

Herd Behavior and Contagion in Financial Markets

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Abstract

Imitative behavior and contagion are well-documented regularities of financial markets. We study whether they can occur in a two-asset economy where rational agents trade sequentially. First, with gains from trade or uncertainty on the proportion of traders with private information, informational cascades arise and prices fail to aggregate information dispersed among traders. During a cascade all informed traders with the same preferences choose the same action, i.e., they herd. Second, sequential trading helps to explain contagion, since the correlation between prices can be higher than between fundamentals. Informational cascades and herds can spill over from one asset to the other, pushing the prices far from the fundamentals.

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During 1997 financial asset prices plunged in most emerging markets, following the financial crisis that hit some Asian economies. This high degree of co-movement across markets that are very different in size and structure and are located in different regions of the world is not a peculiarity of the Asian crisis. Indeed, it is a very common and well documented regularity of financial markets. Since falling asset prices are associated with recession and reduction in growth, it is important to try to understand the source of such co-movement.

There are two main theories explaining why prices in different markets are strongly correlated. The first is based on common aggregate shocks, such as a change in the level of international interest rates or in the price of commodities.

The second theory is based on contagion: co-movements are said to be driven by contagion whenever they cannot be explained by common aggregate shocks. For instance, in Masson (1997) a financial shock in one region can create a self-fulfilling expectation of a crisis in another region. A different mechanism relies on real or informational linkages. In Allen and Gale (2000) liquidity shocks in one region can spread, through the banking sector, to the whole economy. In Calvo (1999) asymmetric information among agents causes a shock to one asset to affect the price of another. In Kodres and Pritsker (1999) idiosyncratic shocks spill over from one market to the other because of cross-market hedging of macroeconomic risk.

Strong co-movements among seemingly unrelated financial assets are often linked to another widely recognized feature of financial markets, i.e., the imitative, herd-like behavior of its participants. Many practitioners would agree that imitative behavior is one reason why a crisis hitting one asset transmits itself to another. We think that herd behavior and contagion are closely related phenomena, both arising from the fact that rational traders with incomplete information trade sequentially.

For years, analyses based on the rational agent assumption have ruled out the presence of imitation, contagion, and other types of seemingly irrational behavior in markets. Indeed, in a scenario in which agents are rational, there seems to be no room for imitative behavior. Recently, many papers have attempted to reconcile the mainstream economics with the empirical and anecdotal evidence and tried to explain imitative behaviors in a world of rational agents. Focusing on the role of knowledge in markets, these papers (see, among others, Banerjee, 1992; Bickhchandani et al., 1992; Chamley and Gale, 1994; Chari and Kehoe, 2000) have studied the social learning effects of

actions taken by agents who act sequentially. When decisions are sequential, the earliest actions may have a disproportionate effect on the choices of the following agents and herd behavior may arise.

With few exceptions (e.g., Avery and Zemsky, 1998), however, this literature studies the decision to buy or to sell a good the price of which is given. Holding the price fixed makes these models unsuitable to analyze financial markets, where asset prices are certainly flexible. Avery and Zemsky study whether herds can arise in a sequential trading model à la Glosten and Milgrom (1985), in which the market maker modifies the price on the basis of the order flow. By allowing the price to react to the traders' decisions, they limit the possibility of herding since agents will always find it convenient to trade on the difference between their own information (the history of trades and the private signal) and the commonly available information (the history only). Therefore, it is never the case that agents neglect their information and imitate previous traders' decisions.

We build our analysis on Glosten and Milgrom (1985), extending their model to a two-asset economy. In doing so, we try to create a bridge between the literature on herd behavior and the literature on contagion in financial markets.

We show that, when agents trade not only for speculative reasons, i.e., they have gains from trade, prices do not converge to the fundamentals. This happens because after a sufficiently long time herding and informational cascades, i.e., situations where agents disregard their private information, arise. Informational cascades also occur when there is an informational asymmetry between the market makers and the traders on the proportion of informed traders in the market. Therefore, the main point of Avery and Zemsky, i.e., that prices destroy the possibility of informational cascades, is valid only when traders act for pure speculative reasons and have the same beliefs as the market makers on the composition of the market. During periods of informational cascades all informed traders will follow the decisions of the previous agents and choose the same action, i.e., they will herd.

Moreover, the history of trades in one market can affect the price path of another permanently and make it converge to a different value from what it would have been otherwise. Informational spillovers are to be expected between correlated asset markets. When there are asymmetries among markets participants, however, these informational spillovers can have pathological consequences. Informational cascades and herd behavior in one market generate cascades and herd behavior in another, pushing the prices, even in the

long run, far from the fundamentals. This long lasting spillover represents a form of contagion: a crisis or a boom in one market transmits itself to the other without regard to the fundamentals.

Finally, we show that the unconditional correlation between market prices is greater than the correlation of fundamentals. Sequential trading when information is incomplete generates excess correlation. Excess correlation is not necessarily the result of irrational behavior, but may be the result of the learning process of rational agents.

The structure of the paper is as follows. Section 1 describes the model and gives some preliminary results about the behavior of prices. Section 2 analyzes the case of gains from trade. Section 3 studies the case of uncertainty on the proportion of informed traders in the market. Section 4 concludes.

1. The model

1.1 The model structure

We base our analysis on Glosten and Milgrom (1985). They consider a market with only one asset. We extend their framework to study a two-asset economy. In our economy there are two assets, A and B , with true values V^A and V^B distributed according to a joint probability distribution function $p(V^A, V^B)$ with support $[m, M]^2$, $0 \leq m < M$. Prices for the two assets are set by two competitive market makers who interact with an infinite sequence of traders. At any time t , a trader is randomly chosen to act either in market A or in market B and can buy, sell or decide not to trade. Each trade consists of the exchange of one unit of the asset for cash. Any trader trades at most once. At time t , market makers and traders know the history of trades until time $t - 1$ (H_t) in both markets.

The assumption that only one unit is traded in each period simplifies the analysis considerably, as the action space of the trader reduces to selling, buying or not trading. This restriction prevents us from studying the importance of quantities in financial markets (see on this Caplin and Leahy, 1996). Nevertheless, we can capture some of the importance of volume by considering the case in which traders are chosen to trade in the same market for more than one period in a row. Indeed, the effect of successive unit trades in the same market can represent the effect of a single trade consisting in the exchange of more than one unit of the asset.

In our economy there are two types of traders: informed and uninformed traders. In both markets the proportion of informed traders is μ . Informed

traders are risk neutral. They receive a private signal on the asset that they are going to trade and maximize their expected profit based on that signal. Informed traders on asset $J = A, B$ observe a signal x^J distributed on $[m, M]$ according to the conditional probability function $q(x^J|V^J)$. Since the two assets are not independent, a signal on one asset gives some information also on the value of the other. Nevertheless, we say, for instance, that a trader is informed on asset A because the distribution of x^A conditional on V^A does not depend on the value of asset B , i.e., $q(x^A|V^A) = q(x^A|V^A, V^B)$. Uninformed (or noise) traders¹ trade for unmodeled liquidity reasons: they buy, sell or do not trade the asset with exogenously given probabilities.

The expected value for an informed trader called to trade on market J at time t is denoted by

$$V_t^J(x^J) = E(V^J|H_t, x^J). \quad (1)$$

On the other hand, the expected value for the market makers will be conditioned only on the public information available at time t , i.e., it will be

$$P_t^J = E(V^J|H_t). \quad (2)$$

We refer to the market makers' expectations as the prices of the assets. It is important to note that, since the action in the other market reveals some information on the value of the asset, each market maker will revise his expectations at time t even when his market was not open at time $t - 1$.

The market makers must take into account the possibility that they may trade with agents more informed than they are. Therefore, they will set a bid-ask spread between the prices at which they are willing to sell and to buy (see Glosten and Milgrom, 1985). Given that the market makers behave competitively, they will make zero expected profits. Hence, the bid and ask prices for asset $J = A, B$ will be

$$B_t^J = E(V^J|H_t, h_t^J = sell^J) \text{ and } A_t^J = E(V^J|H_t, h_t^J = buy^J), \quad (3)$$

¹The presence of noise traders is needed to guarantee that trading occurs. In our baseline model there are no gains from trade. Therefore, in the absence of liquidity traders no one would trade and the market would break down (as shown by the "no-trade" theorem of Milgrom and Stokey, 1982). Note that noise traders are a common feature of financial market models.

where h_t^J is the action in market J taken by the trader who arrives at time t . Note that each market maker computes the expected value conditional on past trades in both markets, since they are public information, and on the action taken in his own market. Each action taken by a trader may reveal some private information, since the actions of the informed traders depend on the signals.

Finally, following Avery and Zemsky (1998), we assume that, for any history, the signal always conveys some information about the value of the asset, i.e., as long as the market has not perfectly identified the values of the two assets, there exists at least one signal realization \hat{x}^J with $\Pr(\hat{x}^J|H_t) > 0$ such that $V_t^J(\hat{x}^J) \neq P_t^J$, for $J = A, B$. Moreover, for any $\delta > 0$, if $|P_t^J - V^J| \geq \delta$, there exists an $\varepsilon > 0$ such that $|P_t^J - V_t^J(\hat{x}^J)| > \varepsilon$. This means that, if the price is bounded away from the fundamental value, there will be a realization of the signal such that the trader's evaluation is bounded away from the price.

Before proceeding to the analysis of herd behavior and contagion, we must provide some results about the behavior of the market prices. These results are an immediate generalization of those obtained by Avery and Zemsky (1998) for the one-asset economy. In our model, at each time t , only one market is open and the history of trades in the other market is public information. Therefore, the computation of bid and ask prices is identical to that of a one-asset economy: at any time t , there exists a unique bid and ask price for the asset $J = A, B$ traded in that period, which satisfies $B_t^J \leq P_t^J \leq A_t^J$. The market maker takes into account that buying or selling orders contain private information and sets a spread between the price at which he is willing to sell and to buy. Equilibrium prices always exist because noise traders are willing to accept any loss and, therefore, markets will never shut down.

Moreover, given that market prices P_t^A and P_t^B are expectations based on all public information, they are martingales. Formally,

$$E(P_{t+1}^J|H_t) = E(E(V^J|H_{t+1})|H_t) = E(V^J|H_t) = P_t^J, \quad (4)$$

where the second equality comes from the law of iterated expectation. Given that prices are martingales they will converge almost surely to a random variable. In particular, it is possible to prove that prices converge almost surely to the true values V^J . We have assumed that a non trivial amount of private information always exists as long as the market does not perfectly identify the values of the two assets. Moreover, given that the market never breaks

down, all private information becomes public as time goes on. Therefore, the market prices must converge to the true values. In the long run, there is no room for herding that misdirects the prices.

