An Empirical Study of the Returns on Defaulted Debt and the Discount Rate for Loss-Given-Default

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Abstract

Prudential management of credit risk and supervisory requirements call for the accurate measurement of loss conditional upon default (LGD). In the case of banks, in order to achieve Advanced Internal Ratings Based (AIRB) compliance under the Basel II minimum regulatory capital framework, loss arising from counterparty default must be estimated. However, the discount rate to be applied to post-default cash flows is a largely unsettled issue, amongst both practitioners and bank supervisors. In this study we survey various methodologies extant in the literature for determining an appropriate discount rate. We propose an approach in which the discount rate is conditional upon the level or undiversifiable risk inherent in the recovery cash flows associated with defaulted facilities. We present a stylized theoretical framework for understanding such an approach. This is followed by an empirical exercise that utilizes a comprehensive and commercially available database of workout recoveries, in which we analyze the returns on marketable bonds and loans, having market prices at default and at the resolution of the default event. We propose alternative empirical measures of the recovery risk inherent in post-default cash-flows: the annualized simple *return on defaulted debt* (RDD) and the *most likely discount rate* (MLDR); and discount rates implied from a structural model of credit risk incorporating systematic recovery risk, a generalization of the asymptotic single risk factor (ASRF) framework (Gordy, 2003). We find our empirically derived estimates to be significantly higher than what has been found in the previous literature, as well as what is used in industry, mean (MLE estimate of) RDD (MLDR) of 29.2% (21.3%); this compares to benchmarks such as the 15% reported by Araten (2004), the 200bps over the risk–free rate suggested by Machlachlan (2004), or rates in the range of 10-15% derived from model-based estimations (one or two factor structural credit models in conjunction with an assumption on the systematic risk factor). Principal findings are that returns on defaulted debt, which can be interpreted as an appropriate discount rate for workout recoveries, vary significantly according to certain different factors. There is some evidence that discount rate metrics are elevated for loans having better collateral quality rank or better protected tranches within the capital structure; and for obligors rated higher at origination, more financially levered or having higher Cumulative Abnormal Returns (CARs) on equity just prior to default. However, the discount rate is *increasing* in market implied loss severity at default. We also find evidence that LGD discount rates vary pro-cyclically, as they increase with industry default rates, but there tends to be some asynchronousity in this relationship; further, they are inversely related to short-term interest rates. However, for other demographics the results are inconclusive, such as the industry group of the obligor. Finally, we conduct an analysis of the impact of the discounting method upon the distribution of estimated LGD and regulatory capital. We find that a regression model based discounting, for a sub-sample of the MULGD database, results in a capital charge 73 bps greater than discounting at a constant punitive rate of 25%, and 113 bps larger than discounting at the contractual coupon rate (where the capital charge ranges in 7-8%). This conservativeness of the risk-sensitive RDD model, as well as the evidence that the risk in recovery cash flows contain a significant non-diversifiable component, supports the appropriateness of this framework for regulatory capital calculations.

1. Introduction and Summary

Financial institutions worldwide are grappling with implementation of the advanced internal ratings-based (IRB) approach under the Basel II minimum regulatory capital framework. Indeed, for many of such institutions this has morphed into a critical activity, often involving a concentration of resources and focus. There are many controversies and unresolved issues, but among one of the most misunderstood and little studied aspects of this surrounds the proper discount factor to be applied to recoveries on defaulted debt, an ingredient in the calculation of the key Basel parameter *economic loss-given-default* (LGD). In the case of banks that qualify to measure LGD from their own reference data-sets, this is defined as the present value reduction in loan value as a proportion of the *exposure-at-default* (EAD).

As workout periods for defaulted loans may be extended over many years, it is necessary to discount cash flows to a common period, the most natural being the time at which the event of default occurs[1]. Banking supervisors, practitioners and academics alike have not been able to agree upon the interest rate to be applied on recovery cash-flows post default, in order to arrive at an estimate of the true economic loss attributable to a defaulted loan. While for some portfolios this estimate can be derived from observing the market price of defaulted debt, in the case of the vast majority of most banks' loan portfolios, the non-marketability of the instruments in question necessitates an actuarial approach that uses a punitive (or risk-adjusted) discount rate.[2]

There exist arguments, potentially supported by certain economic models, that to the extent there may be opportunity costs associated with holding defaulted debt, the discount rate (or, equivalently in this context, the required return on the defaulted instrument) used to risk adjust workout recoveries should be commensurate with this by including an appropriate risk premium. Indeed, this has been the practice of workout specialists since far before the advent of Basel II, as projected recoveries have traditionally been discounted at a punitive rate in order to calculate the expected present value of recoveries for the purpose of managing the workout process. This interpretation of the discount rate for workout recoveries has been the one adopted for purposes of Basel, and indeed is stated as such in official guidance issued on the topic[3] of LGD as well as in the final Rule[4].

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[1] This implies that the calculation of LGD is dependent upon how default is defined, which under the final Basel rule (OCC et al, 2008) includes events such as formal bankruptcy of the firm, out-of-court restructuring or renegotiation of debt at an economic loss, as well as payment arrears in excess of 90 days.

[2] There are two generally accepted methods for estimating LGD within the traditional approach to credit risk management and also under the Basel paradigm. First, one can observe the market prices of defaulted loans at or near default, which implicitly embeds the market perception of recovery risk (Altman et al, 2005). Alternatively, and probably most relevant for most banks holding non-public debt, workout recoveries to come are discounted at an "appropriate", risk adjusted discount rate (Araten et al, 2003). However, even in the case where portfolios consist of traded debt, such exhibits extreme illiquidity conditional upon distress that makes the former impractical. In fact, the latter approach has become standard and is closest to the mandated Basel II IRB formulation.

[3] Early guidance on the topic states: "When recovery streams are uncertain and involve risk that cannot be diversified away, net present value calculations must reflect the time

However, there is wide disagreement on how to think about this, as most of the arguments have approached the issue from a theoretical point of view, and practitioners have tended to adopt this perspective. This strikes us as strange, as common sense, traditional workout practice and even supervisory guidance argue for risk adjustment of some kind. While there are an array of choices that have been proposed, we can divide these arguments into three broad categories. A provocative view, from our standpoint, argues that the discount rate should be taken simply from the risk-free term structure. Indeed, under paradigms of modern finance such as the Arbitrage Pricing Theory (APT), such risk premia should not exist and the only appropriate discount rate is risk-free. However, it may be argued that preconditions for this to hold may not obtain, such as the capability to replicate cash-flows associated with defaulted exposures[5]. Moreover, it is possible that this choice may create an inconsistency between the rate used in quantifying economic LGD, and that used to discount anticipated workout recoveries, which raises "use test" questions. The second argument proposes some kind of opportunity cost of funds, examples being a weighted average cost of capital (WACC), cost of equity capital or some other cost of funds. The theory here is that the bank should account for the opportunity cost of replacing the defaulted loan in its portfolio. Finally, the approach most in line with either workout practice or supervisory guidance recommends using a comparable risky rate of return of some kind. These could include the contract rate at the time of distress (Keisman et al 2001), a rate of return on a distressed index, a rate demanded by vulture investors or simply an appropriately estimated or otherwise imputed punitive rate (Araten et al (2003), Maclachlan (2004) or Davis (2004)).

However, in spite of the intuition and theoretical arguments, in estimating LGD from bank's historical reference data-sets (or other sources) for either risk management or Basel purposes, it is rarely the case that the discount rate is differentiated by the potential recovery risk of the post-default associated cash flows. The implication for a financial institution undertaking compliance with the advanced IRB approach is the potential of not assigning enough regulatory capital to instruments with high recovery risk, and vice versa, assessing too high a charge for those instruments with less recovery risk. However, it has been argued that the cross-sectional variation in recovery cash flows swamps the effect of a differentiated discount rate (Carey and Gordy, 2007). This has not been demonstrated empirically. In the cases where empirical analysis has shown that *overall average* LGD is not sensitive to changes in the *single discount rate* used (Araten et al, 2003, Araten 2004), such analysis has not looked to vary the rate by

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value of money and a risk premium appropriate to the undiversifiable risk." (Basel Committee for Banking Supervision, 2005)

[4] The Basel II Final Rule in the U.S. (OCC et al, 2008) states (Page 450): "Where positive or negative cash flows on a wholesale exposure to a defaulted obligor or a defaulted retail exposure … occur after the date of default, the economic loss must reflect the net present value of cash flows as of the default date using a discount rate appropriate to the risk of the defaulted exposure."

[5] A commonly heard and misleading characterization of this view is that the argument hinges on the observation that in quantifying or estimating economic LGD from historical databases of recovery cash flows, such cash flows are already risk adjusted. However, that need not be the case, as the prevailing version of the APT rests upon hedgibility, as opposed to the diversifiability of risk in other frameworks such as the Capital Asset Pricing Model (CAPM).

segment, nor to study either the effect on the entire distribution of LGD or on economic capital of different discounting methodologies.

In this study, we perform a comprehensive analysis of empirically derived discount rates for LGD, by analyzing the relationship between market and emergence prices of defaulted debt. This can be thought of as analogous to examining the relationship between the value of defaulted assets and that of their associated workout recoveries. We examine alternative methodologies for estimating such empirically derived discount rates. We then propose a model for assigning a proper, risk-adjusted discount rate to defaulted instruments. From this, we quantify the effect of discounting on the distribution of economic LGD, and on estimated regulatory capital, for a hypothetical portfolio.

The data-set that we utilize, Moody's Ultimate Loss-Given-Default (MULGD) data-base, contains the market prices of defaulted bonds and loans near the time of default, and the prices of these instruments (or of the bundle of instruments received in settlement of default) at the resolution of default. We have such data for 550 obligors and 1368 bonds and loans in the period 1987-2007. We develop alternative estimation methodologies to derive the discount rates. First, we examine the distributional properties of the individual annualized rates of return on defaulted debt ("return on defaulted debt" - RDD), across different segmentations in the dataset (e.g., default type, facility type, time period, seniority, collateral, original rating, industry). We then compare this to various alternatives. First, as proposed by Brady et al (2006), an approach that involves solving for a discount rate within a homogenous segment that has the highest likelihood of being the prevalent rate of return in the market at the time of default ("most likely discount rate" – MLDR). Second, we examine the technique of Machlachlan (2003), who develops a risk premium over the risk-free rate that is derived from a structural credit or a market based model.

