Asset Correlation of German Corporate Obligors: Its Estimation, Its Drivers and Implications for Regulatory Capital

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March 2003

Abstract

This paper addresses the gap between the theoretically well-understood impact of systematic risk on the loss-distribution of a credit-risky loan portfolio and the lack of empirical estimates of the default correlation. To this purpose we start with a one-factor model in which the correlation with the systematic risk factor equals the asset correlation between two firms. In the theoretical part of the paper the small sample performance of three different correlation estimators is analysed by Monte Carlo simulation.

In the empirical part asset correlations are estimated from time series of ten years with default histories of 53280 German companies. The sample is divided into categories that are homogenous with respect to default probability (PD) and firm size. In this way we can explore to what extent correlations depend on these two factors. Several economic explanations why asset correlation depends on size and PD are discussed.

The empirical analysis is motivated as well by current proposals for the internal rating based approaches of the new Basel Accord. They suggest that the asset correlation parameter in the formula for the risk weights depends on the PD and on the firm size of the obligor. Our empirical results are compared with this proposal.

Keywords: asset correlation, New Basel Accord, default correlation, firm size, single risk factor model

JEL classification: G 21

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1 Introduction

The objective of this paper is to estimate the asset correlation of German corporate obligors. In the theoretical part the small sample properties of three estimators of the asset correlation are analysed by Monte Carlo simulations. In the empirical part the asset correlation is estimated and its dependency on two factors – firm size and probability of default (PD) – is explored. The analysis of these potential drivers of the asset correlation is inspired by a recent proposal of the Basel Committee for the corporate risk weight function of the internal rating based (IRB) approach of the new Accord.[1] In this proposal a two-dimensional dependency of the parameter asset correlation on the PD and the size of the obligor is introduced.

Our analysis is based on the one-factor model that has been used to derive the IRB capital charge of the new Accord.[2] We refer to this model because it facilitates the comparison with the IRB risk weight functions. In this model there exists a one-to-one mapping between default correlation and asset correlation for a given probability of default. Hence, the analysis provides new results on the level of default correlation which is a key driver of credit risky loan portfolios. Therefore, the results are relevant as well for credit risk modeling in general.

This paper makes the following five main contributions:

First, the asset correlation is estimated from default histories of German firms taken from a database that includes 53280 privately-owned or corporate companies. This database which is maintained by Deutsche Bundesbank allows the calculation of default frequencies for ten years, from 1991 until 2000. A possible sample bias is accounted for by calibrating the default rates to business-sector specific insolvency statistics. To control for the effect of insolvency as a late legal definition of default we calibrate the default rates to a level that is inferred from loan net provisions of German banks.

Second, this study provides theoretical results for a small sample estimation problem. Specifically, we compare the maximum likelihood estimator for the asset correlation which was suggested in Gordy and Heitfield (2000) with two versions of method-of-moments estimators. All these estimators rely on asymptotic theory and their small sample distributions are unknown. By Monte Carlo simulation we can shed some light on the questions how the number of time periods and the size of the portfolio determine a potential estimation bias and which estimator performs best for typical values of these two factors.

Third, this paper explores empirically the *simultaneous* dependency of asset correlation on PD and on firm size. Earlier work by Dietsch and Petey (2002) has focused on the dependency of asset correlation solely on PD. Two recent papers by Lopez (2002) and

[1] See on Banking Supervision (2002).
[2] See Gordy (2001).

Dietsch and Petey (2003) are to the best of our knowledge the only ones that consider the cumulative influence of PD and firm size on the asset correlation. However, both differ from our work in important ways.

The work by Lopez (2002) is based on a vendor model developed by Moody's KMV[3]. However, estimating the asset correlation from default data which is the approach in this paper is more in line with the predominant book-value approach to the management of bank loans than an estimation from a structural model like the one by Moody's KMV that uses equity data. Differences between our work and that by Dietsch and Petey (2003) exist in the use of a quite different sample of German corporates and in that our results are based on a longer time series of ten compared to four years that better covers a full business cycle.

As a fourth contribution we provide tentative answers why dependencies of asset correlation on PD and firm size are observed. To this purpose we take into account earlier work in the literature as well as new empirical results for German corporates.

The fifth contribution is related to the implementation of internal rating systems in Basel II. Our new results for the estimation of the asset correlation as a measure of the impact of systematic risk may help to improve the estimation and the validation of PDs.

Our paper is divided into seven major sections. In section 2 previous theoretical and empirical results on a potential PD- or size-dependency of asset correlation are reviewed. Section 3 describes the model framework and the estimators for the model parameters. A preliminary investigation of their small sample properties is carried out by Monte Carlo simulations in section 4. Section 5 describes the data sample and explains how it is divided by size and by credit quality. The estimation results for the asset correlation and potential implications for the calibration of regulatory capital in the new Basel Accord are discussed in section 6. Section 7 summarizes and concludes.

2 Previous Theoretical and Empirical Results

In this section arguments why we should observe a dependency of asset correlation firstly on firm size and secondly on PD are reviewed, together with empirical results from earlier studies.

In the one-factor model asset correlation measures the exposure against systematic risk, broadly speaking against business cycle risk. The asset correlation is lower for small and medium enterprises (SMEs) than for large corporates if their systematic (idiosyncratic) risk is relatively smaller (higher). This may be the case if the size dependency conceals a dependency on the industrial sector. Different sectors differ in their dependency on the

[3] See Crosbie (1999).

Table 1: **Percentage of small and medium enterprises in Germany, 1997**

business sector	**percentage of small and medium enterprises**
manufacturing	15.6 %
construction	17.6 %
automotive	15.4 %
transport & communication services	31.7 %
health & financial services	27.4 %
other public & personal services	42.1 %

business cycle and in their firm size distribution. Therefore, if sectors which are highly cyclical are dominated by large firms whereas in less cyclical sectors SMEs prevail, then we expect to observe that systematic risk and asset correlation overall increase with firm size. In other words firm size would serve as a proxy for a business sector dependency of the asset correlation.

This hypothesis is supported by the figures in table 1 which show the percentage of small and medium German companies in selected business sectors.[4] The first three sectors which are in general viewed as more cyclical possess a lower share of small and medium companies than the last three sectors which are in general considered as less cyclical. Considering the implication that the cyclical sectors have a relatively higher share of large companies higher asset correlation estimates for large companies may derive from this underlying sector dependency.

A second explanation for a higher asset correlation of large firms may be that they are better diversified than small firms. Because of their better diversification the idiosyncratic risk would be relatively smaller than for small firms and their correlation with the systematic risk factor relatively higher. However, empirical work by Roll (1988) casts some doubt on this hypothesis. He observed that the returns of size-matched portfolios of small firms are better explained by systematic risk factors (have a higher R^2) than the returns of large companies. This result suggests that in the contrary large firms tend to be less diversified than small firms and, therefore, their asset correlation would in general be lower.

