Abstract

We provide a model of the impact of bank mergers on loan competition, individual reserve management and aggregate liquidity risk. Banks compete in rates of differentiated loans, hold reserves against liquidity shocks and refinance in the interbank money market if shocks exceed individual reserves. Mergers can affect market shares, cost efficiency and the distribution of liquidity shocks. In solving the model with and without merger we assess the risk that banking consolidation may drain liquidity away from the interbank money market.

1 Introduction

The last decade has witnessed a substantial number of mergers and acquisitions in the financial services sector of many industrial countries. This ‘merger movement’ has been documented in detail and generally discussed...
in various official reports (see e.g. ECB, 2000; OECD, 2000; Group of Ten, 2001) and research papers (Boyd and Graham, 1996; Berger, Demsetz and Strahan, 1999; Hanweck and Shull, 1999). For example, it was observed that the phenomenon was particularly concentrated among banking firms, that this type of consolidation accelerated during the last three years of the 1990s, that most M&As occurred within national borders and that - as a consequence - many countries (e.g. Australia, Belgium, Canada, France, the Netherlands and Sweden) reached a situation of high banking sector concentration or faced a further deterioration of an already previously concentrated sector, whereas a few others (notably Germany and the United States) remained relatively unconcentrated. The origins of the ‘merger movement’ were found, inter alia, in technical progress (particularly in communication technology), deregulation, general globalisation and the resulting competitive challenges for financial firms and, related to the latter, monetary integration in Europe. Of particular interest for policy makers, market participants and researchers are the consequences of such an extensive consolidation process for the efficiency and competitiveness of bank intermediation, for market liquidity and financial stability and for the working of monetary policy.

Obviously all these important issues cannot be addressed at once. In the present paper we try to provide a theoretical basis for the joint analysis of the impact of mergers on competition among banks and of their effects on individual reserve management and banking system liquidity. The Ferguson ‘Report on Consolidation in the Financial Sector’ pointed out that ‘...by internalising what had previously been interbank transactions, consolidation could reduce the liquidity of the market for central bank reserves, making it less efficient in reallocating balances across institutions and increasing market volatility’ (Group of Ten, 2001, p. 20). Although the central banks contributing to this report did not see any evidence so far that financial sector consolidation had led to this result or voiced any concern that significant problems were looming for the future, they agreed that the situation should be monitored carefully. The model we develop below addresses how bank mergers affect the liquidity situation in the money market, allowing us to discuss within an explicit theoretical framework under which conditions and in which sense mergers may drain liquidity from the money market.

In this model banks compete in prices in a differentiated oligopolistic loan market and hold reserves to protect themselves against liquidity shocks (early deposit withdrawals) occurring before loans mature. These shocks are stochastic in nature, both at the individual bank level and in the aggregate, and are independently distributed across banks. If banks do not have enough reserves to satisfy early withdrawals, they can borrow from a competitive interbank market. Starting from a symmetric status quo, we characterise the
effects of a merger on competition in the loan market, banks’ reserve choices and the probability (liquidity risk) and severity (expected need) of liquidity shortages, both at the individual and at the aggregate (‘systemic’) level. The focus therefore is on relatively ‘large’ mergers that tend to increase the asymmetry of the banking system.

Such a merger has three main effects in the model. First, it enlarges the market share of the merged banks, which leads to upward pressure on loan rates. Second, it can result in efficiency gains, which may lead to a downward pressure on loan rates. Third, the merger can change the structure of liquidity shocks by modifying their distribution within the merged banks. In particular, we consider two cases, either that the merged banks’ shocks remain i.i.d. even after merger or that they become perfectly correlated.

One important result is that the merger always increases the aggregate expected liquidity needs of the banking system in the model, although the probability that the system experiences a liquidity shortage can either increase or decrease, depending on the cost of refinancing relative to the cost of raising deposits. Somewhat paradoxically, the negative effects of the merger are worsened when it is associated with strong efficiency gains. In other words, ‘pro-competitive’ mergers that result in a general decrease in loan rates drain the largest amount of liquidity from the system.

While we are not aware of any other research work addressing the question of how the competitive changes induced by mergers affect reserve holdings and aggregate liquidity risk in the interbank money market, our paper is related to a number of issues that the literature has studied. First of all, aggregate liquidity risk is related to financial stability. Although we are not covering solvency problems in our model, in practice severe liquidity problems may still cause default when there is no adequate intervention. The relationship between competition and bank stability has been studied in the ‘charter value’ literature, basically saying that some monopoly rents (or even quite severe regulations) are desirable in banking, since they avoid the problem of excessive risk-taking (see e.g. Keeley, 1990; Hellman et al., 2000 or Matutes and Vives, 2000).1 Our model can link monopoly rents in loan competition (through concentration) to individual and aggregate liquidity fluctuations, as one factor in banking stability. It suggests that, absent aggregate liquidity management by the central bank, reduced competition through mergers may well worsen liquidity risk and in this sense not be beneficial for financial stability.

Second, the expansion of firms through mergers can lead to the creation of

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1Carletti and Hartmann (2001) provide a more comprehensive review of the literature on competition and stability in banking.
internal capital markets. The literature has related the existence of internal capital markets in large banks to lending activity. For example, Houston et al. (1997) find evidence that loan growth at subsidiaries of US bank holding companies is more sensitive to the holding company’s cash flow than to the subsidiaries’ own cash flow. Our model relates the creation of an internal money market to individual and aggregate liquidity risk. Finally, changes induced by mergers for aggregate liquidity needs may affect the implementation of monetary policy by central banks. They may affect the level or the volatility of liquidity that needs to be provided in monetary operations to stabilise short-term interest rates. Whereas this point was also raised in the Ferguson Report, as far as we know it has not been taken up yet by the literature. Our model may give some indications in which direction the quantities and the variability of operations may have to develop in response to significant merger activity.

The rest of the paper is structured as follows. Section 2 sets up the model, starting with the loan competition part and finishing with the liquidity risk and interbank lending part. Section 3 derives the equilibrium in the model without bank mergers (‘status quo’). Section 4 then introduces bank mergers and studies their effects on loan competition, interest rates, individual reserve management and aggregate liquidity risk. It is split in two sub-sections, one dealing with the case of correlated liquidity shocks between the merging banks and the other dealing with uncorrelated liquidity shocks. The last section concludes.

2 The Model

Consider a three date \( (T = 0, 1, 2) \) economy with \( N \) banks \( (N > 3) \), numerous entrepreneurs and numerous depositors. All agents are risk neutral. As banks have no capital in this model, they need to raise deposits to finance their investments. At \( T = 0 \) each bank raises an amount of deposits \( D_i \) and invests an amount \( L_i \) in loans to entrepreneurs and an amount \( R_i \) in liquid reserves. So the balance sheet accounting identity is

\[
L_i + R_i = D_i. \tag{1}
\]

Banks compete in the loan market and charge an interest rate \( r^L_i \) to borrowers (as specified below). Loans mature at \( T = 2 \) and yield nothing if liquidated before maturity. Reserves are liquid assets with zero interest rate, which banks keep only to protect themselves against liquidity shocks. Deposits are promised a rate \( r^D \) if they do not withdraw before \( T = 2 \). However, a shock in \( T = 1 \) leads a fraction \( \delta_i \) of depositors to withdraw their initial unit of
deposit prematurely. The fraction $\delta_i$ is stochastic, both at an individual and at an aggregate level. This leaves room for a borrowing-lending mechanism at $T=1$. When a bank’s reserves are not sufficient to satisfy depositors’ demands of early withdrawals, it can borrow in an interbank market at a rate $r^I$ (as specified below).

