Integrating credit and interest rate risk: A theoretical framework and an application to banks' balance sheets

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Credit and interest rate risk in the banking book are the two most important risks faced by commercial banks. In this paper we derive a consistent and general framework to measure the riskiness of a bank which is subject to correlated interest rate and credit risk. The framework accounts for all sources of credit risk, interest rate risk and their combined impact. As we model the whole balance sheet of a bank the framework not only enables us to assess the impact of credit and interest rate risk on the bank's economic value but also on its future earnings and capital adequacy. We apply our framework to a hypothetical bank in normal and stressed conditions. The simulation highlights that it is fundamental to measure the impact of correlated interest rate and credit risk jointly as well as on the whole portfolio of banks, including assets, liabilities and off-balance sheet items.

Key words: Integration of credit risk and interest rate risk, asset and liability management of banks, economic value, stress testing

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1 Introduction

Credit and interest rate risk are the two most important risks faced by commercial banks. But as Jarrow and Turnbull (2000) point out *'economic theory tells us that market and credit risk are intrinsically related to each other and not separable'*. In this paper we concentrate on the major source of market risk for commercial banks: interest rate risk in the banking book. First, we derive a general framework to measure the riskiness of banks which are subject to correlated interest rate and credit shocks[1]. In line with the current literature this framework incorporates the integrated impact of interest rate and the credit risk on banks' assets. But, we show that it is important to model the whole portfolio of a bank: assets, liabilities and off-balance sheet items. This not only enables us to assess the impact of credit and interest rate risk on risk adjusted discount rates and, hence, banks' economic value but also on future expected profits and capital adequacy over time. In the second part of the paper, we apply the framework to look at the riskiness of a hypothetical bank in a severe stress scenario. We show that it is fundamental to measure the impact of correlated interest and credit risk jointly and on the whole portfolio of banks, including assets, liabilities and off-balance sheet items.

When setting capital or pricing assets the focus is generally on default risk only. For example, the Merton (1974) model uses the insights from option pricing to derive the likelihood that the value of assets fall below the default threshold and the firm defaults. Similarly, reduced form models in the spirit of Jarrow and Turnbull (1996) only model the default intensity. Basel II and commercial credit portfolio models such as Moody's KMV or CreditRisk$^+$ also account for default risk only[2]. A broader definition of credit risk includes changes in the value of net assets due to changes in credit spreads, downgrades as well as defaults. And all these factors are important when assessing the impact of credit risk on the economic value and profitability of a portfolio.

Interest rate risk is also a broad term and may be attributable to repricing mismatches of assets, liabilities and off-balance sheet net positions, including basis and yield curve risk[3]. For this reason it is important to have a flexible framework to capture the sensitivity of economic value and net interest income to changes in the level and slope of the term structure of the risk-free interest rate as well as the interactions of interest rate and credit risk.

Given the complexity of the underlying sources of interest rate risk empirical papers find it hard to support its importance for banks' riskiness. Following Flannery and James (1984) several papers find a strong negative impact of interest rates on bank stock returns (for a recent study see Fraser *et at* 2002). However, Chen and Chan (1989) argue that this is highly dependent on the actual sample period. A BIS study by English (2002) also concludes that it seems unlikely that interest rate changes are an important factor for the stability of a banking system, even though English acknowledges that interest rate risk may be an important source of volatility of profits. English supports his conclusions by an econometric analysis of annual aggregate net interest income in different countries. He only finds weak support that changes in the slope of the yield curve as well as long- and short-term interest rates impact on net interest income. In a recent study on interest rate risk in the Belgian

[1] By correlated credit and interest rate risk we do not necessarily imply a linear relationship but that we model the two risks' dependence.
[2] Unfortunately, these models also assume non-stochastic interest rates and therefore cannot assess the importance of interest rate risk on credit exposures.
[3] Interest rate risk also arises from differences in embedded options of assets and liabilities. Even though the framework could be extended to capture optionalities we do not consider them in this paper.

banking sector, Maes (2005) argues that interest rate risk is important for banking stability, but again only finds weak empirical evidence when looking at net interest income.

Banks are aware of the importance of interest rate risk and measure their exposure regularly. They do so by undertaking sensitivity tests of parallel shifts or twists in the risk-free yield curve. One of the simplest sensitivity tests often undertaken is gap analysis, where banks or regulators assess interest rate risk by purely looking at the net repricing mismatch between assets, liabilities and off-balance sheet items[4]. Such tests implicitly assume that shocks to the risk-free yield curve have no impact on the credit quality of assets. However, as pointed out already, interest rates risk and credit risk are correlated and we show that ignoring the impact of interest rate risk on credit risk can lead to a severe underestimation of risk. We will also show why annual net interest income may be too aggregated to disentangle the complex effects of interest rates on banks' riskiness. Following an interest rate increase profitability initially falls due to a rise in write offs and a decrease in net interest income as margins between short term borrowing and long term lending are compressed. However, as the bank gradually reprices its assets, net interest income starts to increase because it passes the higher credit risk and interest rates to its borrowers. Over time the second effect outweighs the first until the initial level of profitability is recovered. Given these opposing effects and combined with other fluctuations in the data it is not surprising that an econometric analysis of net interest income using annual data finds it hard to support the importance of interest rate risk for banks.

Jarrow and Turnbull (2000) are among the first to show theoretically how to integrate interest rate (among other market risks) and credit risk in a reduced form model. Their insights are backed by strong empirical evidence that interest rate changes impacts on the credit quality of assets. For example, Jarrow and van Deventer (1998) show that in terms of hedging a bond portfolio, both credit and interest rate risk have to be taken into account. Grundke (2005) finds that significant errors are made when the correlated nature of rating transitions, credit spreads, interest rates and recoveries is ignored. Similarly, interest rates have been identified as the key risk driver of aggregate credit quality in the UK (e.g. see Benito *et al*, 2001, and Whitley *et al*, 2004)

All these papers look at the integrated impact of credit and interest rate risk on assets only, by for example modelling a bond portfolio. However, they do not assess the impact of interest and credit risk on liabilities or off-balance sheet items. In a series of papers, Barnhill and various co-authors (2000, 2001 and 2004) attempt to measure credit and market risk for the whole portfolio of banks. They develop a simulation framework to revalue asset and liabilities depending on the state of several systematic risk factors, such as the term structure of risk-free and risky interest rates, stock indices and property prices. They integrate interest rate and credit risk for corporates by simulating the equity to debt ratio conditional on systematic risk factors and then map it into different rating classes or defaults. Similarly, for loans to individuals they simulate the loan to value (LTV) ratio conditional on systematic risk factors and assume that a borrower defaults if the LTV is below a certain threshold. All other assets and liabilities are valued at the conditional risk-free interest rate.

[4] Generally, gap analysis allocates assets, liabilities and off-balance sheet items to time buckets according to their repricing characteristics and calculates their net difference for each bucket. Because of this netting procedure, gap analysis may fail consider non-linearities and, consequently, underestimate the impact of interest rate risk. For example, some short-term customer deposit rates track the risk-free rate plus a negative spread. Hence, for large falls in the risk-free term structure, banks may not be able to lower deposit rates in line with the risk-free rate because bounded by zero. By modelling the whole portfolio we can capture this compression in banks' net margins.

To assess the stability of a bank or banking system, they focus on the distribution of the economic value of banks, i.e. the market value of assets minus liabilities. They find that credit risk is the most significant risk factor for banks. But their conclusion is likely to be misleading as they ignore one of the most important sources of interest rate risk - repricing mismatches between assets and liabilities[5].

In contrast to Barnhill *et al* and most of the literature we not only look at economic value but also at future earnings and capital ratios. Volatility of earnings is an important focal point for banks because severe falls in profits can pose a threat to banks' capital and may create liquidity problems, either due to a lack of cash or to a downgrade which may lead to funding problems. The measurement of banks' economic value, however, provides a more comprehensive view of the potential long-term exposure to credit and interest rate risk.

Some of the few papers which do take both an economic value and an earnings perspective are Jobst and Zeinos (2003) and Jobst *et al* (2003, 2006). These papers look at optimal portfolio selection in the context of dynamic asset and liability management. They simulate correlated interest rates and credit spreads as well as defaults for a portfolio of corporate bonds and track future portfolio valuations, incorporating all coupon payments. Using this information they compute the optimal portfolio allocation if there is only one investment decision *ex-ante* or if the portfolio can be rebalanced at each point in time.

We cannot use credit spreads from bond data as we look at banks' portfolios which contain a wide range of non-traded assets and liabilities. Instead, we model corporate and household credit risk directly. Further, and more importantly, we model the complex cash flows from liabilities with different maturities rather than assuming a simple cash account as Jobst and his co-authors do. Our approach also takes account of interest rate sensitive off-balance sheet items.

The theoretical set-up of our paper is kept very general. In the empirical part we build on a macro stress testing model as described in Bunn *et al* (2005). This model starts with the general assumption that conditional on systematic risk factors corporate and household sector defaults are independent. And it identifies macroeconomic factors as the key drivers for credit risk. We combine our credit risk model with a dynamic yield curve model linking the risk-free yield curve to developments in macro factors (Diebold *et al*, 2006). By linking credit risk and the risk-free yield curve to the same systematic risk factors we are able to capture the underlying correlation between credit and interest rate risk. This allows us to assess the impact of credit and interest rate risk on risk adjusted discount factors which take account of shifts in the risk-free yield curve as well as the default intensity of borrowers. Expanding the single asset framework of Duffie and Singleton (2003) to a portfolio of assets and liabilities we can therefore compute the economic value of banks which are subject to credit and interest rate risk. But the framework also allows us to project expected profits in future periods. In particular, we are able to calculate expected write-offs and expected net interest income, the latter by explicitly modelling the repricing of assets and liabilities. Hence, we are also able to assess expected earnings, profitability as well as capital ratios conditional on the crystallisation of credit and interest rate risk.

In the simulation we apply our framework to a hypothetical bank which is subject to a combination of stress test scenarios similar to those developed for the IMF Financial Stability Assessment

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[5] The papers also look at a maturity mismatch of +/- one year and conclude that it is important. But +/- one year is clearly too simplistic to capture the full impact of the maturity miss-match on the riskiness of banks.

Programme of the UK in 2002[6]. Even though the stability of the bank is not threatened in the stress scenario, we find strong evidence that interest rate and credit risk have to be assessed simultaneously as well as jointly for assets and liabilities. In line with Bunn *et al* (2005) we show that write-offs increase significantly after the stress. As we also model net interest income we are able to illustrate that the additional margin compression decreases profits even further in the first few quarters. But losses are gradually offset once the bank starts to reprice assets and margins reflect the change in the risk-free yield curve and credit quality again. The offsetting effect of higher net interest income implies that after three years profits are roughly at the same level as in the baseline scenario, even though write-offs peak only in the third year and are significantly higher than in the baseline scenario.