At the same time other work from Bernanke and Gertler (1995) and Bernanke et al. (1996) suggests that asset correlation decreases with increasing firm size.[5] These authors analyse adverse shocks to the economy which are amplified and propagated by changes

[4]The figures have been provided by the "Institut für Mittelstandsforschung" in Bonn and are based on data from the Federal Statistical Agency. They define small and medium companies as those with a yearly turnover of up to 50 Mln. Euro.

[5]See Bernanke et al. (1996).

in credit-market conditions. A key role plays the external finance premium which they define as the difference between the cost of funds raised externally and internally. This premium arises from information asymmetries in the credit markets. It increases when the economic conditions deteriorate and collateral values decline. As a consequence firms have increasing difficulties to obtain funding even for profitable projects. This effect amplifies an economic downturn and is known in the literature as "financial accelerator".[6] The authors expect that the impact of a higher external finance premium will be stronger the more a corporate borrower has to rely on bank loans. Larger firms may to some extent circumvent this effect by tapping capital markets. As a consequence small firms are expected to be more vulnerable against the financial accelerator than large companies. Therefore, macroeconomic shocks should have a stronger impact on SMEs and we should observe a higher asset correlation.

In their empirical work with US data Bernanke et al. (1996) find, that in an economic downturn following tight money small-firm sales drop earlier and their short–term debt drops stronger compared with large firms.[7] This result still holds after controlling for industrial sector composition within size categories and suggests that asset correlation decreases with size.

In summary economic theory proposes two conflicting potential impacts on asset correlation: the business sector argument suggests that asset correlation increases with firm size whereas the financial accelerator works in the opposite direction.

Compared with a potential dependence of asset correlation on firm size the theoretical arguments advocating that asset correlation decreases with PD are less developed. Two theoretical arguments for a PD-dependence are the following:

The first one is a time series argument: If the credit risk of a company increases firm-specific risk factors become relatively more important than systematic risk and, therefore, the correlation with the systematic factor declines. This argument holds only if the cause of the deterioration in credit quality is not the business cycle because otherwise it would have to be attributed to systematic risk. Instead, the argument holds if firm-specific events lower the credit quality and start a downward spiral.

The second argument is a cross-sectional: firms that are more vulnerable to the business cycle may choose a safer capital structure in order to account for this higher risk. Because of the more secure capital structure they have a lower probability of default.

Recent empirical work on the relation between asset correlation and PD has been done by Dietsch and Petey. They estimate the asset correlation in a very similar one-factor model for French corporates. They observe that for homogenous business sectors asset correlation increases with the risk of default with a noticable exception for the highest

[6] See Bernanke and Gertler (1995), p. 35.

[7] See Bernanke et al. (1996), p. 10-11.

risk category.[8] These results contrast with the perception that asset correlation decreases with PD.

After this brief overview of possible reasons for a size- or PD-dependent asset correlation and of earlier work in the literature we introduce in the following section the one-factor model and three estimation methods for the asset correlation.

3 One-Factor Model and Estimation Methodology

Default correlation and *asset correlation* are deeply intertwined. Earlier studies have found strong evidence of correlation in the movements of the credit quality of different obligors.[9] If two obligors belong to a homogenous group sharing the same default correlation, its value can be determined from time series of defaulted and non-defaulted loans of this group without further assumptions.[10] Therefore, estimating correlation is not a problem of methodology. However, in practice we do not know firsthand which obligors build a homogenous group and, even if, estimates may be distorted by a small sample bias because regularly the available time series of default rates are rather short and do not extend 10 to 20 yearly observations. The first problem of homogeneity will be accounted for by splitting our sample into groups according to their credit quality and firm size. The second problem of the small sample performance of the estimators is discussed in the following.

A natural solution to overcome the small sample problem is to pose parametric restrictions. Gordy and Heitfield (2000) improve on the efficiency of the estimation by working in a Merton-type firm value model. This framework is broadly consistent with the widely applied concept of CreditMetrics. Instead of estimating default correlation first and deriving asset correlation thereafter, the authors estimate asset correlation directly from default data. If the model applies there is a one-to-one mapping, conditional on PD, between these two correlations.

In order to estimate asset correlation in the one-factor model we apply three different estimation methods. The first is the maximum likelihood (ML) estimation method put forward in Gordy and Heitfield (2000). Afterwards the model parameters are estimated by two additional methods. These two estimators are based on the method-of-moments and described after a short overview of the model and of the $ML-$approach.

The underlying one-factor model is as follows.[11] The firm value $A_{i,t}$ of obligor i follows a geometric Brownian motion. Under the usual assumptions its log-value at time t can be

[8] See Dietsch and Petey (2002), p. 311–312.

[9] See Carty (1997).

[10] See Lucas (1995) where this methodology is applied.

[11] A more comprehensive description of the model can be found in Schönbucher (2000).

described as follows where μ denotes the drift rate and $Y_{i,t}$ the stochastic error term of obligor i:

$$log(A_{i,t}) = log(A_{i,0}) + \left(\mu - \frac{\sigma^2}{2}\right) t + \sigma \sqrt{t}\, Y_{i,t}. \tag{1}$$

$Y_{i,t}$ follows a Gaussian distribution, i. e. a standard Normal distribution with mean 0 and variance 1. It is decomposed into the return of a systematic risk factor X_t and an idiosyncratic part $\epsilon_{i,t}$.

$$Y_{i,t} = \sqrt{\rho_i}\, X_t + \sqrt{1-\rho_i}\, \epsilon_{i,t}. \tag{2}$$

For every point in time t, X_t and $\epsilon_{i,t}$ are independent for every obligor i and follow a Gaussian distribution. The factor loading $\sqrt{\rho_i}$ of the systematic risk factor can be interpreted either as the sensitivity against systematic risk or as the square root of the asset correlation ρ_i of obligor i. As usual it is assumed that ρ_i does not vary over time.

Company i defaults if $Y_i < \gamma_i$ that means if the asset value falls below the default threshold γ_i. The step-function $L_{i,t}$ with $L_{i,0} = 0$ describes if a credit event has occurred during the target horizon ($L_{i,t} = 1$) or not ($L_{i,t} = 0$) and follows a Bernoulli distribution.
In the following it is important to differentiate between the unconditional and the conditional default probability. The *unconditional* default probability of obligor i for the time span from 0 to t is defined as follows:

$$P(L_{i,t} = 1) = P(Y_{i,t} < \gamma_i) = \Phi(\gamma_i). \tag{3}$$

Let $g(x)$ denote the default probability *conditional* on $X = x$ that is

$$g(x; \rho_i, \gamma_i) = P(L_i = 1|X = x) = \Phi\left(\frac{\gamma_i - \sqrt{\rho_i}\, x}{\sqrt{1-\rho_i}}\right). \tag{4}$$

Equation (4) provides as well the link to the proposed IRB risk weight function in Basel II. These risk weights are defined as the product of

- a factor of 12.5 (to compensate for the solvability coefficient of 0.08)
- the LGD (loss given default)
- the conditional PD given by (4) conditional on an adverse realization (99.9 % quantile) of X and
- an adjustment that accounts for the maturity of the exposure.