For simplicity, the intertemporal discount factor is normalised to one. In the next sub-section we will describe in greater detail the interaction of banks on the asset side and in the subsequent one their links through risks originating from the liability side.

2.1 Competition in the Loan Market

We model competition on the asset side following the standard product differentiation approach (see e.g. Shubik, 1980), instead of regarding credit markets as perfectly competitive. Banks maintain some market power in credit markets, since entrepreneurs cannot freely substitute them against each other. In practice, there may be different sources for the limited substitutability of loans from different banks, including long-term lending relationships, the specialisation of banks in certain types of lending (e.g. lending to small or large firms or lending to different sectors) or certain geographical regions. The analysis of mergers builds on Deneckere and Davidson (1985).

We start from a situation in which all banks are identical (perfect symmetry). Banks compete in prices, i.e. loan rates. Each bank $i$ faces a linear demand for loans given by

$$L_i = A \left(1 - \gamma \left( r^L_i - \frac{1}{N} \sum_{j=1}^{N} r^L_j \right) \right),$$

where $L_i$ is the quantity demanded of bank $i$’s loans, $r^L_i$ is the loan rate charged by bank $i$ and $r^L_j$ is the loan rate charged by banks $j$, with $j = 1...i...N$. The parameter $\gamma \geq 0$ represents the degree of substitutability between the loans offered by banks. The larger (smaller) $\gamma$ the more (less) substitutable are the loans offered by different banks for entrepreneurs. Consequently, the larger (smaller) $\gamma$ the more (less) sensitive the demand for bank $i$’s loans is to the difference between $i$’s own loan rate and the rates charged by the other $N-1$ banks and the smaller (larger) bank $i$’s market

\footnote{This initial symmetry means that a merger will lead to an asymmetric situation. Obviously, in practice also the opposite scenario can be imagined, a merger making a highly asymmetric market more symmetric. We will address the different scenarios in the discussion below.}
power. For simplicity we specify (2) in a way that the aggregate supply of
loans \( L_i = NL \) is constant.\(^3\)

Processing loans involves a per-unit provision cost \( c \) for banks, which can
be interpreted as a set up cost or an on-going monitoring cost.

### 2.2 Individual Liquidity Shocks and Refinancing Needs

We can now turn to liquidity risks originating from banks’ liabilities. Following
standard banking theory at \( T = 1 \) each bank \( i \) is hit by a pure liquidity
shock: A fraction \( \delta_i \) of its depositors turns out to be ‘type-1’ depositors and
withdraws their funds to satisfy early consumption needs. The remaining
\( 1 - \delta_i \) depositors are of ‘type-2’ and leave their funds at the bank until \( T = 2 \).
The fraction of type-1 depositors is stochastic. In particular, \( \delta_i \) is uniformly
distributed between 0 and 1 and shocks are i.i.d. across banks. Hence, we
can define each bank’s gross expected demand for liquidity at \( T = 1 \) as
\[
x_i = \delta_i D_i,
\]
which is uniformly distributed on the support \([0, D_i]\) with density function
\[
f(x_i) = \frac{1}{D_i}.
\]

Each bank uses its reserves \( R_i \) to repay depositors withdrawing at \( T = 1 \). Hence reserve holding can be seen as a bank’s gross supply of liquidity at
\( T = 1 \). However, the stochastic nature of the shocks and \( R_i < D_i \) imply
that banks may suffer liquidity shortages in \( T = 1 \). This happens when
\( x_i > R_i \). We can then express a bank’s liquidity risk – the probability that
it experiences a liquidity shortage – as
\[
\phi_i = \text{prob}(x_i > R_i) = \int_{R_i}^{D_i} f(x_i) dx_i.
\]
A bank’s resiliency against liquidity shocks – the probability that the bank
does not experience a liquidity shortage – can then be expressed as
\[
1 - \phi_i = \text{prob}(x_i \leq R_i) = \int_{0}^{R_i} f(x_i) dx_i.
\]
A bank’s expected need of refinancing (or expected net liquidity demand) at
\( T = 1 \) is given by
\[
\omega_i = \int_{R_i}^{D_i} (x_i - R_i) f(x_i) dx_i.
\]
\(^3\)The specification of the individual demand for loans given in (2) is less cumbersome
than the one with elastic demand and leads to qualitatively the same results.
2.3 Interbank Refinancing and Aggregate Liquidity

In the second stage of $T = 1$ banks with liquidity shortages/refinancing needs ($x_i > R_i$) and banks with liquidity excesses ($x_i < R_i$) are brought together in the interbank money market. (We are thinking here of the ultra-short term interbank deposit market, like the overnight market in the euro area or the Fed funds market in the United States.) This market is assumed to be a large and perfectly competitive market, where lenders break even and borrowers bear the per unit interbank rate $r^I$.

The aggregate liquidity demand of our $N$ banks in the interbank market is given by $\sum_{i=1}^{N} x_i$, and the aggregate reserves are $\sum_{i=1}^{N} R_i$. This means that analogous to individual banks’ liquidity risk $\phi_i$ there can also be an aggregate shortage or excess of liquidity and we can define the aggregate (or systemic) liquidity risk $\Phi$ in our system of $N$ banks as

$$\Phi = \text{prob} \left\{ \frac{\sum_{i=1}^{N} x_i}{\sum_{i=1}^{N} R_i} > \frac{1}{N} \right\}.$$  \hspace{1cm} (5)

In turn, aggregate resiliency against liquidity risk is

$$1 - \Phi = \text{prob} \left\{ \frac{\sum_{i=1}^{N} x_i}{\sum_{i=1}^{N} R_i} \leq \frac{1}{N} \right\}.$$  \hspace{1cm} (6)

and aggregate expected liquidity needs (dependence on outside liquidity) are

$$\Omega = E \left\{ \frac{\sum_{i=1}^{N} x_i}{\sum_{i=1}^{N} R_i} - \frac{1}{N} \right\}.$$  \hspace{1cm} (7)

Since we want to focus on private market liquidity fluctuations only and not on defaults and solvency problems, we assume that central bank operations complete the market in the second stage of $T = 1$ by injecting any amount of liquidity missing and thereby keeping the interbank rate constant at $r^I$.