The empirical exercise highlights two important innovations of our framework. First, in contrast to standard stress testing models[7] we are able to assess both the economic value as well as the profile of the main components of profits during benign and stressed conditions. Second, we can decompose the change in net profits into changes driven by credit risk, interest rate risk as well as correlated credit and interest rate risk. Using our new approach, we show that ignoring the correlations between interest rate and credit risk as well as only modelling the asset side of a bank does severely distort the risks faced by banks.

The remainder of the paper is structured as follows. In Section 2 we discuss our new general framework to integrate credit and interest rate risk in the banking book. In Section 3 we discuss the modelling of the term-structure of interest rate of different assets and liability classes. In Section 4 we present the results of the stress test and in Section 5 we investigate whether interest rate, credit or the interaction between both risks is the key risk driver. Our results are evaluated against a number of sensitivity tests in Section 6. Finally, we summarise the main conclusions of the paper in Section 7.

2 The framework

This section starts by discussing the integration of interest rate and credit risk for a generic asset and then looks at a bank as a portfolio of several assets and liabilities with different risk and repricing characteristics.

2.1 A generic asset

The economic value of a generic asset is simply the risk-adjusted discounted value of future coupon payments and the principal. For simplicity we assume that all assets are equivalent to bullet bonds – i.e. repay the principal only at maturity. For example, such an asset could be a fixed-interest rate bond with no embedded options or a simple variable rate bank loan. Given the information set available at time t, the economic value of asset i with maturity T is:

$$EVA_t^i = \sum_{k=1}^{T} D_{t+k}^i C_0^i A^i + D_{t+T}^i A^i \qquad (1)$$

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[6] For a detail description of the initial IMF stress tests see Hoggarth and Whitley (2003).
[7] For an overview of stress testing models see Sorge (2005).

where A^i is the principal and C_0^i is the constant coupon rate for the generic asset *i* determined at *t=0*. The discount function is given by:

$$D_{t+k}^i = \prod_{l=0}^{k} d_{t+l-1;t+l}^i \tag{2}$$

where the period by period discount rates is:

$$d_{t+l-1;t+l}^i = \frac{1}{1+E_t(R_{t+l-1;t+l}^i)} \tag{3}$$

$E_t(R_{t+l-1,t+l}^i)$ is the time *t* expected risk adjusted nominal interest rate paid by the generic asset between *t+l-1* and *t+l*. To simplify notation, we assume that *t=0* for the remainder of this section. Abstracting from the liquidity premium, $E_0(R_{l-1,l}^i)$ can be decomposed into a risk-free component and a risky component related to the credit risk of the asset:

$$E_0(R_{l-1;l}^i) = E_0(r_{l-1;l}) + E_0(s_{\mathrm{l-1;l}}^{\mathrm{i}}) \tag{4}$$

where $E_0(s_{\mathrm{l-1;l}}^{\mathrm{i}}) = E_0(\lambda_{\mathrm{l-1;l}}^{\mathrm{i}} \cdot LGD_{\mathrm{l}}^{\mathrm{i}})$ is the credit risk premium, $E_0(r_{l-1;l})$ the expected short rate between *l-1* and *l*, $LGD_{\mathrm{l}}^{\mathrm{i}}$ the expected loss given default for borrower *i* at *l* and $\lambda_{l-1;l}^i$ the expected default intensity of borrower *i* between *l-1* and *l*, conditional on survival up to *l-1*. All expectations are taken subject to the information set at time *0*. This decomposition holds in continuous time only.

In discrete time Duffie and Singleton (2003) [8] show that the expected risk adjusted interest rate between *l* and *l+1* can be calculated as:

$$E_0(R_{l-1;l}^i) = \frac{E_0(r_{l-1;l}) + E_0(PD_{l-1;l}^i \cdot LGD_l^i)}{1-E_0(PD_{l-1;l}^i \cdot LGD_l^i)} \tag{5}$$

where $PD_{l-1;l}^i$ is the probability of default of borrower *i* between *l-1* and *l* conditional on surviving until *l-1*. Again, all expectations are taken subject to the information set at time *t=0.*

Arbitrage will ensure that at the time of issuance (*t=0)* the economic value equals the face value of the asset. This implies that $EVA_{t=0}^i = 0$ in equation (1). Solving for $C_{t=0}^i$ we obtain:

$$C_0^i = \frac{1-D_T^i}{\sum_{k=1}^{T} D_k^i} \tag{6}$$

Equations (5) and (6) are crucial for understanding the channels through which credit and interest rate risk are correlated. First, both the expected risk premium and the expected risk free yield curve are dependent on a common set of macroeconomic risk factors. Hence, unexpected changes in these

[8] The above formula holds for coupon bonds if the same LGD applies to both coupons and principal.

risk factors do impact both credit and interest rate risk. Second, unexpected movements in the risk-free yield curve do change borrowers' credit risk[9].

When economic conditions change over time, the expected yield curve, the expected PDs and LGDs of the asset will adjust instantaneously. The discount factors, D_{t+k}, will therefore also adjust immediately. As coupon rates remain fixed the economic value of an asset will diverge from its face value. However, once the asset can be repriced coupon payments will reflect the new economic conditions and the asset's economic value will equal its face value again. For a bank this implies that whist the economic value always reflects instantaneously all future and current economic conditions, interest income will adjust partially as its assets and liabilities gradually reprice.

2.2 The generic bank

A bank can be seen as a large portfolio of assets and liabilities[10]. In particular, we will look at N asset classes A_i and M liability classes L_j. where all exposures in an asset (liability) class *i (j)* have the same risk characteristics. Within each class individual exposures may have different repricing buckets but we assume for simplicity that the maturity of an asset (liability) coincides with its repricing characteristics. According to the repricing and risk characteristics each asset is priced according to formulae 1-5. In theory these formulae also apply to all liabilities using the bank's own PD and LGD. But as we will discuss shortly we will need to treat liabilities differently to match empirical observations.

To assess the vulnerability of a bank to credit and interest rate risk, we adopt both an economic value and an earnings perspective. As discussed earlier, volatility of earnings is an important focal point for the stability of a bank in the short run because a fall in income and an increase in write-offs can pose a threat to banks' capital and possibly liquidity. The measurement of the impact of a shock on a bank's economic value provides a more comprehensive view of the potential long-term exposures to credit and interest rate risk. Clearly, the earnings and economic value perspectives are related as the economic value of a bank should be equivalent to the discounted sum of all future earnings in a risk neutral world.

Before continuing with the theoretical set-up we need to clarify the notation. To enhance readability for a multi-asset and multi-liability bank we drop the expectation operator and will do so for the remainder of the paper. But the reader should keep in mind that all calculations are based on expectations conditional on the information set available at the time of pricing. We will also use subscript t to indicate the flow between $t-1$ and t. To clarify: for stock variables, for example the economic value of a loan, the subscript t indicates the value of the variable at time t. While for flow variables, for example a bank's interest receivables, the subscript t indicates the accrued value of the variable between $t-1$ and t.

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[9] There is also a feedback from credit risk to interest rates. Such effect is partially embedded in the macro-model, which we use to simulate the systematic risk factors. But this channel is hard to quantify formally and we, therefore, do not explicitly consider it in this paper.

[10] More generally, a bank's portfolio also includes off-balance sheet items. In the framework we do not distinguish whether assets and liabilities are on- or off-balance sheet items. But we will model them separately in our application in the next section.

2.2.1 The economic value perspective

A firm's economic value *EVB* is the economic value of its assets *(EVA)* minus the economic value of its liabilities *(EVL):*

$$EVB_t = EVA_t - EVL_t \qquad with \qquad EVA_t = \sum_{i=1}^{N} EVA_t^i \quad and \quad EVL_t = \sum_{j=1}^{M} EVL_t^j \tag{7}$$

A key issue in the treatment of banks' liabilities is the modelling of the banks' own credit risk. One could argue that liabilities should be discounted using the risk-free rate and a bank's *PD* and *LGD.* While this seems to be the case for banks' subordinated liabilities, it is well known that shorter-term customer deposit rates are generally below the risk-free interest rate even when accounting for non-interest costs net of fees (see for example Maes, 2005). This may be the result of deposit insurance schemes or barriers to entry limiting competition. Furthermore, as a bank's credit conditions deteriorate its PD increases and, ceteris paribus, the economic value of its deposits decreases. Hence, a deterioration in asset quality will not be fully reflected in the net economic value of the bank. In some extreme cases the fall in *EVL* can more than offset that in *EVA* with the net result of an increase in the bank's economic value[11].

Although this may be welcomed by shareholders, it is in tension with the concern of depositors, debt-holders and regulators about the ability of banks to repay liabilities at par when due. Given that our concern is to measures banks' financial strength from the perspective of the overall stability of the financial system, we are interested in banks' ability to repay all their liabilities at par when due.

Hence, our first condition to assess the stability of a bank is to see whether the economic value of assets conditional on credit and interest rate risk is greater than the face value of all its liabilities $FVL_t = \sum_{j=1}^{M} L_t^i$. Therefore, the economic value perspective is:

Condition 1 – Economic Value:

$$EVA_t > FVL_t \tag{8}$$

From a regulatory perspective this condition has two benefits. First, it provides a long term view of the bank's ability to repay all its liabilities when due. Second, in severe stressing conditions with hikes in interest rates, it is likely to represent an upper bound in comparison to an economic-value analysis as the face value will be greater than the economic-value of liabilities.

2.2.2 The earnings perspective

Whereas the economic value perspective provides a long term view based on economic fundamentals, the earnings perspective focuses on whether a bank would be sufficiently well

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[11] For example, consider a bank with a positive net value which has to write-off 10 per cent of its assets due to a sudden idiosyncratic adverse shock. And assume that the shock nearly wipes out the bank's capital. As a consequence, the bank's PD is likely to increase dramatically, say from 0.01 to 0.5. If the bank has liabilities with an average maturity of one year (and r_t and LGD are assumed to be constant at 5% and 50% respectively) such an increase would decrease *EVL* of liabilities by around 19%; well above the 10% loss on the asset side. The net result would be an increase in the bank's economic value.

capitalised in all future states of the world. This is an important difference as a particular path of profits may well lead a bank to be undercapitalised in specific periods even though Condition 1 is satisfied. This may occur if a bank incurs severe losses in the short run which are outweighed by sufficient profits in future periods. From an economic value perspective this bank would be solvent but because of market or regulatory constraints it may find it difficult to continue to operate. For example, a bank with a capital adequacy ratio below the minimum requirement could be prone to liquidity runs.