The capital charge is determined by multiplying the risk weight of an exposure with the solvability coefficient of 0.08.

The first estimator of the model parameters ρ_i and γ_i which is called $ML-$estimator in the following uses the fact that the number of defaults D for a homogenous portfolio with n obligors for a certain time period is binomial distributed that is

$$P(D = d|X = x) = \binom{n}{d}\, g(x; \rho, \gamma)^d\, (1 - g(x; \rho, \gamma))^{n-d}. \tag{5}$$

The $ML-$estimator has already been analysed by Gordy and Heitfield and it involves maximizing the following log-likelihood-function $LL(\mathbf{n},\mathbf{d};\rho,\gamma)$. Let $\mathbf{n}$ denote the $(T \times 1)-$vector of total numbers of obligors for T time periods and $\mathbf{d}$ the $(T \times 1)-$vector collecting the number of defaulted obligors. The estimation procedure is carried out only for homogenous segments of obligors so that we can drop the index i.[12]

$$LL(\mathbf{n},\mathbf{d};\rho,\gamma) = \sum_t \log(L_t(\mathbf{n},\mathbf{d};\rho,\gamma)) \tag{6}$$

$$L_t(\mathbf{n},\mathbf{d};\rho,\gamma) = \int_0^1 \binom{n_t}{d_t} g(\Phi^{-1}(x);\rho,\gamma)^{d_t} \, (1 - g(\Phi^{-1}(x);\rho,\gamma))^{n_t-d_t} \, dx. \tag{7}$$

The log-likelihood function LL is maximized numerically.

The second and third estimators are referred to as "method-of-moments"–estimators because the first and second moments of $g(X)$ are matched with the moment estimates from the time series of default rates for $T \to \infty$. The *unconditional PD* is estimated by the average $\bar{p}$ of the time series of default rates.

$$E\left[g(X)\right] = \bar{p}. \tag{8}$$

Let $\Phi_2(.)$ denote the cumulative bivariate Gaussian distribution. The following holds for the second moment:[13]

$$Var\left[g(X)\right] = \Phi_2\left(\Phi^{-1}(\bar{p}), \Phi^{-1}(\bar{p}), \rho\right) - \bar{p}^2. \tag{9}$$

After having estimated $Var\left[g(X)\right]$ from the time series of default rates, the asset correlation ρ can be backed out from (9). The second and third estimator differ in the way how $Var\left[g(X)\right]$ is estimated.

The second estimator which we refer to as the "asymptotic moment estimator" (AMM) estimates $Var\left[g(X)\right]$ from the sample variance $\sigma^2_{\hat{p}}$ of the default frequencies.[14]

The third estimator which is called "finite (sample) moment estimator" (FMM) adjusts the sample variance for the finite number of exposures in the sample from which the default frequencies are taken. This adjustment has already been applied by Gordy (2000) and with this adjustment the estimator of $Var\left[g(X)\right]$ is defined as follows:[15]

$$\widehat{Var}\left[g(X)\right] = \frac{\sigma^2_{\hat{p}} - E\left[1/n\right] \bar{p}\left(1 - \bar{p}\right)}{1 - E\left[1/n\right]}. \tag{10}$$

The finite sample adjustment in (10) accounts for the contribution of idiosyncratic risk to the volatility of the empirical default rates. However, when default correlations are low and the sample size is small this estimator can produce a negative estimate of $Var\left[g(X)\right]$.

Since all three estimators rely on asymptotic theory it is unclear which performs best in small samples. This issue is explored further in the next section.

[12] This is the estimator called *MLE 1* in Gordy and Heitfield (2000).
[13] See Gordy (2000), appendix C.
[14] See Blum et al. (2003), p. 118–119.
[15] See Gordy (2000), p. 146–147.

4 Small Sample Performance of Estimators

Since the small sample performance of the three estimators presented in section 3 is unknown Monte Carlo simulations are carried out to provide a guideline which estimation technique is more accurate for a relevant small sample size. In the first part of this section the design of the simulation exercise is described. In the second part we explore how the estimation bias depends on the length of the time series and in the third part how it depends on the number of exposures in the sample.

At the outset of the simulation experiments the parameters PD and ρ of a data generating process (DGP) are defined. The simulation exercise consists of two nested loops. The outer loop runs over S simulation runs. The inner loop runs over T observation periods. Therefore, the s-th simulation step for the t-th time period, that is described in the following, is repeated $S \cdot T$ times:

In the $s-$th step in period t a realization of the systematic factor $X_t^{(s)}$ is drawn from a standard Normal distribution. This is repeated N times for the idiosyncratic risk component $\epsilon_{i,t}^{(s)}$ of N obligors which is collected in the $(N \times 1)-$vector $\epsilon_t^{(s)}$. Then, the $(N \times 1)-$asset return vector $\mathbf{Y}_t^{(s)}$ is determined as follows where $\mathbf{1}_N$ denotes an $(N \times 1)-$vector of ones.

$$\mathbf{Y}_t^{(s)} = \sqrt{\rho_0}\, X_t^s\, \mathbf{1}_N + \sqrt{1-\rho_0}\, \epsilon_t^{(s)}. \tag{11}$$

The default rate at time t is determined as

$$\hat{p}_t^{(s)} = \frac{d_t^{(s)}}{N_t^{(s)}} \tag{12}$$

where $d_t^{(s)}$ denotes the number of defaults which occur whenever $Y_{i,t}^{(s)} \leq \gamma_0$ and $N_t^{(s)}$ refers to the number of not yet defaulted obligors at the beginning of the time period.

After iterating this simulation step in the inner loop $T-$times the two parameters can be estimated by the $ML-$, the $AMM-$ and the $FMM-$estimators from section 3.

Finally, after iterating in the outer loop we can estimate bias, standard error and root mean squared error (RMSE) for the $ML-$, the $AMM-$ the $FMM-$estimates of ρ.

With these Monte Carlo simulations the small sample properties of the estimators are explored for a specific DGP. We differentiate between two types of small sample bias:

1. the bias that is caused by the finite number of time periods and
2. the bias that is due to the finite number of exposures from which the default frequencies are taken.

We explore the first type of bias next and the second type afterwards.

