Liquidity channels and stability of shadow banking

Sofia Priazhkina[*†]

Abstract

Using financial networks modeling, I ask whether severe liquidity conditions of shadow banks during a crisis can be improved with a support from regulated banks. In equilibrium, financial markets endogenously develop a core-periphery network structure generating heterogeneity in size, interconnectedness, and riskiness of banks. Core regulated banks form long-term relationships with core shadow banks by channeling liquidity to them from periphery banks. These long-term relationships are accompanied by implicit liquidity guarantees during a crisis and make core regulated banks systemically unstable. I provide intuition for why financial markets develop these liquidity channels and examine policies to control systemic risk.

Keywords: Systemic risk, Financial crisis, Financial networks, Shadow banking, Bank run, Central bank

JEL: D85, G01, G21, G23, G28, E58

[*]Financial Stability Department, Bank of Canada, Ottawa, Ontario, Canada K1A 0G9, pria@bankofcanada.ca

[†]The views and findings of this paper are entirely those of the author and do not necessarily represent the views of the Bank of Canada. I am thankful to Jason Allen, Toni Ahnert, Charles Khan, Itay Goldstein, Ben Craig, and Andrew Ellul for their suggestions of how to improve the paper.

1 Introduction

This paper studies the liquidity channels from regulated banks to shadow banks, the importance of endogenous network formation in establishing these channels, and their role during a crisis. Understanding the interaction between shadow and regulated banks is important for the following reasons. First, it explains why government intervention might be necessary to solve the liquidity problems of stressed shadow banks, and what category of shadow banks would benefit the most from this intervention. Second, it shows how the instability of shadow banks propagates to the regulated banks and money market investors. Third, it describes how the presence of shadow banks contributes to the origination of too-big-to-fail banks when financial markets form endogenously.

The research is motivated by the sudden lack of liquidity available to shadow banks during the 2007-2009 financial crisis.[1] Before the crisis, shadow banks originated as an alternative to traditional (regulated) banks to perform similar functions of liquidity, credit, and maturity transformation (Pozsar et al. (2010)). The growth of shadow banking has been largely driven by tightening in the regulatory requirements of banks and financial innovation (Acharya et al. (2013) and Adrian and Ashcraft (2012)). While the assets of regulated banks were relatively safe and financed with stable deposits, the assets of most shadow banks were risky and financed with short-term money-market instruments. Given that shadow banks do not have access to either insured deposits or the central bank discount window, money market investors quickly withdrew liquidity in response to asset price shocks.[2] Illiquidity pushed shadow banks to seek support from their sponsoring regulated banks and other financial institutions.[3] However, the liquidity was passed

[1] While this paper is motivated by the Great Recession, it is relevant to any financial market with the separation of a banking system (see a report on shadow banking worldwide at IMF (2014) and Board (2014)).

[2] For example, Gorton and Metrick (2010, 2012) have showed that runs on bilateral repurchase agreements were at the heart of the financial crisis. Krishnamurthy, Nagel and Orlov (2011) claimed that the money market funds reduced the liquidity provision in the market of asset backed commercial papers. Copeland, Martin and Walker (2014) found a large and precipitous decline in a tri-party repo funding to Lehman Brothers.

[3] Some regulated banks also provided liquidity to shadow banks under indirect liquidity provision program organized by the Federal Reserve. For example, Asset-Backed Commercial Paper Money Market Mutual Fund Liquidity Facility (AMLF) is an indirect liquidity provision program under which the Federal Reserve provided liquidity support to struggling money market mutual funds (MMMFs) indirectly by lending to regulated financial institutions so that they could purchase asset-backed commercial paper (ABCP) from the MMMFs.

onto the shadow banks only when this was in the best interest of the regulated banks. Moreover, a miscalculation of the implicit guarantees of regulated banks to shadow banks during the crisis led to the underestimation of risks that shadow banks were taking pre-crisis (Tarullo (2013), Lane (2013) and Chant (2009)). Therefore, understanding the interplay between regulated and shadow banks before, during, and after periods of turmoil is crucial for understanding systemic risks and liquidity channels in the financial system.

In this paper, I build a three-period network model with strategic interactions between regulated banks, shadow banks, and money-market investors. I analyze under which scenarios regulated banks may be willing to rescue troubled shadow banks by channeling central bank liquidity. The two-tiered liquidity provision scheme considered in this paper is different from direct liquidity support from the central bank because it reallocates credit risk from taxpayers to the regulated banks and, as a result, requires regulated banks to monitor shadow banks. In addition, by considering pre-crisis relationships between banks, I account for the cases when a regulated bank may be more willing to provide liquidity support to a shadow bank given a long-term exposure between the two.

I show that money market investors strategically withdraw their funds from shadow banks as a response to a significant asset shock. Different from the bank run model of Diamond and Dybvig (1983), the liquidity run on shadow banks is inevitable even if a liquidity support is provided to the shadow bank by a regulated bank. In the model, all money-market investors of some shadow banks could be better off by not running. However, the coordination failure occurs because each shadow bank keeps some liquid assets which make the initial withdrawals profitable. Therefore, during a severe financial crisis, shadow banks require significant liquidity inflows from outside of the shadow banking sector.

I first determine the nature of interbank relationships between one shadow bank and one regulated bank. In the equilibrium, there are two regimes under which the regulated bank may provide liquidity support to the shadow bank: vulnerable and immune. The vulnerable regime is characterized by a larger shadow bank size, greater money market exposure, and a higher money market rate. In this regime, the regulated bank contagiously defaults with positive probability if the shadow bank does not recover. The model predicts that the regulated bank charges a minimum rate for the liquidity support during the crisis to reduce pressure on the recovering shadow bank, but is compensated for this bailout service with a higher lending rate on the exposures pre-crisis.

Under the immune regime, the regulated bank stays solvent during and after the market crash in all market events and the shadow bank defaults if the risky asset turns out to be a bad investment. In this regime, the shadow bank is smaller, less exposed to money market investors, and the money market rate is lower. This regime occurs because a highly profitable depository bank limits the amount of liquidity support. It also demands a higher support rate – the rate on emergency liquidity support– in order to compensate itself from a contagious default that may follow if the shadow bank does not recover.

Although the immune regime illustrated for the two banks is more favorable from the viewpoint of systemic risk, this regime is not observed in a network of many banks. The reason for this is that a large coalition of banks profit from delegating the role of rescuing one shadow bank to one regulated bank. In turn, the rescuing regulated bank serves as an intermediary between the depository banks and the shadow bank. As a result, the rescuing bank becomes overexposed to the shadow bank, which leads to a systemic instability and possible contagion. The condition of limited liability and the restricted access to the lender of last resort are crucial for this result to take place. First, limited liability mitigates counterparty risk of the overexposed regulated bank. Second, the indirect liquidity support from the central bank makes the rescuing regulated bank more risk-seeking before the crisis, offering the shadow bank more generous liquidity guarantees. Both phenomena are observed in practice. These findings are also supported empirically: Kacperczyk and Schnabl (2013) find that the sponsors of money market funds during the crisis were large financial institutions that exposed to their money market funds before the crisis. The authors notice that even though the liquidity support is implicit, the sponsoring banks find it optimal to do so because the costs of not providing support is large.

Market-wise, the economic incentives of banks induce a financial network with core-periphery structure.

Core regulated banks grow to be large and highly interconnected; they provide rescuing liquidity channels to core shadow banks and transfer liquidity from regulated banks to the supported shadow banks. Non-supported shadow banks grow smaller than the supported ones and promise lower returns to the money market investors. Given that the initial balance sheets of all regulatory banks are identical, in equilibrium, the core regulated banks choose to generate larger profits in the states of market stability and become exposed to higher default risk in the states of crisis. The results of my paper are consistent with theoretical and empirical findings in the literature. For example, the network model of traditional banks of Leitner (2005) also captures the trade-off that the regulated banks face when they get exposed to each other: networks induce private sector bailouts and systemic risk at the same time. Likewise, Acharya et al. (2013) empirically document that the regulated banks that used conduits to transfer risk off balance sheet become more exposed to systemic risk due to the credit guarantees that they provided to the conduits.

I consider a number of mechanisms to maintain financial system stability. The driven result of the comparative statics is that an asset risk of shadow banks is an amplifier of the overall systemic risk in the financial network of regulated and shadow banks.[4] An increase in the quality of the risky assets leads to overall financial stability and more shadow banks being supported by depository banks. The asset quality also determines the success of other policies. First, a regulation comparable to the Volcker rule— one that imposes a cap on the exposure between shadow banks and regulated banks—improves financial stability only if shadow banks trade assets of a low quality. If shadow banks pose less threat to the solvency of regulated banks, the cap on the exposure between two banking sectors may discourage indirect liquidity support to relatively stable shadow banks and turn liquidity problems of shadow banks into their major vulnerabilities. Second, stricter redemption gates lead to more financial stability only if the quality of the risky assets is low. Moreover, redemption gates and fees favor shadow banks that do not have liquidity guarantees from the regulated banks and increase the funding cost of all shadow banks. I show that the rate at which the central bank provides liquidity support should also be sensitive to the quality of the

[4] I define asset quality as the sensitivity of risky asset returns to an exogenous market crash.

assets traded by shadow banks. This is consistent with the Bagehot (1873). In particular, the model predicts that the central bank should impose a punitive rate for shadow banks with higher asset and liquidity risks and a moderately low rate for more stable shadow banks. The optimal rate of central bank liquidity support is sensitive to the money market entry/exit costs and the riskiness of the shadow banks' assets.

The results of this paper provide intuition about the origins of too-big-to-fail problem. The model shows that in markets with many banks, the function of liquidity support can only be performed by large core banks. Therefore, a regulation that would control the number of large banks would implicitly control the liquidity channels from regulated banks to shadow banks during a crisis. This contributes to the discussion of endogenously formed core banks and precisely to the research question of Farboodi (2014), that provides another explanation for the existence of a core-periphery network structure. The author focuses on the network formation based on risk sharing incentives, while I provide a model of network formation and financial contagion due to both liquidity risk and asset risk. Castiglionesi and Navarro (2007) also find that efficient and stable networks of depositors, banks and their shareholders have core-periphery structure. My results are also consistent with the empirical work of Craig and Von Peter (2014) that show the bank specialization and balance sheet characteristics determine the banks position in the network and lead to the core-periphery structure of the market.

The remainder of the article is structured as follows. Section 2 supplies details of the model. Section 3 defines the notion of equilibrium. Section 4 presents theoretical results. Section 5 contains discussion about different policy mechanism to combat systemic risk. Section 6 concludes the paper. Appendices contain the proofs.

