Macro Stress Testing and Worst Case Analysis of Loan Portfolios

Thomas Breuer       Martin Jandačka
Klaus Rheinberger   Martin Summer

Workshop ”Stress Testing of Credit Risk Portfolios”
Basel Committee on Banking Supervision
and De Nederlandsche Bank
Amsterdam, March 7, 2008
Macro Stress Testing Methods
- Plausibility of scenarios
- Standard stress testing with partial scenarios
- Worst Case Search
- Identify key risk factors

Application to Foreign Currency Loan Portfolios
How to find extreme adverse events in Macro Stress Testing?

- The IMF’s definition requires to use macroeconomic scenarios that are "exceptional but plausible" but there is no clear definition of what exceptional and plausible ultimately mean.
How to find extreme adverse events in Macro Stress Testing?

- The IMF’s definition requires to use macroeconomic scenarios that are "exceptional but plausible" but there is no clear definition of what exceptional and plausible ultimately mean.
- The present paper is an attempt to suggest a methodology that gives a precise operational definition to what it means to find "exceptional but plausible" scenarios in macro stress testing of a portfolio.
How to find extreme adverse events in Macro Stress Testing?

- The IMF’s definition requires to use macroeconomic scenarios that are "exceptional but plausible" but there is no clear definition of what exceptional and plausible ultimately mean.

- The present paper is an attempt to suggest a methodology that gives a precise operational definition to what it means to find "exceptional but plausible" scenarios in macro stress testing of a portfolio.

- The example portfolio which we use and develop in the paper is a portfolio of foreign currency loans to private households.
Outline

Macro Stress Testing Methods
- Plausibility of scenarios
- Standard stress testing with partial scenarios
- Worst Case Search
- Identify key risk factors

Application to Foreign Currency Loan Portfolios
The abstract framework

Given is a portfolio of loans. The value of the portfolio is a function \( v \) of \( n \) macro risk factors \( r = (r_1, \ldots, r_n) \) and of \( m \) idiosyncratic risk factors \( \epsilon_1, \ldots, \epsilon_m \), one for each counterparty.
The abstract framework

- Given is a portfolio of loans. The value of the portfolio is a function $v$ of $n$ macro risk factors $\mathbf{r} = (r_1, \ldots, r_n)$ and of $m$ idiosyncratic risk factors $\epsilon_1, \ldots, \epsilon_m$, one for each counterparty.
- The macro risk factor changes are distributed elliptically with covariance matrix $\Sigma$ and expectations $\mu$. The idiosyncratic risk factors may be continuous or discrete.
The plausibility of macro scenarios will be measured by the Mahalanobis distance:

$$\text{Maha}(\mathbf{r}) := \sqrt{(\mathbf{r} - \boldsymbol{\mu})^T \cdot \Sigma^{-1} \cdot (\mathbf{r} - \boldsymbol{\mu})},$$

where $\mathbf{r}$, $\boldsymbol{\mu}$, and $\Sigma$ only refer to the macro risk factors fixed by the scenario.
The plausibility of macro scenarios will be measured by the **Mahalanobis distance**:

\[
\text{Maha}(r) := \sqrt{(r - \mu)^T \cdot \Sigma^{-1} \cdot (r - \mu)},
\]

where \( r, \mu, \) and \( \Sigma \) only refer to the macro risk factors fixed by the scenario.

Interpretation: \( \text{Maha}(r) \) is (the multivariate analogue of) the size of the move measured in standard deviations.
Partial scenario: Specify the value of some but not all macro risk factors. What about the other risk factors?

$r_A$ Fixed risk factors: value specified by the partial scenario, other macro risk factors: last observed value

$r_B$ fixed risk factors: value specified by the partial scenario, other macro risk factors: unconditional expectation value.

$r_C$ fixed risk factors: value specified by the partial scenario, other macro risk factors: conditional expected value given the values of the fixed risk factors.

$r_D$ fixed risk factors: value specified by the partial scenario, other macro factors not fixed: distributed according to the marginal distribution given the values of the fixed risk factors.
Complete partial scenario so as to maximise plausibility

**Proposition 1**: Assume the distribution of macro risk factors is elliptical with density strictly decreasing as a function of Maha.

\[ \text{Maha}(r_C) = \text{Maha}(r_D) \]

Among the scenarios with the given values of the fixed risk factors, these are the macro scenarios with the highest plausibility.
Complete partial scenario so as to maximise harm

We measure the harmfulness of a stress scenario by looking at conditional expected profits (CEP).