1.2 One interesting feature of the behavior of prices

At a first glance, one can believe that, if there is a positive correlation between the fundamentals, the price in one market should rise after a history of buys in the other. On the contrary, even in our simple economy, this need not be the case. One can construct examples where a buy in market A leads to a fall in the price of asset B . Suppose that assets A and B take values $\{0, \frac{3}{4}, 1\}$ according to the following joint probability distribution:

	$V^B = 0$	$V^B = \frac{3}{4}$	$V^B = 1$	
$V^A = 0$	0.03	0.02	0.01	0.06
$V^A = \frac{3}{4}$	0.04	0.78	0.04	0.86
$V^A = 1$	0.02	0.03	0.03	0.08
	0.09	0.83	0.08	

Furthermore, the proportion μ of informed traders is 0.1 and signals are perfectly informative, i.e., $\Pr(x^J = i | V^J = i) = 1$, for $J = A, B$ and $i = 0, \frac{3}{4}, 1$. Noise traders buy and sell with probability $\frac{1}{2}$. From the joint probability distribution of the asset values we can calculate their correlation, which is 0.18.

In the first period the ask price for asset A , $E(V^A | buy_0)$, is 0.89. Suppose that there is a buy order in market A . It can come either from a noise trader or from an informed trader who knows that the true value of this asset is 1. Therefore, the market maker will update upward the probability that the value of asset A is equal to 1, and downward the probability that the value is 0 or $\frac{3}{4}$. Although the market maker's assessments of the value of asset A being $\frac{3}{4}$ and 1 move in opposite direction, the expected value of asset A after a buy must increase (because it is equal to the ask price, which is always greater than the expected value of the asset): in fact, it rises from 0.78 to 0.89.

Given that the joint probability distribution of the two asset values implies that $\Pr(V^B = v | V^A = v) \geq \frac{1}{3}$, the market maker's belief that B takes values 0, $\frac{3}{4}$ or 1 moves in the same direction as that of asset A . In the case of asset B , however, the increase in the probability of $V^B = 1$ is more than

offset by the decrease in the probability of $V^B = \frac{3}{4}$ and the expected value of asset B goes down, from 0.660 to 0.658.

Therefore, although the two assets are positively correlated, their prices need not move in the same way. Paradoxically, it may happen that a sequence of buys in a market generates a crisis in another market, even if the fundamentals are positively correlated.

This characteristic of Bayesian updating makes the analysis of the covariance between two asset prices difficult. Thus, in some cases, we will restrict our attention to situations where the two assets move in the same direction. In particular we will find the following lemma useful:

Lemma 1 Suppose that assets A and B are distributed on $\{m, M\}^2$ and are positively correlated. Then, the prices of the two assets always move in the same direction.

Proof. See the Appendix.

When assets can take only two values, one “low” and one “high”, only an informed trader with the information that the asset value is high will buy. The probability of the high value is revised upward. Because of the positive correlation, the market maker of the other asset also updates in the same way the probability of the high value. Therefore, both expected values increase.

1.3 Contagion

In a one-asset economy it is possible to prove that the volatility of the price is bounded by the fundamental uncertainty about the value of the asset (see Avery and Zemsky, 1998). This result can easily be generalized in our two-asset economy. Therefore, in our model it will not be possible to generate volatility in excess of that of the fundamentals.

The covariance between asset prices cannot exceed the fundamental covariance either. We study the covariance between prices, conditional on the information available at time 0, and compare it with the covariance between the fundamentals. For tractability, we restrict our analysis to an economy where the assets take only two values: in this case, as we know by Lemma 1, after a trade asset prices move in the same direction. We can prove the following proposition:

Proposition 1 *Suppose that assets A and B are distributed on $\{m, M\}^2$. If the assets are positively correlated, the covariance between prices is always lower than the covariance between the fundamentals, i.e., at any time t ,*

$$0 \leq \text{Cov}(P_t^A, P_t^B) \leq \text{Cov}(V^A, V^B). \quad (5)$$

Moreover, the covariance between prices is non decreasing over time, that is, at each time t ,

$$\text{Cov}(P_t^A, P_t^B) \leq \text{Cov}(P_{t+1}^A, P_{t+1}^B). \quad (6)$$

Conversely, if the assets are negatively correlated,

$$\text{Cov}(V^A, V^B) \leq \text{Cov}(P_t^A, P_t^B) \leq 0, \quad (7)$$

$$\text{Cov}(P_t^A, P_t^B) \geq \text{Cov}(P_{t+1}^A, P_{t+1}^B). \quad (8)$$

Proof. See the Appendix.

The covariance between the two asset prices at time t is bounded by the covariance between the fundamentals. When the fundamental covariance is greater (smaller) than zero, the covariance between the two asset prices is monotonically increasing (decreasing) over time. Given that prices converge almost surely to the fundamentals, the covariance itself will converge asymptotically to the covariance between the fundamentals.

The fact that the covariance is bounded, however, does not imply that the two assets co-move less than the fundamentals. Relative to their variance, the asset prices will covary more than the fundamentals would imply. This is shown in the following proposition:

Proposition 2 *Suppose that assets A and B are distributed on $\{m, M\}^2$ and are positively correlated. At time $t = 1$ the correlation between prices is 1. For $t \rightarrow \infty$, the correlation converges to the fundamental correlation.*

Proof. See the Appendix.

Moreover, through simulation², we can show that the unconditional correlation is greater than the correlation of fundamentals not only at time $t = 1$, but for any time t (see Figure 1).

Intuitively, each asset value is the sum of two components, one common and one idiosyncratic. The market learns the realized values of the two components over time. At the beginning, agents learn the common component

²The parameters of the simulation are reported in the Appendix.

and only over time will they discover the idiosyncratic ones. Therefore, the correlation of prices will be higher than the correlation of fundamentals. Over time, while the idiosyncratic component is being discovered, the correlation will converge to the fundamental correlation.

Sequential trading when information is incomplete helps to explain excess correlation. As long as there is uncertainty about the fundamental values of financial assets, the correlation between prices will be more extreme than the fundamentals would imply. Many empirical studies of financial markets show that asset prices are strongly correlated. Our model suggests that this correlation is not necessarily the result of irrational behavior or frictions in the markets, but may be the result of the learning process of rational agents.

1.4 Herd behavior and informational cascades

In our baseline model prices converge over time to the fundamentals. This happens because the market makers set the bid and ask prices in such a way that at least some of the informed traders will find it convenient to act upon (and therefore reveal) their private information.

It is important to understand whether in this framework there can be imitative behavior, i.e., a situation in which a trader finds it optimal to follow the decision of the previous agents. The concept of herd behavior is closely related to that of informational cascade (see Gale, 1996), a situation where the action taken by a trader is independent of the values of the assets and, therefore, does not reveal any information on them.

We give the following definitions of informational cascade:

Definition 1 *An informational cascade on asset J arises in period t when $\Pr(h_t^J | V^A, V^B, H_t) = \Pr(h_t^J | H_t)$ for all $V^A, V^B \in [m, M]$ and for all h_t^J .*

An informational cascade requires that the actions be independent of the asset values³. If this happens, all informed traders will choose the same action, that is, they will herd⁴.

³In the literature, the standard definition of cascade is that the action taken by a trader does not depend on the signal that he received. Our definition is equivalent to that. If a trader is making use of his signal, his action depends on the asset value (given that the signal itself depends on it). On the other hand, if the action is independent of the asset value, then the trader is not making use of the signal that he received.

⁴Our notion of herd behavior is the one usually adopted in the literature. It differs from Avery and Zemsky's (1998), according to which an informed trader engages in herd

Now we prove that in this baseline model not only do prices always converge to the fundamentals, but herd behavior and informational cascades never happen.

Proposition 3 *An informational cascade will never arise in either market A or market B.*⁵

Proof. See the Appendix.

Informed traders and the market maker compute the expected value of the asset conditional on the same history of trades. Informed traders, however, have an additional source of information, i.e., the signal on the asset value. By assumption, this signal is informative enough to make traders' expectations different from the market maker's and, thus, traders will act upon it. Since the signal depends on the realized value of the asset, the action of the traders cannot be independent of it.

Given that an informational cascade never occurs, the probability of an action will always depend on the realized value of the asset. Therefore, there cannot exist a time t and a history H_t after which all informed traders buy or sell with probability 1. The absence of informational cascades makes herd behavior impossible.

Although our baseline model generates correlation in excess of fundamentals, it is unable to explain why rational traders may engage in imitative, herd-like behavior. As long as the market maker sets a correct bid and ask price, imitation is ruled out. In the next sections of the paper we will argue that this result depends on two crucial simplifying assumptions, namely, that traders act only for speculative reasons and that there is no uncertainty on the number of traders who have private information. We will show that in a more complex environment both herd behavior and informational cascades can arise.

2. Gains from trade

In our benchmark model the only asymmetry between the traders and the market makers is due to the fact that informed traders observe a signal on

behavior at time t if he buys when $E(V^J|x^J) < P_0^J < P_t^J$ or if he sells when $E(V^J|x^J) > P_0^J > P_t^J$. E.g., there is herd buying when a trader upon receiving his signal sells, but after seeing the price rise changes his mind and buys.

⁵This proposition is a straightforward generalization of Proposition 2 of Avery and Zemsky (1998).

the realized values of the assets. In this section, we introduce an additional source of asymmetry: gains from trade. In the market there are traders who enjoy an extra utility and agents who suffer a disutility from holding the asset. Formally, agents with an extra utility $g > 1$ from asset $J = A, B$ find it convenient to buy whenever

$$gE(V^J|H_t, x^J) > A_t^J, \quad (9)$$

and find it convenient to sell whenever

$$gE(V^J|H_t, x^J) < B_t^J. \quad (10)$$

On the other hand, agents with a disutility $0 < l < 1$ from the asset find it convenient to buy whenever

$$lE(V^J|H_t, x^J) > A_t^J, \quad (11)$$

and to sell whenever

$$lE(V^J|H_t, x^J) < B_t^J. \quad (12)$$

The type is a random variable whose distribution is common knowledge independent of the distribution of the assets and of the signal. Only the trader knows his own type.

The first type of trader buys even when his expected value of the asset is lower than the ask, because he enjoys extra utility from the asset, which is not enjoyed by the market makers. The second type, on the contrary, sells even when his expected value is higher than the bid, because he receives a disutility from holding the asset. This is consistent with what happens in financial markets, where agents do not buy or sell only for speculative reasons. E.g., a German importer may buy dollars not because he thinks that the dollar is undervalued, but because he needs that currency to buy goods in the US market. On the other hand, a US importer may be willing to sell dollars because he needs foreign currency. In their seminal paper, Glosten and Milgrom (1985) study a market with a similar kind of asymmetric valuation of the asset. As they do in their paper, we can also interpret the parameters g and l as stemming from different preferences of agents for present and future consumption, different hedging strategies or different assessments of the distribution of the asset value.

Agents with gains from trade are somehow in between pure speculative traders and noise traders. They choose their actions by taking into account their private information (as speculators do) and the exogenously given gain from trade (as noise traders do).

In the previous section we observed that noise traders are necessary for trade to take place. The presence of gains from trade makes trade possible even without noise traders by allowing the market maker to set a bid and ask spread at which he makes zero profit. At a given ask those traders with a negative signal and whose gains from trade are high enough to offset the expected loss would buy. Therefore, it is not true that the market makers suffer an expected loss each time they buy or sell to an uninformed trader. Although we could build a model without noise traders, we keep them to maintain a setup as close as possible to that presented in the previous section.

The presence of gains from trade alters the properties of the price path substantially. In the next sections we will show that with gains from trade herds and informational cascades arise and prices fail to converge to the fundamental values.

