Solvency regulation and credit risk transfer∗

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Abstract

This paper analyzes the optimality of credit risk transfer (CRT) in banking. In a model where banks’ main activity is to monitor loans, we show that a combination of CRT instruments, loan sales and credit derivatives, might be optimal to insure banks against shocks and to optimally redepone capital when new investment opportunities arise, without impairing incentives. We derive implications for the optimal design of capital requirements.

JEL classification: G21; G38.

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1 Introduction

In the latest years larger banks have steadily increased their market share in credit risk transfer activities - credit derivatives and loan sales - as extensively documented by ECB (2004), BIS (2005), Minton et al. (2006) and Duffie (2007), among others. When transferring credit risk for risk management purposes, banks reduce their stake in the return from lending, impairing their incentives to monitor loans. If monitoring is important for bank credit, then credit risk transfer (CRT, hereafter) may reduce the value of intermediation and increase the risk in the banking sector.

The aim of this paper is to explore the impact of different CRT activities on bank monitoring incentives and its implications for banks’ solvency regulation. We put forward an approach to prudential regulation that differs markedly from the approach followed (more or less implicitly) by banking authorities. Instead of a buffer which aim is to limit the probability of a bank’s failure to some predetermined threshold (this is what we call the Value at Risk approach) we defend the view that bank’s capital is needed to provide bankers with appropriate incentives to monitor borrowers (this is what we call the incentives approach). These two approaches have different implications for the prudential treatment of CRT activities. In the VaR approach, basically all CRT activities justify a reduction in regulatory capital requirements (for a given volume of lending) because they reduce the probability that losses exceed any given threshold. By contrast, in the incentives approach, CRT activities allow to reduce capital requirements only in so far as they maintain bankers’ incentives to monitor. Following Holmström’s exhaustive statistics approach, bankers should be, as much as possible, insured against exogenous risks (such as macroeconomic shocks) but should bear a sufficient fraction of all the risks they can influence by their monitoring activities.

In the paper we develop a simple model of prudential regulation where a banker might exert a monitoring effort in order to reduce entrepreneurs’ opportunism. Depositors cannot observe banker’s effort and condition their funding to the commitment to monitor. Bank capital fosters banker’s monitoring incentives. Prudential regulation is achieved by setting a minimum capital requirement and a fair deposit insurance premium.

To this basic setup we add a solvency shock and new lending opportunities at
an interim stage. Once banks have extended loans their portfolio might be hit by an aggregate shock - corresponding to an economic downturn - negatively affecting future returns from loans. After the realization of this shock new lending opportunities occur with positive NPV. Given binding capital adequacy requirements, those opportunities cannot be pursued unless new liquidity is provided to the bank through access to financial markets at a fair price. However since the banker has to be motivated in her monitoring effort, providing liquidity is efficient only in upturns. We show that the optimal incentive scheme can be implemented, in addition to new capital requirements and fair deposit insurance, through a combination of CRT instruments, loan sales to provide state-contingent liquidity and credit derivatives to insure loan losses.

CRT markets respond to different functions here: loan sales supply interim liquidity in upturns to undertake, with the proceeds of the sales, the new lending opportunities, while credit derivatives provide state-contingent insurance. Given that the objective of the prudential regulator is to improve bank solvency and to avoid sub-optimal under-investment, CRT is part of the optimal incentive scheme. In the model the tension between insurance and incentives is driving the results about optimal solvency regulation and use of CRT: when expected loan losses are dominant the bank must buy protection (sell credit default swaps) to shed risk in downturns, while when the rate of growth of new lending opportunities is large the bank may even take on more risk (buy credit default swaps) in downturns to be able to undertake expansion. In conclusion, our simple model of prudential regulation shows that, when taking into account banker’s incentives in the different states of the economy, capital requirements should not be designed with the unique objective to insure for loan losses in down-turns but also with the concern for under-investment in up-turns.

We also confront the optimal solution to the alternative of saving liquidity at the initial stage in order to face latter lending opportunities. In the liquidity hoarding case there is a mis-management of liquidity, because the banker is allowed to expand also in downturns, although it is ex-ante sub-optimal. Therefore, the solution with CRT is preferred as it entails a lower capital ratio and greater lending.

Several results in our paper are in line with the empirical evidence in Cebenoyan and Strahan (2004), Goderis et al.(2006) and Minton et al. (2006) showing that banks accessing CRT markets tend to increase their lending and hold less capital.
In addition, they provide evidence that larger banks engaging in CRT transactions, participate in several markets at the same time, showing that loan sales and credit derivatives are complementary activities. Finally, when looking at participants in credit derivatives transactions, banks are both buyers and sellers of protection (as documented by Minton et al., 2006, ECB, 2004, and Duffie, 2007, among others).

The paper is related to the growing literature on CRT (see the survey in Kiff et al., 2002) and, in particular, on the impact of CRT instruments on banker’s monitoring incentives and bank solvency.

The paper builds on Holmstrom and Tirole (1997) applied to banks as delegated monitors where monitoring incentives are provided through capital regulation, as diversification opportunities are scarce. Along this line of research Chiesa and Bhattacharya (2007) show that CRT insuring for aggregate risks improves monitoring incentives. Their argument is based on the result that contingent transfers, such as credit derivatives insuring loan portfolio for common shocks, are optimal mechanisms to achieve maximum effort when monitoring is more valuable in downturns. In contrast in our model since monitoring effort enhances portfolio outcomes in all states, while new interim liquidity is valuable only in upturns, shedding risk in downturns might create under-investment and reduced monitoring incentives. The optimal balance of insurance and incentives is achieved through a combination of different CRT instruments, loan sales and credit derivatives.

The coexistence of loan sales and credit derivatives in the optimal solution is common to other papers (see for instance Duffee and Zhou, 2001, Thompson, 2006, and Parlour and Winton, 2007). For instance Parlour and Winton (2007) analyze the coexistence of loan sales and credit derivatives for monitoring incentives; however they focus on the impact on loan quality when banks have superior information compared to investors and disregard prudential regulation. Nicolò and Pelizzon (2006) analyze the impact of capital regulation on the incentives to issue different CRT instruments, and show how specific forms of credit derivatives could emerge as an optimal signaling device for better quality banks in response to exogenous capital regulations. Our objective is to analyze the implications of CRT on monitoring incentives together with optimal capital regulation: therefore we assume that monitoring is exerted after issuing CRT instruments, while dis-regarding the implications of private information
There are other papers centered on the impact of CRT on monitoring incentives. For instance, Arping (2004) argues in favor of credit derivatives since credit protection fosters the commitment to liquidate and as a result entrepreneurial effort, although the dilution of the claim with investors reduces banker’s monitoring incentives. Also Morrison (2001) shows that single-named credit derivatives impact negatively on monitoring when mainly loans financed by a mix of direct and bank credit are affected: since banks are risk-averse they benefit from greater insurance on loan losses, but they lose incentives to monitor.