Table 2: **Sample statistics of $ML-$, $AMM-$ and $FMM-$estimates of the asset correlation ρ**

sample size	**bias**			**standard error**		
	ML	AMM	FMM	ML	AMM	FMM
5	-0.020	-0.012	-0.017	0.056	0.055	0.056
10	-0.010	-0.006	-0.011	0.042	0.044	0.044
20	-0.004	-0.002	-0.006	0.032	0.035	0.035
31	-0.003	< 0.000	-0.004	0.025	0.029	0.029

The number of issuers N is set to 1000, the number of simulation runs S is set to 1500 and the number of time periods T varies between 5 and 31 periods. In order to base the DGP for the simulation runs on estimates from a realistic sample we select default histories of bond data, rated by Moody's and aggregated over all speculative grade rating classes.[16] The $ML-$estimates $\rho_0 = 0.098$ and $\gamma_0 = -1.805$ (corresponds to an unconditional PD of 3.55 %) serve as starting values for the simulation runs.

The relevant statistics of the estimation error are given in table 2. The bias is defined as the difference between the mean of the simulation estimates and the true value ρ (here 0.098). A positive value indicates, therefore, an upward bias of the estimator. In general the figure of 31 observation periods is far beyond the sample size that is typically available for bank loan loss data. To explore how shorter time series of default frequencies affect the estimates of ρ we gradually reduce T from 31 to 20, to 10 and finally to 5 observations. The figure of 5 observations is consistent with the number of years of data that will ultimately be required by banks following the IRB approach of the new Basel accord.[17]

Table 2 shows how the accuracy of the estimates increases with sample size. For the $ML-$estimates with 31 observations the standard is 2.5 bp compared with 5.6 bp for 5 observations in time. This means that for a Basel II-compliant period of 5 years the standard error is as high as 57 % of the true value of $\rho_0 = 0.098$. These numbers reveal the wide margin of error if asset correlation has to be estimated from small samples.

We omit here and in the following the estimation results for the second parameter, that is the unconditional PD (or equivalently the threshold value γ). The estimates for a sample size of 10 and higher are pretty accurate and the standard error is below 1 bp. Only for a sample size of 5 the standard error increases to 1.3 bp for both estimation methods. Considering the absolute level of the unconditional PD which is 3.55 % the estimates are still rather accurate.

[16] The default rates are given in appendix 7.1.

[17] For a transition period a shorter time series of 2 years will be deemed sufficient.

Table 3: **Small sample downward bias and RMSE of asset correlation estimates**

estimator	**error statistic**	**Gordy/Heitfield** (? exposures)	**new simulations** (1000 exposures)
AMM	bias	n.a.	0.0003
	RMSE	n.a.	0.032
FMM	bias	-0.020	-0.009
	RMSE	n.a.	0.035
ML	bias	-0.016	-0.004
	RMSE	n.a.	0.032

Next we compare the results of our simulations with those of the previous study by Gordy and Heitfield (2000). The DGP is adopted from Gordy and Heitfield (2000) ($\rho = 0.09$ and PD=1 % corresponding with an average default rate for BB-rated obligors[18]) in order to facilitate comparison. Following their example we generate 1500 random draws of 20 yearly observations. Table 3 summarizes the sample bias for the $AMM-$, the $FMM-$ and the $ML-$ estimator. According to this table the $FMM-$ and the $ML-$ estimator produce a considerably lower small sample bias in the new simulations than in Gordy and Heitfield (2000).[19] A possible explanation may be a lower number of exposures in their samples. The difference between the estimated bias of the $FMM-$estimator (-0.009) and the $ML-$estimator (-0.004) in the new simulations is small in relation to the RMSE.

An at first glance surprising result is the observation that the $FMM-$ estimator does not outperform the $AMM-$estimator given this DGP with 1000 exposures. The bias of the $AMM-$estimates (0.003) is by the order of 10 smaller than those of the other two estimators and it has a different sign. The fact that the $FMM-$estimator explicitly controls for the number of exposures motivates an analysis of the dependance of the bias on this variable.

Next we analyse for a fixed length of the time series of default rates how the performance of the three estimators depends on the number of exposures. Default histories are sampled for 20 time periods with the same DGP (i. e. PD of 1 %, $\rho = 0.09$) as in Gordy and Heitfield (2000) but with varying exposure numbers between 100 and 1000. The results of the 5000 simulation runs for each number of exposures are summarized in table 4. They show that the $AMM-$estimator is essentially unbiased and outperforms the $FMM-$estimator in terms of a smaller bias for 500 or more exposures. For smaller samples of 100 or 250 exposures, however, it is notably upward biased and is outperformed by the $FMM-$estimator. Overall the $FMM-$estimator underestimates the true asset

[18] We assume that this value for the PD has been used in Gordy and Heitfield (2000), too.

[19] A comparison of the $AMM-$estimator is not possible because Gordy and Heitfield haven't applied this estimation method.

Table 4: **Bias and RMSE of the $AMM-$ and $FMM-$estimator dependent on the number of exposures**

estimator:	AMM		FMM	
statistic:	bias	RMSE	bias	RMSE
exposure number				
1000	-0.0004	0.033	-0.010	0.036
500	0.009	0.035	-0.010	0.039
250	0.024	0.044	-0.011	0.044
100	0.066	0.079	-0.004	0.053

correlation in all four cases. Its bias does not vary much with the number of exposures which was expected because it controls for this variation.

With 100 exposures a negative adjusted variance was estimated in 12 % of the simulation runs. This high number casts some doubt on the usefulness of this estimator for small sample sizes even if the bias is relatively small.

The RMSE of both estimators are overall similar and relatively high compared with the estimates of ρ. They notably differ only for a sample size of 100 exposures when the $AMM-$estimator has a notably higher RMSE. So for this DGP we conclude that except for relatively small samples both $MM-$ estimators perform similarly.

Further simulations have shown that this result depends heavily on the parameters of the DGP. For instance with $\rho = 0.03$ the $FMM-$ outperforms the $AMM-$estimator for portfolio sizes up to 1000 exposures. From these results we conclude that the estimators can produce biased estimates but that there is no evidence that one of the three estimators outperforms the others for the whole parameter space. These results clearly warrant further investigation of the small sample bias and its relation to the model parameters including the number of exposures. The next step that is still left to future research will be a response surface analysis where the bias of the estimators is analysed for the whole relevant parameter space that is spanned by the length of the time series T, the number of exposures N, the asset correlation ρ and the unconditional PD. Applying a response surface will enable us to greatly reduce the problem of specificity[20].

In the empirical analysis in section 6 we control for differences in the small sample bias of the estimators by applying both, the $ML-$ and the $AMM-$estimator.

[20]See Hendry (1984), p. 955.

