Benchmarking Deutsche Bundesbank's Default Risk Model, the KMV® Private Firm Model® and Common Financial Ratios for German Corporations

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November 2000

Abstract:

By comparing Gini curves and Gini coefficients that are determined on the same underlying dataset, we assess the discriminative power of Deutsche Bundesbank's Default Risk Model, KMV®'s Private Firm Model® and common financial ratios for German corporations. While the purpose of the Bundesbank Default Risk Model is to decide whether a collateral is eligible for refinancing purposes, the model does this by assessing the creditworthiness of the individual borrowing company. Likewise, the goal of KMV's Private Firm Model is to determine probabilities of default. However in both cases a best possible discriminative power is desirable. In this paper we show that both the statistical model (discriminant analysis) that is the first step in the Bundesbank's system and the structural model of KMV (Private Firm Model) provide

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[1] The views expressed here are those of the authors and not necessarily those of the Deutsche Bundesbank. The authors thank Stefan Hohl (Deutsche Bundesbank), Jing Zhang (KMV) and Stephen Kealhofer (KMV) for comments and recommendations. Special thanks is extended to Francis Lim (KMV) for his many hours of assistance on providing testing analytics and guidance on testing approaches.

powerful approaches to credit analysis with similar results. When incorporating additional information gained from other sources than the financial statements and market trends, power of discrimination can further be improved as demonstrated by an expert system that is the second step of the Deutsche Bundesbank’s system.

The focus of the paper is that of testing the performance of the models not to compare the model approaches in detail. The model construction and features are briefly described rather than exhaustively analysed.

1. Introduction

In an ideal (theoretical) world, probabilities of default (PDs) could directly be assigned to obligors. In such a world the model builder would know the probability distribution of future defaults within the population of borrowers. This information is, however, unknown to the model builder a priori. Due to this data restriction, however, usually a two-step approach is carried out. First, default risk models assign a credit score for each corporate observation, which leads towards a ranking between the contemplated corporations. Second, given the ranking, corporations are mapped to an internal grade for which a PD has to be estimated.

The ranking of the corporations under analysis by a model can be considered a separate issue from that of mapping to a PD estimate. Although the mapping of a default measure to a particular default probability is needed to achieve necessary quantitative properties for further analysis (for example as an input to portfolio management tools, risk pricing etc.), the comparison of model power across different models and modelling approaches, does not need to incorporate the additional step of mapping to a default probability. In essence a good ranking of companies by credit quality can be mapped to different databases of default experience and thus provide different answers in terms of PD. The calibration of the model to a PD does not affect the power of the model as tested in this paper.

The testing of rating systems involves two different aspects: First, the rating methodology, which establishes the ranking between corporations and second, the test of PDs itself. One has to keep in mind that these estimated PDs are usually based on historical default observations, as forecasting default rates requires empirical trend studies. It is usually much easier to recalibrate a more powerful model than to add statistical power to a calibrated model. For this reason, tests of power are more important in evaluating credit models than tests of calibration. This does not imply that calibration is not important, only that it is easier to carry out. Therefore, we focused our study on testing the discriminative power by using Gini curves[i].

2. Gini curves

In the following we discuss the theoretical background of Gini curves[ii], which according to our view are a valuable and simple tool to determine the discriminative power of rating systems.

The Gini coefficient, respectively Gini concentration ratio, belongs to the well-known and frequently used measures of inequality, such as variance, coefficient of variation, standard deviation of logarithms, entropy measure, etc. Its popularity may be related to the fact that the Gini coefficient is derived from the Gini curve[iii] that can be graphed and thus conveys considerable intuitive feelings for the degree of inequality. A precise definition of Gini curve, Gini coefficients and their determination is given in appendix 1.

In a statistical framework we face two possible kinds of errors: A Type I error, which indicates low default risk when in fact the risk is high, and a Type II error, which conversely indicates high default risk when in fact risk is low. From a supervisory viewpoint, Type I error is more problematic, as it produces higher costs[iv]. There are different proposals for objective measures to compare the performance of default risk measures, e.g. Gini curves, Conditional Information Entropy Ratios or Mutual Information Entropy[v].

All of these measures aim to determine the power of discrimination that a model exhibits in warning of default risk over a given horizon. They are also all limited to the same data, namely a sample of defaults and a sample population of model outputs. Regardless of the approach taken, true testing will only be effective when done out-of-sample on a non-biased dataset. The choice of testing tools must come second to the "rationalization" of the data and the data selection criteria. As a quantitative method for minimum standards, the usage of Gini curves has the appealing advantage due to their simplicity. Gini curves test the power of discrimination a model offers across a population for different lead times. That is, they measure the model's ability to identify the firms that are going to default for chosen lead times (e.g. 12 months or 24 months). This allows for the simultaneous comparison of a variety of credit scores regardless of the respective metric used to construct each credit score.

The determination of Gini curves depends on the rating process. That process assigns risk scores to companies and allows for ranking between these companies. To obtain Gini curves, companies are first ordered by their risk (e.g. risk score or rating class). For a given fraction x% of the total number of companies, a Gini curve is constructed by calculating the percentage y(x) of the defaulters whose risk score is equal to or lower than the one for fraction x. In other words, for a given x, y(x) measures the fraction of defaulters (out of total defaulters) whose risk scores are equal or lower than those of the fraction x (out of total companies) at the bottom of the rating.

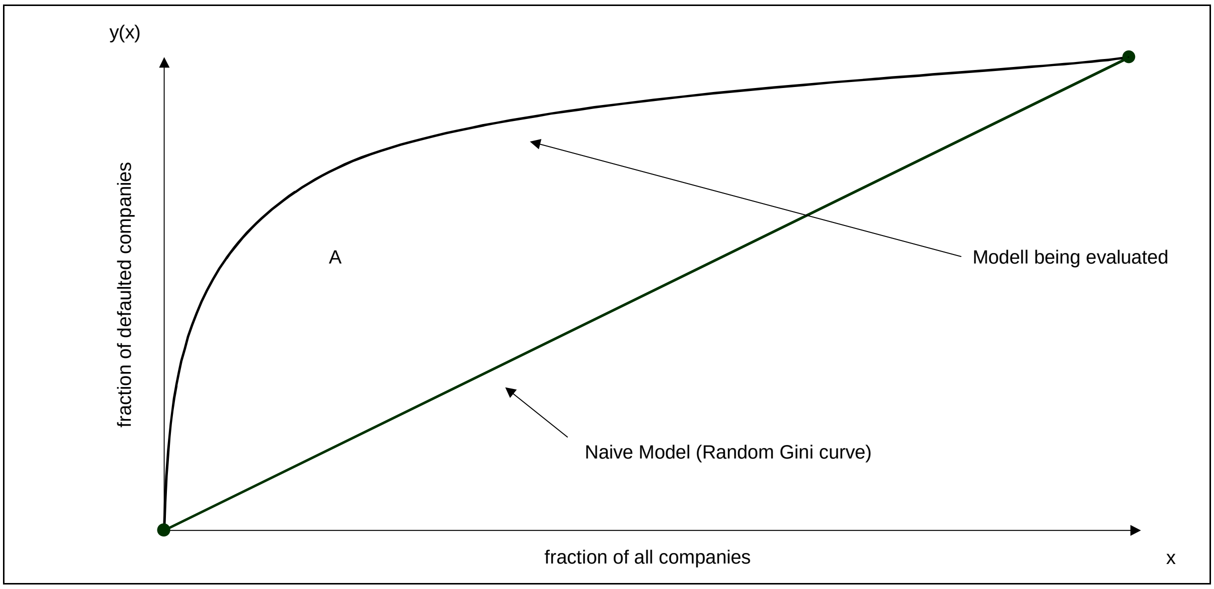


Figure 1

One would expect a concentration of defaults at the riskiest scores and non-defaults at the lowest scores. If the -- one would expect to capture a proportional fraction, i.e. generating a straight line (random Gini curve). A perfect model would produce the ideal Gini curve (Figure 2), which is a straight line capturing 100% of the defaults within a fraction of the population equal to the default rate of the sample.

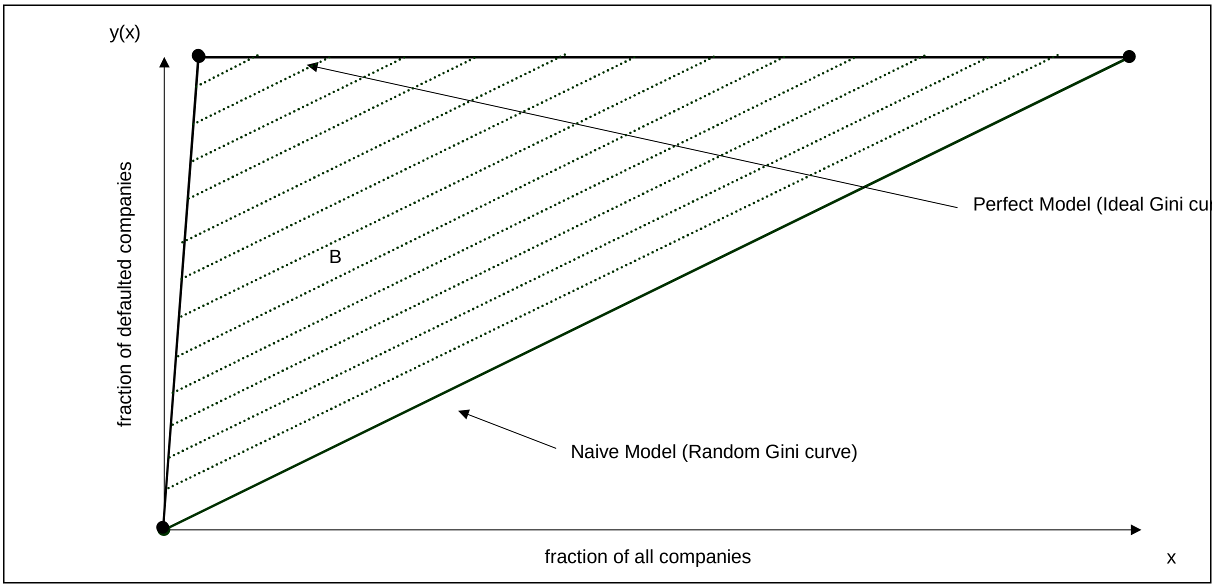


Figure 2

A possible performance measure is the **Gini coefficient** (GC). It is defined as the ratio of the area between a model's Gini curve and the random Gini curve to the area between the perfect Gini curve and the random Gini curve (Figure 3). The Gini coefficient is a fraction between zero and one. Risk measures with Gini coefficients that approach zero have little advantage over a random assignment of risk scores while those close to one display good predictive power.

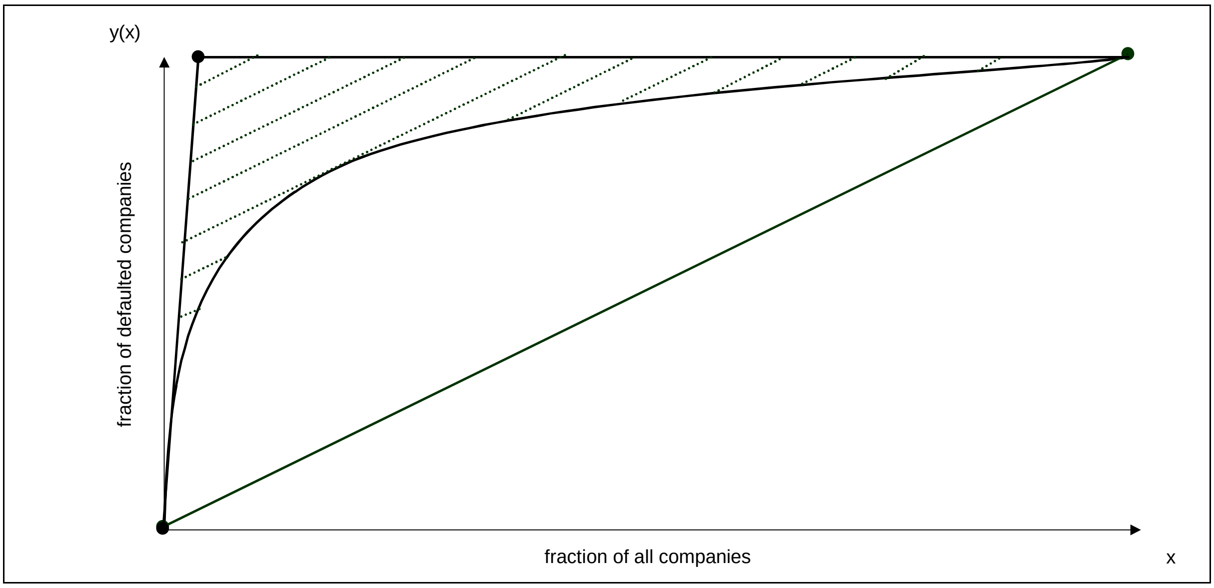


Figure 3

Although different Gini curves can provide similar Gini coefficients (Figure 4), Gini curves are certainly possible criteria for comparing internal rating systems.

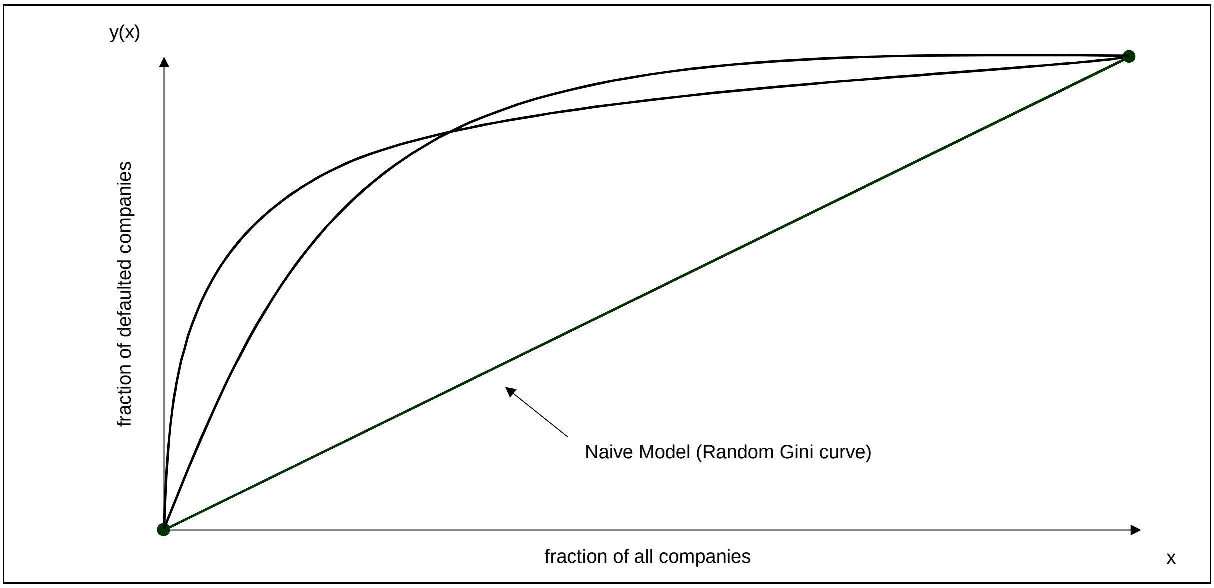


Figure 4

There is a mathematical relation between Gini curves and the default frequency curves. Given a default frequency curve, one can generate a Gini curve and vice versa. More powerful Gini curves correspond to steeper default frequency curves. Mathematically speaking, the default frequency equals the leftsided derivative of the Gini curve multiplied by the average default rate of the portfolio. And conversely, if the default rate is given, the Gini curve can be obtained by integration.
More details are given in appendix 1.

