The Internal Ratings Based Approach for Capital Adequacy Determination: Empirical Evidence from Sweden^{*}

Kenneth Carling Tor Jacobson Jesper Lindé Kasper Roszbach

June 2001, preliminary

Abstract

The Internal Ratings Based approach for the determination of required buffer capital is one of the cornerstones in the proposed revision of the Basel Committee rules for bank regulation. This paper is an attempt to empirically evaluate the IRB approach using historical business loan portfolio data from 1994 to 2000 for a major Swedish bank. In particular, we study how the bank's risk weighted assets change over time (had the bank been subject to the proposed rules). In order to better interpret the calculated risk-weighted capital as given by the new Accord, we have estimated a credit risk model. A VaR-type credit risk measure derived by simulation from the credit risk model allows us to better judge how adequate IRB-determined buffer capital is.

Key words: Internal Ratings Based approach, relative risk weights, credit risk models

^{*}Research Department, Sveriges riksbank, 103 37 Stockholm. Preliminary, incomplete, and with errors included. Thanks to David Cesarini for excellent assistance. Comments and suggestions are welcome, please contact tor.jacobson@riksbank.se

1 Introduction

In January 2001, the Basel Committee on Banking Supervision released its second, revised, proposal for the future capital adequacy rules, i.e. the new Basel Accord¹. The proposal is organized around three so-called pillars. The first one describes the rules for determination of bank's required buffer capital, intended to cover unexpected credit-losses. The second pillar concerns the supervisory review process of the bank internal procedures for capital determination with respect to risk profile. The purpose of the third pillar is to increase the transparency of bank's risk profiles for market participants through disclosure requirements, i.e. to promote market disciplinary effects towards sound banking practice.

The guiding principle of the new accord is that the size of the buffer capital is made much more risk sensitive compared with the current accord. For instance, corporate sector loans are now given a constant risk weight of 100 per cent in the summation of a bank's risk exposed assets, irrespective of factual counterpart credit risk. In the future these risk weights will be contingent on counterpart risk.

The first pillar proposes two main routes for banks to follow when determining risk weights. First, a base-line "standardized approach" designed to be applicable for every bank. In this approach a portfolio of bank loans will be characterized by a relatively small number of risk categories, and the risk weight associated with a given category is based on an external rating institution's evaluation of counterparty risk. Second, a more elaborate model: the so-called Internal Ratings Based (IRB) approach. The underlying idea of the IRB approach is to make further use of the information collected and processed in the bank's internal counterparty rating operation. Since banks make it a business to evaluate risks, these evaluations ought to be a reasonable basis for risk-contingent capital adequacy determination. Each internal rating category in a loan portfolio is characterized by an estimate of its average probability of default, calculated by the bank itself. By means of an estimated function, the supervisory authority provides a mapping from the estimated probability of default to a relative risk weight. The products of relative risk weight, exposure at the time of default (usually taken as the face value of the loan), and the 8 percent absolute capital requirement,

¹The proposal can be found on the homepage of Bank for International Settlements at: http://www.bis.org/publ/bcbsca.htm.

summed over the loans of the portfolio give the bank's required buffer capital. The current proposal suggests that the banks may choose to apply the IRB-approach at either of two levels of sophistication. The more advanced requires bank internally generated inputs on loss given default and exposure at default, whereas the simpler only requires the bank to provide estimates of probability of default..

This paper aims at examining several aspects of the IRB approach for capital adequacy determination. The method is quantitative and empirical. To this end we have collected a historical data set for the corporate sector loan portfolio of an internationally active Swedish bank. The data is in panel data format, i.e., 24 cross-sectional downloads of the portfolio on the last day of each quarter for the period 1994 to 2000. Moreover, the bank data has been augmented with real time information on the characteristics of the firms in the portfolio. The latter data have been acquired from Upplysningscentralen AB, a leading credit bureau in Sweden.

The following questions and analyses will be attempted:

- The bank makes use of an internal rating system comprising 15 classes, 1-14 for non-defaulted counterparts (credit risk postulated to increase with rating class), and class 15 for defaulted ones. An intuitive starting point is to examine if credit risk is really monotonically increasing over the classes. And are classes consistent over time? Are the transitions from relatively risky classes to relatively safe ones in accordance with the general improvement of Swedish economic conditions and the reduction in bankruptcy incidents for this time period?
- The new Accord opens up for several possibilities for the important calculation of the average default probability meant to characterize a rating class. The obvious approach is to base the estimate on the bank's own long-term default experience. Alternatively the bank can use the default experience of external ratings, or the predictions of statistical default risk models. We examine the magnitude of variation in probability of default estimates based on historical frequencies, as well as in model-based ones.
- How will the IRB calculated risk-exposed assets vary with probabilityof-default-estimation method? How do they relate to the business cycle?

• Last but not least, we intend to model the bank data using statistical duration analysis of the survival time of bank loans until default. One purpose is to explore and quantify factors that drive default behavior in the corporate sector. We are specifically interested in macro-economic effects over and above idiosyncratic risk as reflected in variables such as the output-gap and the yield curve. Such a credit risk model can be put to further use through a simulation-based credit risk measure of Value-at-Risk type. Insights can be gained by calculating VaR over time and relate it to macroeconomic development. Moreover, if found reasonably accurate, the VaR-measure can be calculated for the current loan portfolio of the bank and be used as an indicator of future corporate sector credit risk. Finally, it is, of course, appealing to compare capital adequacy determination based on a credit risk model with that of the IRB-approach. Such a comparison would implicitly involve an evaluation of the mapping function between the average probabilities of default and the risk weights for the rating classes.

A fundamental assumption underlying much of the historical IRB-analyses that we undertake is that the bank's credit policy remains unchanged under the new Accord. This is not very realistic, but difficult to avoid. Another serious caveat is that we will abstract from both granularity and maturity issues.

The paper is organized as follows. The next section deals with a description of the data set. In Section 3 we formulate and estimate the credit risk model. In Section 4 we attempt the issues regarding the IRB-approach. The paper ends with some concluding remarks.

2 Data

This section describes in detail the data set that has been used for the estimation of the model in Section 3. The final data set is a panel consisting of 576768 observations covering six years of quarterly data on all 53383 companies Swedish aktiebolag companies that had a loan outstanding at one specific Swedish bank (of the big four) at some time point between April 1, 1994, and March 31, 2000. Aktiebolag are by approximation the Swedish equivalent of US corporations and UK limited businesses. Swedish law requires every aktiebolag has at least SEK 100.000 (approximately US \$ 9,300) of equity, to be eligible for registration at the Patent och registreringsverket (PRV), the Swedish patents and registration office, and deposit an annual report at PRV. Although we have annual report data on small firms such as general partnerships, limited partnerships and sole proprietors, these will disregarded because we could not dispose of the relevant credit histories. This implied that we deleted approximately 20% of all companies. Observe, however, that a large part of the sample still consists of small enterprises: 65% of all spells concerns businesses with 5 employees or fewer.

The data on these companies has been made obtained from two different sources: the bank and by Upplysningscentralen AB (UC), the major credit bureau in Sweden. The bank supplied a full history of internal credit related data, including variables like the amount of credit granted, actual exposure, the types of credit, the amount granted per credit type, collateral, payment status, an internal risk classification. These data were available at a quarterly frequency. Upplysningscentralen provided us with non-bank specific data for each company in the bank's portfolio, which it collects from the PRV annual report data. For example, balance sheet and income statement data from the annual report were provided, but also historical data on payment remarks - dummy variables for credit history and payment behavior related events - for the company and its principals. These data were available at different frequencies, varying from daily for payment remarks to annually for accounting data. We will discuss the specifics of both data sources in greater detail in Sections 3.1 and 3.2 below.

2.1 Bank data

As mentioned earlier, as part of its risk management system the bank that we study maintains a internal credit rating scheme, that requires each business customer to be assigned to one out of 15 credit risk classes. Risk class 1 represents the highest credit quality and risk class 15 stands for the lowest credit quality, actual default, with the intermediate credit risk classes intended to imply a monotonically increasing risk profile. At a minimum, the bank updates the 'credit rating' of each firm in its portfolio every 12 months. We refer to Section 4.1 for a more elaborate description of the rating scheme. For the purpose of this study we will use the banks definition of a default: a loan that is assigned to risk class 15 by the bank. The criteria for such an assignment is that principal or interest payments are 60 days overdue. A comparison with data from the credit bureau shows that risk class 15 is nearly perfectly correlated with (the offiacally registered) bankruptcy. Generally risk class leads the latter by one or more quarters, most likely due to the length of legal procedures that have to be completed before bankruptcy is officially invoked.

The bank provided us with complete time series of credit history of each business customer at the bank. The most important credit variables are: the size of the loan, actual exposure, the risk class, the industry code, and a number of variables splitting up total credit in different types of loans. Appendix A contains a full list of the variables provided to us by the bank. We reduced a total of 19 types of credit to 5 broader groups, also used by the bank for certain analytical purposes: short term lending, long term lending, mortgages, guarantee loans and the remainder, mixed loans. Of all observations, 67% involved short term loans while 32% concerned long run loans, 5% mortgages, 17% guarantee loans and 20% mixed loans (the remaining credit types). More than 40% of all spells involved at least two types of credit. In 21% of all spells, businesses had both a short and a long term loan, implying that about two thirds of the businesses that borrow long, also borrow short term. Other credit type combinations that have a frequency of 5% or more are: short term and guarantee loans, short term and mixed loans and guarantee and mixed loans. The average (censored) spell length for a company is 10.8 quarters. If split up according to credit type, the average (censored) number of spells for a short term loan is 9.9 quarters, whereas a long term loan has an average duration of 10.7 quarters.

