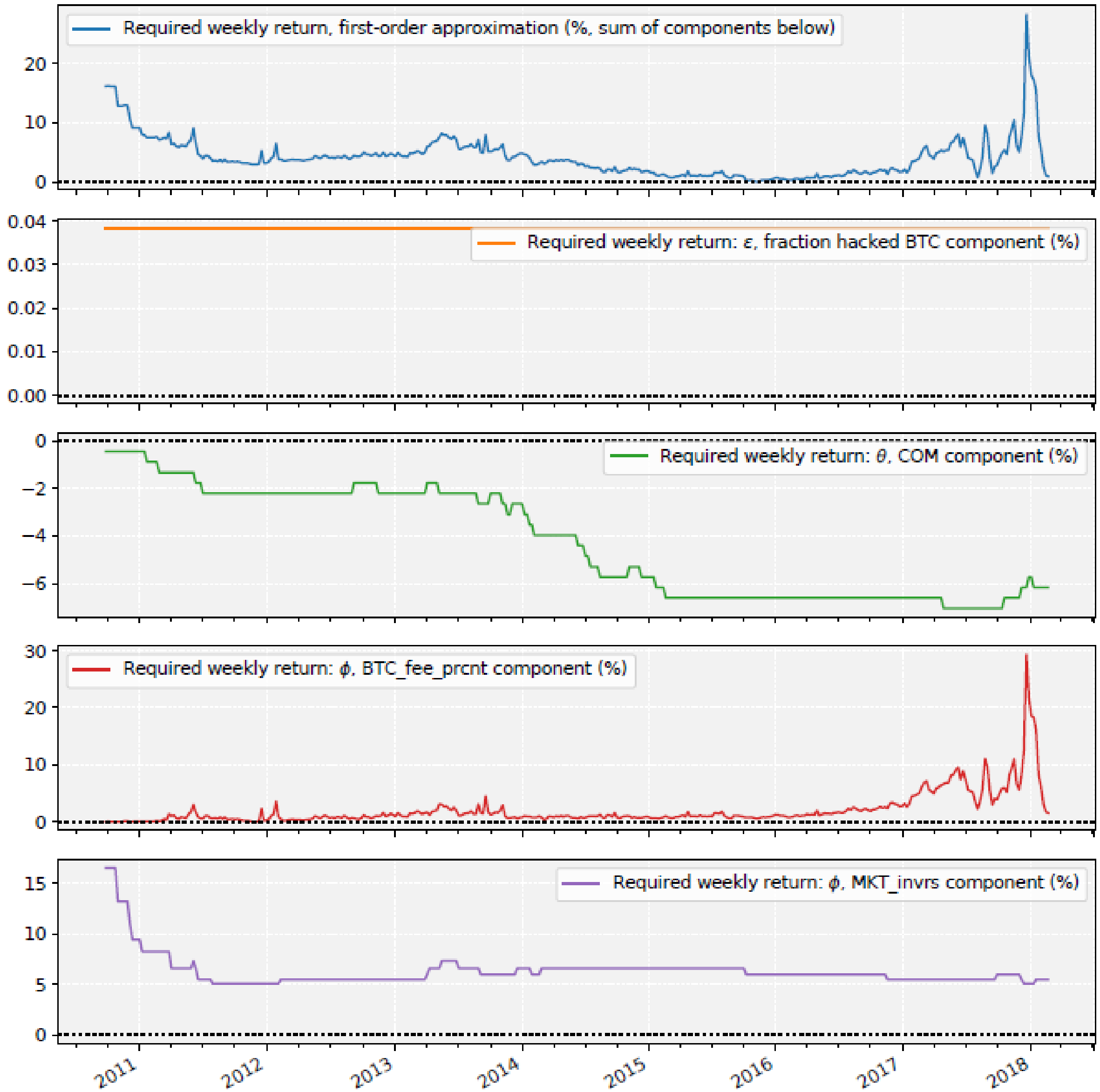
Estimates & t stat (in parentheses)

Intercept	-0.0583 (-1.06)
COM (θ)	0.00364 (2.65)
Transaction fees (φ)	1.60 (1.94)
MKT (φ)	1.20 (2.06)

In line with theory, required E(return) significantly decreasing in transaction benefits (θ) and increasing in unease to exchange btc for dollar (φ)

Transactions fees have right sign, but not really significant (maybe too small)

$$E_t\left[\rho_{t+1}\right] \approx \varphi_t'(X_t) + E_t(h_{t+1}) - E_t(\theta_{t+1}).$$



Changes in fundamentals vs non-fundamental noise

standard deviation observed btc weekly return: 18.0%

standard deviation estimated btc required weekly return: 4.4%

implied R2: (0.044^2)/(0.180^2) = 6.0%

Fundamentals explain part of btc fluctuations

But large fraction of fluctuations reflects exogenous noise (sunspots, changes in beliefs, …)

Tentative conclusion

Theory:

fundamental of currency (including btc) = transaction services

multiplicative structure => exogenous volatility in REE (no Shiller bounds)

Econometric analysis:

preliminary…

fundamentals (e.g., ease to use btc to buy goods and services) seem to explain some of btc fluctuations, but large part reflects exogenous noise