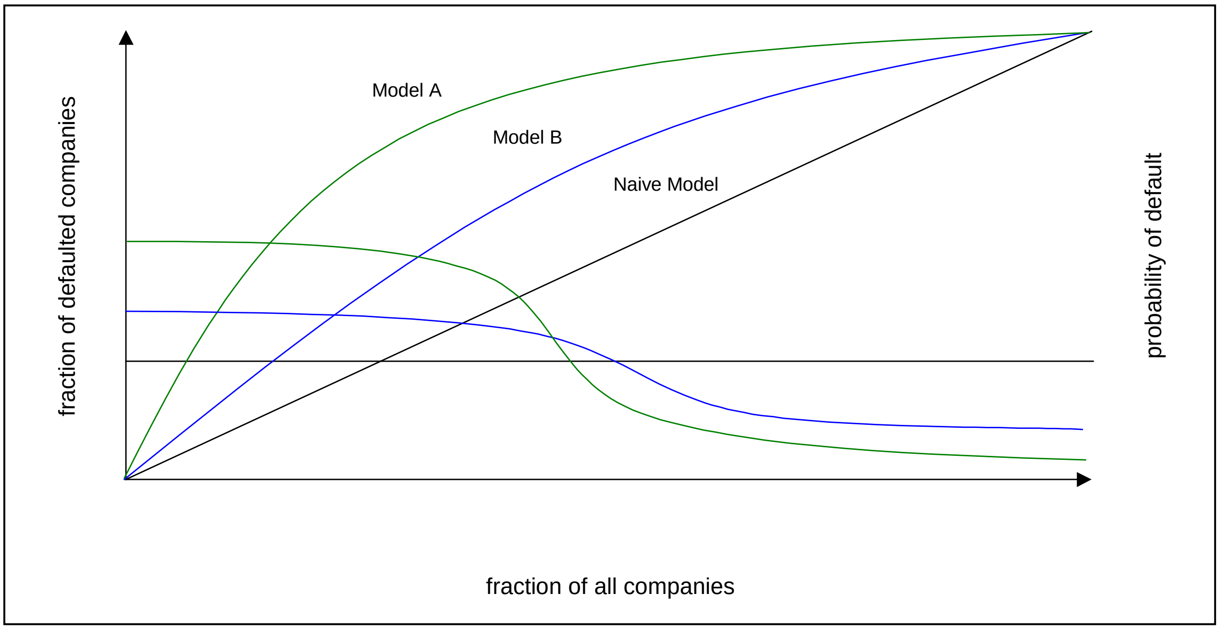


Figure 5

The Gini curves and Gini coefficient that are measured using the approaches described above are always wholly relative to the population of companies and defaulting company subset that is used in the performance test. The test will therefore only be meaningful when either results from testing two or more models performance on the same data or when comparing the same model across multiple populations and default sets.

3. Study Overview

The study of discriminative power in models as presented in this paper brings forth added dimensions through the cooperation of Deutsche Bundesbank with KMV Corporation. Although all testing has been done on the same underlying dataset, the models that have been tested have characteristics that set them apart from one another, which begs the need for careful interpretation of the testing outcome. Four types of models were put to the test. In one dimension, there are traditional credit rating techniques (e.g. financial ratios and subjective input) that are compared to models built from powerful statistical studies and the harvesting of market trend information. A second dimension is the origin of the model power. The statistical based models draw power from being partially fit on the sample dataset, whereas, the financial ratios and the KMV Private Firm Model® are devised independently of the underlying dataset on which they are tested. A summary of the model dimensions is included in figure 6 below.

Model	**Deutsche Bundesbank Discriminant Analysis**	**Deutsche Bundesbank Expert System**	**KMV® Private Firm Model®**	**Common Financial Ratios**
Model Type	Statistical (best fit model)	Statistical plus subjective input	Causal model using a structural approach (subjective input possible)	Quantitative ratio
Data inputs	2 years of financial statements	2 years of fin. statements (at minimum) plus expert opinion	1 year of financial statement plus market trend information provided by KMV	1 or 2 years of financial statement information

Experience the model was built on	Bundesbank database 1994-1996 (fitting) and credit analyst experience (constructing the model)	Bundesbank database 1994-1996 and credit analyst experience	Causal framework developed by KMV and behaviour observed amongst publicly quoted firms	Credit analyst experience

Figure 6

Common to all models is that their purported discrimination between good and bad credits. The testing has been focused on large corporate borrowers because corporations have the highest default risk and provide for the most onerous losses. This focus indicates that the cost of not using a model will be significant. In these tests, companies at minimum had to have greater than five million Euros in turnover. The high cost of default with a firm this size justifies a creditor using several models in the credit assessment process. The utilization of quantitative models in this manner allows for formal empirical validation of the model and encourages the credit institution to develop an internal benchmarking process.

4. Data

German companies are required by law to produce full financial accounts. There is, however, no requirement to share this information with the public. The result is that only the very largest firms that have a public interest in disclosing their financial standing will be available to the public will share information. The difficulty in accessing company balance sheets is exacerbated by bank information protection laws and policies. The Bundesbank is in a special position.

The Bundesbank purchased "fine trade bills" at the discount rate under its discount credit facility in order to provide liquidity to German banks. This activity meant that the Bundesbank had the need to evaluate the credit quality of the companies that the banks presented to the Bundesbank for refinancing.[vi] The collection of financial accounts for these companies was necessary in order for this system to work. The Bundesbank required very high data quality and would investigate the financial statements in great detail. Often this would result in requiring adjusting balance sheet items in order to present a conservative view of the companies.

To evaluate private sector assets, the Bundesbank introduced a standardised creditworthiness assessment procedure from the 1970s onwards. As the Bundesbank has no direct business dealings with the issuers or debtors of such collateral, this creditworthiness assessment procedure is based on firms' annual accounts data. These data are collected by the Bundesbank's branch offices and are fed into a central corporate balance sheet database. Depending on the extent of private collateral assets offered by Bundesbank's counter-parties, there can be between 50,000 and 60,000 balance sheets from firms of various size categories that are recorded in this database each year.

This corporate balance sheet database also contains information on defaults. As the Bundesbank has no direct commercial links with firms, the database only contains details of defaults that have become publicly known and noted by its branch offices. An enterprise is deemed to have defaulted if insolvency proceedings have been initiated against it. The legal preconditions for the initiation of such proceedings are laid down in the Insolvency Code, specifically the inability to meet due payments (section 17), over-indebtedness (section19) or the anticipated inability to meet due payments (section 18)[vii]. The above definition of default can be interpreted as conservative. A more liberal definition of default would necessarily include forced restructurings, voluntary asset sales because of economic distress and cases when banks may accept lenient credit terms to help companies through bad times. Inclusion of such definitions would change the observed default rates in the populations.

The present study covers all firms in the database whose turnover exceeded five million Euros at least once during the period 1993 – 1999. These are between 25.000 to 30.0000 companies. A yearly default distribution from the database and a breakdown of the study dataset by sector and turnover size category are shown in appendix 2.

The Bundesbank database of financial information is representative for German middle market borrowing.

5. The Bundesbank's Default Risk Model

To ascertain whether an issuer or debtor of business credits meets the Eurosystem's minimum credit standard, a preliminary credit assessment is prepared prior to the final credit decision using a standardised procedure ("eligible" or "not eligible") which is taken by the managers of the relevant Bundesbank main branch office in the Land Central Bank (LCB). This assessment procedure is based on the annual accounts data collated and processed by the relevant branch offices. Figure 6 provides a schematic overview of the entire creditworthiness assessment procedure.

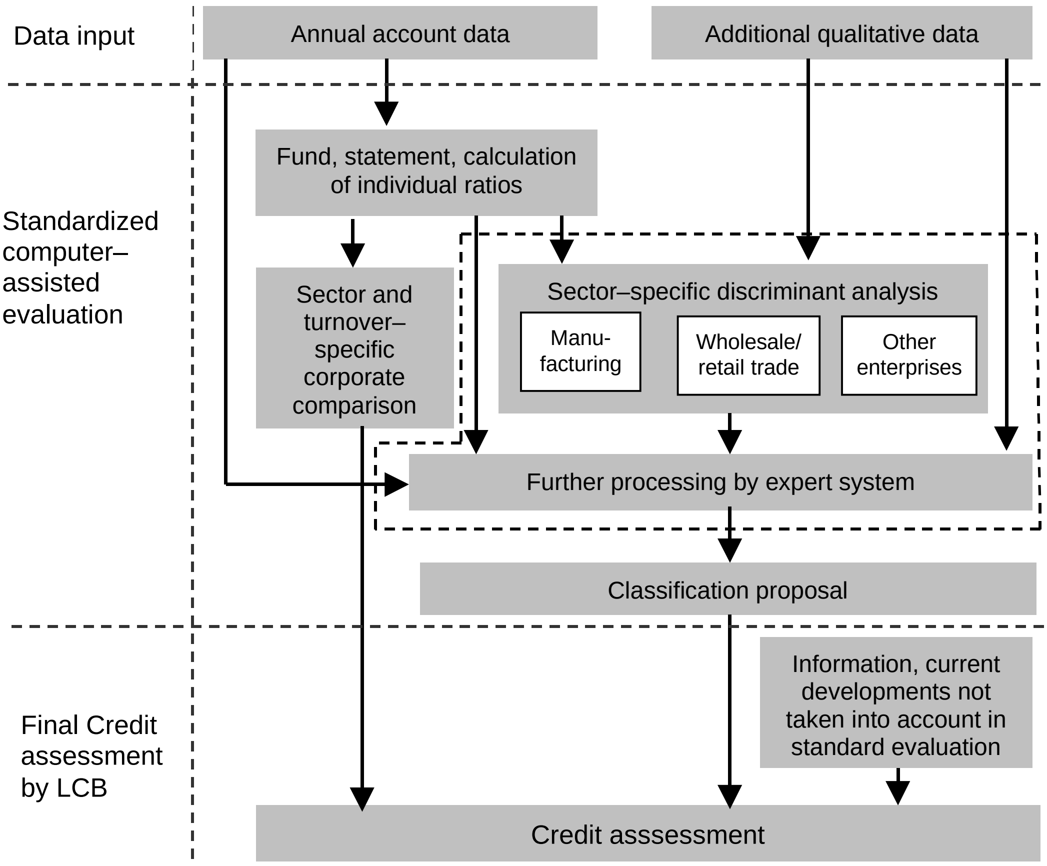


Figure 6: Schematic view of the creditworthiness assessment procedure introduced by the Bundesbank in 1998. The outlined area in the centre of the chart marks the modular system examined in this study.

The core of the creditworthiness assessment procedure is a two-step modular system that assigns an overall ratio Z to every firm whose credit standing is to be assessed. Depending on the value of this overall ratio, the modular system generates one of three classification proposals: "eligible", "not classifiable" and "not eligible". On the basis of this proposal, the managers of the relevant main office branch must then make their final credit decision of either "eligible" or "not eligible"; they have the discretionary power to alter the classification proposal if they have good reasons for doing so. Whatever their decision is, they must document it.

Individual ratios for calculating discriminant functions

Sector	Ratios in the discriminant function
Manufacturing	Equity/pension provision ratio[1] Return on total capital employed [2] Return on equity [3] Capital recovery rate [4] Net interest rate [5] Accounting practice
Wholesale/ retail trade	Equity ratio [6] Return on equity Capital recovery rate Accounting practice
Other Enterprises	Equity ratio Return on equity Capital recovery rate Accounting practice

1 Adjusted equity capital and pension provisions as % of total capital employed. – **2** Profit/loss before taxes on income and before interest paid as % of total capital employed. – **3** Profit/loss before taxes on income as % of adjusted equity capital. – **4** Net receipts/net expenditure as % of capital invested. – **5** Net interest result as % of turnover/total output. – **6** Adjusted equity capital as % of total capital employed.

The first step[viii] in the creditworthiness assessment procedure consists of a sector-specific discriminant analysis[ix] covering four to six individual business performance ratios derived from the processed annual accounts. For the purpose of this sector-specific discriminant analysis, the firms are allocated to one of three sectors: "manufacturing", "wholesale/retail trade" or the omnibus group "other enterprises" – which comprises all other sectors. This division was made in order to take due account of the peculiarities of the different sectors and to improve the informative value and accuracy of the discriminant functions by forming homogeneous groups. A deeper division would have been desirable, but this was not possible as the data material was not comprehensive enough[x].

As section 264 of the German Commercial Code states that a firm's annual accounts must present a "true and fair view of the net worth, financial position and results of the company", every discriminant function contains at least one business performance ratio from each of these three areas. The adjacent chart lists the business performance ratios used for each of the three sectors. As German accounting law allows firms a large measure of interpretative latitude when compiling the balance sheet, the discriminant functions include the qualitative feature "accounting practice". Via this feature, which the Bundesbank has reduced to a standardised choice, the firm's accounting practice is categorised as understating the earnings level (conservative), overstating the earnings level (progressive) or as neither understating nor overstating the earnings level (neutral). In the first case the value of Z is increased compared to a discriminant function, which does not contain this feature, in the second case Z is reduced and in the third case Z is virtually unchanged. All three discriminant functions have the same cut–off–point Z_{co}.

As the accuracy of allocation of a given firm to one of the two groups is deemed to increase with the distance of its overall ratio Z from the cut–off–point Z_{co} [xi]. Every firm can be allocated precisely to one of three creditworthiness groups after the first step, depending on its overall ratio. These three groups are

- "firms with a good credit standing", known as the A group (for Z much larger than Z_{co}),
- "firms with an indifferent credit standing", known as the B group (for Z around Z_{co}) and
- "endangered firms", known as the C group (for Z much smaller than Z_{co}).

Although the dataset can be classified quite well on the basis of the discriminant functions, the result is still not sufficiently clear-cut. For this reason, the overall ratio may be regarded as a variable that enables the dataset to be "pre-sorted" and which yields a rough corporate rating with the three grades A, B and C. In a next processing step, attention can then be focused on the B-grade firms in order to allocate as many as possible to either the A or C groups.

A so-called expert system[xii] is employed at this second processing step – primarily to further process these B-grade firms but also to obtain a more complete picture of A- and C-grade firmsxiii. It processes all the information that has not been considered so far; this includes such features as the legal form, firm size and age, the method used to finance the acquisition of fixed assets and the annual rates of change of various ratios as an indicator of corporate development.