Figure 13 shows that there is quite some movement over time in the aggregate default rate of the bank's portfolio. Although we cannot exclude the possibility that it peaked before 1994 Q2, the maximum quarterly rate of default within the sample period was reached in the second quarter of 1995 at a level of 2%. Over the whole of 1995, 4% of all the bank's loans defaulted,

compared to an average annual rate over the whole sample period of 2.6%. After 1995 the default rate declines, reaches two smaller peaks, in the second quarter of both 1996 and 1997, of 1.0% and 1.5%, and then steadily falls to a zero level in 2000 Q1.

Figures 23-27 show that the default rate not only significantly varies over time but also loan types and between (and even within) industries. For most of the sample period, short term loans are associated with the highest default rates. One exception is 1994 Q4, when mortgage defaults reach a peak of 3.1% and 1997 Q3 - 1998 Q3 when the long term loan default rate slightly exceeds the short term rate. The four largest industries in terms of total average exposure are multi-family real estate, manufacturing of machinery & equipment, commercial real estate and wholesale. Together they accounted for on average 48% of the bank's loan portfolio. The quarterly default rates in three of these industries peaked simultaneously in the third quarter of 1995, although at highly varying levels. The fourth industry, multi-family real estate, reaches its 'top' in the same quarter as mortgage default rate: 1994 Q4. Commercial and multi-family real estate had the highest quarterly shares of defaults, 6.0% and 4.1% respectively. Wholesale and machinery & equipment default rates only reached top levels of 1.6% and 1.4%. After 1995, all four industries more or less follow the economy-wide pattern, their peaks in 1996 Q2 and 1997 Q2 ranging from .9% to 1.6%. Most other industries display a similar pattern over time. Two exceptions are the services industry, where the default rate appears to be more persistent, and the financial services industry, which displays a more erratic behavior, probably because of the smaller number of loans. In terms of average default rates, the commercial and multi-family real estate sectors rank first and second with quarterly rates of over 1%, followed at short distance by mining & quarrying, wood, pulp & paper and hotel & restaurants with rates between .8 and .9%. The best performing industries were electricity/gas and banking, with average rates of .2 and 0%. Chemicals, machinery & equipment and transport are the only other sectors with average default rates below 0.5%.

We suffice here with noticing that the main sources of the trend and fluctuations in the default rates were the Swedish real estate crisis in the early 1990's, the following recession which struck the Swedish economy during the first half and middle of the 1990's, and the accompanying banking crisis. Against this background, the zero average default rate in the banking sector may appear somewhat surprising. It can be completely explained, however, by the fact that the Swedish government de facto granted a non-bankruptcy guarantee to all banks in 199X and founded a national banking emergency authority in 199X. Bad loan portfolio's of banks that were in risk of collapse were taken over and managed by this authority. For a more elaborate discussion of the macroeconomic backgrounds, we refer to Englund [9]. The bank in our sample did, however, not enjoy any government support.

The last variable of interest to be discussed here is the risk class. Figures 14-22 display the default rates over time among companies in the risk classes for three different horizons: 1, 4 and 8 quarters ahead.² The general picture that is brought forward by these graphs is that default risk is not constant within risk classes over time. Figures 14 shows, for example, that the one quarter default rate in risk class 5 follows the movements of the business cycle and varies between 0 and .5% - even within short time intervals of 2 years. Figures 15 and 16 confirm the cyclical default pattern for risk classes 6-14. Roughly, default rates appear to increase groupwise, with companies in classes 6-10 exhibiting higher default risk than classes 1-5, and classes 11-14 representing the riskiest counterparts. Observe, however, that no risk monotonicity in any strict sense exists between the 14 classes. For example, class 10 counterparts are clearly less risky then those in rating classes 8 and 9. In Section 4.1 we discuss the rating classes more extensively.

2.2 Credit bureau data

The data set that from the credit bureau contained information on most standard balance sheet and income statement variables. Some examples of balance sheet entries are cash, accounts receivable and payable, current assets and liabilities, fixed - and total assets, total liabilities and total equity. Some examples of the income statement entries that were available are total turnover, earnings before interest, depreciation and amortization, depreciation, financial income, extraordinary income and taxes. Appendix B contains a complete list of all annual report variables. In addition to the annual report data, which is collected by PRV, we also a number of data series on companies that are collected by their banks but stored and updated by UC. Time series are available on remarks for 61 different credit and tax related events. Two types of remarks exist. The first type are non-payment remarks, the storage and usage of which are regulated by the Credit Information Act,

 $^{^{2}}$ Zero default rates for some risk classes, like 11 and 14, in the first quarters are due to missing values.

the Personal Data Act and overseen by the Swedish Data Inspection Board. Examples of events that are registered are: delays in tax payments, the repossession of delivered goods, the seizure of property, the resettlement of loans and actual bankruptcy. In practice, with a record of non-payment remarks individuals will not be granted any new loans and businesses will find it very hard to open new lines of credit. The second type are bank remarks, which give an image of a firm's payment behavior at banks. All Swedish banks participate in this scheme and report any abuse of a bank account or a credit card and slow loans (loans of which repayment is considered questionable) to UC. Their storage and usage is only regulated by the Personal Data Act. Whereas a bank remarks may have the same consequences as having a non-payment remarks, this is not generally the case. Their effect on credit availability works mainly through the accumulation of negative indicators. Appendix C contains the complete list of non-payment and bank remarks.

As can be seen in Table 1, all descriptive statistics for accounting ratios and other credit bureau variables, such as non-payment and bank remarks and sales, were calculated based on different numbers of observations. For various reasons and depending on the specific variable up to 28,000 observations per variable could not be used in the estimation of the model. This could be due to incorrect entering of data by the credit bureau (unreasonable or negative values for non-negative balance sheet and income statement variables like total liabilities, total assets, inventories and sales), because of the nature of the ratio (a zero in the denominator), or simply the absence of any value. In all, this would have implied the deletion of approximately 10% of the sample. To avoid such reduction of our sample size, we replaced missing data on any variable by the mean value calculated on the basis of the available sample. As a result, the final estimation could be done with the full sample of 576,768 observations.

				Stati	istic			
Spell type	N	μ	σ	\min	1%	50%	99%	max
Performing	573170							
TS (mn SEK)	560540	61.8	765.00	0	0	2.87	912.00	82600
EBITDA/TA	559525	.06	16.52	-8041	79	.11	.70	2946
TL / TA	559678	2.77	439.83	0	.09	.76	2.36	154051
I / TS	548862	.45	67.88	0	0	.03	1.92	24844
AMTYP25 (%)	573170	.20	.20	0				1
NA_AM (%)	573170	.90	.90	0				1
Defaulted	3598							
TS (mn SEK)	3077	8.58	36.50	0	0	1.80	120.00	810
EBITDA/TA	3062	36	14.20	-663	-2.39	.04	1.03	184
TL / TA	3063	19.24	552.13	0	.05	.93	9.70	19783
I / TS	2971	4.87	248.57	0	0	.05	4.34	13549
AMTYP25 (%)	3598	9.90	8.90	0				1
NA_AM (%)	3598	20.3	16.2	0				1

Table 1. Descriptive statistics for the credit bureau data

As annual reports typically become available with a significant time lag, it cannot in general be assumed that accounting data over year t were available during or even at the end of year t to forecast default risk in year t + 1. To account for this, we have lagged all accounting data by 4 quarters. For most companies, who report balance sheet and income data over calendar years, this means that data over year t are assumed to have been available in quarter 2 of year t + 2. Of course, for non-payment and bank remark data, for which the exact dates were available, no such lagging was applied. For a number of companies some transformation had to be applied to the accounting variables to adjust for reporting periods that did not coincide with the calendar year, to assure that each variable was measured in identical units for all companies. Some companies, for example, report accounting information over three month or four month periods during one or more years. In such cases, annual balance sheet figures were calculated as weighted averages of the multiple period values. In other cases companies did report over a 12-month periods, but the period did not coincide with the calendar year. The 1995 figures, for example, could refer to the period 1995-04-01 until 1996-03-31. In these cases, such 'deviations' were accounted for by adjusting the '4 quarter lag' (and thus the date at which information is assumed to have become available) correspondingly.