We use the idea that loan sales provide liquidity when other funds are scarce as Gorton and Pennacchi (1995), although we depart by adding credit derivatives and optimal capital regulation. In Parlour and Plantin (2007) loan sales provide liquidity for new investment opportunities, however, since monitoring is exerted before selling loans, investors cannot distinguish the true motive of the sale, in contrast with our model where information is symmetric, and therefore there could be scarcity of liquidity in the loan sales market.

Another strand of the literature analyzes the impact of CRT on the allocation of risks across sectors in the economy. For instance Wagner and Marsh (2006) show that CRT involving transfer of risk from banks to other sectors enhances welfare, when the banking sector is less diversified compared to other non-banking sectors, as the greater diversification achieved through CRT compensates the reduction in monitoring and is beneficial for financial stability. Allen and Carletti (2006) show that under some conditions on the distribution of liquidity shocks across banks, transfers of risks from the banking sector to the insurance sector do increase financial stability. In this paper instead we focus on monitoring incentives at a representative bank level, while we do not consider the impact on the allocation of risks across banks or across sectors when banks have access to CRT markets.

Wagner (2007) analyses the consequences of credit derivatives for bank solvency. He shows that, although CRT improves loans liquidity diminishing the likelihood of bank runs, when taking into account ex-ante incentives the greater liquidity induces greater risk-taking by the bank, reversing the positive effect on bank stability.

Finally, we share with Kashyap and Stein (2004) the idea that optimal capital
regulation serves both to insure loan losses and to reduce under-investment problems in the different states of the economy, although our conclusions on the optimal capital regulation are affected by the introduction of CRT instruments.

The remainder of the paper is organized as follows. Section 2 describes the model of prudential regulation; we start from a static benchmark model and then extend this model by introducing two additional features, new lending opportunities and a solvency shock. We study the impact of these new features on the optimal capital ratio. In Section 3 we show that this optimal solution can be implemented by a combination of loan sales and credit derivatives together with a solvency regulation. Section 4 discusses the implications for liquidity management and capital regulation of possible alternatives to CRT as for instance liquidity hoarding. In Section 5 we discuss the empirical predictions of the model. Concluding remarks are in Section 6.

2 A model of prudential regulation

In this section we model the need for capital regulation in the banking sector. We start by introducing a static benchmark model where minimum capital requirements are justified for prudential regulation. In a context where banks, whose main function is to monitor borrowers, have an incentive to exploit their informative advantage to shift portfolio losses on depositors, minimum capital requirements provide correct incentives to monitor. We then add to this simple model two ingredients, a negative solvency shock on loan returns and the possibility to undertake new lending opportunities at a latter stage. These two ingredients add into the benchmark model a problem of inter-temporal liquidity management which, we claim, can be resolved using credit risk transfer (CRT) instruments. The aim of the model is to analyze in a tractable way the impact of CRT on the monitoring function of banks and the implications for prudential regulation.

2.1 A static benchmark model

Our starting point is the simple prudential regulation model of Rochet (2004) adapted from Holmstrom and Tirole (1997). Consider a two-date economy \((t=0,1)\). At date 0 a bank, with capital \(E_0\), raises deposits \(D_0\) from dispersed investors and extends
loans $L_0$ to some entrepreneurs. Depositors’ alternative return per unit invested is 1.

Entrepreneurs rely on banking finance to undertake a risky project: each project requires 1 unit of investment at date 0, and yields a return $R > 1$ at date 1 with probability $p \in [0, 1]$ and 0 otherwise. The success probability of the loan portfolio is affected by the banker’s monitoring effort: un-monitored loans’ success probability falls to $p - \Delta p > 0$, while the banker saves a private cost $B > 0$ per unit lent. We assume constant return to scale for loan returns and private benefits. Loans’ returns are perfectly correlated as the bank, facing limited opportunities for diversification, holds some non-diversifiable risk in its portfolio.\(^1\)

Further, we assume that only monitored finance is viable\(^2\)

\[
pR > 1 > (p - \Delta p)R + B
\]

(A1)

Given that the monitoring effort is non-observable, the bank is subject to moral-hazard. For the banker to monitor the portfolio of loans the following incentive compatibility condition must be fulfilled

\[
p (RL_0 - D_0) \geq (p - \Delta p) (RL_0 - D_0) + BL_0
\]

which can be rewritten as

\[
D_0 \leq \left( R - \frac{B}{\Delta p} \right) L_0.
\]

(1)

Given that depositors do not observe the monitoring effort while the banker derives a private benefit from not monitoring, she cannot credibly promise to repay depositors an amount greater than the maximum pledgeable income defined by the right-hand side in the previous expression.

We further assume that a deposit insurance fund (DIF) is in place: by paying a premium $\pi_0$ depositors are fully insured against the risk of bank failure at date 1.

Date 0 bank’s balance sheet is defined as

\[
L_0 + \pi_0 = E_0 + D_0.
\]

(2)

\(^1\)There is a literature on the benefits of diversification of loans for banker’s incentives to monitor (see Diamond, 1984, and Cerasi and Daltung, 2000, where the result of Diamond is applied to a context similar to the one in this paper). However in that case inside equity fully restores incentives eliminating the need for diversification. Holmstrom and Tirole (1997) show indeed that capital - inside equity - strenghten monitoring incentives when opportunities for diversification are scarce. In this context the assumption of perfect correlation is not crucial for the results while it simplifies computations.