To summarise the model and its timing, as also illustrated in figure 1, at $T = 0$ banks compete in prices in the loan market, choose reserve holdings

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4The assumption of perfect competition in the interbank market implies that when banks decide between loans and reserves in $T = 0$ they do not invest in reserves to make profits in the interbank market. One way to think of this is to imagine the interbank market as a relatively ‘passive’ market, in the sense that ‘short’ banks demand what they need and ‘long’ banks offer the funds they have without serious attempts to make money. This picture seems to be a fairly good description of banks’ behaviour in the euro overnight market for example. (Alternatively, $r^I$ can also be thought of as an interbank deposit processing cost, covering e.g. search costs, transaction costs charged by market makers or brokers or a disutility of borrowing, etc.)
and raise the necessary deposits, such that the accounting identity (1) is fulfilled. At $T=1$ banks are hit by the liquidity shocks and borrow or lend in the interbank market. At $T=2$ loans mature and profits realise.\(^5\)

<table>
<thead>
<tr>
<th>T=0</th>
<th>T=1</th>
<th>T=3</th>
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<tbody>
<tr>
<td>price competition in the loan market, choice of $R_i$,</td>
<td>shocks $\delta_i$ realise, interbank refinancing takes place</td>
<td>loans mature and profits realise</td>
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$D_i = L_i + R_i$ are raised

Figure 1: Structure and timing of the model

3 The Status Quo

In this section we line out the problem to be solved and derive the equilibrium without merger. Each bank $i$ will choose at $T=0$ its loan rate $r^L_i$ and liquid reserves $R_i$ to maximise its total expected profits

$$\Pi_i = (r^L_i - c)L_i - Z_{D_i}^\infty r^I(x_i - R_i)f(x_i)dx_i - r^D D_i(1 - E(\delta_i)), \quad (7)$$

where the first term represents profits from the loan market, the second term is the expected cost of refinancing on the interbank market, and the third term is the expected repayment to deposits.

Using Leibiniz’s rule and the accounting identity (1), we obtain the first order conditions with respect to the choice variables $r^L_i$ and $R_i$:

$$\frac{\partial \Pi_i}{\partial r^L_i} = L_i + (r^L_i - c) \frac{\partial L_i}{\partial r^L_i} - \frac{r^I L_i^2 + 2 L_i R_i}{2(L_i + R_i)^2} + \frac{r^D}{2} \frac{\partial L_i}{\partial r^L_i} = 0, \text{ for } i = 1...N, \quad (8)$$

$$\frac{\partial \Pi_i}{\partial R_i} = r^D (L_i + R_i)^2 - r^I L_i^2 = 0, \text{ for } i = 1...N. \quad (9)$$

\(^5\)Letting reserves to be chosen after competition in the loan market, or before but being not observable, leads to the same results. Letting reserves being chosen before loan market competition and being observable would lead to the standard precommitment effect where, since banks compete in strategic complements, they reduce reserves (or deposits) to strengthen capacity constraints and soften loan market competition. Since this effect is well known, we avoid precommitment considerations to focus on liquidity choices. Also, it does not seem plausible from a practical perspective that short-term reserves are chosen before long-term loan decisions are made.
Equation (8) tells us that banks set the loan rate to balance marginal profits from granting loans with the marginal costs of increasing the expected refinancing need and of raising deposits. Equation (9) tells us that banks set their reserve holdings to balance the marginal revenue from reducing refinancing needs with the marginal cost of increasing deposits. Solving (9) for $R_i$, we obtain:

$$R_i = \frac{\tilde{A}r}{r^f} - 1 L_i.$$  

(10)

Equation (10) gives the optimal reserve holding of each bank $i$ as a function of the cost of refinancing relative to the cost of raising deposits and of the quantity of loans $L_i$. We exclude the case of $r^f < r^D$ in which equation (10) implies zero (or negative) reserves. Since usually interbank rates are much higher than retail deposit rates we focus in what follows only on the case $r^f > r^D$ for which equation (10) implies positive bank reserves.6

Proposition 1 summarises the equilibrium for the model without merger.7

**Proposition 1**  The symmetric status quo equilibrium with $r^L_i = r^L_{sq}$ for $i = 1 \ldots N$ is characterised as follows:

1) $r^L_{sq} = \frac{l}{\gamma(N-1)} + c + \sqrt{r^fr^D}$, $L_{sq} = l$, $\Pi_{sq} = \frac{l^2}{\gamma(N-1)}$;

2) $R_{sq} = \frac{\mu q}{r^f} - 1 l$; $D_{sq} = l \frac{q}{r^D}$

3) $\phi_{sq} = \frac{\phi}{r^f}$, $\omega_{sq} = \frac{\omega}{2r} D_{sq}$.

The results appear intuitive. Starting with 1) the expression for $r^L_{sq}$ shows that the equilibrium loan rate is higher than the perfectly competitive rate, as it is higher than the total marginal cost $c + \sqrt{r^fr^D}$. The price distortion (or mark up), $\frac{l}{\gamma(N-1)}$, is decreasing in the number of banks $N$ and in the substitutability parameter $\gamma$, since both reinforce competition, and increasing in the equilibrium demand for loans $l$. The same applies to bank profits of course.

Turning to 2) both the amount of liquid reserves and the deposits held in the status quo equilibrium increase with the demand for loans $l$ and the cost of refinancing $r^f$, while they decrease with the cost of raising deposits $r^D$.

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6In the euro area the overnight deposit rate is usually several times larger than rates paid on demandable deposits. For example, since the introduction of the euro the EONIA rate has been between 3.5 and 5 times larger than average rates paid on sight deposits.

7All formal proofs are in the appendix.
Comparing reserves to loans, we have $R_{sq} < L_{sq}$ if $r^L < 4r^D$ and $R_{sq} > L_{sq}$ otherwise. Note that in the symmetric status quo equilibrium the intensity of competition in the loan market, as parametrised by $\gamma$ and $N$, affects the loan rate $r^L_{sq}$ through the mark up, but it does not affect the equilibrium quantities of loans, reserves and deposits.\footnote{With an elastic loan demand, for example $L_1 = l - \alpha(r^L + 1 - \gamma(\gamma + 1)^N \prod_{j=1}^{N} r^L_j)$, an increase in $\gamma$ would increase individual and aggregate loans. This would increase equilibrium reserves and deposits, but all proportionally so that the ratios $\frac{L_{sq}}{D_{sq}}$ and $\frac{R_{sq}}{D_{sq}}$ remain unchanged. Analogously, an increase in $\alpha$ would decrease equilibrium loans and therefore reserves and deposits, but their ratios would still be constant.}

Finally, in 3) the equilibrium liquidity risk $\phi_{sq}$ is increasing in the cost of raising deposits $r^D$ and decreasing in the cost of refinancing $r^f$. The reason is that both a higher cost of deposits and a lower cost of refinancing leads banks to reduce both deposits and reserves, but the effect of the reduction in reserves on liquidity risk dominates the effect of the reduction in deposits on $\phi_{sq}$ since the expected gross liquidity demand $x$ at $T = 1$ is only a fraction $\delta$ of deposits.

Substituting the value of $D_{sq}$ we have $\omega_{sq} = \frac{q}{r^D}$. Thus, a bank’s expected liquidity need also increases with $r^D$ and decreases with $r^f$ in equilibrium, again because a higher $r^D$ (or a lower $r^f$) reduces reserves more than proportionally to the reduction in the expected liquidity demand. Note that the expected need of liquidity increases with the demand for loans $l$, while liquidity risk does not.\footnote{The intuition is that our liquidity risk $\phi$ is a probability, determined by the ratio of individual reserves to individual deposits. Since both reserves and deposits increase linearly in loan demand, $l$ cancels out in the probability. In contrast, the expected liquidity shortfall is rather a difference between deposits and reserves, so that the size factor $l$ is maintained.}