From the set of balance sheet and income statement variables in Appendix B, a number of commonly used accounting ratios was constructed. We selected 17 ratios that were employed in a number of frequently cited articles studying bankruptcy risk. See Altman [1], [2][3] [4], Frydman, Altman and Kao [12], Li [15], and Shumway [17]. Most of them are closely related liquidity measures, two are leverage ratio and the remainder are profitability ratios. Appendix D contains the full list. In our empirical model, we employ three accounting ratios: earnings before interest, depreciation, taxes and amortization over total assets (earnings ratio), total liabilities over total assets (debt ratio) and inventories over total sales (the inverse of inventory turnover). These three ratios were selected from the original list of 17 variables following a two-step procedure. First, the univariate relationship between the ratio and default risk was investigated. By visual inspection, ratios that displayed a clearly non-monotonic relation or lacked any correlation with default risk were deleted from the set of candidate explanatory variables. Figures 2, 5, 8 and 11 illustrate this for the three selected ratios and for total sales, which is used as a proxy for firm size. Default rates in these figures are calculated as averages over an interval of +/-2500 observations. Figures 2 and 5 reveal a positive relationship between default risk on the one hand and both the leverage ratio and the inverse of inventory turnover. Figures 8 and 11 strongly suggest that a negative relationship exists with both sales and the earnings ratio. We also checked if any significant differences in the average and median ratios existed between healthy and defaulting firms. Table 1 and Figures 3, 6, 9 and 12 contain some additional information on the distribution and the time series properties of the financial ratios and non-payment and bank remarks. Table 1 shows that defaulting firms consistently, that is: for each percentile, have lower earnings, lower sales, higher inventories and a higher level of indebtedness. Figures 3, 6, 9 and 12 confirms this picture and suggests that these differences between (the median financial ratios of) healthy and defaulting firms are persistent, although possibly varying, over time. The median earnings of healthy enterprises, for example, are consistently more than twice as high as for defaulting ones. The difference in leverage ratio varies from approximately 15 percentage points in the midnineties to 25% in early 2000. On average, inventory turnover seems to be higher for defaulting firms, although there is quite some variation over time. Total sales differ in two respects between the two groups of businesses: they are strictly lower and vary more for defaulting firms than for healthy ones.

The above process led to the selection of six candidate variables: the three described above, and three other liquidity measures: cash over total assets, current assets over current liabilities, and accounts payable over sales. In the second step, their multivariate properties were studied by estimating a number of permutations of the empirical model. Neither of them turned out to make any significant contribution in the duration model.

For the non-payment and bank remark variables the same procedure was followed. An intuitively reasonable starting point was to find remark events that (i) lead default as much as possible and (ii) are highly correlated with default. As it turned out, quite some remark variables are either nearly perfectly correlated with default or lack a significant correlation with default behavior. Examples of the first category the start or completion of a company reconstruction. The most likely cause of this is the existence of a reporting lag. Tax related variables are typical examples of the second category. Of the remaining variables, many create a multicollinearity problem. For our final model, we selected two explanatory remark variables. One is a composite dummy of three events: a bankruptcy petition, the issuance of a court order - because of absence during the court hearing - to pay a debt, and the seizure of property. The other variable is "having a non-performing loan".

Finally, Figures 27 and 28 provide an interesting description of the default behavior by firm size. Table 1 shows that 10 respectively 20 % of the defaulting firms has a slow loan or a record of non-payment, in sharp contrast with the less than 1% among companies with performing loans. Figure 27 confirms the common perception that smaller firms, such as small businesses without employees, run a higher risk of defaulting. At nearly every bankruptcy peak, these companies fail at a higher rate than other businesses. Surprisingly, however, and in contrast with the commonly held opinion that large companies are less likely to fail, the category of businesses where rate of failure to repay loans is by far the highest rate, is the one consisting of companies with more than 500 employees. This holds during the whole sixyear sample period except for the last quarter of 1999, close to the top of the business cycle, when default rates in nearly all categories are close to zero.

2.3 Macro data

The importance of macro-economic effects for credit risk is a virtually nonexisting topic in the empirical literature. In all likelihood due to a lack of suitable historical credit data. We hope to contribute to this area using the bank data described above.

Figure 33 shows the developments of the growth rate in real GDP, in 1995 prices, and the output-gap, given by the estimated difference between actual and potential GDP, for the period Q1 1980 to Q2 2000. The series for the output-gap is computed using an unobservable components method due to Apel and Jansson [7]. The deep recession in the beginning of the 1990's can be clearly seen from the figure, with negative growth figures (over 4 per cent at most) and a negative output gap of over 8 per cent. The strong economic improvement of 1994-1996 is also evident. In Figure 30, the yield curve and the output-gap series are related to the default rate for all loans in each quarter. There is a strong downward trend in the default rate over the sample period, reflecting the general improvement of the macroeconomic environment. Finally, in Figure 31 we show the Swedish households expectations of the future macroeconomic development, with a lag of 2 quarters, together with the aggregate default rate.

A priori, we think that these three macroeconomic variables should have a measurable impacts on the default risk of a given firm. Starting with the output gap, it may supposedly work as an indicator of economic activity, increased economic activity reducing default risk. Figures 33 and 31 seem, at large, consistent with this view, although there are some big spikes in the default rate that clearly have to be attributed to other variables. Apart from firm-specific factors, we believe that the two other macroeconomic variables presented above might be important. Recent research, see, e.g., Estrella and Hardouvelis [10] and Estrella and Mishkin [11] suggests that the yield curve can be an important indicator of future real activity; i.e., a positively sloping yield curve signalling higher future economic activity and vice versa. Therefore, we expect that an increase in the spread between a short- and longterm interest rate is associated with decreasing default rates, since banks and firms will act upon this information. Banks will have stronger incentives to renegotiate loan terms with firms at the brink of bankruptcy. Firms will likewise have incentives to prevent the firm from defaulting, given prospects of increased future demand. By similar arguments, we expect that higher household expectations about future economic activity also reduce the default rate today.

We use the difference between the nominal interest rates (annualized) on 10 year government bonds and 3 month treasury bills as the measure of the spread. The index of household expectations about the future stance of the macroeconomy is taken from the survey data produced by Statistics Sweden. In the credit risk model, we will enter the series for the output-gap and the household expectations with a lag of two quarters, since they are available for forecasting purposes with approximately that time delay. However, we will not lag the series for the yield curve spread, since it is accessible in real time.

3 The credit risk model

The aim for this section is to develop a reduced form statistical model for estimation of probabilities of default for counterparts of the bank's corporate sector loan portfolio. The general idea is to enter factors into a model that are determinants of the probability of default and analyze how these contribute towards predicting default realizations. Knowledge about the probability of default can then be used to calculate expected losses per counterpart exposure, given by the product of exposure size and estimated default probability. In a second step, expected losses per exposure can be used to derive total expected losses for a portfolio, and, thus, enable a calculation of the loss-ratio, i.e. the total expected losses in relation to the total value of the outstanding loans. In a third step, the estimated model can be used as a basis for simulating, or bootstrapping if you like, an estimate of the distribution of losses, which, in turn, will allow for a Value-at-Risk-type measure of portfolio credit risk. Moreover, under various assumptions about the future development of, e.g., the macro economy, or the bank's credit policy in terms of portfolio composition, the estimated model can be used for stress-testing experiments where conditional VaR is calculated. Hence, the model should be informative about various aspects of future portfolio credit risk.

3.1 Outline of the statistical model

As discussed earlier, we will, due to data limitations, model counterpart default risks, and not individual loan default risks. Nevertheless, for the sake of simplicity, we will in this section discuss the model in terms of loans.

We begin by assuming the following:

- (i) the loans are revolving
- (ii) default risk is constant over time
- (iii) default is the only absorbing state
- (iv) zero recovery rate for a defaulted loan

The four assumptions (i-iv) are not strictly required, they are imposed to simplify the exposition below, where we discuss the implications of relaxing them.

Let τ denote calendar time and let the random variable D take on unity if a loan defaults at the following point in time, i.e. at $\tau + 1$, and zero otherwise. We seek to identify the following parameter,

$$\Pr[D=1 \mid \tau] \tag{1}$$

which is the probability of a default for the corporate loan at calendar date $\tau + 1$. This parameter can be used to answer the question of the present risk of default and the expected losses at the following time-point, i.e., at $\tau + 1$.

However, empirical identification of the probability parameter in (1) is not feasible unless a fundamental assumption (v) is imposed:

• (v) factors that determine default risk, and vary over time, will repeat themselves

If so, τ can be substituted for by such factors. We will apply the following notation: x refers to factors specific to the loan, y to factors specific to the operating environment of the firm, and z to factors specific to the general operating environment of all firms. Hence, x may represent variables like loan size, firm size, and various performance measures based on accounting data, as well as historical payment records on the payment behavior of the firm. The purpose of x is to capture idiosyncratic risk. y may represent information about the performance of the industry to which the firm belongs, as well as measures of economic activity in the geographical region in which the firm operates. Finally, z is supposed to capture business cycle effects and may be represented by variables such as measures of the yield curvet and the outputgap, and the rates of inflation and unemployment. In order to condition on these factors, we now consider the following parameter,

$$\Pr[D = 1 \mid x, y, z, x(\tau), y(\tau), z(\tau)],$$
(2)

where $x(\tau), y(\tau), z(\tau)$ indicate that the factors vary over time. The effect of x, y, and z will be identified by cross-sectional variation in the probability of default and the effect of $x(\tau), y(\tau)$, and $z(\tau)$ will be identified by cross-sectional variation in the default probability at different calendar times.

3.1.1 Dropping assumption (iv)

Assumption (iv), nothing recovered from a defaulted loan, is only required if the focus is on the question of expected losses and no information on the recovery rate r, say, is available. In our case, however, such information is available, and we can identify the parameter,

$$\mathbf{E}[r \mid x, y, z, x(\tau), y(\tau), z(\tau)], \tag{3}$$

which is the expected recovery rate. By combining the probability parameter in (2), the loan size, and the parameter in (3), it is thus possible to calculate the expected loss giving due account to the fact that in some cases substantial amounts of defaulted loans are recovered.

