A Market Based Macro Stress Test for the Corporate Credit Exposures of UK Banks

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Abstract

This paper presents a stress test for corporate exposures of UK banks. The default process is modelled via a Merton model and several macroeconomic as well market factors are identified as systematic risk factors. We then simulate the expected loss distribution for UK banks conditional on drawings of macroeconomic risk factors. The overall conclusion of our simulation is quite reassuring as even in the worst macroeconomic conditions expected losses of banks corporate exposures are not high enough to cause a bank failure. A key finding of our work is that systematic factors have a non-linear and non-symmetric impact on credit risk and that these effects are most important for highly adverse scenarios which are the main interest from a stress testing perspective. We also argue that this model can be a step towards an integrated approach of stress testing market and credit risk.

The views and analysis expressed in this paper are those of the author and do not necessarily reflect those of the Bank of England or the Monetary Policy Committee members. Mark Manning provided invaluable input into this paper as a building block of this stress test is based on work which I undertook jointly with him. I am also very grateful to Andrew Patton who provided input in the set-up of the simulation and some thought provoking comments. I also would like to thank Alastair Cunningham, Glenn Hoggarth, Ibrahim Stevens, Merxe Tudela, Nicholas Vause and Garry Young for helpful comments and suggestions.
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1. Introduction

In recent years stress tests have become a well established risk management tool. They are frequently used by banks to assess the impact of severe but plausible events on their exposures. For market risk, stress tests are generally undertaken by banks and are used to complement VaR measures (see BIS 2005). However, quantitative stress tests for credit risk are not yet as far developed even though a lot of banks undertake stress tests on a mostly qualitative basis extensively. In the future, this is likely to change as stress tests have to be undertaken for banks to be eligible for the internal ratings approach under Basel II.

In the last few years stress tests also gained increased prominence as a tool to assess the financial stability of banking systems. To date, more than 90 ‘stress tests’ have been completed/are on the way to being completed as part of IMF’s Financial Stability Assessment Programmes, so called FSAPs.

Sorge (2004) provides an excellent overview of the current state of literature of stress tests for financial systems. Simple models are often based on time series or panel-analysis which link write-offs or provisions to macroeconomic factors. These reduced form equations are then used to assess how severe macro scenarios impact on provisions or write-offs of banks. Pain (2004) constructs such a model for the UK and shows that in particular real GDP growth, real interest rates and lagged aggregate lending growth have a strong impact on banks’ provisioning.

Another class of models which is extensively used is based on the idea of CreditPortfolioView (see Wilson, 1997a and1997b). Here, the default process it modelled as a probit process which relates macroeconomic factors to the probability of default of companies. In this spirit Boss (2002) develops a stress testing model for the aggregate Austrian banking sector, whereas Virolainen (2004) applies such a model to the Finnish banking system.

So far, few structural models for stress testing have been developed. Such a model is at the core of the Bank of England’s stress testing agenda. Hoggarth and Whitley (2003) describe an earlier version of this model, which feeds shocks to the macroeconomy through the Bank’s structural macroeconomic model, a structural satellite model linking macroeconomic variables to arrears and liquidation rates and finally a reduced form model assessing the impact of liquidations rates and arrears on banks' write-offs. A later version includes a reduced form relation between profits and shocks to the macroeconomic environment. DeBandt and Oung (2004) describe such a model for France. These models are very useful from a central bank's perspective as they are tractable and conform to the way central bankers are used to communicate. Hence, they provide an ideal framework to discuss risks. In general, these discussions form an important part of the actual assessment how a severe but plausible scenario would impact on banks as residual adjustments have to be undertaken in any model to accommodate possible structural breaks and/or poorly estimated equations.

But, structural models also have limitations. They are by design restricted as equations are generally estimated in log-linear form. Therefore, the impact of shocks will be linear and symmetric. However, credit risk[1] is inherently non-linear - a company is either in default or not. Furthermore, some defaults will always occur because of idiosyncratic risk factors even in the best macro conditions. Hence, the up-side of a very benign macro environment might not be as large as the downside of very severe shocks. This might imply a non-symmetric distribution.

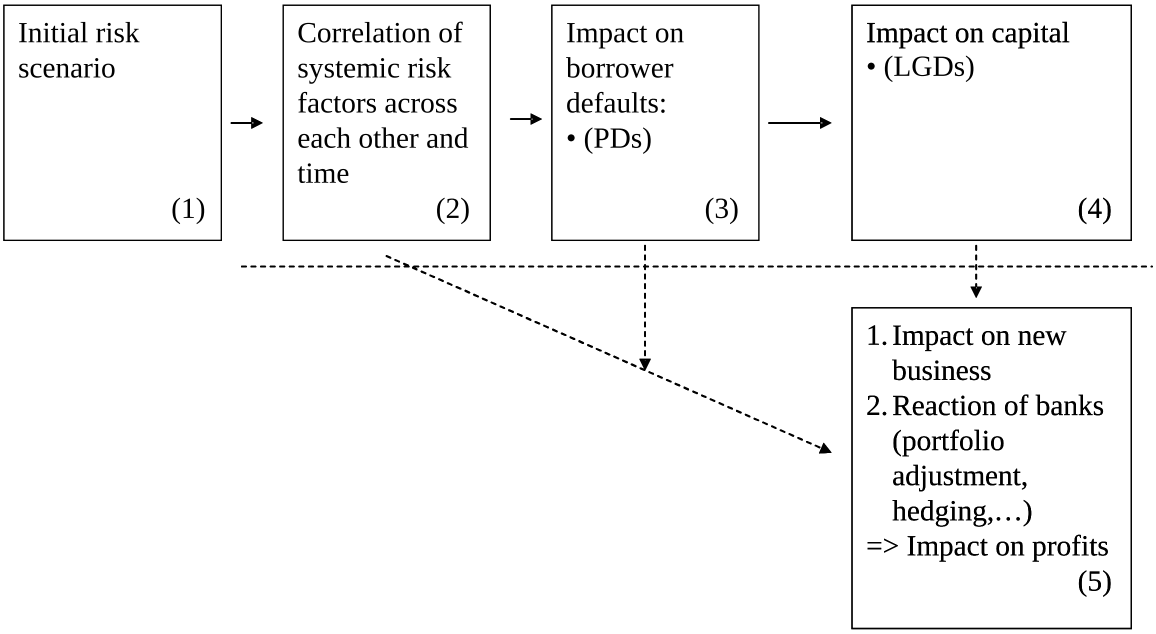
By explicitly modelling the non-linearity of the default process via the Merton model, this paper highlights that the impact of shocks on expected losses of banks is neither linear nor symmetric. Hence, ignoring this might lead to an underestimation of the impact of a severe, but plausible risk scenario on the financial stability of a country.

Figure 1 provides a schematic overview of the general structure of stress tests. First, a meaningful and interesting initial shock to some specific risk factors has to be selected and second, it has to be understood how changes in these risk factors interact/correlate with other systematic risk factors and across time. Third, it has then to be assessed how the overall scenario – ie the initial risk factor change and all systematic risk factor changes following from this – affects PDs of borrowers, as well as fourth their LGD which gives the impact of the scenario on banks' capital.

[1] Credit risk in this paper is used to describe the risk of a company defaulting or not. We do not discuss spread-risk or migration risk.

The overview is presented as a chain, but clearly there may be feedbacks at all stages. For example, were banks to incur material losses they might cut back lending or restructure their risk profile with attendant consequences for household and corporate balance sheets and ultimately for macroeconomic variables. Furthermore, in case of a bank failure this may spread through the banking system via interbank linkages. However, if the initial shock does not have significant effect on bank balance sheets as it is the case in our simulation we might not expect any feedbacks to prove material. In line with most other stress tests we, therefore, do not incorporate them in our analysis.

Figure 1: A schematic overview of stress tests



The key building block of this stress test is a Merton model to capture the default process of banks' borrowers. In his seminal paper, Merton (1974) showed that the probability of default can be determined ex ante. His idea is based on the observation that conditional on some assumptions – most importantly that markets are complete and efficient – the equity value of the firm is equivalent to a call option on the value of the assets with the level of debt as the strike price. By inverting the well known option pricing formula one can therefore derive the value of the underlying assets, their drift and their volatility based on observables (ie the equity price and the level of liabilities). Given this information, one can easily calculate the probability of default, which is equivalent to calculating the probability that the value of assets fall below liabilities. One assumption in Merton's original work is that default can only occur at the maturity of the debt. This implies that the probability of default and, hence, credit risk will be understated as default can occur at any time before maturity. Therefore, we use a Merton type model as developed by Tudela and Young (2003). This model is based on a barrier option pricing approach that takes into account that default occurs at the moment when the default barrier is hit.

This approach to modelling the default process has a benefit as well as a drawback which are both linked to the fact that the key input into the Merton is the equity price. As a benefit it implies by construction the underlying value of assets is measured in a market-to-market fashion. Hence, credit risk is measured with the same frequency as market risk. Furthermore, the systematic factors we identify in the paper are partially identical with factors typically used to stress test market risk in the trading book such as for example interest rates or exchange rates. This implies that one could easily integrate both stress tests which is one of the key challenges for stress tests (see BIS 2005). Unfortunately, there is not enough publicly available data as we do not have any information about the exposures in the trading book of banks or any idea about the magnitude of interest rate risk in the banking book. Hence, we focus on credit risk – and especially default risk - for corporate exposures of banks for the moment. This is in essence the drawback of this method. Equity prices are needed to calculate probability of defaults via a Merton model. Hence, industry PDs/recovery rates are based on data from (relatively) large listed coporates. Implicitly, we, therefore, assume that the average industry PDs are representative for the risk in the overall sector including smaller companies, to which banks are heavily exposed. It is unclear, how valid this assumption is.

The mapping from equity prices into asset values implies that determining the systematic risk drivers for equity returns is equivalent to determining the systematic risk drivers of asset values. We do the former by using the multifactor model as described by Drehmann and Manning (2004) which identifies a set of macroeconomic and market factors as systemic drivers of equity returns. It also shows that the impact of changes in the underlying risk factors differs across industries, the business cycle and whether the Bank of England followed an inflation targeting regime or not.

The Merton model is based on an efficient market assumption and we carry this through to the estimation of the multifactor model. In its simplest form, the efficient markets assumption implies that in a risk neutral world the response of equity returns to a shock should be equivalent to the total impact of the shock on all future discounted profits. In this sense, the key determinant for equity returns should be innovations in systematic risk factors and returns should not be predictable as otherwise arbitrage should be possible. There is an active debate in finance whether returns are indeed predictable or not (eg see Campbell et al, 1997). However, there is no consensus, so far, and it is also clear that the key impact on returns will be the actual innovations in systematic risk factors. Therefore, we consider only innovations of our systematic risk factors as independent variables in the multifactor model.

It is known that recovery rates fluctuate over the business cycle (e.g. see Altman et al 2002). To our knowledge recovery rates are either assumed to be fixed or follow draws from an independent distribution in all stress tests in the literature. But, expected recovery rates are nothing else than the expected value of assets, conditional on default. Given the assumption of the Merton model the value of assets, their volatility and drift is known and, hence, the expected asset values conditional on default can be calculated. Unfortunately, in reality some frictions exist as there are some deadweight costs from bankruptcy, such as eg lawyer fees, loss of expertise with respect to handling certain machines and so forth. A mechanical application of the model can not incorporate this. Therefore, we have to calibrate the mean expected recovery rate to the average observed recovery rate.

To derive the distribution of losses conditional on macro factors we simulate the risk factors identified by the state and industry dependent multifactor model over a one year horizon. The efficient market assumption implies that we only consider innovations of macro factors as systematic risk factors. This has the benefit that these innovations are not correlated across time. Hence, the interdependence between risk drivers can be fully captured by the variance covariance matrix of systematic factors. The multivariate normal is then used to draw the scenarios. We then assume that once a scenario is drawn it is announce to the market. Again, we impose an efficient market view as assume that the market incorporates the effects of the scenario immediately. Based on the stressed equity prices, industry PDs and LGDs are then derived.

To assess the impact of the stress on capital we are restricted by the data. We only know the aggregate industry exposure of UK banks but neither their individual components nor their quality distribution[2]. Given these data limitations, we have to assume that all banks hold a fully diversified portfolio in each industry with an average PD/recovery rate equal to the average industry PD/recovery rate as observed for stocks traded on the London Stock Exchange in this industry. We show that this assumption might be on average not too bad as the unconditional expected PD in our model is only slightly higher to the PD of an average portfolio of G10 banks.