In any of these approaches, we interpret such an estimate as the expected rate of return that should reflect the uncertainty of the recovery cash flows associated with the defaulted bond or loan. We can imagine that a rational investor prices the defaulted instrument after default in accordance with expected future cash flows, or the price of the asset at emergence from bankruptcy (or otherwise the resolution of the default event), which is akin to a workout cash-flow in the setting of a private bank loan. The correct pricing of a defaulted loan requires that an investor estimates potential future recoveries, as well as the timing of them, and then discounts the expected cash-flows using the proper discount rate, which presumably includes the required LGD risk premium. A general approach to estimating the latter is through the former, and while expected future recoveries are not observable, it may be argued that if pricing is rational then such realized cash flows should on average coincide with their expectations. However, this may only hold for a reasonably large, and in terms of recovery risk, homogenous segment of defaulted instruments. We can approach this in two ways: first, by studying rates of return on defaulted instruments from default to emergence, either by looking at averages over such homogenous segments, or through multivariate regression; alternatively, under the assumption of homogeneous segmentations and a certain the rate of information diffusion through the resolution process, we can solve for the discount rate that equates the discounted average realized recoveries to the average market price (Brady et al, 2003). In this exercise, we empirically identify the critical determinants of the estimated discount rates for workout recoveries, as well as provide theoretical explanations and practical insights for the results.

Our principle results are as follows. We find our empirically derived estimates of the appropriate discount rate for workout recoveries, both the RDD and the MLDR, to be significantly higher than what has been found in the previous literature, as well as what is used commonly in industry and for Basel 2 purposes. Mean (MLE estimate of) RDD (MLDR) is 29.3% (21.3%), as compared to benchmarks such as the 15% reported by Araten (2004), or the 200bps over the risk–free rate suggested by Machlachlan (2004). However, discount rates implied from theoretical models of credit risk, including extensions of the asymptotic structural risk factor framework (Gordy, 2003) that incorporate systematic recovery risk, are found to be significantly lower than the RDD or MLDR. These fall into the range of 10% to 15%, depending upon the specification of the factor structure, single vs. multiple or latent vs. observable proxy. Returns on defaulted debt, which can be interpreted as an appropriate discount rate for workout recoveries, vary significantly according to contractual, obligor, equity / debt market and economic factors. At the facility structure level, There is some evidence that discount rate metrics are elevated for loans having better collateral quality rank or better protected tranches within the capital structure. At the obligor or firm level, discount rate measures are elevated for obligors rated higher at origination, more financially levered firms at default or firms having higher Cumulative Abnormal Returns on Equity (CARs) prior to default. However, the discount rate is increasing in market implied loss severity at default. We also find evidence that LGD discount rates vary pro-cyclically, as they vary directly with industry default rates, but there tends to be some asynchronousity in this relationship. Further, the macroeconomy is found to be a determinant, as discount rate measures are inversely related to short-term interest rates. However, for other demographics results are inconclusive, such as the industry group of the obligor. Finally, we conduct an analysis of the impact of the discounting method upon the distribution of estimated LGD and regulatory capital. We find that a regression model based discounting, for a sub-sample of the MULGD database, results in a capital charge 73 bps greater than discounting at a constant punitive rate of 25%, and 113 bps larger than discounting at the contractual coupon rate (where the capital charge ranges in 7-8%). We conclude that this conservativeness of the risk-sensitive RDD model, as well as the evidence that the risk in recovery cash flows contain a significant non-diversifiable component, supports the appropriateness of this framework for regulatory capital calculations.

This study will proceed as follows. Section 2 reviews the relevant literature. Section 3 outlines the theoretical basis for this study. Section 4 presents our empirical methodology. Section 5 summarizes our discount rate measures according to various segmentations of the data. Section 6 presents summary statistics of our available covariates, and presents univariate correlation analysis of these with respect to RDD. Section 7 discusses the results of multiple regression analysis of RDD. Section 8 analyzes the influence on regulatory capital of differing choices of the discount rate for LGD calculation. Section 9 investigates alternative, or benchmark, approaches to estimating the discount rate for workout recoveries (structural and market models). Section 10 concludes and provides possible directions for future research.

2. Review of the Literature

In the generally accepted taxonomy for LGD measurement methodologies, corresponding to the three approaches either proposed by researchers or regulators, are clear implications for the discounting procedure required. First, in the method for inferring LGD from observation of the prices of defaulted instruments in the market at (or soon after default), the discounting is implicit. Examples of this include Carty and Lieberman (1996) or Gupton and Stein (2005), who analyze the prices of defaulted marketable loans and bonds one month after default to their par values. Furthermore, in the various papers which have calibrated Merton structural credit risk models to default rate and recovery data on defaulted instruments[6], they have typically relied upon LGD estimates derived from this near-to-default market price measure. Examples of this burgeoning literature are bracketed by the seminal works of Frye (2000 a,b,c, 2003) and more recently Barco (2007) .

A related method, which some have taken to be an analogue to the workout approach practiced by banks, looks at the secondary market prices of restructured assets at emergence from bankruptcy (or from the default event defined more broadly), which various studies have shown to average roughly 18 months after default (Araten et al, 2003). This can be seen in the work of Keisman et al (2000), and subsequently this measure has been dubbed the "ultimate recovery" (Emery et al, 2007)[7]. The commonality with the workout approach lies in that such values need to be discounted from the point of resolution back to the default date. In the studies cited herein, based upon commercially available databases of large corporate defaulted loans and bonds, this rate has been taken to be the coupon rate on the debt just prior to default (called the "pre-petition rate).

The third method, considered most appropriate for banks by both bankers and supervisors, involves relating realized post-default cash flows to defaulted balances of loans: "workout LGD". These cash flows are supposed to incorporate material credit related losses as well as direct and indirect costs of the workout process. In this context, discounting becomes a critical consideration, and the specification of a cash-flow model for defaulted debt becomes necessary. While many institutions have implemented this approach internally for some time, and this has become increasingly common with the advent of Basel 2, there are limited published studies; notable exceptions include Asarnow and Edwards (1995), Eales and Bosworth (1998) and Araten et al (2003). While the implementation of workout LGD is fraught with difficulties, mainly centered around data integrity and measurements issues, if executed to a reasonable degree of reliability has many advantages. Aside from compliance with supervisory requirements, the benefits to internal risk management include

[6] These are extensions of the asymptotic single factor model (ASRF) framework of Vasicek (1987) and Gordy (2003), models in the Merton (1973) structural model framework, which have become the basis for the Basel II Advanced IRB capital framework. The extensions cited herein allow for recovery, in addition to default rates, to vary systematically; in the ASRF framework, LGD is exogenous and fixed.

[7] Also see Friedman and Sandow (2002), which forms the basis of the S&P LossStats™ model, which produces predictive conditional distributions of LGD by the "maximum expected utility" method. The vendor model allows users to model the LGD at default or at resolution, in contrast to the counterpart Moody's LossCalc™, which is a regression based model built expressly to forecast LGD at the time of default.

applicability to non-marketable debt (the bulk of most banks' loan portfolios), the ability to estimate loss rate distributions under actuarial measure, and the ability to perform verification of historical cash-flow data obtained during the workout process.

An alternative perspective on defaulted debt as an asset class, as given in Guha (2003), gives rise to yet another proposal for the correct discount rate for LGD. The author documents a convergence in market value as a proportion of par with respect to bonds of equal priority in bankruptcy approaching default. This holds regardless of contractual features, such as contractual rate or remaining time-to-maturity. The implication is that while prior to default bonds are valued under uncertain timing of and recovery in the event of default, that varies across issues according to both borrower and instrument characteristics, upon default such expectations become one and the same for issues of the same ranking. There is cross-sectional variation in yields due to varied perceived default risk as well as instrument structures, but as default approaches the claim on the debt collapses to a common claim on the expected share of emergence value of the firm's assets due to the creditor class. Therefore, the contract rate on the debt pre-default is no longer the relevant valuation metric with respect to restructured assets. This was predicted by Merton (1974), who predicted in his theoretical framework that credit spreads on a firm's debt approaches the expected rate of return on the firm's assets, as leverage increases to the point when the creditors become the owners of the firm. Schuermann (2003) echoed the implications of this argument by claiming that cash flows post-default represent a new asset,

Machlachlan (2003), building upon the latter evidence and theoretical insights, outlines a framework that is motivated by a classic single factor CAPM model. First, a reasonable model for LGD (taken to mean the deficit of actualized cash-flows relative to those contractually agreed upon) should involve a diminution of expected cash-flows under physical measure. Second, to the extent that cash-flows occurring post-default are systematically correlated, the factor loading of the return on the firm's unlevered assets with respect to the market (e.g., the "Beta") gives rise to a risk-premium. However, to the extent that recoveries are dependent upon collateral, which may vary differentially with the systematic risk factor, the market "Beta" of the defaulted asset may not coincide with the firm's asset "Beta". As a special case, such premium may be zero in the case that the defaulted loan is fully cash-secured; or something lesser than a "distressed" premium, but still positive, in the case that security is some other kind of liquid and default-free asset (e.g., a long-term treasury bond). Finally, an implication of this is that the proper discount rate for workout recoveries may not be singular, as we may expect it to vary according to sources of repayment in cases were a defaulted asset is secured by multiple collateral types.

In light of this framework, we may evaluate several proposals for the discount rate to compute LGD that have been put forth. The suggested practice probably on the weakest footing among those to be considered herein, is to use the contractual rate on the loan. This could be either some kind of average rate in the portfolio, or perhaps the distressed rate on the loan near default, the rationale being that this represents the opportunity cost of replacing the defaulted loan. The problem with this is that it fails to consider the transformative nature of the default event, as the bank is an investor in a new financial claim, which is no longer a financial claim having promised payments subject to default risk, but rather an asset dependent upon the recoverability of a defaulted firm's assets or of collateral values. The appropriate discount rate depends upon the degree of undiversifiable risk inherent in this new asset, and it may be *less or more* (and by no means necessarily equal to) than that on the pre-existing claim.

A related approach uses some measure of the lenders cost of funds in the capital market. This could either be an average cost of debt, cost of equity, or weighted average cost of capital (WACC). The rationale is that the defaulted instrument would be replaces with funds from some class of the bank's claimants, either debt-holders, shareholders or both. As with the contractual rate of return on the defaulted loan, either on the particular instrument or some average over a portfolio, this confounds the systematic risk associated with two different investments: in this case, that of the bank with that of a defaulted asset. The use of the lender's cost of funds violates the principle that the valuation of a financial asset should not depend upon what a seller wishes to receive in order to repair the balance sheet, but rather the market clearing price that a rational buyer would pay for the expected stream of returns expected on the asset. Approaches along these have been proposed recently by several banks internally for the purposes of satisfying supervisory requirements under Advanced Basel IRB in the U.S.[8]

A proposal which has given rise to some confusion is to use the risk-free rate. This is clearly at odds with the pile of evidence – see Frye (2000), Altman et al (2001) or Gupton and Stein (2002) - that LGD varies systematically with the state of the economy. However, when we are in the context of pricing expected recoveries under *risk neutral (or pricing) measure* – when such cash flows are already adjusted for the investor's risk aversion – then the default-free term structure is proper for discounting. But this is not the context under which we are constructing loss distributions under *physical (or actuarial) measure* – in that case a risk-adjustment to the discount rate is appropriate. The latter is of relevance for purposes of risk management or Basel II.