3.1.2 Dropping assumptions (i) and (ii)

Assumption (i), revolving loans, is technical and harmless, it can easily be dropped if information is available on the repayment schemes for the loans.

Assumption (ii), a constant default risk over time, is implausible and it is likely that the model can be improved by dropping this assumption. Specifically, if the risk of defaultness is not constant and the portfolio, at each instant, consists of a stock of loans with varying durations, then the parameter in (3) will produce biased predictions. Hence, it is *a priori* reasonable to control for loan duration.

Let T be the duration of the loan until it defaults. We then seek the duration analogue to the parameter in (1) given by

$$\Pr[T = s \mid T > s - 1, \tau],\tag{4}$$

which is the probability of default at calendar date τ given that the loan had survived to the previous time-point. Further extensions of the parameter in (2) to the duration setting yields

$$\Pr[T = s \mid T > s - 1, x, y, z, x(\tau), y(\tau), z(\tau)].$$
(5)

The requirement for the data collection adds the following to enable a duration analysis; loans drawn from a time-window of several years and the measurement of the duration of the loans, the factors and the time-path for the time-varying factors.

3.1.3 Dropping assumptions (iii) and (v)

Assumption (iii), default being the only absorbing state, is not problematic if the intention is to identify the default probability parameter in (2). However, if the duration parameter in (4) is of interest and the aim is to calculate expected losses, then assumption (iii) might be too restrictive. Extensions to multiple absorbing states is feasible, but not without invoking additional non-testable assumptions. It is an empirical matter to determine whether it will be necessary to consider multiple absorbing states.

Assumption (v) is fundamental and non-testable. Sensitivity analysis, simulations and goodness-of-fit measures will be applied to determine the sensitivity of the final models to assumption (v).

3.2 The empirical model

Identification of the parameters in (2), or in (5), from a sample as the one described above is theoretically straightforward. The principles of Maximum Likelihood estimation can be applied for this purpose. In what follows we will discuss the estimation procedure and present the resulting model.

The data contains a total of 54,603 firms and 69,249 loan spells, which means that some firms are recorded with multiple loan spells. Thus, there are 69,249 potential observations of T, t_i say. However, only 3,598 spells were observed to default. The remaining did not default, either because the observation-period ended, or because the loan was redeemed. Let the censoring indicator, c_i , indicate with unity if the loan was observed to default and zero otherwise. Moreover, the set of variables pertaining to the *i*:th loan will be indexed by *i*, and let $\lambda_i(t)$ be short hand for the parameter in (5), that is the probability of a default in the *t*:th quarter for a loan with characteristics x_i, y_i, z_i and a time path of $x_i (\tau - t : \tau), y_i (\tau - t : \tau), z_i (\tau - t : \tau)$.

In the specification of the function linking the determinants to the parameter $\lambda_i(t)$ it is desirable to be as flexible as possible. However, as a baseline specification we postulate that the duration dependence is restricted to equality for all loan spells, yielding,

$$\lambda_{i}(t) = \lambda_{0}(t) \exp\left(m\left[x_{i}, y_{i}, z_{i}, x_{i}(t), y_{i}(t), z_{i}(t)\right]; \alpha, \beta, \gamma, \alpha^{t}, \beta^{t}, \gamma^{t}\right), \quad (6)$$

where m[] is some function and the parameters α, β, γ pertains to the x, y, z variables, respectively (a superscript refers to the time-varying variables). For a given choice of m the maximum likelihood estimates of the parameters are obtained by maximizing

$$\ln L\left(\lambda_0\left(t\right), \alpha, \beta, \gamma, \alpha^t, \beta^t, \gamma^t\right) = \sum_{i=1}^n \left(c_i \ln \lambda_i\left(t\right) - \int \lambda_i\left(t\right) dT\right).$$
(7)

The duration is measured quarters and is thus treated as being discrete. At this point it might seem that estimation of the parameter is straightforward. This is an illusion; several additional steps are required. First, there is the issue of determining m.³ Second, a choice of which variables to include must be made. Here we have been pragmatic and left ourselves to be guided by *inter alia* a pseudo $-R^2$ value.

The choice of variables has also been discussed in the data section, and clearly our choice is influenced by previously published work. However, the uniqueness and richness of the data has permitted us to explore, rather freely, an additional number of potentially important variables. Naturally, multicollinearity often restricts a too opulent set of variables. The guiding principles are (in order of importance); previously proposed and theoretically justified variables, stability of the model - both in terms of predictions and in the estimates, simplicity of the model, statistical significance, and the

³Defining the *m*-function is a non-trivial matter for continuous regressors. We have made use of a version of regression smoothers for censored or discrete response variables. We have simplified the matter, though, by adding one regressor at a time, thereby abstracting from the 'curse of dimensionality' problem. We start by defining the response variable as the logarithm of the ODDS of default conditional on non-default in the previous quarter and then use a regression smoother for the relation between logODDS and the regressor (see in particular Hastie and Loader [19], as well as Härdle [20], Gray [18] and Kooperberg, Stone, and Troung [21]). Thereafter, we have sequentially added regressors and determined the functional form of the linking function.

pseudo $-R^2$. The first point (i) is discussed in the data section, whereas the fourth point (iv) is a conventional principle, although in part less meaningful for very large data sets like the present. Nevertheless, we have been reluctant to include variables with t-ratios smaller than two, unless the non-significance of the variable is of interest per se. The pseudo- R^2 measure (v) is supposed to resemble the conventional R^2 measure of linear regression models. It is can be interpreted as the degree to which the distribution of predicted probability of default for performing loans does not overlap the distribution of predicted probability of default for loans that actually defaulted. The smaller the degree of overlap, the better the model discriminates defaulted loans from non-defaulted ones. And hence, the better the predicitive power of the model. The third principle (iii) means that we have avoided complicated transformations or interaktions of various variables, unless a substantial improvment has been achieved. Finally, stability (ii) has been checked by excluding the following subsets; a 90 % fraction of the performing loans, loans after the second quarter of 1997, loans with missing values on at least one of the variables, and loans having values of the variables outside the 10 to 90 percentile range. Moreover, the stability of the estimates has been checked, in addition to above mentioned checks, by including competing variables.

	Coefficient	Standard error
Duration		
1:st year	0	_
2:nd year	.026	.051
3:rd year	194	.057
4:th year	.236	.070
5:th year	104	.110
6:th year	.274	.175
Credit type ^{a}		
Long-term	0	_
Mix of short- and long-term	.511	.057
Short-term	.761	.049
Remarks with credit bureau ^{b}		
No remarks	0	_
Category 25 remarks	.903	.137
Category 8, 11, 16, 25, 31 remarks	2.638	.080
Accounting $data^{c}$		
TS (mn SEK)	079	.023
EBITDA/TA	.038	.026
I/TS	.436	.124
TL/TA	2.829	.114
Macroeconomic variables ^{b}		
Output-gap (lagged 2 quarters)	341	.018
Household expectations	163	.016
(first differences lagged 2 quarters)		
Yield curve, $(10Y - 3M)$	232	.025

 Table 2: The estimated coefficients in the credit risk model

Notes: ^avariables taken to be constant over time, ^bvariables taken to be time-varying, with quarterly variation, ^cvariables taken to be time-varying, with yearly variation.

Table 2 presents the estimated model. First, there is very weak evidence of a duration dependence. For instance, the estimate implies that the risk of

default increases by roughly three percent in the second year of the loan compared with the first year, though the difference is far from being significant. Second, the risk of default is markedly higher for short-term credits compared with long-term ones, the risk is about twice as high for the short-term credits.

The strongest determinant of default is, however, registered remarks during the preceding four quarters. Any such remarks implies that the risk of default increases by 14 times, i.e., 1,400 per cent. Add to this a remark of category 25 and the risk increases by about 34 times. In contrast, the predictive power of the accounting data is modest; although, the liability-toassets ratio (TL/TA) is quite useful. It should be noted, however, that the accounting data provides decent predictions of default occurrences whenever remark data is excluded from the model: it is the inclusion of the remark data in the model that makes the account data seem almost superfluous.

We have evaluated a number of macroeconomic variables and we find that the output-gap and the yield curve indeed are reliable indicators of the evolution of default risk over time. Additional improvement in the fit is achieved by using the households' expectations of the Swedish economy. One way of appreciating the importance of the macroindicators is to consider the output-gap. It varies from a low -7 per cent in the early part of the observation-period, to a zero gap between actual and potential GDP in the later part. This implies that the change in the output-gap yields predicted default rates for the later part of the sample that are roughly 10 times smaller than those for the early part of the observation period. The estimated parameters for the macroeconomic variables have the expected signs and enter significantly in the model. Presumably, the big spikes in the average default rate that occur during the years 1995 and 1997 are very helpful in distinguishing the effects of firm-specific and macroeconomic variables on default risk in the model. Among the macroeconomic variables, the current real economic activity seems most important for the default rate. So, although we do not have an estimation period that covers a complete business cycle (unless one would label the low economic growth during 1996 as a recession, see Figure 33), the estimation results are encouraging. The model is able to accurately distinguish between contributions from firm-specific variables on the one hand, and macroeconomic variables on the other. This is illustrated in Figure 32. Indirectly, the estimation results supports the idea of a credit channel in the monetary policy transmission mechanism, since the short-term nominal interest rate set by the central bank will influence the output gap, the spread and household expectations of the future. It remains as an interesting future challenge to further investigate the existence and quantitative importance of this transmission mechanism in the data.