The focus of our simulation is to derive the distribution of expected losses conditional on the underlying distribution of the macro environment. Hence, we do not calculate the full distribution of losses which would take account of idiosyncratic risk factors. The rationale for doing so is that from a financial stability perspective the key risks are not idiosyncratic risk factors of individual obligors in banks' portfolios. For well diversified large banks, let alone the banking system as a whole, idiosyncratic factors should not have a significant impact. A good example of this are the large corporate defaults in recent years like the defaults of Enron or WorldcCom, which clearly hit banks' profits but did not threaten the financial stability of the whole system. What matters for financial stability are large shocks hitting all obligors in all banks simultaneously (see Elsinger et al 2002)

So far, no stress test uses a Merton model to model the default process. But, our paper is very closely related to Pesaran et al (2004) in that they follow a similar 4 stage approach. Their focus is on a portfolio of international active firms. To capture correlations amongst systematic factors and across time they use a global VAR (GVAR) as the model describing the interrelations of systematic macroeconomic factors. They include output, inflation, stock market indexes, real exchange rates, interest rates and money balances for 11 countries/regions. As a second step they

[2] With more detail information the presented framework could be easily extended to accommodate a more disaggregated portfolio, even down to individual exposures.

estimate a multifactor model for 119 large international active firms. Originally, all factors from the GVAR are included, ie. for each firm they include the differenced series of 6 domestic and 5 foreign variables, which are the trade weighted average of all foreign variables. Some variables turn out to be insignificant or have the wrong sign. The final factor selection is, therefore, based on the significance of the mean group estimator, but the coefficients used in driving defaults are firm specific. Coefficients in the final specification include the home or the foreign stock market index (given multicolliniarity problems only one is included), the dollar real exchange rates, the domestic interest rate, domestic inflation and the oil price. A crucial assumption in their model is that default occurs if the equity price falls below a certain threshold. They assume that the threshold is the same for firms with the same rating and derive it by using historically observed transition matrixes.

Our paper differs in several aspects. First of all, we use a fully fledged Merton model to derive PDs, which assumes that default occurs once the value of asset falls below the default point – not the level of equities by a certain percent. Secondly, we use the insight from the Merton model to model recovery rates. Hence, recovery rates are driven by the same systematic factors as defaults, whereas Pesearn et al assume that recovery rates follow an independent distribution. Thirdly, in the Pesaran et al. work much emphasis is given to the GVAR. This implies that the actual impulse response functions, not the innovations in macro variables, are an important determinant of returns in their stress test. This contrasts to our efficient market set-up.

The overall conclusion from our paper is that the UK banking system seems robust with respect to macroeconomic shocks affecting the credit risk of corporate lending. This confirms previous analysis of the UK banking system (see Hoggarth and Whitley, 2003) and is similar to the results of stress tests discussed in the literature (see Sorge, 2004). More importantly, several interesting observations are highlighted with our approach. First of all, the impact of systematic factors on PDs and to a lesser extent recovery rates is neither linear nor symmetric. Secondly, we show that time is an important dimension for credit risk. Expected losses over a 1 year horizon are much more than twice the expected losses over a 1/2 year horizon. Again, the difference is not symmetric around the mean. In the most adverse macroeconomic conditions the increase from the 1/2 to 1 year PD is much greater than for the mean which in turn is greater than for the most benign conditions. This is an important result from a stress testing perspective. It also reinforces results from earlier stress tests in the literature which also explicitly model the underlying discontinuity of default/non-default like models based on CreditPortfolioView (eg see Virolainen, 2004) . As the essence of a stress test is highly adverse events which occur in the tail of the distribution basing stress tests on symmetric and linear distributions might lead to a severe underestimation of the risk associated with the stress scenario.

The remainder of the paper is structured as follows. In Section 2, we discuss the Merton-type model we use to model borrower defaults and explain how we derive expected recovery rates conditional on macroeconomic factors. Section 3 describes our approach to identify systematic factors of borrower defaults and recovery rates and Section 4 derives the correlation of systematic factors across time and each other. Section 5 describes the mapping from PDs and expected recovery rates to expected losses and Section 6 shows the simulation results. Section 7 concludes.

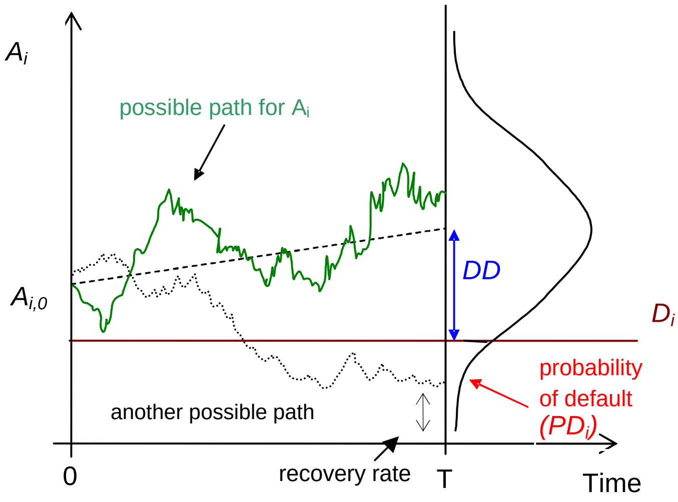
2 The Merton model

We follow Tudela and Young (2003), henceforth TY, in modelling corporate defaults by a Merton type barrier option approach. The intuition behind a Merton model is straight forward and Figure 2 illustrates it graphically. It assumes that the value of assets A_i of a firm i follows a stochastic process with the trend μ_i and volatility σ_i

$$dA=\mu_i A_i dt+\sigma_i A_i dz \tag{1}$$

where $dz=\varepsilon\sqrt{dt}$ and ε~N(0,1). As dz follows a Brownian motion one can easily calculate the probability PD_i that the value of assets A_i falls below the default point D_i and company i goes bankrupt. Often PDs are also expressed in terms of the distance to default DD. This is the number of standard deviations the value of assets is away from the default point taking the trend into account.

Figure 2



Unfortunately, neither the value of assets, their trend nor their volatility are observable. However, assuming efficient markets Merton (1974) showed that the value of a firm's equity is equivalent to the value of an option on its assets with the default point as strike price. Using the well known options formula, one can therefore derive the unknowns from observable equity data by either maximum likelihood (see Duan 1994, 2000) or using theoretical restrictions from the Merton model (see Hull, 2000). TY do not follow the original set up of Merton as he assumed that default occurs only at maturity T. This underestimated credit risk. A company will go bankrupt at the point in time when its assets fall below the default point independent of whether this is at or before maturity. To account for this, TY use a barrier option approach. The formula for the probability of default is described in the Appendix A. For a technical derivation, the reader should go directly to TY.

A problem of Merton models in general is that the debt structure of companies is more complex than simply one liability D with maturity T. In reality, companies have several different debts outstanding with different maturity dates. Furthermore, a firm must not necessarily default, when the value of assets falls below the value of debt as long as it is able to pay the required interest rate. Therefore, TY assume that the default point is all the short term debt plus half the amount of long term debt outstanding, which is in line with commercial models such as Moody's KMV.

In their paper, TY show that their PDs are strong predictors of firms' default one year ahead. The average PD of defaulted firms is nearly 50% whereas of nondefaulted ones it is just around 5%. Furthermore, the model predicts the right rank ordering of firms and captures turning points in the market well. However, in contrast to commercially available models TY do not calibrate the PDs or DDs to observed default data. This may induce some bias. Research for the US (see Kamakura, 2004) indicates that the mapping from PDs derived by a pure Merton model to actual PDs is not one-to-one. It rather seems that Merton model PDs are too low in comparison with actual PDs when Merton-model PDs are low and that they are too high at the other end. Unfortunately, we can, therefore, expect that our results are biased to an unknown degree when we compute an industry average. However, as this paper is concerned about periods of stress, ie when PDs are high, we expect that the results in general are up-ward biased, which is a desirable feature from a stress testing perspective.

2.1 Recovery rates

From the Merton model we can derive the value of assets at default (see Figure 2). Therefore, we can also calculate the expected recovery rates which are nothing else than the expected value of assets conditional on default – as long as the bankruptcy process is frictionless. The underlying distributional assumption of the Brownian motion in Equation 1 implies that changes in asset values and, hence, the level of future asset values are normally distributed. To calculate expected recovery rates we can apply the known formula for the mean of a normal distribution, conditional on assets being less than liabilities (see Appendix 2). We restrict the analysis to measuring the expected recovery rate at the horizon over which we compute the PD[3]. By doing this, we implicitly assume that the asset values are realised at the end of the computation period. This is not in line with the observation that the recovery process might take years. However, it is not clear which horizon should be picked a-priori and restricting the calculation of recovery rates to the same horizon as PDs makes the analysis computational more easily[4].

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[3] Another restriction is that we assume that, once default occurred, the firm will stay bankrupt for ever and the maximum recovery rate is 1.

[4] Simulating recovering rates for an extra two year horizon would imply further 10.000 x 24 simulations for all of the 10.000 scenario considered. As we need to track the business cycle, this is computational very intensive.

The biggest problem for the computation is that in reality, bankruptcy is costly and not frictionless. It is not clear how to incorporate this in a consistent fashion. To overcome this we will calibrate the average recovery rate to observed average recovery rates as will be explained in Section 6.2.

3 Systematic risk factors

In this section, we determine the systematic factors driving PDs and recovery rates. This is equivalent to determining the systematic risk factors of asset returns. As discussed above, assets are unobservable but can be derived from observable equity prices. Hence, by understanding systematic components of equity returns we can derive responses of asset returns to shocks of systematic factors.

Looking at systematic components of equity returns is a long standing question going back to the 1970s (eg Nelson, 1976, or Fama and Schwert, 1977). More recently, much of this work was undertaken within the APT literature and in particular in the context of tests for multifactor models of stock-valuations. Much of this work goes back to the seminal study by Chen, Ross and Roll (1986). Drehmann and Manning (2004) (henceforth DM) look at a first stage of an APT for UK equity returns. This section is based on their work.

The starting point in finance is always the fundamental pricing equation

$$p=E(mx)$$

stating that the price of an asset is its future discounted income stream, with discount factor m. Cochrane (2001) shows how this discount factor model can be mapped into the APT framework as long as the law of one price holds – ie that markets are efficient and complete – and the variance of the discount factor is finite. However, it is not clear what factors should drive the APT process. Empirically, the APT translates into the basic regression:

$$R_{j,t} = \alpha_{I,S,M} + \beta_{I,S,M} \Delta X_t + \varepsilon_t \tag{2}$$

where $R_{j,t}$ is the period-t return on a stock, and X_t is a vector of factors purported to impact upon its dividend expectations or the discount factor. The constant term, α, may be interpreted as the risk-free return. It can be shown that this is also equivalent to the drift of assets in equation (1).

The approach of DM constitutes something of a departure from other research undertaken in this field. First, while researchers have tended to work with returns on stock market aggregates (market indices) or composite portfolios, they adopt a panel estimation methodology. Clearly, the overall explanatory power of the regression is lower, given that the left-hand side returns comprise both systematic and idiosyncratic risk, but greater precision in coefficient estimates could be expected, especially as coefficients are state and time dependent as will discussed later.

Second, the focus is on the UK market, whereas much of the literature has examined US stock returns. The core of the dataset is an unbalanced panel of monthly observations of total return indices (i.e. taking into account, not only the increase in stock prices in a given period, but also dividend income received) for each of the 556 firms (excluding banks and investment trusts) currently in the FTSE All Share index, with at least 12 observations during the period January 1980 - October 2003 inclusive. Due to data constraints for certain explanatory factors (and given that we work with a six-month lead of our activity factor), however, the effective sample period for the empirical work becomes April 1982 – December 2002. Although the noise-to-information ratio in stock returns may be higher at a monthly than quarterly frequency (as observed by Schwert, 1990), the advantage is that it introduces greater variability in the systematic factors and increase the within-groups degrees of freedom which are needed to be able to recognise.

Third, in general the *βs* are assumed to be constant over the estimation period, which is likely to be too restrictive especially from a stress testing perspective. Therefore, after statistically testing for it, DM allow coefficients to differ across monetary regimes (M), industries (I) and states of the business cycle (S). The intuition why these changes should be allowed is relatively straight forward.

There are strong economic priors, why the regime change in the monetary regime needs to be taken into account. After the UK adopted inflation targeting in October 1992 both the level and volatility of inflation as well as the volatility of other macro factors dropped sharply. DM show that macro factors have a significantly different impact pre and post inflation targeting. Therefore, we only take the coefficients post October 1992 for our simulation.

Taking industry differences into account is beneficial from the stress testing perspective as we have information on banks' industry exposures. However, intuitively one would also expect responses to vary across industry groups, due to possible differences in cyclicality, international orientation and dependence on factors such as oil inputs. The data are therefore split into six industry groups, broadly defined according to their SIC codes.[5] The groups ultimately employed in the analysis and the stress testing application are as detailed in Table 1.