**Proposition 2**: If the portfolio value function $v$ is concave in the non-fixed macro risk factors, then

$$CEP(r_D) \leq CEP(r_C).$$

If $v$ is convex in the non-fixed risk factors the opposite inequality holds. If $v$ is neither concave nor convex $CEP(r_D)$ may be higher or lower than $CEP(r_C)$. 
Contrary to the standard practice of stress testing we suggest to search systematically for those macro scenarios in some domain of given plausibility which is most harmful to the portfolio.
Worst case search

- Contrary to the standard practice of stress testing we suggest to search systematically for those macro scenarios in some domain of given plausibility which is most harmful to the portfolio.

- By such a systematic search over an admissible domain we do not miss any harmful yet plausible scenarios.
Contrary to the standard practice of stress testing we suggest to search systematically for those macro scenarios in some domain of given plausibility which is most harmful to the portfolio.

By such a systematic search over an admissible domain we do not miss any harmful yet plausible scenarios.

The search can be formulated as an optimization problem: We look for macro scenarios in the set

\[ Ell_k := \{ r : \text{Maha}(r) \leq k \} \]  

(1)

minimizing the conditional expectation of the profit distribution.
Advantages of worst case search over standard stress testing

- Worst case scenarios are superior to the standard stress scenarios in the sense that they are more severe and more or equally plausible.
Advantages of worst case search over standard stress testing

- Worst case scenarios are superior to the standard stress scenarios in the sense that they are more severe and more or equally plausible.
- Worst case scenarios reflect portfolio specific dangers.
Advantages of worst case search over standard stress testing

- Worst case scenarios are superior to the standard stress scenarios in the sense that they are more severe and more or equally plausible.
- Worst case scenarios reflect portfolio specific dangers.
- Worst case scenarios allow for an identification of the key risk factors which contribute most to the loss in the worst case scenario.
Key risk factors are the risk factors with the highest Maximum Loss Contribution (MLC).

\[
MLC(i) := \frac{CEP(\mathbb{E}r_1, \mathbb{E}r_2, \ldots, r_{i}^{WC}, \mathbb{E}r_{i+1}, \ldots \mathbb{E}r_n) - CEP(\mathbb{E}r)}{CEP(r^{WC}) - CEP(\mathbb{E}r)}.
\]

- \(MLC(i)\) is the loss if risk factor \(i\) takes its worst case value and the other risk factors take their expected values, as a percentage of MaxLoss.
- The \(MLC\) of the risk factors in general do not add up to 100%.
Macro Stress Testing Methods
- Plausibility of scenarios
- Standard stress testing with partial scenarios
- Worst Case Search
- Identify key risk factors

Application to Foreign Currency Loan Portfolios
Credit risk model

- Portfolio of foreign currency loans with $N$ obligors, one period.
- Payment obligation to the bank at time 1 in home currency is
  \[ o_i = l_i(1 + r) \frac{f(1)}{f(0)} + l_i s \frac{f(1)}{f(0)} \]
- The payment ability of obligor $i$ is distributed according to
  \[ a_i(1) = a_i(0) \cdot \frac{GDP(1)}{GDP(0)} \cdot \epsilon, \]
  \[ \log(\epsilon) \sim N(\mu, \sigma) \]
  where $m$ and $a(0)$ are constants, and $\mu = -\sigma^2/2$ ensuring
  $E(\epsilon) = 1$. The realizations of $\epsilon_i$ are independent
- The profit bank makes with obligor $i$ is
  \[ v_i := \min(a_i, o_i) - l_i(1 + r)f(1)/f(0). \]
Calibrating the idiosyncratic risk distribution

Let $p_i$ be the annual default probability

$$p_i = P[a_i(\sigma) < o_i(s)]$$

Spreads are set to achieve some target expected profit for each loan:

$$E(v_i(\sigma, s)) = EP_{\text{target}},$$

where $v_i$ is the profit with obligor $i$ and $EP_{\text{target}}$ is some target expected profit.

The two free parameters $\sigma$ and $s$ are determined from these two conditions.
Payment ability distribution under GDP shifts

foreign loan, B+ client, GVAR, logn

payment ability in 1000 of EUR

pdf of PA

8 x 10^-4

Breuer, Jandacka, Rheinberger, Summer
Macro Stress and Worst Case Analysis
Portfolio

- Portfolio consists of 100 loans with principal \( l = 10000 \) Euro for each obligor.
Portfolio consists of 100 loans with principal $l = 10000$ Euro for each obligor.