It is therefore important to assess whether a bank's expected capital adequacy given the expected future path of earnings remains above the regulatory minimum *k* for all periods in the medium term W[12]. Hence our second condition is:

Condition 2 – Capital adequacy

$$\frac{SF_t}{RWA_t} > k \qquad \forall t < W \tag{9}$$

where *RWA* denotes expected risk weighted assets and *SF* expected shareholder funds, which are assumed to be the only capital of the bank.

Risk weighted assets are calculated under two different approaches. We first take risk weights to be constant over time: 0.5 for inter-bank lending, 0.35 for mortgage lending, 0.75 for unsecured lending and 1 for corporate loans in our simulation. This could be seen as an approximation of Basel I framework currently in use. Under this approach, risk weighted assets are simply the weighted sum of exposures to asset *i* at time *t* with risk weights $\overline{w}^{\,i}$ differing across asset classes. Hence Condition 2 under this approach is:

Condition 2a – Capital adequacy - constant risk weights

$$\frac{SF_t}{RWA_t^{CRW}} > k \quad \forall t < W \quad with \;\; RWA_t^{CRW} = \sum_i \overline{w}^{\,i} A_t^i \tag{10}$$

As we are especially interested in severe manifestations of credit risk, the constant-risk-weight approach described above may not be suitable as it may underestimate the risks to the capital adequacy of the bank. We therefore also use the Basel II internal rating based approach which accounts for time-varying risk-weights w_t^i for different asset classes (see Bank for International Settlements, 2004). Hence Condition 2 under this approach becomes:

Condition 2b – Capital - internal rating based approach

$$\frac{SF_t}{RWA_t^{IRB}} > k \quad \forall t < W \quad with \;\; RWA_t^{IRB} = \sum_i w_t^i A_t^i \tag{11}$$

[12] We will only look at Tier 1 capital (proxied by shareholder funds) in our simulation for which the current minimum capital requirement relative to risk weighted assets is 4%. And we will consider three years as the medium horizon *W*.

2.2.3 Assessing shareholder funds and future cash flows

Simulating expected shareholder funds *SF* involves tracking expected net profits which either grow by retained earnings (i.e. profits after taxes and dividend payouts) or decrease by losses in which case no taxes and dividends are paid[13]. Hence, shareholder fund can be computed as

$$SF_t = \theta \max(0; NP_t) + \min(0; NP_t) + SF_{t-1} \tag{12}$$

with $\theta<1$ given that the bank pays taxes as well as dividends.
For a representative bank, expected net profits (NP_t) between period $t-1$ and t are the sum of expected net interest income plus expected other income (OI_t) minus expected write-offs (WR_t) and expected costs ($Cost_t$). Expected net interest income in turn is the sum of the expected total cash flows the bank receives from its assets (CFA_t), minus expected total cash flows it has to pay on its liabilities CFL_t.

$$NP_t = (CFA_t - CFL_t) - WR_t + OI_t - Cost_t \tag{13}$$

For simplicity we assume that other income and costs are driven by a constant exogenous process calibrated to historical data for an average UK bank. Our approach thus concentrates on simulating expected write-offs and expected net interest income, generally the main source of income for commercial banks.

The contribution of a single asset i to net interest income is simple to calculate: in the case of no default the contribution is $C^i_t A^i_{,t-i}$ while if the borrower defaults the contribution is $(1-LGD^i)C^i_t A^i_{t-1}$. Furthermore, in the period of default the bank will write off its losses: $LGD^i A^i_{t-1}$. Hence, the expected contribution to net interest income one period ahead is:

$$CFA^i_t = [(1-PD^i_t) + PD^i_t(1-LGD^i)]C^i_t A^i_{t-1}$$

and expected write-offs are $WR^i_t = PD^i_t LGD^i A^i_{t-1}$. In this section, we assume that LGDs are not time dependent.
Even if conceptually the same, deriving *CFA* for a portfolio can become computationally very demanding as all *N* assets classes have different PDs and LGDs as well as different repricing buckets *b*. In addition, we want to assess expected cash flows not only one period ahead but into the medium term *W*. Therefore, we have to make an assumption on the (re-)investment behaviour of banks after the assets mature.

To keep our base line simulation as simple as possible we assume that independent of the initial maturity or PD bucket all borrowers have continuing financing needs. Therefore, unless there is a default every borrower will roll over her loan with the same maturity bucket as before or, equivalently, the bank continues to invest into new projects with the same repricing and risk characteristics as the matured assets. This implies that the bank's portfolio composition changes only in line with defaulted assets.

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[13] This equation implicitly assumes that the bank pays dividends proportionally to its income and that the bank is paying dividends as long as it is able to do so. Furthermore, it is assumed that losses cannot be carried forward to offset future taxes. However, it is easy to incorporate different tax and dividend regimes in our simulation as we will show in our sensitivity analysis.

Given the behavioural assumption, the expected evolution of each asset class adjusting for default is:

$$A_t^i = A_{t-1}^i (1 - PD_t^i \cdot LGD^i) \qquad and \qquad A_0^i = A^i \tag{14}$$

and the total expected cash flow between *t-1* and *t* is:

$$CFA_t = \sum_{i=1}^{N} \left(\sum_{b=t}^{T} C_0^{i,b} A_{t-1}^{i,b} \left[(1 - PD_t^i) + PD_t^i (1 - LGD^i) \right] + \right. \tag{15}$$
$$\left. + \sum_{b=1}^{t-1} \sum_{l=1}^{t-1} I_l \, C_l^{i,b} A_{t-1}^{i,b} \left[(1 - PD_t^i) + PD_t^i (1 - LGD^i) \right] \right)$$

with

I_l=1 in period l when assets in bucket b have been repriced the last time prior to t
I_l=0 otherwise

The interpretation of equation (15) is relatively straightforward. The first part in the big brackets sums the expected coupon payments asset classes which have not been repriced at time *t* and the second part sums expected coupon payments of asset classes which have been repriced the last time in period *l* prior to time *t*. In order to ensure consistency with equation (5) we assume that in case of default the coupon in that period can be partially recovered. This is shown in equation (15) by means of the two LGD terms. Finally, equation 15 sums over the *N* different asset classes.

Given the evolution of expected assets, expected future write-offs are given by:

$$WR_t = \sum_{i=1}^{N} LGD^i PD_t^i A_{t-1}^i \tag{16}$$

Equation (15) and (16) highlight how profits are driven by changes in write-offs, exposures and cash flows contributions to net interest income. For example, if economic conditions deteriorate expected write-offs will increase. Such an increase will also decreases A_t^i and in turn *CFA* collected between time *t-1* and *t*, ultimately reducing NP_t. On the other hand, the bank also receives higher coupon payments from non-defaulted assets which have been repriced to reflect the increase in credit risk and risk-free interest rates.

To estimate expected net profits we also need to forecast the future cash flows that the bank needs to pay on its liabilities. As for *CFA* we have to make an assumption on the re-investment behaviour, this time whether depositors are willing to roll over their funds. In line with our previous reasoning we assume that depositors are willing to roll over their deposits unless the bank defaults on its obligations. Given we assume no strategic default this is only the case if either the earnings or the economic value condition is not met. Similar to equation (15), *CFL* evolves in line with

$$CFL_t = \sum_{j=1}^{M} \left(\sum_{b=t}^{T} C_0^{j,b} L_{t-1}^{j,b} + \sum_{b=1}^{t-1} \sum_{l=1}^{t-1} I_l C_l^{j,b} L_{t-1}^{j,b} \right) \tag{17}$$

with:

I_l=1 in period l when liabilities in bucket b have been repriced the last time prior to t
I_l=0 otherwise

As for assets, equation (17) sums over all liability classes with the first part in brackets summing the coupon payments of liability classes which have not been repriced at time *t* and the second part summing coupon payments of liability classes which have been repriced the last time in period *l* prior to time *t*.

Taking equations (14) to (17) allows us to forecast net profits and hence the evolution of shareholder funds. Rewriting Equation (12) the change in shareholder funds is given by:

$$\begin{aligned}\Delta SF_t = \theta \cdot \max\left[0; (CFA_t - CFL_t - WR_t + OI - Cost)\right] \\ + \min\left[0; (CFA_t - CFL_t - WR_t + OI - Cost)\right]\end{aligned} \tag{18}$$

Whereas shareholder funds change in line with write-offs and income, assets will only vary in line with write-offs (as shown in equation (14). To balance the balance sheet we assume that the bank will pay back deposits with the free cash flows and in cases where shareholder funds decrease by more than write-offs we assume that the bank is able to attract new deposits. Therefore,

$$\Delta FVL_t = \Delta L_t = \Delta \overline{A}_t - \Delta SF_t = -WR - \Delta SF_t \tag{19}$$

Our behavioural assumptions are to a certain degree arbitrary. But we restrict ourselves to specific investment rules rather than looking at a bank which re-optimises its portfolio in a mean-variance sense in each period as this would be beyond the scope of this paper.

3 Stress testing credit and interest rate risk for a stylised bank

The theoretical framework outlined above is flexible enough to accommodate standard credit and interest rate risk models as long as different building blocks are mutually consistent. It is essential that underlying correlations are captured - between PDs of different asset classes but also between PDs and the risk-free yield curve. A portfolio model based on Jarrow and Turnbull (2000) would for example fit these requirements. In this paper we use the stress testing model described in Bunn *et al* (2005) amended by the yield curve model of Diebold *et al* (2006), estimated on UK data by Kubelec (2006). Before turning to the results this section describes the composition of the balance sheet of the hypothetical bank and discusses the stress testing and yield curve model in more detail.

3.1 The hypothetical bank

As an example for this paper we construct a hypothetical bank with a highly stylised balance sheet with five asset classes, three liability classes, shareholder funds and interest rate swaps as off-balance sheet items (see Table A1 in the Appendix). We allocate assets, liabilities and off balance-sheet items into five repricing buckets and we refer to the repricing mismatch between them as interest rate sensitivity gaps[14].

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[14] For off-balance sheet items we assume no counterparty risk and therefore model them as risk-free instruments.

Although our balance sheet is a hypothetical construct we ensure that the interest rate sensitivity gap, shareholder funds, profitability (in terms of return on equity and on assets), operating costs and the interest rate sensitivity gap roughly matches a UK bank.

As shown in Table A1 the repricing buckets are not of equal size with respect to the length of time. We assume that the frequency of time is quarterly and that the exposure of the bank to an asset in a particular repricing bucket is equally split between the number of quarters within the bucket. The upper limit of the last repricing bucket is five years and above. To enable us to allocate assets to quarterly buckets we assume that the maximum time-to-repricing is ten years and thus divide the last time-bucket into 20 quarters between the 5th and 10th year to repricing.