5 Classification into PD- and Size Categories

The simulation runs in section 4 illustrate the small sample problem that occurs with the estimation of the asset correlation. The Deutsche Bundesbank has provided us access to a database that contains the credit history of 53280 German firms from whom the Bundesbank has purchased fine-bills between 1987 and 2000.[21] Before the introduction of the Euro these fine bills were purchased at a special rate below the discount credit facility and this financing procedure has been much favored by German companies. The bank of the corporate obligor purchases a fine-bill from its client that is afterwards submitted to a branch office of the Bundesbank and included in the database. In this way there is no direct credit relationship between the Bundesbank and the firm. Therefore, contrary to the definition in the new Basel Accord, the default event is defined solely by legal insolvency.[22] Accordingly, the number of defaults is always conservative in the sense that it would be higher under the Basel definition of default.

The corporates included in the Bundesbank database are only a sample of all German corporates. Arguably this sample may be biased in the sense that the sector distribution in the database differs from the sector distribution of all German firms. Therefore, the default rates are calibrated in the first step to default frequencies which are representative for the German industry in order to remove a potential sector bias. In order to explore if the results from the first step are biased by a late definition of default, a second calibration is carried out. In this second step the default rates are inferred from loan net provisions of German banks. In the following, both steps are explained in more detail.

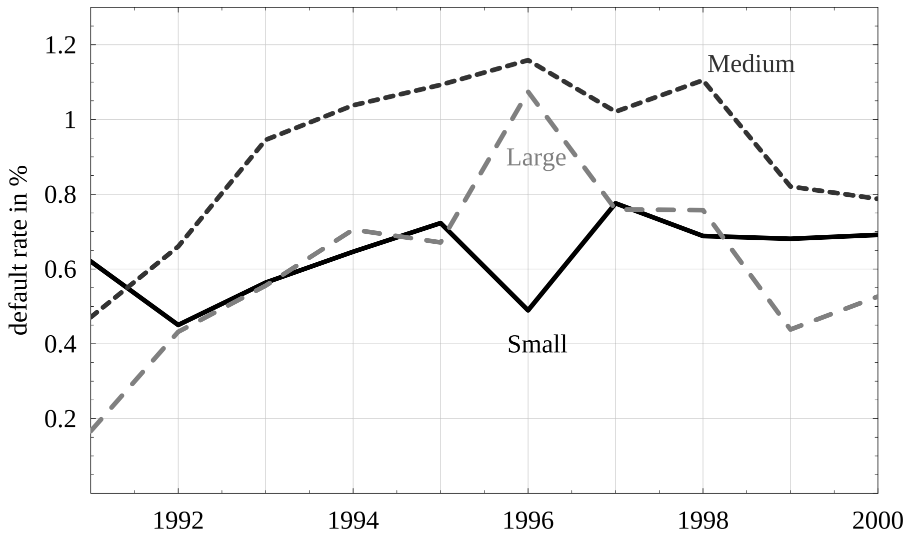
The first step in the calibration of the default rates uses the business-sector specific insolvency statistics of the Federal Statistical Office "Statistisches Bundesamt". A default rate is always determined by dividing the number of defaulted obligors in year t by the total number of solvent obligors at the end of year $t-1$. The numerator of the default rate always refers to the number of all defaulted companies in the Bundesbank database. The number of firms in the denominator, however, is adjusted by the calibration to ensure that the ratio reflects the default rate from the insolvency statistics. This number of firms is then drawn from the Bundesbank database and used in the analysis of a PD- and/or size-dependency of the asset correlation. In this calibration step we account for different insolvency statistics of the three business sectors manufacturing, trade and a third sector that comprises the rest, namely firms in the service business.

The firms that are drawn from the Bundesbank database in the calibration are assigned

[21] This is the same data base that is used in Hamerle et al. (2002). We thank Stefan Blochwitz who has provided very helpful SAS-routines for the data preparation.
Due to data constraints and the need to have two years of balance sheet data for the calculation of a score value the time series of default rates only covers the years 1991–2000.

[22] This is laid down in the German Insolvency Code.

Figure 1: **Corporate Default Frequencies between 1991 and 2000 in Percent**



to three different size categories according to their yearly turnover. The boundaries are selected in order to ensure that the firms are relatively equally distributed among these categories. The respective upper boundaries are 5 mln EUR for smaller and 20 mln EUR for medium size companies. The rest is assigned to the third bucket of large companies.

Figure 1 shows the time series of default rates in the three rating categories for small, medium and large firms.[23] The highest average default rate of 0.91 % (over time) is observed for medium size firms compared with 0.63 % for small firms and 0.61 % for large firms. This result is not fully consistent with other empirical evidence that with decreasing firm size the default probability increases. However, small and medium companies together have on average a higher default risk than large companies.

The observation that size categories have different PDs may distort the estimates for the asset correlation if PD is a driver of this correlation. Therefore, we assign obligors with similar PDs to three different PD-categories which we refer to as *rating grades*. This allows estimating the asset correlation in the three categories small, medium and large firms separately and conditional on their PD (or rating grade).

A time discrete probit hazard model was estimated to assign the firm years into PD categories. The explanatory variables of this model are a constant, a firm specific credit score SC_{it}[24] and a macro-economic variable Z_t. The firm- and time-specific hazard rate HR_{it} is determined as follows:

$$HR_{it} = \Phi(\beta_0 + \beta_1\ SC_{it} + \beta_2\ Z_t). \tag{13}$$

[23]See also tables 14 to 16 in appendix 7.3.

[24]See appendix 7.2.

Table 5: **Distribution of companies by size and sector in percent**

size	manufacturing	trade	services	total
small	31.6 %	30.3 %	38.1 %	100 %
medium	44.5 %	37.8 %	17.7 %	100 %
large	55.4 %	29.4 %	15.2 %	100 %
total	40.1 %	32.4 %	27.5 %	100 %

The boundaries of the rating categories are fixed to ensure that the defaults are relatively equally distributed among the three categories. The respective intervals of hazard rates for the three categories are $0 < HR \leq 0.01$, $0.01 < HR \leq 0.015$ and $HR > 0.015$. The PD–categories are assigned rating grades from 1 to 3 to facilitate referencing. Grade 1 denotes the rating grade bearing the lowest and grade 3 the grade with the highest credit risk.

Table 5 summarizes the distribution of companies in the sample among three sectors: manufacturing, trade and a residual sector that mainly comprises of service companies. From small to large companies the percentage share of manufacturing increases from 31.6 % to 5.4 % and presumably the cyclicality. The contrary holds for the service sector whose weight in the sample decreases from small to large companies. The share of the trade sector shows no monotonic dependency on size.

The results in table 5 for the sector distribution would be consistent with the hypothesis that a higher asset correlation is observed for large companies because of the relatively higher share of manufacturing and, therefore, more cyclical firms. If this hypothesis is supported by empirical results is discussed in the following section.