An approach that we will consider in this paper, and compare to various alternatives, involves examining the ex post return on defaulted debt. Araten (2003, 2004) advocates this punitive rate approach, citing the use of 15% by JP Morgan in its computation of economic LGD, which is supported by referenced to returns demanded by "vulture" investors of distressed debt. Support for this choice is offered by reference to the historical performance of the Moody's Corporate Bond index (Hamilton and Berthault, 2000), which returned an annualized 17.4% in the period 1982-2000. However, this return has been extremely volatile, as most of this gain (147%) occurred in the period 1992-1996. This has lead to the suggestion that, assuming that banks can diversify away the idiosyncratic component of this, the proper metric relates these returns to a market index. Following this argument, Hamilton and BerthaultI (2000) and Altman and Jha (2003) both arrive at estimates of a correlation to the market on this defaulted loan index of about 20%, implying a market risk premium of 216 bps. Davydenko and Strebuleav (2002) report similar results for non-defaulted high-yield corporate bonds (BB rated) in the period 1994-1999. Machlachlan (2003) obtains similar results in two empirical exercises. First, regressing Altman-

[8] This is based upon review of confidential bank documents by the author, hence there are no citations.

NYU Salomon Center Index of Defaulted Public Bonds in the period 1987-2202 on the S&P 500 equity index, a 20% correlation also obtains, implying a market risk premium of 216 bps. Second, he looks at monthly secondary market bid quotes for the period April 2002-August 2003, obtaining a beta estimate of 0.37, which according to the Frye (2000) extension of Basel single factor framework implies a recovery value correlation of 0.21 and an MRP of 224 bps.

Finally, we make note of an approach that is analogous to the option adjusted spread (OAS) methodology of option pricing by Kupiec (2007). The author argues that as not only recoveries are uncertain in value, but also their timing of recoveries is subject to long and variable lags, in order to estimate LGD it is necessary to estimate the market value of an uncertain recovery stream at the time a credit defaults. Such calculation requires a forecast for the expected recovery stream as well as an estimate of the risk-adjusted discount rate for discounting expected recovery values. The methodological note discusses the quantitative issues and methods that are associated with estimating recovery distributions and their associated market risk premia. The author argues that the approach of Brady et al (2006), which suggests that an empirically derived rate of return measures on defaulted debt, may be severely biased.

3. Theoretical Framework

In this section we lay out the theoretical basis for the problem of determining an appropriate discount rate for LGD, as well as propose various empirical strategies for achieving this goal. While the measurement of LGD is a standard problem in finance, the valuation of a future stream of risky cash flows, this is complicated by the fact that upon default such proceeds are no longer the contractual promised payments. Instead, we are dealing with cash flows from either the liquidation of collateral or extracted from a defaulted entity in a workout process, so that the risk profile of the amounts and their timing is fundamentally different from that of the original instrument: not only is the risk in magnitude probably greater, but timing is random as well.

Let us denote the stochastic post-default cash-flow at time s by $\tilde{c}_s$, the random time of resolution by $\tilde{\tau}$, and the joint distributions of these by $F^{c,\tau}$. Then we can write the expectation (under physical measure P) of loss-given-default (LGD), the expected LGD or ELGD, as the complement of the present value of recovery cash flows at time of default t normalized by the *exposure-at-default* (EAD):

$$E_t^P\left[LGD_\tau\right] \equiv ELGD_t = 1 - \frac{\int_{c=\underline{C}}^{\bar{C}} \int_{s=t}^{\tau} c_s e^{-sr_s^D} dF^{\tilde{c},\tau}(c,s)}{EAD_t} \qquad (3.1)$$

Where $\left[\underline{C}, \bar{C}\right]$ is the support of the cash flow relation and r_s^D is the instantaneous discount rate for post-default cash- flows, which is a mapping $\left[0,\infty\right] \rightarrow \left[0,1\right]$. In general r_s^D is not only time-varying, but dependent on covariates and could be stochastic. It is clear that equation (3.1) is far too general to be of any practical use, and it follows that under the workout method typically LGD is estimated from reference data as averages across homogenous segments[9]:

$$LGD_t = 1 - \frac{1}{EAD_t} \sum_{s=t}^{T} \frac{c_s}{\left(1 + r_s^D\right)^s} \tag{3.2}$$

Where c_s are either realized ex-post recovery cast flows when quantifying LGD in a reference data-set and t = 1,..,T are the known times of receipt. If one if taking the approach of applying an estimated LGD or recovery rate to a an exposure currently in a portfolio, then if forming the estimate based upon observed ex-post cash-flows, the discount rate used in that calculation should be risk-adjusted. This is in contrast to an alternative approach, which is also valid, of forming expectations of magnitudes and timings of dollar cash-flows on a defaulted exposure under a pricing measure (i.e., risk-adjusted), in which case discounting will be performed according to the risk-free term structure. We consider the former approach in what follows.

We now turn our attention to the determination of the discount rate in (3.2), r_s^D. First, we may take a theoretical perspective, and follow the capital formulae (BIS, 2003), as developed by Gordy (2000) and Vasicek (2000), based upon the Merton (1974) structural modelling framework. In an intertemporal version of this framework, we may write the stochastic process describing the instantaneous evolution of the i[th] firm's (or segment's) asset return at time t as:

$$\frac{dV_{i,t}}{V_{i,t}} = \mu_i dt + \sigma_i dW_{i,t} \tag{3.3}$$

Where $V_{i,t}$ is the asset value, σ_i is the return volatility, μ_i is the drift (which can be taken to be the risk-free rate r under risk-neutral measure), and $W_{i,t}$ is a standard Weiner process that decomposes as (this is also known as a *standardized asset return*):

$$dW_{i,t} = \rho_{i,X} dX_t + \sqrt{1 - \rho_{i,X}^2}\, dZ_{i,t} \tag{3.4}$$

Where the processes (also standard Weiners) X_t and $Z_{i,t}$ are the systematic risk factor (or standardized asset return) and the idiosyncratic (or firm-specific) risk factor, respectively; and the factor loading $\rho_{i,x}$ is constant across all firms in segment i (or across time for the representative firm)[10]. It follows that the

[9] Note that this highly simplified methodology, while still widely used, is only one choice among many and far from the most elaborate or statistically rigorous of the frameworks in existence. Alternatives include predictive regressions (Gupton and Stein, 2005) or conditional distribution of LGD estimations (Friedman and Sandow, 2002). See Jacobs et al (2007) for a comparison of different methods.

[10] Vasicek (2002) demonstrates that under the assumption of a single systematic factor, an infinitely granular credit portfolio and LGD that does not vary systematically, a closed-form solution for capital exists that is invariant to portfolio composition

instantaneous asset-value correlation amongst firms (or segments) i and j is given by:

$$\frac{1}{dt} Cor_{i,j}^{V}\left[\frac{dV_{i,t}}{V_{i,t}},\frac{dV_{j,t}}{V_{j,t}}\right]=\rho_{i,x}\rho_{j,x} \qquad (3.5)$$

As in Basel regulatory capital framework, it is common to assume that the factor loading in (3.4)-(3.5) is constant amongst firms within specified segments, so that the asset-value correlation for segment i is given by $\rho_{i,x}^{2} \equiv R_i$[11], where R_i represents the Basel notation. If we take the further step of identifying this correlation with the correlation to a market portfolio - arguably a reasonable interpretation in a single-factor, ASFM world – then we get $\rho_{i,x}^{2} = \rho_{i,M}^{2}$ It then follows from the standard CAPM that the relationship between the firm and market rates of return is given by the beta coefficient:

$$\frac{Cov_{i,M}\left[\frac{dV_{i,t}}{V_{i,t}},\frac{dV_{M,t}}{V_{M,t}}\right]}{Var_{M}\left[\frac{dV_{M,t}}{V_{M,t}}\right]}=\beta_{i,M}=\frac{\sigma_i\sqrt{R_i}}{\sigma_M} \qquad (3.6)$$

Where σ_M is volatility of the market return. We may now conclude that in this setting the proper (time invariant) discount rate for LGD on the i^{th} exposure (or segment), r_i^D, is equal to the expected return on the defaulted firm's assets, which is given by the risk-free rate r_{rf} and the firm-specific risk-premium δ_i :

$$r_i^D = r_{rf} + \frac{\sigma_i\sqrt{R_i}}{\sigma_M}\left(r_M - r_{rf}\right) = r_{rf} + \beta_{i,M} MRP = r_{rf} + \delta_i \qquad (3.7)$$

Where the *market risk premium* is given by $RMP \equiv r_M - r_{rf}$ (also assumed to be constant through time) and the firm-specific risk premium is given by $\delta_i = \beta_{i,M} MRP$. In the context of Basel II, this approach identifies the systematic factor X with the standardized return on a market portfolio R_M, from it follows that the asset correlation to the former can be interpreted as a normalized "beta" in a single factor CAPM (or just a correlation between the firm's and the market's return), which is given by $\rho_{M,x} \equiv \sqrt{R_i}$. In order to achieve internal consistency in an Advanced IRB modeling framework, the asset return correlation used in the regulatory capital formula should be the same as that used to discount workout recoveries for the purpose of quantifying economic LGD.

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[11] Indeed, for many asset classes the Basel II framework mandates constant correlation parameters equally across all banks, regardless of particular portfolio exposure to industry or geography. However, for certain exposures, such as wholesale non-high volatility commercial real estate, this is allowed to depend upon the PD for the segment or rating (BIS, 2003).

In a more general and more realistic framework, returns on a defaulted loan may be governed by a stochastic process distinct from that of the firm, as in the case when the collateral securing the asset is secured by cash, third party guarantees or assets not used in production. In these situations it is possible that there are two notions of asset value correlation, one driving the correlation amongst defaults, and another driving the correlation between collateral values and the discount rate for LGD in equilibrium. This reasoning implies that it is entirely conceivable that, especially in complex banking facilities, cash flows associated with different sources of repayment may be discounted differentially according to their level of systematic risk. In not distinguishing how betas may differ between defaulted instruments secured differently, it is highly likely that bank capital will be overstated, as the impact of PD-LGD correlation is given undo weight. This leads naturally to an extension of the Vasicek (2002) and Gordy (2000) framework in which LGD varies systematically. In this framework, a common systematic factor drives both default and expected recovery rates in the economy, and the correlation between the two; however, the factor loadings may differ between the asset value and LGD processes.