2.1 Informational cascades and herd behavior

The presence of gains from trade destroys the convergence result of our baseline model. In particular, we can show that there will be a moment when no further information reaches the market and, as a consequence, prices become constant. Moreover, there is no guarantee that the levels at which prices settle will be anywhere near the fundamental values of the two assets.

We can show that, under a technical assumption, any history of trades will go through periods of informational cascades. After a sufficiently large number of trades, the valuation of the traders and of the market makers will be so close that all informed traders with an extra utility will decide to buy, independently of their signal, in order to enjoy the extra utility from the asset. On the other hand, all the traders with a disutility from holding the asset will sell. The probability of an action will be independent of the signal that the trader receives and, thus, of the realized value of the assets. This result is proven in the following proposition:

Proposition 4 *If $m > 0$ and if there does not exist a realization of the signal \hat{x}^J and of the asset values \hat{V}^J such that $\Pr(\hat{x}^J|\hat{V}^J) = 0$, an informational cascade occurs with probability 1.*

Proof. See the Appendix.

The condition $m > 0$ is needed only to avoid the case in which the gain and loss from trade $-gE(V^J|H_t, x^J)$ and $lE(V^J|H_t, x^J)$ —vanish because the expectations of the traders converge to zero. The condition $\Pr(\hat{x}^J|\hat{V}^J) \neq 0$ rules out too informative signals, that is, signals that tell the traders that some realizations of the asset value cannot have occurred. Upon receiving one of these signals a trader would disregard all past history and his expectation would diverge from the market maker's. Note that $\Pr(\hat{x}^J|\hat{V}^J) \neq 0$ is only a sufficient condition for the informational cascade to arise, by no means a necessary one. Example 1 illustrates a case in which an informational cascade does arise even though the signal is perfectly informative.

An informational cascade can be misdirected, that is, it can occur when the price is far away from the realized value of the asset. In the following example, the asset can take values 1 and 2 and the realized value of the asset is 2. However, a history of sells triggers an informational cascade and the price remains stuck at a value close to 1.

Example 1 *For simplicity's sake, we consider an economy with only one asset J that takes only values 1 and 2 with equal probability. Informed traders receive perfectly informative signals. The proportion of informed traders is $\frac{1}{2}$. Noise traders buy or sell with equal probability $\frac{1}{2}$. Finally, there is a gain from trade $g = \frac{6}{5}$, whereas no one suffers disutility from holding the asset. Suppose that the realized value of the asset is 2 and that in the first period there is a sell order.*

At time 0, the price is equal to the unconditional expectation, i.e., $P_0^J = \frac{3}{2}$. The bid and ask prices are $A_0^J = \frac{7}{4}$ and $B_0^J = \frac{5}{4}$.

At time 1, in the absence of gains from trade, after observing the sell order, the market maker would update the price to $P_1^J = \frac{5}{4}$ and the ask and bid prices to $A_1^J = \frac{3}{2}$ and $B_1^J = \frac{11}{10}$. With gains from trade, however, all informed traders would buy irrespective of the signal that they receive, since their gain from trade is greater than the ask price. Therefore, trading becomes uninformative and the market maker will not post different bid and ask prices. Instead, the posted price will be $P_1^J = \frac{5}{4}$, equal to the expected value of the asset conditional on the history at time 1.

Note that, during an informational cascade, informed agents herd, that is, all agents of the same type act alike. We can prove the following corollary:

Corollary 1 During an informational cascade, there is herd behavior, i.e., all informed agents with a gains from trade g buy and all informed agents with a loss from trade l sell.

Proof. See the Appendix.

Given that agents have different gains from trade, in the corollary a herd is not characterized as the case where all agents make the same decision. Indeed, what is relevant is not the traders' decisions per se, rather the use that they do of their own private information in making this decision. Therefore, following Smith and Sørensen (2000), we say that agents herd when, *conditional on their types*, they choose the same action.

Gains from trade cause a blockage of the information flow because they introduce a wedge between the utility that the market makers and the traders obtain and forgo when they exchange the assets. In Ho Lee (1998) shows that a similar wedge would exist if one introduces trade costs. Even if trade costs were relevant in financial markets, however, they could not explain herd behavior by themselves. With trade costs, when the traders' and the market makers' expectations converge, traders stop trading. Information ceases to flow to the market only because the market shuts down. Herd behavior would never be observed.

It is important to note that in a one-asset economy an informational cascade never ends. After the cascade has started, no information arrives at the market and agents face the same decision problem in each period. On the other hand, in a two-asset economy, even when there is an informational cascade on one asset, the history of trades of the other reveals some information. When there is a cascade only in market A , traders will still act in market B according to their own signals. If the two assets are not independent, the market learns not only on asset B , but also on asset A . The price of asset A will move despite the informational cascade. Moreover, the trades of asset B , by moving the price of asset A , can make the valuation of traders and market maker diverge, thus breaking the cascade.

Proposition 5 *The history of trades on one asset can break an informational cascade on the other.*

Proof. We prove this through an example. Assets A and B take values 0 and 1. The distribution of the values of the assets is the following:

	$V^B = 0$	$V^B = 1$	
$V^A = 0$	$\frac{7}{16}$	$\frac{1}{16}$	$\frac{1}{2}$
$V^A = 1$	$\frac{1}{16}$	$\frac{7}{16}$	$\frac{1}{2}$
	$\frac{1}{2}$	$\frac{1}{2}$	1

Note that the two assets are not independent and, therefore, the history of asset A will reveal some information on asset B and vice versa. The proportion of noise and informed traders is $\frac{1}{2}$. Informed traders on asset A receive a signal distributed as follows: $\Pr(x_A = k|V^A = k) = \frac{5}{8}$ and $\Pr(x_A = k|V^A \neq k) = \frac{3}{8}$, for $k = 0, 1$. Informed traders on asset B receive a perfectly informative signal, i.e., $\Pr(x_B = k|V^B = k) = 1$, for $k = 0, 1$. Noise traders buy or sell with probability $\frac{1}{2}$. Finally, there is a gain from trade $g = 1.7$, whereas no one has disutility from the asset. Suppose that at time 0 market A is open and at time 1 market B is open.

At time 0, prices are equal to the unconditional expectations, i.e., $P_0^A = \frac{1}{2}$ and $P_0^B = \frac{1}{2}$. The ask and bid prices for asset A are $A_0^A = B_0^A = \frac{1}{2}$.⁶ Due to the gain from trade, every informed trader is willing to buy the asset and the market is in a situation of informational cascade. Given that the action at time 0 is not informative, prices at time 1 will remain unchanged, i.e., $P_1^A = \frac{1}{2}$ and $P_1^B = \frac{1}{2}$. The bid and ask prices for asset B are $A_1^B = \frac{3}{4}$ and $B_1^B = \frac{1}{4}$. Suppose that a buy order arrives in market B . At time 2, the price of asset B becomes $P_2^B = \frac{3}{4}$. Given that the two assets are correlated, also the price of asset A will move to $P_2^A = \frac{11}{16}$. This movement is indeed sufficient to break the informational cascade in market A . The expected value of informed traders with a low signal is lower than $\frac{11}{16}$, even taking into account the gain

⁶The computation of bids and asks is done as follows:

$$A_0^A = E(V^A|buy_0^A) = \Pr(V^A = 1|buy_0^A).$$

By Bayes rule, this is equal to:

$$\frac{\Pr(buy_0^A|V^A = 1)\Pr(V^A = 1)}{\Pr(buy_0^A|V^A = 1)\Pr(V^A = 1) + \Pr(buy_0^A|V^A = 0)\Pr(V^A = 0)}.$$

If only informed traders with a good signal were to buy, this expression would be equal

$$\text{to } \frac{(\frac{1}{2}\frac{1}{2} + \frac{1}{2}\frac{5}{8})\frac{1}{2}}{(\frac{1}{2}\frac{1}{2} + \frac{1}{2}\frac{5}{8})\frac{1}{2} + (\frac{1}{2}\frac{1}{2} + \frac{1}{2}\frac{3}{8})\frac{1}{2}} = \frac{9}{16}.$$

With a ask price of $\frac{9}{16}$, however, informed traders with a bad signal would buy as well, since $g\frac{3}{8} > \frac{9}{16}$, so this cannot be an equilibrium ask price. If the market maker expects all types of informed trader to buy, then he sets the ask equal to $\frac{1}{2}$. With this ask, all informed trader buy and so this is the unique equilibrium ask price. Given that only noise traders may want to sell, the bid price will be equal to the unconditional expectation, i.e., $\frac{1}{2}$.

from trade. Therefore, there will exist a bid price at which they sell. The informational cascade is broken. Q.E.D.

Our model gives some insights on how financial markets may recover. After a crisis, frictions in a market can make trading completely uninformative. Without observing trading in the other market, the price of the asset would remain undervalued even though traders receive new positive information. Trading in the other markets, however, by revealing some new information, can help the market to recover. A positive history of trades in the other markets can lead to an increase in the price of the asset. After the price starts to rally, frictions cease to be binding and the normal flow of information to the market resumes.

Given the previous result, it is important to distinguish the case of an informational cascade in only one market from the case of an informational cascade in both markets. In the latter case, no new information will reach the markets and the cascades will last for good. We refer to the case of informational cascades in both markets as an “informational breakdown”.

Definition 2 *An informational breakdown arises when there is an informational cascade in both markets.*

We can show that an informational breakdown will happen with probability one and that it will be misdirected with positive probability. The argument is similar to that for the informational cascade. The implication, however, is much more far-reaching. Whereas an informational cascade blocks the flow of information only temporarily, the informational breakdown, once arisen, never ends. Therefore, the prices can remain stuck forever at levels far from the realized values. If the informational breakdown is misdirected, the markets will never correct their valuations and will never learn the true values of the assets. We can prove the following proposition:

Proposition 6 *If $m > 0$ and if for $J = A, B$ there does not exist a realization of either signal \hat{x}^J and of either asset value \hat{V}^J such that $\Pr(\hat{x}^J | \hat{V}^J) = 0$, an informational breakdown occurs with probability 1.*

Proof. See the Appendix.

Above we have shown that informational cascades can be misdirected. By the same argument an informational breakdown can happen when prices are far away from the fundamentals. In this case, prices will remain stuck

forever at those levels and the market will never learn the true values of the assets.

The different valuation of the assets among agents makes the price mechanism informationally inefficient. It is important to note, however, that if in the market there are some speculators, i.e., agents without gains from trade, the prices will converge to the true values. Even when the bid and ask prices become close, a trade reveals some information, as it can come from an agent who acts purely because of his signal. The thrust of our result, however, would still hold, because the informational content of trade would be small given that traders with a gain from trade would behave as uninformed traders, acting independently of their private information.

Avery and Zemsky (1998) argue that, when prices aggregate private information, informational cascades cannot occur. This is also the result of our baseline model. This result is no longer true when we allow the valuation of the trader and the market makers to diverge. By adding a small friction in the market, the convergence result is destroyed and the information contained in the signals does not flow to the market. Even in a market in which the price of the asset is not fixed, informational cascades can arise, as long as traders do not trade only for speculative reasons.