Table 1. Industry groups employed in the analysis

Industry group
Manufacturing; electricity gas and hot water
Construction; real estate
Wholesale and retail trade; hotels and restaurants
Transport, storage and communication (TSC)
Mining and quarrying
Other business activities; education; community

Finally, DM allow coefficients to vary according to the state of the business cycle. This is especially important from a stress testing perspective as it is not clear whether the response of equity returns to systematic shocks is the same for severe periods of stress – the periods we care most about in a stress test – and normal times. If differences are not allowed for but exist in reality, then the latter will dominate the sample and results for the stress test will be biased. McQueen and Roley (1993) were

[5] To the extent possible, MD have sought to match FT industry classifications to SIC codes, upon which UK bank exposures are based, and then amalgamated certain SIC groups. This procedure necessarily entails a trade-off between homogeneity and degrees of freedom; i.e. greater disaggregation to allow for greater heterogeneity reduces the degrees of freedom within each group.

the first to explore state dependent coefficients. They argued that a “positive surprise to industrial production in a recession could indicate the end of the depression and higher forecasts for firms’ cash flows.…” However in a more favourable state, “with low unemployment and factories running near full capacity, a positive surprise in industrial production may result in fears of an overheating economy, inflation and possible efforts of policy makers to increase the real interest rate….” (p. 684, McQueen and Roley, 1993).

DM define ‘states of the economy’ in terms of the extent of deviation from a trend growth path as measured by the Hodrick Prescott filtered path of ln(GDP). Following McQueen and Roley (1993), the ‘low state’ comprises observations below the 25th percentile of the distribution of deviations from trend – i.e. those in which the deviation from trend is large and negative – and the ‘high state’ comprises observations above the 75th percentile.

3.1. Selection of factors

DM do not follow a theoretical model to derive the factors for their analysis but rather derive a set of factors by general-to-specific. Tables 2 shows the final selection of macro factors which are economically intuitive.

Table 2. Macroeconomic factors included in the specification

Macroeconomic factor		**Calculation and transformation**	**Exp. sign**	$\rho_{t=1}$*	$\rho_{t=2}$*	**Std. Dev**$_{t=1}$**	**Std. Dev**$_{t=2}$**
Innovation in expected GDP growth***	GDP^e	Monthly observations are residuals from regression of 6 month forward change in log real GDP on two lags of itself	+	0.074 (0.40)	0.003 (0.97)	0.002	0.001
Change in real 3mth T-Bill rate	TB3	Nominal T-Bill rate less preceding 12 month log difference in RPI index. 1 month percentage point difference	-	-0.030 (0.73)	-0.063 (0.48)	0.782	0.328
Change in 2yr/3mth yield spread	SPR	Difference between 2 year gilt spot rate and 3mth T-Bill rate. 1 month percentage point difference	-	0.045 (0.61)	0.097 (0.28)	0.470	0.262
Innovation in current RPI inflation***	RPI	Monthly observations are residuals from regression of 1 month log first difference of RPI on two lags of itself	-	-0.032 (0.72)	-0.034 (0.70)	0.005	0.004
Change in real £ effective exchange rate	EER	Real (deflated by RPI) 1 month log first difference	-	0.332 (0.00)	0.097 (0.28)	0.017	0.017
Change in US$ oil price	OIL	Nominal 1 month log first difference of monthly average of unweighted composite of WTI, Brent Crude and Dubai Light	-	0.414 (0.00)	0.111 (0.21)	0.091	0.076

*First-order autocorrelation coefficient for transformed series. t=1 is the period April 1982 – Sep. 1992 (126 observations); t=2 is the period Oct. 1992- Dec. 2002 (123 observations). P-value from Q-test in parentheses.
** Standard deviation calculated over the period April 1982 – Sep. 1992 (t=1), and the period Oct. 1992- Dec. 2002 (t=2).
*** We generate the innovations by regressing each differenced series on two lags of itself. Given that our later empirical tests exploit a structural break in the series at October 1992, we allow for this also in these regressions. In each case, we find significant differences in the autoregressive properties of the series pre- and post-October 1992.

As discussed in the introduction, DM work with the strong assumption that markets are efficient and that asset prices are not predictable. Therefore, highly autocorrelated macroeconomic factors are transformed so as to capture only surprises, or innovations. Following Chen, Roll and Ross (1986), however, it is argued that factors originating in market prices (such as interest rates, commodity prices and exchange rates) are "sufficiently uncorrelated that one can treat [them] as unanticipated."[6].

Some studies (e.g. Pesaran et al., 2003) include a broad market index as a systematic factor. However, the 'market' return is merely a weighted average of individual equity returns and in the panel context DM are essentially working with an unweighted average of individual equity returns on the left-hand side. Thus, although each individual company might be considered small relative to the market as a whole, this might be expected to introduce endogeneity into the specification.

Although this argues against the inclusion of the market index itself, some market proxies should be included in the specification. This is intuitive, as stock prices reflect long-horizon expectations for dividends to shareholders, which are likely to embody expectations for systematic macroeconomic factors extending beyond the horizon that can be captured by our short-term proxies. Furthermore, the market will embody information about time-varying risk premia, liquidity and capital flows that constitute common factors, but cannot be captured by macroeconomic proxies. Market factors are presented in Table 3, below.

Table 3. Market factors included in our specification

Market factor		**Calculation and transformation**	**Exp. sign**	**$\rho_{t=1}$***	**$\rho_{t=2}$***	**Std. $Dev_{t=1}$****	**Std. $Dev_{t=2}$****
Change in volatility	VOL	24-day annualised standard deviation of FT All Share price index. 1 month percentage point difference	-	-0.341 (0.00)	-0.234 (0.01)	7.038	5.629
Change in risk premium	ERP	Implied equity risk premium for FTSE 100 index, applying a 1-stage DDM. 1 month percentage point difference	-	0.035 (0.69)	-0.098 (0.27)	0.269	0.189
Change in valuation	PE-US	Price/Earnings ratio for US S&P 500 composite. 1 month percentage point difference	+	0.067 (0.45)	-0.004 (0.96)	0.965	1.737

*First-order autocorrelation coefficient for transformed series. t=1 is the period April 1982 – Sept. 1992 (126 observations); t=2 is the period Oct. 1992- Dec. 2002 (123 observations). P-value from Q-test in parentheses.
**Calculated over the full sample period.

[6] Chen, Roll and Ross (1986) argue (p.386) that there is a trade-off between the introduction of an errors-in-variables problem if the autocorrelated factor is included directly, and error introduced by misspecification of the estimated equation for determining the expected movement.

3.2 Results for the multifactor model

All results are estimated by GLS, correcting for heteroscedasticity. DM undertake a battery of tests to see whether coefficients are different across industries, states of the business cycle and monetary regimes. This is indeed the case. It can also be seen in Table A3 in the Appendix 3 which shows the estimation results used in the stress test. Overall, the predictions of the impact of market as well as macroeconomic factors seem to be confirmed. Especially pre October 1992 coefficients in extreme states tend to be larger and more significant.

The majority of coefficients (approximately three-quarters) are statistically significant at conventional levels, the bulk of those measured imprecisely may be found in the second sub-period; and disproportionately in extreme states of the economy.[7] Unfortunately, this most likely reflects the much greater incidence of extreme states of the economy in the first sub-period. Only a third of observations fell in the 'normal' state in the pre-October 1992 period, while in the second sub-period GDP remained much closer to trend, with more than 60% of observations in the normal state. Furthermore, extreme observations in the second sub-period are concentrated early in the period. Most of the low state observations are located at the tail-end of the 1990-1992 recession and in the early stages of the recovery; the high state observations occur almost exclusively in the 1994/95 period, as activity gathered pace following sterling's ERM exit and the associated monetary easing.

It is also interesting to assess the explanatory power of the specification (see Table 4). For a comparison we report the R^2 of an alternative specification where the excess returns are regressed on a series of time dummies, which will capture the average variation across firms in each period. The R^2s of the above specification are generally lower than those from regressions on time dummies only. However, this is to be expected as the time dummy regressions capture the maximal systematic variation, but without specifying the driving factors. Hence, given that our R^2s are generally between half and two-thirds of these values, the relatively parsimonious specification

[7] Some 151 of the 189 coefficient estimates is statistically significant at conventional levels in the pre-October 1992 period, compared with 125 in the second sub-period. Of the 64 coefficients estimated imprecisely in the second sub-period, 56 occur in extreme states of the economy.

appears to be very successful and to include the most important systematic determinants.

Table 4. Explanatory power* of industry-level regressions, with state- and time-dependent coefficients

	Pre Oct. 1992		Post Oct. 1992	
	Basic reg. *	**Time dummies**	**Basic reg.** *	**Time dummies**
MANUF.	0.174	0.244	0.059	0.096
CONSTR.	0.206	0.353	0.112	0.225
RETAIL	0.127	0.216	0.081	0.148
TSC	0.175	0.264	0.077	0.142
MINING	0.145	0.249	0.125	0.230
BUS. SERV	0.171	0.249	0.090	0.152

*Within-group R^2s reported. R^2s in columns (1) and (3) relate to the regressions reported in the Annex. The R^2s in columns (2) and (4) relate to regressions on a series of time dummies, one for each month, which capture all systematic variation in each period.

Overall, Table 4 indicates that most risk is not driven by systematic factors but rather by idiosyncratic factors. This is important for our stress test as we will only simulate on the impact of systematic factors but do not look at idiosyncratic ones. Hence, a lot of variation is not picked up in our simulations, where we only look at the impact of systematic factors on expected losses.

4 Correlation of systematic risk factors

The third important building block for a stress tests is an understanding how systematic factors are correlated between each other and across time.

DM's approach to identify systematic factors in an efficient markets context is helpful at this stage as they only analyse the impact of innovations of macroeconomic and market variables on equity returns. Theoretically, these innovations should not be autocorrelated and there should be no correlation between factors across time. Autocorrelation coefficients in Table 2 and 3 show that this is indeed the case for nearly all variables we look at. Exceptions are the trade weighted exchange rate and the oil price in the first period and volatility across both samples. DM investigate whether coefficients would change a lot if these variables would be transformed into innovations as well. However, this is not the case except for the oil factor in the mining industry pre-October 1992. Given that the oil industry is not the main focus and transforming variables into innovations might add more noise, we think that these factors are the right ones to use in the stress test. Furthermore, looking at correlations across factors between different periods we also observe some weakly significant correlations. However, the relationship is only very weak and not consistently significant across different monetary regimes. Given this weak evidence we do not model them in the current paper.

Hence, given no correlation across time we only need to model correlation between systematic factors at each period. We do this by simply using the variance / covariance matrix of the factors when there is significant correlation. Table 5 shows the correlation coefficients used for the stress test.

Table 5. Correlation matrix – post-October 1992

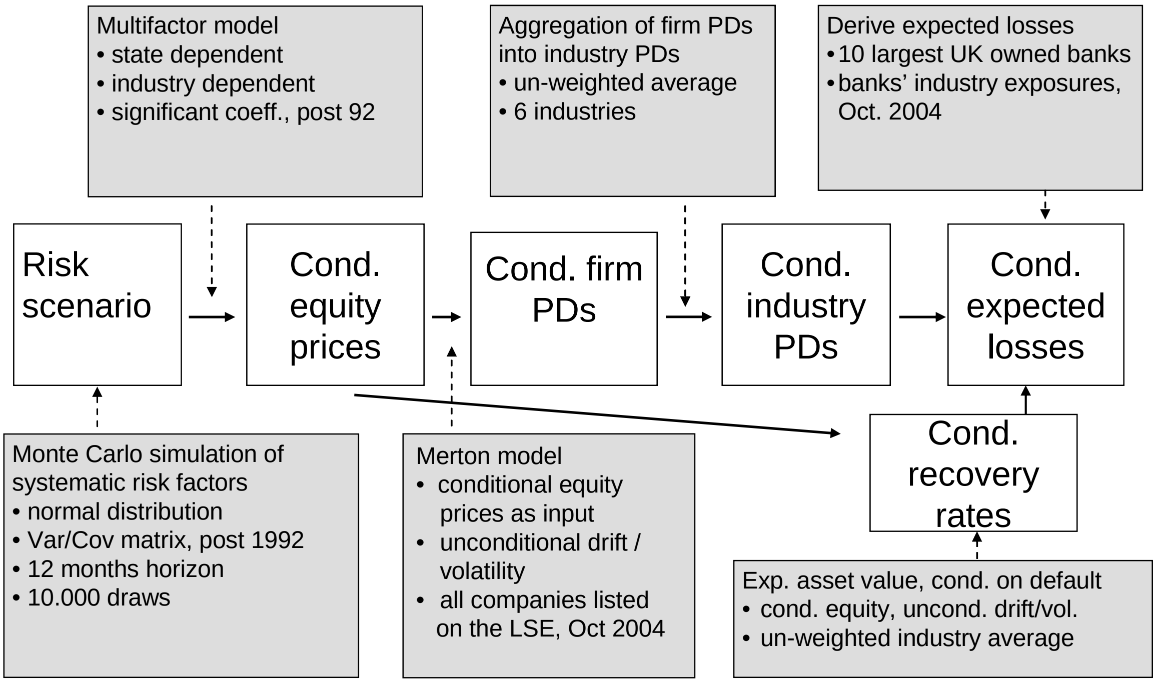
	GDP^e	**TB3**	**SPR**	**RPI**	**EER**	**OIL**	**VOL**	**ERP**	**PE-US**
GDP^e	1.000								
TB3	0.055	1.000							
SPR	0.062	-0.041	1.000						
RPI	0.019	-0.136	-0.050	1.000					
EER	0.027	0.287***	0.002	-0.327***	1.00				
OIL	-0.008	-0.077	0.026	0.197**	-0.166*	1.000			
VOL	-0.135	0.027	-0.191**	-0.085	0.157*	0.064	1.000		
ERP	-0.180**	-0.251***	-0.444***	0.036	-0.199**	-0.001	0.183**	1.000	
PE-US	0.029	-0.005	0.245***	-0.044	0.052	-0.129	-0.212**	-0.361***	1.000

Correlations estimated on a pairwise basis over the period over the period April 1982-September 1992 (Table 5a) and October 1992-December 2002 (Table 5b). ***=pairwise correlation coefficient significant at 1%; **=pairwise correlation coefficient significant at 5%; *=pairwise correlation coefficient significant at 10%.