2 Model

2.1 Model overview

The banking sector is populated by N regulated depository banks and N^s shadow banks. Depository banks are regulated because of the central role that they play financing the real economy. This leads to three key differences between regulated and shadow banks. First, regulated banks are required to only hold high quality (here assumed risk-free) assets, while shadow banks can make risky investments. Second, depositors of regulated banks are protected by government insurance, while the liquidity contributions of money market investors to shadow banks are not insured. Third, regulated banks can count on the central bank in its role as a lender of last resort, while shadow banks do not have direct access to the central bank. These regulatory differences lead to markedly different behaviors of the two types of financial institutions and to a complex interplay between them.

In particular, shadow banks are less stable than regulated banks. The instability comes from both asset risk and liquidity risk. The asset risk of shadow banks is higher due to their specialization in risky activities. Their liquidity risk is magnified by uninsured short-term funding, which is subject to runs. As a result, a significant shock applied to the risky assets returns may incentivize money market investors to massively withdraw funding from shadow banks. Given that shadow banks' assets are of longer maturity than money market instruments, shadow banks may experience severe liquidity problems. Liquidity problems of these institutions may turn to solvency problems if shadow banks are unable to find liquidity support from private financial institutions.

Despite the inherent instability of shadow banks, regulated banks still find it profitable to invest in shadow banks since they have access to higher return projects. The interconnectedness between regulated banks and shadow banks, in turn, makes regulated banks less stable. Runs on shadow banks may propagate to regulated banks through long-term exposures between the two and lead to defaults of both bank types.

In this study, I characterize the network of interbank relationships with strategically formed links and determine how the network properties affect stability of shadow banks during a crisis.

2.2 Timeline

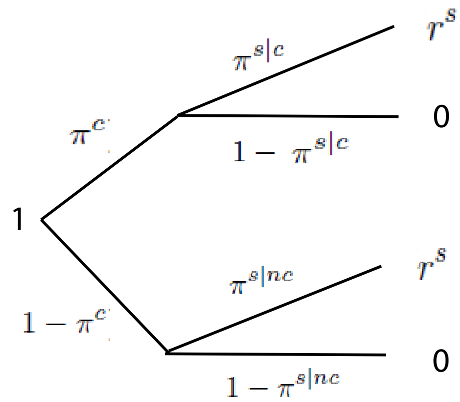
The model has three periods. All market participants are risk-neutral and there is no time discounting. At time $t = 1$, banks make investment and funding decisions. Money market investors make investment decisions. At time $t = 2$, the market of risky assets crashes with a certain probability. In case of a market crash, risky assets become more likely to default, which may force a subset of investors to withdraw shadow bank funding. In the case of liquidity withdrawals, regulated banks decide if they want to provide liquidity support to the illiquid shadow banks. At time $t = 3$, all assets pay off, profits are delivered to the players, and the ultimate set of defaulted banks is determined.

2.3 Asset payoffs

At time $t = 1$, each shadow bank $i \in B^s$ invests in risky asset $a_i \in A^s$ that pays off at time $t = 3$. Each a_i generates gross return r^s at time $t = 3$ in the event of *success* and zero return in the event of *failure* (or *no success*). The probability of *success* of investment a_i is determined by market conditions at time $t = 2$. I assume that a major market event—called financial market *crash*—can happen at $t = 2$ with probability π^c and affect the return distributions of all risky assets. If asset market conditions are *regular*, meaning the market crash is avoided, *success* of asset a_i happens with conditional probability $\pi^{s|nc}$. If the market crash happens, the conditional probability of *success* decreases to $\pi^{s|c}$, such that $\pi^{s|c} < \pi^{s|nc}$ (see Figure 1). Therefore, the returns of different risky assets are dependent random variables with non-trivial covariance matrix. At the same time, I assume that risky assets $A^s = (a_1, ..., a_{N^s})$ are not identical:

Assumption 1. *Any two risky assets a_i and a_j, $i \neq j$, generate different asset returns with a positive probability.*

Figure 1: The contingent claim structure of risky asset returns.



This assumption implies that a default of one shadow bank is not necessarily accompanied by the default of another shadow bank; however, all assets are more likely to default following a crash.

To shorten mathematical expressions, I will use joint probabilities of events

$$\pi^{s,c} = \pi^{s|c}\pi^c, \qquad \pi^{s,nc} = \pi^{s|nc}(1-\pi^c),$$
$$\pi^{ns,c} = (1-\pi^{s|c})\pi^c, \qquad \pi^{ns,nc} = (1-\pi^{s|nc})(1-\pi^c).$$

and marginal probabilities of events

$$\pi^s = \pi^{s,nc} + \pi^{s,c}, \qquad \pi^{ns} = \pi^{ns,nc} + \pi^{ns,c},$$
$$\pi^c = \pi^{s,c} + \pi^{ns,c}, \qquad \pi^{nc} = \pi^{s,nc} + \pi^{s,nc},$$

where superscript s stands for *success*, superscript c stands for *crash*, and superscript n stands for *no*.

We make two assumptions about the distributions of asset payoffs. First, we assume a sufficiently high expected return on a risky asset in the event of *regular* market conditions:

Assumption 2. *The expected rate of return of risky assets exceeds the rate of return on deposits even if* *a market crash completely destroys the asset value:*

$$r^s \pi^{s,nc} > r^{dep}.$$

Therefore, Assumption 2 is crucial for risky assets to be traded ex-ante the crisis.

Second, I assume that the market shock is severe, which makes investors unwilling to keep the asset ex-post the crash:

Assumption 3. *Following a market crash, the net present value of a risky asset is negative:*

$$r^s \pi^{s|c} < 1.$$

By considering a severe crisis, I restrict the set of all possible outcomes to the cases with full runs of money market investors (see Lemma 1), which simplifies equilibrium search and makes the model more intuitive.

2.4 Regulated banks and the real economy

The main function of regulated banks B is to facilitate intermediation between depositors and borrowers in the real economy. Therefore, I will sometimes refer to these banks as the depository banks. I assume that each depository bank generates profit v from the real economic activities. Profit v is generated from investing in safe assets and loans; however, the risk of real economy loans is reduced to zero, which is consistent with the logic that risky assets are managed by shadow banks (Gennaioli et al. (2013)). Regulated banks finance their safe assets with deposits at the competitive rate r^{dep}. In this model, I also do not consider a risk of runs on deposits due to the deposit insurance provided by the regulator.

At time $t = 1$, depository banks may provide funding to shadow banks or to each other expecting repayment at $t = 3$. I designate lending from bank $i \in B$ to bank $j \in B \cup B^s$ as q_{ij} and the corresponding interest rate as r_{ij}. At time $t = 2$, depository bank i may provide additional liquidity support to shadow bank j using central bank liquidity. Liquidity support amount from i to j is denoted q^b_{ij} and the corresponding interest rate is denoted r^b_{ij}. Central bank liquidity can be borrowed in the unlimited amount at rate r^{cb}:

Assumption 4. *Central bank lends at a positive net interest rate in the event of market crash:*

$$r^{cb} > 1.$$

It immediately follows from Assumptions 3 and 4 that the central bank does not profit in expectations from providing indirect liquidity support: $r^s \pi^{s|c} < r^{cb}$. Therefore, I consider this liquidity support as an emergency measure aimed at keeping shadow banks liquid.

I make an assumption that all regulated banks have an unlimited liquidity access during a crisis to simplify the model. Alternatively, the depository banks will experience additional liquidity constraints, which will make them less willing to support shadow banks. The two-tiered liquidity provision scheme considered in this paper is different from the discount window of the central bank because it reallocates credit risk from taxpayers to the regulated banks (Freixas and Rochet (2008)). As a result, regulated banks are required to monitor shadow banks. The pressure on taxpayers is only released partially because the central bank will still face losses if a supporting depository bank defaults.

Depository banks maximize expected profit from interaction with both financial market and the real economy. If a regulated bank is not able to repay its debt to the central bank or any other counterparty – it defaults. Limited liability is imposed on the defaulted banks, meaning that the utility payoff of a bankrupted bank is zero. In this model, I keep the default procedure simple[5]: when repaying a debt, regulated bank favors short-term debt to long-term debt and depositors to banks and central bank. If the amount of available liquidity is not sufficient to compensate insured depositors, the rest is covered by the deposit insurance institution. If multiple regulated and shadow banks demand liquidity, the defaulting

[5]In reality, a default procedure of a regulated bank may be complicated because bank's default costs are not zero and deposit insurance companies may get involved in the debt settlement process.

bank pays them on pro-rata basis according to the order above.

A financial regulator can control for the degree of exposure of regulated banks to shadow banks by imposing a cap $\bar{q}$ on the amount of risky investments that regulated banks make if they trade on their own behalf. This regulation is similar to the Volcker rule that aims to limit proprietary trading activity of regulated banks. Further in the paper, I will consider different regulatory regimes depending on the tightness of cap $\bar{q}$.

2.5 Shadow banks and money market investors

Shadow banks maximize expected profit generated over three periods. Each bank finances assets using funding from depository banks and money market investors. Shadow bank j may borrow from depository bank i in the amount of q_{ij} at rate r_{ij} if these terms satisfy bank j. To receive funding from the money market, shadow bank j chooses money market rate r_j^m and the amount of money market funding is determined by the market supply schedule $\phi_j(r_j^m)$.

To gain more intuition about the money market, I derive supply schedule ϕ_j from behaviors of an infinite pool of small heterogeneous investors surrounding bank j. I assume that every investor in the pool maximizes expected profit at $t = 3$ by making investment decisions at $t = 1, 2$. At $t = 1$, each investor μ decides either to invest a unit of liquidity in bank j with nominal rate r_j^m or invest in an alternative option at risk-free rate $r^{inv}(\mu)$ specific to the investor. If investor μ decides to lend to the shadow bank, it makes an additional decision in the event of market crash: either to withdraw or to keep liquidity in the bank. I assume that the individual risk-free rates have cumulative exponential distribution function:

$$F(r^{inv}) = F_0 \exp\left(\frac{r^{inv} - r^{dep}}{\lambda}\right), \tag{1}$$

Equivalently, function $F(r^{inv})$ defines the amount of liquidity that will be provided by the money market investors to the shadow bank promising expected rate of return r^{inv}. Parameter λ represents the stickiness of investor's preferences towards alternative option. Therefore, I will use λ as a proxy measure of entry and exit costs to the money market. The distribution $F(r^{inv})$ also assumes that there is a non-empty pool of investors F_0 indifferent between lending to a shadow bank and keeping funds as a deposit at rate r^{dep}: $F(r^{dep}) = F_0$.

An important feature of the model is the maturity mismatch in the operations of shadow banks.[6] While the risky asset is held until $t = 3$, the money market investors can withdraw their liquidity at $t = 2$. I assume that withdrawing the funds has an opportunity cost. In particular, if one unit of liquidity was invested at time $t = 1$, a maximum of one unit can be withdrawn at time $t = 2$. The exact amount withdrawn depends on the amount of cash available at the bank at time $t = 2$. A liquid bank $j \in B^s$ repays debt to running creditors in full. If bank $j \in B^s$ does not have enough liquidity to repay the creditors at $t = 2$, it either borrows the liquidity or defaults and distributes available liquidity among those who withdraw.