6 Estimates of the Asset Correlation

From the default frequencies of tables 14 to 16 in appendix 7.3 the PD and the asset correlation ρ are estimated by the methods described in section 3. We employ the $AMM-$estimator and the $ML-$estimator (assuming a large number of 3000 exposures). These "asymptotic" estimators for an infinite or at least large number of exposures are more appropriate because the underlying default frequencies have been calibrated to insolvency rates or loan net provisions that have been observed for the whole universe of German firms.

The estimates of ρ for the selected size- and PD-categories are summarized in table 6 for the $AMM-$ and in table 7 for the $ML-$estimator. The estimation results show a relatively low absolute level of the asset correlation. The strongest increase is observed for

Table 6: **Parameter estimates (with standard errors) calibrated to insolvency rates ($AMM-$estimator)**

	$\hat{\rho}_{AMM}$		
rating	1	2	3
small	0.005	0.011	0.004
	(0.006)	(0.010)	(0.006)
medium	0.007	0.012	0.018
	(0.006)	(0.012)	(0.014)
large	0.021	0.015	0.064
	(0.011)	(0.021)	(0.036)

Table 7: **Parameter estimates (with standard errors) calibrated to insolvency rates ($ML-$estimator)**

	$\hat{\rho}_{ML}$		
rating	1	2	3
small	0.002	0.010	0.005
	(0.001)	(0.005)	(0.002)
medium	0.007	0.011	0.016
	(0.003)	(0.005)	(0.007)
large	0.013	0.016	0.045
	(0.006)	(0.007)	(0.020)

the lowest rating category 3. This is consistent with estimation results for asset correlation in Dietsch and Petey (2002) for French corporates.[25]

The estimates of ρ in tables 6 and 7 increase with size for all three rating grades and under both estimation methods. This result supports the hypothesis that asset correlation is positively correlated with firm size. It contrasts, however, with the observation in Dietsch and Petey (2003) that inside the SME-segment asset correlation decreases with firm size. But consistent with our results they observe the highest asset correlations for large corporates.

The standard errors in table 7 are determined by bootstrapping for the $AMM-$ and analytically for the $ML-$estimator. Especially for small correlation estimates the standard errors from bootstrapping are considerably higher than the asymptotic ones. This suggests that the asymptotic standard errors may be severely underestimated. Due to the short time series of default data, the standard errors are relatively high, especially for the $AMM-$estimates. Although the ranking of the estimates for ρ in table 7 supports the hypothesis of a size-dependency, it is, therefore, difficult to prove this relation statistically.

The low level of the estimates may be driven by the late legal definition of default. To account for this effect in the next step the level of the default rates is calibrated to default frequencies that are inferred from loan net provisions of banks.

The calibration to loan net provisions, however, cannot control for a potential bias that is induced by size-dependent differences in the distribution of default events. Legal default, for instance, is more common for small firms than for large companies.

The data on loan net provisions are extracted from the OECD-report on 'Bank Profitability - Financial Statements of Banks'. They are available for different banking groups. For the purpose of calibration we use the group 'commercial banks' which comprises the former category of installment sales financing institutions which have developed into universal banks.

The transformation of the loan net provisions of year t into an inferred default-rate is carried out by the following formula in which LGD denotes the loss given default:

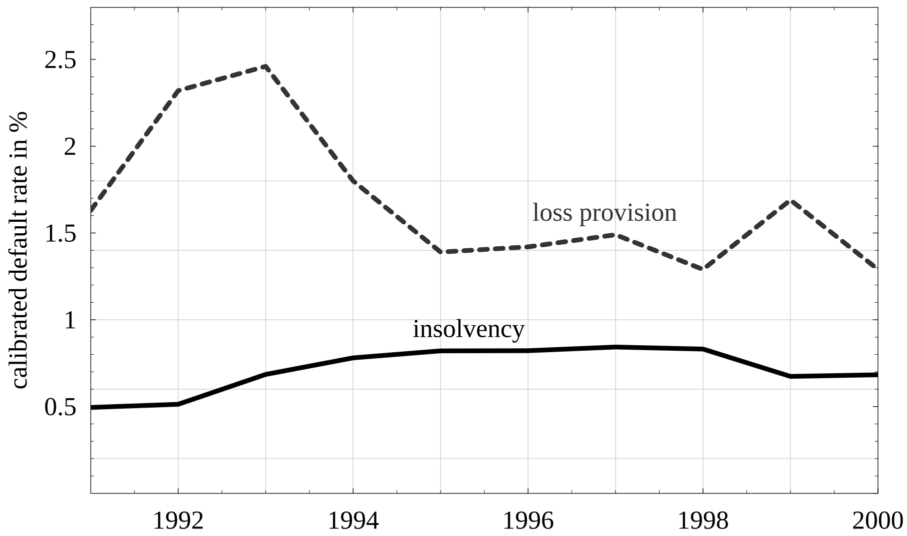
$$DR_{OECD}(t) = \frac{Loan\ Net\ Provisions(t)}{Loan\ volume(t-1) \times LGD}. \tag{14}$$

The calibrated default rate $DR^{cal}(g,t)$ for rating grade g in year t is determined from the default rates of all obligors in that year, $DR(t)$, the obligors of this rating grade, $DR(g,t)$, and the default rate from the OECD-data, $DR_{OECD}(t)$:

$$DR^{cal}(g,t) = \frac{DR_{OECD}(t)}{DR(t)}\, DR(g,t). \tag{15}$$

[25] See Dietsch and Petey (2002), p. 311–312.

Figure 2: **Corporate Default Frequencies between 1991 and 2000 in Percent**



The LGD is set to 50 % which equals the LGD-assumption of uncollateralised loans in the 2nd consultative paper of the new Basel accord. This assumption is made only to avoid severe empirical problems with estimating the LGD separately. Note that recent research suggests that LGD should be modeled as a stochastic variable that is correlated with the PD.[26] In our case keeping LGD constant means that all variability is transferred into the PD so that the PD behaves more volatile than in the case where the LGD is stochastic. This may distort the estimates of the asset correlation.

Figure 2 shows the default rates calibrated to insolvency statistics aggregated over all size- and PD-buckets and compares them with the default rates inferred from the loan net provisions. Two observations are noteworthy:

First the default rates from loan net provisions are overall on a higher level. This may be explained by the fact that loan net provisions provide in general an "earlier" default criterion than insolvency and not all obligors for whom specific provisions have been made enter insolvency at a later stage.

Second the default rates determined by loan net provisions appear to be be more volatile. This observation may suggest that they are more sensitive to the business cycle and estimates of the asset correlation from these data may be higher than those from insolvency rates.

Based on the calibrated default rates $DR^{cal}(g,t)$ the asset correlation is estimated again with the $AMM-$estimator. The results are given in table 8. The level of the estimated asset correlation is higher for the calibrated data in table 8 than in table 6. The maximum

[26] See Altman et al. (2002) and further references given there.