If we define the recovery rate on the defaulted asset as $R_{i,t} \equiv 1 - \frac{LGD_{i,t}}{EAD_{i,t}}$, then in an intertemporal version of this framework, we may write the stochastic process describing the instantaneous evolution of the recovery on i[th] defaulted asset (or LGD segment, or seniority class, i) at time t as:

$$\frac{dR_{i,t}}{R_{i,t}} = \mu_i^R dt + \sigma_i^R dW_{i,t}^R \tag{3.8}$$

Where μ_i^R is the drift (which can be taken to be the expected instantaneous return on collateral under physical measure, or the risk-free risk-neutral measure), σ_i^R is the volatility of the collateral return and $W_{i,t}^R$ is a standard Weiner process that decomposes as:

$$dW_{i,t}^R = q_{i,x} dX_t + \sqrt{1 - q_{i,X}^2}\, dZ_{i,t}^R \tag{3.9}$$

Where the processes (also standard Weiners) X_t is the systematic risk factor (or standardized asset return, same as in the firm value process), $Z_{i,t}^R$ is the idiosyncratic defaulted asset (or collateral-specific) risk factor, and the factor loading $q_{i,x}$ is constant across all loans in segment i (or across time for the seniority class). Various further extensions this framework have appeared in the literature subsequent to Frye (2000), which have in common that they allow the recovery process to depend upon a 2[nd] systematic factor, which may be correlated with the macro (or market) factor X_t.[12] We choose to focus on recent model in this stream of literature, Barco (2008), who introduces the following process for the standardized return on the defaulted asset:

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[12] See Pyktin (2003), Dullman and Trapp (2004), Giese (2005), Rosch and Scheule (2005) and Hillebrand (2006).

$$dW_{i,t}^{R} = q_{i,x} dX_t + \sqrt{1 - q_{i,X}^2}\, dY_t^R \qquad (3.10)$$

Where Y_t^R is a second systematic factor that influences the return on the loan collateral. Note that in this framework, the idiosyncratic component of the defaulted asset return disappears, and is replaced by this additional source of undiversifiable risk, which can be interpreted as something related to the market for the collateral. Additionally, Barco (2003) distinguishes the standardized return on the defaulted asset from the *loss ratio per unit* exposure, which is the positive part of the return to the creditor on these assets:

$$L_{i,t}^{R} = \left[1 - \exp\left(\mu_i^R + \sigma_i^R W_{i,t}\right)\right]^{+} \qquad (3.11)$$

It follows that the LGD is unconditionally distributed as a truncated log-normal random variable, and it can be analyzed in a framework that gives rise to "option-theoretic-like" analytic formulae. In this paper, we consider a further extension to this, where the default and recovery side each have a systematic risk factor, which are correlated, and each have their own idiosyncratic factor. We therefore re-write (3.10) as:

$$dW_{i,t}^{R} = \rho_{i,X^R} dX_t^R + \sqrt{1 - \rho_{i,X^R}^2}\, dZ_{i,t}^R \qquad (3.12)$$

Where the two-systematic factors are bivariate standard normal, each standard normal, but with correlation q between each other:

$$\left(dX_t, dX_t^R\right)^T \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & r \\ r & 1 \end{pmatrix}\right) \qquad (3.13)$$

We can estimate the vector of parameters $\left(\mu_i, \mu_i^R, \rho_{i,X}, \rho_{i,X^R}, r\right)^T$ by a straightforward full-information maximum likelihood FIML), given a time series of default rates and realized LGD rates. The algorithm for FIML in the context of this model is outlined in Section 15 (Appendix 2). The resulting estimate $\hat{\rho}_{i,X^R}$ can be used in equation (3.7) - in conjunction with estimates of the market volatility σ_M, firm-specific volatility σ_i ,the MRP $\left(r_M - r_{rf}\right)$ and the risk-free rate r_{rf} - in order to derive the theoretical discount rate for LGD within this model. Alternatively, we may pursue alternative estimates of $\hat{\rho}_{i,X^R}$, through regressing actual defaulted debt returns on some kind of market factor or other measure of systematic risk (e.g., aggregate default rates). We consider these in Section 9.

4. Empirical Methodology

We now sketch the alternative empirical strategies that will be employed. First, we consider model-free measures of the discount rate for workout recoveries, which involve only the observation of post-default prices of debt. A simple measure, motivated in part by the availability of (and the form of what is available in) of a rich data-set of defaulted bonds and loans available to us, involves analyzing the observable market price of debt two points in time: the default event (e.g., bankruptcy or other financial distress qualifying as a default) and the resolution of the default event (e.g., emergence from bankruptcy under Chapter 11 or liquidation under Chapter 11). This can be interpreted as an estimate of the discount rate for each segment modeling as the expected rate of return on the investments in defaulted instruments belonging to that segment. This assumes that all instruments within a particular segment are identical in terms of their LGD risk, and thus share the same expected rate of return and the fact that the realized recovery deviates cross-sectionally from the expected recovery is solely because of LGD uncertainty during the recovery process. We can calculate the annualized rate of return as on the i[th] loan in segment s the as:

$$r_{i,s}^{D} = \left(\frac{P_{i,s,t_i^E}^{E}}{P_{i,s,t_i^D}^{D}} \right)^{\frac{1}{t_{i,s}^E - t_{i,s}^D}} - 1 \qquad (4.1)$$

where $P_{i,s,t_i^D}^{D}$ ($P_{i,s,t_i^E}^{E}$) are the prices of debt at time of default $t_{i,s}^{D}$ (emergence $t_{i,s}^{E}$). An estimate for the discount rate appropriate for the s[th] "LGD segment" (seniority class of collateral type) can then be formed as arithmetic averages across loans:

$$\overline{r}_s^{D} = \frac{1}{N_s^D} \sum_{i=1}^{N_s^D} \left[\left(\frac{P_{i,s,t_i^E}^{E}}{P_{i,s,t_i^D}^{D}} \right)^{\frac{1}{t_{i,s}^E - t_{i,s}^D}} - 1 \right] \qquad (4.2)$$

where N_s^D is the number of defaulted loans in the recovery group s. A measure of the recovery uncertainty in recovery class s is given by sample standard deviation:

$$\overline{\sigma}_{\overline{r}_s^D} = \sqrt{\frac{1}{N_s^D - 1} \sum_{i=1}^{N_s^D} \left[\left\{ \left(\frac{P_{i,s,t_i^E}^{E}}{P_{i,s,t_i^D}^{D}} \right)^{\frac{1}{t_{i,s}^E - t_{i,s}^D}} - 1 \right\} - \overline{r}_s^{D} \right]^2} \qquad (4.3)$$

We also pursue an alternative to this simple annualized rate of *return on default debt* ("RDD"), the *most likely discount rate* ("MLDR"), as introduced by Brady et al (2006). This involves a consideration of the price of defaulted debt $P_{i,s,t_i^D}^{D}$ as the expected, discounted recovery $P_{i,s,t_i^E}^{E}$ at rate $r_{i,s}^{D}$ over the resolution period $t_{i,s}^{E} - t_{i,s}^{D}$:

$$P_{i,s,t_i^D}^{D} = \frac{E_t^P \left[P_{i,s,t_i^E}^{E} \right]}{\left(1 + r_{i,s}^{D}\right)^{t_{i,s}^E - t_{i,s}^D}} \qquad (4.4)$$

where expectation is taken with respect to physical measure P, and it is assumed that the time-to-resolution $t_{i,s}^{E} - t_{i,s}^{D}$ is known. In order to account for the fact that we cannot observe expected recovery prices ex ante, as only by coincidence would they coincide, we invoke market rationality to postulate that for a segment homogenous with respect to recovery risk the difference between expected and average realized recoveries should be small. Following Brady et al (2006), we formulate this by defining the normalized pricing error as:

$$\tilde{\varepsilon}_{i,s} \equiv \frac{P^{E}_{i,s,t^{E}_{i}} - P^{D}_{i,s,t^{D}_{i}} \times \left(1 + r^{D}_{i,s}\right)^{t^{E}_{i,s} - t^{D}_{i,s}}}{P^{D}_{i,s,t^{D}_{i}} \times \sqrt{t^{E}_{i,s} - t^{D}_{i,s}}} \tag{4.5}$$

This is the pricing error as a proportion of the debt price at default (a "unit-free" measure of recovery uncertainty) and the square root of the time-to-resolution. This is a mechanism to control for the likely increase in uncertainty with time-to-resolution, which effectively puts more weight on longer resolutions, increasing the estimate of the discount rate. The idea behind this is that more information is revealed as the emergence point is approached, hence a decrease in risk. Alternatively, we can analyze the error $\varepsilon_{i,s} \equiv \frac{P^{E}_{i,s,t^{E}_{i}}}{P^{D}_{i,s,t^{D}_{i}}} - \left(1 + r^{D}_{i,s}\right)^{t^{E}_{i,s} - t^{D}_{i,s}}$ that is non-time adjusted, and argue that its standard error is proportional to $\sqrt{t^{E}_{i,s} - t^{D}_{i,s}}$, which is consistent with an economy in which information is revealed uniformly and independently through time (Miu and Ozdemir, 2005). Assuming that the errors $\tilde{\varepsilon}_{i,s}$ in (4.5) are standard normal[13], we may use maximum likelihood, by maximizing the log-likelihood (LL) function:

$$\hat{r}^{D}_{i,s} = \arg\max_{r^{D}_{i,s}} LL = \arg\max_{r^{D}_{i,s}} \sum_{i=1}^{N^{D}_{s}} \log\left[\phi\left(\tilde{\varepsilon}_{i,s}\right)\right] = \arg\max_{r^{D}_{i,s}} \sum_{i=1}^{N^{D}_{s}} \log\left[\phi\left(\frac{P^{E}_{i,s,t^{E}_{i}} - P^{D}_{i,s,t^{D}_{i}} \times \left(1 + r^{D}_{i,s}\right)^{t^{E}_{i,s} - t^{D}_{i,s}}}{P^{D}_{i,s,t^{D}_{i}} \times \sqrt{t^{E}_{i,s} - t^{D}_{i,s}}}\right)\right] \tag{4.5}$$

This turns out to be equivalent to minimizing the squared errors:

$$\hat{r}^{D}_{i,s} = \arg\min_{r^{D}_{i,s}} \left\{ \sum_{i=1}^{N_{s}} \frac{1}{t^{E}_{i,s} - t^{D}_{i,s}} \left(\frac{P^{E}_{i,s,t^{E}_{i}} - P^{D}_{i,s,t^{D}_{i}} \times \left(1 + r^{D}_{i,s}\right)^{t^{E}_{i,s} - t^{D}_{i,s}}}{P^{D}_{i,s,t^{D}_{i}}} \right)^{2} \right\} = \arg\min_{r^{D}_{i,s}} \left\{ \sum_{i=1}^{N_{s}} \tilde{\varepsilon}_{i,s}^{\;2} \right\} \tag{4.6}$$

We may derive a measure of uncertainty of our estimate by the ML standard errors, which are derived from the Hessian term: deviation:

$$\hat{\sigma}_{\hat{r}^{D}_{i,s}} = \left[-\frac{\partial^{2} LL}{\left(\partial \hat{r}^{D}_{i,s}\right)^{2}} \right]^{-\frac{1}{2}} \Bigg|_{r^{D}_{i,s} = \hat{r}^{D}_{i,s}} \tag{4.7}$$

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[13] If the errors are i.i.d. and from symmetric distributions, then we can still obtain consistent estimates through ML, which has the interpretations as the quasi-ML estimator.