It is assumed that a shadow bank always has a limited amount of cash l available. An additional amount of emergency liquidity can be obtained by borrowing from depository banks. A shadow bank first uses its available cash to repay current liabilities, and defaults with no cash in hand if the amount of liquid assets is not sufficient. In this case, the bank's debt will be repaid to creditors proportionally to their debt size. These simplifying assumptions allow us to proceed without modeling the bankruptcy procedure, which is not the focus of this study.

I next determine the set of money market investors that choose to provide liquidity to shadow bank j at a given rate r^m_j. First, notice that when an investor μ does not withdraw liquidity from the shadow bank, the expected return is:

$$E[r^\mu] = \begin{cases} r^m_j \pi^s, & \text{if bank } j \text{ stays solvent given } crash, \\ r^m_j \pi^{s,nc}, & \text{if bank } j \text{ defaults given } crash, \end{cases}$$

When investor m withdraws liquidity and bank j stays solvent, the expected return is

[6] Calomiris and Kahn (1991) suggest that this mismatch is common among financial institutions and may arise in the environment of asymmetric information to prevent banks from fraudulent activities.

$$E[r^{\mu}] = \begin{cases} r_j^m \pi^{s,nc} + \pi^c, & \text{if bank } j \text{ stays solvent given } crash, \\ r_j^m \pi^{s,nc} + \frac{l}{\phi(r_j^m)} \pi^c, & \text{if bank } j \text{ defaults given } crash, \end{cases}$$

such that the expected return of a running investor depends on the amount of cash c available at the shadow bank at time $t = 2$. This result is consistent with the predictions of Ahnert (2016) that an amount of liquidity that a bank holds determines its attractiveness to the wholesale investors.

Because investors are rated according to their risk-free rates r^{inv} and the expected return is a linear function of the rate, all investors with the risk-free rate above a certain threshold will run on the shadow bank. This result follows from the fact that our model generates strategic complementarities among investors – withdrawal of one investor lowers the expected return of other investors and increases incentives for them to withdraw (see Chen et al. (2010) for the empirical evidence of this mechanism). I prove that the all investors will withdraw their funds if the asset shock is severe.

Lemma 1. *Given Assumption 3, there is a full run of money market investors in the event of market crash.*

The proof of Lemma 1 is provided in Appendix A.

Given this result, I can find the exposure of bank $j \in B^s$ to the money market when the liquidity support is provided:

$$\phi_j(r_j^m) = F(r_j^m \pi^{s,nc} + \pi^c), \tag{2}$$

and when the liquidity support is not provided:

$$\phi_j(r_j^m) = F(r_j^m \pi^{s,nc} + \frac{l}{\phi_j} \pi^c). \tag{3}$$

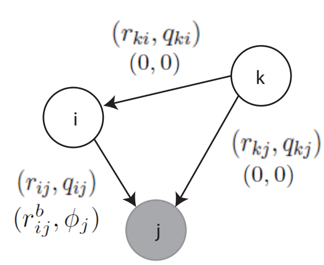
Therefore, an increase in liquid assets of shadow banks provides more confidence for investors and boosts the amount of provided funds.

3 Notion of network stability

The theoretical approach of this paper is consistent with the network formation literature assuming that agents act strategically (see Jackson (2008) for a review). I mention just a few of the most related findings. Allen and Gale (2000) were among the first to show that typical financial networks are fragile and subject to financial contagion. Acemoglu et al. (2015) found that exogenously given networks with dense structure are more robust to small shocks, while less robust to large market shocks. **?** showed that density network has a concave effect on the fragility of the network. Allen et al. (2012) developed an endogenous model of network formation and showed that the network structure plays an especially important role for the economic welfare in the market with the short term financing. Freixas et al. (2000) analyzed the coordination role of the central bank in the financial networks with contagion.

In this model, the interbank market is modeled as a directed multi-layered network with each connection representing a set of contracts between the two banks. Denote a set of multi-layered networks by **G**. Each possible network contains two layers: one layer for investment decision at $t = 1$, and one layer for liquidity support decision following market crush at $t = 2$. For each time period, a network link specifies interest rate and lending amount.[7] For example, in Figure 2, the exposure from bank i to bank j is measured as a loan size q_{ij} for layer at $t = 1$ and as a liquidity support $q^b_{ij} = \phi_j$ for layer at $t = 2$.

Figure 2: Example of a multilayered network with full market run and a single channel of liquidity support.



[7] For consistency in notation, I assume that the rate is zero when the exposure is zero and the existence of one contract is enough for the connection to exist.

The model is a cooperative game: an interest rate and a loan volume between a lender and a borrower are not set unilaterally and require a consent of both counterparties. Therefore, I proceed with the cooperative concept of equilibrium – core. To define the equilibrium concept, I first assume that the set of feasible bank coalitions $\mathbf{S}$ is a set of all subsets of banks. The rules of link formation between banks are consistent with the pairwise network formation rules defined by Jackson and Wolinsky (1996). This means that if a pair of banks forms a new lending relationship, the interest rate is determined by both parties of the contract, while a breach of the contract can be done unilaterally by either lender or borrower.

Definition 1. In the interbank network, the following deviations are *feasible*:

a) a unilateral decision of either $i \in B \cup B^s$ or $j \in B \cup B^s$ is sufficient to breach the contract by deviating to $(r_{ij}, q_{ij}) = (0, 0)$, and a bilateral decision of coalition (i, j) is necessary to breach the contract by deviating to $(r_{ij}, q_{ij}) \neq (0, 0)$;

b) a unilateral decision of either $i \in B$ or $j \in B^s$ is sufficient to breach the contract by deviating to $(r^b_{ij}, q^b_{ij}) = (0, 0)$, and a bilateral decision of coalition (i, j) is necessary to breach the contract by deviating to $(r^b_{ij}, q^b_{ij}) \neq (0, 0)$;

If a bank (pair of banks) belongs to coalition $s \in \mathbf{S}$, the actions allowed to this bank (pair of banks) are allowed to coalition s.

The main advantage of using the approach of cooperative game theory is that it allows to model the bargaining between a lender and a borrow without specifying the sequence of offers and counter-offers. Instead, I determine the sets of bargaining outcomes that – if happen – will not be changed by any coalition. Allen et al. (2016) demonstrate the advantages of the core notion in the empirical estimation of market efficiency and bargaining power of banks.

To strictly define the notion of stability, I first define dominance relationship.

Definition 2. Network $g_2 \in \mathbf{G}$ *dominates* network $g_1 \in \mathbf{G}$, $g_2 \succ g_1$, if there is coalition $s \in \mathbf{S}$, such that a deviation from g_1 to g_2 is feasible for s, and each coalition member $i \in s$ benefits from this deviation:

$$E[u_i(g_2)] > E[u_i(g_1)].$$

In order to understand cooperative deviations, consider a simple example: lender $i \in B$ and borrower $j \in B^s$ deviate to favorable terms of liquidity support $(r_{ij}^b,\ q_{ij}^b)$, such that the regulated bank agrees to deviate only if the shadow bank reduces money market exposure ϕ_j. Then, the pairwise deviation should include the adjustments in the money market terms by bank j and in the liquidity support terms by coalition (i, j).

I now give the formal definition of network stability given that the money market supply is determined by equations (2)–(3) for each shadow bank j.[8]

Definition 3. The interbank network $g \in \mathbf{G}$ is *stable* if it is not dominated by any other network $g' \in \mathbf{G}$.

4 Stable interbank network

4.1 Two banks

In this section, I find stable equilibrium for one regulated and one shadow banks, $i \in B$, $j \in B^s$ under Assumptions 1 and 4 take place. Proposition 1 summarizes the main properties of the stable equilibrium, while the detailed findings are provided in Appendix B.

Proposition 1. *In the market with two banks, $i \in B$ and $j \in B^s$, a stable interbank network is characterized in the following way (see Figure 3):*

1) Regulated bank lends to the shadow bank amount $q_{ij} = \bar{q}$ even if the liquidity support is not provided.

2) The liquidity support can be provided in one of the two regimes:

a) The"vulnerable" regime is characterized by larger money market volume

[8] Although I use a version of the notion of core from the cooperative game theory, I will refer to equilibrium concept as network stability to avoid confusion between the core equilibrium and the core banks.

$$\phi^{high} = F(r^{m,high}\pi^{s,nc} + \pi^{c}).$$

and higher money market rate

$$r^{m,high} = r^{s}(1 + \frac{\pi^{s,c}}{\pi^{s,nc}}) - r^{cb}\frac{\pi^{s,c}}{\pi^{s,nc}} - \frac{\lambda}{\pi^{s,nc}},$$

Under this regime, the liquidity support is provided with zero interest spread

$$r^{b}_{ij} = r^{cb}$$

and the regulated bank contagiously defaults with positive probability $1-\pi^{s|c}$ *at time* $t = 3$ *if the shadow bank does not recover.*

b) The "immune" regime is characterized by smaller money market volume

$$\phi^{low} = F(r^{m,low}\pi^{s,nc} + \pi^{c}),$$

and lower money market rate

$$r^{m,low} = r^{s}(1 + \frac{\pi^{s,c}}{\pi^{s,nc}}) - r^{cb}\frac{\pi^{c}}{\pi^{s,nc}} - \frac{\lambda}{\pi^{s,nc}},$$

Under this regime, the liquidity support is provided with a positive spread:

$$r^{b}_{ij} = \frac{r^{cb}}{\pi^{s,c}}.$$

and the default of the shadow bank at $t = 3$ *does not trigger a contagious default of the regulated bank.*

3) Liquidity support under "immune" regime is only possible when depository bank generates high profits $v \geq r^{dep}\bar{q} + v^{*}$ *and is overexposed to the shadow bank,* $q_{ij} = \bar{q} \geq \bar{q}_{supp}$.