### 4 The Effects of Bank Mergers

Consider now the situation in which two banks (for example bank 1 and bank 2) propose to merge, while the other $N - 2$ banks continue to act independently.\footnote{For the moment we analyse only the case in which two banks merge. The analysis can however be easily generalised to the case in which $M$ banks merge and $N - M$ banks continue to act independently.} If the merger takes place, the behaviour of the merged banks is different from the combined behaviour of the two separate banks pre-merger. In particular, the merger has three main effects. First, it enlarges the market share (and thereby enhances the market power) of the merged banks, which leads to upward pressure on loan rates. Second, the merger can result
in efficiency gains in terms of a reduction of loan provision costs, which may lead to downward pressure on loan rates. Hence, in our framework a bank merger can either result in a general increase or in a general decrease of loan rates, depending on the relative importance of these two effects.\footnote{There is a long debate about whether many of the mergers observed turn out to be efficient or rather inefficient. Our framework considers that the merger either has no effects on efficiency or causes an efficiency gain. However, the analysis can be easily extended to the case of inefficient mergers.} Third, it can change the structure of liquidity shocks by modifying their distribution within the merged banks. Maintaining the independence of shocks across separate banks, we consider two distinct assumptions for the merged banks:

a) the merger does not change the relationship between $\delta_1$ and $\delta_2$, so that they remain i.i.d. even after merger;

b) the merger leads to a stronger relationship between $\delta_1$ and $\delta_2$. For simplicity, we assume that $\delta_1$ and $\delta_2$ become perfectly correlated.

Assumption a) can be thought of as reflecting cases in which the merger between bank 1 and 2 is not followed by any change with regard to the deposit customer basis (the two banks just adjust the amount of deposits to the new demand for loans but the type of depositors do not change). For example, this could happen if two banks merged but each one maintained its previous customer services, its branches and its name. Assumption b) can be thought of as reflecting cases in which at least one of the two banks adjusts its customer policies towards the one of the other after merging. The idea is that the two banks do not only adjust the amount of deposits but also make their deposit bases more similar in terms of liquidity shocks. For example, this could occur if one of the two banks adopted the services and name of the other or moved its retail operations to the same region.

In what follows we provide first an in-depth analysis of what happens to loan rates and liquidity risks in case b) (see sub-section 4.1) and then we discuss what becomes different in case a) (sub-section 4.2).

4.1 The Case of Correlated Liquidity Shocks

We start by analysing the case when the shocks between the merged banks become perfectly correlated.

4.1.1 The Post-Merger Equilibrium

For simplicity, we can consider that the merged banks are hit by a common shock $\delta_m$, which affects their total deposit base $D_m = D_1 + D_2$. Hence, we...
can define the merged banks’ gross expected demand of liquidity at $T = 1$ as

$$x_m = \delta_m D_m,$$

which is uniformly distributed on the support $[0, D_m]$ with density function $f(x_m) = \frac{1}{D_m}$. The merged banks use their joint reserves $R_m = R_1 + R_2$ to repay depositors withdrawing at $T = 1$.

The combined demand for loans of the merged banks is given by

$$L_m = L_1 + L_2 = l - \gamma (r^L_1 - \frac{1}{N} \sum_{j=1}^{N} r^L_j) + L - \gamma (r^L_2 - \frac{1}{N} \sum_{j=1}^{N} r^L_j),$$

where $L_1$ and $L_2$ are the quantities demanded of bank 1’s and bank 2’s loans and $r^L_1$ and $r^L_2$ are the corresponding loan rates. The cost of providing loans for the merged banks is equal to $c_m = \beta c$, with $0 \leq \beta \leq 1$ representing potential efficiency changes in the form of reduced loan provision costs induced by the merger. The larger $\beta$, the weaker the efficiency gains produced by the merger. At the limit, when $\beta$ approaches one, the merger produces no efficiency gains.

The combined profits of the merged banks are then given by

$$\Pi_m = \Pi_1 + \Pi_2 = (r^L_1 - c_m) L_1 + (r^L_2 - c_m) L_2 - \int_{R_m} r^f(x_m - R_m) f(x_m) dx_m - r^D D_m (1 - E(\delta_m)).$$

The merged banks set the loan rate $r^L_1$ and $r^L_2$ and choose $R_m$ to maximise the combined profits.\footnote{We are keeping the choice of loan rates separate for the merged banks to emphasize that the loan market is segmented, while deposits and reserves are a common pool. Since we are focusing on symmetric equilibria, we could also have set $r^L_1 = r^L_2 = r^L_m$ in the profit function (12) and solved the problem with respect to $r^L_m$. The two approaches lead to the same result.} Using Leibniz’s rule and the balance sheet identity $D_m = R_m + L_1 + L_2$, the first order conditions are

$$\frac{\partial \Pi_m}{\partial r^L_h} = L_h + (r^L_1 - c_m) \frac{\partial L_1}{\partial r^L_h} + (r^L_2 - c_m) \frac{\partial L_2}{\partial r^L_h} \cdot \frac{r^f (L_1 + L_2)^2 + 2 (L_1 + L_2) R_m}{(L_1 + L_2 + R_m)^2} + \frac{r^D}{2} \frac{\partial L_1}{\partial r^L_h} + \frac{\partial L_2}{\partial r^L_h},$$

with $h = 1, 2,$ and

$$(13)$$
\[ \frac{\partial \Pi_m}{\partial R_m} = r^D (L_1 + L_2 + R_m)^2 - r^I (L_1 + L_2) = 0. \]  

(14)

By inspecting (13) one sees that in setting the loan rate, each bank involved in the merger now takes also into account the effect that a change in its own loan rate has on the demand for loans of the other bank involved and the change in the expected cost of refinancing occurring through \( r_1^L \) on \( L_2 \) and through \( r_2^L \) on \( L_1 \). For example, if bank 1 increases \( r_1^L \) then \( L_2 \) decreases but \( L_2 \) increases and analogously for \( r_2^L \). In other words the merged banks can keep higher loan rates than before without losing business. So, the internalisation of these externalities enhance the market power and together with the efficiency gains in the loan provisions costs modify the behaviour of the merged banks.

The other \( N - 2 \) banks (the competitors marked with subscript \( c \)) are still all identical. They have the same cost structure as before merger and behave independently. Their individual demand for loans is given by (2) and their profits by (7). As before the merger, each of them sets its loan rate and reserve holding to maximise its individual profits. The first order conditions are still given by (8) and (9). We have then the following proposition describing the post-merger equilibrium with symmetric behaviour within the 'coalition' (merger) and among competitors.

**Proposition 2** The post-merger equilibrium with \( r_i^L = r_i^L = r^L_m \) and \( r_i^L = r^L_c \) for \( i = 3...N \) is characterised as follows:

1) \( r_i^L = \frac{i 2N - 1}{N} \frac{\xi}{c} \frac{l + h(N-1)(N+1)l}{2N} i c + \sqrt{r^I r^D}, \) \( r_c^L = \frac{i N - 2}{N} \frac{\xi}{\gamma} \frac{h(N-1) + \beta}{N} i c + \sqrt{r^I r^D}; \)

\( L_m = \frac{i 2N - 1}{N} \frac{\xi}{l} \frac{l + \frac{(N-1)(N-2)(1-\beta) c}{2N}}{L_m} = i \frac{N - 1}{N} \frac{\xi}{N - 2} \frac{l + \frac{(1-\beta) c}{N}}{N} \frac{i}{c} \)

\( \pi_m = \frac{(1-2N)(1-\gamma)(N-1)(N-2)(1-\beta)c^2}{2N(N-2)}, \) \( \pi_c = \frac{(N-1)(N-1)(1-\gamma)(N-2)(1-\beta)c^2}{2N(N-2)}; \)