\(^2\)Given that investors are dispersed, they do not have incentives to monitor. Monitored finance is thus provided by banks.
The break-even condition for the DIF is that the expected repayment to depositors, when the banker monitors, must not exceed the premium, that is

$$\pi_0 \geq (1 - p)D_0,$$

and substituting from (2)

$$L_0 \leq E_0 + pD_0.$$  \hspace{1cm} (3)

In this simple model we derive the optimal prudential regulation as the contract between the DIF and the banker that maximizes expected social surplus.\(^3\)

**Proposition 1** *The optimal contract between the DIF and the banker can be implemented by a combination of a fair premium on deposit insurance, \(\pi_0 = (1 - p)D_0\), and a capital adequacy requirement limiting banks’ lending to a certain multiple of their equity, that is

$$L_0 \leq \frac{E_0}{k_S}$$ \hspace{1cm} (4)

where \(k_S \equiv 1 - p \left( R - \frac{B}{\Delta p} \right) \) is the (static) capital ratio.*

**Proof.** The optimal contract between the DIF and the banker requires choosing the level of loans \(L_0\) and deposits \(D_0\) that maximize expected social surplus

$$ES = (pR - 1) L_0$$

subject to incentive compatibility constraint (1) and break-even condition (3). The optimal solution is obtained by saturating the two constraints. In particular, setting:

$$D_0 = \left( R - \frac{B}{\Delta p} \right) L_0$$

Substituting into (3), we obtain

$$E_0 \geq \left[ 1 - p \left( R - \frac{B}{\Delta p} \right) \right] L_0.$$  \hspace{1cm} \(\blacksquare\)

For this result to hold we need to assume that banks need capital, i.e.

$$p \left( R - \frac{B}{\Delta p} \right) < 1.$$  \hspace{1cm} (A2)

If (A2) was not fulfilled, then banks could be 100% financed by deposits. By contrast, when (A2) applies, there is a maximum to the amount of deposits that the bank can

\(^3\)The idea is that the regulator acts in the interest of depositors (see Rochet, 2004, for a detailed discussion of the optimal prudential regulation).
raise: it is given by the maximum pledgeable income in the right hand side of (1). For each unit of loan, the maximum repayment to depositors is \((R - \frac{B}{\Delta p})\) in case of loan success, thus the difference between the maximum amount of money depositors are willing to supply \(p(R - \frac{B}{\Delta p})L_0\) at date 0 and loans \(L_0\), must be covered with own capital \(E_0\).

From Proposition 1 it follows that banks can expand their lending to a maximum of \(1/k_S\) of their equity: the static capital ratio \(k_S\) is increasing in the severity of moral hazard, measured by \(\frac{B}{\Delta p}\), while decreasing in the expected return of the project, \(pR\), as the maximum pledgeable income to depositors decreases accordingly. A greater capital ratio implies tighter credit conditions.

### 2.2 The relation with the credit risk literature

Our benchmark model is extremely stylized, and makes several unrealistic assumptions for the sake of tractability. In particular, we assume that returns on bank loans are perfectly correlated, which is of course a very strong assumption. We show in this section that the logic of our model is preserved if we adopt specifications that are closer to those used in the credit risk literature. This will also allow us to clarify the difference between the VaR approach and the incentives approach to prudential regulation.

Assume indeed that the return on bank loans has a continuous distribution, derived from a standard credit risk model. Suppose for example that each bank loan returns either \(R\) or 0 (zero recovery rate in case of default) but that default is driven by a combination of a common factor \(\tilde{f}\) and an idiosyncratic shock \(\tilde{\epsilon}_i\). Default of loan \(i\) occurs when

\[
\sqrt{\rho \tilde{f}} + \sqrt{1 - \rho^2} \tilde{\epsilon}_i \leq s(e)
\]

where \(\rho\) is a correlation parameter and \(s(e)\) is a threshold that depends on the monitoring effort \((e = 0, 1)\) exerted by the banker, with \(s(0) > s(1)\). We assume that conditionally on the common factor \(\tilde{f}\), idiosyncratic shocks \(\tilde{\epsilon}_i\) are i.i.d. with a cumulative distribution function \(\Phi\). By the law of large numbers, the average loss \(\tilde{l}\) on the
loan portfolio is completely determined by the realization of the common factor $\tilde{f}$:

\[
\tilde{\ell} = R \Pr \left( \tilde{e} \leq \frac{s(e) - \sqrt{\rho} \tilde{f}}{\sqrt{1 - \rho}} \right),
\]
\[
\tilde{\ell} = R \Phi \left[ \frac{s(e) - \sqrt{\rho} \tilde{f}}{\sqrt{1 - \rho}} \right].
\]

The cumulative distribution function of losses is thus determined by the level of effort of the banker:

\[
F_e(\ell) \equiv \Pi \left( \tilde{\ell} \leq \ell | e \right) = \Pr \left( \sqrt{\rho} \tilde{f} \geq s(e) - \sqrt{1 - \rho} \Phi^{-1} \left( \frac{\ell}{R} \right) \right).
\]

We assume that the c.d.f. of $\tilde{f}$ satisfies MLRP. By adapting the arguments of Innes (1990), it is easy to see that the optimal contract\(^4\) is similar to that obtained above:

- The bankers gets a remuneration $I(\ell^* - \ell)$ whenever losses $\ell$ do not exceed the threshold $\ell^*$ and 0 above this threshold.

- The bank’s default threshold $\ell^*$ is determined by the incentive compatibility condition:

\[
\int_0^{\ell^*} (\ell^* - \ell)[dF_1(\ell) - dF_0(\ell)] = B.
\]

- The minimum capital ratio is equal to the net expected shortfall:

\[
\frac{E}{I} \geq \int \max(\ell, \ell^*) dF_1(\ell) - (R - 1).
\]

As before, the deposit insurance premium is actuarial:

\[
\pi = I \int (\ell - \ell^*)_+ dF(\ell).
\]

There are two fundamental differences with the VaR approach to prudential regulation. First, the capital requirement is meant to cover not the Value at Risk, but the net expected shortfall. This means that it covers the expected losses above the default threshold $\ell^*$, net of the nominal excess return $(R - 1)$ on loans.

\(^4\)Like Innes (1990), we restrict attention to contracts such that the marginal remuneration of the banker (as a function of loans’ returns) is always between 0 and 1.
The second difference is that the default threshold \( \ell^* \) is not given by an exogenously determined probability of default \( \varepsilon \) but by the incentive compatibility condition. The default threshold \( \ell^* \) is the minimum value that provides the banker who exerts a monitoring effort (distribution of losses \( F_1(\cdot) \)) with an incremental expected gain (with respect to the case where he shirks, and the distribution of losses \( F_0(\cdot) \)) at least equal to the benefit \( B \) from shirking. This has important implications on the prudential treatment of CRT. In order to capture these differences we go back to our initial benchmark model (with perfect correlation of loan returns) and extend it to include uncertainty.

### 2.3 The dynamic model with uncertainty

We now add two new ingredients to the simple benchmark model to generate an intertemporal problem of liquidity management for the bank. The first ingredient is a negative shock (a credit loss) affecting the expected return on the portfolio of loans. In particular we assume that, at date 1/2, an observable shock occurs with probability \( q \in [0, 1] \) and that in this event the loan portfolio return in case of success is reduced to \( (R - \alpha) \) per unit lent, instead of \( R \). We assume that \( 0 < \alpha < R \).