This information is processed by the expert system on the basis of verbally formulated, statistically significant rules derived from business performance trends. The rules have the syntactic structure "If property 1 and property 2 and … and property N apply, then increase (or decrease) Z. The properties in the If part of the rule are quantifiable, initially imprecise, verbally formulated features (high, medium, low or decreasing, unchanged, increasing etc.). Thus each of these rules alters a firm's overall ratio if the firm satisfies the relevant rule conditions: if in general the rule conditions apply to sound firms then the overall ratio is increased, otherwise it is decreased. The amount of this increase or decrease depends on two factors:

1. How significant is a given rule? An "absolute rule weight" assigned to each rule determines the significance of this rule vis-à-vis other rules. The greater this absolute rule weight, the more a rule changes Z.
2. To what extent is a given rule satisfied in a given case? To ascertain this, membership functions are ranked[xiv] according to the extent to which properties 1 to N are fulfilled on a scale from 0 (meaning "not fulfilled") to 1 (meaning "completely fulfilled"), thus determining each rule's "individual degree of fulfilment".

The product of "absolute rule weight" and "individual degree of fulfilment" is known as rule weight and is directly proportional to the change in Z that is caused by a given rule.

After the dataset has been processed by the expert system a new – adjusted – overall ratio Z^{new} is calculated using the formula $Z^{new} = Z + \Delta Z$ where ΔZ is the adjustment contribution of the expert system dependent on the rule weights of the applied rules:

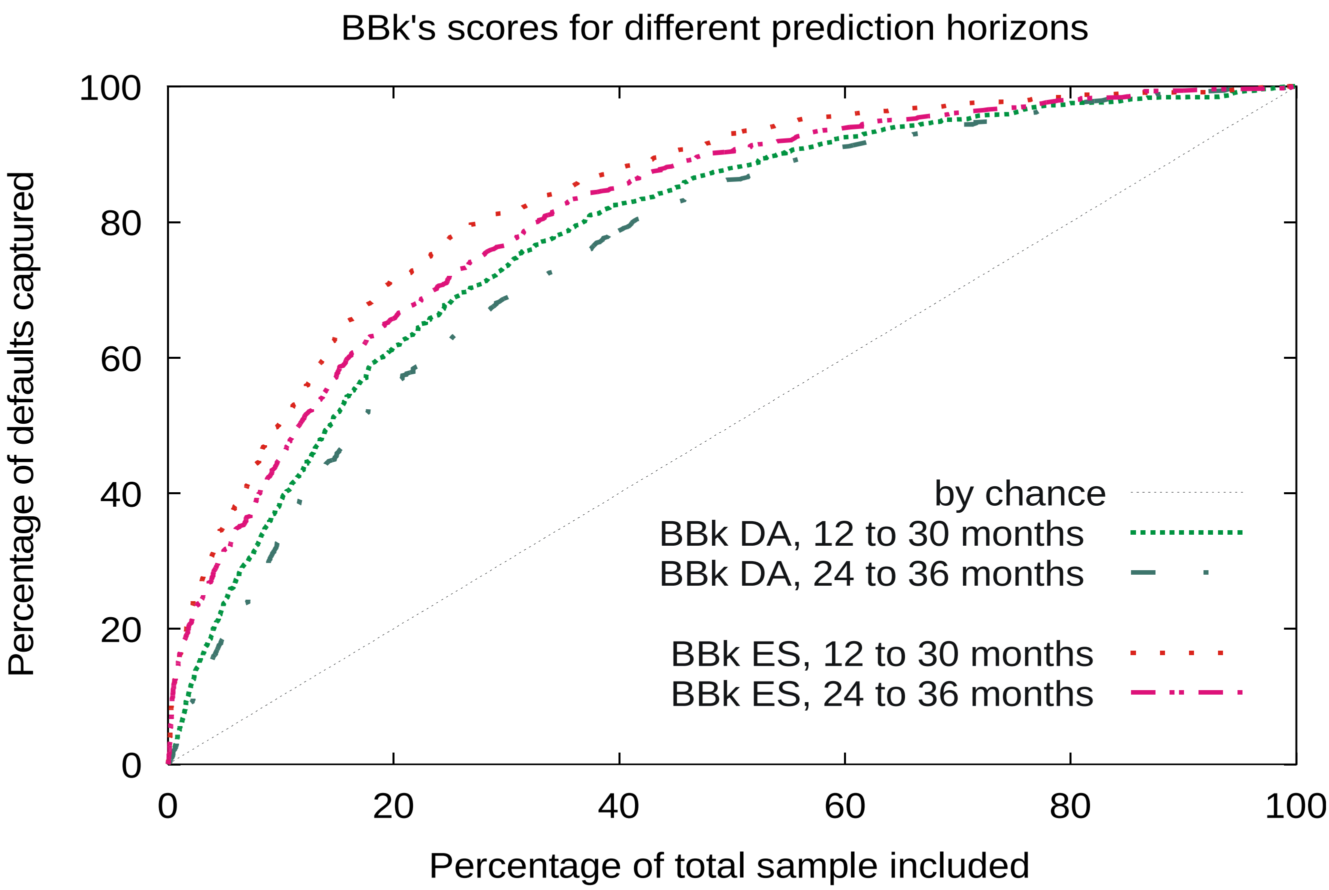
$$\Delta Z = \frac{Z^{+} \sum_{\text{increase Z}} \gamma_i + Z^{-} \sum_{\text{decrease Z}} \gamma_i}{\sum_{\text{all rules}} \gamma_i}$$

where γ_i denotes the rule weight of the i–th rule and Z^{+} and Z^{-} denote the maximum possible increase or decrease in the overall ratio Z. This adjustment contribution is so defined that no firm can be reclassified from the A group to the C group or vice versa. Some of the expert system's variables are free parameters that are unknown from the outset, namely the absolute rule weights, Z^{+}, Z^{-} and two position parameters for each membership function. The expert system can be rendered more objective by these free parameters because they can be adapted to a sample in a non-linear optimisation process.

The result of the two-step process is therefore a continuous overall ratio Z^{new} that is then – purely due to the Bundesbank's specific requirements – scaled down to the various system-generated classification proposals. Z^{new} has the same properties as the overall ratio Z from the discriminant analysis; hence it is possible to determine Gini curves and all the variables derived there from on the basis of both Z and Z^{new}.

Figure 7 shows power curves for various forecasting horizons – i.e. the period between the balance sheet date and the default date. It can be seen that the expert system markedly increases the discriminatory power of the discriminant functions. The two-step system currently in use extends the forecasting horizon by about one year compared with a one-step system consisting solely of discriminant functions.

Figure 7: Power curves for the overall ratio Z determined by discriminant analysis and Z^{new} determined by expert system for various forecasting horizons.



6. Private Firm Model® (KMV®)

KMV® constructs a structural model for assessing the credit risk of individual firms. The KMV model of credit risk for individual firms provides the framework for measuring the default probability, or EDF™ - Expected Default Frequency™, based on a relationship between the franchise value of the business (the market value of its assets[xv]), the volatility of this value and the liabilities due which are at risk. Like the Merton approach (1974), it is based on two theoretical relationships. First, the option pricing model, where the value of equity can be viewed as a call option on the value of a firm's assets. Second, on the theoretical link between the observable volatility of a firm's equity value and its unobservable asset value volatility. Merton modeled equity as a call option on the assets of the firm with strike price (default point) equal to the debt due at expiration. KMV has further advanced these approaches to provide a proven model that converts information taken from the equity markets into a measure of default risk that has shown predicitve power in empirical testing. The KMV Credit Monitor® EDF credit measure is in use by over 120 credit institutions globally.

KMV® has extended its market based structural model to calculate EDF for private firms without quoted equity through its KMV Private Firm Model[xvi]. In case of private companies, for which stock price data are generally not available, KMV uses essentially the same approach as for quoted firms. However, the market value of assets and asset volatility are estimated from the firm's observed characteristics (e.g. industry mix, size and geographic region) and are based on observations of trends amongst public firms. This allows the KMV Private Firm Model to reflect underlying changes in industry and country credit cycles. The approach incorporates forward looking information about future credit quality expectations as reflected by the actions of investors dealing in quoted firms. The Private Firm Model is updated by KMV with monthly parameters harvested from the publicly quoted firms.

The structural approach of the Private Firm Model builds on three drivers that are in turn estimated using balance sheet, income statement, industry and country information on the individual firm level and by observing the asset value and asset volatility behavior of comparable firms that are publicly quoted. The basic drivers are the market value of assets, the volatility of these assets and the amount of liabilities that are due within a given horizon. The basic idea is that when the value of a firm falls below its liabilities due it will not have any recourse but to default on its liabilities. Forecasting the likelihood of these events happening means that the expected behavior of the firm's market value of assets must be known. This is done by using a measure of the volatility of the market value of assets. The resulting metric is one that captures the number of standard deviations the firm is from its default point. Although the number of standard deviations from default is a valuable ranking of the creditworthiness of firms, it does not aid in telling us what the corresponding probability of default (PD) would be. The PD and KMV's EDF Credit Measure are both measures of the likeliness of default in decimal terms. KMV uses an empirical default database to convert the distance to default measure to an EDF. For more detail on the structural approach employed by KMV Private Firm Model please see appendix 3.

The structural approach employed by the KMV Private Firm Model to measure EDFs for non-listed companies is summarized in Figure 8

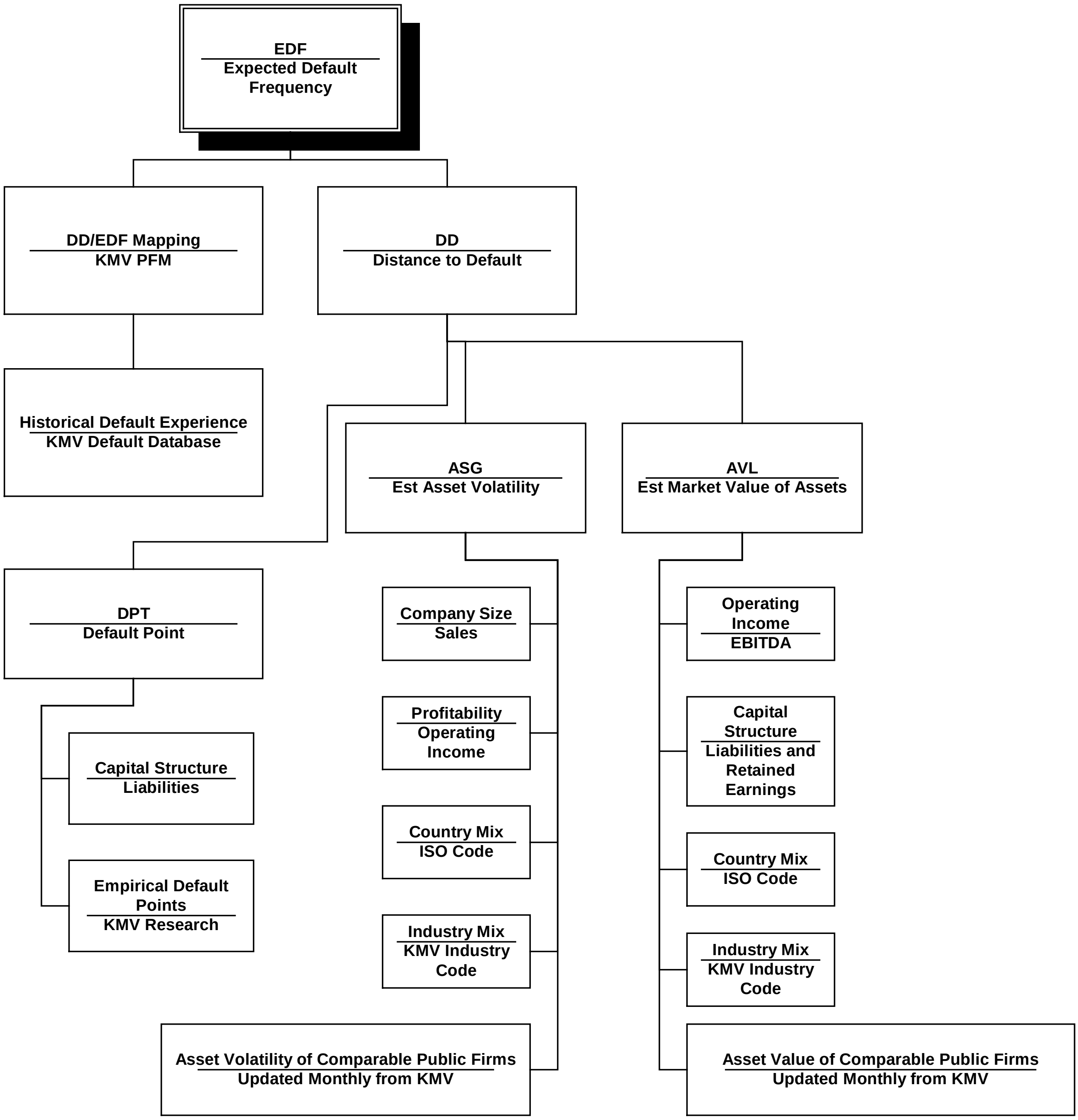


Figure 8

The structural approach combined with a store of monthly historical parameters allows for easy back testing of the Private Firm Model. KMV has specifically designed the Private Firm Model to operate with very few accounting inputs. The model does not require multiple years of history, merely the most recent financial statement, nor is it sensitive to local accounting rules and customs. On this background, KMV and Deutsche Bundesbank have collaborated to compare the discriminative power of the KMV Private Firm Model with more traditional modeling approaches, including the Deutsche Bundesbank's own internal credit scoring system.

The Private Firm Model was constructed with large publicly quoted firms in mind and has never, prior to this test, been formally tested for companies with a turnover less than five million Euro. This is however, the size cutoff used in the testing of the Private Firm Model in this paper. Private Firm Model is well suited to testing as the market influences in the model can be recreated using empirical information. The structural nature of the model allows for easy out-of-sample testing of the model as time figuratively can be turned back to reflect market influences for any month back to the mid-eighties.

This same structural design of the model gives the user a unique view into the cause and effect relationships that drive the model results. Any change in the EDF produced by the model can fully be explained by the asset valuation, asset value volatility or default point estimated by the model. Thus a user will update the financial information that serves as model input with pro-forma statements and get an updated EDF. Making this sort of update is equally important when the user is modeling a loan to the frim being modeled. The change in capital structure is thus reflected in the EDF, providing a more accurate measure of credit risk that simply processing the firm, pre-transaction through the model. The advantage of the structural approach is that this firm retains its individual characteristics, such as size and profitability while having its capital structure altered. Hence, the model proceeds to choose the same ‚comparable firms' from the quoted firm database as grounds for comparison. A statistical approach, by contrast would be using data on different firms when changing the underlying firm capital structure.