2) \( R_m = \frac{q}{\sigma} - 1 \) \( L_m, R_c = \frac{q}{\sigma} - 1 \) \( L_c, D_m = L_m \frac{q}{\sigma}, D_c = \frac{q}{\sigma}; \)

3) \( \phi_m = \phi_c = \frac{q}{\sigma}, \omega_m = \frac{\sigma}{\sigma} D_m, \omega_c = \frac{\sigma}{\sigma} D_c. \)

The results are again quite intuitive. The first terms in the expressions for \( r_m^L \) and \( r_c^L \) represent the mark up that banks can charge. As the merger reduces
the number of banks, the markups for all banks are now higher than in the pre-merger equilibrium (see $r_{sq}^L$ in proposition 1). Furthermore, as a result of the increased market power coming from the internalisation of externalities described above, the merged banks can charge an even higher mark up than the competitors. The last terms in the expressions for $r_m^L$ and $r_c^L$ represent the efficiency effect of the merger on the loan rates. Since efficiency gains are associated with a reduced $\beta$, the loan rates decrease ceteris paribus in response to the efficiency gains produced by the merger. Since banks compete in strategic complements, the post-merger equilibrium loan rates of all banks move in the same direction and we have the following corollary.

**Corollary 1** In the post-merger equilibrium $r_m^L > r_c^L > r_{sq}^L$ if $\beta > \beta^*$, and $r_m^L < r_c^L < r_{sq}^L$ if $\beta < \beta^*$, where

$$\beta^* = 1 - \frac{l}{\gamma c N (N - 1)(N - 2)}.$$ 

The merger raises equilibrium loan rates when efficiency gains are small relative to the increase in market power and reduce equilibrium loan rates when efficiency gains are sufficiently large. Note that since $\beta^* \in [0, 1]$, when $\frac{1}{\gamma c (N-1)(N-2)} > 1$ it can never be $\beta < \beta^*(N)$ and the merger always increases loan rates. The necessary condition for the merger to reduce loan rates $\frac{N}{\gamma c (N-1)(N-2)} < 1$ is more easily satisfied when the loan market is competitive, that is the larger are $\gamma$ and $N$.

The behaviour of the merged banks with respect to the loan rate determines banks’ market shares in the post-merger equilibrium and their balance sheet.

**Corollary 2** In the post-merger equilibrium, $L_m > L_c, L_{sq}$; $R_m > R_c, R_{sq}$ and $D_m > D_c, D_{sq}$. Moreover,

i) if $\beta > \beta^*$, then $L_m < 2L_{sq} < 2L_c$, $R_m < 2R_{sq} < 2R_c$ and $D_m < 2D_{sq} < 2D_c$;

ii) if $\beta < \beta^*$, then $L_m > 2L_{sq} > 2L_c$, $R_m > 2R_{sq} > 2R_c$ and $D_m > 2D_{sq} > 2D_c$.

When efficiency gains are small, the merged banks charge a higher loan rate than competitors (corollary 1) and their loan market share is smaller than both their pre-merger joint market shares and the post-merger loan market.
shares of two competitors. By proposition 2, this induces an analogous relationships for reserves and deposits of merged banks and competitors. Conversely, when efficiency gains are large, the loan market share of the merger banks grows more than proportionally and so do their reserves and deposits.

It can be easily verified using the corollaries and equilibrium profits given in proposition 2 that the merged banks always benefit from merging. Both when they increase the loan rates (and decrease the quantity of loans supplied) and when they decrease the loan rates (and increase the quantity of loans), their combined profits are higher than their individual profits before the merger. The other banks instead do not always benefit from the merger. They are better off when loan rates increase after merger, as they both expand their loan supply and increase their loan rate (in this case, the competitors take a ‘free-ride’, as they gain even more than the banks involved in the merger). However, they are worse off when loan rates decrease after merger, as they reduce both their loan quantity and the rate at which it is supplied.

Part 3) of proposition 2 shows that the post-merger equilibrium liquidity risk (the probability of facing an individual liquidity shortage) of the two merged banks together and of each competitor is the same and is unchanged with respect to the pre-merger equilibrium (compare to proposition 1). That is the merged banks are as resilient as any other individual bank. This is a consequence of the perfect correlation between the liquidity shocks of the merged banks, which prevents them from enjoying gains from mutual insurance, i.e. from exchanging reserves internally. In other words, the correlation of the shocks makes the internal reserves market created by a merger worthless. However, the expected liquidity needs of the merger banks $\omega_m$ do differ from those of competitors and from the status quo, since they are proportional to equilibrium deposits and therefore also to loan market shares. Interestingly, from bringing together proposition 2 and corollary 2 one sees that the inefficient mergers ($\beta > \beta^*$) have a lower joint expected liquidity need than two non-merged competitors, whereas efficient mergers ($\beta < \beta^*$) always have a higher expected liquidity need than two competitors in the post-merger equilibrium, since $\frac{\omega_m}{\omega_c} = \frac{D_m}{D_c} = \frac{L_m}{L_c} > 2$.

4.1.2 The Effects on Interbank Liquidity

We consider now how our $N$ banks’ aggregate (or systemic) liquidity risk $\Phi$, as defined in (5), and expected aggregate liquidity needs $\Omega$, defined in (6), are affected by the merger. Since this will be different in the case of uncorrelated post-merger liquidity shocks, we start by noting that in the case of post-merger correlation of deposit withdrawals the aggregate amount of
reserves — the aggregate gross supply of liquidity does not change with the merger. Aggregate reserves are always equal to
\[
\sum_{i=1}^{N} R_{sq} = R_m + \sum_{i=3}^{N} R_c = NL - \frac{r_I}{r_D} - 1
\]
in this case. In fact, from propositions 1 and 2 we know that both in the pre-merger and in the post-merger equilibria, each bank chooses reserves according to condition (10), while the inelastic aggregate demand for loans leads to
\[
\sum_{i=1}^{N} L_{sq} = L_m + \sum_{i=3}^{N} L_c = NL.
\]
Clearly, this implies that with correlation the aggregate amount of deposits does not change either with the merger, i.e.,
\[
\sum_{i=1}^{N} D_{sq} = D_m + \sum_{i=3}^{N} D_c = NL - \frac{r_I}{r_D}.
\]
The aggregate liquidity demand instead is affected by the merger. Before the merger it was simply
\[
X_{sq} = \sum_{i=1}^{N} x_i = \sum_{i=1}^{N} \delta_i D_{sq}
\]
whereas after the merger it becomes
\[
X_m = \sum_{i=1}^{N-1} x_i = \delta_m D_m + \sum_{i=3}^{N} \delta_i D_c.
\]
This implies that in the status quo the aggregate liquidity risk is
\[
\Phi_{sq} = \text{prob} \left( \sum_{i=1}^{N} \delta_i D_{sq} > NL - \frac{r_I}{r_D} - 1 \right),
\]
while the post-merger aggregate liquidity risk is
\[
\Phi_m = \text{prob} \left( \sum_{i=1}^{N-1} \delta_i D_m + \sum_{i=3}^{N} \delta_i D_c > NL - \frac{r_I}{r_D} - 1 \right).
\]
The aggregate gross liquidity demands and \( \delta_i D_m \) and \( \delta_i D_c \) are both sums of uniform random variables, but they differ in both the number of the summed variables and in their individual distribution. The distribution of the aggregate liquidity demands are in both cases symmetric around the same mean \( \frac{NL}{2} - \frac{r_I}{r_D} \). \( ^{14} \)

We are interested in understanding how the aggregate liquidity risk is affected by the merger.