4) When the depository bank is not overexposed to the shadow bank, $\bar{q} < \bar{q}_{supp}$, the liquidity support may not be provided. In this case, the shadow bank receives money market funding in the amount of ϕ^{non-c} at rate $r^{m,non-c}$:

$$r^{m,non-c} = r^s - \frac{l}{\phi^{non-c}} \frac{\pi^c}{\pi^{s,nc}} - \frac{\lambda}{\pi^{s,nc}},$$

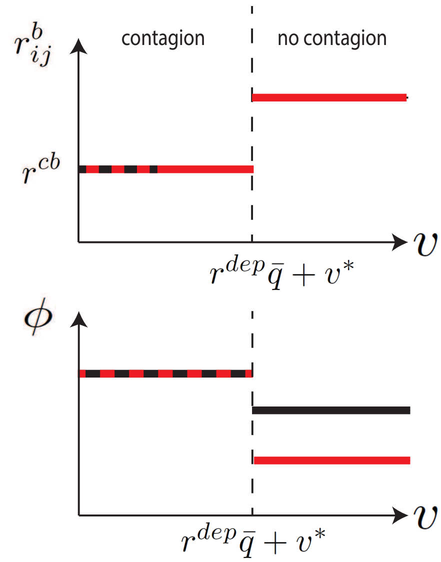
$$\phi^{non-c} = F(\pi^{s,nc} r^s - \lambda).$$

According to Proposition 1, the depository bank channels liquidity support to the shadow bank either because it is highly exposed to the shadow bank pre-crisis or the default of the shadow bank does not lead to a sufficient profit loss for this depository bank. Moreover, a more profitable depository bank would provide less support and demand a higher compensation to avoid a contagious default that may follow if the shadow bank does not recover.

The proposition also states that a lender-borrower relationship does not necessarily impose additional liquidity support during the crisis. In particular, credit risk is a natural barrier for the regulated bank. However, when the shadow bank invests in the asset of a relatively high quality, as presumed in Assumption 2, the regulated bank gets exposed to the shadow bank even when the crisis support is not provided.

Figure 3: Equilibrium relationships between one depository and one shadow banks.

	Dep. bank robust to contagion $v \geq r^{dep}\bar{q} + v^*$	Dep. bank exposed to contagion $v < r^{dep}\bar{q} + v^*$
Strict regulation $\bar{q} < \bar{q}_{supp}$	- No contagion - No support - Medium sized shadow bank	- Contagion - Selective support (low rate) - Large sized shadow bank
Tolerent regulation $\bar{q} \geq \bar{q}_{supp}$	- No contagion - Support provided (high rate) - Small sized shadow bank	- Contagion - Support provided (low rate) - Large sized shadow bank



The figure shows how equilibrium support rate r_{cb} and money market exposure ϕ depend on the profit v of the depository bank when the exposure between regulated and shadow banks is more regulated, $q < q_{supp}$ (in black) and less regulated $q \geq q_{supp}$ (in red). The table provides a description for the graph and Proposition 1.

It is an interesting result that, when both regimes are supported, there is a discontinuity in the comparative statics of the equilibrium. In particular, money market size and money market rate increase with a jump if profit v decreases. This occurs because there is a threshold on profit v below which the bank is no longer robust to contagion. Once the profit of the depository bank becomes insufficient to battle contagion, the bank becomes driven by the limited liability condition. In particular, it encourages the supporting shadow bank to enlarge its balance sheet size by promising the lowest rate of financial support. We summarize the comparative statics as follows:

Corollary 1. *Ceteris paribus, equilibrium exposure ϕ_j of the shadow bank to money market increases with a decrease in profit v and the central bank's support rate r^{cb}.*

The comparative statics result is driven by the trade-off that a regulated bank faces: higher return is accompanied by possible contagion. Moreover, the second part of the corollary can be reversed and extended: a sufficient increase in the central bank's support rate r^{cb} leads to the termination of the liquidity support.

In Appendix B, I also show that the equilibrium outcome is consistent with an idea of monitoring. If the shadow bank is subsidized by the solvent bank, it is required to keep a limited balance sheet size. Therefore, in the cases when liquidity support is expensive (high r^{cb} and high v), the shadow bank may prefer to expand its balance sheet size at the cost of staying unsupported during the crisis.

4.2 Large number of banks

I consider a market with many banks $B = (1, ..., N)$ and $B^s = (N+1, ..., N+N^s)$ and focus on the case where banks have sufficient profits from interactions with the real economy: $v \geq r^{dep}\bar{q}$.[9] I show that when banks set relationships cooperatively, they develop a core-periphery market structure with larger risky banks becoming the core and smaller solvent banks becoming the periphery. Proposition 2 summarizes the main properties of the stable network, while the detailed findings are provided in Appendix C.

Proposition 2. *In the game with many banks, a stable equilibrium network takes a core-periphery structure, such that*

1) The liquidity support is only provided by core regulated banks $B^{core} \subset B$ to core shadow banks $B^{supp} \subset B^s$ according to the vulnerable regime specified in Proposition 1.

2) Each core regulated bank serves as an intermediary from a subset of periphery regulated banks to the only supported shadow bank and benefits from a positive spread between the lending rate r^{core} and the borrowing rate r^{per}:

$$r^{per} < r^s < r^{core}.$$

Each regulated bank lends to shadow banks net amount $\bar{q}$ either directly or indirectly.

[9]Alternatively, all regulated banks, directly or indirectly exposed to the shadow banks, default if the crisis hits, which makes the network case uninteresting to consider. I also assume that the number of depository banks is sufficiently large, such that the profit from safe activities v is small relative to the total liquidity flows $\bar{q}N$ from depository banks to shadow banks, and the number of shadow banks is sufficiently large, such that not all of them are supported by the depository banks.

As a result, balance sheet sizes and default risks of core regulated banks exceed those of the periphery regulated banks.

3) Supported shadow banks promise a higher money market rate and attract more money market investors than non-supported shadow banks:

$$r^{m,high} > r^{non-c} \text{ and } \phi^{high} > \phi^{non-c}.$$

3) Both types of depository banks get the same expected profit with the core banks getting higher profits when their supported shadow banks succeed.

4) Both types of shadow banks get the same expected profit with the supported shadow banks getting higher expected profits in the events of market crash.

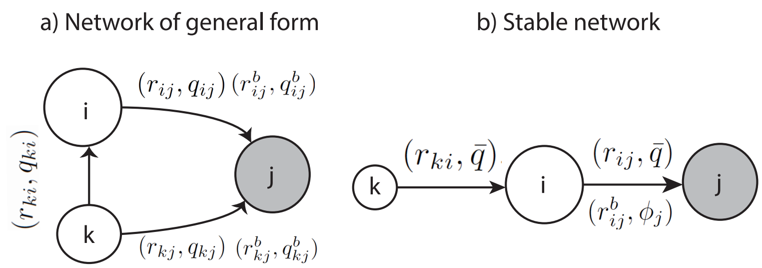
In the remainder of the section I provide intuition of why financial markets develop the heterogeneity in balance sheets and relationships described in Proposition 2. First, notice that in the market with many banks, different from the market with only two banks, liquidity support is only provided in the vulnerable regime. The reason for this is that a large coalition of banks profit from delegating the role of rescuing one shadow bank to one regulated bank. This core regulated bank becomes overexposed to the shadow bank and encourages the supporting shadow bank to enlarge its balance sheet size by promising the lowest rate of financial support r^{cb}. This result is consistent with the empirical funding of Allen et al. (2016) that the riskier borrower banks were able to obtain surprisingly low rates during the financial crisis.

Interestingly, the supporting bank is still compensated for the risk of providing liquidity guarantees even though the interest spread on the liquidity support is zero. In particular, the shadow bank feeds its sponsor with higher interest returns before the crisis. In fact, the shadow bank mostly loses from interactions with the supporting depository bank during the regular market conditions, and benefits in the events of crisis—when the liquidity support is provided. The shadow bank also compensates the supporting bank for the positive reputation effects that the presence of liquidity support has on investors. As stated in Proposition 2, money market investors are more attracted to supported shadow banks and require low rate of return on funding.

The next example explains why banks endogenously create a source of financial instability by channeling the liquidity flows through core regulated banks.

Example 1. Consider a general network of two regulated banks and one shadow bank illustrated in Figure 4a.[10] I aim to show that the special case network in Figure 4b is the only network stable to deviations in the case of the three banks.

Figure 4: Finding stable flows in the network of three banks



Given Assumptions 1 and 4 , total utility of banks i, j and k is maximized when the regulated banks get exposed to the shadow bank completely at $t = 1$: $q_{ij} = q_{ki} + \bar{q}$ and $q_{ki} + q_{kj} = \bar{q}$. This result is based on the convex properties of the expected utilities and can be proved in the same way as in Proposition 1.

We would like to know what is the most optimal bail out scheme (q^b_{ij}, q^b_{kj}) for the coalition of three banks when the exposure to the money market is fixed at level ϕ_j. Ceteris paribus, we choose the scheme that generates the highest total profit in the event of market crash followed by asset failure, because, in other market events, the banks' expected profits do not depend on how the supporting liquidity is channeled.

When both banks stay solvent following the crash, banks get the following profit:

$$2(v - r^{dep}\bar{q}) - r^{cb}\phi_j. \tag{4}$$

[10]This network has the most general form in the market with two regulated banks and one shadow bank: bank i lends amount $q_{ij} = q_{ki} + \Delta q_{ij}$ to bank j and does not lend to bank k given that loops are not beneficial to the regulated banks.

When bank k defaults, bank i also defaults, because both banks generate the same revenue and bank i incurs larger cost. Therefore, zero total profit is generated.

When bank k stays solvent and bank i defaults, the profit changes to

$$v - r^{dep}\bar{q} - r^{cb}q^b_{kj} + p_{ki}, \tag{5}$$

where p_{ki} is the payment made by defaulting bank i to regulated bank k; it is determined according to the pro-rata rule:

$$p_{ki} = \frac{v - r^{dep}\bar{q}}{r^{cb}(\phi_j - q^b_{kj}) + r_{ki}q_{ki}} r_{ki}q_{ki}.$$

With first order conditions it can be shown that profit (5) increases with a decrease of q^b_{kj}, so that it is beneficial for the banks when bank k does not provide liquidity support to the shadow bank: $q^b_{kj} = 0$.

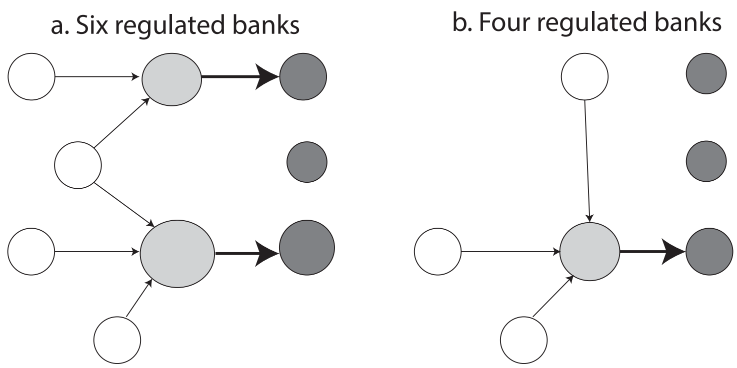
Comparison of (4) and (5) also indicates that the joint utility is maximized when one depository bank defaults in the event of market crash followed by asset failure, when each regulated banks transfer sufficient liquidity amount $\bar{q}$:

$$v - r^{dep}\bar{q} + p_{ki} = 2(v - r^{dep}\bar{q}) - p_{cb,i} \geq \ 2(v - r^{dep}\bar{q}) - r^{cb}\phi_j.$$

The proposed network structure is beneficial for the banks because the depository banks pass some debt burden onto the central bank and keep enough profits within the banking system. The assumptions of limited liability and long-term relationships between regulated banks are crucial for this result to take place. First, the limited liability mitigates counterparty risk of the core regulated bank k. Second, the lending rate pre-crisis is conditional on the loss-given-default and the default probability of the core bank k, which allows two regulated banks to price and share the additional risk from providing liquidity support.