The second ingredient is the occurrence of new lending opportunities after date 1/2, that is after the realization of the shock. In particular, the bank has the possibility to finance new loans of the same quality of the old ones in proportion to \( L_0 \): loans can be increased up to \( L_1 = (1 + x)L_0 \) with \( x \in [0, \beta] \). This ingredient captures the idea that new valuable projects may become available once the bank has already extended loans and is constrained by the capital requirement. Since we assume rigidities in the deposit market, the banker has to raise money from investors to fund these new projects. This new ingredient requires solving for the optimal amount of new funds, in addition to the optimal lending capacity determined at \( t = 0 \).

At date 0 the bank raises \( E_0 + D_0 \), lends \( L_0 \), and pays a premium \( \pi_0 \) to the DIF as before. At date 1/2 the negative shock occurs with probability \( q \) and right afterwards new lending opportunities arise up to \( \beta \) of extended loans \( L_0 \). After the realization of the shock and new loans are funded, the banker may monitor loans. Figure 1 might help to clarify the sequence of events. The upper branch variables (no credit loss) are denoted by a superscript +, while the lower branch variables (solvency shock) are
denoted by a superscript $-$.  

[Insert Figure 1]

For the banker to monitor loans in both states, the following incentive compatibility constraints must hold:

$$
R_B^+ \geq \frac{B}{\Delta p} L_1^+, \quad R_B^- \geq \frac{B}{\Delta p} L_1^-,
$$

(5)

where $R_B^+$ (respectively, $R_B^-$) denotes the revenue in case of loan success in the upper (resp., lower) branch and $L_1^+ = (1 + x^+) L_0$ (resp., $L_1^- = (1 + x^-) L_0$) total lending in the upper (resp., lower) branch.

From an ex-ante perspective, investors are willing to commit to inject new funds at date $t = 1/2$ if and only if the expected return is greater than the opportunity cost of their capital:

$$
p \left\{ \begin{array}{l}
(1 - q) [RL_1^+ - D_0 - R_B^+] + q [(R - \alpha)L_1^- - D_0 - R_B^-] \\
\geq (1 - q) x^+ + qx^- \end{array} \right\} L_0.
$$

(6)

The optimal contract between the DIF and the banker is defined as the vector $(\pi_0, L_0, x^+, x^-)$ that maximizes expected social surplus

$$
ES = (1 - q) [pR - 1] (1 + x^+) L_0 + q [p(R - \alpha) - 1] (1 + x^-) L_0
$$

(7)

under constraints (5), (6) and of course $x^+, x^- \in [0, \beta]$. It is derived in the following Proposition.

**Proposition 2** The optimal contract between the DIF and the banker is characterized by a fair premium $\pi_0 = (1 - p) D_0$, and an initial volume of lending limited to

$$
L_0 = \frac{E_0}{k_0}
$$

where $k_0 = k_S[1 + \beta(1 - q)] + q\alpha$ denotes the modified capital ratio at date $t = 0$. Moreover $x^+ = \beta, x^- = 0$: the bank is only allowed to exploit new lending opportunities at date $t = 1/2$ in state + (boom) but not in state $-$ (recession).

**Proof.** See the Appendix.  ■
The prudential regulator has two instruments to achieve the optimal solution: the first is given by the two lending growth rates \( \{x^+, x^-\} \) allowing to reward the banker for her effort differently in the two states; the second is the scale of activity, that is the level of loans \( L_0 \) constrained by the maximum pledgeable income to depositors at date 0 for a given capital \( E_0 \). Both instruments affect the reward of the banker in the two states fulfilling the incentive conditions (5). The optimal solution requires setting \( x^- = 0 \) while \( x^+ = \beta \) for a given \( L_0 \). In other words the banker is not allowed to grant new loans in state \(-\), while she can lend at full capacity in state \(+\), which allows to maximize her incentives to monitor.\(^5\) This is due to the fact that monitoring is more valuable in state \(+\), as its marginal benefit is greatest while its marginal cost is constant.\(^6\) To foster banker’s incentives the regulator leaves a greater rent in state \(+\) while rewarding effort the least as possible in state \(-\).

However by doing this the maximum pledgeable income to depositors is affected, as it takes different values in the two states. Total deposits, and as a consequence lending, are constrained by the minimum of the pledgeable income across states, leaving an extra-rent to the banker in one of the two states. There is scope for insuring the maximum pledgeable income to depositors in order to boost lending. As a result, the optimal capital ratio is greater compared to that in the static model, due to the insurance cost, and thus credit conditions are tighter. We analyze the precise measure of this effect in the next section.

3 Optimal prudential regulation and CRT

In this section we show that there is an optimal mix of CRT instruments (a combination of loan sales and credit derivatives) and prudential regulation that implements the optimal solution characterized above. Define \( k_0 \) to be the capital ratio at date 0, implying that loans \( L_0 \) have to be at most a multiple \( 1/k_0 \) of the bank’s capital \( E_0 \) (which is exogenously given by assumption) (minimum capital requirement). The DIF premium is set, as before, to \( \pi_0 = (1 - p)D_0 \), since the probability of default on the face value of deposits is unchanged.

At date 1/2 in state \(+\) the banker can raise new funds \( \beta L_0 \) by selling a fraction \( y \)

\(^5\)Note that this is true even when the NPV of loans is positive in state \(-\).

\(^6\)This is different in Chiesa and Bhattacharya (2007) where instead monitoring is more valuable in state \(-\) and the MLRP property does not apply to the monitoring effort.
of its old loans. Denote the date 1 repayment to investors buying these loans as $YL_0$. The banker’s incentive to monitor in state $+$ is preserved whenever

$$p(RL_0 - YL_0) \geq (p - \Delta p) (RL_0 - YL_0) + BL_0$$

The banker can promise to repay at maximum $Y = (R - \frac{B}{\Delta p})$ when loans succeed, therefore the unit price of a loan in state $+$ is

$$P = p\left(R - \frac{B}{\Delta p}\right).$$

This is the maximum price at which the banker retains incentives to monitor the loans she sells. To raise enough liquidity to extend $\beta L_0$ new loans, the banker has to sell $y$ such that $yP L_0 = \beta L_0$, therefore

$$y = \frac{\beta}{p\left(R - \frac{B}{\Delta p}\right)}.$$

Note that, due to assumption (A2), $P < 1$; the bank has to sell more loans than it grants new ones, in order to maintain its incentives, that is $y > \beta$. If state $+$ prevails, the banker receives $(1 + \beta)\frac{B}{\Delta p} L_0$ at date 1. By contrast, if state $-$ prevails, the reward to the banker is only $\frac{B}{\Delta p} L_0$, as in the static model.