**Proposition 3** If a merger induces correlation between the liquidity shocks of the merged banks, then in equilibrium \( \Phi_m < \Phi_{sq} \) if \( r_I < 4r_D \) and \( \Phi_m > \Phi_{sq} \) if \( r_I > 4r_D \).

\(^{13}\)The random variables \( \delta_i D_h \) are uniformly distributed on the support \([0, D_h]\) with density \( f(\delta_i D_h) = \frac{1}{D_h} \) and \( D_{sq} \neq D_c \neq D_m \).

\(^{14}\)Given that aggregate deposits are constant and equal to \( NL - \frac{r_I}{r_D} \) and aggregate liquidity demand is the sum of symmetric uniform distributions, such a sum will also be symmetrically distributed on the support \([0, NL - \frac{r_I}{r_D}]\) both before and after the merger.
The merger affects the resiliency of the interbank market against liquidity shocks \((1 - \Phi)\) in two ways: First, it alters the sizes of balance sheets by causing different loan demands across banks (‘size channel’); second, it changes the structure of liquidity shocks by modifying their distribution within the merged banks (‘shock distribution channel’). The ‘size channel’ alters banks’ reserves holdings and deposit sizes and, in turn, the total liquidity demand in the interbank market. The ‘shock distribution’ channel implies different joint probability distributions of shocks across banks and, thereby, alters the effective realisations of liquidity demands across banks.

The two effects both increase the variance of the joint distribution of the aggregate liquidity demands. The size effect changes the variance of aggregate liquidity demand by eliminating the uniform character of weights in the sum of individual liquidity demands (see corollary 2). These weights are identical to the individual deposit or balance sheet sizes, which according to proposition 2 depend on the size of loans. Moving from a uniformly weighted sum of random variables (in the status quo) to a heterogenously weighted sum of random variables (after merger) always increases the variance of the total sum. The more (less) heterogenous the weights are the larger (smaller) the variance. Regarding the distribution effect, the variance increases because in the present case the assumption of correlation between liquidity shocks within the merged bank enters a covariance term in the variance of the sum of random variables, which is not the case in the status quo.

Figure 2 illustrates the consequences of the increase in the variance of the aggregate liquidity demands. The merger increases the likelihood of extreme events, that is of very low and very high aggregate liquidity demands. In figure 2 we display the case when the cost of interbank refinancing is not too high \((r_I < 4r_D)\) and therefore reserves (indicated by the vertical line) are relatively low \((R_h < \frac{1}{2}D_h)\). A larger probability of having a very low liquidity demand obviously increases the resiliency of the system, as illustrated by the larger size of the area \(1 - \Phi_m\) as compared to the shaded area \(1 - \Phi_{sq}\), since for given interbank and deposit rates the aggregate reserves are constant here. Now it can be easily seen that for the case of high interbank refinancing costs \((r_I > 4r_D)\) and therefore high reserves \((R_h > \frac{1}{2}D_h)\), exactly the opposite happens.

Although interesting in itself, our measure of aggregate liquidity risk \(\Phi\) does not take into account how ‘severe’ the liquidity shortages created are. Hence, it is only a partial indicator of the liquidity risk in our bank system. A complementary indicator of how the merger modifies the liquidity risk in our
system is the expected aggregate liquidity need $\Omega$, as defined in (6). It measures the expected size of liquidity needed, given that a shortage occurred. In light of the above discussion, this measure is given by

$$\Omega_{sq} = \mathbb{E} \left( \sum_{i=1}^{N} \delta_i D_{sq} - Nl \right) \frac{r_I}{r_D} - 1 \sum_{i=1}^{\delta_i D_{sq} > Nl} \delta_i D_{sq} - Nl \frac{r_I}{r_D} - 1$$

in the status quo, while after the merger it becomes

$$\Omega_m = \mathbb{E} \left( \delta m D_m + \sum_{i=3}^{\delta_i D_c - Nl} \delta_i D_c - Nl \right) \frac{r_I}{r_D} - 1 \sum_{i=3}^{\delta_i D_c > Nl} \delta_i D_c - Nl \frac{r_I}{r_D} - 1$$

We now have the following result.

**Proposition 4** If a merger induces correlation between the liquidity shocks of the merged banks, then in equilibrium $\Omega_m > \Omega_{sq}$.

Perhaps somewhat surprisingly after having seen proposition 3, the expected aggregate liquidity needs (conditional on a shortage) are always higher after the merger than before. In this sense the merger always worsens the liquidity dependence of the banking system. The reason for this result is that in contrast to liquidity risk $\Phi$, the expected aggregate liquidity need $\Omega$ depends not only on the frequency of events in which aggregate liquidity demand exceeds aggregate supply (total reserves) but also on the magnitude by which aggregate liquidity demand exceeds aggregate supply in each of these events.

The merger increases the frequency of extreme events, those where aggregate liquidity demand is very low and those where it is very high. When banks do not hold reserves these increases offset each other, so that aggregate liquidity demand has the same mean before and after the merger. When banks hold positive reserves, the extreme events in which the aggregate liquidity demand is very small drop out from the calculation of the liquidity needs because reserves cover the aggregate liquidity demand. Hence, when reserves are positive the increased frequency of extreme events with high aggregate liquidity demand induced by the merger is not outweighed any more by the increased frequency of low demand events and net expected liquidity demand grows.

### 4.2 The Case of Uncorrelated Liquidity Shocks

We turn now to the case when the shocks $\delta_1$ and $\delta_2$ hitting the two merged banks remain i.i.d. even after the merger. The main difference with respect
to the case of post-merger correlation is that now there is room for an internal money market between the merged banks. That is, the two banks can pool their reserves and reshuffle them as needed without paying the refinancing rate $r^I$. This mutual insurance mechanism changes the merged banks’ behaviour with respect to both the choice of reserves and competition in the loan market. We sketch here the most important effects at play and discuss whether and in which fashion the results on the aggregate liquidity risk and the expected aggregate liquidity needs are modified.

The presence of an internal market with its mutual insurance effects reduces the merged banks’ expected costs of refinancing for any given levels of reserves and loans. To maximise this portfolio effect and minimise refinancing costs, the two merged banks will be symmetric in terms of deposits raised.

A lower cost of refinancing implies that the reserve-deposit ratio that was optimal in the status quo and in the case of the merger with correlated shocks is now too high. Hence, with independent shocks, the merged banks will set $\frac{R}{D_{m}} < 1 - \frac{r}{q}$ while competitors still find it optimal to keep reserves according to the ratio $\frac{R}{D_{c}} = 1 - \frac{r}{q}$. The lower cost of refinancing also implies that the merged banks enjoy a competitive advantage with respect to their non-merged rivals, which changes the equilibrium in the loan market. The merged banks will become more aggressive in lowering loan rates. This adds to any potential efficiency gains with respect to loan provision costs and, ceteris paribus, always reduces the equilibrium loan rates compared to the case of mergers with correlation. Another consequence of this competitive advantage is that the merged banks will always have a larger share of the loan market with independent shocks than with correlated ones.