In Example 1 one of the two regulated banks grows large enough to provide liquidity guarantees. In the case with many regulated banks, the number of core regulated banks is endogenously determined in the network (see Appendix C). Because the core banks perform intermediary and support functions on behalf of other banks, the need for their existence is limited. Consider Figure 5: in the market with six traditional banks, two regulated banks are the core banks, and four regulated banks are periphery banks. In Figure 5b, the number of regulated banks is not sufficient to support all shadow banks in the case of market crash, therefore only one shadow bank is supported. In particular, only one bank can generate sufficient profit from the margin on rates on lending and borrowing to get interested in becoming a core bank and a sponsor of a shadow bank.

Figure 5: Endogenous number of core banks for different numbers of regulated banks.



Out of six regulated banks, two regulated banks grow large enough to become core banks and provide liquidity guarantees to shadow banks. Out of four regulated banks, only one bank grows large enough.

5 Policy implications

5.1 Measure of financial stability

The equilibrium characteristics given in Proposition 2 can be used to derive implications about financial stability. In this paper, improving financial stability is equivalent to minimizing the expected number of bank defaults. When measuring financial instability, it is important to include an after-crisis period, because temporary liquidity support does not always lead to long-term financial stability: sometimes the asset risk is not mitigated during the crisis but rather hidden until maturity date.

The absolute stability of regulated banking is achieved when the regulated banks do not get exposed to shadow banks. This means that the stability of regulated banks alone can be improved by limiting exposure between regulated banks and shadow banks (decrease of $\bar{q}$), encouraging traditional banking activities as an alternative to money market instruments (increase of v and increase of λ), and increasing the cost of liquidity support r^{cb} when exposure $\bar{q}$ is sufficiently low. While these measures prevent regulated banks from default, they also lead to more defaults of shadow banks in the states of market crash.

In reality, financial regulator may account for the stability of both shadow and regulated banks. This is especially the case for the macroprudential regulator that in contrast to microprudential regulation aims to mitigate risk to the financial system as a whole. To account for overall financial market, we use the expected number of defaults as objective function of the regulator:

$$E[N^{defaults}] = (2\pi^{ns,c} - \pi^{c})N^{core} + N^{s}\pi^{c}. \tag{6}$$

Expression (6) is derived based on the result of Proposition 2 that the connected core depository bank and core shadow bank default if the liquidity support is not successful.

In the remainder of this section, I present a set of comparative statics results[11] and find the effects of different regulations on the expected number of bank defaults (6) and other equilibrium characteristics.

5.2 Asset quality control

It is beneficial to label the quality of the risky assets according to the following rule:

Definition 4. An asset is of *high quality* if it is more likely to pay off a positive net return following the crash: $\pi^{s|c} \geq 1/2$, and the asset is of *low quality* if it is less likely to pay off a positive net return following the crash: $\pi^{s|c} < 1/2$.

As formula (6) implies, when the quality of risky assets is low, the market stability can be improved

[11] The proofs of these results are straightforward and can be sent to a reader by request.

by discouraging regulated banks from rescuing shadow banks. Alternatively, when the quality of the risky assets is high, the liquidity support should be encouraged by the regulators.

By performing the following comparative statics exercise on $\pi^{s|c}$, I conclude that the asset risk is an amplifier of the overall systemic risk. Therefore, increasing the quality of the risky assets leads to higher financial stability:

Corollary 2. *Ceteris paribus, an increase in the quality of risky assets leads to a lower expected number of defaults, a larger size of supported shadow banks, and more shadow banks being supported.*

5.3 Control of exposure between shadow and regulated banks

Another way to increase the financial stability of all banks is to control the maximum net exposure $\bar{q}$ that regulated banks have to shadow banks. This result is also sensitive to the asset quality:

Corollary 3. *Ceteris paribus, the expected number of defaults decreases with an increase in exposure $\bar{q}$ if and only if the risky assets held by the shadow bank are of high quality.*

The quality of risky assets can be improved in different ways depending on the asset types. For example, a quality of long-term risky assets, such as mortgages, can be improved by tightening mortgage origination rules or requiring an insurance on risky mortgages. These policies would increase the probability $\pi^{s|c}$ of mortgage repayment following a severe economic shock. Alternatively, a quality of short-term financial loans can be improved by requiring loans to be collateralized by safe and liquid instruments.

While improving the quality of risky assets seems natural, this is not always possible in a free market environment. For example, consider an origination of highly risky debt that exists in the economy to provide credit to entrepreneurs or low income population. If there is a risk-reducing regulation on the origination of this instrument, it is likely to be imposed on certain financial institutions or financial instruments. Therefore, in the financial markets with free entry, the new regulation may lead to an origination of alternative types of financial entities and instruments that are not subject to this regulation.

These entities will be exposed to the same high asset risk but in the shadow of the new regulatory regime. This will only strengthen the separation between shadow and regulated banks.

5.4 Liquidity holdings of shadow banks

I determine how an increase in the cash holdings of shadow banks, measured by proxy l, affects stability of the financial system. When the market experiences a severe asset shock, an increase in the amount of liquid assets l is not sufficient to stop liquidity withdrawals. However, more investors will be attracted to the non-supported shadow banks ex-ante the crisis as they are more likely to be paid off in the case of withdrawal. Therefore, liquidity support becomes less needed for the shadow banks, which leads to the following results.

Corollary 4. *Ceteris paribus, an increase in the cash reserves of shadow banks leads to lower funding cost* $r^{non,c}$ *for non-supported shadow banks, higher expected profits of all shadow banks, and fewer shadow banks being supported.*

The cash reserves in Corollary 4 should be understood as required cash reserves, because in the model, banks do not profit from keeping cash and prefer investing in risky assets that deliver higher expected returns.

5.5 Redemption gates and liquidity fees

In this section, banks are allowed to impose redemption gates and liquidity fees on running investors. In modeling terms, a redemption gate is a partial or full restriction on the withdrawals of money market investors, and liquidity fee is a payment charged by a shadow bank to a running investor. Both of these measures aim to solve liquidity problems of shadow banks and, as a result, protect shadow banks from an immediate default during the crisis. However, because investors are aware of these regulations ex-ante the crisis, they will affect not only the stability of the financial system during the crisis but also investor's willingness to keep funds in the shadow banks before the crisis.

Partial redemption gates. I first consider the effect of partial redemption gates on liquidity withdrawals and overall systemic risk. Suppose money market investors of a solvent bank are limited to a withdrawal of only γ share of liquidity following the crash. Because the utility function of investors is additive, investors still prefer to withdraw as much funds as possible when the market conditions deteriorate as indicated in Assumption 3. Therefore, the money market rate and volume of a supported bank is conditional on how easy it is to withdraw liquidity:

$$r_{\gamma}^{m,high} = r^{s}(1 + \frac{\gamma\pi^{s,c}}{\pi^{s,nc} + (1-\gamma)\pi^{s,c}}) - r^{cb}\frac{\gamma\pi^{s,c}}{\pi^{s,nc} + (1-\gamma)\pi^{s,c}} -$$

$$-\frac{\lambda}{\pi^{s,nc} + (1-\gamma)\pi^{s,c}}.$$

$$\phi_{\gamma}^{high} = F^{inv}(r_{\gamma}^{m,high}(\pi^{s,nc} + (1-\gamma)\pi^{s,c}) + \gamma\pi^{c}). \tag{7}$$

Therefore, according to Assumption 3, more restrictive redemption gates discourage investors to invest in the supported shadow banks and lead to a higher funding cost for these banks.

Interestingly, partial redemption gates do not change the amount of funding ϕ^{non-c} provided to the non-supported shadow banks, but make the funding more expensive:

$$r^{m,non-c} = r^{s} - \frac{\lambda}{\pi^{s,nc}} - \frac{l}{\gamma\pi^{s,nc}\phi^{non-c}}\pi^{c}.$$

Moreover, short-term investors still withdraw funding from the shadow banks following a significant drop in the banks' asset values as presumed in Assumption 3 .

Lemma 2. *Partial redemption gates discourage investors to invest in the supported shadow banks and lead to a higher funding cost for all shadow banks.*

Full redemption block. When the liquidity withdrawals are completely prohibited by the regulator:

$\gamma = 0$, all shadow banks are equally funded on the money market independent of their exposures to depository banks. In this case, money market rates and money market exposures for shadow bank j are:

$$r_j^m = r^s - \frac{\lambda}{\pi^s}. \tag{8}$$

$$\phi_j = F^{inv}(\pi^s r_j^m) = F^{inv}(\pi^s r^s - \lambda). \tag{9}$$

Therefore, shadow banks that lack liquidity support from depository banks become more attractive to the investors after the full redemption gates are imposed.

Given Assumptions (1)-(4), the effects of redemption block is summarized as follows:

Lemma 3. *Full redemption block decreases money market rate and exposure of shadow banks.*

I also find how redemption gates and block affect total number of expected defaults in the equilibrium network:

Lemma 4. *Stricter redemption gates and redemption block lead to more financial stability if and only if the quality of the risky assets is low.*

Redemption fees. I next focus on the policy implications of liquidity fees on withdrawals. I assume that banks do not impose liquidity fees to raise additional liquidity from investors and only use them to stop the runs. The expected utility of investors is linearly dependent on the liquidity fees. Moreover, investors of the same shadow bank face equal trade-offs following the crash. Therefore, the fees become effective only when they are not to be repaid but are sufficient to stop the withdrawals completely. If this is the case, the equilibrium money market exposure and the rate of shadow banks are equivalent to the ones in equations (8)–(9). In this case, the redemption fee is equal to $1 - r^s \pi^{s|c}$ and the indirect liquidity support is not required.