We show that the implementation of the optimal solution requires, in addition to loan sales at date 1/2 to finance new loans in state $+$, state-contingent transfers at date 1. In other words it implies an optimal transfer of funds across the two states through an insurance contract where the bank pays investors $S^+$ if state $+$ occurs and $S^-$ in state $-$. For this insurance to be fairly priced, the contingent transfers $S^+$ and $S^-$ must fulfill the condition

$$q S^- + (1 - q) S^+ = 0.$$

The maximum pledgeable income to investors in state $-$ at date 1 is

$$S^- = (R - \alpha)L_0 - D_0 - \frac{B}{\Delta p} L_0,$$

while the maximum pledgeable income to investors in state $+$ at date 1 is

$$S^+ = RL_0[1 + \beta - y] - D_0 - \frac{B}{\Delta p} L_0(1 + \beta - y).$$

\footnote{The banker’s private benefits in state $+$ are given by $(1 + \beta - y)\frac{B}{\Delta p} L_0$ on the loans retained in the portfolio until their maturity at date 1; in addition the banker earns $y\frac{B}{\Delta p} L_0$ implicit in the price $P$ paid by investors at date 1/2.}
After easy computations, we find indeed that the expressions of \( S^- \) and \( S^+ \) can be simplified into:

\[
S^- = -(1 - q) \left[ \alpha - \frac{\beta k_S}{p} \right] L_0,
\]
and

\[
S^+ = q \left[ \alpha - \frac{\beta k_S}{p} \right] L_0.
\]

We can state the following result.

**Proposition 3** The optimal contract between the regulatory authority and the banker can be implemented by the following series of state contingent capital ratios:

\[
k_0 = (1 - q)(1 + \beta)k_S + q(k_S + p\alpha) = (1 + \beta)k_S + pqW
\]  
(8)

at date 0, and at date 1/2: \( k_S \) in state + and \( k_S + p\alpha \) in state −. Given that the bank increases its volume of loans by a fraction \( \beta \) in state +, regulatory capital must equal (at least) \( k_S(1 + \beta)L_0 \) in state + and \( (k_S + p\alpha)L_0 \) in state −. Regulatory capital at date 0, \( k_0L_0 \) equals the expected value of regulatory capital at date 1/2. New loans are financed by loan sales at date 1/2 in state +, of a fraction

\[
y = \frac{\beta}{p \left( R - \frac{B}{p} \right)}
\]  
(9)

of initial loans. In state −, the banker is not allowed to issue new loans. Finally adjustments in regulatory capital are provided by state contingent transfers (contracted upon at date 0 and interpreted as CDS)

\[
S^+ = qWL_0, \quad S^- = -(1 - q)WL_0
\]  
(10)

with \( W \equiv \left[ \alpha - \frac{\beta k_S}{p} \right] \).

In order to get the intuition behind the results in this Proposition, let us consider first the case without loan losses and without new lending opportunities, that is \( \alpha = \beta = 0 \). In this case the optimal capital ratio is the static capital ratio \( k_S \) and there is no role for credit risk transfer, neither loan sales at date 1/2 nor state contingent transfers at date 1 as

\[
y = S^+ = S^- = 0.
\]

15
This result shows that the two ingredients, solvency shock on the value of loans and interim lending opportunities, are crucial to justify CRT instruments for optimal prudential regulation.

When $\alpha > 0$, but $\beta = 0$ (no new lending opportunity at $t = 1/2$), the bank does not need to sell loans in state $+$, in fact $y = 0$, but it uses state contingent transfers to insure against its credit losses through a Credit Default Swap (CDS, hereafter). The capital ratio at date 0 is augmented relatively to the static model by $q\alpha$, representing the CDS premium paid when the portfolio of loans succeeds with probability $p$. To understand why there is need for insurance, we compute the maximum amount of deposits without the CDS in the two states. The pledgeable income to depositors in state $-$ without CDS is

$$D_0^- = \left(R - \frac{\beta}{\Delta p}\right) L_0 - \alpha L_0,$$

while in state $+$ without CDS it is

$$D_0^+ = \left(R - \frac{\beta}{\Delta p}\right) L_0.$$

Total deposits are given by the minimum of these two pledgeable incomes

$$D_0 = \min(D_0^-, D_0^+) = \left(R - \frac{\beta}{\Delta p}\right) L_0 - \alpha L_0$$

that is state $-$ maximum pledgeable income. Loan losses are 100% on the shoulders of depositors in state $-$, while the banker earns an extra-rent in state $+$, since

$$R_B^+ = \frac{\beta}{\Delta p} L_0 + \alpha L_0.$$

There is scope for smoothing income across states: by paying an insurance premium $q\alpha L_0$ to recover the losses $\alpha L_0$ in state $-$, the maximum pledgeable income would be reduced in state $-$ just by the amount of the CDS premium and not by the full 100% of loan losses

$$D_0 = RL_0 - \frac{\beta}{\Delta p} L_0 - q\alpha L_0.$$

This allows to boost total deposits and increase lending: thus the solution with CDS dominates the one without CDS. In this case the combination of debt at date 0 and a
state-contingent insurance contract at date 1 is optimal.\(^8\) In the solution with CRT the banker’s reward in state + is reduced to eliminate the extra-rent, in order to increase lending, while preserving monitoring incentives.

When \(\alpha, \beta > 0\) things are a little bit more complicated. To foster banker’s incentives new liquidity is injected and loans are extended up to \(\beta L_0\) in state +. Further loan losses occur in state – at date 1. We compute the maximum pledgeable income in state + without CDS

\[
D_0^+ = RL_1^+ - R_B^+ - y \left( R - \frac{B}{\Delta p} \right) L_0 = \left( R - \frac{B}{\Delta p} \right) L_0 - \frac{\beta}{p} k_S L_0
\]

and in state – without CDS

\[
D_0^- = \left( R - \frac{B}{\Delta p} \right) L_0 - \alpha L_0
\]

Total deposits are given by the minimum of the two expressions above, that is

\[D_0 = \min(D_0^-, D_0^+) = \left( R - \frac{B}{\Delta p} \right) L_0 - \max \left\{ \frac{\beta}{p} k_S, \alpha \right\} L_0 \tag{11}\]

Assume that \(p\alpha > \beta k_S\), the banker’s reward in state + is given by the following expression

\[R_B^+ = RL_1^+ - D_0 - y \left( R - \frac{B}{\Delta p} \right) L_0\]

and after substituting total deposits from (11) it is easy to derive that

\[R_B^+ = (1 + \beta) \frac{B}{\Delta p} L_0 + W L_0\]

Since in this case \(W = \left( \alpha - \frac{\beta}{p} k_S \right) > 0\), deposits are determined by state – pledgeable income, this leaves an extra-rent \(W\) to the banker in state +. There is scope to smooth income to depositors across the two states by selling a CDS (to buy protection) which in exchange of a premium \(q W\) insures \(W\) in state – (which amounts to insure only a fraction \(1 - \frac{\beta k_S}{\alpha p}\) of loan losses). In other words the optimal solution requires a transfer of resources from state + to state –.

