Collateral, Financial Intermediation, and the Distribution of Debt Capacity

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Who Has Debt Capacity?

Research question

• Which investors have debt capacity available to take advantage of investment opportunities due to temporarily low asset prices?
Main Results

Three main substantive results

• Result 1: Productive and/or poorly capitalized borrowers may exhaust their debt capacity rather than conserve it.
  
  • Cost of conserving debt capacity is opportunity cost of foregone investment.

• Result 2: Borrowers who exhaust debt capacity may be forced to contract when asset prices and cash flows are low.
  
  • Capital less productively deployed in such times.

• Result 3: Intermediary capital may be more scarce in such times forcing borrowers to contract by more.
Main Results (Cont’d)

Two main theoretical results

• Result 4: Collateral constraints due to limited enforcement
  • Link between two classes of models

• Result 5: Model of financial intermediaries as collateralization specialists
  • Role for intermediary capital
Additional Results

Additional implications of the model

- Debt capacity
  - Definition
  - Endogenous and jointly determined with investment

- Role of long term debt

- Implementation with loan commitments

- Higher collateralizability may make contraction more severe
  - “Financial innovation”

- Dynamics of minimum down payment requirements
  - “Lending standards”
Abridged Literature Review

Dynamic models of collateral

• Kiyotaki and Moore (1997)
  • ... motivated by incomplete contracting (à la Hart and Moore 1994)

• Kehoe and Levine (1993)
  • ... motivated by limited contract enforcement/limited commitment

Models of financial intermediary capital

• Holmström and Tirole (1997) and Diamond and Rajan (2000)
Model

Borrowers

• 3 dates: 0, 1, and 2. Let $\mathcal{T} \equiv \{1, 2\}$

• Continuum of borrowers with measure 1. Types $n \in \mathcal{N}$. Density of type $n$: $\psi(n)$. Suppress types whenever possible.

• Preferences

\[ E \left[ d_0 + \sum_{t \in \mathcal{T}} d_t \right] \]

• Endowment $w_0$ at time 0 and no other endowment.

• Return $E[Af(k)]$ in cash flow at time $t + 1$ for investment of $k$ at time $t$. Capital depreciates at rate $\delta$.

Lenders

• Risk neutral and discount the future at rate $\beta < 1$.

• Large endowment of funds in all dates and states.

• Cannot operate the technology.

• ... willing to lend in state-contingent way at expected return $R \equiv 1/\beta > 1$. 
Model (Cont’d)

Collateral constraints due to limited enforcement

• Limited enforcement
  • ... agents can abscond with cash flows and fraction $1 - \theta$ of capital (default); agents not excluded from borrowing
  • ... implies collateral constraints
  • ... agents can borrow up to $\theta$ times the resale value of capital against each state.

Price of capital

• Consumption goods can be transformed into capital goods (and vice versa) at rate $\phi_0$ and $\phi_t(s)$ at time $0$ and $t$ in state $s$, $s \in S \equiv \{H, L\}$.

Figure 1: Time Line

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Figure 1: Time Line
0   1   2

\pi(H) \rightarrow s = H

\pi(L) \rightarrow s = L
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Model with Limited Enforcement

Contracting problem with limited enforcement

\[ \max \left\{ d_0, d_t(s), l_0, l_1(s), k_0, k_1(s), b_{t-1}(s) \right\}_{s \in S, t \in T} \]

\[ d_0 + \sum_{s \in S} \pi(s) \left\{ \sum_{t \in T} d_t(s) \right\} \]  \hspace{1cm} (1)

subject to the budget constraints at date 0, 1, and 2,

\[ w_0 + l_0 \geq d_0 + \phi_0 k_0 \]  \hspace{1cm} (2)

\[ A_1(s) f(k_0) + \phi_1(s) k_0 (1 - \delta) + l_1(s) \geq d_1(s) + \phi_1(s) k_1(s) + Rb_0(s), \ \forall s, \]  \hspace{1cm} (3)

\[ A_2(s) f(k_1(s)) + \phi_2(s) k_1(s)(1 - \delta) \geq d_2(s) + Rb_1(s), \ \forall s \in S, \]  \hspace{1cm} (4)

lender’s ex ante participation constraint at date 0,

\[ \sum_{s \in S} \pi(s) \left\{ \sum_{t \in T} R^{-(t-1)} b_{t-1}(s) \right\} \geq l_0 + \sum_{s \in S} \pi(s) R^{-1} l_1(s), \]  \hspace{1cm} (5)