We also undertook some preliminary investigation whether modelling dependency structures via copulas would improve the results. However, it turns out no significant changes are observed. This is mainly due to the fact that actual correlations are relatively weak and so the approximation by the multivariate normal works quite well. Therefore, we do not report these results.

5 The Simulation

Figure 2: A schematic overview of the set up of the simulation



Integrating the three building blocks enables us to stress test UK banks' portfolios. Figure 2 provides a schematic overview of the simulation set-up. The simulation starts by taking a random drawing of systematic factors based on the assumption that factors are jointly normally distributed with N(0, Σ) where Σ is the observed variance/covariance matrix post October 1992. A scenario consists of 12 independent random drawings of the nine factors for 1 to 12 months. During each scenario we track the output gap and apply the state depended multifactor model to calculate equity returns conditional on the scenario. In line with general stress test practices, we assume that the development of each scenario is known at the starting period of the stress test. Keeping with the assumption of efficient markets asset prices are, therefore, assumed to react immediately to incorporate the effect of the whole scenario. The equity prices, conditional on the scenario are then fed into the TY Merton model, which generates asset values. Using the TY model also generates the drift and volatility of assets, which we use to calculate PDs. Based on the conditional asset prices, we also calculate conditional expected recovery rates. We do this for all stocks on the London stock exchange taking October 2004 as a starting point.

To derive expected losses for banks we assume that expected losses on banks' industry exposures behave in line with the average expected losses in this industry, conditional on the scenario. This assumption is mainly driven by data limitations. Only the total value, but not the quality distribution, of exposures of banks to a certain industry is known to the Bank of England. We look at the exposures of the ten largest UK banks[8] as of October 2004. This is repeated 10,000 times.

In the simulation, we do not stress the drift or the volatility of assets but take them from the unconditional TY model. As discussed above, the drift is theoretically equivalent to the estimated constant in equation (2), adjusted for dividend payouts which we however not include in the prediction of equity returns. Using unconditional volatilities should induce some up-ward bias in PDs as well as a downward bias in recovery rates. This is the case as we actually simulate already part of the volatility of asset returns by simulating systematic risk factors. Hence, the variance we use to calculate PDs as well as recovery rates is too high but not massively so as the explanatory power, especially post October 1992, of the multifactor model is quite low. Given, that the variance might increase in a stress environment in the first place, this assumption might not induce a massive distortion. Using the unconditional variance is also beneficial as it capture idiosyncratic risk factors of obligors which are not simulated in our analysis.

6 Results

6.1 PDs

The simulated median PD over a one year horizon is over 8%, which is above the average probability of default for BB bonds as observed from Moody's default data(see Table 6). This implies that the average PD of our hypothetical portfolio is higher than the average PD of a portfolio of an average G10 bank which is similar to a BB rating (see Catarineu-Rabell et al, 2003). This was to be expected. As discussed in Section 2 the Merton model employed does not map measured PDs into actual PDs

[8] The banks are Abbey, Alliance and Leicester, Bradford and Bingley, Barclays, HSBC, HBOS, Lloyds, Northern Rock, RBS, Standard and Charters.

and this introduces some up-ward bias. As we are not concerned about pricing but downward risk in stress environments, the up-ward bias in PDs is a desirable feature[9].

Table 6: Average PD for different ratings and average portfolio distributions of credit quality for corporate exposures for G10 banks.

	Average PD* **(%)**	**Quality distribution of average G10 bank** (%)**
AAA	0.00	4
AA	0.02	6
A	0.01	27
BBB	0.15	30
BB	1.21	29
B	6.53	4
CCC	24.73	1

* reported by Moody's for all defaulted bonds between 1970-2001
** reported by Catarineu-Rabell et al (2003)

Chart 1 shows the evolution of the distribution of PDs over different horizons. A pattern which will be seen in all simulated results emerges: The distribution is neither symmetric nor linear. It is clear that for all forecast horizons the PDs in the best macroeconomic environment (ie the 1st percentile) are closer to the median PD than the PDs in the worst macroeconomic conditions (ie the 99th percentile). Furthermore, Chart 1 shows that time is an important dimension as the one year ahead PD is greater than twice the 6 month PD. Again, the difference is not symmetric around the median. In the most adverse macroeconomic conditions the increase from the 1/2 to 1 year PD is much greater than for the median which in turn is greater than for the most benign conditions. This is an important observation as most other stress testing models are based on linear approximations, where such effects can not occur.

Industry probabilities of default show similar characteristics, even though non-symmetry and non-linearity is far more pronounced for some industries (see Charts 2-7). To a certain degree, this might be an artefact as there are much fewer companies in some industries and hence the distribution is more prone to outliers. Interestingly, Industry 5 (see Chart 6) seems to show an inverse relation to other industries. This is

[9] As a robustness check we artificially restricted the maximum PD to 50% in the spirit of KMV's restriction to PDs to 20%. Even though this changes the shape of the distribution of some industry PDs it hardly influences the aggregate results.

mainly driven by the fact that PDs in mining and quarrying are extremely low and that there are only few companies in this industry.

6.2 Expected Recovery rates

As discussed in Section 2.1 the assumption of normality allows us to calculate the expected recovery rate, conditional on default. The distribution of simulated expected recovery rates for a one year horizon is shown in Chart 8. Unfortunately, the simulated results show quite a poor fit with actual data. The observed historical average expected recovery rate (for the US) is around 40.34%[10] whereas the unconditional average expected recovery rate over a one year horizon is just under 90% in our simulation. This is clearly a significant difference, which might be explained by several factors. Firstly, looking at expected recovery rates over the 1 year horizon (see Chart 9) it can be seen that expected recovery rates decline over time. Given that recovery can take years, it might be the case that the market already anticipates this and looks at the expected recovery rate say 3 years ahead. However, this can also not explain our high recovery rates. In another simulation based on the same model but without changing coefficients in the multifactor model and with data from December 2003 we simulated recovery rates over a 24 month horizon. Even then the lowest percentile of the simulated recovery rate across all companies is only around 80%. Another explanation might be the most plausible one, in that default implies some form of bankruptcy costs and hence a step change in the value of assets is observed once there is a default. The market clearly incorporates this into its assessment of recovery rates, whereas our simulations do not. To accommodate this, we calibrate the mean expected recovery rate over a one year horizon to equal 40.34%, which is equivalent to assume that bankruptcy costs are around 58% of the asset value. Informal discussion with UK banks revealed that this restriction might be too severe and that actual recovery rates lie between bond recovery rates (which are unsecured) and our initial simulations. However, by imposing a severe recovery rate we increase expected losses which is justified from a standpoint of conservatism. This

[10] Generally, data on recovery rates is poor and the most commonly used proxies are prices of bonds three months after default. These can be seen as expected recovery rates as the market is forward looking. The reported recovery rate is based on Moody's data of all US bonds defaulted between 1982 and 2003. Including UK bonds does not change this, as only 66 bonds defaulted in the UK in this period.

might also capture some liquidity problems which may arise if several banks would need to liquidate assets simultaneously.

Chart 9 shows the simulated aggregate expected recovery rate over time. It is interesting to note that the dispersion of recovery rates is far less pronounced than on PDs and the impact of shocks does not seem to be strongly non-linear and non-symmetric. Furthermore, recovery rates do not fall as much over time as PDs increase over time.

6.3 Distribution of expected losses conditional on macroeconomic factors

Chart 10 shows the distribution of total expected losses of UK banks corporate exposures, conditional on macroeconomic scenarios. It is worth stressing again that we do not simulate idiosyncratic risk factors. Our main interest lies in the impact of systematic risk factors driving correlated losses. However, it should be noted that idiosyncratic risk is captured in the general PD. The chart, therefore, does not look at the whole distribution of losses but only at expected losses, conditional on macro stresses as macro risk is the important aspect from a financial stability perspective. The 99th percentile, for example, in Chart 10 shows the amount of expected losses, given the worst macroeconomic outlook. Looking at the 1 year horizon expected losses conditional on the worst macroeconomic environment would be less than 20% of capital (Chart 11).

The underlying non-linearity of the default process is the driver for the non-symmetric shape of the expected loss distribution. It is clear that the difference between normal conditions and the most benign ones is not that pronounced whereas there is a significant increase in expected losses for the most adverse macro scenario. It is clear that due to idiosyncratic risk implicitly captured in PDs, there are always some expected losses, even in the best macro conditions. But, macro factors really kick in when general conditions are severe. Unsurprisingly, this implies that there is little up-side from good macro conditions but a severe down-side from highly adverse conditions in the systematic factors.

Overall, this stress test indicates that the UK banking system is rather robust. This conclusion is strengthened when looking at expected losses relative to past profits as they are the fist buffer against losses. Even the most adverse conditions total expected losses do not exceed total profits. Hence, it would not be necessary for banks to use up some of their capital to cover unexpected losses (Chart 12) in the first place.

However, the above graphs might be misleading in several aspects. First of all, so far we only simulated losses for total UK corporate exposures of banks relative to total capital. Clearly, banks capital holding is determined by their total exposures across all different asset classes in the UK as well as internationally. To adjust for this, we assume as a robustness check that banks capital holdings against risks in different portfolios are proportional to their exposures in these portfolios. We call these capital holdings 'relative capital'. This is obviously a very simplistic approach as it ignores potential diversification benefits between different asset classes. However, it seems likely that these are less important than the fact that the risk in corporate lending is generally much higher than for household and especially mortgage lending. For example, over the period from 1993 to 2004 the average aggregate write-off rate for corporate loans was 0.19 with a variance of 0.02 in contrast to the average aggregate write-off rate of 0.09 with a variance of 0.001 for secured household lending. Therefore, the mapping of expected losses to relative capital should overstate risks to the financial system.

Whereas the aggregate might imply stability, it must not necessarily follow that all banks are sufficiently profitable and well capitalised to withstand the shocks. Therefore, we look at the 25% percentile of banks' expected losses over relative capital in the best macro conditions and the 75% percentile of banks' expected losses over relative capital in the worst state of the world. Chart 13 nicely illustrates the difference between upside and downside risk. Overall, it indicates that even in the worst macro conditions and for the worst affected bank expected losses are only about 120% of capital. This overstates the true risks as the analysis does not take any future profits into account which would be the first buffer against losses. More importantly, banks where the worst scenario impacts more than a 100% relative to relative capital have all very small corporate portfolios relative to their household lending activities. Hence, the adjusting for relative capital is especially severe for these institutions.

The observation that the impact of shocks is neither linear nor symmetric can also be summarized by looking at the mean[11], the standard deviation and the skewness of the distribution of returns, PDs, LGDs and total expected losses as in Table 7.

Table 7: Summary statistics of the distribution of returns, PDs, LGDs and total UK expect losses.

	Average Returns			PDs		
horizon	Mean	Std	Skew	mean	std	skew
1	0.000	0.094	-0.021	0.011	0.001	0.286
3	0.000	0.163	0.056	0.024	0.003	0.674
6	-0.001	0.224	0.086	0.042	0.008	0.956
9	-0.001	0.274	0.055	0.062	0.015	1.030
12	-0.002	0.316	0.045	0.083	0.022	0.990

	Recovery rates			Total Expected Losses		
horizon	mean	std	skew	mean	std	skew
1	0.989	0.000	-0.211	293.91	18.0	0.29312
3	0.969	0.002	-0.258	1254.4	119.9	0.42855
6	0.944	0.004	-0.259	3443.0	446.2	0.55481
9	0.920	0.006	-0.288	6354.3	1015.1	0.67489
12	0.899	0.009	-0.285	9869.8	1799.1	0.77066

As expected, average returns have a mean, which is not significantly different from 0 and the standard deviation increases linearly with square root of T= $\sqrt{12} = 3.5$. Even though, shocks to returns are normally distributed we observe some skewness, which might be driven by the fact that coefficients change over the course of the simulation. The picture of PDs is in stark contrast to this as the standard deviation increases by around a factor 22 from 1 to 12 months instead of a factor 3.5. Furthermore, significant skewness can be observed, with a fat right tail[12]. The distribution of LGDs shows a similar pattern but not so strongly. Skewness this time is negative, which implies that the left tail - ie low recovery rates - is more pronounced than for a normal distribution. For total expected losses the increase in the standard deviation is also highly non-linear and the distribution shows significant skewness, which, however, is not as pronounced as for the distribution of PDs.