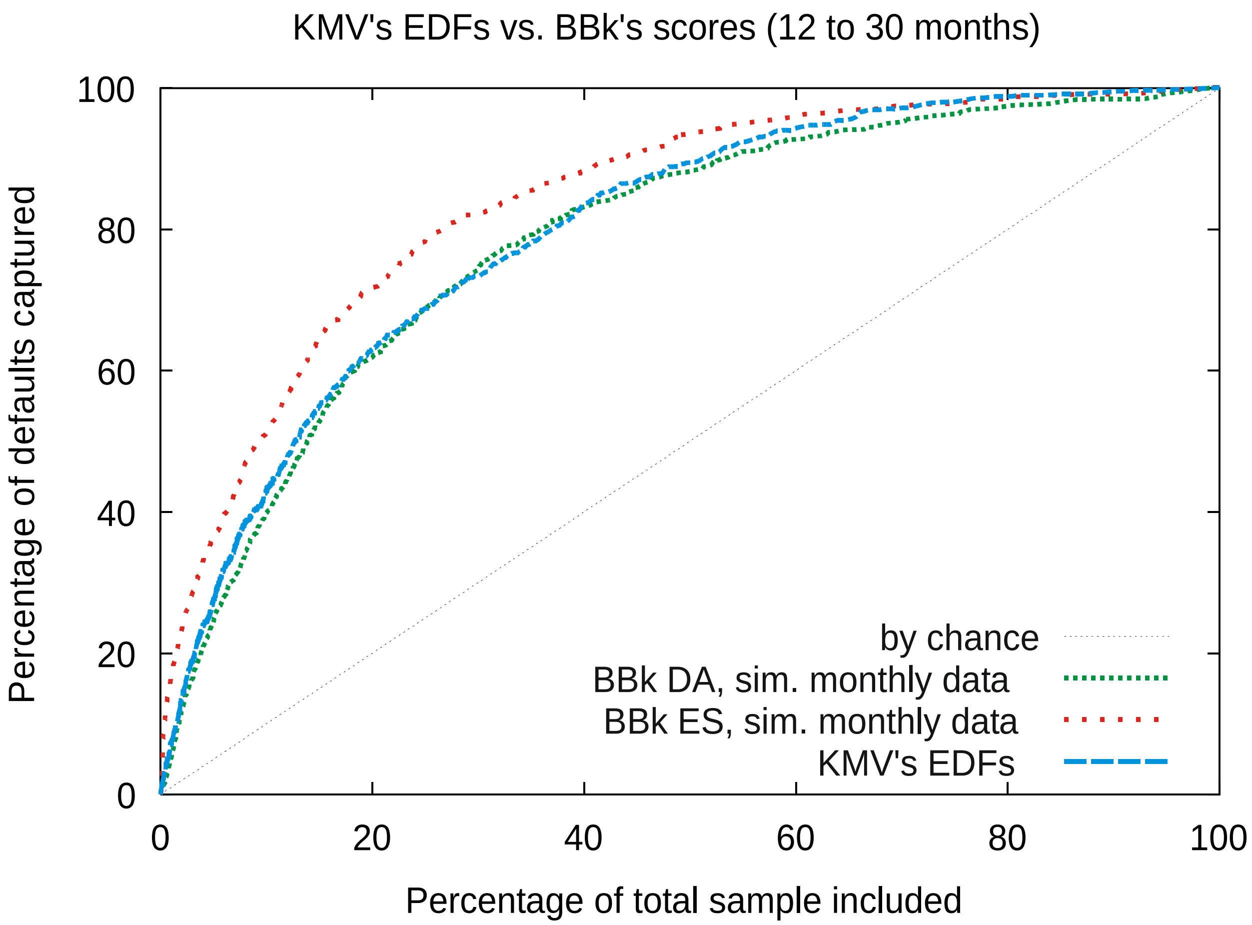
The KMV Private Firm Model is compared to the Bundesbank Default Risk Model by calculating Gini curves and coefficients on the same population and defaults. The application of Gini analysis is only valid for comparing models against each other.

7. Benchmarking Deutsche Bundesbank's Discriminant analysis and Deutsche Bundesbank's expert system with the KMV® Private Firm Model®

Benchmarking different models by their Gini curves and their Gini coefficients permits a comparison about their ability to discriminate between the defaulting vs non-defaulting firms for a given lead time and a given portfolio resp. database.
For our study we used the database of Deutsche Bundesbank which is described in part 2 and appendix 2. The underlying dataset includes more than 125 000 balance sheet from December 1992 through December 1999. We calculated the Gini curve and the Gini coefficient for a lead time of 12 months and a lead time of 24 months.
Because of the monthly change of KMV EDF estimates we calculated the risk scores for both the KMV Private Firm Model and Bundesbank's discriminant analysis resp. Bundesbank's expert system on a monthly basis.
To prevent falsification because of missing balance sheet data we had to carry out a selection within the defaulted companies. The default selection[xvii] for the Gini curve with lead time of 12 months (24 months) is based on the criteria that at least one balance sheet existed between 12 months (24 months) and 30 months (42 months) before default. If there is more than one balance sheet in this period we used the latest one. The population and subset of defaults is the same for all models in the testing presented in this paper.

For the Gini curve with lead time of 12 months (24 months) we used 761 (672) out of 950 defaults which are in the Bundesbank default database for the observed period.

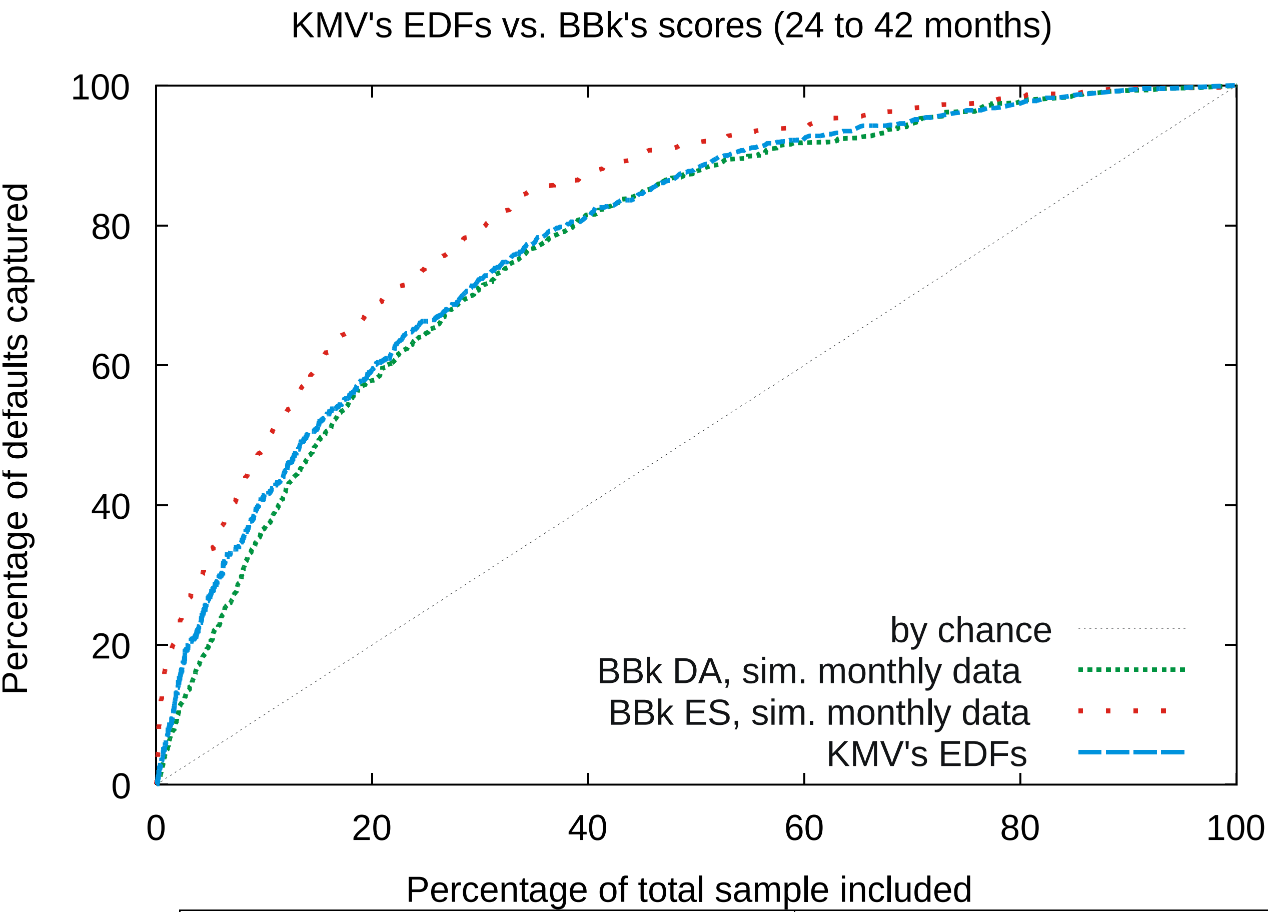
The graphs below show the Gini curves and the Gini coefficients for Deutsche Bundesbank's discriminant analysis and Deutsche Bundesbank's expert system matched with the KMV Private Firm Model model. Figure 9 (10) shows the Gini curve and the Gini coefficient for a lead time of 12 months (24 months).



12 months lead time	Gini Coefficient
Bundesbank discriminant analysis	0.5740
Bundesbank expert system	0.6800
KMV® Private Firm Model®	0.5970

Figure 9

By increasing the lead time from 12 months to 24 months we can observe the increase in predictive power that is being extracted from the underlying financial statements and in the KMV Private Firm Model additionally by the market information which the model is incorporating. The underlying data sample is identical to that used in the above depiction of the 12 month Gini-curve. However, the number of defaults used differs by the time horizon employed as already discussed.



24 months lead time	Gini Coefficient
Bundsbank discriminant analysis	0.5430
Bundesbank expert system	0.6490
KMV® Private Firm Model®	0.5660

Figure 10

Although these tests give us an absolute picture of the effectiveness of using a quantitative model on a sample population[xviii] it is illuminating to put the testing in a relevant context.

The relative position of the curves indicate that Private Firm Model is giving a similar discrimination beween defaulting and non-defaulting firms as found by the Deutsche Bundesbank discriminant analysis model. The expert system records an improved discrimination over the two other models.

The structural nature of the Private Firm Model can accomodate the subjective input of a credit officer by having the officer adjust the market value of assets or asset volatility components of the model. The ability to make these adjustments allows the end user to incorporate similar information as evidenced by the expert system. In the testing prepared in this paper the Private Firm Model has been run without adding information beyond what is contained in the financial accounts and in the market comparable information supplied by KMV.

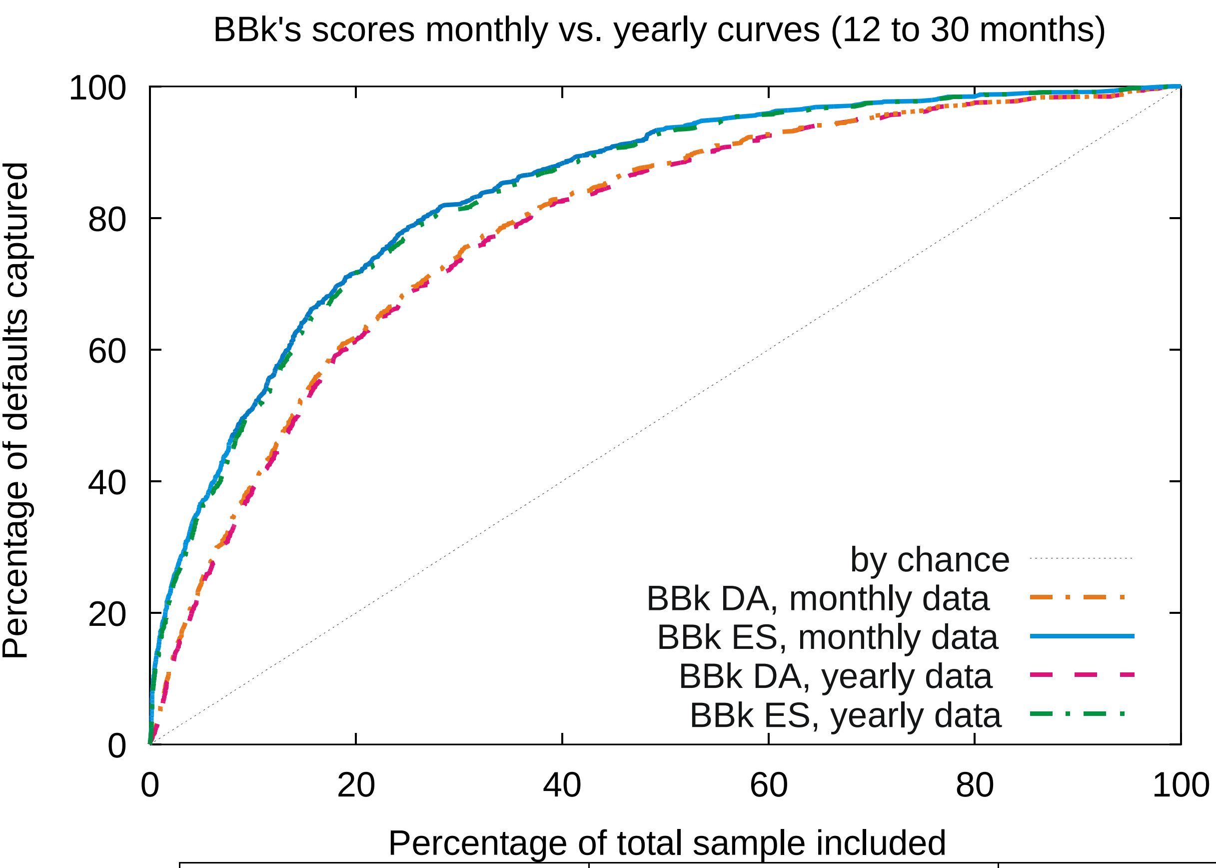
The similarity between KMV Private Firm Model EDF credit measures and the Bundesbank discriminant analysis model is all the more striking when noting that the Private Firm Model EDFs are being tested entirely out of sample. No data originating from the Bundesbank databases has been used in building the KMV Private Firm Model. As described previously in this paper, the discriminant anlysis model was built on a small subset of the population sample that is now being tested on[xix].

8. Analysing common financial ratios

In addition to the benchmarking of Deutsche Bundesbank’s discriminant analysis and Deutsche Bundesbank’s expert system with the KMV® Private Firm Model® we analysed for the three sectors trade, manufacturing and other corporates some of the financial ratios which are used in the discriminant analysis. We determined the Gini curves and the Gini coefficients.

Private Firm Model is excluded from the comparison with common financial ratios because these are ratios used in the Bundesbank discriminant analysis model. KMV does not use common financial ratios in its structural approach. The ratio analysis depicted in these test are for the industry sub-categories given by the Bundesbank Default Risk Model.