2.2 Contagion

Gains from trade can generate a long lasting spillover effect from one asset to the other. The very fact that traders and market makers are able to see the history of trades in another market can cause the price mechanism to fail. The presence of more than one market can make the flow of information stop early and the market price remain stuck at a wrong level. We regard this as a form of contagion. More precisely, we say that there is a *contagious spillover* when there is an informational cascade on one asset that would not have occurred if agents were able to observe only the history of that asset. That is, the informational cascade happens only because agents are able to observe the history of both markets.

The contagious spillover can have permanent effects. If the informational cascade on one asset, caused by the spillover effect, happens together with an informational cascade on the other asset, an informational breakdown arises. The price will remain stuck at a wrong valuation forever. Gains from trade make it possible for the history of one asset to have everlasting effects on the price path of the other.

Let us denote H_t^B the history of trades on asset B until time $t - 1$. We give the following definition of contagious spillover:

Definition 3 *There is a contagious spillover from market A to market B at time t when there is an informational cascade in market B and there exist $\hat{V}^{B'}$ and $\hat{V}^{B''}$ such that $\Pr(h_t^B | \hat{V}^{B'}, V^A, H_t^B) \neq \Pr(h_t^B | \hat{V}^{B''}, V^A, H_t^B)$ for some V^A and some h_t^B .*

We now present an example of a history of trades in which a contagious spillover arises. If traders in market B were not able to observe the history of asset A , the price of asset B would converge towards its fundamental value. Given that traders are able to observe the history of both assets, however, the initial sells on A cause an informational cascade on both assets A and B , i.e., an informational breakdown of the market. The price of asset B is stuck for ever at a level below the fundamental value and its initial fall is not reversed, not even in the long run.

Example 2 *Assets A and B can take values 1 and 2. The distribution of the asset values is the following:*

	$V^B = 1$	$V^B = 2$	
$V^A = 1$	$\frac{7}{16}$	$\frac{1}{16}$	$\frac{1}{2}$
$V^A = 2$	$\frac{1}{16}$	$\frac{7}{16}$	$\frac{1}{2}$
	$\frac{1}{2}$	$\frac{1}{2}$	1

The proportion of noise and informed traders is $\frac{1}{2}$. Informed traders receive a perfectly informative signal on the value of the asset, i.e., $\Pr(x^J = k | V^J = k) = 1$, for $k = 1, 2$ and $J = A, B$. Noise traders buy or sell with probability $\frac{1}{4}$ and decide not to trade with probability $\frac{1}{2}$. Finally, there is a gain from trade $g = \frac{10}{7}$, whereas no one suffers a disutility from holding the asset.

Suppose that the realized value of asset B is 2 and that market A is open in periods 0, 1, and 3 and market B is open in period 2. Moreover the first two trades in market A are sells.

At time 0, prices are equal to the unconditional expectations, i.e., $P_0^A = \frac{3}{2}$ and $P_0^B = \frac{3}{2}$. The bid and ask prices for asset A are $A_0^A = \frac{11}{6}$ and $B_0^A = \frac{7}{6}$.

At time 1, after observing the sell order in market A , market makers update the prices to $P_1^A = \frac{7}{6}$ and $P_1^B = \frac{5}{4}$. The ask and bid prices for asset A are $A_1^A = \frac{3}{2}$ and $B_1^A = \frac{27}{26}$. After the new sell order in market A , at time

2 prices become $P_2^A = \frac{27}{26}$ and $P_2^B = \frac{11}{10}$. If only traders receiving a positive signal bought asset B , the ask price for this asset would be $A_2^B = \frac{19}{14}$. At this ask price, however, traders with a negative signal would buy too, since $g = \frac{10}{7} > \frac{19}{14}$. Given that all informed traders want to buy, there is an informational cascade in this market. Since the action taken in market B at time 2 does not reveal any information, prices do not move, i.e., $P_3^A = \frac{27}{26}$ and $P_3^B = \frac{11}{10}$. At time 3, the ask price for asset A would be $A_3^A = \frac{7}{6}$. Every informed, however, trader would buy at this price, independently of the signal: market A is under an informational cascade as well. The first sell orders in market A have led to an informational breakdown. If the history of trades in market A were not observable, and there were buy orders in market B from time 2 onwards, the price of asset B would converge to the fundamental value. It is the observable history of sells in the other market that causes the price of asset B to remain forever stuck at $\frac{11}{10}$, far away from the fundamental value of 2.

Figure 2 shows a simulated path of the price of asset B for two different cases: when the agents in market B are not able to observe the history of market A (solid line) and when they are able to do so (dotted line)⁷. When both histories of trades are observed, the fall in the price of asset A makes the price of B fall below the level at which an informational cascade arises and remain stuck there (far below the fundamental value of 2). On the other hand, if the market maker does not observe the history in market A , the price of B remains above the level at which an informational cascade arises and eventually converges to $\frac{19}{10}$, a value close to the fundamental.

The previous example also gives some hints on the way the volume traded in one market may affect the other. As the example shows, when the same market is open for more than one period, the transactions in that market can have a stronger impact on the other. A long sequence of buys or sells in the same market can be thought of as a large amount bought or sold in that market. Our results suggest that the exchange of large quantities in a market can have long lasting effects on another.

In the previous section we have shown that the unconditional variance and covariance of prices are bounded by the covariance of fundamentals. On the contrary, the unconditional correlation of asset prices is greater than the fundamental correlation. In the long run, however, without gains from

⁷The parameters of the simulation are shown in the Appendix.

trade the correlation between the prices converges to the one between the fundamentals. By simulating the model, we are able to show that gains from trade generate an unconditional correlation that is higher than the fundamental correlation even asymptotically.

As figures 3 and 4 show, both the variances and the covariance of prices converge to values lower than the variances and the covariance of fundamentals. With gains from trade, prices may not converge and will tend to stay closer to their unconditional expected values: therefore, they will vary and covary less. For example, if the parameters g and l were equal to $\frac{M}{m}$ and $\frac{m}{M}$, no information would ever be revealed because traders with a gain would buy irrespective of their expectation and traders with a loss would sell. Prices would remain stuck at their unconditional expected values and their variances and covariance would be zero. The simulation of the model shows that when the gain and loss from trade are lower the asymptotic variances and covariance are greater than zero, but lower than if there were no gains from trade.

As for the case of no gains from trade, the correlation between prices is higher than the fundamentals' correlation (figure 5). However, with gains from trade it does not converge to the fundamentals' correlation, but remains permanently greater. In the baseline model we showed that, as long as the true values are not known, the correlation between prices is higher than that between the fundamentals. Since with gains from trade the true values are not discovered, the correlation is in excess of that of the fundamentals even in the long run: the fact that informational cascades arise and prices do not converge to the fundamentals causes the correlation between prices to be permanently higher than the correlation between the fundamentals. Therefore, with sequential trading contagion can occur also as a long run phenomenon.

3. Informational uncertainty

In the previous section we have shown that an asymmetry between the market makers' and the traders' valuations of the asset can generate an informational cascade and a breakdown of the market mechanism. Moreover, informational cascades can spill over from one asset to the other, a phenomenon of contagion. In this section we show that a similar result can be obtained when there is uncertainty on the proportion μ of informed traders in the market. With this informational uncertainty, however, contrary to the

case of gains from trade, informational cascades and herding do not imply a long run misalignment of prices with respect to the fundamentals, that is, over the long run, markets will learn the realized values of the assets.

In our baseline model and in the model with gains from trade, the proportion of informed traders (μ) is known to the traders as well as to the market makers. In this section we relax this assumption. In financial markets the presence of informed traders changes over time, even during a single trading day. The chances that the counterpart in a trade is an informed trader are rarely known. This is especially true in emerging markets or in markets for newly issued securities, where the amount of information spread among market participants can at best be guessed. Informed traders can have a different (often more accurate) belief than the market maker on the likelihood that a trade comes from other informed traders. For example, a trader informed about a new event, e.g., a merger of two firms or a technological innovation, may have a better idea than the market maker of how much this information is shared by other market participants.

When there is uncertainty about the proportion of noise traders, the behavior of prices is significantly altered. This happens because the uncertainty will generate asymmetric beliefs between the market makers and the traders on the degree of informativeness of the market. In particular the informed trader updates his belief that the market is informed, taking into account the fact that he himself is informed. For example consider the case where μ takes two values, μ^H and μ^L , with probability q and $(1 - q)$. The distribution of μ is common knowledge. At time 0, the probability that the market maker assigns to $\mu = \mu^H$ is q . On the other hand, the probability that the trader assigns to $\mu = \mu^H$ is

$$Pr(\mu = \mu^H | \text{trader is informed}) = \frac{\mu^H q}{\mu^H q + \mu^L (1 - q)}. \quad (13)$$

It is immediate that $Pr(\mu = \mu^H | \text{trader is informed}) > q$. Therefore, the beliefs of the trader and of the market maker on the probability of the market being informed are different. It is easy to show that the same asymmetry arises if μ is a continuous random variable.

The additional informational asymmetry generated by the fact that μ is not known will cause the market maker and the traders to update their valuations of the assets in a different way. For instance, if there are many buy orders arriving at the market, the market maker, who thinks that only

few traders are informed, revises the price very little. On the other hand, the traders believe that the market is better informed and interpret these buy orders differently. Therefore, they think that the price posted by the market maker is too low. In this situation, they may find it convenient to buy the asset even if they have negative private information and informational cascades can arise. Moreover, informational cascades can spill over from one asset to the other, generating an informational breakdown. The result is similar to what we have shown in the case of gains from trade. As soon as we introduce an asymmetry between the traders and the market maker, be it a gain from trade or an informational asymmetry, informational cascades can arise and spillovers from one asset to the other can have long-lasting consequences. Thus, the presence of informational cascades seems to be a robust feature of financial markets. Different kinds of asymmetries among financial market participants make the price mechanism unable to offset the incentives of agents to withhold their private information and follow the behavior of the previous agents.

There is, however, an important difference between gains from trade and informational uncertainty. With gains from trade the informational breakdown, once arisen, never ends and prices fail to converge to the fundamentals. On the contrary, in the case of informational uncertainty, even during an informational breakdown the market is still learning something, namely, the actual proportion of informed traders. After a certain number of trades the beliefs of the market makers and of traders on μ will be so accurate-and, therefore, so close-that the market will work as in the case analyzed in the baseline model: the informational breakdown will end and prices will resume converging to the fundamentals.

For simplicity's sake, in the remainder of this section we assume that there are two states of the world, U^0 (uninformed markets) and U^1 (informed markets). In U^0 no trader is informed ($\mu = 0$), whereas in U^1 there are both informed and uninformed traders. Nature chooses the state of the world U^0 with probability q , $0 < q < 1$. If U^1 is realized, there is an informational asymmetry between informed traders and the market makers, given that informed traders know that the state of the world is U^1 , whereas the market makers do not. This characterization of informational uncertainty is not new in the literature. For example, Easley and O'Hara (1987) adopt it in a Glosten and Milgrom type of model to study the role of trade size. Avery and Zemsky (1998) use a similar structure to show, through simulation, that there can be histories of trades in which bubbles arise.

The fact that informed traders know for sure that they are in U^1 (because $Pr(informed|U^0) = 0$) makes our model simple, as only the beliefs of the market makers will change over time. As we have explained above, however, our analysis is more general, because the mere fact that there is uncertainty on μ is enough to generate asymmetric beliefs between the traders and the market makers.