enforcement constraints at date 1 and 2,

\[ d_1(s) + d_2(s) \geq \hat{d}_1(s) + \hat{d}_2(s), \ \forall s \in S, \]  \hspace{1cm} (6)

\[ d_2(s) \geq A_2(s) f(k_1(s)) + \phi_2(s) k_1(s)(1 - \theta)(1 - \delta), \ \forall s \in S, \]  \hspace{1cm} (7)

and non-negativity constraints on dividends and capital,

\[ d_0 \geq 0, \quad d_t(s) \geq 0, \quad k_0 \geq 0, \quad k_1(s) \geq 0, \quad \forall s \in S \text{ and } t \in T, \]  \hspace{1cm} (8)

where \( \{\hat{d}_t(s)\} \) dividends that the borrower can achieve after default.
Model with Limited Enforcement (Cont’d)

Outside option: No exclusion after default

- \( \{ \hat{d}_t(s), \hat{k_1(s), \hat{b}_1(s) \} \}_{t \in T} \) maximize

\[
\sum_{t \in T} d_t(s) \tag{9}
\]

subject to

\[
A_1(s)f(k_0) + \phi_1(s)k_0(1 - \theta)(1 - \delta) + b_1(s) \geq d_1(s) + \phi_1(s)k_1(s), \tag{10}
\]

borrower’s net worth after default

(4), (7), and (8).

- Difference to Kehoe and Levine (1993): production and outside option
  - ... they assume exclusion from intertemporal trade after default.

- Recursive structure: agent’s problem after default is identical to continuation problem when agent keeps promises, except net worth is different.

- Similar outside option in Lustig (2007) (endowment economy) and Lorenzoni and Walentin (2007) (constant returns to scale).
Role for Long Term Debt?

Irrelevance

● **Lemma 1** *Considering state-contingent one period debt is sufficient.*

● No gains from long term contracting.
  
  ● If the borrower promises to pay $Rb_0(s)$ in state $s$ at time 1, he receives an amount of funds $\pi(s)b_0(s)$ at time 0.

● Intuition:
  
  ● Enforcement constraints restrict credible promises to payments with present value less than or equal to the value of capital that borrower cannot abscond with.

  ● Any contract satisfying this restriction can be implemented with one period debt contracts.

● In contrast:
  
  ● Long term contracts not irrelevant with outside option as in Kehoe and Levine (1993).

  ● No borrowing at all with outside option as in Bulow and Rogoff (1989).
Collateral Constraints due to Limited Enforcement

Equivalence

• Lemma 2 Enforcement constraints (6) and (7) are equivalent to collateral constraints

\[
\phi_1(s) \theta k_0(1 - \delta) \geq Rb_0(s), \quad \forall s \in S, \quad (11)
\]

\[
\phi_2(s) \theta k_1(s)(1 - \delta) \geq Rb_1(s), \quad \forall s \in S. \quad (12)
\]

• Advantages:
  
  • Simple decentralization of optimal dynamic lending contract
    
    • ... implementation with state contingent secured loans
    
    • ... equilibrium with collateral constraints (similar to constraints in Kiyotaki and Moore (1997), but state contingent)

  • related: equilibrium with solvency constraints (Alvarez and Jermann (2000))

  • Constraint set in problem with collateral constraints is convex.

• Notion of (state contingent) debt capacity: \( R^{-1} \phi_1(s) \theta (1 - \delta) \)
Role for Loan Commitments?

Definition

• Binding agreement to provide loan at specific date for fee paid up front.

Why take out a loan commitment?

• So far all loans have zero $NPV$ when extended, that is $l_1(s) = b_1(s)$ and

$$NPV_1(s) = -l_1(s) + R^{-1}Rb_1(s) = 0.$$ Then loan commitments are unnecessary (and fees are zero).

• Suppose for fee $c_0(s) > 0$ at time 0, the lender agrees to provide loan $l_1(s) > b_1(s)$ in state $s$ at time 1 such that (due to competitive pricing)

$$c_0(s) + \pi(s)R^{-1}\{-l_1(s) + R^{-1}Rb_1(s)\} = 0.$$ 

• Suppose borrower conserves debt capacity $b_0(s) < R^{-1}\phi_1(s)\theta k_0(1 - \delta)$.