The lack of transparency for the non-linear model we apply is a serious drawback. Model checking is therefore of critical importance. We find a pseudo- R^2 of about 60 %, which is quite respectable considering the predominantly cross-sectional nature of the data. Figure 32 provides further insights in the functioning of the model. The figure depicts the actual and the predicted default rate quarter by quarter. Included in the figure is also the output gap (scaled to fit in the window). The actual default rate is quite erratic, whereas it is obvious that the output gap may capture only the smooth changes in the default rate over time. The predicted rate follows quite well the short-term variation in the actual default rate, although it fails somewhat to capture two of the later peaks. To sum up, the estimated model demonstrates the need to take account of both idiosyncratic risk factors, as captured by payment remark data and accounting data, as well as macroeconomic effects.

For clarity and for future reference note that the predicted default probability, $\hat{p}_{i,\tau}$ say, for loan *i* at quarter τ is given by (5) where the determinants are set at the value corresponding to the *i*:th loan. The predicted default rate is simply the sum of all $\hat{p}_{i,\tau}$.

3.3 Value-at-Risk

Having identified the prediction model, it can serve many purposes, however two immediately spring to mind. First, the model can quantify the subportfolio risk at each time-point, e.g. a portfolio of loans as defined by a particular internal rating category. Such estimates of loss distributions could then provide estimates of required capital for given estimated probability of default, and hence admit, e.g., estimation of a relative-risk-weight mapping function for use in the IRB-approach. Secondly, the model may provide answers to questions like; what happens to portfolio risk and relative risk weights if the bundle of loans in the portfolio is changed? And what happens to the portfolio risk and relative risk weights if the, e.g., the interest rate spread increases? In other words, the model can be used to simulate the consequences of a hypothetical future change in the environment or a hypothetical change in the portfolio strategy. However, for the purposes of this study, an evaluation of the IRB-approach, we will use the estimated model for simulating credit risk measures, both for the portfolio and for the individual risk classes. These risk measures can function as standard, or basis, when evaluating the outcomes of calculated buffer capital under proposed Accord.

Consider, as a first step in a derivation of a Value-at-Risk-measure, the following simple observation. As stated in the previous subsection, the modelpredicted probability of default is denoted by $\hat{p}_{i,\tau}$. If $S_{i,\tau}$ denotes the utilized size of a loan *i* in quarter τ , it follows that the expected loss for that particular loan in the quarter of interest equals $\hat{p}_{i,\tau} \times S_{i,\tau}$. Summing over all loans would readily yield the expected losses for that quarter. Value-at-Risk requires a somewhat more sophisticated procedure, as it refers to the potential loss in a worst case scenario.

Calculation of Value-at-Risk, $VaR(\tau)$ will be done for one quarter at a time, so in the following the τ index is dropped. We suggest the following algorithm:

i) Draw a uniform random variate, u_i , and define $D_i = I(p_i > u_i)$ for all i.

ii) Define $VaR_r = \sum D_i \times S_i$.

iii) Repeat R times.

iv) Let $VaR(\tau)$ equal the 99:th percentile of the distribution of VaR_r .

Figure 34 shows the expected and actual losses on the bank's loan portfolio and three (90th, 95th and 99th) Value-at-Risk percentiles for the whole sample period.⁴ For this purpose, we have defined the credit loss in case of a predicted default as the utilized amount of credit (not the granted amount of credit) times one minus the recovery rate.⁵ The recovery rate that we use

⁴The x-th percentile Value-at-Risk is defined as the amount in SEK (alternatively the share of the portfolio) that will be lost by the bank with a maximum probability of x percent. Another way to interpret this is: with a probability of (100-x) percent, the loss by the bank will not be greater than some SEK amount (alternatively some share of the portfolio).

⁵For the purpose of calculating Value-at-Risk, we made one change to the data material. First, we merged risk classes 14 and 15. As we already mentioned in Section 2, firms that were assigned to risk class 15 in two or more subsequent quarters were assumed to have defaulted and exited from the sample in the first of this series of quarters. The only exception we made to this rule was for firms that had different types of loans in two subsequent quarters. This occurred for only a very small number of firms. For the purpose of VaR, we considered it more useful to treat classes 14 and 15 jointly, both because of the small number of firms with subsequent spells in risk class 15 and because of the uncertainty about the causes of such multiple spells.

here was calculated by the bank as a non-time-varying sample average for each loan loan type. Although this loss rate is implicitly affected by collateral that businesses provide, any individual differences in loss rates between firms due to variations in the available collateral are not (yet) taken into account. The expected loss rate has the same weak trend that is similar to that of the actual oss rate, , with the expected quarterly loss declining from approximately 2% in 1994-Q2 to .4% in 1995-Q4. From 1996 and onward the expected loss rate remains below .5%. Although the *expected* loss appears to capture the general trend in the actual loss rate actual and the macro series, it does not indicate if the portfolio risk increases at any stage. The peak in the actual oss rate in 1995 is missed completely. Therefore, we have also calculated distribution of Value-at-Risk for the bank's loan portfolio over time. The upper three lines in figure 34 are the 90th, the 95th and the 99th VaR percentiles. These clearly show that an expected credit loss measure fails to capture any (variations in) downward risk that the bank is exposed to. In fact, the same appears to be the case for the 90th and the 95th VaR percentiles, as they move more or less parallel with the expected loss rate, although at a somewhat higher level. The 99th percentile however, shows much more variation than any of the other three credit risk measures. In the second and third quarter of 1995, for example, the 99th VaR percentile rises to 3.4% of the portfolio, an increase of .6% compared to the preceding quarter. Expected credit losses as well as the 90th and 95th percentiles remain unchanged over the same period, however. Between the first and fourth quarter of 1997, the relative growth in VaR is even bigger as the 99th percentile rises from 1.6% to 2.9%, while the expected loss merely increases from .3% to .5%. In general we can conclude from Figure 34 that one level of expected credit losses is associated with widely varying levels of risk. For expected loss rates between .3% and .5%, 99th percentile Value-at-Risk actually ranges from 1.6% to 2.9%. For loss rates between .2% and .5% the VaR interval widens by another .5%. Consequently, any risk weight mapping function that maps expected loss rates into relative risk weights will fail to account fully for variations in portfolio risk!

Figures 35 - 48 contain the outcomes of similar calculations as those underlying Figure 34 for each separate rating class. ELR and VaR-values are displayed only for those quarters in which at least 10 companies were assigned to the rating class in question.⁶ A general property that the risk

 $^{^{6}}$ A miminum of 10 observations guarantees a minimum of at least 2^{10} possible different

classes appear to share with the portfolio is that the variance of the Valueat-Risk measures is much higher than of the expected loss rate. Whereas it can be seen in Figure 48 that the expected loss rate varies between 0% and 25% for risk class 14, its 99th percentile ranges from 2% to 61%. Although such fluctuations may be expected for companies that are (as) close to default (as possible), similar movements would appear more surprising for the 'safest' debtors. Figure 36 shows, however, that even the low risk part of the portfolio displays much variation in VaR and little variation in the expected loss rate. While the ELR ranges from 0 to 3% over all 15 risk classes, VaR at the same time takes values between 0 and 36%. Over all risk classes, the ratio between the ELR and 99% VaR varies from a factor 1 to a factor 46 for risk class 14. Although a strictly monotonic relationship between class size and variance in VaR does not exist, it is worthwhile to observe that 99th percentile VaR's for rating class 9, the biggest group, is quite smooth over time and has a maximum value of merely 6.2%. This compares more or less to the maximum of 4% for the whole portfolio. By comparison, of the 'safer' risk classes 1-8, only number 1 and four have lower VaR maxima. This reveals one of the less attractive features of an internal rating system with a large number of 'finer' risk classes. In general, given a predetermined average default rate, small rating classes (in the sense of number of companies) will tend to have higher Value-at-Risk peaks than big rating classes. If buffer capital is to reflect not only the first moment but also the second moment of portfolio credit risk, attempts in a finitely sized portfolio to refine risk estimates may actually lead to higher capital requirements. When designing an internal rating system group size will consequently have important consequences for the corresponding risk weight mapping function.

Finally, Figures 49 - 57 compare the ELR and VaR measures for the same three groups of internal risk classes as in Figures 14-22. When drawing some preliminary conclusions from them, we keep in mind that the above mentioned group size effects may be distorting the 'risk monotonicity 'properties between groups. Although for example example risk class 4 appears to display a strictly lower expected loss rate than class 2 for the second half of the sample period, and class 10 has lower risk than 8 and 9, not major and persistent inconsistencies with the monotonicity proprty are found. For the VaR measures, this is somewhat different. Risk class 2, for example, despite the relatively large number of spells, displays a steep increase in 99% VaR

outcomes in the VaR simulations.

in 1997 (35%), whereas 3-5 experiences only minor rises (below 10%). Similarly, in Figure 56, risk classes 6 and 8 show large increases in VaR (15-25%) while classes 9 and 10 stay at levels below 10%. Thus although monotonicity roughly appears to hold between groups for expected losses, this is much likely for a Value-at-Risk measure.

4 Bank's internal ratings

In this section we will analyze the bank's internal ratings by focusing on four issues. We begin by providing a set of stylized facts for the ratings and then carry on with the issue of estimating the average probability of default characteristics of rating classes. Thereafter we attempt the heart of the matter; size comparisons of the bank's risk-weighted assets according to the current and the proposed Accord. Finally, we will, equipped with the estimated credit risk model and associated VaR-measures, check the appropriateness of the proposed mapping function from average probability of default estimate to a relative risk-weight for a given rating-category.