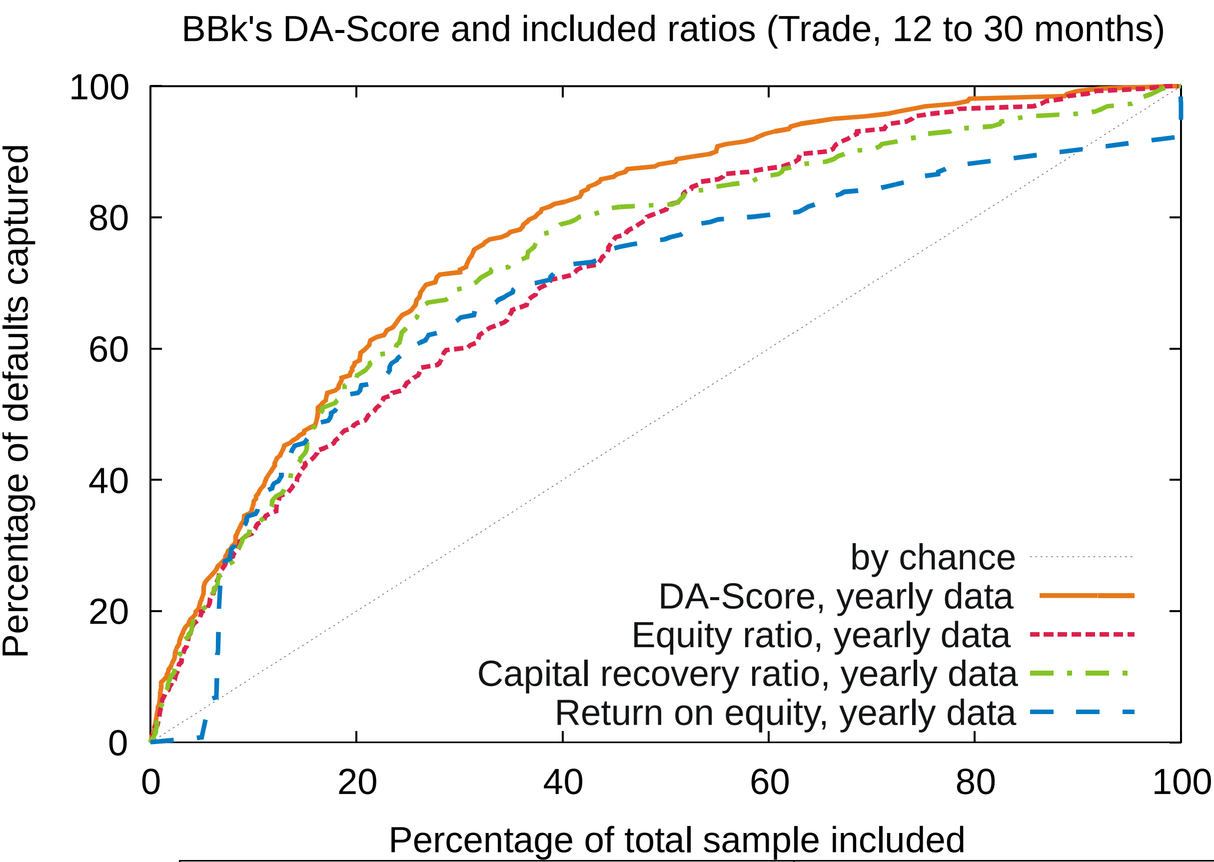
Since the risk scores for Deutsche Bundesbank’s discriminant analysis, Deutsche Bundesbank’s expert system and the financial ratios only change when a new balance sheet arrives, a yearly calculation of the risk scores is sufficient. When comparing the Gini curves and Gini coefficients between a monthly and a yearly calculation for the discriminant analysis and the expert system for which is given an example in Figure 11 for a lead time of 12 months. There is only a small difference between these both methods observable. The reason for this difference is that the ranking between the companies can differ from month to month, though the risk score of one particular company stays constant until the next balance sheet arrives.



12 months lead time	Gini coefficient (yearly basis)	Gini coefficient (monthly basis)
Discriminant analysis	0.5650	0.5740
Expert system	0.6690	0.6800

Figure 11

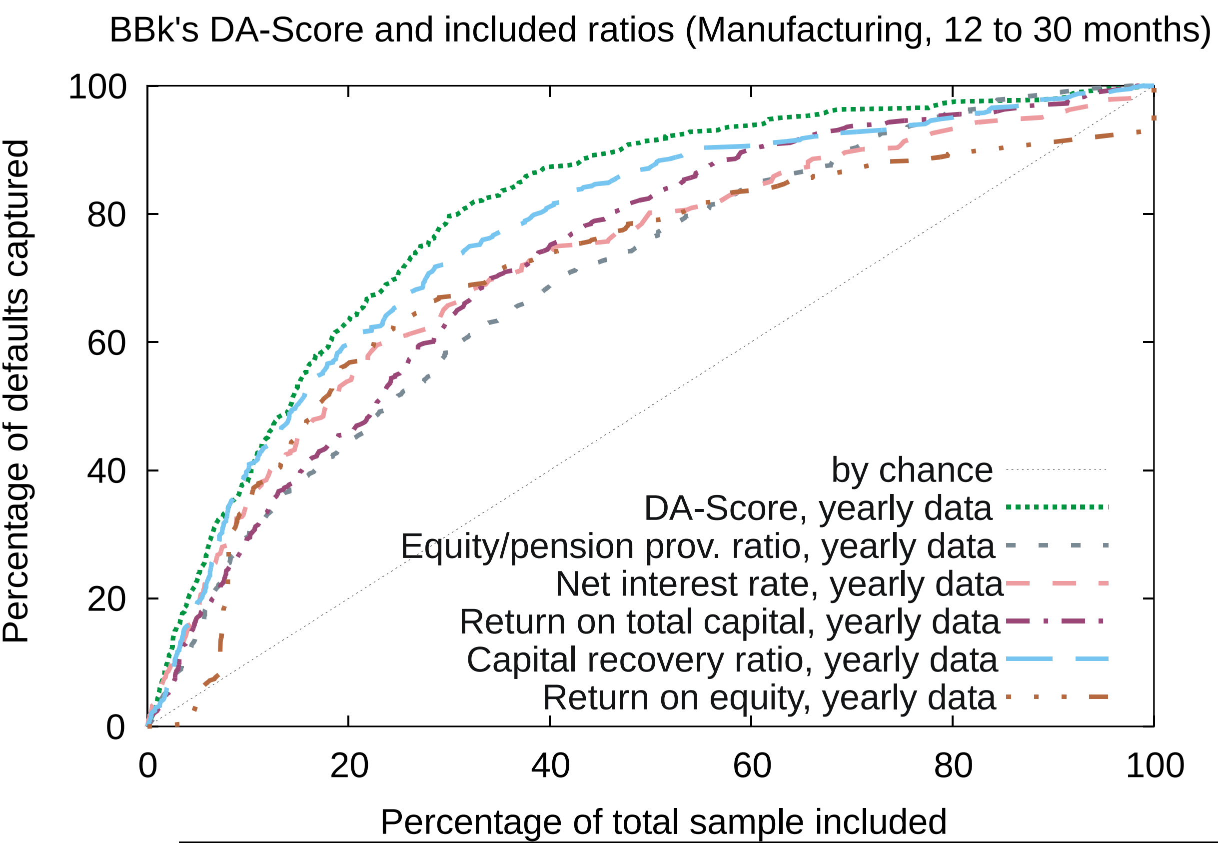
For the trade–sector we determined the Gini curve and the Gini coefficient for the equity ratio, the capital recovery ratio and the return on equity[xx] and compared it with the results of the discriminant analysis. As expected the Gini coefficient of the discriminant analysis is higher than the Gini coefficients of the individual financial ratios (see Figure 12).



12 months lead time	Gini coefficient
Discriminant analysis Trade sector	0.5580
Equity ratio	0.4480
Capital recovery ratio	0.4790
Return on equity	0.3750

Figure 12

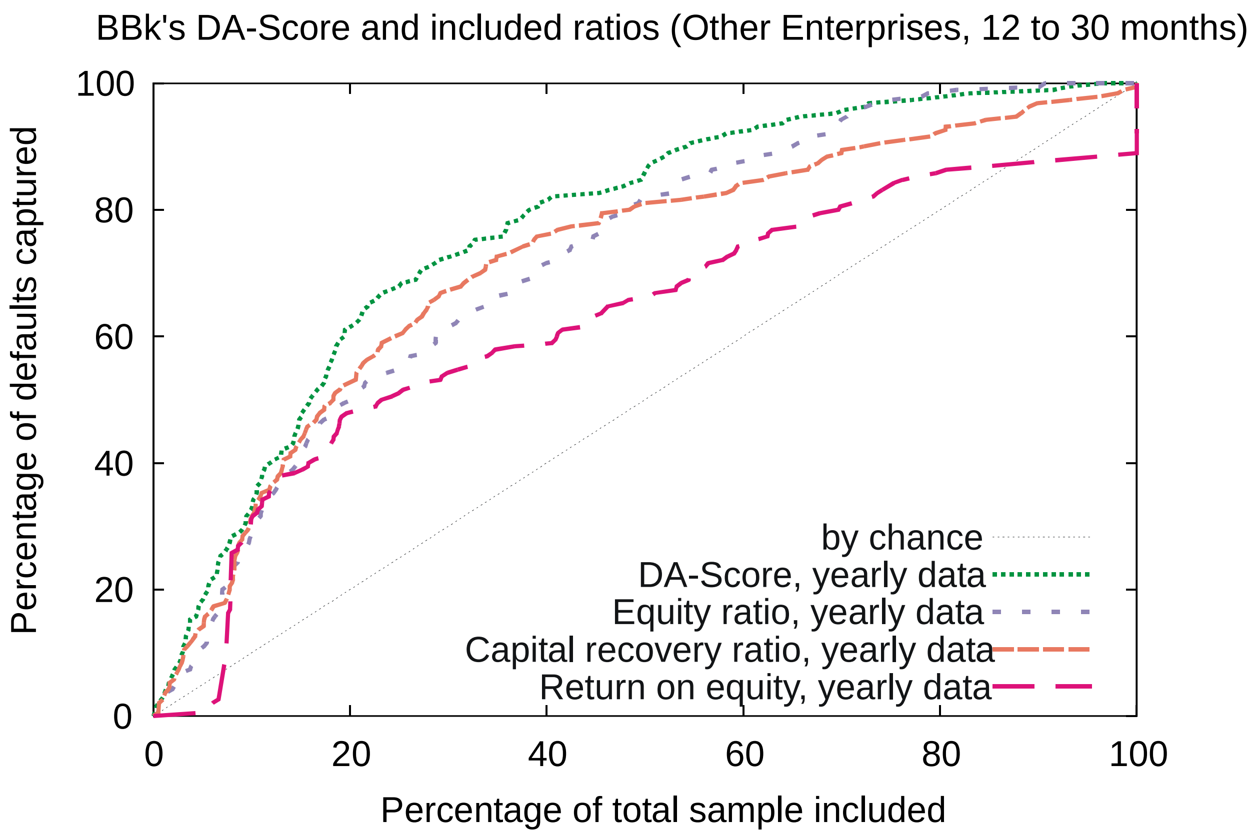
Figure 13 shows the results for the manufacturing–sector and the financial ratios equity/pension provision ratio, net interest rate, return on total capital employed, capital recovery ratio and return on equity.



12 months lead time	Gini coefficient
Discriminant analysis Manufacturing sector	0.6000
equity/pension provision ratio	0.3960
Net interest rate	0.4440
return on total capital employed	0.4520
capital recovery ratio	0.5370
Return on equity	0.4040

Figure 13

Figure 14 shows the results for other enterprises and the financial ratios equity ratio, capital recovery ratio and return on equity.



12 months lead time	Gini coefficient
Discriminant analysis Other companies	0.5440
Equity ratio	0.4480
Capital recovery ratio	0.4400
Return on equity	0.2490

Figure 14

9. Concluding remarks

Significant power of discrimination can be achieved through the use of quantitative models of credit analysis. As shown in this paper the direct application of both statistical (discriminant analysis) and structural (KMV® Private Firm Model®) models both provide powerful approaches to credit analysis with similar results. The simple reliance on traditional approaches of analysis, as demonstrated in this paper by examining the power of using ratio analysis can easily be eclipsed by the application of quantitative models in the credit process. When adding information – judgement of the firm's situation that is not included in the financial reporting – in addition to the application of quantitative techniques, the power of discrimination can further be improved as evidenced by the Deutsche Bundesbank's expert system.

The use of the expert system exemplifies an efficient credit scoring system that combines both the efficiency of a quantitative approach with including the skills of experienced credit officers. Certainly, the costs of the application of an expert system must be weighed against the additional discriminatory power gained over a pure quantitative system that may require far less resources.

The application of a model in a credit process is premised on that the model works in a consistent manner over time while forewarning of instances of default.The testing of the various models presented in this paper has centered on the view that an efficient credit rating system will accurately rank firms by a risk score from best to worst. In a second step the mapping from the risk score to probability of default is carried out by using historical default data. The accuracy of this step depends primarily on data quantity and quality and is less dependent on methodology.

The Gini curve and coefficient has been applied to different modelling approaches included in this study and is a stable way to measure the discriminatory power and monitor the quality of models over time. Although customary care must be attributed to sampling issues, the Gini approach gives a visual representation of the power of different models in a format that is attractive and easy to understand for a wide audience.

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[i] Similar performance measures exist under a variety of different names (power curves, cumulative accuracy profiles, lift-curves, dubbed-curves, receiver-operator curves, etc.).

[ii] Please refer to Appendix 1 for a technical description of the approaches discussed in this chapter.

[iii] The Gini curve maps the fraction of companies with the highest risk score onto the fraction of defaulted companies by that group.

[iv] Usually the cost of a default is higher than the loss of prospective profits.

[v] For more details see „Rating methodology, Benchmarking Quantitative Default Risk Models: A Validation Methodology“, Moody’s Investors Service, March 2000

[vi] Until the start of stage three of European Monetary Union (EMU) on January 1, 1999, the Bundesbank purchased "fine trade bills" at the discount rate under its discount credit facility; that facility was abolished with the start of EMU. Since January 1, 1999, trade bills or bank loans to enterprises (business credits) may collateralise all monetary policy operations of the Eurosystem[vi] provided they meet the Eurosystem's stringent credit standard. The Eurosystem consists of the European Central Bank and all european national central banks, that take part in stage three of the European Monetary Union.

[vii] Before the Insolvency Code entered into force on January 1, 1999 the default criterion applied by the Bundesbank was the initiation of bankruptcy or composition proceedings, which was an identical procedure to that used in the Insolvency Code except for the new concept of the anticipated inability to meet due payments.

[viii] For a more detailed description of the underlying principles of the first step of the Bundesbank’s system cf. Blochwitz / Eigermann (1999) and Blochwitz / Eigermann (2000b).

[ix] Discriminant analysis is a statistical method that creates a classification rule on the basis of business performance ratios and thus can categorize sound firms and unsound firms. In the case of the linear discriminant function used by the Bundesbank, firms are classified according to the following rule: The discriminant value (known as the overall ratio) Z is specified as $Z = \mathrm{a}_1 \cdot x_1 + \cdots + \mathrm{a}_\mathrm{n} \cdot x_\mathrm{n}$, where $x_1, x_2, \cdots, x_\mathrm{n}$ are the input ratios and $\mathrm{a}_1, \mathrm{a}_2, \cdots, \mathrm{a}_\mathrm{n}$ are their respective weights. If Z is greater than the cut–off–point Z_{co} the firm in question is allocated to the category of sound firms, otherwise it is allocated to the category of unsound firms. Z_{co} is set such that the chance of misclassification is minimized.

[x] The number of unsound firms limits the size of the sample because it is better for calibration reasons to have roughly the same number of sound and unsound firms. The sample on which the Bundesbank's modular method was developed totalled just over 1,500 firms of which around 700 were drawn from the manufacturing sector and around 400 each were classified as belonging to the categories wholesale/retail trade and other enterprises.

[xi] It can be shown that, under certain circumstances, a one-to-one mapping exists between the overall ratio Z and the misclassification probability – and hence the default probability can be determined. Using this approach the probability of allocation can be interpreted as a continuous measure of risk.