When facing a buy or a sell order, the market makers update their expectations that U^0 was realized conditioning on the history of trades and on the buy or sell order itself; that is, they will compute $Pr(U^0|H_t, h_t)$.⁸ Using this updated probability, they will be able to compute the bid and ask prices.

With informational uncertainty, as in the baseline model, prices are martingales with respect to the past history of trades. In addition, also $Pr(U^i|H_t)$ is a martingale, since, for $i = 0, 1$,

$$E\left(\Pr\left(U^i|H_{t+1}\right)|H_t\right) = \sum_{h_t} \Pr\left(U^i|H_t, h_t\right) \Pr\left(h_t|H_t\right) = \Pr\left(U^i|H_t\right). \quad (14)$$

The market makers have to learn not only the true values of the assets, but also the level of informativeness of the market. In the learning process, they use all the information provided by the trades. Therefore, both the prices and the probability of an informed market are martingales with respect to the history of trades.

With this result, we can prove the following proposition:

Proposition 7 *If the market is informed, prices converge almost surely to the realized values of the assets and the market makers' belief, $Pr(U^1|H_t)$, converges almost surely to 1.*

Proof. See the Appendix.

The new dimension of uncertainty does not prevent the market from learning the true values of the assets. Eventually the uncertainty is resolved, that is the market makers will learn whether they are in an informed or uninformed market with probability one. At this point, the market makers face the same problem as in the baseline model and prices will converge to the fundamentals.

⁸For simplicity's sake, we assume that markets are either both informed or both uninformed. All our results, however, only require that the events that market A and B are informed be not independent.

Although prices converge to the fundamental values, an informational cascade can arise. This happens because the market makers do not know whether the markets are informed, whereas informed traders do. After a trade, the market makers update the prices less than informed traders do. Therefore, traders might decide not to follow their signals and trade according to the past history. In the following example we show how an informational cascade arises and is eventually broken.

Example 3 *For simplicity's sake, let us consider an economy with only one asset A that takes values 0 and 1 with equal probability. The probability of being in the state of the world U^0 is 0.99, but the state U^1 has been realized.⁹ The proportion of noise traders in the informed market is $\frac{1}{4}$. Noise traders buy, sell or do not trade with equal probability. Informed traders receive a signal according to the following conditional distribution: $\Pr(x^A = k|V^A = k) = 0.9$, for $k = 0, 1$. We consider a history of four buy orders, all in market A .*

At time 0, the price is equal to the unconditional expectation, i.e., $P_0^A = \frac{1}{2}$. Conditional on receiving a buy or a sell order, the market maker will update downward the probability that the market is uninformed. In an uninformed market a buy and a sell happen each with probability 0.33, while in an informed market they happen with a higher probability, given that at time 0 informed traders never decide not to trade. Therefore, the probability of being in an uninformed market will become $\Pr(U^0|\text{buy}_0^A) = \Pr(U^0|\text{sell}_0^A) = 0.986$. The ask and bid prices will be $A_0^A = 0.51$, $B_0^A = 0.49$. The spread stems from the usual fact that informed traders will buy only if they receive a positive signal and will sell only if they receive a negative one.

At time 1, after receiving a buy order, the market maker will update his belief and the price in the same fashion. This time, conditional on receiving a buy, he will revise down his belief that the market is uninformed more than conditional on receiving a sell; that is, $\Pr(U^0|\text{buy}_0^A, \text{buy}_1^A) = 0.97 < \Pr(U^0|\text{buy}_0^A, \text{sell}_1^A) = 0.98$. The ask and bid prices will be $A_1^A = 0.52$, and $B_1^A = 0.51$. Note that the bid and ask prices barely move. This is because the market maker has a strong a priori that the market is uninformed. Moreover, the probability of being in an uninformed state of the world declines slowly, as only the lower probability of no trades in an informed than in an uninformed market makes it decline.

⁹We choose an extreme value for $\Pr(U^0)$, 0.99, just to make the example short, with an informational cascade arising after only two periods.

At time 2, if the market maker updated the price as before, the ask and bid prices would be 0.57 and 0.54. However, an informed trader with a negative signal, facing this ask price, would decide to buy. Following the previous two buys, this trader raises his valuation of the asset more than the market maker, since he knows that the market is informed. The market maker realizes that both types of informed traders would buy and that the trade reveals nothing on the value of the asset. Therefore, an informational cascade starts and the market maker will post both the bid and the ask price at the same level, 0.56. The probability of being in an uninformed market, however, will be revised downwards conditional on a buy order, because the market maker knows that in an informed market buys are very likely, given that all informed traders would buy. In period 3, after receiving another buy order, the informational cascade continues and the probability that the market maker attaches to the uninformed market declines further to 0.87.

In period 4, if the market maker receives another buy, the expectation of being in an uninformed market declines again. This in turn raises the expected value of the asset for the market maker, as he now knows that the past history of buys is more likely to have come from informed traders with a positive signal. Therefore the market maker will raise his ask price to a point (0.86) where traders with a negative signal are not willing to buy, as their expected value is lower (0.72). The informational cascade is therefore broken.

After a sequence of buy orders, when the informational cascade arises, all traders-irrespective of their own private signal-evaluate the asset more than the ask price that would have been posted in the absence of a cascade. Therefore, they all buy, thus imitating the behavior of the previous informed traders. On the other hand, after a sequence of sell orders, all traders evaluate the asset at less than the bid price, irrespective of their own private signal: therefore, they all sell.

Thus, when there is uncertainty on the proportion of noise traders, an informational cascade will generate herd behavior. After a history of buys, when the informational cascade arises, the valuation of all informed traders will be greater than the ask price and $Pr(h_t^J = \text{buy}|H_t) = 1$. Similarly, after a history of sells, the valuation of all informed traders will be below the bid price and $Pr(h_t^J = \text{sell}|H_t) = 1$. This result is proven in the following corollary:

Corollary 2 With informational uncertainty, if there is an informational

cascade, informed traders all act alike, that is, they engage in herd behavior.

Proof. See the Appendix.

With informational uncertainty the traders and the market maker interpret the past history of trades in a different manner. The market maker updates his belief too slowly and, therefore, traders may find it convenient to disregard their signals. The informational cascade that arises leads to herd behavior. After a sequence of buys (sells), all informed agents may be willing to take advantage of the slow movement of the ask (bid) price to buy (sell) the asset. The actions taken by the previous agents affect the decisions of the following traders.

A history of asset A can also induce an informational cascade (and therefore herd behavior) on asset B . Upon observing the history of asset A , traders of B will decide to take an action irrespective of their signal. This is the phenomenon of *contagious spillover*, which we have defined in the previous section. We can show it by studying what happens to market B in the previous example.

Example 4 Consider the economy of the previous example. Assume that asset B is distributed on $\{0, 1\}$ as follows: $\Pr(B = 1|A = 1) = \Pr(B = 0|A = 0) = 0.85$. Traders of asset B receive a signal x^B distributed according to the following probability function: $\Pr(x^B = k|B = k) = 0.9$. Suppose that, as in the previous example, in the first two periods there are trades only on market A and that they happen to be buys.

The sequence of ask prices calculated assuming that only traders with a positive signal buy is $A_0^B = 0.505$, $A_1^B = 0.52$, $A_2^B = 0.57$. However, at time 2 an informed trader receiving a negative signal would have an expected value 0.70 and therefore would also buy, should he be drawn to trade¹⁰. Therefore, the decision of the trader in this period is independent of his signal. An informational cascade has arisen just because of the history of buys of asset A .

Therefore, exactly as in the case of gains from trade, the history of one asset can have long-lasting consequences for the price path of the other.

¹⁰Indeed, the price that the market maker will post at time 2 will take into account that the informed trader will buy whatever his signal is. The posted price will therefore be lower than 0.57.

Moreover, a boom in one market can also generate a period of euphoria in the other market because herd buying is transmitted from the first market to the second. Note that also in our baseline model the fundamental correlation between the assets imply that the history of trades of one asset affects the price path of the other. When we introduce an asymmetry between the market makers and the traders, however, this spillover between markets can cause an informational cascade to arise.

4. Conclusion

In this paper we have obtained two main results. The first result is on informational cascades. In a financial market in which there are some gains from trade, informational cascades arise and the information stops flowing to the market. Since traders enjoy an extra utility (or suffer a disutility) from buying the asset, the market is unable to infer traders' private information and to discover the true values of the assets. The asset prices can remain at levels different from those of the fundamentals forever. Informational cascades can also arise when there is uncertainty on the proportion of informed traders in the market. Informational cascades imply that all informed traders will choose the same action, i.e., they will herd. Thus, we can explain the presence of herd behavior in financial markets.

These findings indicate that the results of the Social Learning literature on herd behavior and informational cascades are robust under the introduction of flexible prices. Avery and Zemsky(1998) question the robustness of these results as they show that, when the price incorporates the information coming to the market, informational cascades are impossible. We have shown that, when agents have different valuations of the assets or different opinions on the degree of informativeness of the market, both herd behavior and informational cascades arise.

The second result is on contagion. Although the prices of the two assets vary and covary little, the correlation between prices can be higher than that between the fundamentals. This explains the high correlation among financial assets that we observe in the data. Moreover, with gains from trade or informational uncertainty, the history of trades on one asset can significantly affect the price of the other. Informational cascades and herd behavior on one asset generate cascades and herd behavior on the other asset, pushing the prices far from the fundamental, even in the long run.

Appendix

Proof of Lemma 1: We prove the lemma for the case in which the assets are distributed on $\{0, 1\}^2$; it is immediate to generalize the proof to the case $\{m, M\}^2$. Let us consider the case of a buy order. We know that the ask is always greater than the price, i.e., after a buy the price is always revised upward. In order to show that prices move in the same direction we need to prove that also the price of asset B after the buy in market A moves upward.

We prove this by means of two claims.

Claim 1: The price of B is updated upward if and only if $\Pr(buy_t^A | V^B = 1, H_t) > \Pr(buy_t^A | V^B = 0, H_t)$.

Proof of claim 1:

$$E(V^B | H_t, buy_t^A) = \Pr(V^B = 1 | H_t, buy_t^A) = \quad (A1)$$

$$\frac{\Pr(buy_t^A | V^B = 1, H_t) \Pr(V^B = 1 | H_t)}{\Pr(buy_t^A | V^B = 1, H_t) \Pr(V^B = 1 | H_t) + \Pr(buy_t^A | V^B = 0, H_t) \Pr(V^B = 0 | H_t)}$$

Hence,

$$\frac{\Pr(V^B = 1 | H_t, buy_t^A)}{\Pr(V^B = 1 | H_t)} > 1 \text{ if and only if} \quad (A2)$$

$$\frac{\Pr(buy_t^A | V^B = 1, H_t)}{\Pr(buy_t^A | V^B = 1, H_t) \Pr(V^B = 1 | H_t) + \Pr(buy_t^A | V^B = 0, H_t) \Pr(V^B = 0 | H_t)}$$

is greater than 1.