4.1 Stylized facts

The bank's internal rating system comprises 15 classes. Table 3 shows the bank's appreciation of how these 15 rating classes relate to the well known rating categories of Moody's and Standard & Poor's. We have also included estimates of long-run average default rates for the ratings from Moody's and Standard & Poor's.

-				
Bank rating	Moody's	Def.rate	S& P's	Def.rate
1	Aaa, Aa1	0.00	AAA, AA+	0.00
2	Aa2, Aa3	0.03	AA, AA-	0.00
3	A1, A2	0.01	A+, A	0.04
4	A3	0.01	A-	0.04
5	Baa1	0.15	BBB+	0.22
6	Baa2	0.15	BBB	0.22
7	Baa3	0.15	BBB-	0.22
8	Ba1	1.34	BB+	0.92
9	Ba2	1.34	BB	0.92
10	Ba3	1.34	BB-	0.92
11	B1	6.50	B+	4.82
12	B2, B3	6.50	В, В-	4.82
13	Caa, Ca	26.16	CCC, CC	20.39
14	С		С	
15	D		D	

Table 3: Rating classes and correspondingratings from Moody's and S&P

Remark: Default rates are given by average one-year transitions for the periods 1980-1998, Moody's, and 1981-1998, S&P's, as reported by BIS (2000) on p. 149.

The assignment of an internal rating class to a new loan, or the reevaluation of a counterparty rating in connection with the annual review process of all counterparties, is performed according to a set of quantitative and qualitative criteria. There are two quantitative measures. First, an external rating performed by the credit bureau UC, for details and an evaluation of their model based approach, see Jacobson and Lindé [14]. UC provides an assessment of counterparty bankruptcy risk for the next 8 quarters. Second, the bank runs a calibrated risk model where one input is the rating from UC and other inputs consist of internal information. Unfortunately, we have no information on the details of this model. The qualitative criteria are summarized in a counterparty risk classification handbook. The handbook provides verbal descriptions of the properties of firms in a given rating class along a number of dimensions. Tables 4a-b is an attempt to capture the essentials of the handbook's characterization of the rating classes. It should be noted that the three criteria are not weighted according to some formal "scoring" procedure for the rating decision, they are used as independent inputs.

Risk rating	Ownership	Industry	Management
1	listed shares, easy access to additional capital	industry leader, recession resistant counter-cyclical industry	highly respected and experienced
6	acceptable structure, may have difficulty to raise new capital	well-established in cyclical industry, small market shares	adequate to above average
9	structure just adequate, doubts whether new capital can be raised	in cyclical industry recovering from recession, or newly established	adequate
14	weak owners, cannot access new capital shares tradin suspended	negligble market shares in a trobled industry, small chanses of continued operation	little experience in tough decision- making, significant management turnover, no plan for financial crisis

 Table 4a:
 Characterization of a selection of rating classes

Table 4b:	Characterization	of a	selection	of	rating	classes
-----------	------------------	------	-----------	----	--------	---------

Risk rating	Financial status	General
1	steady sales growth, very conservative balance sheet ratios, very solid cash flow, excellent debt service capacity	only a handful of large corporates make it to this class
6	moderate potential growth in sales, adequate balance sheet ratios, volatile cash flow, at times thin debt service coverage	unlikely that well established firms in solid markets fall beyond this class
9	little or no potential to change mediocre sales growth, possible over-capacity problems, great volatility in cash flow	_
14	negative sales growth outlook, balance sheet ratios give rise to serious concern, cash flow shows extreme volatility, may be in process of distressed selling of critical assets	marked increase or unacceptable level of delinquency in payment to trade creditors

To get a better understanding of the dynamics in the bank's internal rating system, we have calculated the empirical transition frequencies between the 15 risk classes. The frequencies after respectively 1, 4 and 8 quarters in Tables 5a, 5b and 5c have been obtained in the following way. For any transition horizon h, we compared and counted the internal rating of each company that was part of the bank's portfolio at both time t and t + h, for t = 1, 2, ..., 24 - h. Any companies that defaulted end ended up in the absorbing state of risk class 15 between t and t + h were also taken into account.

When looking at the diagonal entries in Table 5a, we see that all risk classes display a high degree of persistence at short horizons. Excluding class 15, the percentage stayers after one quarter varies from 61% for class 4 to 92% for class 1. Three quarters later, these percentages have dropped to 78 and 14, and after 8 quarters only 64 and 5% of the original companies are

left in their original rating class.

The second column of the tables contains the average relative shares of each risk class in the whole portfolio, in terms of numbers of loans. Risk class 9 has by far the largest share, with 31.8% of all companies. at quiet some distance followed by classes 8, 11 and 12 with shares of 8-10%. From Table 5a we can also see which rating classes are the most important sources of defaulting firms. Of those ranked class 14, 7.2% defaults after one quarter, compared with 3.9%, 1.2% and 5% for classes 13, 12 and 11. If we take the relative sizes of different rating classes into consideration, this picture changes, however. In absolute numbers, risk class 13 produces most defaults: almost .2% of the total portfolio per quarter. Risk class 9, which on average accounts for approximately one third of the portfolio, contributes with .14% of the portfolio per quarter. Risk class 14 ranks only fourth with a share of .07%, after class 12 which has a share of .10%.

Finally, it's worth noting that row 15 actually contains non-zero entries, reflecting the fact that some companies that are rated 15 actually obtain *new* loans next period. Although we do not know the exact causes, this could be the consequence of some renegotiation process leading to new terms for old credit lines. As we see in the tables, a substantial share of them end up in risk class 9 after one quarter, suggesting that their creditworthiness has improved.

									То							
\mathbf{Fr}	%	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	.0	92	1	0	0	0	2	0	0	2	1	0	0	0	0	.0
2	3.3	0	62	0	4	0	0	4	6	17	4	0	2	1	0	.1
3	0.3	0	1	91	3	0	2	0	0	2	0	0	0	0	0	.0
4	5.7	0	14	0	61	0	0	2	7	13	1	0	2	1	0	.2
5	7.7	0	0	0	0	74	8	2	3	5	3	3	1	0	0	.1
6	6.7	0	0	0	0	7	81	2	1	4	2	1	0	0	0	.0
7	5.7	0	0	0	0	5	11	71	2	5	2	2	1	0	0	.1
8	9.6	0	0	0	0	4	1	1	79	7	1	3	2	1	0	.4
9	31.8	0	1	0	4	3	1	2	4	77	2	2	3	1	0	.4
10	5.7	0	0	0	0	3	1	2	3	5	84	1	1	0	0	.1
11	9.0	0	0	0	0	1	0	0	3	2	0	89	2	1	0	.5
12	8.2	0	1	0	3	1	0	0	3	16	0	4	66	4	1	1.2
13	4.6	0	0	0	1	0	0	0	1	7	0	3	6	74	3	3.9
14	1.0	0	0	0	0	0	0	0	0	1	0	2	3	10	76	7.2
15	.7	0	0	0	0	0	2	1	1	35	0	3	2	3	5	47.5
	100.0															

Table 5a: Internal ratings' transition matrix, average 1 quarterforward movements, in per cent

									То								
\mathbf{Fr}	%	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
1	.0	78	4	0	5	1	7	0	0	4	1	0	0	0	0	.0	
2	3.3	0	10	1	2	18	2	14	12	22	8	4	4	2	1	.3	
3	0.3	1	5	75	7	1	6	1	0	3	1	1	0	0	0	.0	
4	5.7	0	13	1	14	9	2	9	13	24	5	3	5	2	1	.03	
5	7.7	0	0	0	0	24	24	4	9	18	11	8	2	0	1	.2	
6	6.7	0	0	0	1	25	37	8	5	12	6	4	1	0	0	.1	
7	5.7	0	0	0	1	10	38	13	6	16	7	6	2	0	1	.1	
8	9.6	0	0	0	0	8	3	2	46	14	4	12	6	2	2	.5	
9	31.8	0	2	0	3	6	4	4	11	46	5	7	5	3	4	.5	
10	5.7	0	0	0	0	8	2	3	7	16	56	5	2	1	0	.5	
11	9.0	0	0	0	0	2	2	0	10	8	2	65	6	2	3	.6	
12	8.2	0	3	0	6	3	1	2	8	18	1	12	31	7	6	1.2	
13	4.6	0	1	0	1	1	0	1	4	10	1	9	13	37	18	2.8	
14	1.0	0	0	0	0	0	0	0	2	1	0	6	8	12	65	3.6	
15	.7	0	0	0	0	1	2	0	1	27	0	4	1	2	61	1.6	
	100.0																

Table 5b: Internal ratings' transition matrix, average 4 quartersforward movements, in per cent