[xii] For details see Blochwitz / Eigermann (2000a).

[xiii] The expert system thus has three aims: (i) to reduce the uncertainty of the status of firms in the B–group by reclassifying them – naturally as accurately possible – into the A or C group, (ii) to facilitate the inclusion in a standardised form of additional information that has so far not been taken into account during the evaluation so as to obtain a more complete picture of the situation of the firm being analysed, (iii) to provide assistance to the credit officer in making his final credit decision.

[xiv] The properties 1 to N comprise an imprecise or "fuzzy" set (in the jargon of fuzzy logic) $\tilde{A} := \left\{ (x_i, \mu_{x_i}(x)) \mid x_i \in X \right\}$ in relation to property X with x as its observed value and the verbal features $\left\{ x_1, x_2, \ldots \right\}$, and the real membership function $\mu_{x_i}(x\ ) := x\ \ \rightarrow [0,1]$. For example, if X is turnover, and its change in two consecutive years x may have the value of 10% and the verbal features could take the form { x_1 =“declining“, x_2 =“unchanged“, x_3 =“increasing“}. The membership functions then may constitute a fuzzy–set $\tilde{A} = \{(x_1, 0); (x_2, 0.5); (x_3, 0.2)\}$

[xv] Market value of assets = market value of equity + market value of liabilities

[xvi] See Appendix 3 for details on the KMV Private Firm Model®

[xvii] i.e. the defaulted companies which we considered for the Gini curve resp. Gini coefficient

[xviii] See chapter 2 for a description of the characteristics of the Bundesbank sample population

[xix] The number of defaults which are used both for calibration and for testing is small (88 defaults), i.e. approximately 10% of the defaults which are used for testing have been also used for the calibration of the Deutsche Bundesbank discriminant analysis model.

[xx] For the ratio return on equity we wish to note, that the Bundesbank uses a very conservative definition of equity, that yields to a lower value of equity as allowed by German law. Due to the small denominator the return on equity may result in relatively high absolute values. To avoid numerical problems in the discriminant analysis in the Bundesbank’s system return on equity values of higher (lower) than 99% (-99%) are cut to 99% (-99%). Due to that fact a significant part of defaulted as well as not defaulted enterprises have the same (extreme) value of return on equity. That fact explains the somewhat strange behaviour of the Gini–curve for that variable on their boundaries.

Appendix 1: Definition of Gini curve and Gini coefficient, Relation between Gini curve and default frequency curve

Definition of Gini curve

Let $T=\{t_1, t_2,...,t_k\}$ be the set of observation points and C^t the set of all companies which are in the database at time $t\in T$. The observation points are chosen equidistant, i.e. $t_{i+1}-t_i=c$ for all i=1,...,k-1. Usually c equals one month or one year. Due to defaults or foundations the set of companies differs from year to year.

The mapping $r^t : C^t \rightarrow IR$ is called the risk score at time t. Define $r_c^t := r^t(c)$.
The risk score reflects the model's asessment about the riskiness of a company, i.e. a higher risk score means a higher riskiness.

Let $J = \{ c_1,...,c_n \}$ be the set of all defaulted companies and d_j the time of default of company $c_j\in J$.

(1) $$x_j = \frac{\left|\{c\in C^{d_j-l} \,\middle|\, r_c^{d_j-l} \geq r_{c_j}^{d_j-l}\}\right|}{\left|C^{d_j-l}\right|} \qquad ,c_j \in J$$

x_j is the percentage of all companies which are in the database at time d_j-l, i.e. at time of default minus lead time l, with a risk score at time d_j-l higher or equal than the risk score of the defaulted company c_j at time d_j-l.
In other words x_j is the percentage of all companies at time d_j-l which are in the model's view riskier than the defaulted company c_j at time d_j-l .

Order the defaulted companies such that $x_1\leq x_2 \leq .... \leq x_n$.
The Gini curve $G(\cdot)$: $[0,1] \rightarrow [0,1]$ and the default frequency curve $p(\cdot)$: $[0,1] \rightarrow [0,1]$ are given by

(2) $$G(x)= \begin{cases} \dfrac{m-1}{n}+\dfrac{r-m+1}{n}*\dfrac{x-x_{m-1}}{x_r-x_{m-1}} & \text{if } x_{m-1} \leq x < x_r \\ \\ 1 & \text{if } x\geq x_n \end{cases}$$

(3) $$p(x)= \begin{cases} \dfrac{1}{x_r-x_m-1}\cdot\dfrac{r-m+1}{n}\cdot p_A & \text{if } x_{m-1} \leq x < x_r \\ \\ 0 & \text{if } x\geq x_n \end{cases}$$

where p_A is the average default rate and $x_0:=0$, $1\leq m\leq n$,
$r:=\max\left\{ j \in \{m,...,n\} \,\middle|\, x_j=x_m\right\}$

Perfect Gini Curve

The Gini curve of a perfect model, i.e. a model which discriminates perfectly between defaulting and non-defaulting companies, is called **perfect Gini curve**. In this perfect model the worst risk scores are assigned to the defaulted companies.

The perfect Gini curve is given by

$$(4)\quad G_p(x) = \begin{cases} \dfrac{x}{x_n} & \text{if } x < x_n \\ 1 & \text{if } x \geq x_n \end{cases}$$

with $x_n = \max_t \dfrac{\left|\left\{ j \in J \middle| d_j = t \right\}\right|}{\left|C^t\right|}$

x_n is the maximal default rate of the sample at any point in time.

Definition of Gini coefficient

The Gini coefficient (GC) is defined as the ratio of the area between a model's Gini curve and the random Gini curve to the area between the perfect Gini curve and the random Gini curve.

$$(5)\quad GC = \frac{\int_0^1 G(t)dt - \frac{1}{2}}{\int_0^1 G_p(t)dt - \frac{1}{2}}$$

The area between the perfect Gini curve and the random Gini curve is usually very close to 0.5. Therefore 2* (the area between a model's Gini curve and the random Gini curve) is a good approximation for GC. Mathematically speaking

$$GC \approx 2 \cdot \int_0^1 G(t)dt - 1$$

In our study we used this approximation for GC.

Determination of the Gini curve

The risk scores (e.g. EDF, Bundesbank ratio Z) are determined on a monthly basis. While the EDF measure of KMV's Private Firm Model changes monthly, the Bundesbank ratio Z stays constant until a new balance sheet arrives.
First all companies are ranked monthly by their risk score. In the next step the quantile x_j , the Gini curve G(x) and the Gini coefficient GC are calculated according to the formulas above.

Relationship between Gini curve and default frequency curve

The default frequency curve p(x) is the right-sided derivative of G(x) multiplied with the average default rate, i.e.

$$p(x) = p_A \cdot \lim_{t \to x^+} G(t)$$

Appendix 2: Characteristics of the database

Table 1 shows the yearly default distribution in the database. Annual accounts for different years are available for most of the firms – both defaulted and non-defaulted. To complete the picture of the dataset structure, Table 2 and Table 3 show a breakdown of the dataset used in the study by sector and turnover size category.

	Defaulted firms; years until default												Non-defaulted firms		Total	
Acctg.	1 .. 2		2 .. 3		3 ..4		4 .. 5		5 ..6		> 6					
Year	#	in %	#	in %	#	in %	#	in %	#	in %	#	in %	#	in %	#	in %
1994	30	0.09	187	0.59	225	0.70	151	0.47	250	0.78	50	0.16	31057	97.18	31959	100.00
1995	24	0.08	159	0.51	155	0.50	278	0.90	53	0.17			30291	97.82	30965	100.00
1996	19	0.06	130	0.44	260	0.87	41	0.14					29379	98.48	29833	100.00
1997	18	0.07	168	0.63	39	0.15							26510	99.14	26741	100.00
1998	18	0.09	15	0.08									19008	99.81	19045	100.00
1999													1142	100.00	1142	100.00

Table 1: Defaulted firms in relation to the balance sheet date. Some firms are counted more than once (for different accounting years) as balance sheets for several successive years are available for most firms.

		Defaulted		Non-defaulted		Total	
Acctg. Year	Sector	#	in %	#	in %	#	in %
1994	Manufacturing	425	3.0	13658	97.0	14083	100.0
	Wholesale and retail trade	271	2.0	13141	98.0	13412	100.0
	Construction	154	7.8	1834	92.3	1988	100.0
	Agriculture, fishing, hunting and forestry	2	1.3	158	98.8	160	100.0
	Other business	50	2.2	2266	97.8	2316	100.0
1995	Manufacturing	312	2.3	13362	97.7	13674	100.0
	Wholesale and retail trade	209	1.6	12900	98.4	13109	100.0
	Construction	122	6.7	1708	93.3	1830	100.0
	Agriculture, fishing, hunting and forestry	3	1.9	154	98.1	157	100.0
	Other business	28	1.3	2167	98.7	2195	100.0
1996	Manufacturing	212	1.6	12865	98.4	13077	100.0
	Wholesale and retail trade	150	1.2	12638	98.8	12788	100.0
	Construction	75	4.6	1553	95.4	1628	100.0
	Agriculture, fishing, hunting and forestry	3	2.0	144	98.0	147	100.0
	Other business	14	0.6	2179	99.4	2193	100.0

1997	Manufacturing	115	1.0	11568	99.0	11683	100.0
	Wholesale and retail trade	75	0.7	11499	99.4	11574	100.0
	Construction	29	2.2	1316	97.8	1345	100.0
	Agriculture, fishing, hunting and forestry	2	1.4	137	98.6	139	100.0
	Other business	10	0.5	1990	99.5	2000	100.0
1998	Manufacturing	22	0.3	8487	99.7	8509	100.0
	Wholesale and retail trade	12	0.2	8139	99.9	8151	100.0
	Construction	3	0.3	867	99.7	870	100.0
	Agriculture, fishing, hunting and forestry	.	.	88	100.0	88	100.0
	Other business	.	.	1427	100.0	1427	100.0
1999	Manufacturing	.	.	569	100.0	569	100.0
	Wholesale and retail trade	.	.	460	100.0	460	100.0
	Construction	.	.	40	100.0	40	100.0
	Agriculture, fishing, hunting and forestry	.	.	10	100.0	10	100.0
	Other business	.	.	63	100.0	63	100.0
1994 .. 1999	Manufacturing	1086	1.8	60509	98.2	61595	100.0
	Wholesale and retail trade	717	1.2	58777	98.8	59494	100.0
	Construction	383	5.0	7318	95.0	7701	100.0
	Agriculture, fishing, hunting and forestry	10	1.4	691	98.6	701	100.0
	Other business	102	1.0	10092	99.0	10194	100.0

Table 2: Defaulted and non-defaulted firms by economic sector

		Defaulted		Non-defaulted		Total	
Acctg. year	Turnover in η million	#	in %	#	in %	#	in %
1994	0 .. 10	404	3.2	12385	96.8	12789	100.0
	10 .. 20	271	3.5	7557	96.5	7828	100.0
	20 .. 50	152	2.5	5912	97.5	6064	100.0
	50 .. 100	50	2.1	2334	97.9	2384	100.0
	100 .. 250	19	1.2	1550	98.8	1569	100.0
	> 250	6	0.5	1319	99.6	1325	100.0
1995	0 .. 10	288	2.5	11422	97.5	11710	100.0
	10 .. 20	206	2.7	7520	97.3	7726	100.0
	20 .. 50	129	2.1	5973	97.9	6102	100.0
	50 .. 100	34	1.4	2387	98.6	2421	100.0
	100 .. 250	13	0.8	1610	99.2	1623	100.0
	> 250	4	0.3	1379	99.7	1383	100.0
1996	0 .. 10	203	1.8	10856	98.2	11059	100.0

	10 .. 20	129	1.7	7314	98.3	7443	100.0
	20 .. 50	92	1.5	5902	98.5	5994	100.0
	50 .. 100	19	0.8	2291	99.2	2310	100.0
	100 .. 250	7	0.4	1625	99.6	1632	100.0
	> 250	4	0.3	1391	99.7	1395	100.0
1997	0 .. 10	90	1.0	8924	99.0	9014	100.0
	10 .. 20	73	1.1	6733	98.9	6806	100.0
	20 .. 50	53	0.9	5556	99.1	5609	100.0
	50 .. 100	11	0.5	2241	99.5	2252	100.0
	100 .. 250	4	0.2	1635	99.8	1639	100.0
	> 250	.	.	1421	100.0	1421	100.0
1998	0 .. 10	10	0.2	5498	99.8	5508	100.0
	10 .. 20	16	0.3	4726	99.7	4742	100.0
	20 .. 50	9	0.2	4322	99.8	4331	100.0
	50 .. 100	1	0.1	1825	100.0	1826	100.0
	100 .. 250	1	0.1	1388	99.9	1389	100.0
	> 250	.	.	1249	100.0	1249	100.0
1999	0 .. 10	.	.	244	100.0	244	100.0
	10 .. 20	.	.	215	100.0	215	100.0
	20 .. 50	.	.	279	100.0	279	100.0
	50 .. 100	.	.	143	100.0	143	100.0
	100 .. 250	.	.	129	100.0	129	100.0
	> 250	.	.	132	100.0	132	100.0
1994 .. 1999	0 .. 10	995	2.0	49329	98.0	50324	100.0
	10 .. 20	695	2.0	34065	98.0	34760	100.0
	20 .. 50	435	1.5	27944	98.5	28379	100.0
	50 .. 100	115	1.0	11221	99.0	11336	100.0
	100 .. 250	44	0.6	7937	99.5	7981	100.0
	> 250	14	0.2	6891	99.8	6905	100.0

Table 3: Defaulted and non-defaulted firms by turnover size category

Appendix 3: Modeling Default Risk: Private Firm Model

Modeling Default Risk

Private Firm Model

K·M·V

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Release Date: 15-August-1997

Revised: 09-October-1998

Table of Contents

1. Introduction

As discussed in *Modeling Default Risk*, there are three basic types of information that are relevant to the default probability of a firm: financial statements, market prices of the firm's debt and equity, and subjective appraisals of the firm's prospects and risk. Financial statements, by their nature, are inherently backward looking. They are reports of the past. Prices, by their nature, are inherently forward looking. Investors form debt and equity prices as they anticipate the firm's future. In determining the market prices, investors use, amongst many other things, subjective appraisals of the firm's prospects and risk, financial statements, and other market prices. This information is combined using the investor's own analysis and synthesis and results in their willingness to buy and sell the debt and equity securities of the firm. Market prices are the result of the combined willingness of many investors to buy and sell and thus prices embody the synthesized views and forecasts of many investors.

The Credit Monitor model, for publicly traded companies, uses the information embodied in market prices to calculate Expected Default Frequencies (EDFs). As of May 1997, Credit Monitor covered over 18,000 public companies globally. This breadth, however, translates to coverage of from 40% to 50% of the corporate portfolio of most large banks.

KMV Corporation has extended the model of default probability to cover private non-financial firms. Private Firm Model (PFM) uses public market information, in particular, share prices, on peer companies plus the firm's financial statement data to estimate the firm's asset value and volatility. As such, it is a hybrid relying on the powerful forward-looking information from market prices and of the financial statement data that underpin traditional credit analysis.