\(^8\)The optimality of state-contingent transfers in combination to initial debt for providing incentives when information is revealed before the effort is exerted is similar to Chiesa (1992) where the solution is debt cum warrants. Our model introduces the possibility to inject new liquidity at an interim stage which complicates the solution.
Assume instead that \( p \alpha < \beta k_S \), the banker’s reward in state \(-\) is given by the following expression

\[
R_B^- = RL_1^- - D_0 - \alpha L_0
\]

and after substituting total deposits from (11) it easy to derive that

\[
R_B^- = \frac{B}{\Delta p} L_0 - W L_0
\]

Since in this case \( W = \left( \alpha - \frac{\beta}{p} k_S \right) < 0 \), deposits are bound by state + maximum pledgeable income, which leaves an extra-rent to the banker of \((-W) > 0\) in state \(-\). There is scope to smooth income to depositors across the two states by selling a CDS (to sell protection) which in exchange of a premium \( qW \) promises to pay \( W \) in state +. In other words the optimal solution requires a transfer of resources from state \(-\) to state +.

The sign of the term \( W \) captures two contrasting effects. When \( W > 0 \) the maximum pledgeable income is smaller in state \(-\) due to loan losses. The optimal solution requires to redistribute funds from state + (the lucky state) to state \(-\) (the unlucky state). To achieve this the banker could buy protection through a CDS insuring for loan losses in the event of the negative shock; for each unit of premium \( qW > 0 \) the banker receives a refund of \( W > 0 \) in state \(-\). When \( W < 0 \) instead the maximum pledgeable income is smaller in state + due to funding of new loans by investors. To boost lending capacity in state + the solution requires to redistribute funds from state \(-\) to state +. To achieve this the banker takes on more risk by selling protection through a CDS: the banker receives the premium \( qW \) in both states and pays \( W < 0 \) when state \(-\) occurs. This maximizes the resources in state + to fund all new lending opportunities.

To understand the optimal capital ratio, we compute interim (i.e.date \( \frac{1}{2} \)) capital ratios, denoted by \( k^+ \) and \( k^- \). In state +, the bank is allowed to sell a fraction \( y \) of its initial loans, in order to finance a fraction \( \beta \) of new loans. As already noted, the banker’s incentive to monitor the loans that she sells are only maintained if she keeps an equity position \( E_1 = (y - \beta)L_0 = k_S y L_0 \) in these loans. Moreover the (unconsolidated) balance sheet equation of the bank in state + is

\[
L_0(1 + \beta - y) + \pi_0 = E^+ + pS^+ + D_0,
\]
which gives after simplification:

$$E^+ = k_S L_0 (1 + \beta - y)$$

On total the bank is required to maintain total capital $E^+ + E_1 = k_S (1 + \beta) L_0$, that is a capital ratio $k^+ = k_S (1 + \beta)$ in state $+$. However when defining the consolidated balance sheet, i.e. the balance sheet of the bank and that of a Special Purpose Vehicle (SPV) in which sold loans and CDS payments are accounted together, the consolidated capital ratio is equal to the static capital ratio $k_S$. Thus there is no change to the static capital ratio, provided that the bank maintains a sufficient equity stake in the loans that have been sold, and that the solvency ratio is also satisfied at the consolidated level.

By contrast, in state $-$ the capital ratio has to be increased, due to the deterioration of profitability. Indeed, the balance sheet equation of the bank in state $-$ is:

$$L_0 + \pi_0 = E^- + pS^- + D_0,$$

which gives after simplification $E^- = (k_S + p\alpha) L_0$ implying a capital ratio $k^- = (k_S + p\alpha)$ in state $-$. This higher capital ratio prevents the bank from increasing its lending in state $-$, which would destroy the banker’s incentive to monitors her loans.

The optimal capital ratio at date 0 is the expected value of the two interim capital ratios $k^+$ and $k^-$, that is

$$k_0 = (1 - q)k^+ + qk^- = (1 - q)k_S (1 + \beta) + q(k_S + p\alpha)$$

from which the expression of the modified capital ratio follows.

We can now derive the following results on the effect of changes in the parameters on the optimal capital ratio at date 0.

**Proposition 4** The optimal capital ratio at date 0 increases with $\alpha$ and $\beta$. The effect of an increase in the probability of a shock $q$ on the capital ratio is positive (resp. negative) when $W > 0$ (resp., $W < 0$).
Proof. It is easy to derive from the optimal capital ratio in (8) the following results:

\[
\frac{\partial k_0}{\partial \alpha} = qp > 0; \\
\frac{\partial k_0}{\partial \beta} = (1-q)k_s > 0; \\
\frac{\partial k_0}{\partial q} = pW.
\]

Capital requirements must be tighter the larger the loan losses and the greater the rate of growth of new lending opportunities. Finally, the impact of a larger probability of a solvency shock on the optimal capital ratio depends on the relative strength of the two opposite motives captured in the sign of $W$.

To conclude, the mix of CRT instruments together with capital regulation is explained by the tension between incentives and insurance. The tension is resolved in two different ways according to the sign of $W$. When the solvency shock is dominant ($W > 0$) insurance helps to restore incentives, while when new lending opportunities dominate ($W < 0$), there is the usual trade-off between insurance and incentives. More specifically, in the first case in the optimal solution buying insurance restores banker’s incentives, by reducing the banker’s extra-rent in state $+$: insurance fosters incentives to monitor. In the second case instead, in the optimal solution the bank has to take on more risk by selling insurance on loan losses to transfer funds from state $-$ to state $+$, in order to reduce the banker’s extra-rent in state $-$: therefore incentives are restored by reducing insurance.