By simple algebra, this is equivalent to:

$$\Pr(buy_t^A | V^B = 1, H_t) [1 - \Pr(V^B = 1 | H_t)] >$$

$$\Pr(buy_t^A | V^B = 0, H_t)(1 - \Pr(V^B = 1 | H_t)), \quad (A3)$$

which is true if and only if $\Pr(buy_t^A | V^B = 1, H_t) > \Pr(buy_t^A | V^B = 0, H_t)$.

Claim 2: $\Pr(buy_t^A | V^B = 1, H_t) > \Pr(buy_t^A | V^B = 0, H_t)$.

Proof of claim 2:

Let us denote H_t^J the history of trades on asset J until time $t - 1$.

$$\begin{aligned}
\Pr(buy_t^A | V^B = 1, H_t) &= \frac{\Pr(buy_t^A, H_t | V^B = 1)}{\Pr(H_t | V^B = 1)} = \\
&= \frac{\Pr(buy_t^A, H_t^A | V^B = 1) \Pr(H_t^B | V^B = 1)}{\Pr(H_t^A | V^B = 1) \Pr(H_t^B | V^B = 1)} = \\
&= \frac{kA + hB}{A + B}, \tag{A4}
\end{aligned}$$

$$\begin{aligned}
\Pr(buy_t^A | V^B = 0, H_t) &= \frac{\Pr(buy_t^A, H_t | V^B = 0)}{\Pr(H_t | V^B = 0)} = \\
&= \frac{\Pr(buy_t^A, H_t^A | V^B = 0) \Pr(H_t^B | V^B = 0)}{\Pr(H_t^A | V^B = 0) \Pr(H_t^B | V^B = 0)} = \\
&= \frac{kC + hD}{C + D}, \tag{A5}
\end{aligned}$$

where:

$$k = \Pr(buy_t^A | V^A = 1),$$

$$h = \Pr(buy_t^A | V^A = 0),$$

$$A = \Pr(buy_t^A | V^A = 1)^{\#buys^A} \Pr(sell_t^A | V^A = 1)^{\#sells^A} \Pr(V^A = 1 | V^B = 1),$$

$$B = \Pr(buy_t^A | V^A = 0)^{\#buys^A} \Pr(sell_t^A | V^A = 0)^{\#sells^A} \Pr(V^A = 0 | V^B = 1),$$

$$C = \Pr(buy_t^A | V^A = 1)^{\#buys^A} \Pr(sell_t^A | V^A = 1)^{\#sells^A} \Pr(V^A = 1 | V^B = 0),$$

$$D = \Pr(buy_t^A | V^A = 0)^{\#buys^A} \Pr(sell_t^A | V^A = 0)^{\#sells^A} \Pr(V^A = 0 | V^B = 0), \tag{A6}$$

and $\#buys^A$ and $\#sells^A$ denote the number of buys and sells on asset A until time t . Note that we omitted the term referring to the probability of no trades. Given that only noise traders would ever decide not to trade, the probability of a no trade is always independent of the value of both assets and therefore would cancel out.

To prove the claim we need to prove that:

$$\frac{kA + hB}{A + B} > \frac{kC + hD}{C + D}. \quad (\text{A7})$$

This is true if and only if $(k - h)(AD - CB) > 0$, which is true given that $k > h$ and $AD > CB$.

The proof for the case of a sell is analogous. Q.E.D.

Proof of Proposition 1: We prove the proposition in three steps.

Step 1: $E(\Delta P_i^A \Delta P_i^B) \geq 0$.

Proof of Step 1: Let us assume, without loss of generality, that at time t market A is open. Then

$$E(\Delta P_i^A \Delta P_i^B) = \quad (\text{A8})$$

$$\sum_{h_t^A} \Pr(h_t^A | H_t) [E(V^A | H_t, h_t^A) - E(V^A | H_t)] [E(V^B | H_t, h_t^A) - E(V^B | H_t)].$$

Note that if $E(V^A | H_t, h_t^A) \geq E(V^A | H_t)$ then, by Lemma 1, $E(V^B | H_t, h_t^B)$ is greater than $E(V^B | H_t)$. Therefore, all of the addenda in the formula will be greater than zero and $E(\Delta P_i^A \Delta P_i^B) \geq 0$.

Step 2: $Cov(P_t^A, P_t^B)$ is greater than zero and increasing.

Proof of Step 2:

$$Cov(P_t^A, P_t^B) = Cov(P_0^A + \sum_{i=1}^t \Delta P_i^A, P_0^B + \sum_{i=1}^t \Delta P_i^B) =$$

$$E \left[(P_0^A + \sum_{i=1}^t \Delta P_i^A)(P_0^B + \sum_{i=1}^t \Delta P_i^B) \right] -$$

$$E(P_0^A + \sum_{i=1}^t \Delta P_i^A) E(P_0^B + \sum_{i=1}^t \Delta P_i^B) =$$

$$\begin{aligned}
&= E\left(\sum_{i=1}^t \Delta P_i^A \Delta P_i^B\right) = \\
&= \sum_{i=1}^t E(\Delta P_i^A \Delta P_i^B), \tag{A9}
\end{aligned}$$

where the second equality holds because prices are martingales and, therefore, $E(\Delta P_t^A) = 0$, $E(\Delta P_t^B) = 0$, $E(\Delta P_t^A \Delta P_{t+k}^B) = 0$.

Given that, in the first part of the proof, we showed that all the addenda in the sum are greater than zero, $Cov(P_t^A, P_t^B)$ will be greater than zero and increasing.

Step 3: $Cov(P_t^A, P_t^B) \leq Cov(V^A, V^B)$ for all t .

Proof of Step 3: Given that, as $t \rightarrow \infty$, prices converge almost surely to the true value, $Cov(P_t^A, P_t^B) \rightarrow Cov(V^A, V^B)$. Therefore, $Cov(P_t^A, P_t^B)$ is smaller than $Cov(V^A, V^B)$ for all t . Q.E.D.

Proof of Proposition 2: The proof for $t = 1$ is merely algebraic. The expression for the correlation between prices at time 1 is the ratio of two long polynomials. After lengthy simplifications, this ratio turns out to be equal to 1 in absolute value and its sign depends only on the sign of the correlation between the fundamentals. Giving the full proof would be cumbersome. For presentational purposes we provide an illustrative example.

Suppose the distribution of the values of assets A and B is the following:

	$V^B = 0$	$V^B = 1$	
$V^A = 0$	$\frac{3}{8}$	$\frac{1}{8}$	$\frac{1}{2}$
$V^A = 1$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{2}$
	$\frac{1}{2}$	$\frac{1}{2}$	1

Informed traders on asset A receive a perfectly informative signal, i.e., $\Pr(x^A = k | V^A = k) = 1$, for $k = 0, 1$. Noise traders buy or sell with probability $\frac{1}{2}$. The correlation between the two asset values is

$$\frac{Cov(V^A, V^B)}{\sqrt{Var(V^A) Var(V^B)}} = \frac{\frac{1}{8}}{\sqrt{\frac{1}{4} \frac{1}{4}}} = \frac{1}{2}. \tag{A10}$$

At time 0 there will be a buy or a sell with equal probability. Therefore, at time 1, prices will be, with probability $\frac{1}{2}$, either $P_1^A = \frac{3}{4}$ and $P_1^B = \frac{5}{8}$ or

$P_1^A = \frac{1}{4}$ and $P_1^B = \frac{3}{8}$. The correlation between the two prices will be:

$$\frac{Cov(P_1^A, P_1^B)}{\sqrt{Var(P_1^A) Var(P_1^B)}} = \frac{\frac{1}{32}}{\sqrt{\frac{1}{64} \frac{1}{16}}} = 1. \quad (\text{A11})$$

As $t \rightarrow \infty$ prices converge to the true values and the correlation between prices converges to the correlation between the fundamentals. Q.E.D.

Parameters for the simulated model: Asset A can take values 0 and 1 with probabilities 0.3. and 0.7 Asset B takes value 0 and 1 according to the following conditional distribution: $P(V^B = 1|V^A = 0) = 0.3$, $P(V^B = 1|V^A = 1) = 0.7$.

The proportion of informed traders is $\frac{1}{2}$. Informed traders on asset $J = A, B$ receive a signal x^J that can take values 0 and 1 according to the following conditional distribution: $\Pr(x^J = 1|V^J = 0) = 0.3$, $\Pr(x^J = 1|V^J = 1) = 0.7$. Uninformed traders buy and sell with equal probability $\frac{1}{2}$.

The price path is calculated for 1,000 periods. The unconditional correlation is estimated with 100,000 runs.

Proof of Proposition 3: The proposition is proven by contradiction. First, recall that, because of noise traders, any history of trades happens with positive probability. Hence, at time t the true realization of the asset is not known with certainty. By assumption, whenever the true realization of the asset is not known with certainty, in the market there is always a minimum amount of useful information. In other words, there is at least one trader with a realization of the signal \hat{x}^J whose expected value is $V_t^J(\hat{x}^J) \neq P_t^J$. Now, suppose that at time t there is an informational cascade in market $J = A, B$. In an informational cascade the action of any traders does not depend on the value of the asset, therefore also for the realization of the signal \hat{x}^J it has to be that $\Pr(\hat{x}^J|V^J, H_t) = \Pr(\hat{x}^J|H_t)$, for all V^J . Thus,

$$\Pr(V^J|\hat{x}^J, H_t) = \frac{\Pr(\hat{x}^J|V, H_t) \Pr(V^J|H_t)}{\Pr(\hat{x}^J|H_t)} = \Pr(V^J|H_t). \quad (\text{A12})$$

This implies that

$$V_t^J(\hat{x}^J) = E(V^J|\hat{x}^J, H) = E(V^J|H) = P_t^J, \quad (\text{A13})$$

a contradiction. Q.E.D.

Proof of Proposition 4: To prove the proposition it is enough to show that there exists a time t when

$$E(V^J|H_t, buy_t^J) - gE(V^J|H_t, x^J) < 0 \quad \text{for all } x^J, \quad (\text{A14})$$

and

$$lE(V^J|H_t, x^J) - E(V^J|H_t, sell_t^J) < 0 \quad \text{for all } x^J. \quad (\text{A15})$$

In this case, at time t , a trader enjoying a positive utility from the asset buys independently of his signal. On the other hand, a trader suffering a disutility from the asset always sells. That is, the action of the trader is independent of the realization of the value of the asset.

Since $E(V^J|H_t, x^J)$ and $E(V^J|H_t, buy_t^J)$ are martingales and the probability of buying and selling is bounded away from zero by the existence of noise traders, the market maker's and the trader's valuations conditional on all possible x^J given V^J converge to the same random variable with probability one. This means that for any $\varepsilon > 0$ there exists a T such that, for $t > T$,

$$\Pr(|E(V^J|H_t, x^J) - E(V^J|H_t, buy_t^J)| < \varepsilon) = 1 \quad (\text{A16})$$

and

$$\Pr(|E(V^J|H_t, x^J) - E(V^J|H_t, sell_t^J)| < \varepsilon) = 1, \quad (\text{A17})$$

for all possible x^J given V^J . If there does not exist a realization of the signal \hat{x}^J such that $\Pr(\hat{x}^J|\hat{V}^J) = 0$, all realizations of the signals are possible. Therefore,

$$\Pr(|E(V^J|H_t, x^J) - E(V^J|H_t, buy_t^J)| < \varepsilon) = 1 \quad (\text{A18})$$

and

$$\Pr(|E(V^J|H_t, x^J) - E(V^J|H_t, sell_t^J)| < \varepsilon) = 1, \quad (\text{A19})$$

for all x^J . By choosing $\varepsilon = \min\{m(g-1), M(1-l)\}$, the theorem is proven. Q.E.D.