Table 8: **Parameter estimates (with standard errors) calibrated to loan net provisions ($AMM-$estimator)**

	$\hat{\rho}_{AMM}$		
rating	1	2	3
small	0.012	0.046	0.032
	(0.008)	(0.028)	(0.020)
medium	0.016	0.033	0.075
	(0.010)	(0.024)	(0.043)
large	0.021	0.049	0.140
	(0.014)	(0.045)	(0.083)

Table 9: **Estimates of asset correlation and default correlation calibrated to loan net provisions ($ML-$estimator)**

	$\hat{\rho}_{ML}$			$\hat{\rho}_{ML}^{def}$		
rating	1	2	3	1	2	3
small	0.009	0.040	0.025	<0.000	0.007	0.006
	(0.004)	(0.017)	(0.011)			
medium	0.012	0.036	0.057	<0.000	0.007	0.018
	(0.005)	(0.016)	(0.024)			
large	0.016	0.053	0.094	<0.000	0.012	0.033
	(0.007)	(0.022)	(0.040)			

is observed again for large corporates with the lowest rating grade but with 0.14 it is 2.2times higher than in table 6. Again we observe that the asset correlation overall increases with size conditional on the rating grade. The only exception occurs for rating grade 2 where the estimated asset correlation of small companies is slightly higher than for medium size corporates. For the lowest rating grade it increases with size quite strongly from 0.03 to 0.14.

Table 9 shows the estimates of the asset correlation by applying the $ML-$estimator. The estimates overall increase with firm size for all three PD-categories. The only exception occurs again for grade 2 in the transition from small to medium enterprises.

Moving the focus towards a potential PD-dependence of the asset correlation we observe in table 9 that the asset correlation increases with PD for medium and for large enterprises. Both findings reflect previous results from the $AMM-$estimates in table 8. Note, that although both estimators provide similar results as to the ranking of asset correlations of PD- and size categories the level of the estimates differs, particularly for large companies with high PDs (0.094 for $ML-$ vs. 0.140 for $AMM-$estimates). In order to examine if this difference can arise from estimation error we simulate both estimators with a given parameter set of $\rho = 0.1$ and $PD = 0.1$. Whereas the mean estimates (0.094 for $ML-$ and 0.099 for the $AMM-$estimator) are close to the true values, the mean squared error in both cases is with 0.054 and 0.048 rather high. This result indicates that the differences between table 8 and table 9 are caused by estimation error.

In the one-factor model and in a homogenous portfolio an estimate $\hat{\rho}^{def}$ of the default correlation between two firms can be determined directly from the estimates of the PD and the asset correlation.

$$\hat{\rho}^{def} = \frac{\Phi\left(\Phi^{-1}(\hat{p}), \Phi^{-1}(\hat{p}), \hat{\rho}\right) - \hat{p}^2}{\hat{p}\,(1-\hat{p})}. \tag{16}$$

The estimates of $\hat{\rho}^{def}$ are provided in the right section of table 9. Due to the low level of the estimates of the asset correlation the default correlation between firms in the best rating class is less than 0.1 %. In the rating classes 2 and 3 we observe that default correlations increase with size. The highest estimate of 3.3 % is observed for large firms in rating category 3.

Envoking the Wald principle we apply a statistical test in order to answer the question if the observed differences in the estimates of ρ for portfolios of different firm size are statistically significant. If this is the case we should at least observe a significant difference between the estimates of asset correlation of large and of small firms. The following test statistic

$$\frac{\hat{\rho}^{large} - \hat{\rho}^{small}}{\sqrt{(\hat{\sigma}^{large})^2 + (\hat{\sigma}^{small})^2}} \tag{17}$$

asymptotically follows a Gaussian distribution. We test the null hypothesis, that the estimates of ρ are the same for the categories of large and small firms in every rating class. The p-values are given in table 10.

We can reject the null hypothesis on a significance level of 5 % for three out of four estimates in the lowest rating category. For the other two rating categories it is mostly not possible to reject the null hypothesis.

Three out of four rejections in table 10 occur for the ML-estimator. The standard errors for this estimator are overall smaller than for the $AMM-$estimates because they are asymptotic values. This may be the reason that the null hypothesis is more often rejected for the $ML-$estimates.

We conclude that although the estimates indicate that asset correlation depends on size, this relation is only in some cases statistically significant on the usual confidence levels. The latter result may be driven by a relatively high estimation error that is due to the short time series of data.

Next we compare these empirical results with the prospective IRB risk weights of the new Basel Accord. Since the publication of the second consultative document in January 2001

Table 10: **P-values for mean-test of differences between estimates of ρ for large and small firms**

estimator	**default rates** calibrated to ...	**rating** 1	2	3
AMM	insolvency rates	0.114	0.432	*0.049*
	loss Provisions	0.292	0.473	0.103
ML	insolvency rates	*0.035*	0.243	*0.023*
	loss Provisions	0.193	0.320	*0.048*

two notable changes have been made to the risk weights for corporate obligors from which the capital charge can be determined. These two changes involve the parameter asset correlation which is no longer a constant value of 0.2. The risk weights proposed in the third Quantitative Impact Study of the Basel Committee depend on the obligor's PD and – for SMEs[27] – additionally on firm size which is measured by yearly turnover. The new risk weight function assumes that asset correlation decreases with PD and increases with size.

The first modification which assumes that asset correlation declines with PD is not supported by our empirical results. Whereas the results are mixed for the default rates calibrated to the insolvency statistics the results for the calibrated OECD-data suggest that in two out of three size classes asset correlation increases monotonicly with higher PDs. This increase is stronger for larger corporates.

The second modification that asset correlation increases with size is qualitatively backed by the correlation estimates in table 6, 8 and 9 and holds, therefore, independent of the calibration and of the estimation method. The increase is strongest for the category with the highest credit risk. This result may derive from the distribution of companies among business sectors, e. g. that the share of firms in cyclical sectors increases with firm size.

Turning to correlation estimates based on the OECD-statistics we find again that asset correlation overall increases with firm size. Summarizing, our results support a size-dependent capital relief for SMEs in the IRB risk weights.[28]

7 Conclusion

The first and theoretical part of this paper explores the small sample performance of three estimators of the asset correlation from time series of default rates. We find that these

[27]SMEs are defined as firms with a yearly turnover below 50 Mln. EUR.

[28]This capital relief is implemented in the IRB-corporate risk weights by a size dependent reduction of the asset correlation parameter.

estimators are biased in small samples and that in a homogenous portfolio the size of the bias depends on the number of exposures, the PD and the "true" asset correlation.