4 Alternatives to CRT

The solution of the model has implications for liquidity management and capital regulation. In particular in this section we discuss one possible alternative to the use of CRT instruments which is to hoard liquidity at date 0 and use it at date 1/2 to fund new loans. We show that liquidity hoarding is dominated by the solution where banks access CRT markets.
4.1 Liquidity hoarding

The intuition is that liquidity hoarding requires the bank to save a fixed amount of liquidity before the realization of the shock at date $1/2$. At this stage not all information is available. The ex-ante optimal level of liquidity to hold at time $t = 0$ is therefore different from the ex-post optimal level of liquidity and this impairs banker’s incentives. To mitigate this ex-ante incentive problem, capital ratio adjusts to a higher level, reducing total lending in the first stage. On the contrary access to CRT markets provides state-contingent liquidity at $t = 1/2$, that is when uncertainty about the shock is resolved.

Assume that the bank raises $E_0 + D_0$ and lends $L_0$, pays the premium to the DIF as before and hoards $\tilde{L}_0$ as liquidity to be used at date $t = 1/2$. From date 0 bank’s balance sheet, we have:

$$L_0 + \pi_0 + \tilde{L}_0 = E_0 + D_0.$$

At date $t = 1/2$ when new lending opportunities $\beta L_0$ arise, the banker can invest up to $xL_0$ of his hoarded liquidity $\tilde{L}_0$. Notice that this amount cannot be made conditional upon the realization of the shock, since there is no credible commitment not to employ it at time $t = 1/2$. Regardless of the state of the economy the banker funds new loans up to $\beta L_0$ in both states as $\beta$ is the optimal growth rate. Since there is a constant level of liquidity hoarded at date 0, that is $x^+ = x^-$, and given that the expected surplus in (7) is increasing in this constant level of liquidity, the optimal rate of growth is $\beta$ and thus $L^+_1 = L^-_1 = (1 + \beta)L_0$.

Given the (fair) DIF premium, date 0 balance’s sheet becomes

$$(1 + \beta) L_0 = E_0 + pD_0$$

The banker’s expected return at $t = 1$ is thus

$$R^+_B = R(1 + \beta)L_0 - D_0$$
$$R^-_B = (R - \alpha)(1 + \beta)L_0 - D_0$$

At date 1/2 for the banker to monitor the following incentive constraints must hold:

$$R^+_B > R^-_B \geq \frac{B}{\Delta p}L_1$$
from which $R_{B} = \frac{B}{\Delta p} L_{1}$. This sets an upper limit to the amount of deposits the bank can raise, that is state – pledgeable income

$$D_{0} = \left[ \left(R - \frac{B}{\Delta p}\right) - \alpha \right] (1 + \beta) L_{0}. \quad (14)$$

Substituting (14) into the balance sheet in (12) we derive the capital adequacy requirement

$$E_{0} \geq \tilde{k}_{0} L_{0}$$

where $\tilde{k}_{0} = (1 + \beta) [k_{S} + p\alpha]$. It is easy to check that

$$\tilde{k}_{0} > k_{0}$$

that is the capital ratio is greater (tighter credit conditions) compared to the capital ratio when CRT is available, to compensate for the soft-budget constraint given by the liquidity hoarded at time 0. We can state the following result:

**Proposition 5** The solution with liquidity hoarding is sub-optimal compared to the solution with CRT markets.

**Proof.** We can compare the expected surplus in the two cases. From expression (7) substituting the optimal capital ratio and the two optimal levels $x^{+} = \beta, x^{-} = 0$ we derive

$$ES^{*} = \frac{E_{0}}{k_{0}} \{(pR - 1)(1 + \beta) - q\alpha\}$$

While computing the expected surplus in the liquidity hoarding solution, we have

$$ES^{LH} = \frac{E_{0}}{k_{0}} \{(pR - 1) (1 + \beta) - q\alpha (1 + \beta)\}$$

Given that $\tilde{k}_{0} > k_{0}$ and that the term in brackets is smaller in the expression below we conclude that $ES^{*} > ES^{LH}$. ■

For a given level of capital the banker will lend less in this case, and therefore liquidity hoarding implies a sub-optimal solution compared to the case where the bank has access to CRT markets. There are two reasons why this solution is dominated by the solution with CRT. The first reason is that liquidity hoarding does not comply with the tough incentive scheme of $x^{-} = 0$: as a matter of fact the banker is equally rewarded in the two states, but this leaves her a greater rent reducing the maximum
pledgeable income to depositors. As a consequence, the scale of activity of the bank is smaller. The second reason is that liquidity is better provided through state-contingent financial contracts allowing to transfer funds across states once information about the shock is revealed. The solution with CRT allows to implement a better management of the liquidity, by providing funds in the state in which liquidity is worth more.

4.2 Other possible alternatives

There are other possible alternatives other than liquidity hoarding to raise liquidity at date $1/2$, such as collecting new deposits, resorting to inter-bank lending or issuing outside equity to relax capital requirements. We briefly discuss these alternatives in what follows.

**Deposits.** In the model we have ruled out the possibility for the bank to raise deposits at date $1/2$. To increase the volume of deposits the bank has most likely to open new branches, as it is documented in the empirical literature showing the importance of the notion of "distance" in retail banking competition. Furthermore the decision to open a new branch is a long-term decision not easy to reverse. In contrast, financial markets, and in particular CRT markets, provide flexibility for funding at the time when new investment opportunities arise and in the contingencies in which it is desirable. In our model financial markets are better providers of the interim liquidity needed only in state $+$ to implement the optimal solution. If on the contrary banks were able to raise new deposits by opening new branches, this liquidity would be available also in state $-$. But this solution would be equivalent to the liquidity hoarding case discussed in the previous subsection and we know it is sub-optimal.

**Inter-bank lending.** Other banks could in principle supply liquidity at date $1/2$ through the inter-bank market at the same terms as private investors. However other banks should have extra-liquidity when the borrowing bank is short of liquidity. This requires banks to be hit by idiosyncratic shocks, while in the model the solvency shock is a common shock associated with state $-$. Therefore all banks are simultaneously on the same side of the liquidity market and hence the inter-bank market would not be a feasible substitute for CRT.
Outside equity. To finance new loans banks could raise equity in financial markets at date 1/2, relaxing capital requirements and enabling the bank to undertake new loans. However outside equity is costly as any accrued benefits from monitoring has to be shared with outside shareholders impairing insider’s incentives (as shown for instance in Cerasi and Daltung, 2000). While in our context outside equity discourages monitoring effort, inside equity fully restore incentives as shown by Holmstrom and Tirole (1997). As a matter of fact in our model inside equity is the initial capital which we assume it cannot be increased further at a latter stage.

5 Empirical predictions

The model has numerous predictions that can be discussed in the light of the empirical literature.