3.2 The risk-free term structure of interest rates

We look at UK spot interest rates with maturities from 3 months to 10 years extracted from the Bank of England yield curve data set[15]. The yield curve data are estimated by fitting a spline through general collateral REPO rates and conventional government bonds.

We use the term-structure model by Diebold *et al* (2006) with three latent factors and three observable macroeconomic variables. In vector form, the state-space system of the vector of latent and observable variables, f_t, is given by the vector autoregression of order 1:

$$f_t - \mu = \Phi(f_t - \mu) + \eta_t \tag{20}$$

The three latent factors have the usual interpretation as the level, slope and curvature of the yield curve. The vector of yields, y_t, with different maturities is related to the latent and observable macro factors by:

$$y_t = \Gamma f_t + \varepsilon_t, \tag{21}$$

where Γ contains one free parameter and the yields are assumed only to be affected by the three latent factors. Appropriate zero restrictions are thus imposed on Γ. The transition and measurement disturbances are assumed orthogonal to one another with:

$$\begin{pmatrix} \eta_t \\ \varepsilon_t \end{pmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Delta & 0 \\ 0 & \Pi \end{bmatrix} \right) \tag{22}$$

whereas Δ is not constrained, Π is diagonal and hence the innovations across yields are assumed to be independent. The three observable macroeconomic variables are the output-gap, inflation and the Bank of England base rate[16]. The model is estimated on UK data by Kubelec (2006) using monthly data between 1986 and 2005.

The estimated term-structure model enables us to forecast the risk-free yield curves across maturities up to ten years conditional on a given macro scenario. LIBOR is then forecasted by assuming a constant spread over the risk-free term structure of 30 basis points.

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[15] See Anderson and Sleath (1999). Data are available from www.bankofengland.co.uk.

3.3 Modelling PDs and LGDs for different asset classes

The previous discussion highlights that macro factors are important drivers of the yield curve. It has also long been understood that macroeconomic factors are important drivers of credit risk (see Duffie and Singleton, 2003). In contrast to most credit risk models our adopted approach has the benefit that it explicitly models the correlation between the systematic risk drivers of credit risk and interest rate risk as macroeconomic factors. This allows us to undertake a scenario analysis and simulate the economic value as well as profits for normal and highly adverse economic conditions.

To capture the interaction between macroeconomic shocks and credit risk we build on a stress testing model described in Bunn *et al* (2005). The models for corporate and household sector PDs were originally developed by Benito *et al* (2001) but extended work has been undertaken by Whitley and Windram (2003), Bunn and Young (2004) and Whitley *et al* (2004). They use simple regression techniques to link the probability of default to macro economic variables. Hence, for each of the different asset class, the expected probability of default is linked to macroeconomic variables by:

$$E_t(PD^i_{t-1,1}) = \Psi(X_t, \hat{\beta}) \tag{23}$$

The function $\Psi(\cdot)$ indicates that the expected probability of default may be non-linearly related to a vector of explanatory variables, X_t, and a vector of estimated coefficients, $\hat{\beta}$.

The expected corporate probability of default is modelled as a function of own lagged values, changes in the logarithm of GDP, corporate income gearing, the change in commercial property capital values, first difference of the real interest rate and the ratio of net debt of PNFCs to nominal GDP.

Similarly the expected probability of default on mortgage loans is modelled as a function of mortgage income gearing, unemployment, undrawn housing equity and LTV is the loan to value ratio of first time buyers. Finally the expected probability of default on credit card loans is modelled as a function of household income gearing and the number of active credit balances. Seasonal dummies are also found significant in explaining the probability of default on credit card loans.

For all types of household and corporate lending, income gearing – a measure of the ease with which households and firms can cover debt-servicing obligations – is found to be an important driver of the probability of default. Income gearing in turn is highly sensitive to changes in interest rates. This implies that the interest rate will not only determine the net interest income but is one of the key drivers of default risk. GDP and unemployment are additional significant explanatory variables. The probability of default on corporate and mortgage loans is also found to be affected by the prices of commercial and residential property respectively.

In our main simulation we assume that the LGD is fixed and not changing from the baseline to the stress scenario. But we will assess the impact of changing LGDs in our sensitivity analysis. Slightly worse than average industry numbers suggest, we assume that the LGD on inter-bank loans is 40%, the LGD on mortgage loans to be 30%, the LGD on credit cards to be 80% and the LGD on corporate loans to be 60%.

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[16] This type of model does not impose the no-arbitrage condition across yields with different maturities (see, for example Ang *et al*, 2006, or Lildholdt *et al*, 2006). But the model should still be giving a reasonably good forecast of the term structure based on a given macro scenario.

3.4 Modelling liabilities

As discussed in the framework section it is well known (Maes, 2005) that shorter-term customer deposit rates are generally below the risk-free interest rate. This may be a result of deposit insurance schemes, operational costs and barriers to entry limiting competition. While an economic rationalisation of negative spreads can be found for short maturities it is not convincing for medium to long maturities. We assume that as the time-to-repricing increases the interest paid by the bank on deposits gradually converges to the risk-free interest rate. We model the deposit rate on household deposits with one quarter to repricing to be 2% below the REPO rate and the corporate deposit rate to be 1% below the LIBOR rate. The negative spreads are then assumed to decline linearly to be zero in the fourth quarter.

3.5 Forecasting systematic risk factors

To be able to forecasts PDs and yield curves we need a model that forecasts and captures the correlation of systematic risk factors between each other and across time. Rather than using a simple macro VAR model which would fit this requirement we use the Bank of England's macro model. This allows us to use the Bank of England Inflation Report forecasts as baseline scenario.

As discussed above it is necessary to consider the stability of the bank in the short and medium as well as the long term. We choose the medium term to be three years. For a given macro scenario we forecast the dynamics of the macro economy and map theses into PD forecasts over the next three years using the models discussed in Section 3.3[17].

4 Results

In the this section we measure the impact in the baseline and stress scenario of credit risk, interest rate risk and their interaction on the economic value and the profitability of our representative bank over a three year horizon.

4.1 The scenarios

We follow Bunn *et al* (2005) and look at the combination of three shocks originally used for the IMF 'Financial Stability Assessment Programme' (FSAP) in 2002: a 12% decline in UK residential and commercial property prices, a 1.5% unanticipated increase in UK average earnings growth and a 15% unanticipated depreciation in the trade weighted sterling exchange rate. Individual scenarios are described in Appendix A1. Originally, the magnitudes of the individual shocks have been calibrated to extreme events relative to their historical distribution, and combining them begs the question about the likelihood of such an event occurring. One could use the historic covariance matrix to determine the probability of a combination of these shocks happening. But Bunn *et al* (2005) do not

[17] After the third year we assume that the probability of default of each asset class reverts back to its long run level over the following ten years. The quarterly probability of default on corporate loans thus reverts to 0.35%, on mortgage loans to 0.70% and on credit cards to 0.61%. These assumptions are not going to be strongly decisive for the results presented in the next section. Results of this sensitivity test can be provided by the authors on request.

attempt this and approach it as a purely mechanical exercise. Unsurprisingly, the combination of shocks has a greater impact than each of the individual shocks alone.

All our scenarios are run from 2005 Q1 and forecasted over a three year horizon. As base case scenario we use the Bank of England February 2005 Inflation Report's projections where interest rates are assumed to follow market expectations. When running the combination of shocks through the macro model, we do not apply any judgements and we simply apply the shocks mechanically. As will become apparent, and at the heart of this paper, the key macroeconomic variable is the interest rate. Hence, modelling the monetary policy reaction to the initial shock is crucial. In line with general macro stress testing practices we assume a mechanical Taylor rule[18].

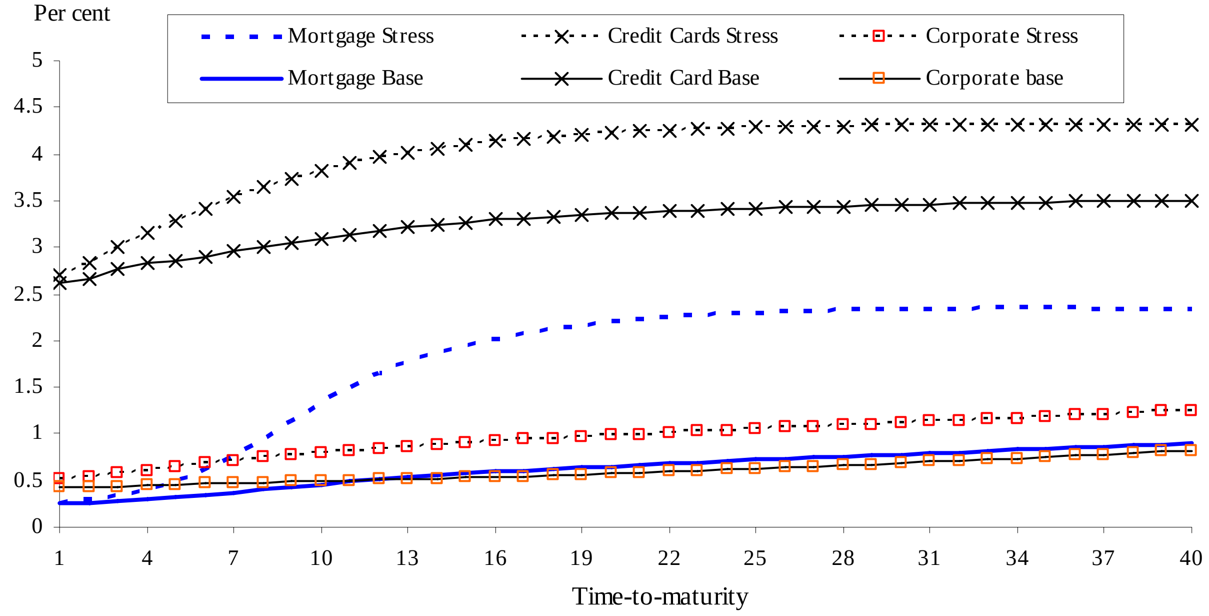
4.2 Risk-free and credit spread yield curves

In Figure A1 in the Appendix we compare the evolution of the risk-free yield curves over the next three years in the baseline and stress scenario. Whereas in both cases the risk-free yield curve is downward sloping, the increase in the level following the stress is evident across all maturities. Furthermore, the yield curve flattens in the stress scenario with the short end of the curve around 5.5% in the first quarter increasing steadily over the three years reaching almost 10% three years after the shock.