[11] When reading these results the reader should keep in mind that so far we focused on the median. In case of positive skewness the mean will be higher than the median.

[12] The skewness will be affected by the fact that Merton model PDs are too high relative to actual PDs at the high end and too low at the low end. However, the skewness is so pronounced that adjusting for this should not impact on the overall qualitative result.

The insight from this table clearly indicates that back of the envelope stress tests based on the normal distribution will produce wrong results. Take the mean and standard deviation of expected losses as given and assume that the stress test would be a plus/minus three standard deviation shock to expected losses. In this stress test expected losses are 15,000 and 4,500 respectively. Especially, in the negative scenario expected losses are underestimated by around 20% which is quite significant.

Clearly, banks' stress tests are more sophisticated than this simple example. However, the analysis above stresses the importance of modelling the underlying non-linearity of credit risk which gives risk to signficiant skewness and non-linearity in increases of the standard deviation over time.

7 Conclusion

This paper presents a stress test for corporate exposures of UK banks. The overall conclusion is quite reassuring as even in the worst macroeconomic conditions expected losses of banks corporate exposures are not high enough to cause a bank failure.

This stress test showed that systematic factors have a non-linear and non-symmetric impact on credit risk and that these effects are most important for highly adverse scenarios which are the main interest from a stress testing perspective. Insofar the results are reassuring because in spite of modelling the non-linearity and upward biasing our estimates at several stages the overall impact of severe macro conditions on UK banks is limited.

However, one should also caveat this methodology. Both the Merton model and the multifactor model are based on the assumption that markets are efficient. Therefore we limit our attention to innovations of macro factors, which might underestimate correlations of risk factors over time. As all market based models we also assume that market prices always reflect true economic fundamentals - a statement some might want to question especially after the latest tech bubble. Furthermore, we implicitly assume that no market disruptions can occur. Again, it is not clear whether this is indeed the case especially when looking at times of severe stress which we do in the tails of our simulation.

Not withstanding these arguments, the developed stress test can be an important surveillance tool to analyse the financial stability of a banking system as it highlights the impact of non-linear impacts of highly adverse scenarios which previously have not been modelled. Furthermore, given that our model is based on market data, the setup can be nicely integrated with stress tests for market risk in the trading book. This is an important step into the direction of a fully integrated approach to stress test all risks faced by banks.

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Appendix 1

The Merton model of Tudela and Young (2003)

Assume that the value of assets *A* of a firm *i* (for clarity we omit the index *i* for this appendix) follows a stochastic process with the trend μ_A and volatility σ_A

$$dA=\mu_A A dt+\sigma_A A dz \tag{A1}$$

where $dz=\varepsilon\sqrt{dt}$ and ε~N(0,1). The level of liabilities *L* follows a deterministic process with $dL=\mu_L L dt$. This implies that the asset liability ratio $k=A/L$ follows a stochastic process with trend $\mu_K=\mu_A-\mu_L$ and volatility $\sigma_K=\sigma_A$. TY derive the probability density function for *k*, which enables the estimation of μ_K and σ_K. This enables them to calculate the probability of a firm *i* defaulting before or at time *T* based on information at time 0

$$PD_{i,0}=1-\{[1-N(u_1)]-\varpi[1-N(u_2)]\} \tag{A2}$$

$$u_1=\frac{K-\left(\mu_i-\frac{\sigma_i^2}{2}\right)T}{\sigma_i\sqrt{T}} \qquad u_2=\frac{-K-\left(\mu_i-\frac{\sigma_i^2}{2}\right)T}{\sigma_i\sqrt{T}}$$

$$\varpi=\exp\left(\frac{2K\left(\mu_i-\frac{\sigma_i^2}{2}\right)}{\sigma_i^2}\right) \qquad K=\ln\left(\frac{1}{k_0}\right)$$

Following Nickel and Perraudin (1999) they derive a mapping from the observable equity-liability ratio, $y=X/L$, to the unobservable asset liability ratio which in the case of a suitable default point is $y=k-1$.

Appendix 2

Moments of a truncated normal distribution

Assume that $x \sim N(\mu,\sigma^2)$ and C is a constant, then

$$E(x|x<C)=\mu+\sigma f(\gamma)$$

$$Var(x|x<C)=\sigma^2(1-g(\gamma))$$

with

$$\gamma=(C-\mu)/\sigma$$

$$f(\gamma)=-\varphi(\gamma)/\Phi(\gamma)$$

$$g(\gamma)=f(\gamma)\,(f(\gamma)-\gamma)$$

where φ/Φ are the normal distribution/cumulative normal distribution function.

Appendix 3

Table A3(a). GLS by industry with state-dependent coefficients – pre-October 1992

	State	MANUF	CONSTR	RETAIL	TSC	MINING	BUS
GDP^e	Low	6.670***	13.057***	4.653***	4.283*	1.263	10.140***
	Normal	0.580	1.979*	3.302***	-1.074	2.230	0.978
	High	2.726***	3.312***	2.304***	1.476	5.395***	3.179***
State diffs. (p-val)		0.000	0.000	0.339	0.175	0.392	0.000
TB3	Low	-0.032***	-0.057***	-0.035***	-0.057***	-0.065***	-0.027***
	Normal	-0.025***	-0.039***	-0.028***	-0.035***	-0.030***	-0.028***
	High	-0.042***	-0.044***	-0.037***	-0.047***	-0.033***	-0.034***
State diffs (p-val)		0.000	0.013	0.088	0.024	0.005	0.256
SPR	Low	-0.051***	-0.024***	-0.030***	-0.061***	-0.042**	-0.036***
	Normal	-0.029***	-0.035***	-0.022***	-0.047***	-0.030***	-0.029***
	High	-0.039***	-0.017***	-0.034***	-0.041***	-0.011	-0.025***
State diffs (p-val)		0.004	0.068	0.221	0.541	0.294	0.517
RPI	Low	0.689	4.965***	1.878***	1.098	-2.199	2.528***
	Normal	0.362	-1.161**	0.646	0.786	1.701*	1.201***
	High	-2.763***	-3.628***	-2.055***	-2.969***	-4.288***	-2.250***
State diffs (p-val)		0.000	0.000	0.000	0.000	0.000	0.000
EER	Low	-0.566***	-0.598***	-0.289*	-0.444	-0.078	-0.429***
	Normal	-0.717***	-0.617***	-0.648***	-0.612***	-0.852***	-0.744***
	High	-0.730***	-0.252*	-0.029	-0.701***	-0.884***	-0.305***
State diffs (p-val)		0.519	0.162	0.001	0.760	0.191	0.015
OIL	Low	0.039**	-0.024	0.029	0.018	0.048	0.057**
	Normal	-0.076***	-0.127***	-0.050**	-0.120***	0.055	-0.099***
	High	0.005	-0.102***	0.006	-0.000	0.146***	-0.040**
State diffs (p-val)		0.000	0.03	0.04	0.018	0.319	0.000
VOL	Low	-0.000	-0.002***	-0.001*	-0.000	0.002*	-0.002***
	Normal	-0.003***	-0.004***	-0.002***	-0.004***	-0.003***	-0.003***
	High	-0.002***	-0.001**	-0.001	-0.003***	0.001	-0.001***
State diffs (p-val)		0.000	0.000	0.009	0.000	0.000	0.000
ERP	Low	-0.053***	-0.076***	-0.046***	-0.049***	-0.095***	-0.036***
	Normal	-0.097***	-0.089***	-0.100***	-0.122***	-0.091***	-0.114***
	High	-0.147***	-0.131***	-0.129***	-0.178***	-0.105***	-0.169***
State diffs (p-val)		0.000	0.007	0.000	0.000	0.896	0.000
PE-US	Low	0.007***	0.004	0.008***	0.001	-0.002	0.009***
	Normal	0.021***	0.021***	0.015***	0.017***	0.026***	0.021***
	High	0.015***	0.010***	0.019***	0.016***	0.021***	0.012***
State diffs (p-val)		0.000	0.000	0.006	0.002	0.000	0.000

The dependent variable in each case is the excess stock return over the risk free rate. Panels are unbalanced, with estimation covering the period April 1982–December 2002. The 'state-diffs' row for each factor presents the p-value from a joint Wald test of coefficient equality across states of the world. Each regression includes a constant term (not reported). ***=coefficient significant at 1%; **=coefficient significant at 5%; *=coefficient significant at 10%.

Table A3(b). GLS by industry with state-dependent coefficients – post-October 1992

	State	MANUF	CONSTR	RETAIL	TSC	MINING	BUS
GDP^e	Low	5.578**	13.741***	-2.735	9.898*	-6.161	3.916
	Normal	8.017***	5.862***	8.351***	9.211***	1.423	9.150***
	High	-1.648	-10.892***	-9.058**	-6.961	-27.932***	10.098**
State diffs (p-val)		0.012	0.000	0.000	0.062	0.006	0.305
TB3	Low	-0.013	-0.022**	-0.009	-0.009	0.015	0.001
	Normal	-0.0214***	-0.035***	-0.0187***	-0.0256***	-0.0436***	-0.0254***
	High	0.001	-0.009	0.015	-0.018	0.016	-0.008
State diffs (p-val)		0.014	0.027	0.008	0.646	0.003	0.019
SPR	Low	-0.009	0.002	-0.007	-0.009	-0.014	0.019
	Normal	-0.0227***	-0.0429***	-0.01074*	-0.02968***	-0.0254**	-0.0196***
	High	-0.0608***	-0.0355**	-0.04314**	-0.07899***	-0.046	-0.04537***
State diffs (p-val)		0.002	0.004	0.167	0.084	0.750	0.001
RPI	Low	-2.15161***	-2.47479***	-1.7008**	-4.31328***	-1.428	-3.62336***
	Normal	-0.59085**	-0.95584***	-0.61012*	-3.43664***	0.405	-1.45758***
	High	-1.62747***	0.502	0.940	-2.25523*	2.744	-3.03796***
State diffs (p-val)		0.025	0.030	0.092	0.514	0.216	0.009
EER	Low	-0.23943***	-0.196	-0.026	-0.311	-0.321	-0.49333***
	Normal	-0.35529***	-0.18197**	-0.32669***	-0.6065***	-0.33061*	-0.83666***
	High	-0.64113***	-0.005	-0.245	0.552	0.277	-0.306
State diffs (p-val)		0.127	0.755	0.187	0.014	0.520	0.015
OIL	Low	-0.003	0.014	-0.013	-0.015	0.166416*	-0.06752*
	Normal	0.010	0.0668***	0.006	0.015	0.097729***	-0.017
	High	-0.11657***	-0.022	-0.076	-0.119	0.127	-0.15377***
State diffs (p-val)		0.016	0.173	0.412	0.333	0.752	0.031
VOL	Low	0.001	0.002444**	-0.00193*	0.001	-0.001	0.002335**
	Normal	-0.00342***	-0.0029***	-0.00333***	-0.00462***	-0.00402***	-0.00419***
	High	-0.00429***	-0.00412***	-0.00487***	-0.00663***	-0.00405***	-0.00507***
State diffs (p-val)		0.000	0.000	0.009	0.000	0.448	0.000
ERP	Low	-0.07848***	-0.07017***	-0.10946***	-0.07563**	-0.057	-0.07352***
	Normal	-0.077***	-0.07439***	-0.04977***	-0.12277***	-0.12621***	-0.08685***
	High	-0.027	-0.011	-0.027	-0.07697*	-0.073	0.032
State diffs (p-val)		0.065	0.080	0.044	0.310	0.298	0.000
PE-US	Low	0.010903***	0.004	0.012324**	0.016175**	0.028416***	0.024238***
	Normal	0.003922***	0.005092***	0.005597***	0.004696***	0.000	0.005166***
	High	0.003	0.002	-0.00737**	0.002	0.000	0.004
State diffs (p-val)		0.119	0.493	0.000	0.249	0.015	0.000

The dependent variable in each case is the excess stock return over the risk free rate. Panels are unbalanced, with estimation covering the period April 1982–December 2002. The 'state-diffs' row for each factor presents the p-value from a joint Wald test of coefficient equality across states of the world. Each regression includes a constant term (not reported). ***=coefficient significant at 1%; **=coefficient significant at 5%; *=coefficient significant at 10%.