5.6 Optimal cost of liquidity support

A central bank can manipulate the liquidity support rate r^{cb} to increase financial stability. When the quality of risky assets is high, the central bank should increase the number of indirect liquidity channels from the central bank to the shadow banks. This can be done by keeping the interest rate low and providing incentives to the regulated banks to support shadow banks. At the same time, when the central bank's policy is determined ex-ante the crisis, and the regulator follows its promises, keeping support rate low may increase risk-taking behavior and lead to higher systemic risk. Therefore, the optimal interest rate of support has an interior optimal value (see Lemma 5). Alternatively, when the shadow banks invest in an asset of low quality, the burden from risky assets is taken by the core regulated banks, which makes them systemically unstable. The rules for the optimal interest rate of the central bank is summarized in the following lemma:

Lemma 5. *When the quality of risky assets is high, the optimal rate of indirect liquidity support is proportional to the money market entry/exit cost λ and the riskiness of the asset $\frac{1}{\pi^{s,c}}$:*

$$r^{cb} = \frac{\lambda}{\pi^{s,c}}.$$

When the shadow banks invest in the assets of low quality, raising support rate r^{cb} is beneficial to decreasing systemic risk.

The lemma above claims that the indirect liquidity support provided by the central bank to shadow banks through regulated institutions can increase financial stability when the quality of the assets traded by the shadow banks is sufficiently high. When banks trade low quality assets, liquidity can be provided at a punitive rate (Bagehot (1873)). Moreover, the central bank should make the liquidity support more expensive when it is more difficult for investors to exit the money market.

5.7 “Too-big-to-fail” considerations

In reality, if core banks grow as a result of the interbank intermediation function, they are also likely to grow in other banking activities. This additional growth in size will only exaggerate the “too big to fail” problem that becomes an issue when the core regulated banks serve as liquidity conduits for non-core regulated banks. Therefore, a regulator may be interested in imposing a cap on the size of banks or discouraging banks from growing by imposing more regulatory requirements on the systemically important institutions. Before imposing this type of regulation, it would be important to consider the reasoning of why these institutions have grown in the first place. The results of this paper provide relevant intuition. In markets with many banks, the function of liquidity support can only be performed by large core banks. Therefore, a cap on the size of banks' balance sheets may not only decrease the size and the riskiness of core regulated banks, but also reduce the balance sheet sizes and increase the riskiness of shadow banks.

6 Conclusion

This paper considers a stable financial network before and after a crisis when regulated banks are allowed to serve as liquidity conduits between the central bank and shadow banks. I characterize the contagion following an asset price shock and determine a set of banks that can default. I also show that financial exposures between banks form a network with the core-periphery structure. The core regulated banks, defined as the most interconnected banks, provide the rescue liquidity channels to shadow banks and transfer liquidity from regulated banking. Although the initial model characteristics of all banks are identical, the core regulated banks make larger profits than periphery regulated banks while the market is stable, but are exposed to higher default risk due to their intermediary function, risk taking, and financial contagion.

I consider a number of mechanisms to maintain financial system stability, which include quality control of shadow banks' assets, minimum cash reserves requirements, redemption gates and liquidity withdrawal fees. I also find the optimal rate of support that the central bank should impose when setting the indirect liquidity channels. Finally, I provide a discussion of too-big-to-fail considerations in the context of liquidity channels between regulated and shadow banks. The main take-away is that the rate of liquidity support and banking regulations should be considered together with the asset quality control. Depending on the quality of assets held by shadow banks, the same policies may either increase or reduce systemic risk.

References

Daron Acemoglu, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. Networks, shocks, and systemic risk. Technical report, National Bureau of Economic Research, 2015.

Viral V Acharya, Philipp Schnabl, and Gustavo Suarez. Securitization without risk transfer. *Journal of Financial Economics*, 107(3):515–536, 2013.

Tobias Adrian and Adam B Ashcraft. Shadow banking regulation. *Annual Review of Financial Economics*, 4(1):99–140, 2012.

Toni Ahnert. Rollover risk, liquidity and macroprudential regulation. *Journal of Money, Credit and Banking*, 48(8):1753–1785, 2016.

Franklin Allen and Douglas Gale. Financial contagion. *Journal of Political Economy*, 108(1):1–33, 2000.

Franklin Allen, Ana Babus, and Elena Carletti. Asset commonality, debt maturity and systemic risk. *Journal of Financial Economics*, 104(3):519–534, 2012.

Jason Allen, James Chapman, Federico Echenique, and Matthew Shum. Efficiency and bargaining power in the interbank loan market. *International Economic Review*, 57(2):691–716, 2016.

Walter Bagehot. *Lombard Street: A description of the money market.* Scribner, Armstrong & Company, 1873.

Financial Stability Board. Global shadow banking monitoring report. 2014.

Charles W Calomiris and Charles M Kahn. The role of demandable debt in structuring optimal banking arrangements. *The American Economic Review*, pages 497–513, 1991.

Fabio Castiglionesi and Noemi Navarro. Optimal fragile financial networks. *SSRN working paper*, 2007.

John Chant. *The ABCP crisis in Canada: the implications for the regulation of financial markets.* Expert Panel on Securities Regulation, 2009.

Qi Chen, Itay Goldstein, and Wei Jiang. Payoff complementarities and financial fragility: Evidence from mutual fund outflows. *Journal of Financial Economics*, 97(2):239–262, 2010.

Adam Copeland, Antoine Martin, and Michael Walker. Repo runs: Evidence from the tri-party repo market. *The Journal of Finance*, 69(6):2343–2380, 2014.

Ben Craig and Goetz Von Peter. Interbank tiering and money center banks. *Journal of Financial Intermediation*, 23(3):322–347, 2014.

Douglas W Diamond and Philip H Dybvig. Bank runs, deposit insurance, and liquidity. *The Journal of Political Economy*, pages 401–419, 1983.

Maryam Farboodi. Intermediation and voluntary exposure to counterparty risk. *SSRN working paper*, 2014.

Xavier Freixas and Jean-Charles Rochet. *Microeconomics of banking.* MIT press, 2008.

Xavier Freixas, Bruno M Parigi, and Jean-Charles Rochet. Systemic risk, interbank relations, and liquidity provision by the central bank. *Journal of money, credit and banking*, pages 611–638, 2000.

Nicola Gennaioli, Andrei Shleifer, and Robert W Vishny. A model of shadow banking. *The Journal of Finance*, 68(4):1331–1363, 2013.

Gary Gorton and Andrew Metrick. Regulating the shadow banking system. *Brookings Papers on Economic Activity*, (2), 2010.

Gary Gorton and Andrew Metrick. Securitized banking and the run on repo. *Journal of Financial Economics*, 104(3):425–451, 2012.

IMF. Risk taking, liquidity, and shadow banking: Curbing excess while promoting growth. *The October 2014 Global Financial Stability Report (GFSR)*, 2014.

Matthew O Jackson. Social and economic networks, 2008.

Matthew O Jackson and Asher Wolinsky. A strategic model of social and economic networks. *Journal of Economic Theory*, 71(1):44–74, 1996.

Marcin Kacperczyk and Philipp Schnabl. How safe are money market funds? *The Quarterly Journal of Economics*, page 010, 2013.

Arvind Krishnamurthy, Stefan Nagel, and Dmitry Orlov. Sizing up repo. *The Journal of Finance*, 69(6): 2381–2417, 2014.

Timothy Lane. Shedding light on shadow banking. *Remarks by Mr. Timothy Lane, Deputy Governor of the Bank of Canada, to the CFA Society Toronto, Toronto, Ontario*, 26, 2013.

Yaron Leitner. Financial networks: Contagion, commitment, and private sector bailouts. *The Journal of Finance*, 60(6):2925–2953, 2005.

Zoltan Pozsar, Tobias Adrian, Adam B Ashcraft, and Haley Boesky. Shadow banking. *SSRN working paper*, 2010.

Daniel K Tarullo. Shadow banking and systemic risk regulation. In *Speech at the Americans for Financial Reform and Economic Policy Institute Conference, Washington, DC, November*, volume 22, 2013.

Appendices:

A. Proof of Lemma 1

I initially show that a non-running investor benefits from a deviation when some other investors run. First, if the shadow bank is insolvent, the investor gets a minimum payoff of $\frac{l}{\phi_i} > 0$ if running comparing to the payoff of zero if not running. Therefore, the investor benefits from the deviation. Second, if the shadow bank managers the withdrawals and stays solvent, it means that the required liquidity has been granted to this bank. In this situation, all running investors get a payoff of 1, while all non-running investors get a payoff of $r^s\pi^{s|c}$. It means that a non-running investor benefits from deviating to running: $1 > r^s\pi^{s|c}$. This deviation will turn the shadow bank into the defaulting bank.

Finally, consider a situation, where investors do not run at all. Then at least one investor has an incentive to deviate to running strategy, which increases the investor's payoff by to $1 > r^s\pi^{s|c}$ and leads to the default of the shadow bank. This finishes the proof that all investors will run on the shadow bank when the asset shock is severe.

B. Proof of Proposition 1

I consider an outcome where regulated bank i is exposed to shadow bank j at time $t = 1$ and the liquidity support is provided at time $t = 2$.

I first assume that the long-term exposure is fixed at level q_{ij}: $0 < q_{ij} \leq \bar{q}$. Therefore, the banks expect profits[12]

$$E[u_i] = ((r_{ij} - r^{dep})q_{ij} + v)\pi^s + \phi_j(r^b_{ij} - r^{cb})\pi^{s,c} + ...$$

$$... + (v - r^{dep}q_{ij})_{\{+\}}\pi^{ns,nc} + (v - r^{dep}q_{ij} - r^{cb}\phi_j)_{\{+\}}\pi^{ns,c},$$

$$E[u_j] = (r^s - r_{ij})q_{ij}\pi^s + \phi_j\left((r^s - r^m_j)\pi^{s,nc} + (r^s - r^b_{ij})\pi^{s,c}\right),$$

such that the money market investors expect return $r^m_j\pi^{s,nc} + \pi^c$ and provide funding in the amount of

$$\phi_j = F^{inv}(r^m_j\pi^{s,nc} + \pi^c).$$

In addition, for the promises of the shadow bank to be credible, their debt must be paid off in the states of asset success:

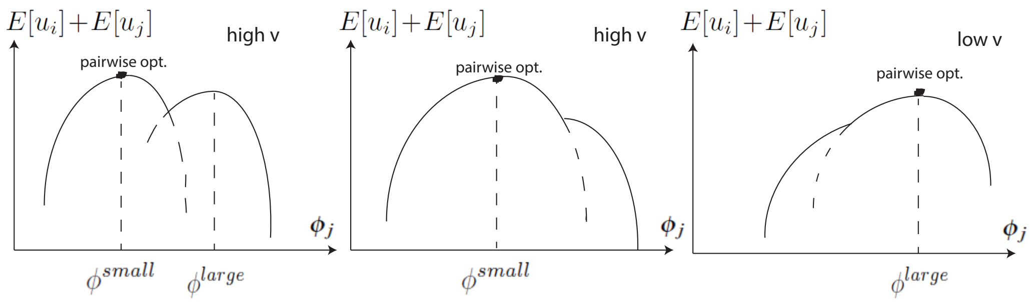
$$(r^s - r_{ij})q_{ij} + (r^s - r^m_j)\phi_j \geq 0, \tag{10}$$

$$(r^s - r_{ij})q_{ij} + (r^s - r^b_{ij})\phi_j \geq 0. \tag{11}$$

I first find the pairwise optimal strategies of the two banks. For this purpose, I determine separately the strategies in the case when regulated bank stays solvent, $v - r^{dep}q_{ij} - r^{cb}\phi_j > 0$, and in the case when a default of the shadow bank triggers a default of the regulated bank, $v - r^{dep}q_{ij} - r^{cb}\phi_j \leq 0$; I next select the strategies that generate higher expected profits for the coalition of banks (see Figure 6 for the illustration).