Proof of Corollary 1: As shown in the proof of Proposition 6, there exists a time t at which all traders enjoying a gain from trade buy independently

of their signal and all traders suffering a loss from trade will sell. Therefore, all informed traders of the same type act alike. Q.E.D.

Proof of Proposition 6: To prove the proposition we must show that there exists a time t when

$$E(V^A|H_t, buy_t^A) - gE(V^A|H_t, x^A) < 0, \quad (A20)$$

$$lE(V^A|H_t, x^A) - E(V^A|H_t, sell_t^A) < 0 \quad \text{for all } x^A, \quad (A21)$$

and

$$E(V^B|H_t, buy_t^B) - gE(V^B|H_t, x^B) < 0 \quad (A22)$$

$$lE(V^B|H_t, x^B) - E(V^B|H_t, sell_t^B) < 0 \quad \text{for all } x^B. \quad (A23)$$

In this case, in both markets a trader enjoying a positive utility from the asset buys independently of his signal. On the other hand, a trader suffering a disutility from the asset will always sell. That is, the action of the trader is independent of the realization of the values of the assets.

By the same arguments given in the proof of Proposition 6, if, for $J = A, B$, there does not exist a realization of either signal \hat{x}^J such that $\Pr(\hat{x}^J|\hat{V}^J) = 0$, for any $\varepsilon > 0$ there will be a t such that, *in both markets*,

$$\Pr(|E(V^J|H_t, x^J) - E(V^J|H_t, buy_t^J)| < \varepsilon) = 1 \quad (A24)$$

and

$$\Pr(|E(V^J|H_t, x^J) - E(V^J|H_t, sell_t^J)| < \varepsilon) = 1, \quad (A25)$$

for all x^J . By choosing $\varepsilon = \min\{m(g-1), M(1-l)\}$ the theorem is proven. Q.E.D.

Parameters for the simulated model with gains from trade: The gain from trade g is equal to 1.1 and l is equal to 1. Asset A can take values 0 and 1 with probabilities 0.3. and 0.7 Asset B takes value 0 and 1 according to the following conditional distribution: $Pr(V^B = 1|V^A = 0) = 0.3$, $Pr(V^B = 1|V^A = 1) = 0.7$.

The proportion of informed traders is $\frac{1}{2}$. Informed traders on asset $J = A, B$ receive a signal x^J that can take values 0 and 1 according to the following conditional distribution: $\Pr(x^J = 1|V^J = 0) = 0.3$, $\Pr(x^J = 1|V^J = 1) = 0.7$. Uninformed traders buy and sell with equal probability $\frac{1}{2}$.

Proof of Proposition 8: First we prove that prices converge. The proof is a generalization of that given by Avery and Zemsky (1998) for the baseline model. It is done in three steps:

Step 1: The bid and ask prices converge.

Proof of Step 1: This is proven by Glosten and Milgrom (1985) for the baseline model. The proof requires only that prices are conditional expectations of the asset values and therefore are martingales. Given that with informational uncertainty this property is satisfied, the proof holds.

Step 2: The beliefs of the traders and of the market maker converge.

Proof of Step 2:

$$\begin{aligned}
A_t^J &= E(V^J | H_t, buy_t) = E(E(V^J | H_t, x_t, U^i) | H_t, buy_t) = \\
&E(E(V^J | H_t, x_t, U^i) | H_t, E(V^J | H_t, x_t, U^1) > A_t) > \\
&E(E(V^J | H_t, x_t, U^i) | H_t, E(V^J | H_t, x_t, U^1) > P_t). \tag{A28}
\end{aligned}$$

Therefore,

$$A_t^J - P_t^J \geq E(E(V^J | H_t, x_t, U^i) - P_t^J | H_t, E(V^J | H_t, x_t, U^1) > P_t^J). \tag{A29}$$

By step 1, we know that $A_t^J - P_t^J$ converges almost surely to 0. Therefore,

$$E(E(V^J | H_t, x_t, U^i) - P_t^J | H_t, E(V^J | H_t, x_t, U^1) > P_t^J) \xrightarrow{a.s.} 0. \tag{A30}$$

By Markov's inequality

$$\begin{aligned}
&E(E(V^J | H_t, x_t, U^i) - P_t^J | H_t, E(V^J | H_t, x_t, U^1) > P_t^J) \geq \\
&\varepsilon Pr(E(V^J | H_t, x_t, U^i) - P_t^J > \varepsilon | H_t, E(V^J | H_t, x_t, U^1) > P_t^J) \geq
\end{aligned}$$

$$\varepsilon Pr(E(V^J | H_t, x_t, U^i) - P_t^J > \varepsilon | H_t). \tag{A31}$$

Therefore, $\varepsilon Pr(E(V^J | H_t, x_t, U^i) - P_t^J > \varepsilon) \xrightarrow{a.s.} 0$ for all possible ε . Thus $Pr(E(V^J | H_t, x_t) - P_t^J > \varepsilon)$ will also converge to 0. By the same argument,

using the bid price, one can show that $Pr(E(V^J|H_t, x_t, U^i) - P_t^J > -\varepsilon)$ also goes to 0. This implies that traders' and the market maker's evaluations converge in probability.

Step 3: Prices converge to the fundamental values.

Proof of Step 3: Suppose not. Then, in each period t there exists a positive probability that the market maker's expectation differs from V^J by some positive value δ . By the assumption made in Section 2.1 on the existence of a minimum amount of useful information in the market, this implies that there is a strictly positive probability that an informed trader has an assessment different from the market maker's by a strictly positive value ε . This contradicts the fact that the expectations of the market maker and of the informed traders converge over time.

Now we prove that, if the price of asset V^J converges to the fundamental value, then $Pr(U^1|H_t)$ converges almost surely to 1.

$$E(V^J|H_t) = E(V^J)(1 - Pr(U^1|H_t)) + E(V^J|H_t, U^1)Pr(U^1|H_t). \quad (\text{A32})$$

By step 3 we know that $E(V^J|H_t)$ converges to the fundamental value. It is straightforward to prove that, if the traders' and the market maker's evaluations converge to V^J , then $E(V^J|H_t, U^1)$ will also converge to V^J . Therefore, $Pr(U^1|H_t)$ converges to 1. Q.E.D.

Proof of Corollary 2: Suppose an informational cascade has arisen after at time t . By definition of informational cascade, all traders will choose an action without regard to their signal. Therefore, either they all will buy or they all will sell.

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Figure 1: Correlation between the prices and between the fundamentals.

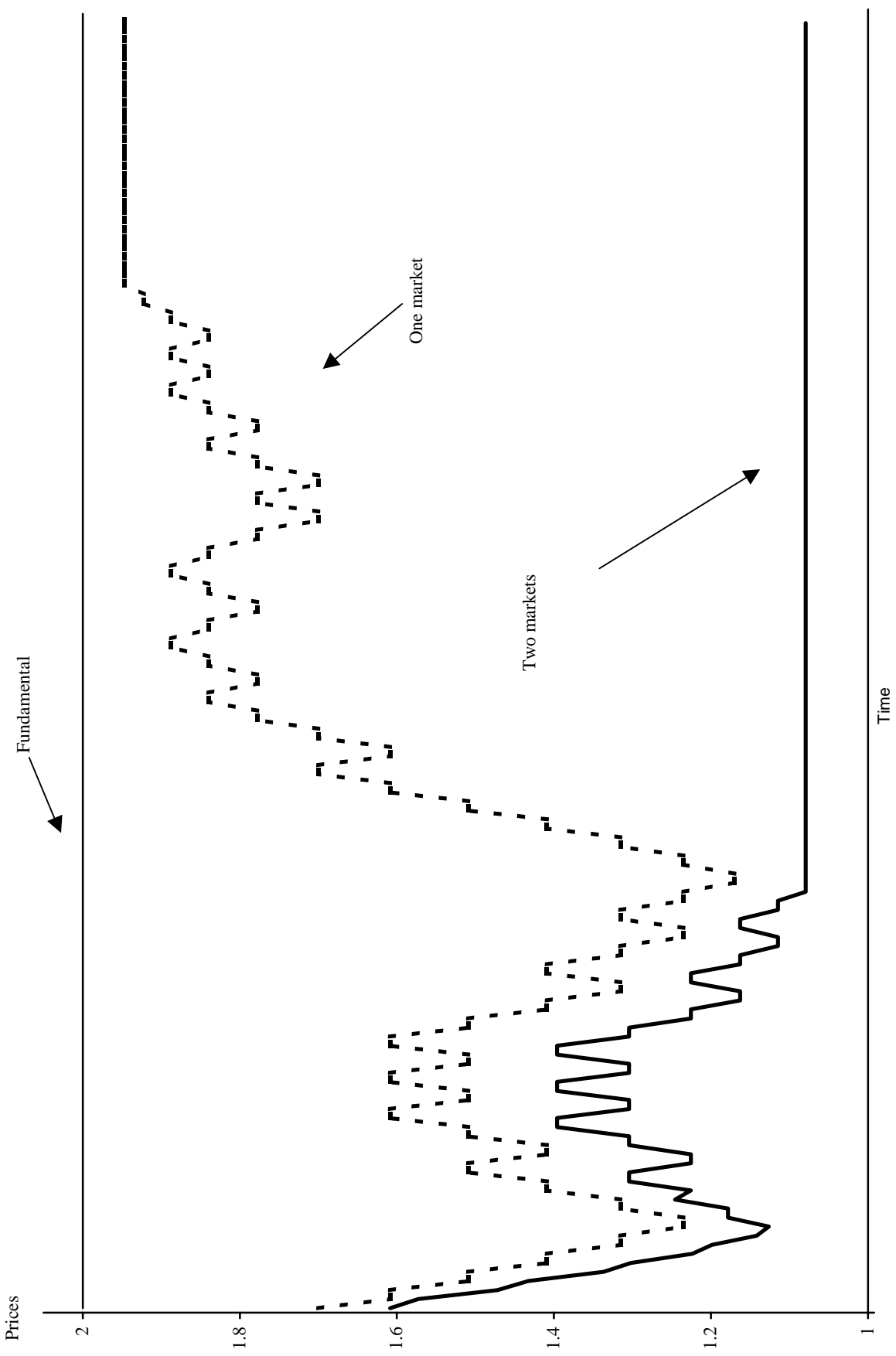


Figure 2: A history of contagion.