  • Alternative and equivalent implementation: loan commitment for $l_1(s) \equiv b_1(s) + R(\hat{b}_0(s) - b_0(s))$ where $\hat{b}_0(s) \equiv R^{-1}\phi_1(s)\theta k_0(1 - \delta)$.

Loan commitments are equivalent to saving contingent debt capacity.

• Borrowers, who choose to exhaust debt capacity, choose not to arrange loan commitments!
Dynamics of Minimum Down Payments

Minimum down payment requirements

• Define minimum down payments $\varphi_0$ (and similarly $\varphi_1(s)$) as

$$\varphi_0 \equiv \phi_0 - R^{-1} \sum_{s \in S} \pi(s) \phi_1(s) \theta (1 - \delta)$$

Expected capital appreciation affects minimum down payment

• Minimum down payment as fraction of the price of capital is low when the price of capital is expected to rise, for example, when

$$\sum_{s \in S} \pi(s) \phi_1(s) / \phi_0$$

is high.

• Consistent with anecdotal evidence on minimum down payment requirements (or lending standards).
Distribution of Debt Capacity

Who conserves debt capacity?

• **Simplifying assumptions**
  
  • Define return $R_1(k_0, s)$ as $R_1(k_0, s) \equiv \frac{A_1(s)f'(k_0)+\phi_1(s)(1-\theta)(1-\delta)}{\varphi_0}$ (and similarly $R_2(k_1(s), s)$).
  
  • With constant returns to scale, $f(k) = k$, $f'(k) = 1$, $R_1(s) \equiv R_1(k_0, s)$ (and similarly $R_2(s)$).
  
  • **Assumption 1** $R_2(s) > R$, $\forall s \in S$.

• **Proposition 1** *Productive borrowers exhaust their debt capacity*, that is, if $\sum_{s \in S} \pi(s) R_1(s) R_2(s) > \max_s \{RR_2(s)\}$, then $k_0 = \frac{1}{\varphi_0} w_0$ and $V_0(w_0) = \sum_{s \in S} \pi(s) R_1(s) R_2(s) w_0$. *Less productive borrowers conserve their net worth*, that is, if the condition is not met, $k_0 = 0$, $k_1(s') = \frac{R}{\pi(s')} w_0$, and $V_0(w_0) = RR_2(s') w_0$, where $s'$ such that $R_2(s') = \max_s \{R_2(s)\}$.

• With constant returns, either exhaust debt capacity or conserve it all.
Relative Contraction of Productive Firms

Can firms contract?

• **Proposition 2** For open set of parameters, borrowers are “forced to” contract, that is, \( \exists s \in S \) such that \( k_1(s) < k_0 \).

• When \( k_0 > 0 \), then

\[
k_1(s) = \left( \frac{A_1(s) + \phi_1(s)(1 - \theta)(1 - \delta)}{\phi_1(s) - R^{-1}\phi_2(s)\theta(1 - \delta)} \right) k_0
\]

• Thus, **productive borrowers contract**
  - ... when cash flows \( (A_1(s)) \) are sufficiently low.

Effect on average productivity

• More productive firms may contract, while less productive firms expand.
  - Productive firms exhaust debt capacity & have low cash flow/net worth.
  - Less productive firms conserve debt capacity and expand.

• Lower average productivity in such times.
Effect of Collateralizability on Contraction

Leverage and severity of contraction

- **Proposition 3** With higher collateralizability, borrowers, who exhaust debt capacity, may be forced to contract by more. Suppose the parameters are as in Proposition 2; then \( \frac{\partial}{\partial \theta} \left( \frac{k_1(s)}{k_0} \right) < 0 \) as long as \( \frac{\phi_1(s)}{\phi_2(s)} > \frac{1}{R} \frac{k_1(s)}{k_0} \).

Two effects of leverage (higher \( \theta \))

- Less “free net worth” since able to pledge larger fraction of funds at time 0.
- Lower minimum down payment requirement due to greater ability to borrow going forward.
- Opposite direction, but as long as price of capital not too much higher at time 2, first effect dominates.
  - Higher leverage due to **higher pledgeability leads to more severe contraction** in capital.
  - “Financial innovation.”
Role of Borrower Net Worth

Effect of borrower net worth on debt capacity

• Simplifying assumptions

  • **Assumption 2** $R_2(k_1(s), s) > R, \forall s \in S$.