In Figure 1 we show the credit spread curves for mortgages, corporate and credit cards. The solid lines represent the spreads after one quarter in the base case and the dashed lines the spreads one quarter after the shock (indicated by 1 in Figure A1). As default rates and LGDs on credit cards are highest, spreads on credit card lending are much higher than for lending to (secured) households or corporates. In the base case spreads on mortgages are in line with average mortgage rates currently observed in the market place. Spreads on corporates compare to a BBB spread which is slightly above the average quality (BB) of the corporate portfolio of a typical G10 bank (see Catarineu-Rabell, *et al*, 2003).

[18] Under a Taylor rule, interest rates are modelled as a linear combination of deviations of inflation from a target rate and output from potential output. This treatment is, of course, not representative of the way in which the Monetary Policy Committee sets interest rates. As has been described by the Bank of England elsewhere, Committee members use a range of models and judgements in forming their assessments.

Figure 1: Annualised credit spread curves before end after the stress



The largest increase in spreads in the stress scenario occurs for mortgages. Although the spread on credit cards does not rise by as much, it remains higher than that for mortgages. The corporate spread is least affected by the macroeconomic shock. The main reason for the subdued rise in the corporate spread is consistent with the relatively high credit quality of the banks' corporate lending book.

4.3 Condition 1: the economic value perspective

As postulated in the framework section we adopt an economic value perspective to measure the potential long-term impact of the shock on the bank. The economic value of our hypothetical bank in the baseline scenario is calibrated to 7.3% of the face value of assets. This equals the book value of assets net of liabilities and net off-balance sheet items. Immediately after the shock crystallises the economic value falls to 5.7% of the face value of assets. Even though this represents a 21% fall in economic value, the long-term combined impact of credit and interest-rate risk is not large enough to threaten the stability of the hypothetical bank.

4.4 Condition 2: the earnings perspective

Given the severe nature of the shocks it may still be the case that in the short or medium term the bank makes losses which could threaten its capital. For this reasons it is important to investigate whether Condition 2 is satisfied, that is whether the bank's expected capital adequacy remains above the regulatory minimum in all periods in the medium term, i.e. the next three years.

As described in the framework section Condition 2 depends, *inter alia*, on the evolution of net profits, shareholder funds and risk weighted assets. In turn the key two determinants of net profits are net interest income and write-offs. In line with Bunn *et al* (2005) write-offs are significantly higher in the stress scenario and peak after around 2 ½ years (dotted lines in Figure 2). This increase in credit risk is also reflected in the increasing credit spreads in Figure 1.

Initially, net interest income falls slightly due to a rise in borrowers defaulting as well as to the margin compression between short term borrowing and long term lending rates (starred lines in Figure 2). However, after 1 ½ years, net interest income starts to increase up to the 12th quarter when it fully offsets the higher write-offs and interest rates. The increase in net interest income follows the gradual repricing of assets reflecting the higher credit risk in the stress scenario[19].

Figure 2: Evolution of quarterly net profits, net interest income and write-offs

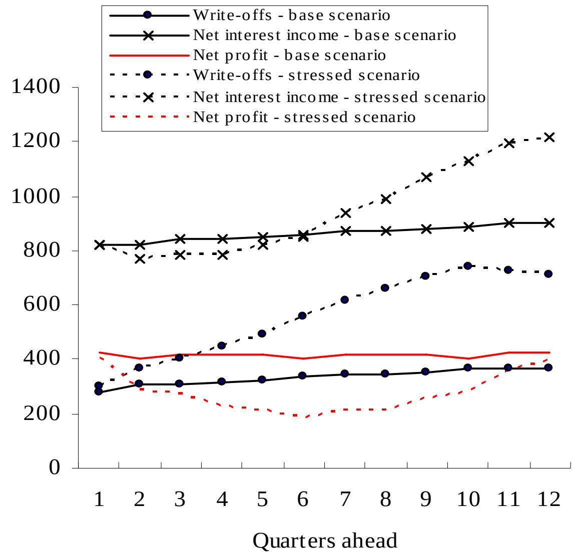
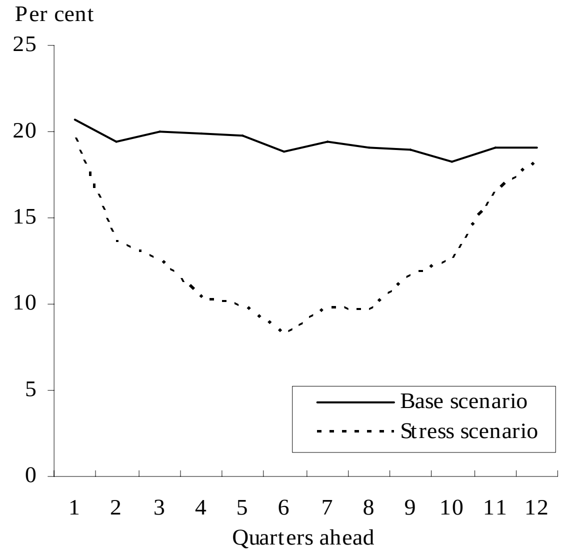


Figure 3: Evolution of annualised return on equity per quarter (RoE)



The combined impact of write-offs and net interest income imply that net profits (solid line in Figure 2) fall by about 50% in the sixth quarter but recover to the base line after 3 years. As we will discuss in more detail in the next section Figure 2 already indicates that interest rate and credit risk have to be assessed jointly. For example, focusing on pure default risk by only looking at write-offs leads to an underestimation of risk in the short term and an overestimation of risk in the long run. To capture risks fully, the initial margin compression and the subsequent repricing has to be taken into account.

The impact of the shock can also be summarized in terms of return on equity (RoE) as illustrated in Figure 3. Compared to a roughly constant RoE of 20% in the base line scenario, the shock halves the bank's RoE in the worst quarter. But it is also evident how the bank remains profitable in every quarter over the three-year horizon. As profits after tax and dividends are retained as capital, shareholder funds increase in each quarter. And given that under the standardised approach risk weights do not adjust to the decrease in credit quality, Condition 2a improves in both scenarios as shown in Figure 4, Panel A.

Conversely, under the internal approach the increase in shareholder funds is more than offset by the increase in risk weights reflecting the rise in credit risk (Figure 4, Panel B). However, the overall fall does not threaten the stability of the bank as the capital ratio always remains well above the regulatory minimum. As well as Condition 2a, Condition 2b is therefore satisfied in all periods.

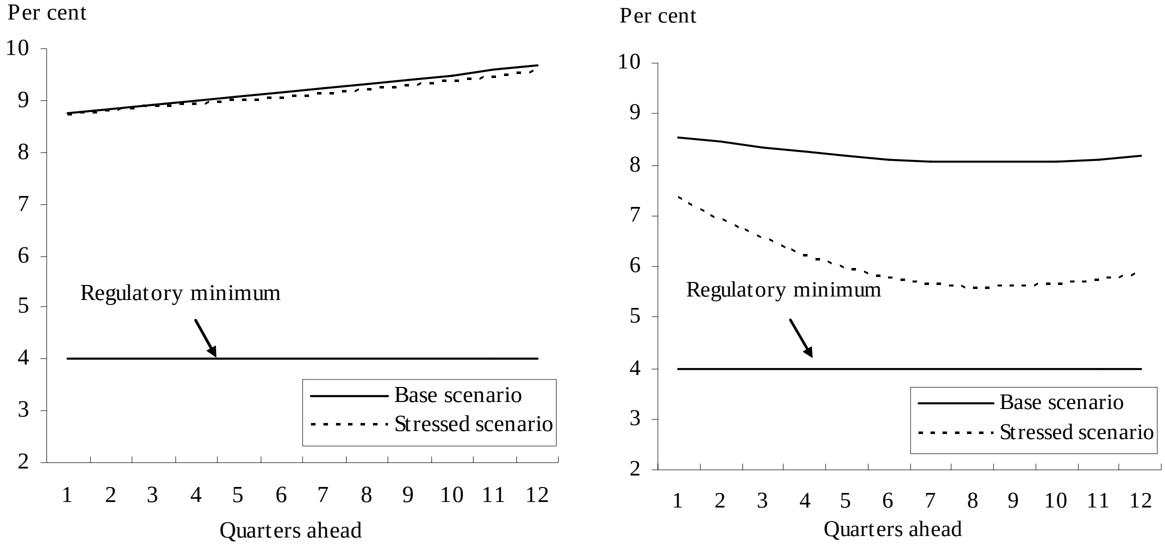
[19] Note that we are assuming that the bank can fully translate the increase in PDs into the premia it charges on borrowers, and that such a rise in premia does not affect write-offs and arrears.

Overall, we can conclude that independently of whether we look at the short or long run indicators developed in this paper, the shock would weaken our hypothetical bank but it would not threaten its stability.

Figure 4: Shareholder funds as a proportion of risk-weighted assets – Condition 2

Panel A: Condition 2a: Constant-risk-weight approach

Panel B: Condition 2b: Internal approach



5 Integration of interest and credit risk

Given interest rate and credit risk are intrinsically related, this section investigates which risk is the main driver of the fall in profits in the stress scenario. For this analysis we break down the impact into three components:

A. The impact of credit risk from non-interest rate factors.

B. The impact of interest rate risk but excluding the effect of changes in interest rates on credit risk.

C. The impact of the interaction of credit risk and interest rate risk.

To assess (A) we calculate PDs conditional on all systematic risk factors changing to their stressed levels but interest rates remaining as in the base case scenario. Hence, (A) highlights the importance of all non-direct interest rate factors. (B) is similar to interest sensitivity analysis run by banks on the whole portfolio. As discussed previously, these tests look at shifts (often only parallel ones) in the yield curve but ignore any implications this may have on credit risk. (C) is calculated as the difference between the impact of the overall shock, as described in the previous section, and the combined impact of (A) and (B).

