Macro Stress Testing and Worst Case Analysis of Loan Portfolios

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Macro Stress Testing Methods

- Plausibility of scenarios
- Standard stress testing with partial scenarios
- Worst Case Search
- Identify key risk factors

Application to Foreign Currency Loan Portfolios

How to find extreme adverse events in Macro Stress Testing?

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- The present paper is an attempt to suggest a methodology that gives a precise operational definition to what it means to find "exceptional but plausible" scenarios in macro stress testing of a portfolio.
- The example portfolio which we use and develop in the paper is a portfolio of foreign currency loans to private households.

Outline

The abstract framework

- Given is a portfolio of loans. The value of the portfolio is a function v of n macro risk factors $\mathbf{r} = (r_1, \dots, r_n)$ and of m idiosyncratic risk factors $\epsilon_1, \dots \epsilon_m$, one for each counterparty.

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- Given is a portfolio of loans. The value of the portfolio is a function v of n macro risk factors $\mathbf{r} = (r_1, \dots, r_n)$ and of m idiosyncratic risk factors $\epsilon_1, \dots \epsilon_m$, one for each counterparty.
- The macro risk factor changes are distributed elliptically with covariance matrix Σ and expectations $\boldsymbol{\mu}$.The idiosyncratic risk factors may be continuous or discrete.

Plausibility of scenarios

- The plausibility of macro scenarios will be measured by the Mahalanobis distance:

$$\mathrm{Maha}(\mathbf{r}) := \sqrt{(\mathbf{r}-\boldsymbol{\mu})^T \cdot \Sigma^{-1} \cdot (\mathbf{r}-\boldsymbol{\mu})},$$

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  where $\mathbf{r}$, μ, and Σ only refer to the macro risk factors fixed by the scenario.
- Interpretation: $\mathrm{Maha}(\mathbf{r})$ is (the multivariate analogue of) the size of the move measured in standard deviations.

Standard stress testing with partial scenarios

Partial scenario: Specify the value of some but not all macro risk factors. What about the other risk factors?

- $\mathbf{r}_A$ Fixed risk factors: value specified by the partial scenario, other macro risk factors: last observed value
- $\mathbf{r}_B$ fixed risk factors: value specified by the partial scenario, other macro risk factors: unconditional expectation value.
- $\mathbf{r}_C$ fixed risk factors: value specified by the partial scenario, other macro risk factors: conditional expected value given the values of the fixed risk factors.
- $\mathbf{r}_D$ fixed risk factors: value specified by the partial scenario, other macro factors not fixed: distributed according to the marginal distribution given the values of the fixed risk factors.

Complete partial scenario so as to maximise plausibility

Proposition 1: Assume the distribution of macro risk factors is elliptical with density strictly decreasing as a function of Maha.

$$\mathrm{Maha}(\mathbf{r}_C) = \mathrm{Maha}(\mathbf{r}_D)$$

Among the scenarios with the given values of the fixed risk factors, these are the macro scenarios with the highest plausibility.

Complete partial scenario so as to maximise harm

We measure the harmfulness of a stress scenario by looking at conditional expected profits (CEP).

Proposition 2: If the portfolio value function v is concave in the non-fixed macro risk factors, then

$$CEP(\mathbf{r}_D) \leq CEP(\mathbf{r}_C).$$

If v is convex in the non-fixed risk factors the opposite inequality holds. If v is neither concave nor convex $CEP(\mathbf{r}_D)$ may be higher or lower than $CEP(\mathbf{r}_C)$.

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- By such a systematic search over an admissible domain we do not miss any harmful yet plausible scenarios.
- The search can be formulated as an optimization problem: We look for macro scenarios in the set

$$\mathrm{Ell}_k := \{\mathbf{r} : \mathrm{Maha}(\mathbf{r}) \leq k\} \tag{1}$$

minimizing the conditional expectation of the profit distribution.

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- Worst case scenarios reflect portfolio specific dangers.
- Worst case scenarios allow for an identification of the key risk factors which contribute most to the loss in the worst case scenario.

Identify key risk factors

Key risk factors are the risk factors with the highest Maximum Loss Constribution (MLC).

$$MLC(i) := \frac{CEP(\mathbb{E}r_1, \mathbb{E}r_2, \ldots, r_i^{WC}, \mathbb{E}r_{i+1}, \ldots \mathbb{E}r_n) - CEP(\mathbb{E}\mathbf{r})}{CEP(\mathbf{r}^{WC}) - CEP(\mathbb{E}\mathbf{r})}.$$

- $MLC(i)$ is the loss if risk factor i takes its worst case value and the other risk factors take their expected values, as a percentage of MaxLoss.
- The MLC of the risk factors in general do not add up to 100%.