  • **Assumption 3** (i) $R_2(k, H) < R_2(k, L)$, for $k$ in the relevant range;
    and (ii) $k_1(H) > k_1(L)$, where $k_1(s) \equiv (A_s(s)f(w_0/\varphi_0)+\phi_1(s)w_0/\varphi_0(1-\theta)(1-\delta))/\varphi_1(s)$ for $w_0$ in the relevant range.

• **Borrowers conserve some debt capacity** for the low state as long as they are not too constrained.

• **Proposition 5** Suppose Assumption 3 holds. Then there exist $\underline{w}_0 < \bar{w}_0$ such that (i) for $w_0 \leq \underline{w}_0$, $\lambda_0(s) > 0$, $\forall s \in S$, $k_0 = \frac{1}{\varphi_0}w_0$, and $k_1(s) = \frac{1}{\varphi_1(s)}(A_1(s)f(k_0)+\phi_1(s)k_0(1-\theta)(1-\delta))$; (ii) for $\underline{w}_0 < w_0 < \bar{w}_0$, $\lambda_0(H) > 0$ and $\lambda_0(L) = 0$; and (iii) for $\bar{w}_0 \leq w_0$, $\lambda_0(s) = 0$, $\forall s \in S$, $R_2(k_1(H), H) = R_2(k_1(L), L)$, and $R = \sum_{s \in S} \pi(s)R_1(k_0, s)$.
Financial Intermediation

Financial intermediaries as “collateralization specialists”

• Financial intermediaries ...
  • ... are lenders with particular ability to collateralize claims, in particular, ability to reduce amount of capital borrowers can abscond with to $1 - \theta^i$ ($\theta^i > \theta$) (similar to monitoring in Diamond (2007))
  • ... have limited capital $w^i_0$,
  • ... and are themselves subject to the same limited enforcement constraints.

• Role for intermediary capital
  • Intermediary capital required to finance extra $\theta^i - \theta$ since cannot in turn borrow against that amount due to limited enforcement constraints.
  • Dynamic model of intermediary capital; net worth of intermediaries is a state variable.
Direct vs. Intermediated Finance

Borrower’s problem

- For exposition, one period problem here; borrower can borrow in state contingent way from direct lenders and financial intermediaries.

- Direct lenders lend to intermediaries, but, to simplify, notation as if providing direct finance.

\[
\begin{align*}
\max_{\{d_0, d_1(s), k_0, b_0(s), b_0^i(s)\}_{s \in S}} & \quad d_0 + \sum_{s \in S} \pi(s)d_1(s) \\
\text{subject to budget constraints,} & \quad w_0 + \sum_{s \in S} \pi(s)\{b_0(s) + b_0^i(s)\} \geq d_0 + \phi_0 k_0 \\
& \quad A_1(s)f(k_0) + \phi_1(s)k_0(1 - \delta) \geq d_1(s) + Rb_0(s) + R_0^i b_0^i(s), \quad \forall s \in S,
\end{align*}
\]

two sets of collateral constraints,

\[
\begin{align*}
\phi_1(s)\theta k_0(1 - \delta) & \geq Rb_0(s), \quad \forall s \in S, \\
\phi_1(s)\theta^i k_0(1 - \delta) & \geq Rb_0(s) + R_0^i b_0^i(s), \quad \forall s \in S,
\end{align*}
\]
and \(d_0 \geq 0, d_1(s) \geq 0, k_0 \geq 0, b_0^i(s) \geq 0 \forall s \in S\) and \(t \in T\).
Dynamics with Limited Intermediary Capital

Limited intermediary capital affects spreads

- Assumption
  - **Assumption 4** \( nl^i_1(L) > 0 > nl^i_1(H) \).
  - Loan demand from borrowers who conserve debt capacity potentially important.

- Highest spread between intermediated and direct finance in state \( L \):

  **Proposition 8** Suppose Assumption 4 holds. Then \( \exists \varepsilon > 0 \) such that \( \forall w^i_0 < w^i_0 \) and \( \varepsilon > w^i_0 - w^i_0 \), \( R^i \equiv R^i_0(H) = R^i_1(L) > R \), and \( R^i_0(L) = R^i_1(H) = R \).

- Define time 0 spread by \( \varsigma_0 \equiv \sum_{s \in S} \pi(s)R^i_0(s) - R \); time 1 spread in state \( s \) by \( \varsigma_1(s) \equiv R^i_1(s) - R \).