5. Empirical Results: Summary Statistics of Discount Rate Measures by Segments

In this section, and the following two, we document our empirical results. These are based upon our analysis of defaulted bonds and loans in the June, 2008 Moody's Ultimate Loss-Given-Default (MULGD) database. This contains the market values of defaulted instruments at near the time of default[14], as well as the values of such pre-petition instruments (or of instruments received in settlement) at the time of default resolution. This database is largely representative of the U.S. large-corporate loss experience, from the mid 1980's to present, including most of the major corporate bankruptcies occurring in this period.

In this section, we discuss various summary statistics, measures of central tendency and dispersion, tabulating observations of our two alternative measures of the discount for workout recoveries. These measures are the simple annualized return on defaulted (RDD) as defined in (4.2)-(4.3), and the optimized segment-wise most likely discount rate (MLDR), as defined in equations (4.5)-(4.7). We investigate segmentations such as default event type (bankruptcy vs. out-of-court restructuring), instrument type (loans vs. bonds), seniority rank, collateral quality rank, proportion debt above & below in the capital structure, duration of distress or of the resolution, annual cohort and industry. Subsequent sections investigate univariate correlation and multiple regression analyses of RDD,

5.1 Summary Statistics by Debt and Default Type

In Tables 1, condensed Table 1.1, and in Figures 1.1 through 4.2, we summarize basic characteristics (measures of central tendency and dispersion) of LGD discount rate measures, RDD and MLDR, by default event type (bankruptcy under Chapter 11 vs. out-of-court restructuring), and instrument type (loans – broken down by term and revolving – vs. bonds).

In the more expansive Table 1, the bottom panel represents the entire Moody's database, whereas the top panel summarizes the subset for which we calculated defaulted instrument return measures. Here we also show the means and standard deviations of four key quantities: LGD measured at default ("DLGD") and discounted from resolution ("ULGD"), time-to-resolution ("TTR"), and outstanding-at-default ("OAD"), for both the RDD / MLDR sample as well as for the entire MULGD database (i.e., including instruments not having trading prices at default). We conclude from this that our sample is for the most part representative of the broader database. For the bankrupt firms, average ULGD is 52.0% (45.7%), TTR is 1.7 (1.6) years and average OAD is $202.0M ($141.8M) for the analysis (broader) samples.

The version of MULGD that we use (June 2008 release) contains 3,886 defaulted instruments, 3,391 (or 87.3%) of which bankruptcies, and the remaining 495 distressed restructurings. On the other hand, in the RDD / MLDR sub-set, the vast majority (94.3% or 1,262) of the total (1,338) are Chapter 11. The reason for this is

[14] This an average of trading prices from 30 to 45 days following the default event. A set of dealers is polled every day and the minimum /maximum quote is thrown out. This is done by experts at Moody's.

two-fold: first, the times-to-resolution of the out-of-court settlements are so short (about 2 months on average, and many are much shorter) that post-default trading prices at 30-45 days are not available; second, many of these were extreme values of RDD, and were heavily represented in the outliers that we choose to exclude from the analysis (35 out of 37)[15].

Overall average of 1,277 annualized RDDs is 29.7%, with a high standard deviation relative to the mean of 117.5%, ranging from -100% to 894%. This says that there were some very high returns – as the 95th percentile of the RDD distributions is 191%, this says that in well over a 100 cases investors would have more than doubled their money holding defaulted debt. On the other hand, overall MLDR is 22.4%s, with an MLE standard error of 107.8%, a smaller estimate than the RDD but showing the same high degree of variation about the MLE.

We observe that the distribution of RDD and MLDR is significantly different in the case of out-of-court settlements as compared to bankruptcies, with respective means and MLEs of 37.3% and 52.4% for the former, and 29.2% and 21.3% in the latter. The standard deviations and MLE SD's are also much higher, 133.3% and 104.0% for out-of-court, versus 116.5% and 114.2% for bankruptcies, respectively. This large difference in distributional properties can be observed in the empirical distributions of RDD in Figures 1. Since there are only 76 of these, they appear to behave so differently, and we have concerns about the degree of recovery uncertainty embedded in the very short resolution time of out-of-court settlements, we make the decision to eliminate that segment from subsequent analysis.

Now focusing upon bankruptcies, we examine facility types. Approximately 30% of the sample consists of bank loans, 379 out of 1338 instruments. Loans appear to behave differently, yet the RDD differs from the MLDR measure in the direction of difference: they have a higher (lower) RDD (MLDR) as compared to the broader sample, 43.3% (14.5%) vs. overall figures for bankruptcies of 29.2% (43.3%). On the other hand, bonds appear slightly riskier and higher returning according to MLDR, with an estimate of 23.9%, yet to have less recovery risk according to average RDD of 23.3% as compared to the broader sample. Finally, in comparing revolving credits to loans, they appear approximately as risky according to MLDR (15.2% and 14.5%, respectively), yet according to RDD loans are riskier than revolvers (43.3% vs. 40.0%).

5.2 Summary Statistics by Seniority Rank and Collateral Code

Table 2 (full), Table 2.1 (condensed), and Figures 5-6, summarize distributional properties of RDD and MLDR by seniority rankings (bank loans; senior secured, unsecured and subordinated bonds; and junior subordinated bonds) and collateral types. We have 2 sets of collateral types: the 19 lowest level labels appearing in MULGD (Guarantees, Oil and Gas Properties, Inventory and Accounts Receivable, Accounts Receivable, Cash, Inventory, Most Assets, Equipment, All Assets, Real Estate, All Non-current Assets, Capital Stock, PP&E, Second Lien, Other, Unsecured, Third Lien, Intellectual Property and

[15] Based upon extensive data analysis in the Robust Statistics package of the S-Plus statistical computing application, we determined these 37 observations to be statistical outliers. The optimal cutoff was determined to be 894%, above which we removed the observation from subsequent calculations.

Intercompany Debt), and a 6 level high level grouping of that we constructed from the (Cash, Accounts Receivables & Guarantees; Inventory, Most Assets & Equipment; All Assets & Real Estate; Non-Current Assets & Capital Stock; PP&E & Second Lien; and Unsecured & Other Illiquid Collateral).

Generally, since this does not hold monotonically across collateral classes or is consistent across recovery risk measures, better secured or higher ranked instruments exhibit higher RDDs or MLDRs. Average RDDs (MLE estimates of MLDRs) are 46.4% for secured vs.17.9% for unsecured (31.7% for secured vs.18.3% for unsecured) facilities. The difference is much accentuated for the RDD as opposed to the MLDR measure. Focusing upon bank loans, we see 44.3% vs. 28.9% (17.9% vs. 9.9%) split for secured and unsecured. However, by broad measures of instrument ranking, RDD and MLDR do not agree to the same extent with regard to this ordering: average RDD (MLDR) is 43.5% and 51.7% (14.5% and 38.4%) for loans and senior secured bonds, as compared to 22.5% and 23.9% (20.9% and 21.9%) for senior secured and senior subordinated bonds, so the pattern is less monotonic for MLDR (lower for loans).

However, in the case of RDD, while unsecured loans have lower measures of recovery risk than secured loans, within the secured loan class we have that these measures increase with the collateral quality rank. Across all seniorities, there is an almost monotonic increase in RDD from 27.8% for Cash, to 43.4% for All Assets & Real Estate, to 54.8% for PP&E & Second Lien. This result does not carry over to the MLDR measure: 33.3% for Cash, down to 30.4% for All Assets & Real Estate, and back up to about the same level of 33.7% for PP&E & Second Lien.

5.3 Summary Statistics by Year of Default

Table 3 and Figures 7.1-7.2 summarize distributional properties of RDD and MLDR by the calendar year in which the instrument went into default. Here we are trying to get an idea of the cyclical properties of a measure of the discount rate for LGD. We observe that there is weak evidence of LGD discount rate measures being pro-cyclical, or elevated during the downturn periods; however, there appears to some asynchronicity, as (especially in the most recent downturn) these tend to peak in the years after the episode is over. With regard to whether and how this quantity varies with the state of the economy, there are some slight differences across the two metrics, RDD vs. MLDR, as well as across the two recessionary episodes, which we identify as the years 1990-91 and 2000-02 (when the Moody's speculative grade default rate is highest). One caution here is the censoring issue: low counts in the beginning and end of the sample, rendering inference problematic.

In the 1st episode, the averages of RDD do appear to be elevated in comparison to the periods immediately preceding and following, 46.4% and 43.8% in 1990 and 1991; while the peaks of the 1980's and 1990's are 29.1 and 27.5%, respectively (and furthermore, in many 2 years during the 1990's the average is in fact negative). But in the 2nd episode, the RDD does not appear elevated until 2002, when it increases to 50.8% in that year from -4.0% and 16.2% in the years 2000 and 2001, respectively; and the peak of the current decade occurs in 2003, with average RDD of 64.2%, and it remains above 40% until 2006.

In the case of MLDR, it is also hard to clearly discern cyclicality. While the estimate does appear elevated in 1991 (39.4%) relative to the 1990's (although it is 35.5% in 1989), and its peak in the 1990s is 20.4% in 1996. While MLDR is elevated in 2001 and 2002 relative to the 1990s (24.5% and 32.8%, respectively), as with the RDD it peaks in the current decade, and even later still in 2005 (50.8%)

Measures of dispersion in discount rates estimates also exhibit this weak pattern of elevation around the downturn periods. The standard deviation of RDD is higher in 1990-91 (143.3% and 117.7%) than in the late 1980s (where it is highest at 108.3% in 1989), yet the local peak occurs at 172.3% in 1992. A similar pattern for this statistic occurs in the second downturn period, generally higher but peaking in the year after, rising monotonically from 53.3% in 1999 to 161.2% in 2003, thereafter falling monotonically. In the case of MLDR, we see a peak in the MLE estimate of the standard error peaking 91.4% in 1991, and the second peak occurring in 2000 at 60.3%; but there is another local maximum in the current decade, 53.2% in 2005.

There are some patterns with respect to other variables in this data-set that are worthy of note. While the credit cycle is clear whether one looks at the count or volume of defaults in this data-set, which correspond to the 5 highest average Moody's speculative grade default rate, it is not clear if loss severity is higher in these periods for this sub-set of MULGD. In the 1st episode, while LGD does have a local peak of an average 75.6% in 1990, it is again elevated in 1992 (58.9%), and has a 1990's peak of 70.6% in 1994. In the 2nd episode, LGD peaks in 1999 and 2000, respective averages of 65.0% and 62.6%, which slightly leads the downturn; however, LGD does remain low on average after 2002. Second, we note that it is difficult to see how average time-to-resolution varies with the cycle. We observe that it is lower on average during the 2 years of the 1st downturn than in the 1980s (1.69 and 1.61 years in 1990 and 1991, respectively), and then having peaks in the mid-90's (2.08 years in 1995), although it bounces around non-monotonically in this period. However, in the 2nd recessionary episode time-to-resolution is higher than the surrounding years, rising from 1.35 years in 1999, to 1.74 and 1.73 years in 2000 and 2001, respectively; and then never rising above 1.12 in 2005.