Let us turn now to the effects of the merger with independent shocks on aggregate liquidity risk and expected liquidity needs. Differently from the case of post-merger correlation, the aggregate amount of reserves — the aggregate gross supply of liquidity — and deposits do change now with the merger. In particular, not only reserves decrease (for the eliminated external refinancing costs) but also deposits, as total loan demands have to be constant. This implies that the merger with independent shocks will tend to increase the liquidity risk in the banking system relative to the merger with correlation and will make it less likely that the resiliency will improve with respect to the status quo. The main intuition is that the ratio between aggregate reserves and aggregate deposits is lower after a merger with independent shocks than in the status quo, which together with the increased variance of the aggregate liquidity demand due to the asymmetry in the deposit sizes, increases the probability of liquidity shortages. The lower ratio
between total reserves and total deposits together with the higher variance of the distribution of aggregate liquidity demand will also unambiguously lead to higher expected aggregate liquidity needs after the merger compared to the status quo, because total reserves decrease more than total liquidity demands.

5 Conclusions

In this paper we tried to provide a theoretical basis for the joint analysis of the impact of mergers on competition among banks and of their effects on individual reserve management and banking system liquidity. For this purpose we developed a model in which banks compete in prices in a differentiated oligopolistic loan market and hold reserves to protect themselves against liquidity shocks. The shocks are stochastic and independently distributed across banks. If reserves are not sufficient to serve withdrawing depositors, then banks can refinance in a competitive interbank market. We characterised the effects of a merger on competition in the loan market, banks’ reserve choices, the probability of individual and aggregate liquidity shortages as well as the expected size of such shortages. The focus therefore is on relatively ‘large’ mergers that tend to increase the asymmetry of the banking system.

One important result we found is that large mergers, which increase asymmetry in bank sizes, always increase the aggregate expected liquidity needs of the banking system in this framework. However, the probability that the system experiences a liquidity shortage can either increase or decrease, depending on the cost of refinancing relative to the cost of raising deposits. Perhaps surprisingly, the liquidity draining effects of mergers are worsened when they substantially enhance cost efficiency in loan provision. So, ‘pro-competitive’ mergers that generally decrease loan rates have the most detrimental effect on bank system liquidity.

On the basis of this analysis we can draw a number of tentative policy conclusions. First, not too surprisingly whether a merger is anti-competitive in terms of increased loan rates or not depends on whether increases in market power more than offset potential cost efficiency gains. However, since improvements in efficiency are very difficult to assess for competition authorities in advance, it may be difficult to exploit this knowledge in practice. Second, concerns voiced in recent official reports that substantial consolidation in banking can worsen the liquidity situation in the money market, in the sense of a larger average size of liquidity shortages, are borne out by our model. In this sense, the convenience of an internal money market for
a large banking firm may come at the expense of other market participants. However, whether the size of detrimental liquidity effects are economically relevant is an empirical question that has to be answered in future research. The relevance of such affects will strongly depend, for example, on the size and depth of the money market considered. Third, and by implication, to the extent that liquidity risk is an important factor in financial stability, the often claimed trade-off between bank competition and stability does not need to hold. If mergers create asymmetry in the banking system, then the increased market power may well go hand in hand with increased aggregate liquidity fluctuations. So, a permissive attitude by competition or supervisory authorities towards bank mergers may not only create competition problems but also expose the financial system to greater liquidity risk. Finally, from the perspective of monetary policy implementation careful monitoring of consolidation tendencies are justified, since changes in liquidity risk may affect aggregate liquidity management by the central bank. Again, how important such affects can become is very much an empirical question.

References


[29] Shih M.S., 2001, ”Banking Sector Crisis and Mergers as a Solution”, mimeo, National University of Singapore and University of Toronto

Appendix

Proof of Proposition 1

Substituting (2) and (10) in (8) and solving for $r^L_i$ in a symmetric equilibrium with $r^L_1 = r^L_2$ for $i = 1...N$ gives the status quo equilibrium loan rate $r^L_{sq}$. Substituting then $r^L_{sq}$ in (2) gives $L_{sq}$. Substituting $r^L_{sq}$, $L_{sq}$ and (10) in (7) and manipulating it, we obtain $\Pi_{sq}$.

From $L_{sq}$ in (10) $R_{sq}$ follows. Substituting $R_{sq}$ and $L_{sq}$ in (1) gives $D_{sq}$.

Solving (3) gives $\phi_i = 1 - \frac{R_i}{D_i}$, from which, given $R_{sq}$ and $D_{sq}$, we obtain $\phi_{sq}$. Solving (4) yields $\omega_i = \left(\frac{R_i}{D_i}\right)^2 \frac{R_i + D_i}{2}$, from which, once inserted $D_i = D_{sq}$ and $R_i = R_{sq} = 1 - \frac{\mu}{\gamma} D_{sq}$, we get $\omega_{sq}$.

Proof of Proposition 2

We look for the post-merger equilibrium where the two merged banks charge $r^L_1 = r^L_2 = r^L_m$ and all competitors charge $r^L_i = r^L_c$. Substituting (11) and (14) in (13) and using $L_1 = L_2 = L_m$, we obtain the post-merger reaction function of the merged banks as

$$r^L_m = \frac{l}{2\gamma \left(\frac{N-2}{N}\right)} + \frac{(\beta_c + \sqrt{r^L} D)}{2} + \frac{r^L_c}{2}. \quad (19)$$

Analogously, substituting (2) and (10) in (8) and using $L_i = L_c$ for $i = 3...N$, we find the post-merger reaction function of the competing banks as

$$r^L_c = \frac{l}{\gamma \left(\frac{N+1}{N}\right)} + (\frac{N-1}{N+1})^3 c + \frac{\sqrt{r^L} D}{N+1} + \frac{2}{N+1} r^L_m. \quad (20)$$

Solving (19) and (20) gives the post-merger equilibrium loan rates $r^L_m$ and $r^L_c$. Substituting $r^L_m$ and $r^L_c$ respectively in (11) and in (2) gives the equilibrium $L_m$ and $L_c$. Solving (14) for $R_m$ and inserting it together with $r^L_m$ and $L_m$ in (12), we get $\Pi_m$. From $r^L_c$, $L_c$ and (10) in (7) it follows $\Pi_c$.

Inserting $L_m$ in (14) and $L_c$ in (10) gives $R_m$ and $R_c$. From (1) we derive $D_m$ and $D_c$.

From $R_m$ and $D_m$ in $\phi_i = 1 - \frac{R_i}{D_i}$ we have $\phi_m$. Similarly, we get $\phi_c$. Substituting $D_i = D_m$ and $R_m = 1 - \frac{\mu}{\gamma} D_m$ in $\omega_i = \left(\frac{R_i}{D_m}\right)^2 - R_i + \frac{D_i}{2}$, it follows $\omega_m$. Analogously, we obtain $\omega_c$. 

Proof of Corollary 1

As banks compete in strategic complements and (19) is steeper than (20) for \( N > 3 \), it is either \( r^L_m > r^L_c > r^L sq \), or \( r^L_m < r^L_c < r^L sq \). It then suffices to show how \( r^L_m \) changes with respect to \( r^L sq \). From propositions 1 and 2, 

\[ r^L_m - r^L sq = \frac{(N + 1)}{2} \left( \frac{1}{(N - 1)(N - 2)} \right) L \gamma - \frac{(1 - \beta)}{N} c, \]

from which \( r^L_m - r^L sq > 0 \) when \( \beta > \beta^* \) and \( r^L_m - r^L sq < 0 \) otherwise, with \( \beta^* = 1 - \frac{L}{\gamma c N (N - 1)(N - 2)} \).