  **Corollary 3** Under the conditions of Proposition 8, \( \varsigma_1(L) > \varsigma_0 > \varsigma_1(H) = 0 \).
Impact of Limited Intermediary Capital on Borrowers

Effect on severity of contraction

- The scarcer intermediary capital, the more borrowers will contract in the state in which intermediary capital is scarce.

- **Proposition 9** Suppose \( w_0^i \) is as in Proposition 8. If \( s \) such that \( nl_1^i(s) > 0 > nl_1^i(s') \), \( s' \neq s \), then \( \frac{d}{dw_0^i} \frac{k^q_i(s)}{k_0^q} > 0 \).

Two reasons why productive borrowers contract

- First: low cash flow and low net worth in state \( L \).
- Second: cost of intermediated funds increases in state \( L \).
Conclusion

Distribution of debt capacity

- Productive/less well capitalized borrowers likely exhaust debt capacity.
- Borrowers who exhaust debt capacity may be forced to contract.
- Scarce intermediary capital may force borrowers to contract by even more.
Outline

Literature

• Models of collateral and debt capacity

Model

• Collateral constraints due to limited enforcement

Results

• Role for long term debt?

• Loan commitments and contingent financing

• Determinants of minimum down payment requirements

• Productivity and distribution of debt capacity

• Implications for firm investment

  • ... effect of collateralizability

• The role of borrower net worth

• Financial intermediation

Conclusion


Literature

Models of collateral

• ... motivated by incomplete contracting (à la Hart and Moore 1994)
  • Kiyotaki and Moore (1997)

• ... motivated by limited contract enforcement/limited commitment

• ... motivated by private information of cash flows
Literature (Cont’d)

Other models of collateral

- Barro (1976) ... affects borrowing rate
- Bester (1985) ... eliminates credit rationing
- Stulz and Johnson (1992) ... reduces underinvestment problem
- Rajan and Winton (1995) ... incentives to monitor
- Dubey, Geanakoplos, and Shubik (2005) and Geanakoplos (1997) ... renders market more market

Models of debt capacity

- Shleifer and Vishny (1992)
Effect of Asset Prices on Contraction

How do asset prices affect contraction?

• Proposition 4 \( \frac{\partial}{\partial \phi_1(s)} \left( \frac{k_1(s)}{k_0} \right) < 0. \)

Two effects of higher price of capital at time 1 in state \( s \)

• Higher “free net worth”
• Higher minimum down payment requirement
• Second effect dominates first:
  • The higher price of capital, the more capital contracts.
• Key: Higher net worth requirements!
Financial Intermediary’s Problem

Model with representative financial intermediary

• Given $R^i_0(s), \forall s \in S$, the intermediary solves

$$
\max_{\{d^i_0, d^i_1(s), l^i_0(s)\}_{s \in S}} d^i_0 + \sum_{s \in S} \pi(s) R^{-1} d^i_1(s)
$$

subject to

$$
\begin{align*}
\omega^i_0 &\geq d^i_0 + \sum_{s \in S} \pi(s) l^i_0(s) \\
R^i_0(s) l^i_0(s) &\geq d^i_1(s), \quad \forall s \in S,
\end{align*}
$$

as well as $d^i_0 \geq 0, d^i_1(s) \geq 0, l^i_0(s) \geq 0, \forall s \in S$, where $l^i_0(s)$ is the amount that the intermediary lends against state $s$.

Comments

• Simplified 1-period problem; clearly $R^i_0(s) \geq R, \forall s \in S$.

• Lemma 5 $R^i_0(H) = R^i_0(L) \equiv R^i_0$ without loss of generality.
Capital Structure: Intermediated vs. Direct Finance

Cross section of capital structure

- Most productive/most constrained firms borrow from financial intermediaries.

**Proposition 6** Suppose $R^i_0 > R$. If $R \geq \sum_{s \in S} \pi(s)(A_1(s) + \phi_1(s)(1 - \delta))/\phi_0$, then $k_0 = 0$ and $V(w_0) = Rw_0$; otherwise, if $R^i_0 \geq \mu_0^* \equiv \sum_{s \in S} \pi(s)R_1(s)$, then $k_0 = (1/\varphi_0)w_0$ and $V(w_0) = \mu_0^*w_0$, and if $R^i_0 < \mu_0^*$, then $k_0 = (1/\bar{\varphi}_0)w_0$ and $V(w_0) = \bar{\mu}_0^*w_0$ where $\bar{\varphi}_0$ and $\bar{\mu}_0^*$ are defined in the proof.