First of all, one of the implications of the liquidity hoarding case is that banks with access to CRT markets tend to hold less capital and increase their lending compared to other banks. This prediction finds support in Cebenoyan and Strahan (2004) as they confront the different behavior of US banks active in the loan sale market and show that they hold less capital and lend more compared to other banks without access to CRT markets. Also Goderis et al. (2006) find evidence on a sample of banks worldwide issuing collateralized loan obligations, which they use as a public signal of access to CRT markets, expand their lending by 50%, while Minton et al. (2006) provide evidence of lower capital ratios for US banks who are net buyers of credit protection.

Second, one of the implications of the model is that CRT instruments are complement more than substitute, as they respond to different needs: while loan sales provide state-contingent liquidity, CDS serve to insure against loan losses. As a matter of fact in the optimal solution banks use a combination of CRT instruments, loan sales and CDS. Cebenoyan and Strahan (2004) show evidence that banks using derivatives are more likely to sell loans, while Goderis et al. (2006) use a similar argument when using loan sales as a proxy for a more wide access to CRT markets. Also Minton et al. (2006) provide evidence that banks selling loans tend to access credit derivative markets more likely than others banks.

Third, the model predicts that banks with access to CRT markets might be on
either sides of the credit derivative markets, both as buyers and sellers of credit protection, according to their need to insure loan losses or to pursue lending expansion. Several papers provide figures in support to the fact that banks are both protection sellers and protection buyers (see among others Minton et al., 2006, ECB, 2004, Duffie, 2007).

6 Conclusions

In a model where bank monitoring is important but non-observable we have shown that the access to CRT markets improve incentives, provided that the capital ratio is adjusted accordingly. The model has implications for solvency regulation, in particular for capital requirements, and for liquidity management.

CRT markets serve as state-contingent providers of liquidity at future dates when the bank is capital constrained. Loan sales supply interim liquidity, while credit derivatives provide state-contingent transfers to balance between incentives and insurance for loan losses. We show that banks accessing CRT markets must hold an equity position in sold loans in order to maintain monitoring incentives. Further, capital ratio should be adjusted in order to let the bank to expand in upturns and forego investment opportunities in down-turns.

The model has implications for the design of capital ratios. According to the literature on risk management in banks, Basel II capital ratios are derived from VaR models where the threshold of bank solvency is set at an exogenous level. In our simple model of prudential regulation we have shown that the optimal capital requirement should be state-dependent as it must account for banker’s incentives in the different states of the economy. Furthermore the model shows that capital ratios should not be designed with the unique objective to insure for loan losses in down-turns but also with the concern for under-investment in up-turns (see Kashyap and Stein, 2004, who make a similar point).

In the model we do not discuss the lemon problem associated with informational asymmetries between CRT sellers and buyers, in particular in the market of loan sales. In our model monitoring takes place after CRT contracts are issued. This eliminates the problem of asymmetries of information between banks and investors as the bank does not have superior information at the time of selling a portion of the loan or an
insurance position on the loan. Furthermore, this assumption eliminates any cost due to the coexistence of credit derivatives and loan sales, as explored in Duffee and Zhou (2001) and Thompson (2006), where the introduction of credit derivatives may cause a break-down in the market of loan sales. However, our model is not fit to explore the implications of loan sales or credit derivatives on loan quality as done for instance in Parlour and Winton (2007).

In the model banks are homogenous with regard to their moral hazard costs and exposure to shocks. We leave for future research the task of exploring optimal solvency regulation in a setting where banks are heterogeneous with regard to the size of loan losses or private benefits. For instance Rochet (2004) explores optimal closure rules and capital regulation when banks are hit differently by the same macroeconomic shock.
Appendix

Proof of Proposition 2

The optimal contract between the DIF and the banker requires choosing the level of loans $L_0$, deposits $D_0$ and a rate of growth of loans in both states $0 \leq x^-, x^+ \leq \beta$, that maximize expected social surplus (7) under the incentive compatibility constraints (5) and the break-even conditions (3) and (6). Define $A^+ \equiv (1 + x^+)L_0$ and $A^- \equiv (1 + x^-)L_0$. The expected surplus is increasing in both $A^+, A^-$, therefore the two incentive compatibility constraints (5) are binding. The optimization problem amounts to maximize

$$ES = (1 - q) [pR - 1] A^+ + q [p(R - \alpha) - 1] A^-$$

given the constraint, once we substitute the break-even condition (3) and the two binding constraints (5),

$$E_0 \geq (1 - q) \left[ 1 - p(R - \frac{B}{\Delta p}) \right] A^+ + q \left[ 1 - p(R - \alpha - \frac{B}{\Delta p}) \right] A^- \quad (15)$$

Define the following parameters $\delta \equiv (1 - q)(pR - 1), \gamma \equiv q[p(R - \alpha) - 1], a \equiv (1 - q) \left[ 1 - p(R - \frac{B}{\Delta p}) \right], b \equiv q \left[ 1 - p(R - \frac{B}{\Delta p}) + p\alpha \right]$. Given our assumptions, when also $\gamma > 0$, all four parameters are positive. Thus, given that the expected surplus is increasing in both $A^+, A^-$ the solution requires the constraint (15) to be saturated. One can derive from the constraint the expression for $A^+$ (resp. $A^-$), substitute it into the ES and compute the derivative w.r.t. $A^-$ (resp. $A^+$)

$$\frac{dES}{dA^-} = \frac{\gamma a - b\delta}{a}; \quad \frac{dES}{dA^+} = \frac{b\delta - \gamma a}{b}.$$ 

It is easy to see that $\gamma a - b\delta = -pq\alpha(1-q)\frac{B}{\Delta p} < 0$, therefore the solution implies choosing $A^-$ as smallest as possible, while $A^+$ as greatest. Note that it would be true even in the case of $\gamma < 0$, namely with a negative net present value of the monitored project in state $-$, as the sign of $\gamma a - b\delta$ would still be negative. Since the optimal solution implies choosing $x^- = 0, x^+ = \beta$ and saturating the other constraints, condition (6) becomes:

$$pD_0 \leq \left\{ (1 - q) \left[ p\left( R - \frac{B}{\Delta p} \right)(1 + \beta) \right] + q \left[ p\left( R - \alpha - \frac{B}{\Delta p} \right) \right] \right\} L_0 - (1 - q)\beta L_0$$

and substituting it into (3), we obtain

$$E_0 \geq [1 + \beta(1 - q)] \left[ 1 - p\left( R - \frac{B}{\Delta p} \right) \right] L_0 + pq\alpha L_0.$$
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Figure 1