									T								
									Ίo								
\mathbf{Fr}	%	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
1	.0	64	5	0	170	10	0	0	3	0	0	0	0	0	0	.0	
2	3.3	0	2	1	2	16	12	10	11	19	7	10	5	2	2	.4	
3	0.3	2	8	60	15	3	6	0	0	4	2	0	0	0	0	.0	
4	5.7	0	0	1	5	17	6	12	13	16	5	10	7	3	3	.5	
5	7.7	0	0	0	0	23	12	3	11	24	10	11	2	1	2	.1	
6	6.7	0	0	0	1	9	21	12	11	23	12	9	2	0	1	.0	
7	5.7	0	0	0	1	16	18	8	9	23	12	9	2	0	1	.1	
8	9.6	0	0	0	0	5	6	1	28	22	7	17	6	2	5	.3	
9	31.8	0	0	0	0	9	6	4	12	32	6	12	7	4	6	.5	
10	5.7	0	0	0	0	4	8	1	9	21	42	9	2	1	1	.1	
11	9.0	0	0	0	0	3	1	0	14	14	3	47	8	2	7	.5	
12	8.2	0	0	0	0	7	2	4	13	13	3	18	18	8	12	.9	
13	4.6	0	0	0	0	2	1	1	7	6	2	16	13	20	31	1.6	
14	1.0	0	0	0	0	0	0	0	3	3	1	8	10	9	64	1.8	
15	.7	0	0	0	0	1	0	2	1	25	0	3	1	0	67	.0	
	100.0																

Table 5c: Internal ratings' transition matrix, average 8 quarters forward movements, in per cent

4.2 The bank's risk-weighted capital requirements

As noted in the introduction, a key input for IRB-determined risk-exposed assets is the average probability of default associated with a given rating class. According to the proposal the estimated PD:s can be determined in three ways; by historical default frequencies, by external ratings, or by means of a credit risk model. Below we will present PD estimates for the 14 nondefaulting rating classes. The purpose of this is exercise is examine this source of variation in IRB capital adequacy, prior to the study of the IRB risk-weighted charges for the portfolio over time. We consider two approaches for the PD-estimates. First, and naturally, the method that most likely will be the one used by most banks: historical default frequencies. In Figures 5861, we present 4-quarter and 12-quarter moving average PD-estimates based on the estimated probability of defaults given by the credit risk model. Likewise, Figures 62-65 show corresponding average PD-estimates using historical default frequencies. Both sets of figures refer to the rating classes 1, 6, 9, and 14. It is clear that the estimated PD:s are not stable over time. Hence, the risk classes cannot be characterized by a fixed, long-run PD, such that would reflect that changes in risk for the portfolio is only manifested in transitions between risk classes. It is therefore worthwhile to consider how much information to make use of when estimating the PD for a risk class, recognizing the trade-off between on the one hand wanting the PD to accurately reflect risk and, on the other hand, avoid short-run, erratic instability in the estimate. Judging by Figures 58-65, a 4-quarter moving average estimate seems to be a reasonable compromise. Moreover, the model-based PD-estimates are *per se* smoother over time, cf., e.g., Figures 61 and 65 that show the two estimation approaches for risk class 14. Another apparent benefit of modelbased PD-estimates is the ensured existence of estimates despite the lack of defaults in a particular risk class. This effect is highlighted in Figure 62.

The next natural step is to study how the calculated IRB risk-weights behave. Figures 66-70 shows risk-weights using the six probability-of-default approaches above, evaluated for risk classes 6, 9, 12, and 14. The weights have been calculated using the following

$$RW_{c,t} = \left(\frac{LGD}{50}\right) \times BRW_{c,t}$$
 or $12.5 \times LGD$, whichever is smaller,

where

$$BRW_{c,t} = 976.5 \times N \left(1.118 \times N^{-1} \left(\widehat{PD}_{c,t} \right) + 1.288 \right) \times \left(1 + 0.0470 \times \frac{\left(1 - \widehat{PD}_{c,t} \right)}{\widehat{PD}_{c,t}^{.44}} \right),$$

and where N is a standard Normal c.d.f. and $\widehat{PD}_{c,t}$ is the estimated probability of default for a risk class in quarter t.

Figure 70 compares the risk-weights for classes 6, 9, 12, and 14, using PDestimates from the credit risk model. First, we see that the weights do reflect the general trend of declining trend in the portfolio. Second, the weights for the different risk classes are, with a few exceptions, distinct from each other. Hence, the differences in credit risk for the risk classes are preserved in the risk weights, as one would expect and hope. Finally, the heart of the matter, Figure 71 and Table 6 present the IRB capital charges for the entire portfolio over time. In order to convey a risk characterization of the portfolio, we have also included the estimated VaR-percentiles. First, the results are re-assuring in so far as being quite reasonable. The turbulent, risky beginning of our sample period is associated with relatively high charges in the range of 10 to 20 per cent. As time progresses, and risks decline, so do the capital charges. It is interesting to note that the charges are actually raised in 1997, thus capturing the temporary worsening in macroeconomic conditions. The agreement of capital charges and portfolio credit risk, as measured by the estimated VaR, is remarkable. Although the charges fall rapidly, so does VaR, to the effect that the portfolio is at all times, but for the last quarter, fully protected. In the last quarter, however, the 99% VaR-estimate is larger than all estimated capital charges. In fact, the model-based charges are inadequate even for the 90% VaR.

			Capita	al charges			VaR-estimates			
	Act.Q1	Act.Q4	Act.Q12	Mod.Q1	Mod.Q4	Mod.Q12	99%	95%	90%	
	4.0.00									
94:2	12.33	-	-	22.73	-	-	3.95	3.13	2.75	
94:3	12.32	-	-	21.87	-	-	3.33	2.59	2.28	
94:4	17.41	-	-	23.41	-	-	3.22	2.42	2.09	
95:1	9.70	11.48	-	17.91	19.15	-	2.76	1.90	1.52	
95:2	11.87	11.66	-	21.00	20.44	-	3.42	1.97	1.54	
95:3	15.46	9.70	-	11.54	11.47	-	3.42	1.90	1.48	
95:4	7.86	8.84	-	10.46	9.16	-	2.19	1.13	0.81	
96:1	8.06	8.84	-	9.04	8.76	-	2.06	1.24	0.84	
96:2	7.11	6.45	-	8.13	6.54	-	2.52	1.35	0.94	
96:3	4.27	5.70	-	5.92	6.73	-	1.85	0.87	0.59	
96:4	2.40	5.03	-	5.38	5.64	-	1.85	0.98	0.66	
97:1	6.34	5.44	5.72	5.72	5.69	5.92	1.63	0.93	0.69	
97:2	10.13	6.48	6.45	6.58	5.87	6.12	1.98	1.15	0.78	
97:3	4.44	6.59	6.77	6.49	6.39	6.62	1.95	1.11	0.73	
97:4	4.04	5.69	5.63	6.90	6.35	6.17	2.92	1.32	0.88	
98:1	4.56	5.57	5.61	6.80	6.75	6.38	2.29	1.16	0.81	
98:2	2.55	3.50	5.81	6.05	6.38	6.15	2.05	0.96	0.71	
98:3	3.58	3.30	4.61	4.55	6.04	6.01	1.24	0.68	0.43	
98:4	2.09	2.76	4.26	4.10	4.99	5.44	1.13	0.65	0.43	
99:1	6.41	3.94	4.65	4.89	5.47	6.53	1.36	0.76	0.47	
99:2	1.76	3.04	3.40	2.17	2.94	3.88	1.15	0.46	0.28	
99:3	1.24	2.41	3.05	1.37	2.16	3.11	0.86	0.28	0.18	
99:4	1.49	2.00	2.62	1.00	1.54	2.54	0.75	0.22	0.15	
00:1	0.23	0.25	0.37	0.06	0.07	0.13	0.82	0.20	0.12	

 Table 6: IRB risk-weighted capital charges and VaR-estimates for the portfolio (%)

5 Conclusions

In this paper we have analyzed a rich data set covering the corporate sector credit portfolio for a large Swedish bank as taken on the last day of each
of 24 quarters from 1994 to 2000. The purpose has been to empirically evaluate the properties of the Internal Ratings Based approach for capital adequacy determination. In order to better interpret the calculated riskweighted capital requirements as given by the proposed new Accord, we have estimated a credit risk model. A VaR-type credit risk measure derived by simulation from the estimated credit risk model allows us to judge just how adequate IRB-determined buffer capital is. Some of our findings are: to be written.....

References

- Altman, Edward I., (1968), Financial ratios, discriminant analysis and the prediction of corporate bankruptcy, *Journal of Finance*, Vol. XXIII No. 4, pp. 589-611.
- [2] Altman, Edward I., (1971), Railroad bankruptcy propensity, Journal of Finance, Vol. XXVI No. 2, pp. 333-345.
- [3] Altman, Edward I., (1973), Predicting Railroad bankruptcies in America, Bell Journal of Economics, No. 4 (1), pp. 184-211.
- [4] Altman, Edward I., (1984), the success of business failure prediction models, *Journal of Banking and Finance*, No. 4, pp. 171-198.
- [5] Altman, Edward I., and Anthony Saunders, (1997), Credit risk measurement: developments over the last twenty years, *Journal of Banking* and Finance, No. 21 (11-12), pp. 1721-42.
- [6] Altman, Edward I., and Anthony Saunders, (2001), An analysis and critique of the BIS proposal on capital adequacy and ratings, *Journal of Banking and Finance*, No. 25 (1), pp. 25-46.
- [7] Apel, Mikael and Per Jansson, (1999), "System Estimates of Potential Output and the NAIRU", *Empirical Economics*, Vol. 24, No. 3, pp. 373-388.
- [8] Blume, Marshall E., Felix Lim, and A. Craig Mackinley, (1998), The declining credit quality of U.S. corporate debt: myth or reality?, *Journal* of Finance, Vol. LIII No. 4, pp. 1389-1413.
- [9] Englund, Peter (1999), The Swedish Banking Crisis: Roots and Consequences, Oxford-Review-of-Economic-Policy, 15 (3) Autumn, pp. 80-97.
- [10] Estrella, Arturo and Gikas A. Hardouvelis, (1991), "The Term Structure as a Predictor of Real Economic Activity", *Journal of Finance*, Vol. 46, No. 2, pp. 555-576.