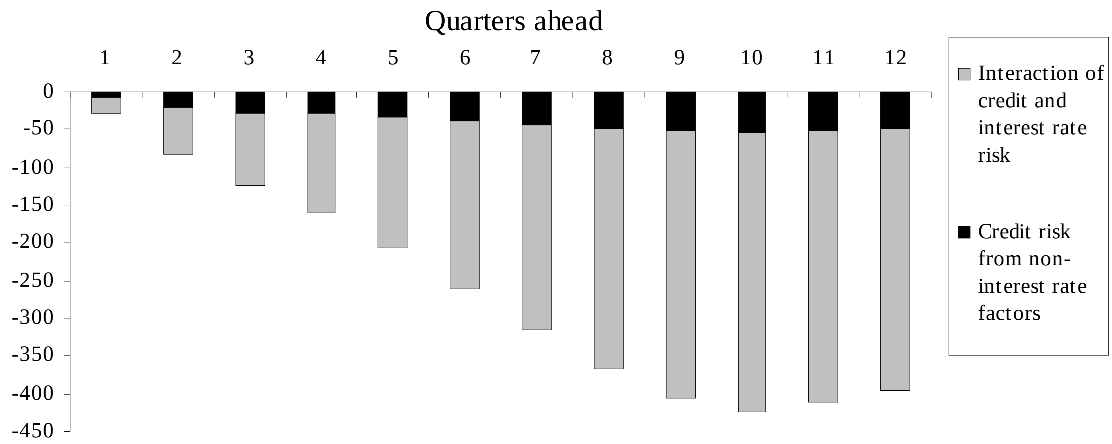
Figure 5 shows that in comparison to other macroeconomic factors interest rates are the key driver of the rise in credit risk in our scenario. Figure 6 disentangles the complex effects of interest rate and credit risk on net interest income. As gap-analysis suggests ‘pure interest rate risk’ decreases net interest income as margins are compressed. However, 'pure interest rate risk' does not take account of the impact of interest rates on credit quality nor the correlation of interest rates and other credit risk drivers in a stressed scenario. The increase in credit risk has two effects on net interest income. First, higher write-offs decrease net interest income as the bank's exposures decline over time. But second, this effect is outweighed by the positive impact of credit risk on net interest income because, over time, banks adjust the credit spread on loans that are repriced.[20]

Looking at the overall impact on profits (Figure 7) it is evident that the rise in interest rates is the main driver of the fall in net profits reflecting the combined effect of the squeeze in net margins and the rise in write-offs caused by the change in interest rates. Hence, the combined impact of correlated credit and interest-rate risk is the key determinant of the banks risk profile.

This discussion highlights why risks have to be assessed jointly. For example, a simple gap analysis is not sufficient for risk assessment as this only looks at the striped area in Figure 6 ignoring all other effects on net profits. Similarly, focusing on credit quality only, for example by projecting expected write-offs, is also misleading. Such an analysis does not account for the initial fall and subsequent increase in net interest income as shown in Figure 6.

[20] Even though net interest income falls in the first quarter due to some loans defaulting this effect is negligible. But the small impact on net interest income in this quarter is driven by our assumption that the shortest repricing maturity is 3 months across all asset and liability classes. Shorter maturities such as overnight bank deposits would only lead to a bigger decrease in net-interest income in the first quarter but would not change the remainder of the analysis.

Figure 5: Impact on write offs[a]



(a) The scale is inverted to visually enable the adding of write-offs and net interest income

Figure 6: Impact on net interest income

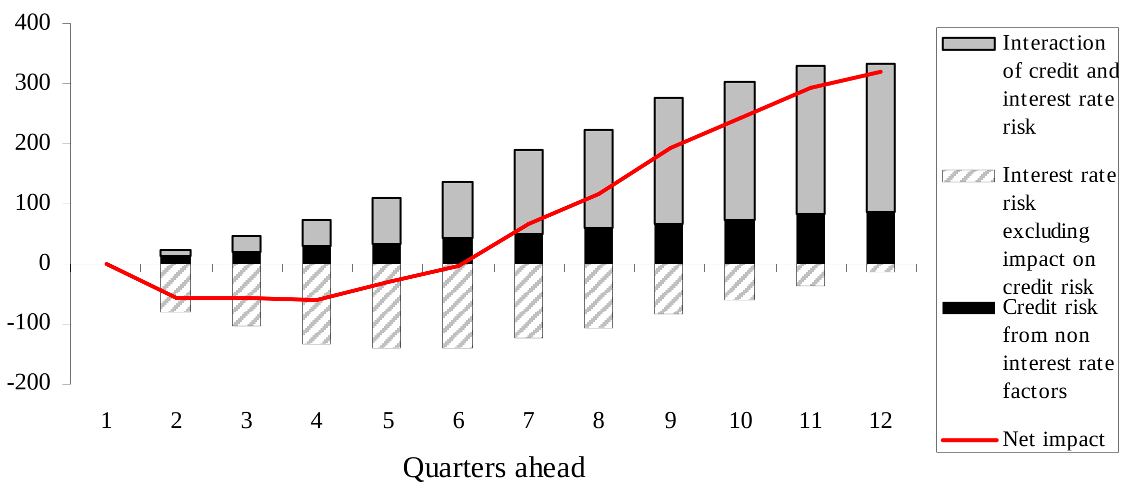
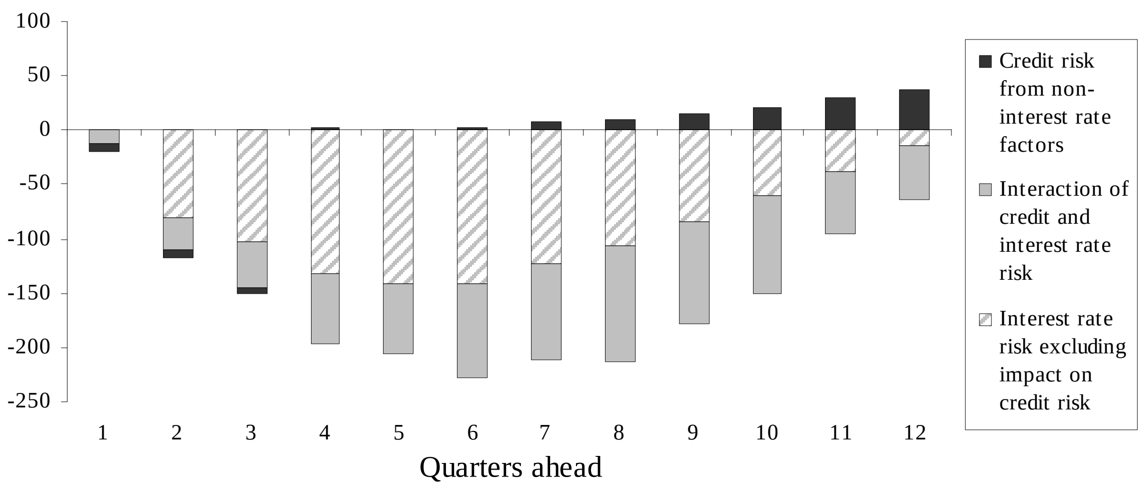


Figure 7: Impact on profits



6 Sensitivity analysis

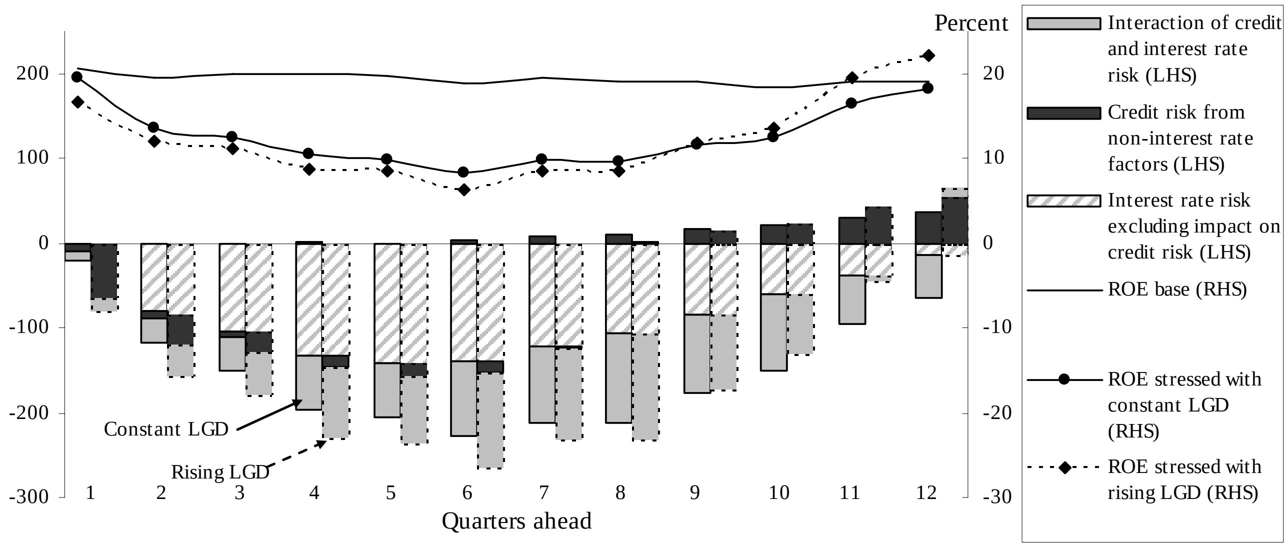
In this section we analyse the sensitivity of the results to some of our main assumptions and to changes in the characteristics of the hypothetical bank balance sheet.

6.1 Cyclical LGD

A recent book edited by Altman *et al* (2005) provides strong evidence that macroeconomic variables have an important effect on LGD and that recovery rates are low when aggregate default rates are high. For example, Schuermann (2005) finds that recovery rates are one-third lower in recessions. Frye (2003) corroborates this evidence by showing that the LGD in high default years exceeds LGD in low default years by around 15 percentage points.

Even though we start with relatively high LGDs in the main section we assume that they are fixed and not changing in the stress scenario. We test the sensitivity of our results by decreasing recovery rates by 1/3 as suggested by Schuermann[21]. As it is unrealistic that LGDs remain at the high level forever we assume that LGDs gradually revert to baseline LGDs over the following 10 years, consistent with our PD assumptions.

Figure 8: Return on equity and impact on net profits; constant LGD versus rising LGD



Interestingly, the additional impact of higher LGDs implies only a further four percentage point decline in economic value from 21% to 25%. The LGD assumption does also not have any material impact on shareholder funds as a proportion of RWA under the constant-risk-weights approach. However, under the internal rating based approach the capital ratio falls to a minimum of 4.2% in the eighth quarter versus a minimum 5.8% with constant LGD in the same quarter. This fall is mainly driven by the rise in risk weights and the fall in net profits (Figure 8). It is also worth noting that the rising LGDs lead to a substantial increase in the impact of credit risk from non-interest factors in

[21] We also used a 15 percentage point increase of LGD as suggest by Frey. This generates results which are slightly less severe than using Schuermann's assumption.

particularly in the first quarter. The rise in LGDs also augments the negative impact of the interaction between credit and interest-rate risk between the second and the eighth quarter and that of pure credit risk in the first six quarters.