Outline

Credit risk model

- Portfolio of foreign currency loans with N obligors, one period.
- Payment obligation to the bank at time 1 in home currency is

$$o_i = l_i(1+r)\ f(1)/f(0)\ + l_i\ s\ f(1)/f(0)$$

- The payment ability of obligor i is distributed according to

$$\begin{aligned} a_i(1) &= a_i(0) \cdot \frac{GDP(1)}{GDP(0)} \cdot \epsilon, \\ \log(\epsilon) &\sim N(\mu, \sigma) \end{aligned}$$

  where m and a(0) are constants, and $\mu = -\sigma^2/2$ ensuring $E(\epsilon) = 1$. The realizations of ϵ_i are independent
- The profit bank makes with obligor i is

$$v_i := \min(a_i, o_i) - l_i(1+r) f(1)/f(0).$$

Calibrating the idiosyncratic risk distribution

- Let p_i be the annual default probability

$$p_i = P[a_i(\sigma) < o_i(s)]$$

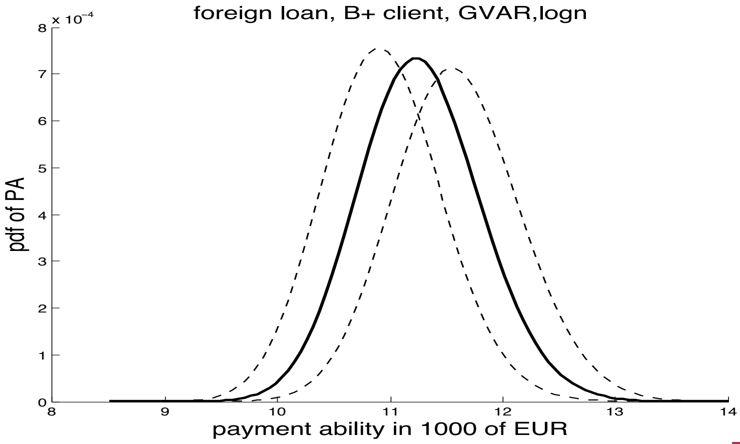
- Spreads are set to achieve some target expected profit for each loan:

$$E(v_i(\sigma, s)) = \mathrm{EP}_{\mathrm{target}},$$

  where v_i is the profit with obligor i and $\mathrm{EP}_{\mathrm{target}}$ is some target expected profit.

- The two free parameters σ and s are determined from these two conditions.

Payment ability distribution under GDP shifts



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- $a_i(0)$=1.2 l
- Spreads are set so expected profit on a loan of 10 000 Euro is 160 Euro. This gives spreads of 158.06 bp for BBB_+ and 163.88 bp for B_+ customers.

Standard stress test with partial scenario

FX scenario: EUR falls by 20% against CHF.

Rating	Type	Maha	CEP
B_+	A	5.587	-64 294
B_+	B	4.979	-56 293
B_+	C	4.905	-53 337
B_+	D	4.905	-54 209
BBB_+	A	5.587	-58 134
BBB_+	B	4.979	-48 225
BBB_+	C	4.905	-44 587
BBB_+	D	4.905	-45 136

Compare to unconditional EP of $+16$ 000.

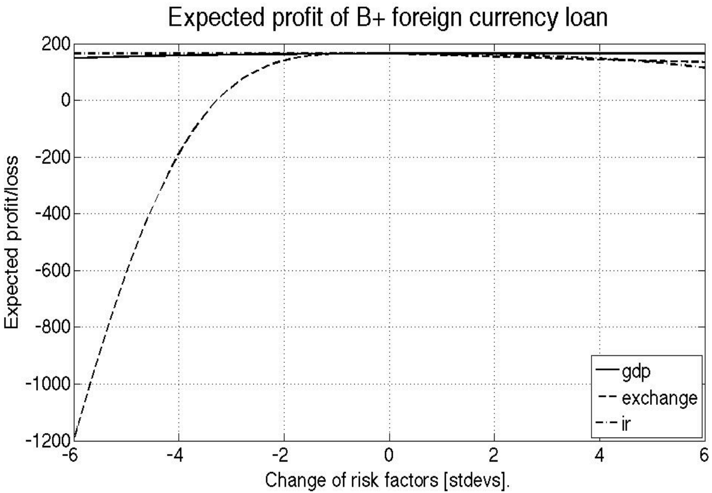
Standard Stress test versus Worst Case Analysis

Rating	Scenario	Maha	CEP
B_+	Stress	4.91	-53 337
B_+	Worst Case	4.91	-68 023
BBB_+	Stress	4.91	-44 587
BBB_+	Worst Case	4.91	-62 139

Worst Case Analysis

	Worst Macro Scenario						
max.	GDP		foreign IR		CHF/€		
Maha	stdv	MLC	stdv	MLC	stdv	MLC	CEP
foreign	**B+**						
2	-0.14	0.5%	0.04	0.4%	2	100.0%	15 400
4	-0.1	0.1%	1.17	0.4%	-3.78	65.3%	-26 084
6	-0.03	0.0%	1.59	0.2%	-5.74	77.0%	-136 000
foreign	**BBB+**						
2	-0.14	0.0%	0.03	0.0%	2	100.0%	14 855
4	-0.27	0.0%	0.07	0.0%	4	100.0%	13 859
6	-0.04	0.0%	1.58	0.0%	-5.74	75.8%	-135 203

Key Risk factors



Conclusions

1. Measure plausibility of scenarios:
   by Maha.
2. Partial scenarios:
   Plausibility maximised if we set the remaining risk factors to their conditional expected values (or leave them unspecified)
3. Maximise severeness of stress scenarios:
   Among the macroeconomic scenarios satisfying some plausibility constraint determine the worst case scenario.
4. Identify key risk factors:
   Risk factors with highest MaxLoss contribution MLC.