The bias can be accounted for in two ways: *directly* by adjusting the estimator, as has been suggested by Gordy (2000) to control for the influence of idiosyncratic risk, or *indirectly* by estimating a functional relation between the bias and the relevant model parameters (response surface analysis) and adjusting the biased estimate afterwards. More worrisome than the bias, however, are the relatively high standard errors of the estimates.

In the second and empirical part the asset correlation of German corporate obligors is estimated from a database of balance sheet information and default histories of German corporates that is maintained at the Bundesbank. A special focus is given to a potential dependence of asset correlation on PD and firm size. The results from default rates, calibrated to German insolvency statistics, show that aggregated over all rating categories as well as for single rating grades the asset correlation increases with size. However, we do not observe an unambiguous dependence on PD. The absolute level of the asset correlation is relatively low (between 0.002 and 0.06). The same estimation carried out with default rates implied by net loan net provisions of German banks show higher asset correlations (between 0.01 and 0.14). Again we find that asset correlation overall increases with firm size but we do not observe an unambiguous relation between asset correlation and PD.

Comparing the results with the currently proposed calibration of the IRB-risk weights for corporate loans in the new Basel Accord needs a word of caution. These risk weights are calibrated from a macro-prudential as well as a micro-economic perspective. The decision to have an asset correlation parameter declining with PD, for instance, is justifiable by the desire to reduce pro-cyclical effects of the New Accord. In this paper instead, we deal only with the micro-perspective and in this case our estimation results indicate a converse but more ambiguous relationship between asset correlation and PD. The modification that asset correlation increases with size which has been introduced into the risk weights but is restricted to SMEs is overall corroborated by our estimates. This relation exists in all of the obligors PD-categories and seems to be stronger for obligors with higher PDs. The empirical results indicating a size- and a PD-dependence of asset correlation hold irrespective of the chosen estimation technique.

Our results suggest that further research is warranted in the following two areas. The first area is an economic explanation for the empirical result that the asset correlation increases with firm size. We find tentative evidence that this result may derive from the fact that the size-distribution of corporates varies between business sectors which in turn differ in their dependence on the business cycle.

The second area where further research seems to be warranted are the small sample properties of the estimators for the asset correlation. The differences in levels between the $ML-$ and the $AMM-$estimates of this parameter may be distorted by the estimation error which has a level component (bias) and a volatility component (standard error). In order to improve our understanding of the impact of the estimation error we intend to analyse the relation between the estimation error and the estimation method with a response surface analysis.

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Appendix

7.1 Bond default rates from Moodys (speculative grade)

Table 11: **Bond default rates**

year	default rate	year	default rate
1970	9.38	1986	5.67
1971	1.14	1987	4.23
1972	1.94	1988	3.47
1973	1.28	1989	6.03
1974	1.35	1990	9.85
1975	1.79	1991	10.52
1976	0.89	1992	4.86
1977	1.35	1993	3.51
1978	1.79	1994	1.93
1979	0.42	1995	3.30
1980	1.62	1996	1.65
1981	0.71	1997	2.03
1982	3.57	1998	3.41
1983	3.88	1999	5.63
1984	3.39	2000	5.71
1985	3.90		

Source: Hamilton et al. (2001), p. 45–46.

7.2 Scoring Function

The parameters of the scoring function depend on the affiliation of the obligor to one of the three business sectors manufacturing, trade or others.[29] The respective scoring function is defined in table 12.

Table 12: **Scoring Function**

industry	function
Manufacturing	$10.1749 + 0.1651 \times CRR + 0.3608 \times ROR - 0.1681 \times APR + 0.0829 \times ER$
Trade	$2.2943 + 0.2080 \times RER + 1.0275 \times RCE - 0.1290 \times ARR + 0.2618 \times ER$
Others	$8.9966 + 0.0718 \times ARD + 0.5746 \times ROR - 0.1233 \times CRR + 0.1830 \times ER$

[29] We are very grateful to Stefan Blochwitz who has provided us with the parameters of the scoring function.

Table 13: **Definition of input factors to the scoring function**

factor	**interpretation**
CRR	capital recovery rate= $\frac{net\ cash\ flow}{invested\ capital} * 100$
ROR	return on revenues= $\frac{earnings\ before\ tax}{total\ turnover} * 100$
APR	$\frac{notes\ payable+trade\ accounts\ payable}{total\ turnover} * 100$
ER	equity ratio=$\frac{equity+accounts\ payable\ to\ owners}{total\ assets} * 100$
RER	$\frac{net\ cash\ flow}{total\ turnover} * 100$
RCE	return on total capital employed= $\frac{earnings\ before\ taxes\ and\ interest\ payments}{total\ assets} * 100$
ARR	$\frac{trade\ accounts\ receivable}{total\ turnover} * 100$
ARD	ability to repay debt=$\frac{net\ cash\ flow}{accounts\ payable-liquid\ assets} * 100$

7.3 Default Rates calibrated to Insolvency Statistics

Table 14: **Default frequencies of small firms for different rating grades between 1991 and 2000 in percent**

year	all	grade 1	grade 2	grade 3
1991	0.62	0.34	1.47	2.52
1992	0.45	0.25	1.18	1.72
1993	0.56	0.26	1.45	2.06
1994	0.65	0.28	1.43	1.90
1995	0.72	0.33	1.11	2.12
1996	0.49	0.28	0.69	1.31
1997	0.78	0.41	1.11	2.17
1998	0.69	0.33	0.83	2.06
1999	0.68	0.27	1.09	2.16
2000	0.69	0.27	1.76	2.04
Average	0.63	0.30	1.21	2.00

Table 15: **Default Frequencies of medium firms for different rating grades between 1991 and 2000 in percent**

year	all	grade 1	grade 2	grade 3
1991	0.47	0.25	1.72	3.31
1992	0.66	0.40	1.78	4.74
1993	0.95	0.54	2.76	5.83
1994	1.04	0.47	3.20	3.62
1995	1.09	0.40	3.19	5.03
1996	1.16	0.55	1.72	6.25
1997	1.02	0.55	2.62	3.53
1998	1.10	0.59	1.83	4.57
1999	0.82	0.39	2.56	2.83
2000	0.79	0.46	1.82	2.81
Average	0.91	0.46	2.32	4.25

Table 16: **Default Frequencies of large firms for different rating grades between 1991 and 2000 in percent**

year	all	grade 1	grade 2	grade 3
1991	0.17	0.15	0.82	0.00
1992	0.43	0.26	1.94	4.55
1993	0.56	0.39	1.50	3.55
1994	0.71	0.30	1.71	4.72
1995	0.67	0.31	1.43	5.52
1996	1.07	0.52	1.30	7.50
1997	0.76	0.38	2.17	4.64
1998	0.76	0.45	2.61	2.82
1999	0.44	0.20	1.79	2.55
2000	0.53	0.35	2.32	1.31
Average	0.61	0.33	1.76	3.72