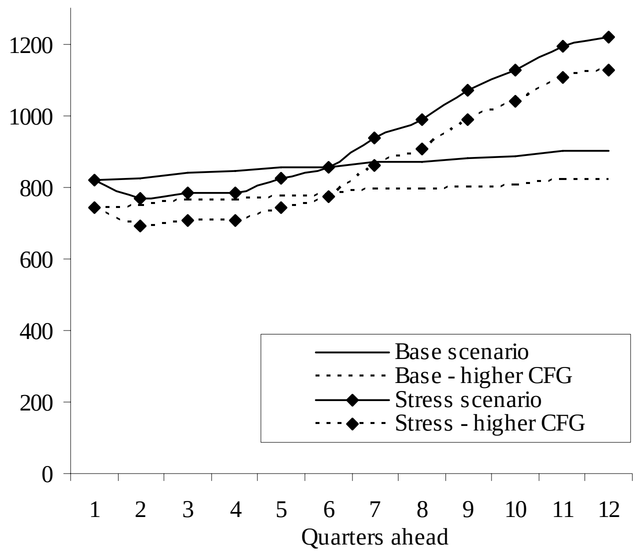
6.2 A rise in the customer funding

In recent years major banks have seen their annual growth rate in lending to non-bank customers outpace the corresponding growth in deposits from this sector at an aggregate UK level. This has created a so called customer-funding-gap (CFG), i.e. the stock of lending to customers exceeds the stock of customer deposits (see Bank of England, 2005). We simulate a rise in the CFG for our hypothetical bank by decreasing customer deposits in the first repricing bucket by 20% and increasing deposits from banks by the same amount.

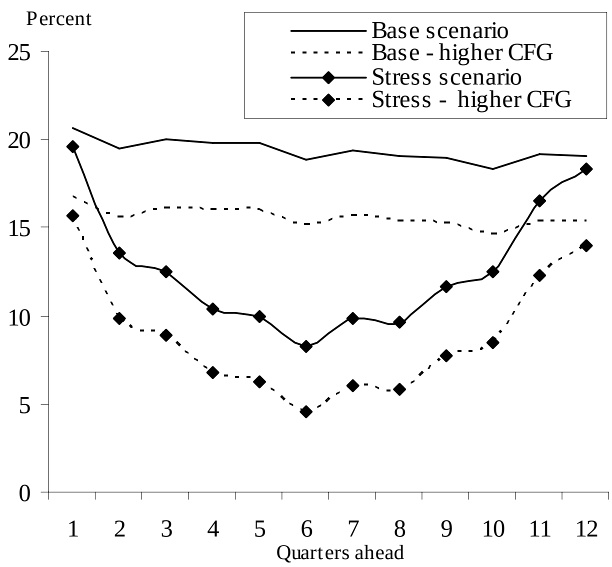
Both in the baseline and stress scenario the rise in CFG has the same negative impact (Figure 10) and is entirely driven by a decrease in net interest income. As the bank has to pay LIBOR on interbank deposits compared to risk-free interest rates with a constant negative spread to household deposits, the increase in the CFG only lowers the net interest margin without affecting the banks interest rate sensitivity gap or its exposure to credit risk.

Figure 10: Net interest income and return on equity with rise in customer-funding-gap

Panel A: Net interest income



Panel B: Return on equity



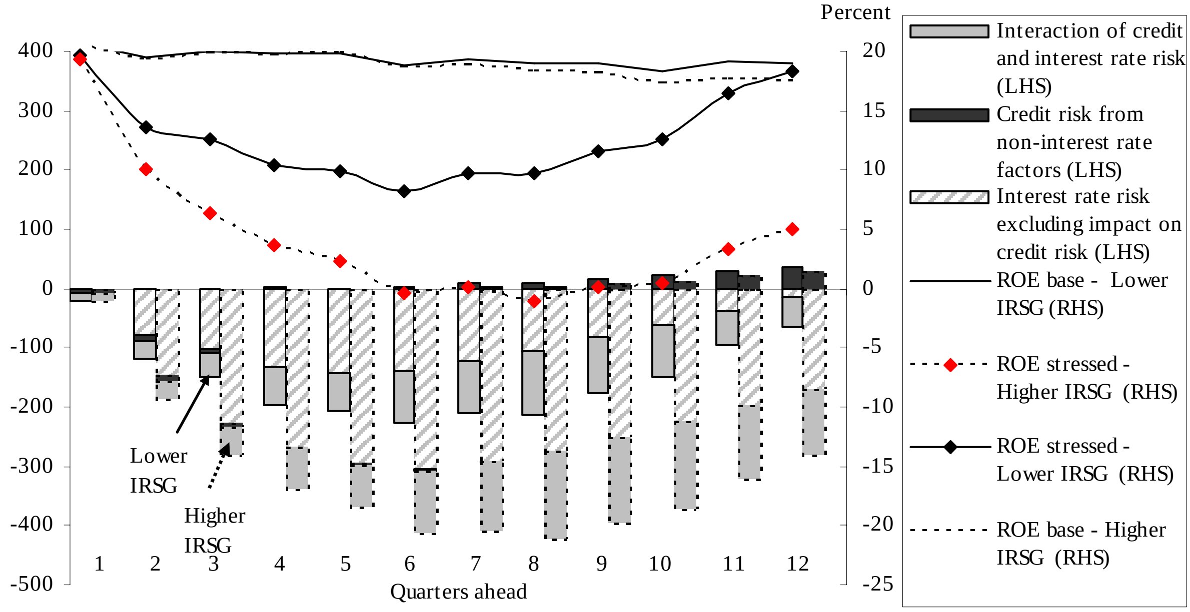
6.3 High interest rate sensitivity gap

While in the UK the majority of mortgages carries variable rate, this is not the case in many other countries where banks have a higher proportion of fixed-interest rate mortgages. A bank with a relatively higher proportion of variable mortgage loans can reprice the mortgage loans more quickly in response to increased credit risk and changes in the risk-free interest rate. Therefore, a bank holding a relatively higher proportion of variable rate mortgages should, ceteris paribus, be less affected by changes in risk-free interest rates than a bank with more fixed mortgages.

To illustrate this point we arbitrarily increase the proportion of fixed mortgages in our hypothetical bank from 24% to 65% and keep the amount of its off-balance sheet items constant. We assume that the proportion of fixed rate mortgages in the repricing bucket from 6-12 months is 20%, from 1-5 years 35% and above 5 years 45%. As a result this bank has a much larger negative interest-rate sensitivity gap (IRSG) in the first repricing bucket.

This is somehow a counterfactual analysis as it is likely that a bank with predominantly fixed rate mortgages would offset the higher interest rate risk by increasing the use of off-balance sheet items. But we undertake this simulation as an example to highlight the importance of the initial interest rate sensitivity gap.

Figure 11: Return on equity and impact on net profits with an higher interest rate sensitivity gap (IRSG)



In Figure 11 we compare the results from our original bank with the bank with the higher IRSG. The figure clearly illustrates the difference between a high and low IRSG as net profits of the high IRSG bank decrease much further. This reflects the substantial fall in net interest income as the high IRSG bank is not able to reprice the mortgage loans to reflect the shift in the risk-free term structure and the increasing credit risk. The lower profitability of the bank with the higher IRSG is also evident when looking at RoE. It falls dramatically and even becomes negative in the sixth and eighth quarter following the stress. Subsequently it recovers gradually as mortgages in the longer repricing buckets are repriced. But given that a large proportion of mortgages is allocated to the greater than five year repricing bucket, RoE is still well below its level in the base case at the end of the third year.

Because of the negative RoE, it is important to ascertain that both Condition 1 and Condition 2 are still satisfied. Although the fall in the banks' economic value increases to 31%, it remains well above zero.[22] Moving to Condition 2, under the internal approach the ratio of shareholder funds to risk

[22] The fall in economic value in the main section was 21 %.

weighted assets falls to a minimum of 5.44% in the 9th quarter which is still well above the regulatory minimum. Hence, the financial stability of the bank is not seriously threatened even though our hypothetical bank has several quarters with a negative RoE.

6.4 Constant spread on variable rate mortgage loans

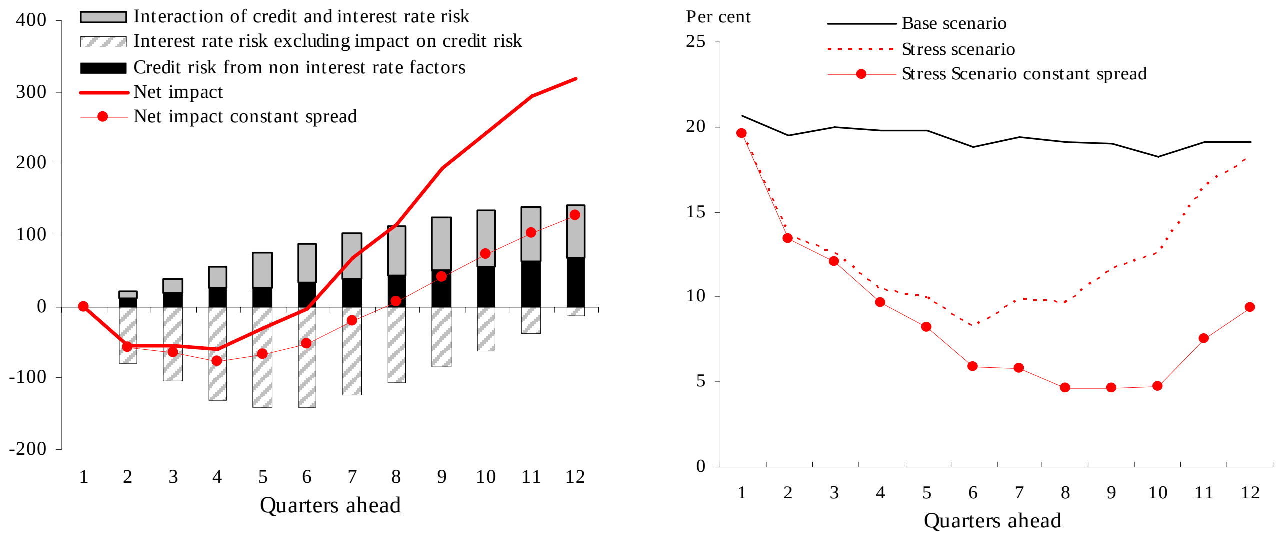
In the main section we assumed that the maturity of assets and liabilities coincide with the time to repricing. This implies that every contract can be rewritten every time a loan is repriced. Hence the bank can change the lending rate on variable rate mortgages reflecting both the changes in the risk-free interest rate and the change in credit risk. If, more realistically, the time-to-maturity is longer than the time-to repricing it is not clear whether banks can modify the credit spread on variable rate mortgages whenever the loans are repriced.