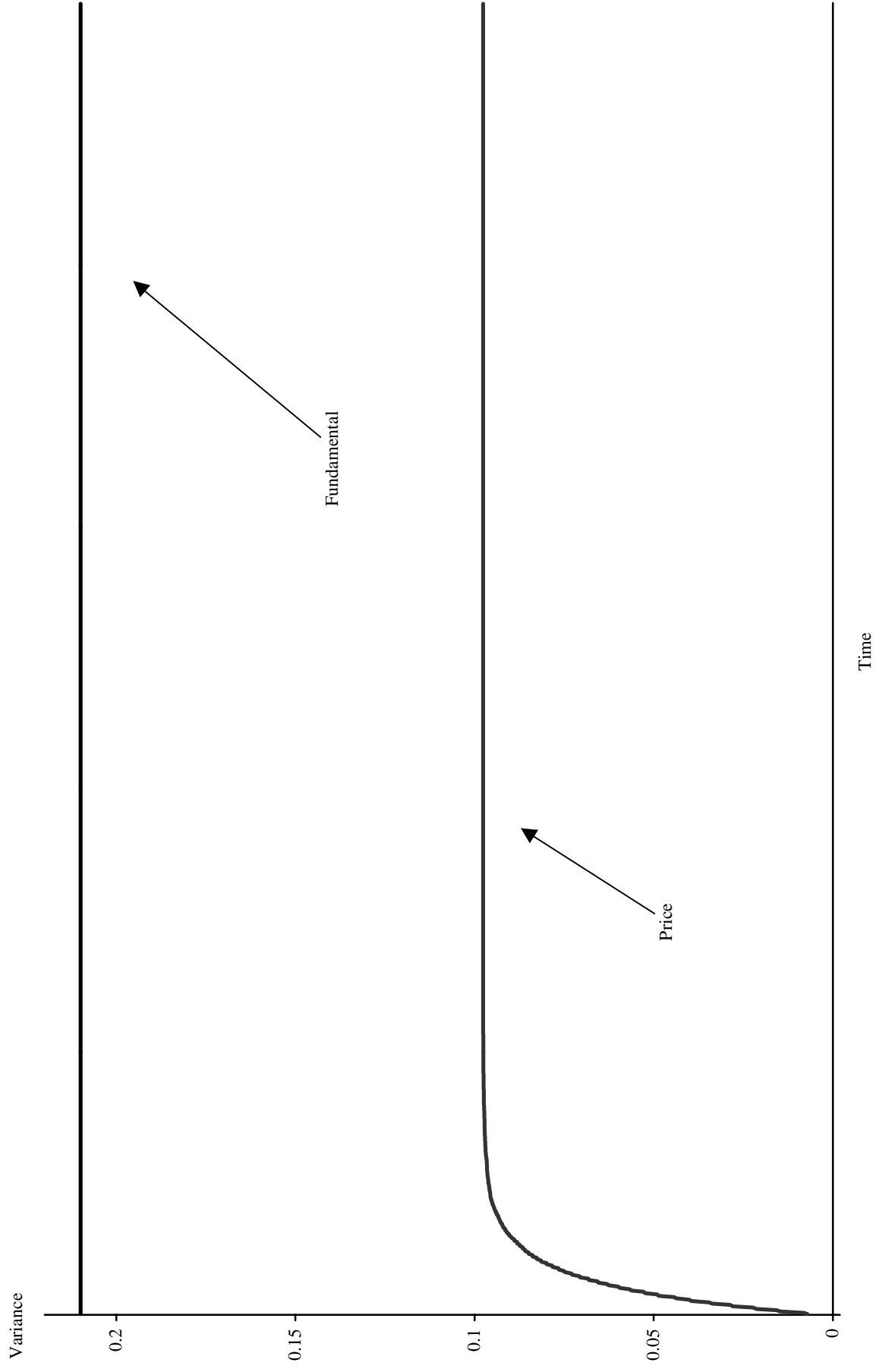


Figure 3: Variance of the price and the fundamental with gains from trade.

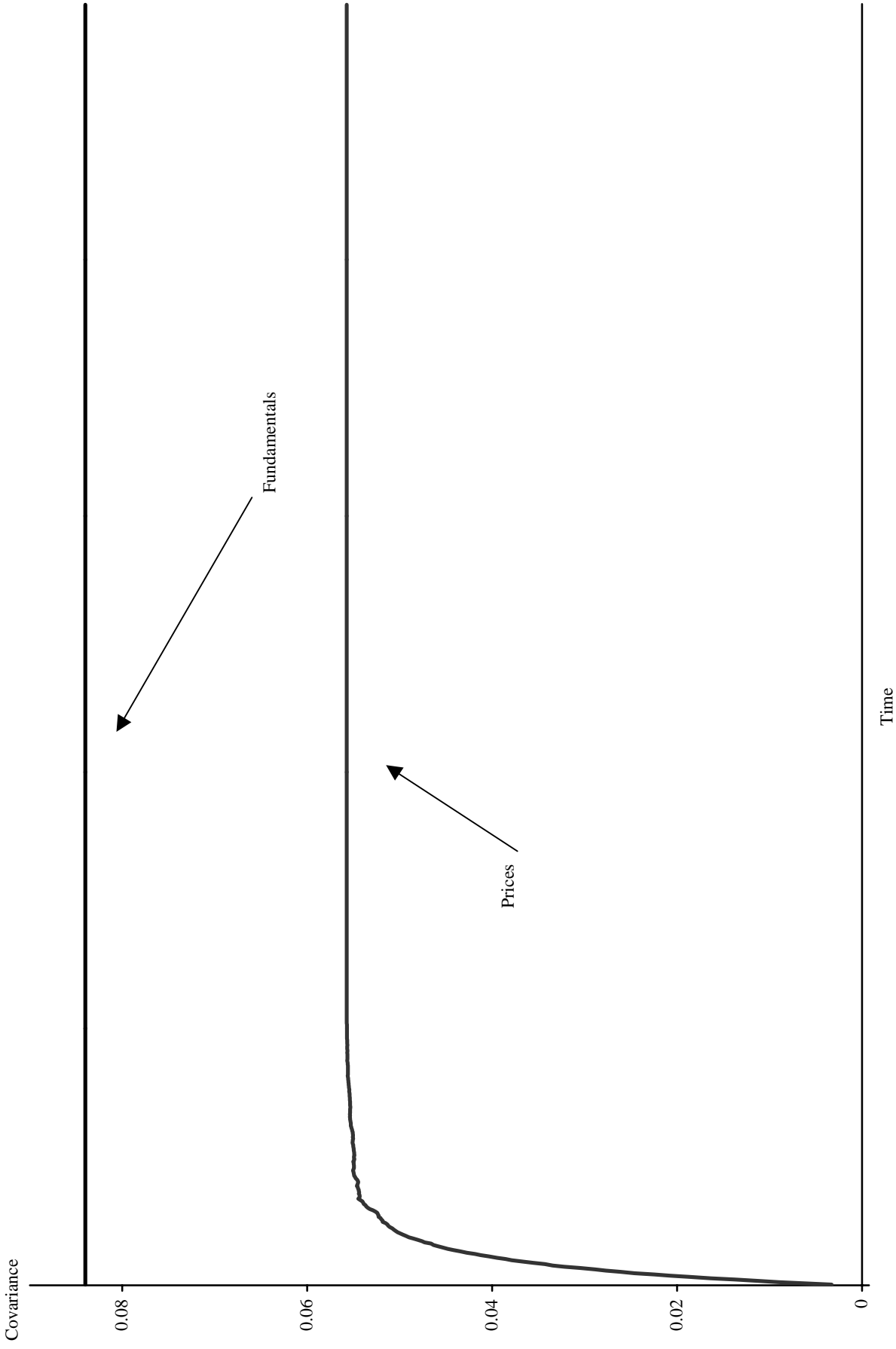


Figure 4: Covariance between the prices and between the fundamentals with gains from trade.

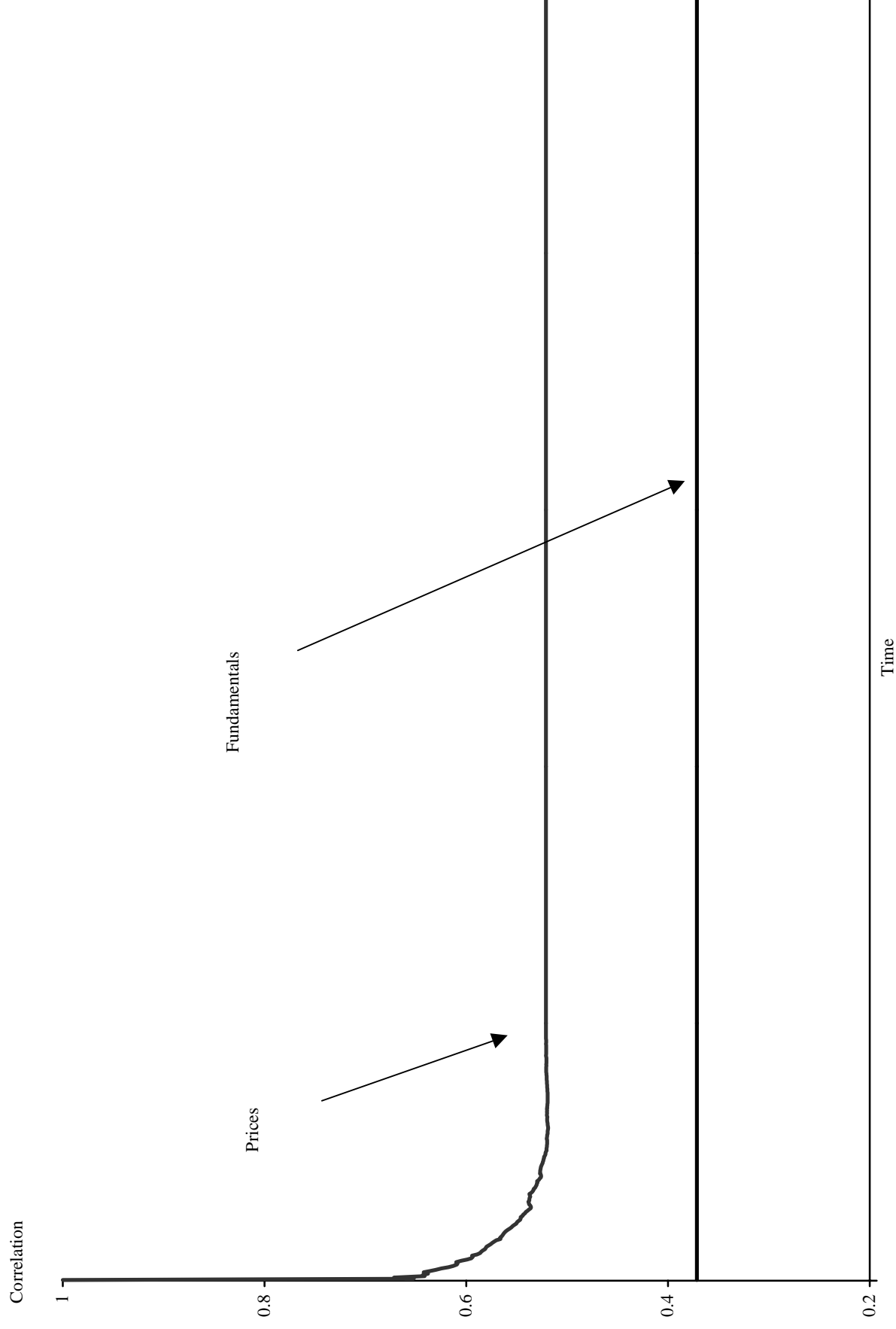


Figure 5: Correlation between the prices and between the fundamentals with gains from trade.