Charts

Chart 1

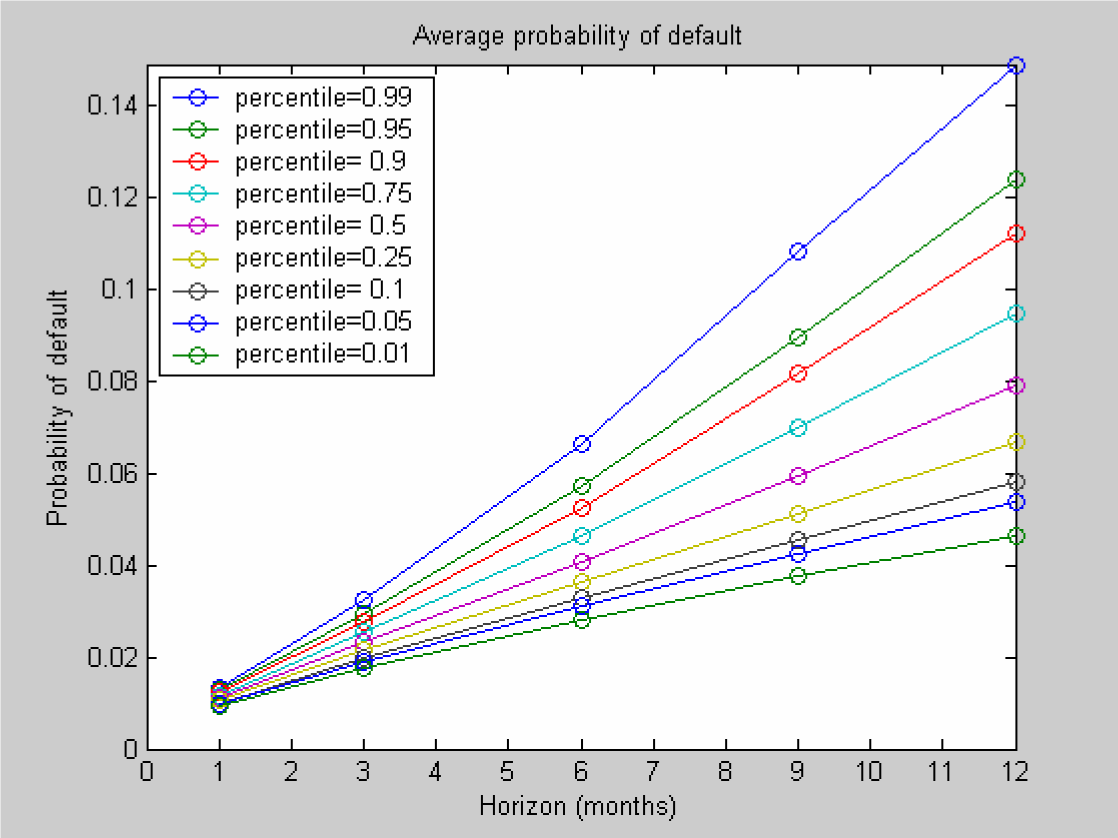


Chart 2

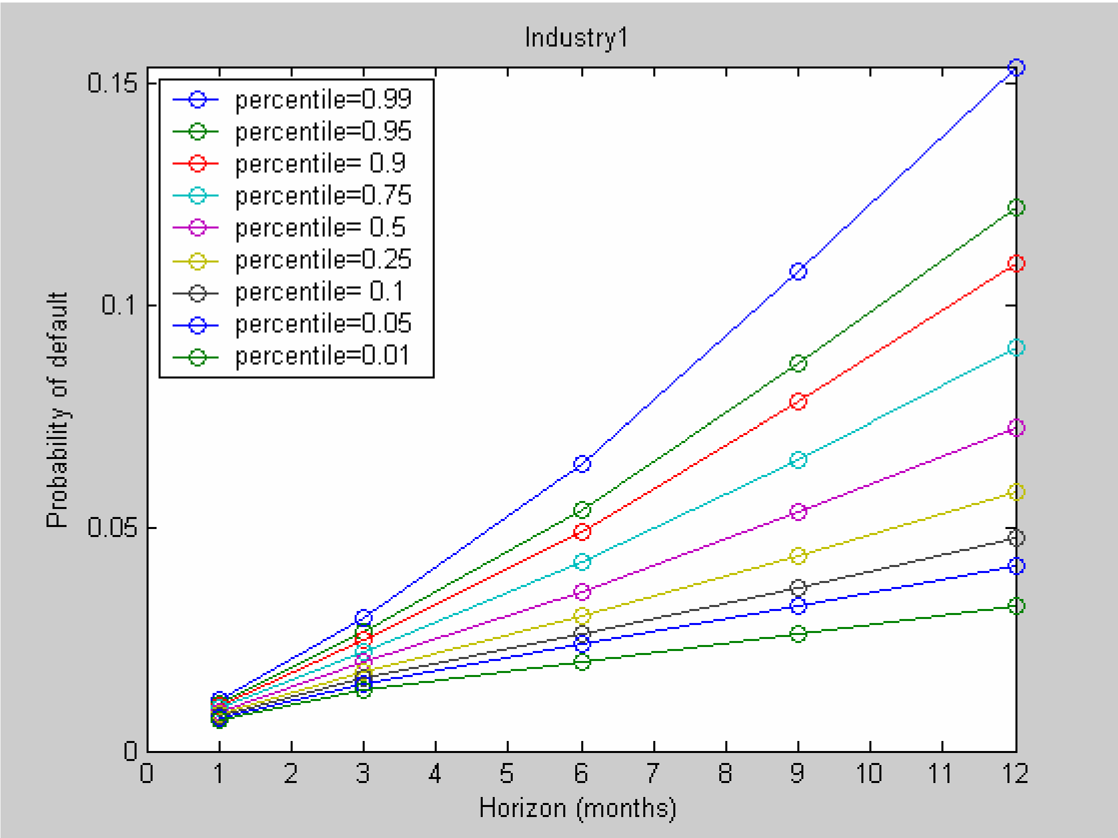


Chart 3

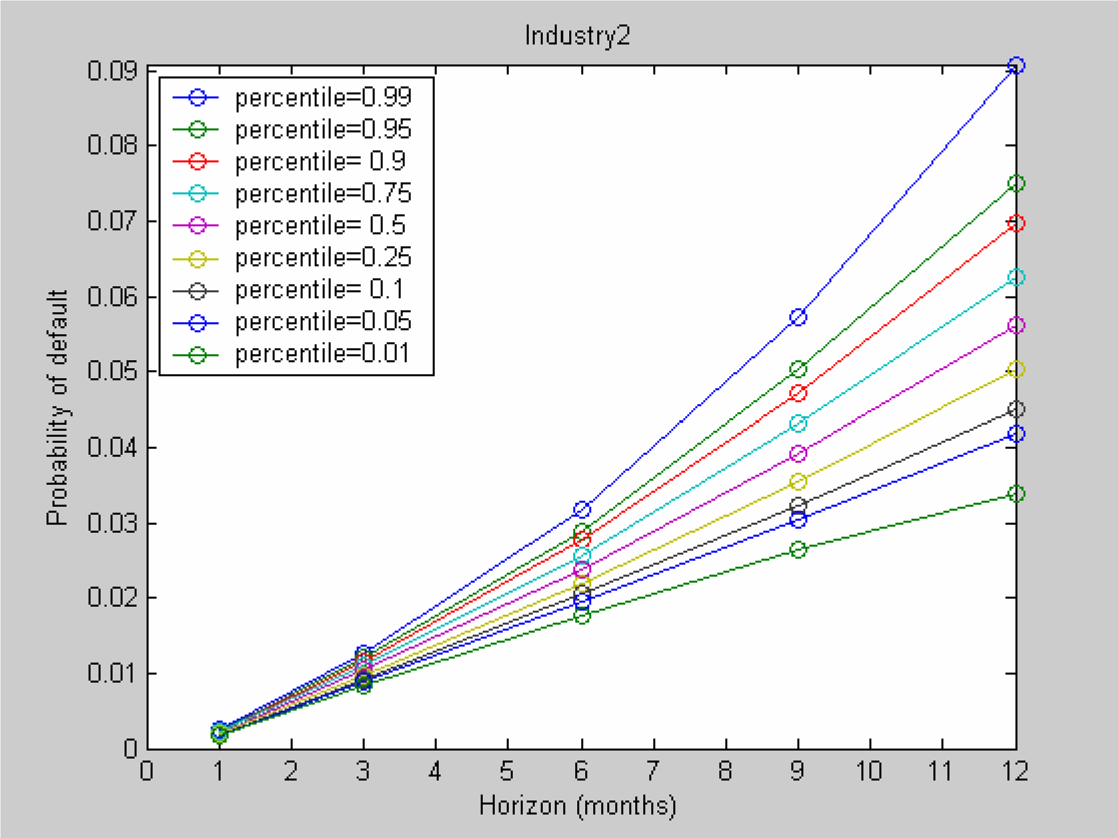


Chart 4

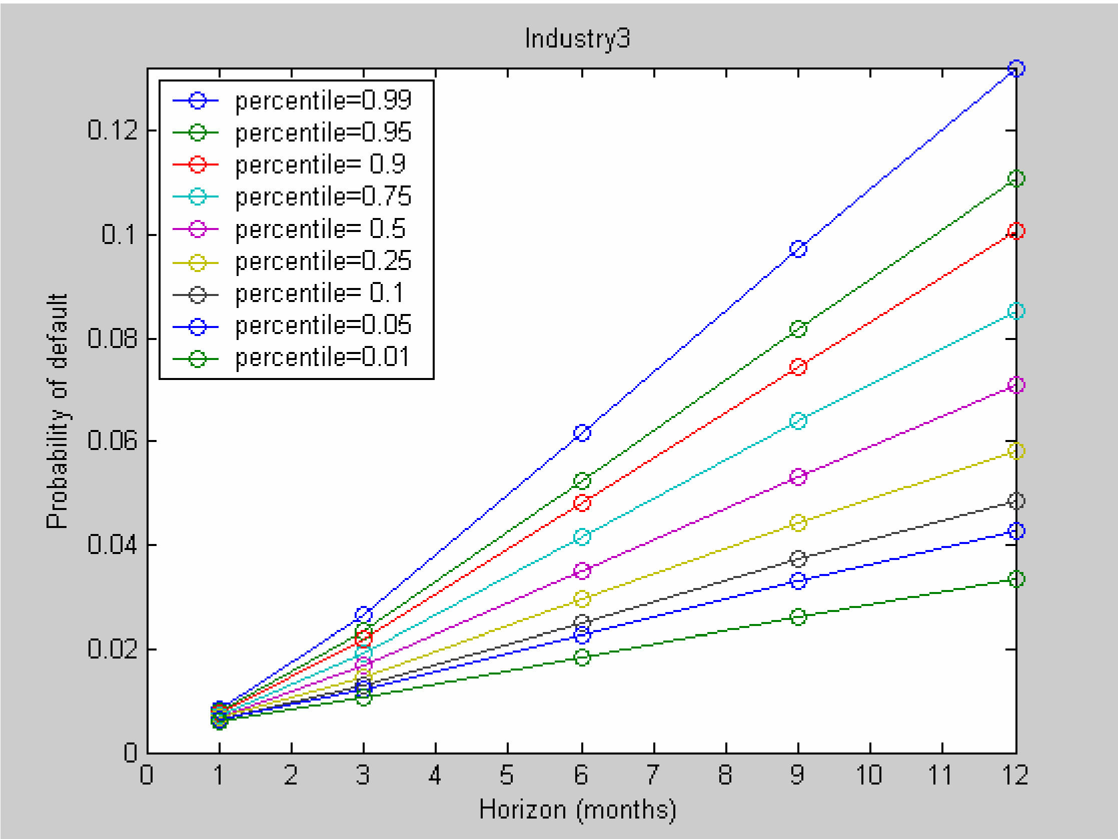


Chart 5

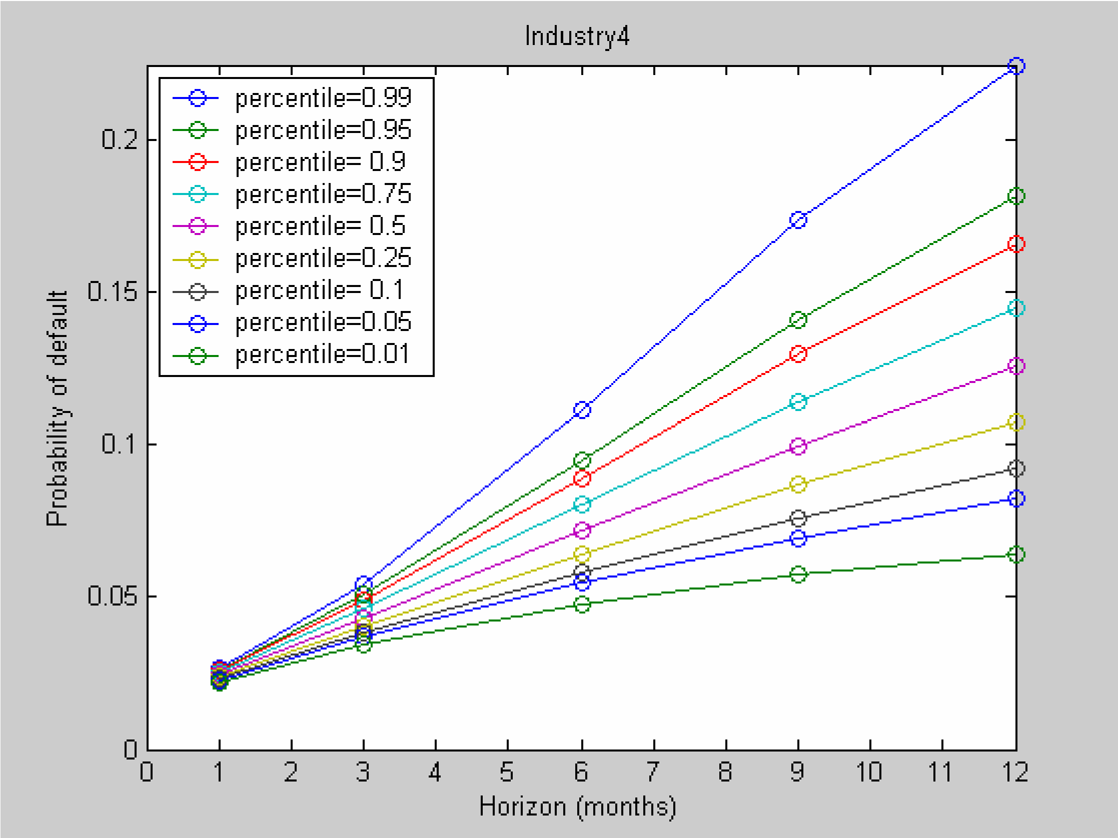