To analyze how sensitive our results are to such an assumption we consider the opposite case. We assume that the bank must hold a constant spread on the variable rate mortgages for at least the first three years (this is equivalent to assume that no variable mortgage matures within three years). In Figures 12 and 13 we illustrate how this impacts our results.

Figure 12: Return on equity and net interest income with constant credit spread on variable mortgages

Panel A: Net interest income

Panel B: Return on equity



First, from Panel A it is clear that, in the stress scenario, the offsetting impact of pure and integrated credit risk via the rise in credit spreads (dotted line) is much smaller than that in the main section (continuous line). Hence, net interest income is substantially lower as a result of the bank not being able to price the additional credit risk in the loan.

The consequence is that following the stress the RoE falls more sharply and for a longer time as shown in Panel B.

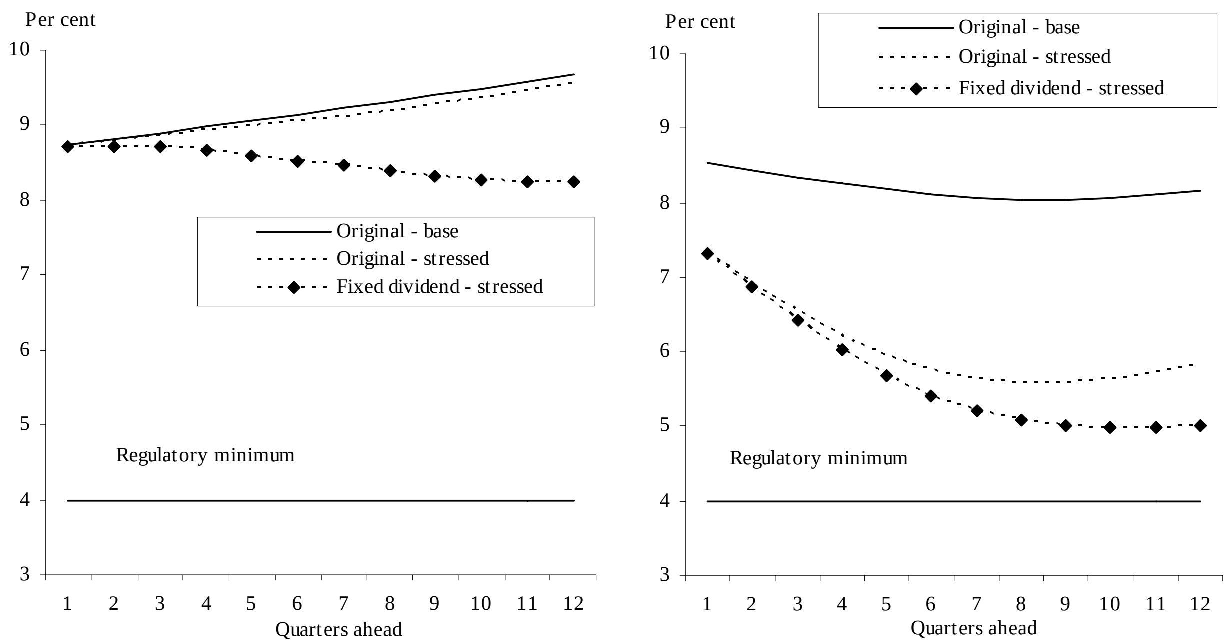
6.5 *Different levels of payout by the bank*

So far we assumed that the variable θ, determined by the tax rate and the pay-out ratio, does not change over time. While it may be reasonable to assume that the tax rate remains constant, a constant pay-out ratio is less convincing as this would imply that dividends are perfectly flexible. Indeed, stickiness in dividends is both empirically documented (Allen *et al*, 2000 or Benito *et al*, 2002) and theoretically explained by the asymmetric response of equity prices to bad and good news (Glosten *et al*, 1993, Nelson, 1991, or Bekaeart and Wu, 2000) and information-signalling (Howe *et al*, 1992, DeAngelo, 2000).

Figure 13 shows the impact on capital adequacy if the bank keeps its dividends at a constant level. In comparison to fully flexible dividends, the bank's capital ratio falls under both the constant-risk-weight and internal approach. This is driven by a pay-out ratio which is greater than one in most quarters. Hence, the bank has to use some of its capital to pay dividends. However, capital adequacy still remains above the regulatory minimum in all periods. Furthermore, if we constrain the pay-out ratio not to be above one, the fall in the bank's capital ratio in Panel B is much less evident.

Figure 13: Shareholder funds as a proportion of risk-weighted assets under a fixed dividend policy

Panel A: Constant-risk-weight approach   Panel B: Internal approach



7 Conclusion

Credit and interest rate risk are the two most important risks faced by commercial banks. And given that they are correlated, they cannot be measured separately. Surprisingly, most studies focus on the correlation between interest rate risk and default risk of assets. But a bank's profitability and net worth depend not only on the default risk but also on the overall credit quality of its assets as well as its liabilities and off balance sheet items. This paper proposes a general framework to compute a bank's economic value as well as its future profitability and capital adequacy over time by assessing the impact of correlated credit and interest rate risk on risk-adjusted discount rates and cash flows contributions to earnings. The essence of our framework is therefore relatively simple but at the same time holistic. Given its generality, we think it represents an important step forward from the current literature.

In the second part of our paper we apply our framework to evaluate the impact of a severe stress on the economic value and capital adequacy of a hypothetical bank. To capture the correlated impact of credit and interest rates we employ a simple stress testing model linking macroeconomic factors to the risk-free yield curve and PDs of companies and households to common macroeconomic factors.

Although the stability of our hypothetical bank is not threatened in the stress scenario, we find that it is fundamental to assess the impact of interest rate and credit risk jointly on assets, liabilities and off-balance sheet items. We show that a simple gap analysis will underestimate the risks to banks. Even though it captures the initial repricing miss-match it will not account for the strong negative impact of interest rates on the credit quality of assets. But purely focusing on default risk by for example projecting expected write-offs is also misleading. Such an analysis does not account for the increase in net interest margins in the medium run which reflect the increase in interest rates and the deterioration in credit quality once banks can reprice their assets.

The qualitative results of our paper are stable across a whole range of sensitivity test. However, we show that the cyclicality of LGDs and the ability of a bank to raise credit spreads can matter. Similarly, the composition of a bank's balance sheet in terms of riskiness and repricing mismatch is crucial for the overall riskiness of the bank.

Obviously the implementation of the framework relies on a particular term structure and credit risk model. These are key inputs to measure the riskiness of a bank. And we stress that it is important to use consistent models, which capture the correlation between yield curves and credit risk by explicitly modelling the underlying systematic risk drivers. We think that this is an area where it may be interesting to undertake more work. First, it could be useful to look at more disaggregated and sophisticated credit risk models such as for example the models of UK corporate PDs proposed by Drehmann *et al* (2006) and the UK household PDs model proposed by May and Tudela (2005). Second, it would be interesting to explore the sensitivity of LGD to systematic risk factors in greater depth. Even though the literature is expanding in this area, data limitations on recovery rates, especially for UK bank loans, could be a potential obstacle. Finally, in this paper we measure expected losses only, whereas it would be useful to generate the full loss distribution during periods of stress. Although we expect that these extensions will refine the exact estimation on the importance of credit versus interest rate risk, we think that they will not alter the main message of this paper: for a complete risk assessment it is fundamental to measure the impact of correlated interest and credit risk jointly and on the whole portfolio of banks.

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Appendix

A1: Stress scenarios

A) Decline of 12% in residential and commercial property prices. This scenario is assumed to result from a general drop in demand for the flow of property services. Since housing accounts for one half of households' net worth, the personal sector's balance sheet deteriorates and household consumption is reduced. Output is lower than otherwise but the adverse effect is a little smaller than under the first scenario.

B) 1.5 percentage point unanticipated increase in average earnings growth (reflecting a step increase in real reservation wages). This supply shock boosts personal incomes and consumption but the transmission to higher inflationary pressure induces a rise in official interest rates under the Taylor rule. Overall there is a marginal decline in GDP compared with the base case.

C) A 15% (initial) unanticipated depreciation in the trade-weighted sterling exchange rate. This scenario entails a fall in the demand for sterling owing to an increase in the perceived relative riskiness of sterling assets (in other words, a rise in the sterling risk premium). Sterling depreciation results in higher inflation and, in response, nominal interest rates increase under the Taylor rule. Nonetheless, since wages and prices adjust only gradually, there is a temporary depreciation in the real exchange rate which in turn boosts net export volumes.

In addition, the IMF FSAP also considered a shock the world equity prices due to a downward revision in corporate earnings, which we do not include in our exercise as the Taylor rule implies a monetary policy reaction offsetting some of the consequences of the initial shock.

A2: Additional tables and graphs

Table A1: balance sheet of the hypothetical bank

	Time buckets					Non-interest bearing funds	Total
	0 - 3 months	3 - 6 months	6 - 12 months	1 - 5 years	> 5 years		
Assets							
Total loans and advances to banks	5,500	1,900	500	100	100	0	8,100
Total loans and advances to customers	86,900	12,200	5,000	17,800	12,300	0	134,200
Total Households	44,700	10,600	2,400	10,600	5,600	0	73,900
Mortgage	24,600	9,800	1,200	7,200	2,400	0	45,200
Fixed Rate Mortgages	0	0	1,200	7,200	2,400	0	10,800
Variable Rate Mortgages	24,600	9,800	0	0	0	0	34,400
Credit Cards+Credit Cards	20,100	800	1,200	3,400	3,200	0	28,700
Total PNFCs/NPISH	42,200	1,600	2,600	7,200	6,700	0	60,300
Treasury bills and other debt securities	6,700	2,300	2,100	3,400	3,200	0	17,700
Total assets	99,100	16,400	7,600	21,300	15,600	0	160,000
Liabilities							
Total deposits by banks	32,300	1,600	600	100	300	0	34,900
Total deposits to customer accounts	98,000	3,400	4,300	4,300	300	6,000	116,300
Total Households	49,000	1,700	2,150	2,150	150	3,000	
Total PNFCs/NPISH	49,000	1,700	2,150	2,150	150	3,000	
Shareholders funds - equity						8,800	8,800
Total liabilities (excl shareholder funds)	130,300	5,000	4,900	4,400	600	6,000	151,200
Total liabilities	130,300	5,000	4,900	4,400	600	14,800	160,000
Off-balance sheet items	13,600	-9,800	-1,100	-2,500	2,600		2,800
Interest rate sensitivity gap	-17,600	1,600	1,600	14,400	17,600		

Figure A1: Evolution of the risk-free term structure over the next 12 quarters in the base and stress scenario respectively

Panel A: baseline

Panel B: stress