5.4 Summary Statistics by Time-to-Resolution and Time-in-Distress

Table 4 and Figures 8.1-8.2 summarize distributional properties of RDD and MLDR by two duration measures: the "time-in-distress" (TID), defined as the time (in years) from the last cash pay date to the default sate, and the "time-to-resolution" (TTR), the duration from the date of default to the resolution or settlement date. These help us under the term-structure of the discount rate for workout recoveries, and answer the question, does the length of time in workout, or under pre-default "watch", influence the uncertainty in recovery cash flows that is implicit in the discount rate measure, RDD or MLDR. We examine features of RDD and MLDR by quartiles of the TTR and TID distributions, where the 1st refers to the bottom fifth of durations in length, and the 5th quartile the top longest.

We can observe some patterns, although they are non-monotonic, and the two discount rate measures do not agree on the shape. First, focusing on the TTR, in the case of RDD both the mean and standard deviations decline in longer duration buckets. Average RDD declines, albeit bumpily, from 70.8% at the 1st quartile, to 17.8% in the middle, and 7.5% at the top quartile. The standard deviation declines monotonically, from 178.5% at the bottom, to 45.0% at the top quartile. This has the interpretation that greater uncertainty in recovery cash-flows is reflected in higher average and volatility of returns on defaulted debt, and as this recovery uncertainty is resolved over time, there is a concomitant decline in return and in volatility of return. However, MLDR is not telling the same story. The MLE estimate peaks at 42.9% in the 4th quartile, while staying in the range of 17-24% in the bottom 2 quartiles, and falls back to this range in the top quartile (18.5%); this is somewhat suggestive of a humped shape, albeit non-monotonic. However, the MLE standard error of MLDR exhibits a dramatically different pattern, dropping off from 96.6% in the 1st quartile to a rather low range for the remaining quartiles, and then bo8uncing around: 5.7% and 7.6% in the 2nd and 3rd quartiles, then peaking again at 18.5% in the 4th, and falling back to 6.7% in the top.

Like with TTR, the RDD and MLDR measures of recovery uncertainty behave a little differently with respect to TID bucket. Mean RDD exhibits a general decline, albeit non-monotonic in middle range, from 41.3% in the 1st quartile, to 31.0% in the middle, and 21.3% at the top. The standard error of the mean RDD has a U-shaped pattern, going from 145.6% at the bottom quartile, bottoming out at 93.1% in the middle, and increasing back to 133.0% at the top. On the other hand, the MLE standard error of MLDR exhibits a general, albeit non-monotonic , hump shape: from 24.7% in the 1st quartile, to 53.3% in the middle, and 26.9% at the top (with inexplicable plunges to 6.6% and 5.5% at the 2nd and 4th quartiles, respectively).

Taken together, this has the interpretation that we see some evidence that the longer in distress prior to default, or in the restructuring process following default, the more uncertainty is resolved and the lower is the appropriate discount rate for workout recoveries that properly adjusts for the risk.

5.5 Summary Statistics by Credit Rating at Origination

Table 5 and Figures 9.1-9.2 summarize distributional properties of RDD and MLDR by the earliest available Moody's senior unsecured credit rating for the obligor. This provides some evidence that LGD discount rate estimates are augmented for defaulted obligors that had, at origination (or time of first public rating), better credit ratings or higher credit quality. This implies that in this sense we have higher recovery uncertainty embedded in the recovery cash-flows of better rated credits, and that the appropriate discount rate for such credits should be higher, controlling for other risk factors. But, as with many of the other results herein, the relationships are not monotonic in these tabular analyses, and the high degree of dispersion in the estimates calls statistical significance of the separation amongst categories into question.

Mean RDD generally declines as credit ratings worsen, albeit unevenly. While the average is 26.4% for the AA-A category, it goes from 48.6% for BBB, down to 32.1% and 19.6% for B and CC-CCC; but note the anomalous dip to 18.1% for BB. The MLE estimate of MLDR exhibits a similar overall downward yet kinky drift: from

111.6% at BBB, 22.1% and 13.2% at BB and B; but there is a strange low value of 20.7% at AA-A (as with RDD), and it peaks up a bit to 18.3% at CC-CCC. However, if we look at the investment grade (IG) vs. non-investment grade (NIG) split, the picture is clearer, as we see mean RDD (MLE of MLDR) as 33.3% and 26.1% (23.7% and 18.5%) for IG and NIG, respectively.

There is much less in the way of recognizable or intuitive patterns with respect to the dispersion measures. In the case of RDD, the standard deviations generally *increase* in worsening rating, and in a double humped pattern. The latter is a bit surprising, and we would associate greater recovery risk with a greater measure in the variation of the estimate. However, for MLDR, there is a general *decline* in the MLE estimate of the standard error, peak at BBB and declining thereafter, which is more what we would expect. In the IG vs. NIG comparison, we see that the standard deviation of RDD increases from 81.8% to 212.8%, while on the other hand the MLE standard error of MLDR declines from 25.9% to 8.1%.

5.6 Summary Statistics by Loan Position in the Capital Structure ("Tranche Safety Index")

Table 6 and Figures 10.1-10.2 summarize distributional properties of RDD and MLDR by measures of the relative debt cushion of the defaulted instrument. MULGD provides the proportion of debt either above ("degree of subordination") or below ("debt cushion") any defaulted instrument, according to the seniority rank of the class to which the instrument belongs. It has been shown that the more debt below, or the less debt above, the better is the ultimate recovery (or the lower is the ultimate LGD) of a defaulted bond or loan (Keisman, 2000). We can also think of this position in the capital structure in terms of "tranche safety" – the less debt above, more debt below, or the thinner the tranche, then the more likely it is that there will be some recovery. However, this is not the entire story, but this measure has been demonstrated to be an important determinant of ultimate LGD; therefore, we suspect that it will have bearing on the performance of defaulted debt.

Here, we offer evidence that returns on defaulted debt measured by RDD (or the appropriate discount rate as measured by MLDR) are increasing in the degree of tranche thickness or relative debt cushion, in the sense of the difference between debt below and debt above. To the end of showing this in tabular form, we define the *Tranche Safety Index* (TSI) as:

$$TSI \equiv \frac{1}{2}\left[\%\,Debt\ Below - \%\,Debt\ Above + 1\right] \qquad (3.6.1)$$

This ranges between zero and 1, where it is near unity the greater the difference between debt below and above (i.e., the thinnest tranche or the most subordinated), and closest to zero when debt below is nil and most of the debt is above (i.e., the thickest tranche or the greatest debt cushion). In Table 6, we examine the quantiles of the TSI, where the bottom 20th percentile of the TTI distribution represents the least protected instruments, and the top 20th percentile the most protected. Additionally, we define several dummy variables in order to capture this phenomenon, as in Brady et al (2006). "No Debt Above and Some Debt Below" (NDA/SDB) represents a group that should be the best protected, while "Some Debt Above and Some Debt Below" (SDA/SDB) and "No Debt Above and No Debt Below" (NDA/NDB) represent intermediate groups, and "No Debt Below and Some Debt Above" (NDA/SDA) should be the best protected group.

There is some mixed in Table 6 evidence, as for quintiles of TTI the results are non-monotonic for both RDD and MLDR. There is a U-shape in average RDD with respect to quintiles of TSI, starting at 33.0% at the bottom quintile, having a minimum in the 2nd of 9.6%, and increasing thereafter to 26.8%, 43.6% and 53.2% at the top. The pattern is also U-shaped for MLDR, but the minimum occurs at the mid quintile (14.4%), having peaks at the lowest (33.8%) and the highest (29.6%) quintiles. The dispersion measures for these are also U-shaped, as with the central tendency measures bottoming in the 2nd and 3rd quintiles of RDD and MLDR, respectively.

In regard to the dummy variables, there is a general decline in discount rate measures from the most to least favourable positions. In the case of RDD, it is highest for NDA/SDB (44.2%), lower and similar for SDA/SDB and NDA/NDB (25.1% and 26.3%), and lowest for the most subordinated (18.1%). But in the case of MLDR, the relationship amongst these dummies is bumpier but for the most part in the same direction, highest for NDA/SDB and SDA/SDB (25.3% and 35.4%), and lowest for NDA/NDB and NDB/SDA (12.0% and 19.7%). The dispersion measures are similar in their patterns for theses dummies, across the two recovery uncertainty measures exhibiting an approximate inverse U-shapes for both.

5.7 Summary Statistics by Industry Groups

Table 7 and Figures 11.1-11.2 summarize distributional properties of RDD and MLDR by industry group. These 8 high level categories were derived from the Moody's 12 industry groupings judgmentally, in consultation with subject-matter experts, with an eye toward finding meaningful groupings with some separation in recovery risk measures. Among all the segmentation considered thus far, we see the greatest disagreement between the RDD and MLDR measures of the discount rate. In the case of mean RDD, we observe that the industry groups Leisure Time / Media, High Technology / Telecommunications and Aerospace / Auto / Capital Goods / Equipment exhibit elevated estimates of 36.6%, 41.0% and 41.3%, respectively. However, in the case of MLDR, the Forest / Building products / Homebuilders category has the highest MLE estimate of 34.9%, while the lowest is Transportation, at 5.8%, which is also substantially below the overall average of RDD at 6.0%.

6. Empirical Results: Distributional Properties of Covariates and Univariate Correlation Analysis

In this section we analyze the independent variables available to us and calculated from MULGD, as well as data attached to this from Compustat and CRSP. Tables 8, 8.1-7 and Figures 12.1-# summarizes the distributional properties of key covariates in our database and their univariate correlations to RDD. We have grouped these in to the following categories: Financial Statement and Market Valuation, Equity-Price Performance and Capital Structure, Credit Quality / Credit Market, Instrument / Contractual, Durations / Vintage and Macro / Cyclical, Capital Structure, Credit Quality / Credit Market, Instrument / Contractual, Macro / Cyclical and Duration / Vintage.

6.1 Summary Statistics and Correlation Analysis: Financial Statement and Market Valuation

In this section we consider the financial variables, alone and in conjunction with and equity market metrics, extracted from Compustat or CRSP. The Compustat variables are taken from the date nearest to the 1st instrument default date of the obligor, but no nearer than on month, and no further than one year, to default. These are shown in the top panel of Table 8, in condensed form in Table 8.1, and in Figures 12.1-12.3. First, We see some evidence that leverage is positively related to RDD, suggesting that firms that were nearer to their "default points" prior to the event had defaulted debt that performed better over the resolution period, all else equal. This is according to an accounting measure, Book Value of Total Liabilities / Book Value of Total Assets (BVTL/BVTA), which has a substantial positive correlation of 17.2%. However, this result does not carry over to a market measure, Book Value of Total Liabilities / Market Value of Total Assets (BVTL/BVTA), which has a rather small (albeit statistically significant) correlation of -1.2%. Note that these results are robust to alternatives, such as Book Value of Total Assets / Book Value of Equity or Book Value of Total Assets / Market Value of Equity, or variations on those. Note the extreme degree of leverage present in this defaulted population, with a median BVTL/BVTA of 1.13, and a maximum of 3.92. Also note the high degree of coverage for these measures, 111 out of 1267. In the multivariate analysis, the variable BVTL/BVTA enters in the favored model for RDD.