Loans are taken in CHF from Austrian customers in rating class $B_+ (p_i = 2\%)$ and $BBB_+ (p_i = 0.1\%)$.
Portfolio

- Portfolio consists of 100 loans with principal $l = 10000$ Euro for each obligor.
- Loans are taken in CHF from Austrian customers in rating class $B_+(p_i = 2\%)$ and $BBB_+(p_i = 0.1\%)$
- $a_i(0)=1.2 \ l$
Portfolio

- Portfolio consists of 100 loans with principal $l = 10000$ Euro for each obligor.
- Loans are taken in CHF from Austrian customers in rating class $B_+(p_i = 2\%)$ and $BBB_+(p_i = 0.1\%)$.
- $a_i(0) = 1.2 \ l$
- Spreads are set so expected profit on a loan of 10 000 Euro is 160 Euro. This gives spreads of 158.06 bp for $BBB_+$ and 163.88 bp for $B_+$ customers.
Standard stress test with partial scenario

FX scenario: EUR falls by 20% against CHF.

<table>
<thead>
<tr>
<th>Rating</th>
<th>Type</th>
<th>Maha</th>
<th>CEP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_+$</td>
<td>A</td>
<td>5.587</td>
<td>-64 294</td>
</tr>
<tr>
<td>$B_+$</td>
<td>B</td>
<td>4.979</td>
<td>-56 293</td>
</tr>
<tr>
<td>$B_+$</td>
<td>C</td>
<td>4.905</td>
<td>-53 337</td>
</tr>
<tr>
<td>$B_+$</td>
<td>D</td>
<td>4.905</td>
<td>-54 209</td>
</tr>
<tr>
<td>$BBB_+$</td>
<td>A</td>
<td>5.587</td>
<td>-58 134</td>
</tr>
<tr>
<td>$BBB_+$</td>
<td>B</td>
<td>4.979</td>
<td>-48 225</td>
</tr>
<tr>
<td>$BBB_+$</td>
<td>C</td>
<td>4.905</td>
<td>-44 587</td>
</tr>
<tr>
<td>$BBB_+$</td>
<td>D</td>
<td>4.905</td>
<td>-45 136</td>
</tr>
</tbody>
</table>

Compare to unconditional EP of +16 000.
## Standard Stress test versus Worst Case Analysis

<table>
<thead>
<tr>
<th>Rating</th>
<th>Scenario</th>
<th>Maha</th>
<th>CEP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_+$</td>
<td>Stress</td>
<td>4.91</td>
<td>-53 337</td>
</tr>
<tr>
<td>$B_+$</td>
<td>Worst Case</td>
<td>4.91</td>
<td>-68 023</td>
</tr>
<tr>
<td>$BBB_+$</td>
<td>Stress</td>
<td>4.91</td>
<td>-44 587</td>
</tr>
<tr>
<td>$BBB_+$</td>
<td>Worst Case</td>
<td>4.91</td>
<td>-62 139</td>
</tr>
</tbody>
</table>
# Worst Case Analysis

## Worst Macro Scenario

<table>
<thead>
<tr>
<th>max. Maha</th>
<th>foreign IR</th>
<th>CHF/€</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>stdv</td>
<td>MLC</td>
</tr>
<tr>
<td>foreign</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 B+</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.14</td>
<td>0.5%</td>
</tr>
<tr>
<td>4</td>
<td>-0.1</td>
<td>0.1%</td>
</tr>
<tr>
<td>6</td>
<td>-0.03</td>
<td>0.0%</td>
</tr>
<tr>
<td>foreign</td>
<td>BBB+</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-0.14</td>
<td>0.0%</td>
</tr>
<tr>
<td>4</td>
<td>-0.27</td>
<td>0.0%</td>
</tr>
<tr>
<td>6</td>
<td>-0.04</td>
<td>0.0%</td>
</tr>
</tbody>
</table>
Key Risk factors

Expected profit of B+ foreign currency loan

- gdp
- exchange
- ir

Change of risk factors [stdevs]
Conclusions

1. **Measure plausibility of scenarios:**
   by Maha.

2. **Partial scenarios:**
   Plausibility maximised if we set the remaining risk factors to their conditional expected values (or leave them unspecified).

3. **Maximise severeness of stress scenarios:**
   Among the macroeconomic scenarios satisfying some plausibility constraint determine the worst case scenario.

4. **Identify key risk factors:**
   Risk factors with highest MaxLoss contribution MLC.