[12] I use notation $x_{\{+\}} = \begin{cases} x & \text{, if } x > 0 \\ 0 & \text{, if } x \leq 0 \end{cases}$ to account for the limited liability condition.

Figure 6: Different shapes of pairwise expected payoff as a function of money market exposure ϕ_j depending on the parameters.



The results of our analysis are as follows: under the regime of low profit $v < v^* + r^{dep}q_{ij}$, the coalition of banks prefers high money market rate $r^{m,high}$ and volume ϕ^{high}:

$$r^{m,high} = r^s(1 + \frac{\pi^{s,c}}{\pi^{s,nc}}) - r^{cb}\frac{\pi^{s,c}}{\pi^{s,nc}} - \frac{\lambda}{\pi^{s,nc}}, \tag{12}$$

$$\phi^{high} = F^{inv}(r^{m,high}\pi^{s,nc} + \pi^c),$$

and under the regime of high profit $v \geq v^* + r^{dep}q_{ij}$, the pairwise optimal choice is low money market rate $r^{m,low}$ and volume ϕ^{low}

$$r^{m,low} = r^s(1 + \frac{\pi^{s,c}}{\pi^{s,nc}}) - r^{cb}\left(\frac{\pi^{ns,c}}{\pi^{s,nc}} + \frac{\pi^{s,c}}{\pi^{s,nc}}\right) - \frac{\lambda}{\pi^{s,nc}}, \tag{13}$$

$$\phi^{low} = F^{inv}(r^{m,low}\pi^{s,nc} + \pi^c),$$

where threshold $v^* + r^{dep}q_{ij}$ is defined as the profit which makes traditional bank indifferent between being overexposed to the shadow bank and preserving its own stability:

$$v^*(q_{ij}) = \frac{\lambda}{\pi^{ns,c}}(\phi^{high} - \phi^{low}). \tag{14}$$

I next relax the assumption that q_{ij} is fixed and determine the pairwise optimal exposure. Because utility function is transferable, the pairwise optimal q_{ij} is the one that maximizes the sum of the two expected utilities. The total expected payoff may take one of the three forms. For low profits $v < r^{dep}q_{ij}$:

$$E[u_i] + E[u_j] = (r^s - r^{dep})q_{ij}\pi^s + \phi^{high}\lambda + v\pi^s,$$

For average profits $r^{dep}q_{ij} \leq v < v^* + r^{dep}q_{ij}$:

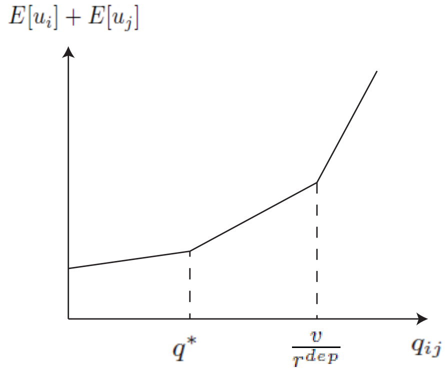
$$E[u_i] + E[u_j] = (r^s - r^{dep})q_{ij}\pi^s - r^{dep}q_{ij}\pi^{ns,nc} + \phi^{high}\lambda + v(1 - \pi^{ns,c}),$$

For high profits $v \geq v^* + r^{dep}q_{ij}$:

$$E[u_i] + E[u_j] = (r^s - r^{dep})q_{ij}\pi^s - r^{dep}q_{ij}\pi^{ns} + \phi^{low}(\lambda - r^{cb}\pi^{ns,c}) + v, \tag{15}$$

Therefore, the pairwise payoff of the two banks, as a function of q_{ij}, is continuous and convex (see Figure 7). It means that q_{ij} that maximizes joint expected payoff is either zero or $\bar{q}$. On the interval with lowest values $q_{ij} \leq q^*$ the slope of (15) as a function of q_{ij} is positive according to Assumption 2. Therefore, the total expected utility increases with an increase in q_{ij}, which proves that constraint $q_{ij} \leq \bar{q}$ is binding: $q_{ij} = \bar{q}$.

Figure 7: Pairwise expected utility as a function of exposure q_{ij}.



For the pairwise optimal outcome (12)–(13) to be stable it is necessary that the support rate r^b_{ij} is chosen such that the utility function of bank i does not depend on the exposure ϕ_j and, as a result, shadow bank j does not deviate unilaterally. Therefore, it is required that

$$r^b_{ij} = \begin{cases} r^{cb}, & \text{if } v < v^* + r^{dep} q_{ij} \\ r^{cb}(1 + \frac{\pi^{ns,c}}{\pi^{s,c}}), & \text{if } v \geq v^* + r^{dep} q_{ij} \end{cases}. \tag{16}$$

To prove that there are no other stable equilibria, we need to show that other strategies are prone to deviations. We first notice that when constraints (10) and (11) are non-binding, negotiable rates $r_{ij} > 0$, $r^b_{ij} > 0$ make it possible for the banks to jointly improve expected utilities. In particular, if shadow bank is underexposed to the money market, banks can increase exposure ϕ, lower support rate r^b_{ij}, and increase pre-crisis rate r_{ij}, such that both counterparties benefit from the change. If shadow bank is overexposed to the money market, the opposite changes can be done.

Banks can also improve when one of the constraints (10) and (11) is binding and money market exposure is not pairwise optimal. Without loss of generality, consider a case when a default of shadow bank triggers a default of the regulated bank: $v - r^{dep} q_{ij} - r^{cb}\phi_j < 0$. First, suppose that constraint (10) is binding. It is sufficient to find a deviation that strictly benefits shadow bank j and keeps utility $E[u_i]$ unchanged. The best rate r^m_j is the one that maximizes profit:

$$\max_{r_j^m} \ (r^s - r_{ij})q_{ij}\pi^{s,c} + \phi_j(r_j^m)(r^s - r_{ij}^b)\pi^{s,c}.$$

Given binding constraint (10), I find that the optimal money market rate is

$$r^{m,high} = argmax_{r_j^m} \ \phi_j(r_j^m)\left(r^s\pi^s - r^{cb}\pi^{s,c} - r_j^m\pi^{s,nc}\right), \tag{17}$$

and the resulting lending rate is

$$r_{ij} = r^s + (r^s - r_j^{m,high})\frac{\phi^{high}}{q_{ij}}.$$

The liquidity support rate r^b is adjusted such that both banks benefit from the deviation.

Similarly, when constraint (11) is binding, the shadow bank maximizes expected profit by re-negotiating rates to maximize bank's profit

$$\max_{r_j^m} \ \phi_j(r_j^m)(r_{ij}^b - r_{ij}^m)\pi^{s,nc},$$

which can be simplified to (17). Therefore, the profits of both banks increase as a result of a deviation to $r_j^m = r^{m,high}$, $\phi_j = \phi^{high}$ and

$$r_{ij} = r^s + (r^s - r_j^b)\frac{\phi^{high}}{q_{ij}}.$$

When a default of shadow bank does not trigger a default of the regulated bank, $v - r^{dep}q_{ij} - r^{cb}\phi_j \geq 0$, we can show in a similar way that the banks are able to increase profits by re-arranging flows and changing rates.

The last condition that we need to check is whether banks i and j have incentives to terminate the liquidity support. For this purpose, I find the pairwise expected utility in the non-cooperative case:

$$E[u_i] + E[u_j] = \phi_j(r^s - r^m)\pi^{s,nc} + v$$

such that the money market exposure is determined as a solution of the fixed point problem

$$\phi_j(r_j^m) = F^{inv}(r_j^m \pi^{s,nc} + \frac{l}{\phi_j}\pi^c). \tag{18}$$

In order to find non-cooperative money market exposure $\phi(r_j^m)$, I differentiate the expression above:

$$\frac{\phi'(r_j^m)}{\phi} = \frac{\pi^{s,nc}}{\lambda} - \frac{l\pi^c}{\lambda}\frac{\phi'}{\phi^2},$$

and re-write this result as

$$\phi'(r_j^m) = \frac{\pi^{s,nc}\phi^2}{l\pi^c + \lambda\phi}. \tag{19}$$

Equation (19) can be used to substitute the marginal money market volume in the first order conditions of shadow bank j:

$$\phi'(r_j^m)(r^s - r^m) - \phi = \frac{\pi^{s,nc}\phi^2(r^s - r^m) - \phi l\pi^c - \lambda\phi^2}{l\pi^c + \lambda\phi} = 0,$$

which results in the solution

$$\phi^{non-c} = \frac{l\pi^c}{\pi^{s,nc}(r^s - r^m) - \lambda}. \tag{20}$$

Finally, I find the optimal exposure of shadow bank j to the money market when the liquidity support is not provided:

$$\phi^{non-c} = F^{inv}(\pi^{s,nc} r^s - \lambda - \frac{l}{\phi_j}\pi^c + \frac{l}{\phi_j}\pi^c) = F^{inv}(\pi^{s,nc} r^s - \lambda),$$

and the corresponding money market rate

$$r^{m,non-c} = r^s - \frac{\lambda}{\pi^{s,nc}} - \frac{l}{\pi^{s,nc}\phi_j}\pi^c.$$

Using expression (20), we find the pairwise expected utility in the non-cooperative case:

$$E[u_i] + E[u_j] = \phi^{non-c}\lambda + \pi^c l + v. \tag{21}$$

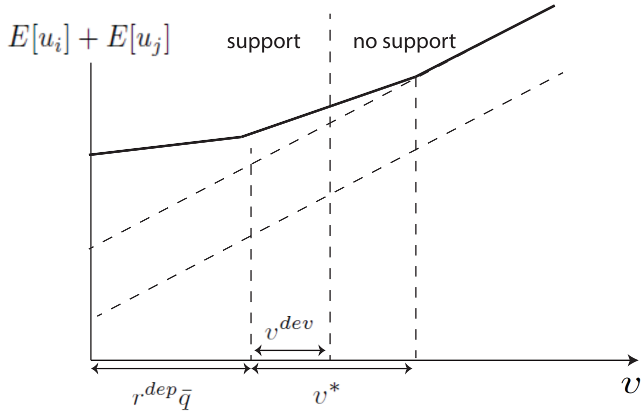
We next show that it is beneficial for the banks to keep lending channel even if the liquidity support is not provided. The formula for the pairwise expected payoff given $q^b_{ij} = 0$ and $q_{ij} = \bar{q}$ is

$$E[u_i] + E[u_j] = (r^s - r^{dep})q_{ij}\pi^{s,nc} + (1 - \pi^{s,nc})(v - r^{dep}q_{ij})_{\{+\}} + \phi^{non-c}\lambda + \pi^c l + v\pi^{s,nc},$$

which is an increasing function of q_{ij} and is greater than (21) according to Assumption 2.