PFM, like Credit Monitor for public companies, has three steps in the determination of the default probability of a firm:

1. **Estimate asset value and volatility**: In this step the asset value and asset volatility of the private firm are estimated from market data on peer companies from Credit Monitor coupled with the firm's reported operating cash flow, sales, book value of liabilities and its industry mix.

2. **Calculate the distance to default**: The distance to default is calculated from the asset value and asset volatility (estimated in the first step) and the book value of liabilities.
3. **Calculate the default probability**: The default probability is determined directly from the distance to default and the default rate for given levels of distance to default.

Steps two and three are not significantly different for public and private firms.

This paper will focus on the first step, estimating asset value and volatility for private firms since the second and third steps are essentially the same for public and private firms and are described in detail *in Modeling Default Risk*.

2. Estimate Asset Value for Private Firms

The best estimate of a firm's asset value comes from the information in the market price of its shares. This represents the present value of the uncertain future cash flows of the firm. Since private firms lack market prices, the model relies on comparisons with similar public firms.

The model was initially built by observing the relationship between cash flow (as measured by earnings before interest, taxes, depreciation and amortization, EBITDA) and asset values for public companies. Although the Credit Monitor model for public firms does not use operating income to estimate asset value (instead relying on the option-theoretic approach to imply it from market equity prices), the resulting asset values do have, as we would expect, a strong relationship with the firm's observed cash flow. This relationship varies by industry. High growth industries tend to have a much higher asset value per dollar of cash flow, since the current cash flow is relatively small compared to expected future cash flows. For more mature industries, the asset values per dollar of cash flow are lower.

For firms in the normal range of positive cash flows, higher cash flow means higher asset value and decreases in cash flow lead to decreases in asset value. However, there is a range of cash flows, negative and close to zero, when a decrease in cash flow does not lead to a decrease in asset value. This flat relationship comes from the phenomenon of "liquidation value". At some point of low cash flows, the valuation of the firm begins to be less a function of its ongoing business value (from cash flows) and more a function of the liquidation value of the assets. Figure 1 shows the relationship between EBITDA and asset market values for firms in the Broadcast Media industry. Both variables are normalized (divided) by book assets to allow comparison between firms of different sizes.

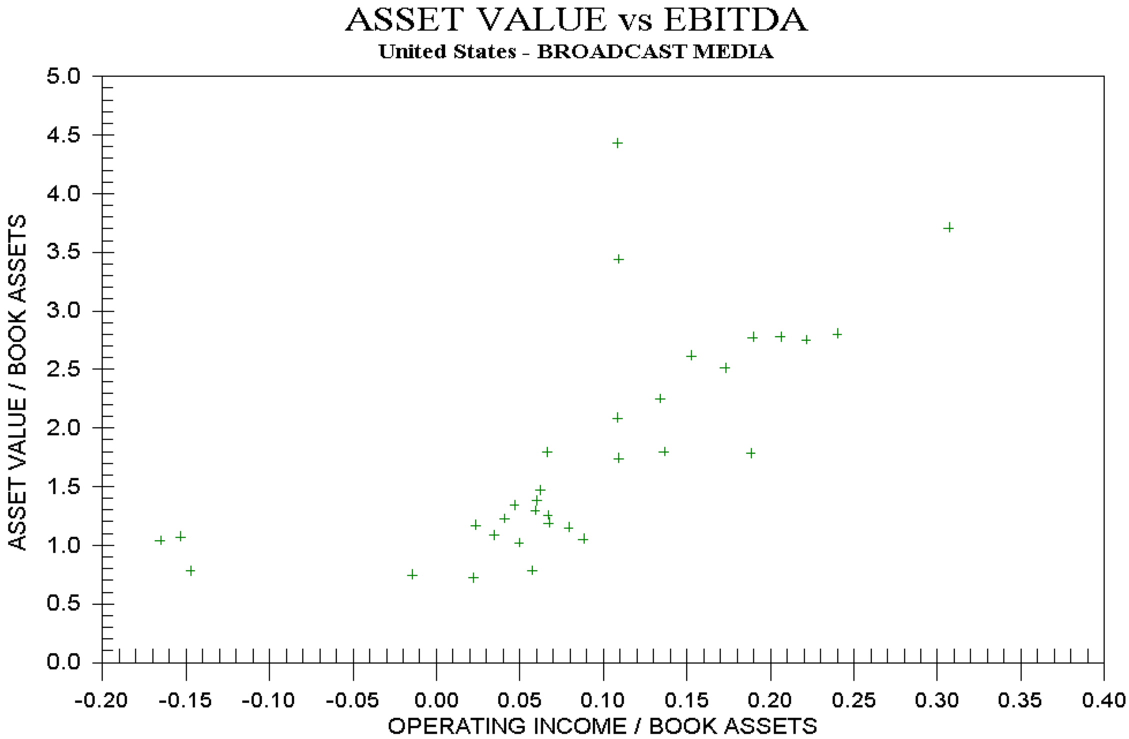

Figure 1

To estimate the value of a private firm's assets, the model uses the median value from firms in the same region and industry that have similar cash flow. The PFM asset values are shown in Figure 2 with the observed asset values for the same companies overlaid.

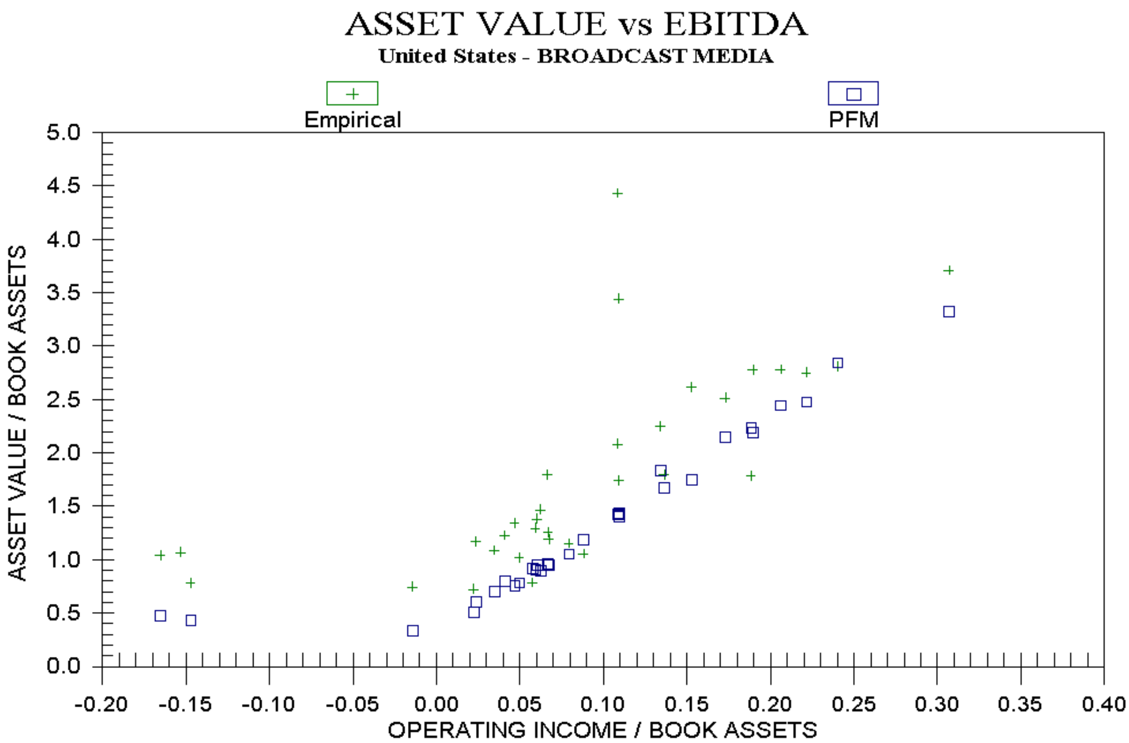

Figure 2

3. Estimate Asset Volatility for Private Firms

Modeling Default Risk details how, if the market price of equity is available, the market value and volatility of assets can be determined directly using an options theoretic approach that recognizes equity as a call option on the underlying assets of the firm. Credit Monitor uses this approach to simultaneously estimate a firm's market asset value and volatility from its observed equity value and volatility.

When we observe a firm's asset volatility, as extracted from market information, it is clear that industry, firm size, and geographic region are primary drivers for volatility. The larger the firm the less the variability in its asset value. Larger firms have more diversification within the firm, so there is less likelihood of a single event, such as a warehouse fire or a fraudulent manager, wiping out the entire firm.

Industry is also closely correlated with asset volatility. For a given size of a firm, banks are less risky then beverage retailers who are, in turn, less risky then biotechnology firms. In general, growth industries have riskier assets then mature industries.

Asset volatility, the measure of risk used in the KMV models, is a measure of the variability in the market value of the firm's assets. The market's valuation of assets changes when there is a change in the firm's earnings prospects, when there is a change in the expectation for economy-wide earnings, and when there is a change in the discount rate at which those future earnings are valued. Discount rate movements account for around 10% of the variability in asset valuations. The remaining variability is explained by revisions in the market's consensus view of the individual firm's prospects.

Some firms' earnings are easier to forecast than others. Regulated utilities, at least before the advent of competition among them, have an externally predetermined rate of return. Barring event risk and discount rate changes, the forecast of their future earnings is not subject to frequent revisions. In contrast, small technology companies might have no prospect of earnings for several years and an expectation that the company's single product will either become an industry standard or disappear. As information arrives that modestly affects the market's evaluation of the probability of one of those outcomes, the impact on the current value can be extremely large. These firms are difficult to forecast and subject to frequent and large revisions. They will have very high asset volatility.

Figures 3 through 6 present the asset volatility (as implied from market information in Credit Monitor) of publicly traded firms in three different industries, banking, chemicals and computer software. We have included two graphs for the chemicals industry, one for the US, the other for Europe, to show regional variation (Figures 4 and 5, respectively). The horizontal axis shows the firm's size on a log scale, which allows us to compare firms that are several orders of magnitude different in size. The vertical axis measures asset volatility in percent.