- [11] Estrella, Arturo and Frederic S. Mishkin, (1998), "Predicting U.S. Recessions: Financial Variables as Leading Indicators", *Review of Economics and Statistics*, Vol. 80, No. 1, pp. 45-61.
- [12] Frydman, Halina, Edward I. Altman, and Duen-Li Kao, (1985), Introducing recursive partitioning for financial classification: the case of financial distress, *Journal of Finance*, Vol. XL No. 1, pp. 269-291.
- [13] Honjo, Yuji, (2000), Business failure of new firms: an empirical analysis using a multiplicative hazards model, *International Journal of Industrial* Organization, No. 18, pp. 557-574.
- [14] Jacobson, Tor and Jesper Lindé, (2000), "Credit rating and the business cycle: can bankruptcies be forecast?", *Sveriges Riksbank Economic Review*, 2000:4, pp. 11-33.
- [15] Li, Kai, (1999), Bayesian analysis of duration models: an application to Chapter 11 bankruptcy, *Economics Letters*, No. 63, pp. 305-312.
- [16] Platt, Harlan D., and Marjorie B. Platt, (1991), A note on the use of industry-relative ratios in bankruptcy prediction, *Journal of Banking* and Finance, No. 15, pp. 1183-1194.
- [17] Shumway, Tyler, (2001), Forecasting bankruptcy more accurately: a simple hazard model, *Journal of Business*, 74 (1), pp. 101-124.
- [18] Gray, R.J., (1996), 'Hazard Rate Regression Using Ordinary Nonparametric Regression Smoothers', Journal of Computational and Graphical Statistics, Vol 5:2, 190-207.
- [19] Hastie, T., and Loader, C., (1993), 'Local regression: Automatic kernel carpentry', *Statistical Science*, 8, 120-143.
- [20] Härdle, W., (1991), 'Smoothing techniques: with implementation in S', New York, Springer Verlag.
- [21] Kooperberg, C., Stone, C.J., and Troung, Y.K., (1995), 'Hazard regression', Journal of the American Statistical Association, 90:429, 78-94.

A Bank variables

EX DAT = Measurement dateBR K = 4 figure industry classification (by bank) KO KU NR =KU KAT =BR GR K = 2 figure industry classification (by bank) UT KRED = Amount of credit utilizedBE KRED = Granted credit RI K = Risk classKONK = Bankruptcy dummySAK BEL =SEBRRKAP = Collateral 1ENARRKAP = Collateral 2KSKVPRGR = Dummy, 1 if short term credit is granted KSKVBEKR = Amount of short term credit grantedLSLVPRGR = Dummy, 1 if long term credit is granted LSLVBEKR = Amount of long term credit grantedS1S4PRGR = Dummy, 1 if mortgage is granted S1S4BEKR = Amount of mortgage grantedGUARPRGR = Dummy, 1 if guarantee loan is granted GUARBEKR = Amount of guarantee loan grantedMIXTPRGR = Dummy, 1 if other mixed credit is granted MIXTBEKR = Amount of other mixed credit granted

B Credit bureau variables

SAOMSTIL = Current Assets SAKORTSK = Current Liabilities SATILLG = Total Assets LIKVID = Cash VARULAG = Inventories SALONGSK = Long Liabilities LEVSKULD = Accounts Payable SAANLTIL = Fixed Assets SAEGETKA = Total Equity OMSAETT = Total Sales

RESFOEAV = Earnings bef. Interest, Depreciation and Amortizations ANTANS = No. employees LOENER = WagesAVSKRIVN = DepreciationFININT = Financial income FINKOST = Financial costsEXTORDIN = Extraordinary costsEXTORDKO = Extraordinary income SKATT = TaxesKUNDFORD = Accounts receivableOVOMSTIL = Other liquid assetsSPAERRKO = Blocked accounts (e.g. escrows)GOODWILL = GoodwillINVENT = Machinery etcOBESRES = Untaxed reservesAKTIEKAP = Nominal equityOVREGBUN = Other EquitySASKOEGE = Sum of taxes and equity (equals total assets)SCBSNIKO = Statistics Sweden industry code SCBSTKL = Statistics Sweden company size code ORGNR = Company's 10 figure identification number PANTER = Total of property pledges for non-mortgage loans) ANSVAR = Total guaranties assumed for third party loans SAFTGINT = Total of property pledges for mortgages in public register



Figure 1: Cumulative distribution function for the leverage ratio



Figure 2: Default rate versus leverage ratio



Figure 3: Median total sales for performing and defaulted firms



Figure 4: Cumulative distribution function for inventory turnover



Figure 5: Default rate versus inventory turnover



Figure 6: Median inventories over sales for performing and defaulted firms



Figure 8: Default rate versus total sales



Figure 9: Median leverage for performing and defaulted firms



Figure 10: Cumulative distribution function for earnings ratio



Figure 11: Default rate versus earnings ratio



Figure 12: Median earnings over assets for performing and defaulted loans



Figure 13: Default rates for the entire portfolio



Figure 14: The one-quarter ahead default rates for rating classes 1-5



Figure 15: The one-quarter ahead default rates for rating classes 6-10



Figure 16: The one-quarter ahead default rates for rating classes 11-14



Figure 17: The 4 quarters ahead default rates for rating classes 1-5



Figure 18: The 4 quarters ahead default rates for rating classes 6-10



Figure 19: The 4 quarters ahead default rates for rating classes 11-14



Figure 20: The 8 quarters ahead default rates for rating classes 1-5



Figure 21: The 8 quarters ahead default rates for rating classes 6-10



Figure 22: The 8 quarters ahead default rates for rating classes 11-14



Figure 23: Default rates in the real estate sector



Figure 24: Default rates in the manufacturing sector



Figure 25: Default rates in the services sector



Figure 26: Default rates for financial services and investment companies



Figure 27: Default rates for different credit types



Figure 28: Default rates for firms with less than 26 employees



Figure 29: Default rates for firms with more than 25 employees



Figure 30: The output-gap and the yield curve



Figure 31: Household expectations concerning general economic conditions



Figure 33: The output-gap and GDP growth in Sweden



Figure 34: Actual and Expected loss rates and Value-at-Risk for entire portfolio



Figure 35: Expected loss and 90-99% VaR, Risk Class 1



Figure 36: Expected loss and 90-99% VaR, Risk Class 2



Figure 37: Expected loss and 90-99% VaR, Risk Class 3 $\,$



Figure 38: Expected loss and 90-99% VaR, Risk Class 4



Figure 39: Expected loss and 90-99% VaR, Risk Class 5



Figure 40: Expected loss and 90-99% VaR, Risk Class 6



Figure 41: Expected loss and 90-99% VaR, Risk Class 7



Figure 42: Expected loss and 90-99% VaR, Risk Class 8



Figure 43: Expected loss and 90-99% VaR, Risk Class 9 $\,$



Figure 44: Expected loss and 90-99% VaR, Risk Class 10



Figure 45: Expected loss and 90-99% VaR, Risk Class 11



Figure 46: Expected loss and 90-99% VaR, Risk Class 12



Figure 47: Expected loss and 90-99% VaR, Risk Class 13



Figure 48: Expected loss and 90-99% VaR, Risk Class 14



Figure 49: Expected loss rates, Risk Classes 1-5



Figure 50: Expected loss rates, Risk Classes 6-10



Figure 51: Expected loss rates, Risk Classes 11-14



Figure 52: 90% VaR, Risk Classes 1-5



Figure 53: 90% VaR, Risk Classes 6-10



Figure 54: 90% VaR, Risk Classes 11-14



Figure 55: 99% VaR, Risk Classes 1-5



Figure 56: 99% VaR, Risk Classes 6-10



Figure 57: 99% VaR, Risk Classes 11-14



Figure 58: Risk class 1: risk model PD:s (actual, 4Q and 12Q moving averages)



Figure 59: Risk class 6: risk model PD:s (actual, 4Q and 12Q moving averages)



Figure 60: Risk class 9: risk model PD:s (actual, 4Q and 12Q moving averages)



Figure 61: Risk class 14: risk model PD:s (actual, 4Q and 12Q moving averages)


Figure 62: Risk class 1: historical PD:s (actual, 4Q and 12Q moving averages)



Figure 63: Risk class 6: historical PD:s (actual, 4Q and 12Q moving averages)



Figure 64: Risk class 9: historical PD:s (actual, 4Q and 12Q moving averages)



Figure 65: Risk class 14: historical PD:s (actual, 4Q and 12Q moving averages)



Figure 66: Estimated IRB risk weights, Risk Class 6



Figure 67: Estimated IRB risk weights, Risk Class 9



Figure 68: Estimated IRB risk weights, Risk Class 12



Figure 69: Estimated IRB risk weights, Risk Class 14



Figure 70: IRB risk weights, model based PD:s, risk classes 6, 9, 12, and 14



Figure 71: IRB determined capital requirement