Therefore, in order to show that there are no breaching deviations from the equilibrium, it is sufficient to check deviations to $q^b_{ij} = 0$ when lending channel is preserved, $q_{ij} = \bar{q}$. I will use a geometrical approach to show when banks benefit from both providing liquidity support $q^b_{ij} = \phi_j$ and keeping the long-term exposure $q_{ij} = \bar{q}$ (see Figure 8). Given that convex properties of cooperative utility $E[u_i] + E[u_j]$ with respect to v, it is sufficient to find when function segment (15) lies above non-cooperative utility $\phi^{non-c}\lambda + \pi^c l + v$ (see Figure 8).

Figure 8: Pairwise expected utility as a function of profit v.



The liquidity support is provided for high-valued v when the regulated bank is highly exposed to the shadow bank $\bar{q} > \bar{q}_{supp}$:

$$\bar{q}_{supp} = \frac{\phi^{non-c}\lambda - \phi^{low}(\lambda - r^{cb}\pi^{ns,c}) + \pi^c l}{r^s \pi^{s,c}}. \tag{22}$$

If condition (22) is not satisfied, the liquidity support is only provided when profit v is sufficiently low:$v \leq r^{dep}\bar{q} + v^{dev}$, such that threshold is defined as

$$v^{dev} = r^s \frac{\pi^{s,c}}{\pi^{ns,c}}\bar{q} + (\phi^{high} - \phi^{non-c})\frac{\lambda}{\pi^{ns,c}} - \frac{l\pi^c}{\pi^{ns,c}}.$$

It immediately follows that the critical value v belongs to the interval where regulated bank defaults if the recovery of shadow bank is not successful (see Figure 8): $v^{dev} < v^*$.

Given the findings, I conclude the stability of the equilibrium rates and volumes (13)–(12) and lending rate (16) in the appropriate range of parameters.

C. Proof of Proposition 2

Consider an equilibrium network candidate, such that a subset of shadow banks get funding from both money markets and regulated banks and the rest of shadow banks get funding only from the money market. The shadow banks of the first group, which we will call core shadow banks, receive funding from money market investors as in the "vulnerable" regime: in the amount ϕ^{large} at rate $r^{m,large}$, and the shadow banks of the second group, called periphery shadow banks, receive funding in the amount $\phi^{non-coop}$ at rate $r^{m,non-coop}$. Therefore, core shadow banks are more exposed to money market and provide a higher return to the investors.

Each core shadow bank j is offered liquidity support from a core regulated bank i at rate $r^b_{ij} = r^{cb}$. Before the crisis, the supporting bank borrows liquidity from a subset of non-core (periphery) banks $B^{per}_i \subset B$ in the total amount of q^{per} at rate r^{per}. All periphery regulated banks transfer liquidity to core regulated banks in the amount of $\bar{q}$. By definition, transfer q^{per} is determined from the equality of incoming and outgoing liquidity flows:

$$N^{core}q^{per} = (N - N^{core})\bar{q}, \tag{23}$$

which delivers

$$q^{per} = \frac{N - N^{core}}{N^{core}}\bar{q}. \tag{24}$$

The liquidity of all periphery banks is distributed equally between core regulated banks, such that the regulated bank defaults if the bail out of the shadow bank is not successful:

$$v - r^{dep}\bar{q} - r^{cb}\phi^{large} < r^{per}q^{per}. \tag{25}$$

Each regulated bank i invests in shadow bank j before the crisis at rate r^{core} by transferring liquidity q^{per} from the periphery banks in addition to its own funds $\bar{q}$.

1) I first prove the non-existence of coalitional deviations to different bail-out scheme for a particular shadow bank. We will do a proof by contradiction: assume there are at least two regulated banks i, k that provide financial support to shadow bank j at $t = 2$. Similarly to the example with three banks in section 4.2, the liquidity support scheme that generates the highest coalitional profit is the one that generates the highest conditional profit in the event of market crash followed by asset failure, because, in other market events, the banks' expected profits do not depend on how the supporting liquidity is channeled. In the event when periphery bank stays solvent and core bank defaults, the conditional utility of the two banks increases if q^b_{kj} is reallocated to bank i:

$$arg \max_{q^b_{kj}} \quad v - r^{dep}\bar{q} - r^{cb} q^b_{kj} + p_{ki}(q^b_{kj}) = 0.$$

Moreover, a default of one bank would be, ceteris paribus, more beneficial for the coalition of banks than the solvency of both in the event when the liquidity support is not successful:

$$v - r^{dep}\bar{q} + p_{ki} = 2(v - r^{dep}\bar{q}) - p_{cb,i} - \sum_m p_{m,i} \geq \ 2(v - r^{dep}\bar{q}) - r^{cb}\phi_j.$$

Generalizing this result, I state that to create an equilibrium stable to deviations, it is necessary to create an unstable core banks that would benefit from the condition of limited liability and pass the share of its profit to the periphery bank following its default.

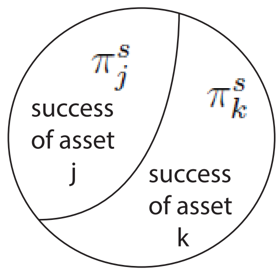
2) I prove that a core regulated bank does not deviate to contracts with multiple core shadow banks. The proof of this result is based on the fact that a minimum revenue

$$R = v - r^{dep}\bar{q} - r^{cb}\phi^{large} - r^{per}q^{per}$$

is required for the regulated bank to cover its expenses in non-crisis times. If the revenue from lending to the shadow bank is below R, the regulated bank defaults and gets zero utility. When a regulated bank splits lending between two shadow banks, its credit risk increases (according to Assumption 1), while its expected revenue stays the same. Therefore, bank i prefers lending to one bank to minimize the probability of counterparty default or is at least indifferent between different counterparties, if a default of one of them does not trigger a contagion.

For better understanding, consider an example with the default events depicted in Figure 9, where the joint probability of default of two assets is zero (an extreme case of Assumption 1) and at least one asset defaults in any event state. Suppose that R is sufficiently large. If the depository bank lends to one shadow bank, it gets a positive payoff $r_{ij}q_{ij} - R \geq 0$ in case of the asset success. However, if the bank allocates the funds to k and j equally, it defaults if at least one of the shadow banks defaults: $r_{ij}\frac{q_{ij}}{2} - R < 0$. Given the distribution of asset returns consistent with Figure 9, I conclude that the depository bank defaults with probability one and expects a payoff of zero.

Figure 9: Event space of defaults of assets a_i and a_j and corresponding probabilities.



3) Consider deviations that break the core-periphery structure of regulated banks. If core bank b deviates by attracting other periphery bank k, the maximum rate that it can offer is $r_{kb} = r^s$; however, even this rate would not be sufficient to cover the decrease in the loss-given-default that periphery bank k faces[13]:

$$r^s \pi^s \bar{q} + p_{bk} \leq r^{per} \pi^s q_{ki} + p_{ki}.$$

That is the case because the equilibrium rate r^{per} is selected to be sufficiently high, precisely as

[13] Similarly, a partial withdrawal will not be beneficial.

$$r^{per} \geq r^s - p_{ki}\frac{r^s}{\pi^s\big(r^{cb}\phi^{high} + r^{per}q^{per} + r^s\bar{q}\big)}, \tag{26}$$

where the payment given default is determined based on pro-rata rule:

$$p_{ki} = \frac{v - r^{dep}\bar{q}}{r^{cb}\phi^{high} + r^{per}q^{per}}q_{ki}$$

to guarantee that these deviations are not beneficial.

Condition (26) is also sufficient to prevent periphery banks from dealing with shadow banks directly, because a direct loan to the shadow bank would have zero loss-given-default, which is not enough to compensate the depository bank.

I also show that a periphery bank k does not have incentives to redirect lending from a core depository bank to another periphery bank m thus creating a longer sequence of intermediaries. If both m and k lend to the same core bank, their deviation can be beneficial only when the core bank accepts a double-sized loan at a rate below r^{per}. However, core bank does not have incentives to do this, because a shadow bank would not accept the loan at this rate. Alternatively, when periphery banks m and k lend to different core banks, one of their shadow bank would be willing to accept additional liquidity at a rate between r^{dep} and r^s. However, the core bank will need to reduce the compensation of both banks in case of default to satisfy the pro-rata rule. According to (26), this reduction would be sufficient to make the periphery banks unwilling to deviate.

4) The shadow bank sets borrowing rate r^{core} at the level that makes him indifferent from breaching contracts with the core bank:

$$E[u_j^{non-c}] = E[u_j^{high}],$$

which determines lending rate of the core depository bank

$$r^{core} = r^s + \frac{(\phi^{high} - \phi^{non-c})\lambda - \pi^c l}{(q^{per} + \bar{q})\pi^s}. \tag{27}$$

It becomes clear that $r^{core} > r^s$, because the provision of liquidity support creates additional benefits for the shadow bank in the form of the confidence of money market investors: $\phi^{high} > \phi^{non-c}$.

5) Core and periphery depository banks expect the same payoff. Therefore, the exposure q^{per} and rate $r^{per}$are determined in such way that the core bank i receives utility equal to

$$\Big((r^{core} - r^{per})q^{per} + (r^{core} - r^{dep})\bar{q} + v\Big)\,\pi^s = (r^{per} - r^{dep})\bar{q} + v)\pi^s + (v - r^{dep}\bar{q})\pi^{ns},$$

which can be simplified to the condition that together with (27) determines all possible pairs r^{per} and q^{per}

$$r^{per} = r^{core} - \frac{(v - r^{dep}\bar{q})}{(q^{per} + \bar{q})}\frac{\pi^{ns}}{\pi^s}.$$

Moreover, the lending terms can be selected to guarantee that $r^{per} < r^s$ and N^{core} defined in (23) is a natural number.

5) Money market exposure ϕ^{high} and rate r^{high} are the equilibrium terms for all supported shadow banks, because the core depository bank gets overexposed to the sponsoring shadow bank and, therefore, encourages higher exposure to risky investments, which becomes possible if the shadow bank borrows higher volume on the money market.

In conclusion, the equilibrium candidate network proposed at the beginning is the stable network.