Regarding variables measuring size of the firm, by either accounting or market values, we see mixed evidence point to a negative relationship to RDD. This holds most true for Market Value of Total Assets (MVTA), which has a reasonably robust correlation of -8.2%. On the other hand, the two accounting measures, Net Sales and Book value of Assets (BVA), show a weaker but still inverse relationship, correlations of -2.7% and -1.8%, respectively. We are not sure what a good story is here – a candidate includes coordination issues in larger bankruptcies that cause the debt of these companies to underperform during the resolution process; but one can be equally credible in positing that larger companies have better wherewithal to emerge successfully from such a process. In none of our regressions did variables in this dimension appear to make a significant contribution.

Next, we consider a set of variables measuring the degree of market valuation relative to stated value, or alternatively the degree of intangibility in assets: Tobin's Q, Market Value of Total Assets / Book Value of Total Assets (MVTA/BVTA or "Market-to-Book"), Book Value of Intangibles / Book Value of Total Assets (BVI/BVTA), and the Price / Earnings Ratio (PE). In this group, there is evidence of a positive relationship to the discount rate for LGD, which is strongest by far for MVTA/BVTA, having a correlation of 18.5%. BVI/BVTA and PE Ratio are significantly weaker, having correlations to RDD of 2.0% and 4.0%, respectively. Tobin's Q makes the least contribution, with a nil coefficient of -0.1%. MVTA/BVTA enters into some of our candidate regression models significantly, but not the final model chosen. We speculate that the intuition here is akin to a "growth stock effect" – such types of firms may have available a greater range of investment options, that when come to fruition results in better performance of the defaulted debt on average.

Next we consider a range of variables that measure the liquidity position of the firm: Current ratio (CR), Interest Coverage ratio (ICR), Working Capital / Book Value of Total Assets (WC/BVTA) and Cash Flow / Current Liabilities (CF/CL). These measures are evenly split in their relation to RDD, which in all cases is weak: positive for CR and WC/BVTA (2.3% and 2.8%), and negative for ICR and CF/CL (-6.6% and 3.0%). The reasonably robust magnitude on the inverse correlation in the case of ICR may not be as puzzling as at first blush, as one can make the argument that a firm with adequate ability to service debt that is nonetheless in default may be more likely to have a fundamental problem, all else equal. However, this variable does not make it to any of our candidate multiple regression models.

We display 3 covariates in Table 8 and Table 8.1 that measure the cash-flow generating ability of the entity: Free Asset Ratio (FAR), Free Cash Flow / Book Value of Total Assets (FCF/BVTA) and the Cash Flow from Operations / Book Value of Total Assets (CFO/BVTA). Results are mixed, a strong (weak) negative relationship for FAR (FCF/BVTA), and a weak positive relationship for CFO/BVTA. The intuition here may be considered strained, as it is natural to think that the ability to throw off cash may signal a firm with an underlying business model that is viable, which is conducive to a successful emergence from default and well performing debt; however, this may also be take to mean an "excess" of cash with not good investments to apply it to and a basically poor economic position. At any rate, note that FAR does make it into one of our three candidate multiple regression models, but not the final one.

Finally for the financials, we have a set of variables that measure some notion of accounting profitability: Net Income / Book Value of Total Assets (NI/BVTA), Net Income / Market Value of Total Assets (NI/MVTA), Retained Earnings / Book Value of Total Assets (RE/BVTA), Return on Assets (ROA) and Return on Equity (ROE). With the exception of the small positive correlation for NI/MVTA (-0.2%), these are generally modest but inversely related to RDD: correlations of -2.9%, -6.7%, -8.1% and -2.8% for NI/BVTA, RE/BVTA, ROA and ROE, respectively. As with other dimension of risk considered here, we resort to a "backward story", relative to the expectation that least-bad profitability mitigates credit or default risk: that is, if already in default, than better accounting profitability may a harbinger of deeper woes for the firm, as reflected in the performance of its debt to emergence.

6.2 Summary Statistics and Correlation Analysis: Equity Price Performance Variables

In this section we consider the equity price performance metrics, extracted from CRSP at the date nearest to the 1st default date of the obligor, but no nearer than on month to default. These are shown in the 2nd from top panel of Table 8, in condensed form in Table 8.2, and in Figure 12.4.

The 1-Month Equity Return Volatility ("1M-ERV"), the standard deviation of daily equity returns in the month prior to default, exhibits small positive correlation of 2.3% to RDD. This sign is explainable by an option theoretic view of recoveries, since the value of a call-option on the residual cash flows of the firms to creditors firm are expected to increase in asset value volatility, which is reflected to some degree in equity volatility. On the other hand, the 1-Year Expected Equity Return ("1Y-EER"), defined as the average return on the obligor's stock in excess of the risk- free rate the year prior to default, exhibits a modest degree of negative correlation (-6.4%). We find this a little puzzling. On the other hand, the Cumulative Abnormal Returns ("CAR") on equity, the returns in excess of a market model in the 90 days prior to default, have a the strongest positive relationship to RDD of the group, 10.9%. This is understandable, as the equity markets may have a reasonable forecast of the firm's ability to become rehabilitated in the emergence from default, as reflected in "less poor" stock price performance relative to the market. Note this is one of two variables in this group that enters the candidate regression models. Market capitalization of the firm relative to the market as a whole ("MCRM"), defined as the logarithm of the scaled market capitalization[16], also has a significant negative univariate correlation to the market of -8.6%, and enters all of the regressions, as with CAR. We have no clear a priori expectation for this variable, perhaps we would expect larger companies to have the "resiliency" to better navigate financial distress, counter to what we are measuring. The Stock Price Relative to the Market ("SPRM"), which is the percentile ranking or the absolute level of the stock price in the market, has a moderate negative correlation to RDD of -5.4%. The purpose of this variable is to capture the delisting effect when a stock price goes very low, and we might expect the opposite sign on this correlation. Finally, the Stock Price Trading Range ("SPTR"), defined as the stock price minus its 3-year low divided by the difference between its 3-year high and 3-year low, is showing only a small negative correlation to RDD of 2.8%. This is another counter-intuitive result, as one might expect that a stock doing better as compared to its recent range to signal a better quality firm whose debt might do better in default, but the data is not showing that, or much less of any kind of relationship here.

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[16] The scale factor is defined as the market capitalization of the stock exchange where the obligor trades time 10,000.

6.3 Summary Statistics and Correlation Analysis: Capital Structure Variables

In this section we consider capital structure metrics, extracted from the MULGD data at the default date of the obligor. These are shown in the 3rd from top panel of Table 8, in condensed form in Table 8.3, and in Figure 12.5.

The two measures of capital structure complexity, *Number of Instruments* ("NI") and *Number of Creditor Classes* ("NCC"), show an inverse relationship to defaulted debt performance. NI (NCC) has a modest negative correlation to RDD of -4.0% (-3.2%). We might expect that a simpler capital structure to be conducive to favorable defaulted debt performance according to a coordination story. Note that neither of these variables enters the final regression models. While most companies in our database have relatively simple capital structures, with NI and NCC having medians of 4 and 2, respectively, there are some rather complex structures (the respective maxima are 80 and 7).

We have three variables that measure the nature of debt composition: *Percent Secured Debt* ("PSCD"), *Percent Bank Debt* ("PSD") and *Percent Subordinated Debt* ("PSBD"). The typical firm in our database has approximately 40% of its debt either secured, subordinated or bank funded. All of these exhibit moderate positive correlation to RDD: 9.2%, 7.3% and 5.6% for PSCD, PBD and PSBD, respectively. The result on PBD may be attributed to either a monitoring, or "optimal foreclosure boundary choice" (Carey and Gordy, 2007) , story.
As with the complexity variable, none of these appear in the regression model.

6.4 Summary Statistics and Correlation Analysis: Credit Quality / Credit Market Variables

In this section we consider credit quality / credit market metrics, extracted from the MULGD database and Compustat at the default date of the obligor. These are shown in the 4rthfrom top panel of Table 8, in condensed form in Table 8.4, and in Figure 12.6.

Two of the variables in this group have, what may seem to be at first glance, counter-intuitive relationships to RDD. First, the *Altman Z-Score* ("AZS"), which is available in Compustat, has a relatively large negative correlation of -11.2% (note that higher values of the AZS indicate lower bankruptcy risk). Second, the LGD implied by the trading price at default – which forms the basis for the RDD calculation – exhibits a moderate *positive* correlation to RDD of 6.88%. As this variable has been shown to have predictive power for ultimate LGD (Emery et al, 2007), at first glance this relationship may seem difficult to understand[17]. But note that the same research demonstrates that LGD at default is also an upwardly biased estimate of ultimate LGD in some sense. Therefore, we might just as well expect the opposite relationship to hold, as Intuitively it may be that otherwise high quality debt may perform better on average if it is (perhaps unjustifiably) "beaten down". Indeed, LGD enters all of our regression models with this sign,

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and as a more influential variable than suggested by this correlation; but AZS does not make it to any of our regression models.

The remaining variables in this group are reflective of the Moody's ratings at the first point that the debt is rated. These are the *Moody's Original Credit Rating Investment Grade Dummy* (MOCR-IG), *Moody's Original Credit Rating - Major Code*(MOCR-MJC; i.e., numerical codes for whole rating classes), *Moody's Original Credit Rating - Minor Code*(MOCR-MNC; i.e., numerical codes for notched rating classes) and *Moody's Long Run Default Rate - Minor Code*(MLRDR-MNC; i.e., empirical default rates associated with notched rating classes). The only meaning univariate result here is the small positive correlation of 2.4% in the case of MOCR-IG, consistent with Brady et al (2006). This variable enters significantly into our candidate regression models.

Two other variables in this group have "intuitive" correlations to RDD, but do not enter the regressions significantly: *Credit Spread* (CS) and *Contractual Coupon Rate* (CCR). These are negatively and moderately associated with RDD, having coefficients of -5.7% and -5.8% for CS and CCR, respectively.

6.5 Summary Statistics and Correlation Analysis: Instrument / Contractual Variables

In this section we consider instrument / contractual metrics, extracted from the MULGD database at the default date of the obligor. These are shown in the 3rrdfrom top panel of Table 8, in condensed form in Table 8.5, and in Figure 12.7.