Proof of Corollary 2

From (2) and (11), the equilibrium \( L_m \) can be expressed as

\[ \frac{L_m}{2} = l - \gamma \left( \frac{N - 2}{N} \right)(r^L_m - r^L c). \tag{21} \]

Using \( L sq = l \) and (21), \( \frac{L_m}{2} - L sq < 0 \) if and only if \( r^L_m > r^L sq \) that holds, given corollary 1, if and only if \( \beta > \beta^* \). Since it is always \( \frac{L}{\gamma c N (N - 1)(N - 2)} \), it follows \( L_m < 2L sq < 2L c \) for \( \beta > \beta^* \). Similarly, \( L_m > 2L sq > 2L c \) if and only if \( \beta < \beta^* \). The other results of the corollary follow immediately.

Proof of Proposition 3

Applying the formula for the distribution of a weighted sum of uniformly distributed random variables in Bradley and Gupta (2001) to our model we obtain the density function of the aggregate liquidity demand in the status quo \( f sq(X sq) \) and after the merger \( f m(X m) \) as

\[
\begin{align*}
f sq(X sq) &= \frac{1}{(N-1)! (D sq)^N} \sum_{i=0}^{\infty} \left( -1 \right)^i \mu_i N \cdot (X sq - i D sq)^{N-1} \\
f m(X m) &= \frac{\prod_{i=1}^{N-2} \left( -1 \right)^i i N-2} {\prod_{i=1}^{N-2} \left( -1 \right)^{i-1} i N-2} (X m - D m - (i - 1) D c)^{N-2} + \frac{\prod_{i=1}^{N-2} \left( -1 \right)^i i N-2} {\prod_{i=1}^{N-2} \left( -1 \right)^{i-1} i N-2} (X m - i D c)^{N-2}.
\end{align*}
\]

The two density functions are plotted in figure 2. The density \( f sq(X sq) \) is more concentrated around the mean than \( f m(X m) \). To verify that this is always the case, we compare the variances of \( X sq \) and \( X m \). Since the variance of a weighted sum of \( N \) random variables \( z_i \sim [\mu_i, \sigma_i] \) is \( V ar \prod_{i=1}^{N} k_i z_i = \).
\[
\mathbb{P}_{i=1}^N k_i \sigma_i^2 + 2 \mathbb{P}_{i<j} k_i k_j \text{cov}_{ij} \sigma_i \sigma_j, \text{ and } X_m = \delta_m D_m + \mathbb{P}_{i=3}^N \delta_c = \delta_m \frac{D_m}{2} + \delta_m \frac{D_m}{2} + \mathbb{P}_{i=3}^N \delta_c D_c, \text{ we have}
\]

\[
\text{Var}(X_{sq}) = \sum_{i=1}^N D_{sq} \text{Var}(\delta_i)
\]

\[
\text{Var}(X_m) = \frac{D_m^2}{4} \text{Var}(\delta_m) + \frac{D_m^2}{4} \text{Var}(\delta_m) + \sum_{i=1}^N \delta_c \text{Var}(\delta_i) + \frac{D_m^2}{2} \text{Var}(\delta_m)
\]

\[
= \frac{D_m^2}{2} \text{Var}(\delta_m) + \sum_{i=1}^N \delta_c \text{Var}(\delta_i) + \delta_c \text{Var}(\delta_i)]
\]

since \(\text{Var}(\delta_i) = \text{Var}(\delta_i). \) Given that \(\frac{D_m^2}{2} \text{Var}(\delta_i) > 0\) and \(\mathbb{P}_{i=1}^N \delta_c = \mathbb{P}_{i=1}^N \delta_{sq}, \) by Lagrangian maximisation one immediately obtains \(\frac{D_m^2}{2} + \sum_{i=3}^N D_c^2\). Hence, it is always \(\text{Var}(X_m) > \text{Var}(X_{sq})\). Since \(f(X_{sq})\) and \(f(X_m)\) are well behaved, in the sense that they approach a normal distribution, they intersect only in two points. This, together with the symmetry of the two density functions around the same mean \(E[X_m] = E[X_{sq}] = \frac{Nl}{2} \frac{r^l}{r^D}\) and \(\text{Var}(X_m) > \text{Var}(X_{sq})\) imply

\[
\text{Pr}(X_{sq} > k) > \text{Pr}(X_m > k) \text{ for any } k < \frac{Nl}{2} \frac{r^l}{r^D},
\]

and vice versa

\[
\text{Pr}(X_{sq} > k) < \text{Pr}(X_m > k) \text{ for any } k > \frac{Nl}{2} \frac{r^l}{r^D},
\]

from which the statement of proposition 3 follows.

**Proof of Proposition 4**

For simplicity, we normalise to 1 the support of the distribution of aggregate liquidity demands both before and after the merger. That can be achieved dividing \(X_{sq}\) and \(X_m\) by the sum of deposits. Let denote as

\[
X'_{sq} = \frac{\mathbb{P}_{i=3}^N \delta_i D_{sq}}{ND_{sq}} \tag{22}
\]
and
\[ X'_m = \delta_m D_m + \frac{P}{D_m + (N - 2)D_c} \] (23)
the normalised aggregate liquidity needs before and after the merger and as \( f'_n(X'_{sq}) \) and \( f'_n(X'_m) \) the corresponding normalised density function. Accordingly, let
\[ \rho = \frac{\rho}{ND_{sq}} = \frac{R_m + \sum_{i=3}^N R_c}{D_m + (N - 2)D_c} = 1 - \frac{rD}{r^D} \] (24)
be the share of aggregate reserves both before and after the merger. From (6), (22), (23) and (24), we have:
\[ \Omega_m - \Omega_{sq} = Z_1 (X'_m - \rho) f'_m(X'_m)d(X'_m) - Z_1 (X'_{sq} - \rho) f'_n(X'_{sq})d(X'_{sq}) \] (25)
\[ Z_1' \rho = X'_m f'_m(X'_m)d(X'_m) - X'_{sq} f'_n(X'_{sq})d(X'_{sq}) - \rho (1 - F'_m(\rho)) + \rho (1 - F'_{sq}(\rho)). \] (26)
Deriving (25) with respect to \( \rho \) gives
\[ \frac{d(\Omega_m - \Omega_{sq})}{d\rho} = -\rho f'_m(\rho) + \rho f'_n(\rho) - (1 - F'_m(\rho)) + \rho f'_m(\rho) + (1 - F'_{sq}(\rho)) - \rho f'_n(\rho) = F'_m(\rho) - F'_{sq}(\rho). \]
From proposition 3, \( F'_m(\rho) - F'_{sq}(\rho) > 0 \) for \( \rho < \frac{1}{2} \) and \( F'_m(\rho) - F'_{sq}(\rho) < 0 \) for \( \rho > \frac{1}{2} \). Also, \( F'_m(0) = F'_{sq}(0) = 0 \) and \( F'_m(1) = F'_{sq}(1) = 1 \). This implies \( \Omega_m - \Omega_{sq} > 0 \) for all \( \rho \in [0, 1] \).
Figure 2: Aggregate liquidity risk before and after merger