Chart 6

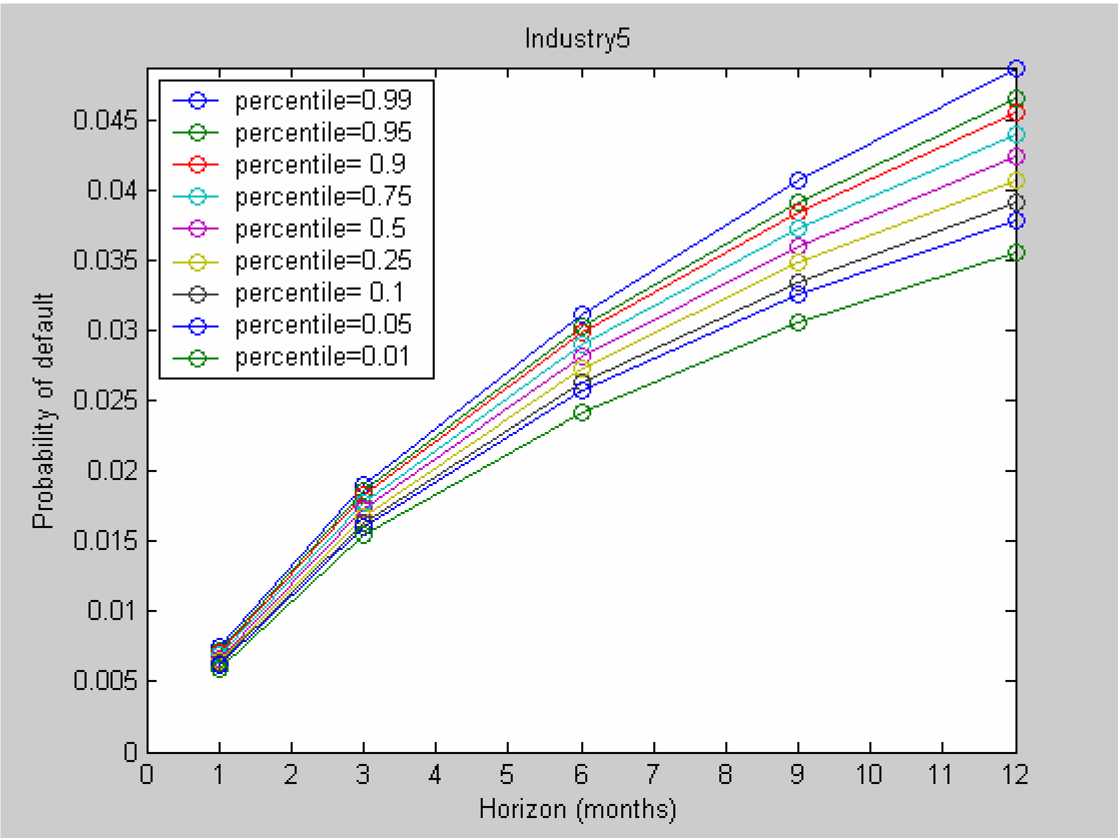


Chart 6

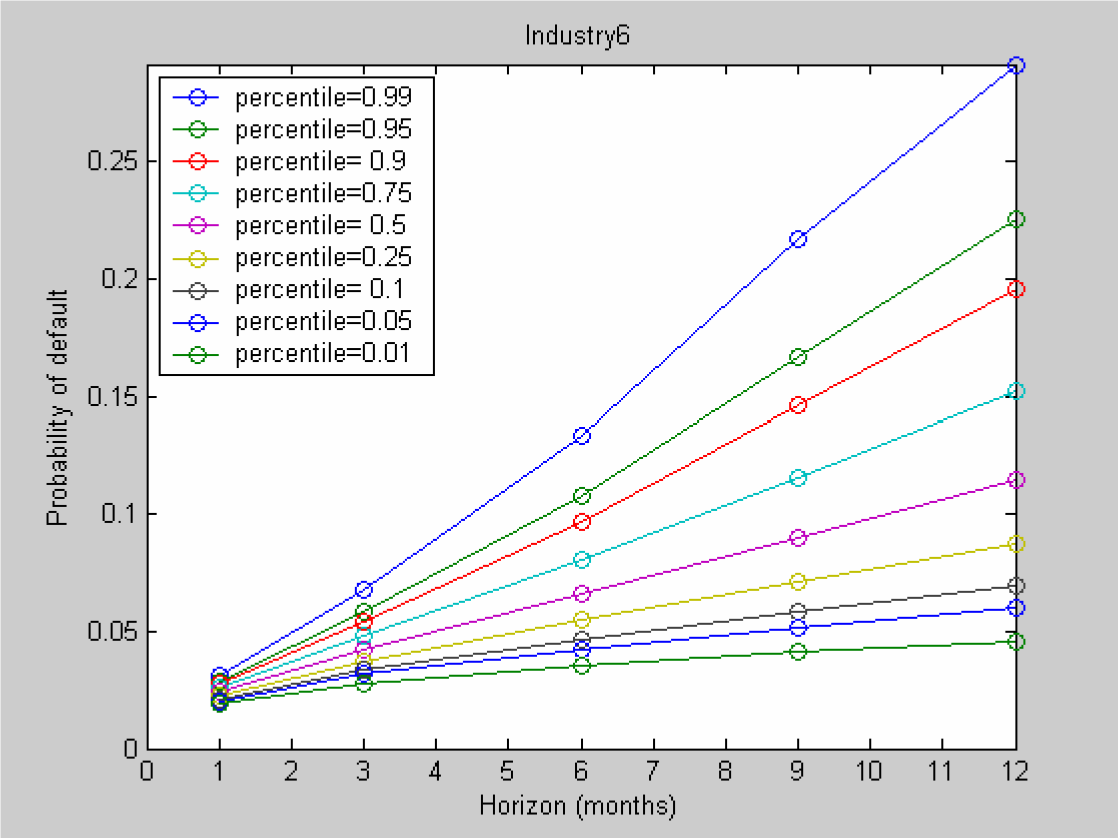


Chart 8

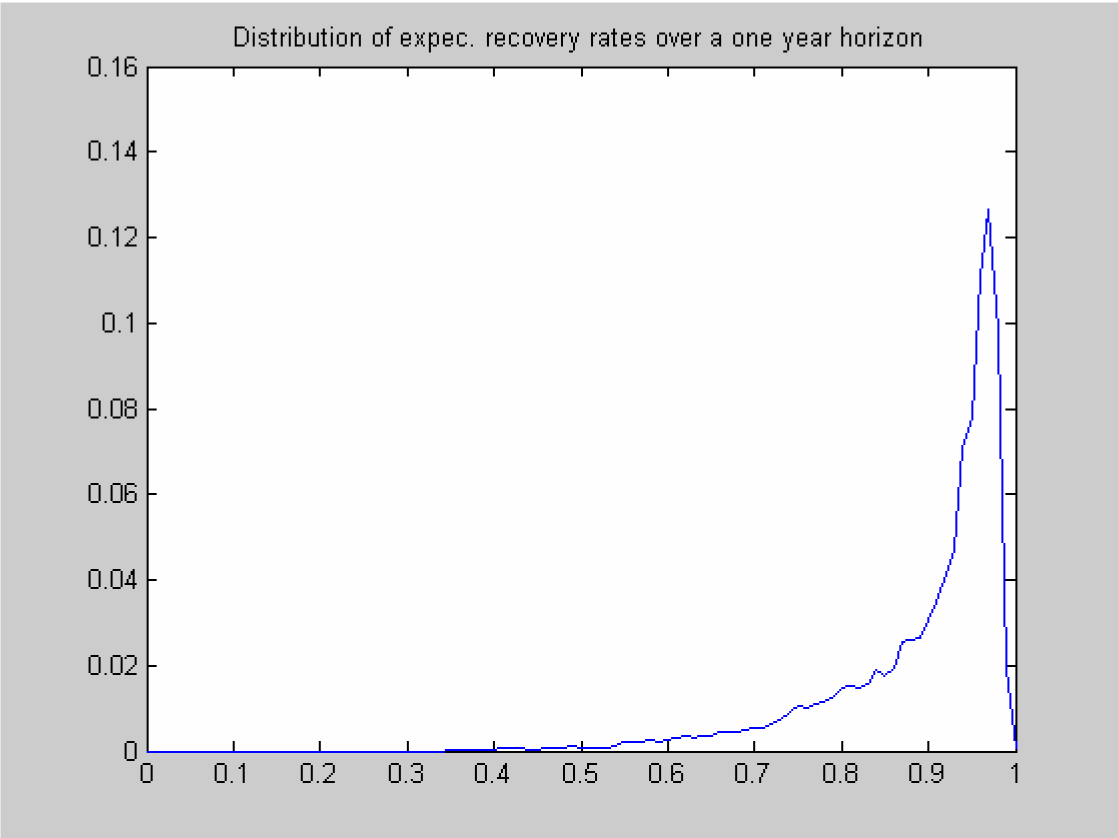


Chart 9

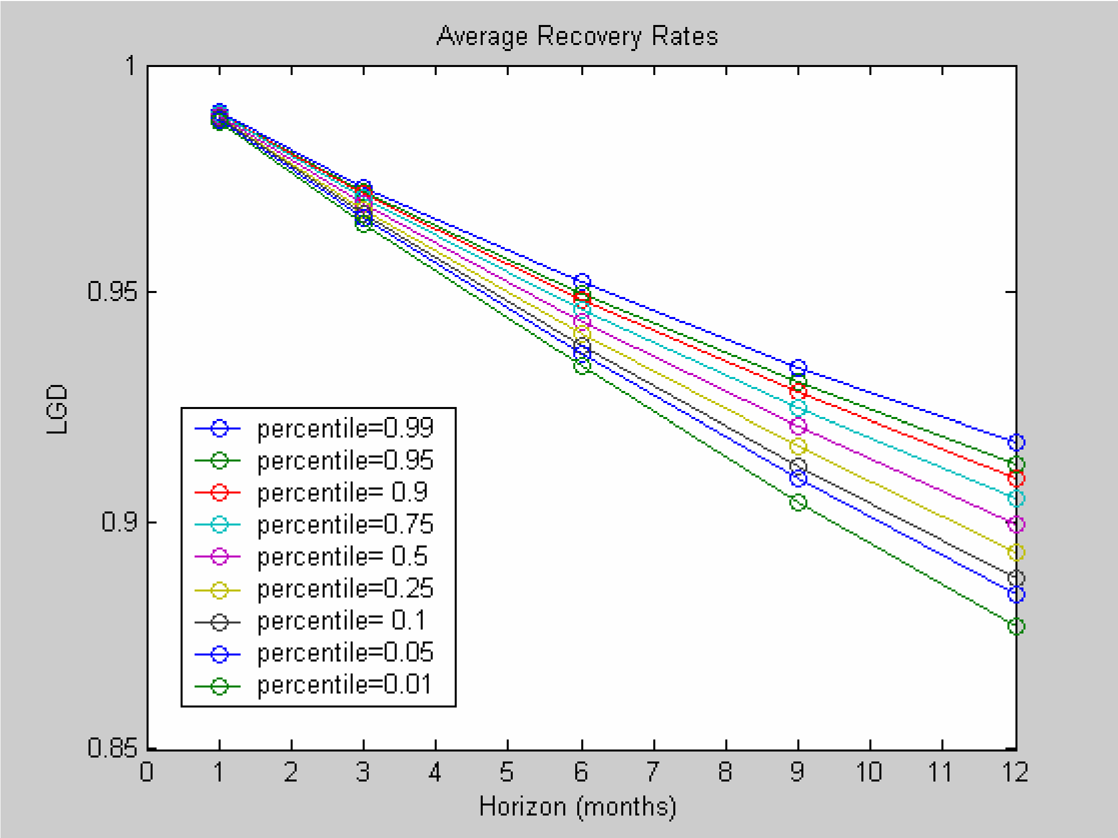


Chart 10

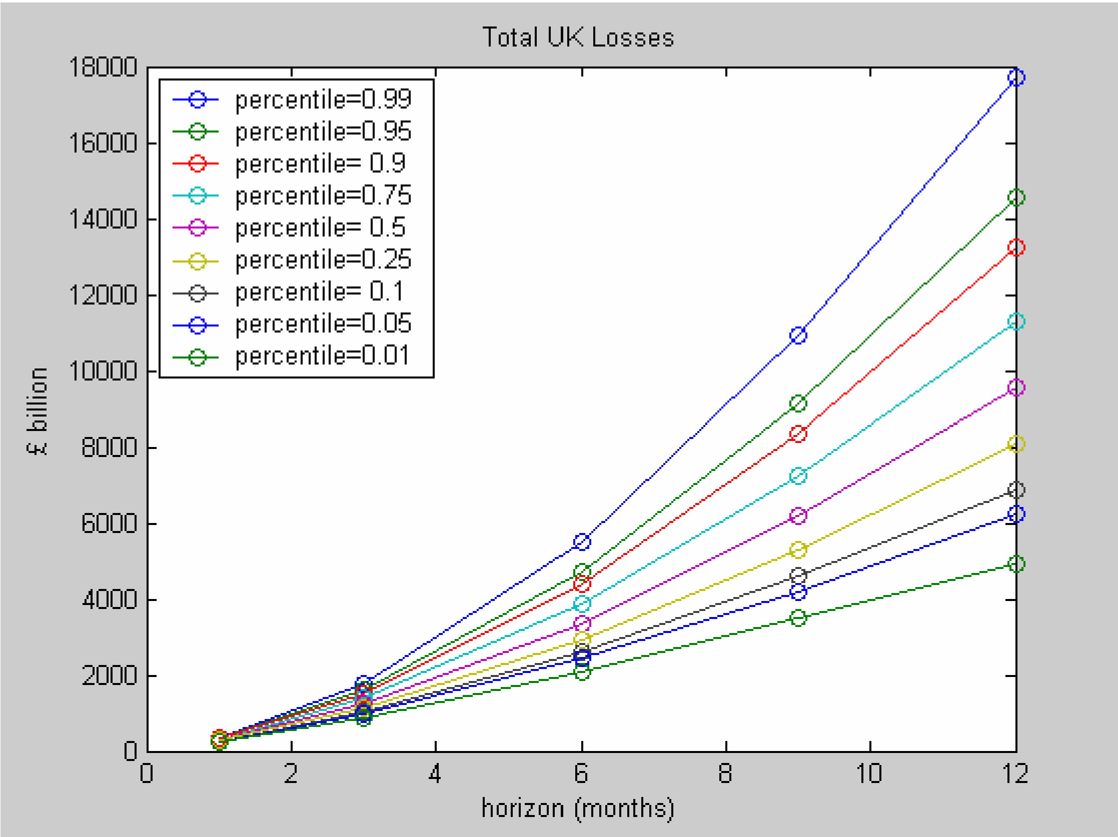


Chart 11