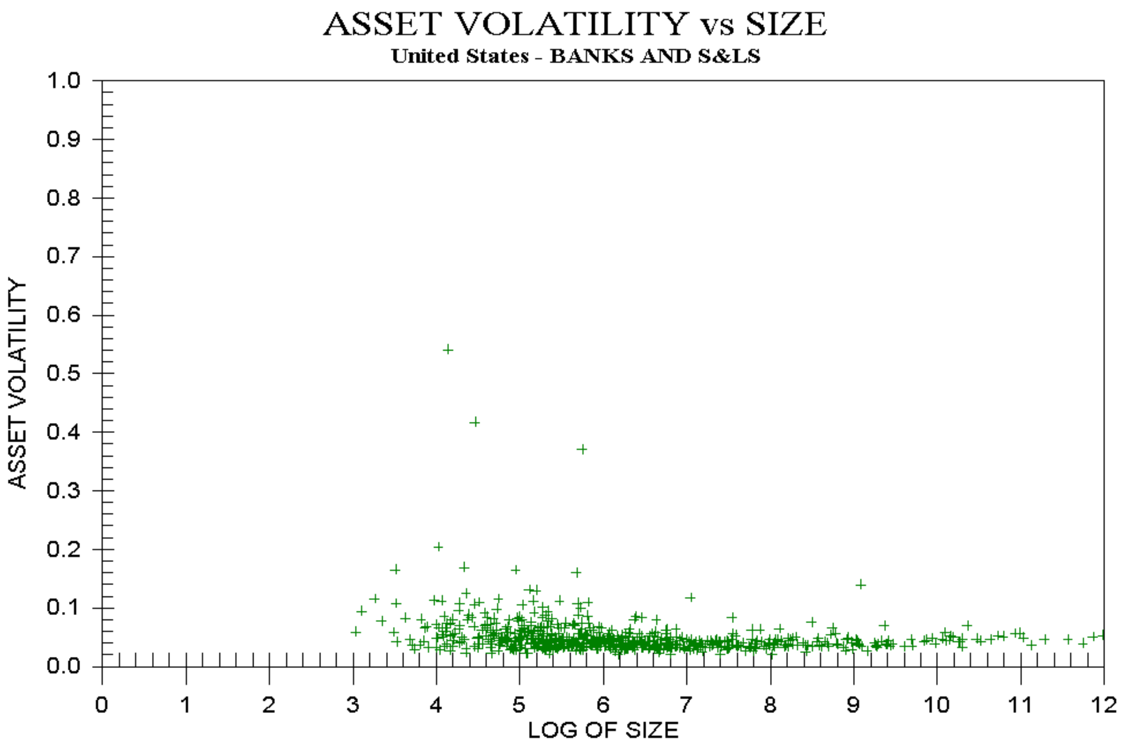

Figure 3

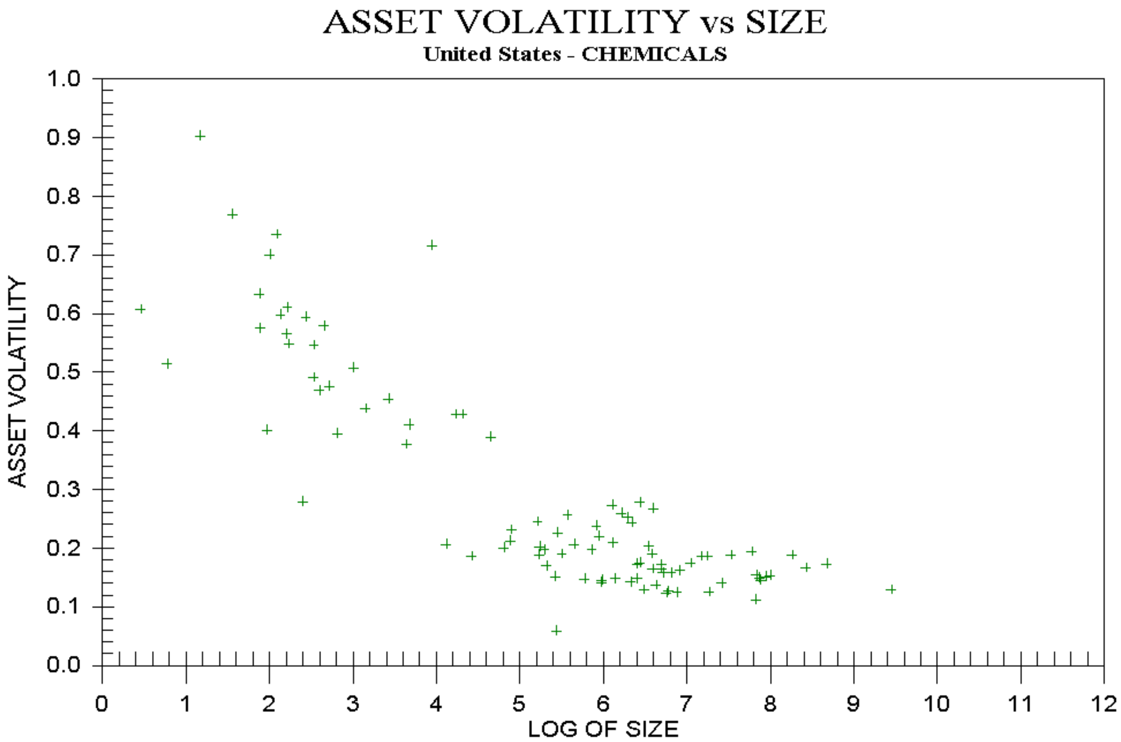

Figure 4

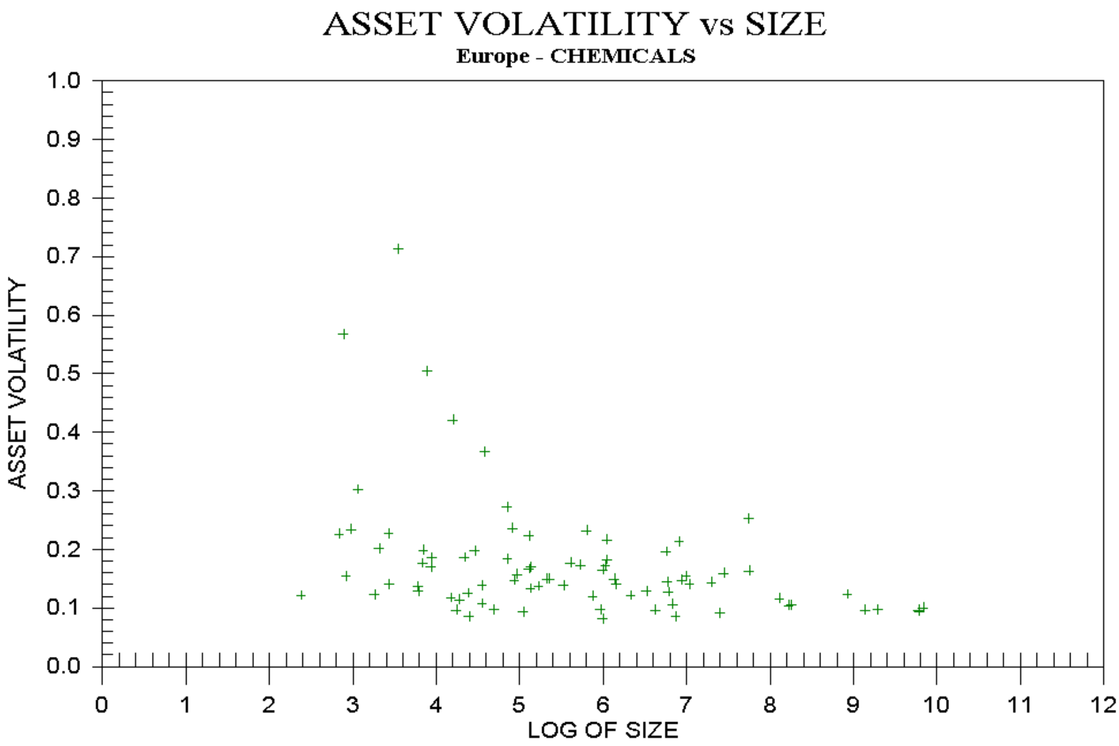

Figure 5

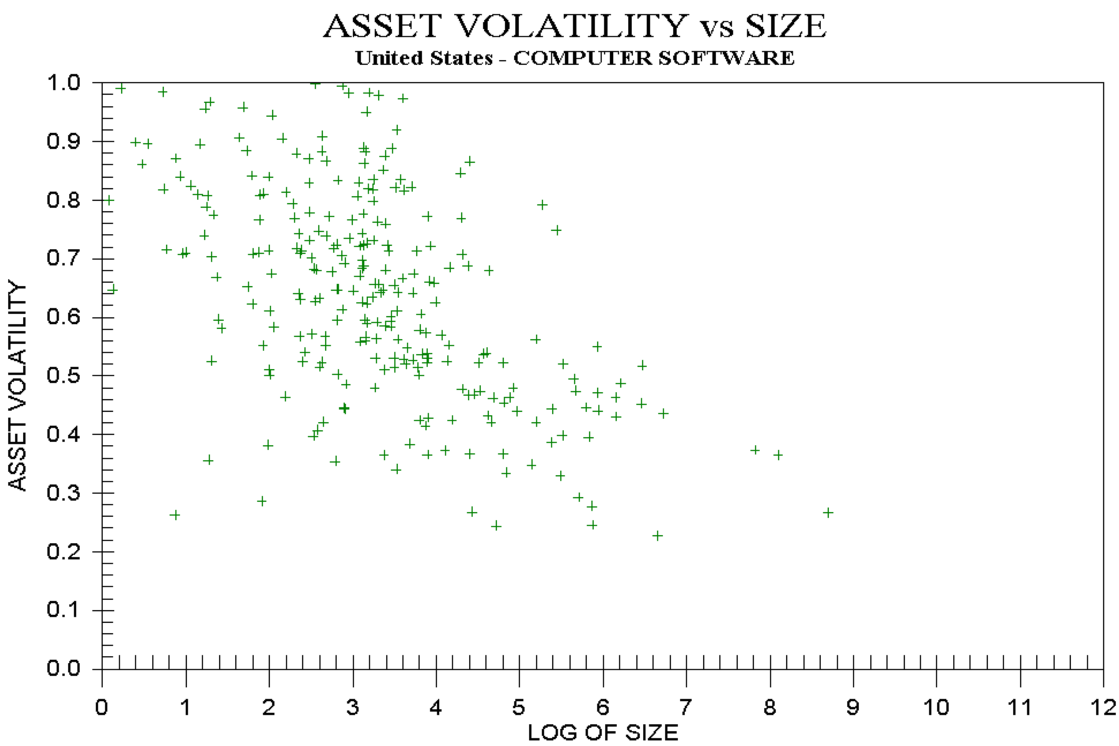

Figure 6

For each region, we estimate from the public market data a non-linear relationship between increasing asset size and decreasing asset volatility by industry. This is somewhat like drawing a curve through the center of the data points above. This line represents the median firm's asset volatility, given its size, industry and geographic region.

This median volatility is termed "modeled volatility", and it turns out to be a powerful tool in creating a predictive measure of future volatility. For public companies, where we have an actual measure of asset volatility from the market, we blend the actual volatility with the modeled volatility to sharpen our estimate by reducing the impact of noisy data. For private firms, we have only the modeled estimate of asset volatility, since we cannot observe market prices.

Volatility varies between different geographic regions. We observe that asset values in Japan are more stable (though over time this effect is lessening) than asset values in the United States. This observed lower variability is perhaps an effect of government stabilization policies. The influence of size on volatility is also lower outside North America. Smaller firms outside the US are less different than their larger peers in volatility.

Given the industry and region in which a firm operates and a measure of its size, the modeled asset volatility can be calculated. This volatility is further modified by a few characteristics specific to the firm. Looking at public companies, we have observed that companies with very high or very low EBITDA relative to their industry tend to be more volatile. Intuitively, we can understand that these are the "high-flyers" or the "dogs" and that their assets are less stable.

4. The Impact of Estimation Errors

In the absence of market data, using median data for highly similar firms is a good predictor of asset value and volatility. However, the median values are going to differ from the actual values for the individual firms that compose the median. That difference is the estimation error.

The model was first tested on public companies. We test on public companies by ignoring the market information, and treating the company as private by using only financial statement data available. When we use only the median comparable volatility instead of blending it with the asset volatility estimated from market prices, the estimate will be too high for firms that are below-median volatility and too low for firms with above-median volatility. Likewise, the median asset value derived from comparables will overstate value for some firms and understate it for others. An example is the observation labeled "G" in Figure 7 below, where estimated asset value would be much lower than the observed asset value. Within each industry the model will be accurate on average, but there will be "outliers".

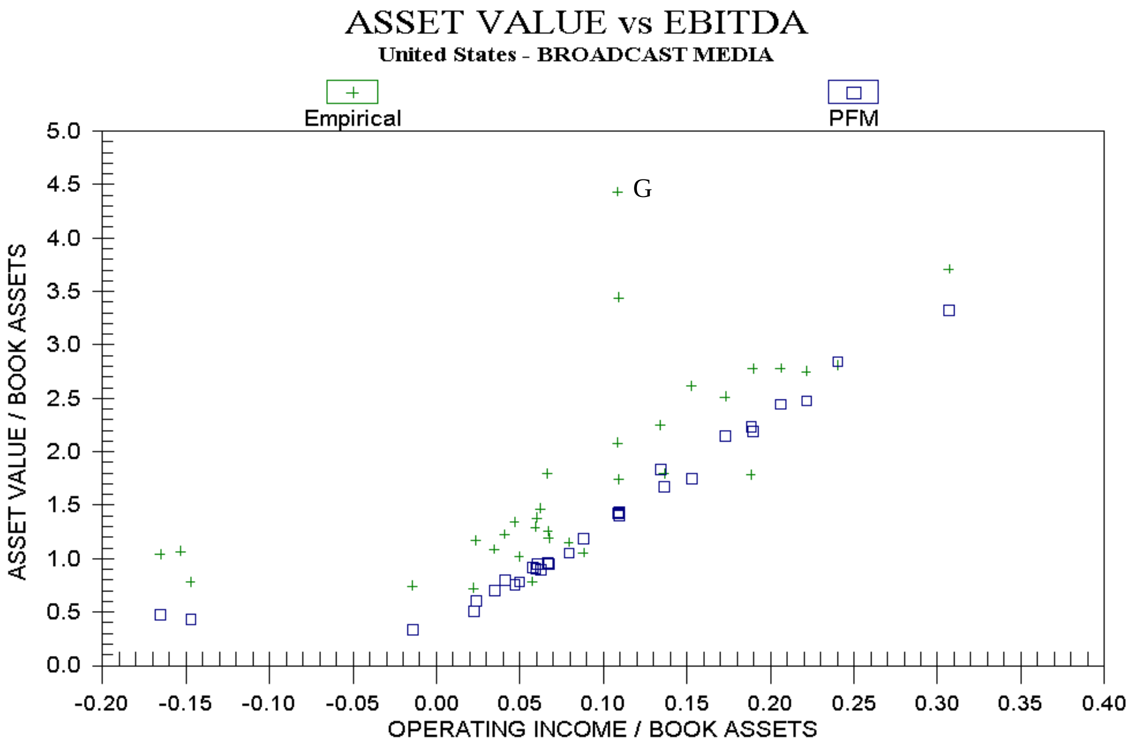

Figure 7

Errors in estimation could potentially have a measurable impact on the estimate of credit quality for the private firm since overstating asset value or understating asset volatility will lower the EDF. Understating asset value or overstating asset volatility will increase the EDF. Our research on public companies, however, has given us some comfort on this point. We have observed that the companies that have above-median asset values tend to be companies with higher growth opportunities. The model will underestimate the asset value by choosing the median value, thereby increasing EDF. Those same companies, however, tend to have above-median volatilities – a normal characteristic of growth companies. The median model will underestimate the asset volatility by selecting the median. These effects are offsetting. This characteristic of the model implies that the resulting credit quality measure, EDF, will be more accurate than the individual estimates of asset value and asset volatility that go into its calculation.

5. Testing the Private Firm Model

In tests on public companies, the PFM underperforms the Credit Monitor public model. This is an expected result, since to treat a public company as a private company is to throw away useful data about the firm.

Figure 8 below shows graphically the power of the Private Firm Model to predict default in North American public companies. The horizontal axis measures the percent of the portfolio excluded, in the effort to exclude firms that will default. Plotted on the vertical axis is the percent of actual defaults that are excluded. The goal of any credit measure is to eliminate all of the defaults while excluding just a tiny fraction of the portfolio. An extremely tall first bar on the left side of the chart would represent that best case scenario.

The grey bars in Figure 8 below represent the random case, where credits are excluded without any predictive measure that distinguishes if they will default. In the absence of any information, randomly eliminating 10% of the portfolio will eliminate just 10% of the defaults. Both the Credit Monitor and Private Firm models contain significant predictive power. Eliminating the worst 10% of the portfolio based on the Credit Monitor public EDF would eliminate 53% of defaults, as shown by the first white bar. Using the Private Firm Model EDF to eliminate credits would eliminate 44% of defaults, as shown by the first black bar. The Credit Monitor advantage over PFM is strictly due to having more information, in the form of market prices.

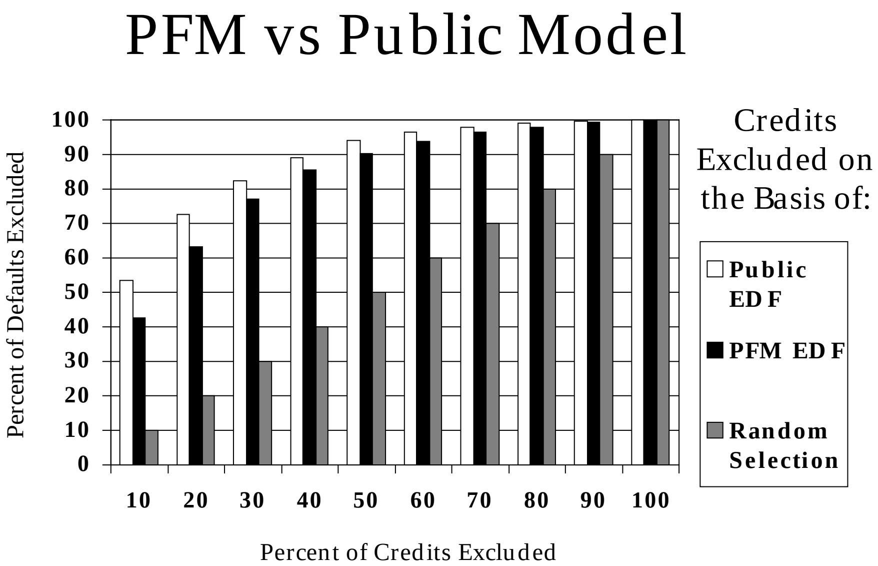

Figure 8

Users of the Beta version of the Private Firm Model in the US and Europe have tested on actual private firms and generally confirmed our testing results from US public firms. They have also found it outperforms regression type models.

Private companies compete with public companies and face the same pressure from suppliers and customers, so we expected them to be fairly similar. One notable difference came to light, however, for the very small public firms, those companies with less than $10 million per year in sales. Very small public firms tend to have much higher volatility than do the same-sized private firms. Why? The reason lies in why a tiny company would go public in the first place. Tiny software companies go public. Tiny fish-and-chips shops do not. The difference is that the software firm has extreme growth prospects that make it appealing to the equity market. Such growth prospects, as we noted above, also imply a much higher volatility. So the volatility of tiny public firms is a biased proxy for the asset volatility of similar private companies. In the production release of the PFM, we correct for this effect.

6. Using the Private Firm Model

The private firm model offers a structured approach to private firm credit analysis, but there is still a greater degree of “art” in using PFM than Credit Monitor. The user must be more vigilant about the quality of the data inputs to the private firm. The operating income figure (EBITDA) is critical in determining asset value, so if it were significantly unrepresentative of future earnings for some reason, the analyst might improve the model result by modifying that input. Sales and industry data are critical to estimating the volatility and the analyst must check for accuracy there as well.

The Private Firm Model is a predictive and objective platform for analyzing credit risk for private firms in the large corporate and middle market portfolio. This consistent treatment and objective framework should not replace traditional credit analysis for private firms, but it provides an advanced starting point that ensures greater consistency in analysis across firms, industries and analysts.