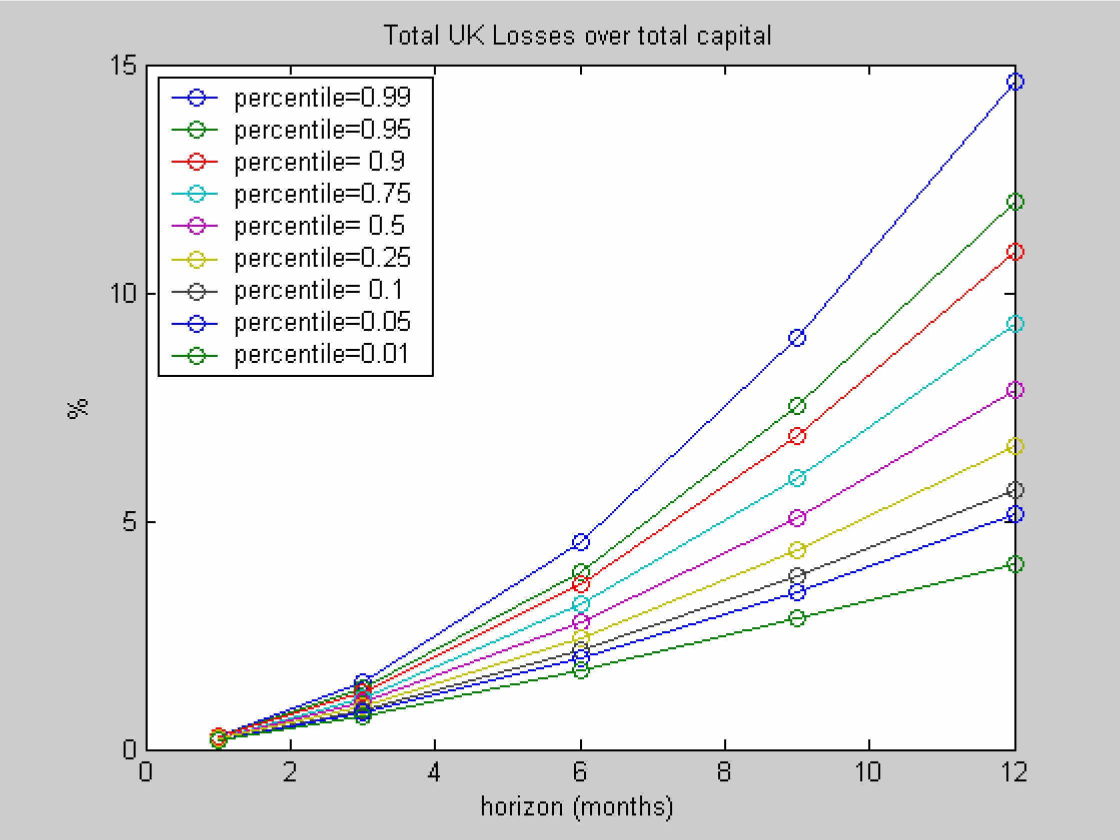


Chart 12

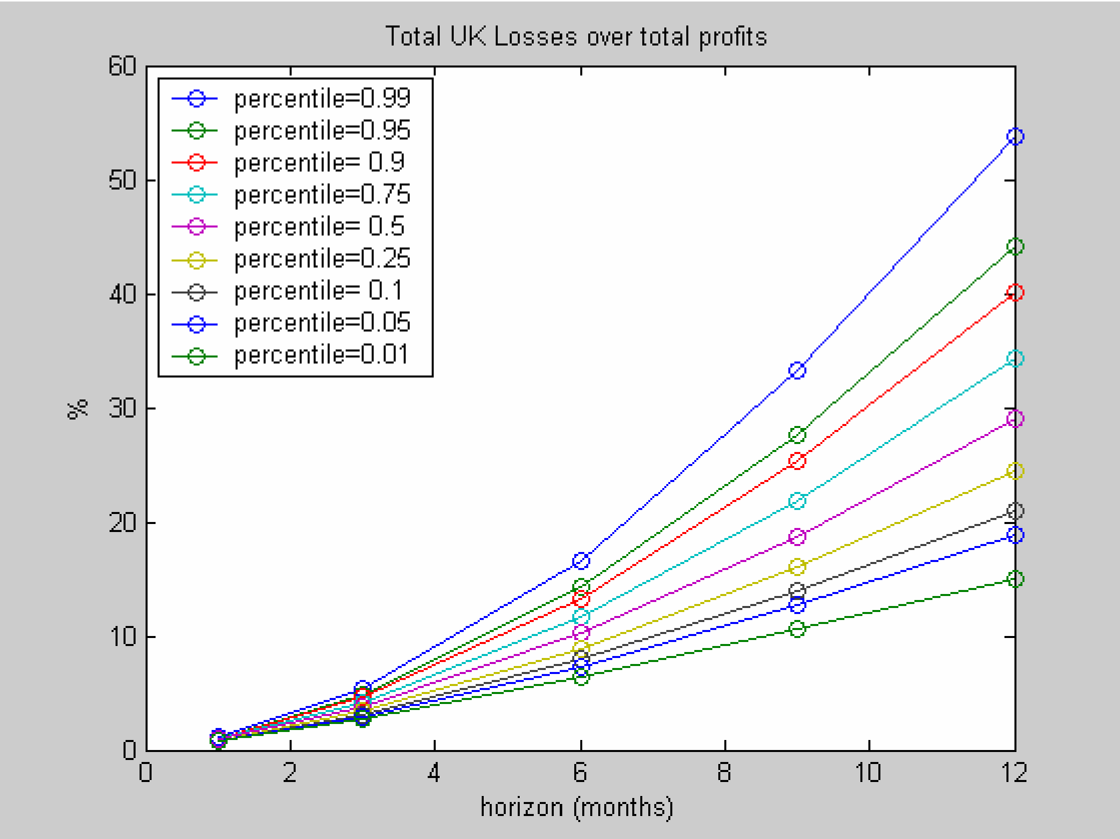
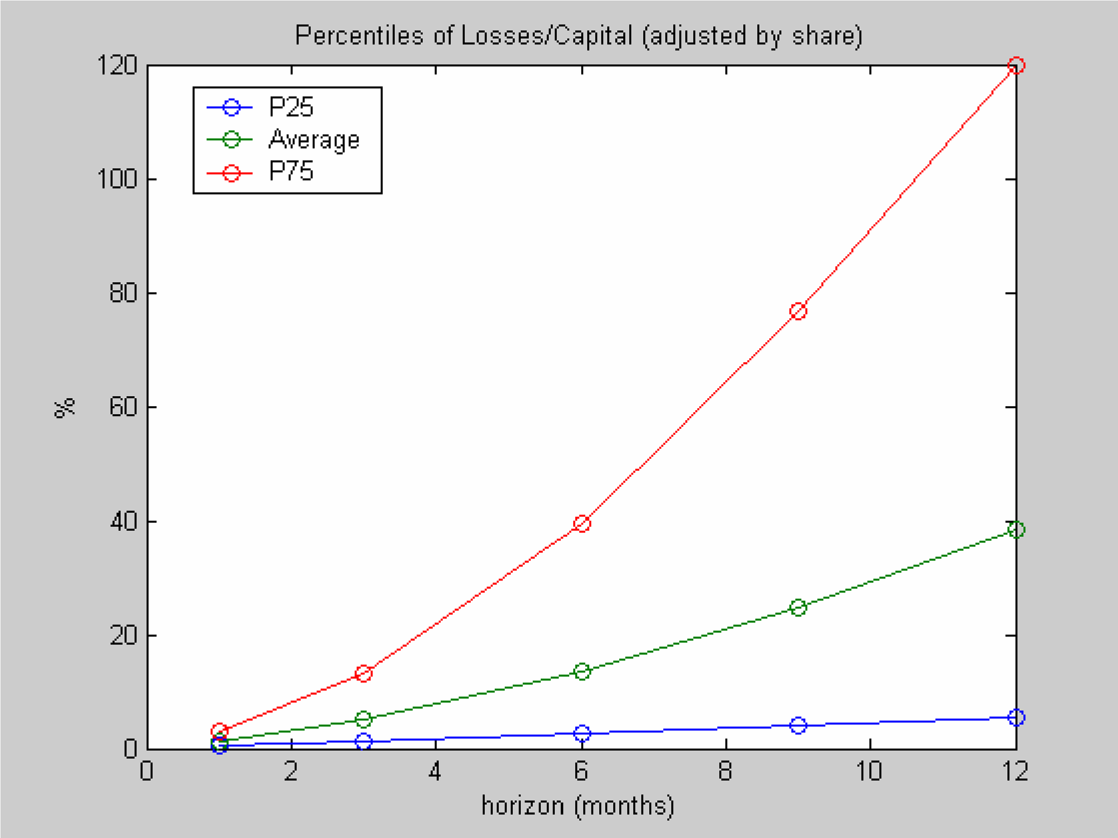


Chart 13



Percentiles of Losses/Capital (adjusted by share)
P25
Average
P75
%
120
100
80
60
40
20
0
0
2
4
6
8
10
12
horizon (months)