Consistent with the analysis of the previous section, the correlations with RDD in this group reflect that more instruments more senior, better secured or in a safer tranche experience better performance of defaulted debt. The *Senior Rank* (SR and *Collateral Rank* (CR) codes both have negative and reasonably sized correlation coefficients with RDD, -9.6% and -10.0% for SR and CR, respectively. *Percent Debt Below* (PDB) and *Percent Debt Above* (PDA) are positively (negatively) correlated to RDD, coefficients of 10.5% (-6.5%). And the *Tranche Safety Index* (TSI), constructed from the latter two variables as detailed in the previous section, has a significant positive correlation with RDD of 9.7%. This is consistent with our understanding that there is in fact more recovery risk associated with low expected LGD segments.

6.6 Summary Statistics and Correlation Analysis: Macroeconomic / Cyclical Variables

In this section we consider macroeconomic / cyclical metrics, extracted from the MULGD database at the default date of the obligor. These are shown in the 2nd from bottom panel of Table 8, in condensed form in Table 8.6, and in Figure 12.8-12.9. Confirming the more casual analysis of Section 5.3, where we analyzed LGD discount rate measures RDD and MLDR b averages over annual cohort, through analyzing correlations we find that these measures vary procyclically. That is, debt defaulting in downturn periods tends to perform better, implying that a higher discount rate for recovery cash-flows is warranted to adjust for elevated recovery risk.

We have measures of the aggregate default rate, extracted from Moody's Default rate Service (DRS) database. These are lagging 12-month default rates, with cohorts formed on an overlapping quarterly basis (e.g., the default rate for the 4th quarter of 2008 would represent the fraction of Moody's rated issuers in the beginning of 4Q07 that defaulted over the subsequent year[18]). The four versions of this are the *Moody's All-Corporate Quarterly Default Rate* ("MACQDR"), the *Moody's Speculative Grade Quarterly Default Rate* ("MSGQDR"), the *Moody's All-Corporate Quarterly Default Rate by Industry*[19] ("MACQDRI"), *Moody's Speculative Grade Quarterly Default Rate by Industry* ("MSGQDRI"). All of these have a mild, albeit significant, positive linear correlation with RDD: 5.7%, 5.4%, 7.4% and 6.7% for MACQDR, MSGQDR, MACQDRI and MSGQDRI, respectively. In our regression results of the next section, we will see that MACQDRI is the systematic risk variable to enter the candidate regression models, in spite of not having the highest univariate correlation.

The next set of variables represent measures of aggregate equity market performance, the *Fama and French* (FF) portfolio returns, which are commonly used in the finance literature[20]. These are *Excess Return on the Market* ("FF-ERM"), *Relative Return on Small Stocks*[21] ("FF-RRSS") and the *Relative Return on Value Stocks*[22] ("FF-ERVS"). We measure these on a monthly basis, in the month prior to instrument default. We see that RDD is not related to aggregate return on the market factor FF-ERM, as the correlation -0.1%[23]. On the other hand, RDD seems to have a small positive (negative) relation to FF-RRSS (FF-RRVS), with correlations of 2.4% (-3.6%). We have one more aggregate equity market return variable, *2-Year Stock Market Volatility* (2Y-SMV), defined as the standard deviation of the S&P 500 return in the 2-years prior to default. This variable shows a negligible negative linear correlation to RDD of -0.4%. Note that none of these aggregate equity market variables are significant in any of the multiple regression models and do not appear in any further analysis of RDD.

Finally, we consider aggregate interest rates, the *1-Month Treasury Bill Yield* ("1M-TBY") and the *10-Yearf Treasury Bond Yield* ("10Y-TBY"). Both of these exhibit moderate negative correlation to RDD, of -10.4% and -6.7% for 1M-TBY and 10Y-TBY, respectively. However, only the 1M-TBY appears in the final regressions. The intuition here may be that defaulted debt performs better in low interest rate environments, which is associated with lower aggregate economic activity, as well as a higher marginal utility of consumption on the part of investors.

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[18] We follow the practice of adjusting for withdrawn ratings by subtracting one-half the number of withdrawn obligors from the number of available-to-default (or the numerator of the default rate).
[19] We use our high level 8 categories discussed in Section 5.7.
[20] These can be downloaded from Kenneth French's website:
[21] This is more commonly termed the *Small Minus Large* (SML) portfolio (see Fama and French, 1992).
[22] This is more commonly termed the *High Minus Low* (HML) portfolio, meaning high vs. low market-to-book ratio (see Fama and French, 1992).
[23] Results for the S&P 500 return, not shown, are very similar.

6.7 Summary Statistics and Correlation Analysis: Duration / Vintage Variables

In this section we consider duration / vintage metrics, based on calculations from extracted dates in the MULGD database. These are shown in the bottom panel of Table 8, in condensed form in Table 8.7, and in Figure 12.10.

We can conclude from this section that the duration / vintage measures that would be in one's information set at the time of instrument default are largely uninformative regarding the performance of defaulted debt. The variables that we have chosen to display include *Time from Origination to Default* ("TOD"), *Time from First Rating to Default* ("TFRD"), *Time from Last Cash-pay Date to Default* ("TID" or "Time in Distress"), *Time from Default to Origination*("TTR" or "Time-to-Resolution") and *Time from Origination to Maturity* ("TOM"). We see that the two vintage measures, TOD and TFRD, have negative but negligible correlations to RDD, -0.7% and -0.5%. Counter to the analysis of TID quintiles in Section 5.4, where we found some evidence of a negative relationship with RDD, here we see only a very small positive correlation of 0.2%. However, we do see results consistent with that analysis for TTR, as the correlation to RDD is negative and a sizable 10.6%; but note that this variable does not significantly enter any of the candidate multiple regression models discussed in the subsequent section. Finally, the TOM variable is inversely related to RDD, but this is rather weak, a correlation of -1.3%.

7. Empirical Results: Multiple Regression Analysis of the Returns on Defaulted Debt

In this section we discuss the construction and results of multiple regression models for RDD. In order to cope with the highly non-normal nature of the nature of the RDD distribution, we turn to the various techniques have been employed in the finance and economics literature to classify data in models with constrained dependent variables, either qualitative or bounded in some region. However, much of the credit risk related literature has focused upon qualitative dependent variables, which the case of PD estimation naturally falls into. Maddala (1981, 1983) introduces, discusses and formally compare the different *Generalized Linear Models* ("GLMs"). Here we consider the case most relevant for RDD estimation, and that least pursued in the GLM literature. In this context, since we are dealing with a random variable in a bounded region, this is most conveniently modelled through employing a beta distribution. Therefore, we follow Mallick and Gelfand (1994), in which the GLM *link function*[24] is taken as a mixture of cumulative beta distributions, which we term the beta-link GLM (BLGLM). We can solve for the parameters of the model through maximum likelihood, which is detailed in Section 14 (Appendix 1).

The coefficient estimates and diagnostic statistics for our "leading" three models are shown in Table 9. These are determined through a combination of

[24] In the terminology of GLMs, the link function connects the expectation of some function of the data (usually the random variable weighed by density, in the case of the expected value) to a linear function of explanatory variables.

automated statistical procedures[25] and expert judgment, where we try to balance to sometimes competing considerations of in-sample fit with the sensibility of the models. Essentially, the three models shown in Table 9 had the best fit to the sample data, while spanning what we thought was the best set of risk factors, based upon prior expectations as well as the univariate analysis. Note that there is much overlap between the models, as Model 2 differs from Model 1 by two variables (it has MV / BV instead of TL / TA, has RSIZ), and Model 3 from Model 2 by two variables as well (FAR in lieu of TSI and LGD).

Across the 3 candidate models, we observe that all coefficients estimates attain a high degree of statistical significance, in almost all cases at better than the 5% level[26], and in many cases at much better than the 1% level. The number of observations for which we had all of these explanatory variables is close for Models 1 and 2, 959 and 958, respectively; but there is a sizable drop-off for Model 3, only 791 observations. In all cases, the likelihood functions converged to stable global maxima.[27] Model 3 achieves the best in-sample fit by McFadden pseudo r-squared of 41.7%, followed by Model 2 (38.8%) and Model 1 (32.5%). In terms of maximized log-likelihood, Model 3 is far better than the others (-504.0), and Model 1 is only slightly better than Model 2 (-592.3 vs. -594.7) in spit of having one less explanatory variable, but as these models are not nested this may not be so meaningful a comparison. Overall, we deem these to signify good fit, given to non-linearity of the problem, the relatively high dimension as well as the high level of noise in the RDD variable.

We now turn to the signs and individual economic significance of the variables, note that we report *partial effects* ("PE"), which are akin to straight coefficient estimates in an ordinary least squares regression. Roughly speaking, this represents a change in the dependent variable for a unit change in a covariate, holding other variables fixed at their average sample values[28].

First, we consider the systematic risk variables. In the case of the Moody's speculative default rate by industry, that appears in all models. we see from the PE's ranging in 2.05-2.25. This implies that a percentage point elevation in aggregate default rates adds about 2.15% in return on defaulted debt on average, all else equal, which can be considering highly significant in an economic sense. For example, the near quadrupling in default rates between 1996 and 2001 would imply an increase in expected RDD about 12%. On the other hand, the PE's on the 1-Month Treasury yield are in the range of -0.49 to -0.37, so that debt defaulting when short-term rates are about 2% higher will experience about 0.8% deterioration in performance, ceteris paribus.

Next, we consider the contractual variables. The dummy variable for secured collateral has PE's ranging in 0.24-0.25 across all models, suggesting the presence of any kind of security can be expected to augment RDD by about 25%. The TIS, appearing only in Models 1 and 2, has a PE ranging in 0.43-0.45, suggesting that going up a single decile in this measure can increase RDD by about 4.5%.

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[25] To this end, we employ an alternating direction stepwise model selection algorithm.

[26] Moody's investment grade rating in model 3 is on the borderline, having a p-value of 0.06, just shy of significance at the 5% level.

[27] The estimation was preformed in S+ 7.0 using built-in optimization routines.

[28] See Greene () for a discussion of this concept in the context of probit and logit regressions.

Turning to the credit quality / market variables, for LGD at default, only in Models 1 and 2, PE's are about 0.28-0.33, implying that a 10% lower expected recovery rate by the market at default can lead to about a 3% lower ultimate LGD. The dummy variable for a Moody's investment grade rating at origination, appearing in all models, has PE's ranging from 0.15 in Model 3 to 0.21 in Model 1. This tells us that "fallen angels" are expected to have about 15-20% better return on their defaulted debt.

On the other hand, the single relative stock price performance variable, CAR, is in all 3 models with PE's ranging in 0.37-0.40. This says that, for example, a firm with 10% better price performance relative to the market in the 90 days prior to default will experience about 4% better return on its defaulted debt.

In the case of the financial ratios, TL / TA appears only in Model 1, having a PE of 0.27. This means that a the debt of a defaulted firm having 20% higher leverage will have about 3% greater return on its debt. MV / BV appears in Models 2 and 3, with PE's of 0.14-0.19, so that a 10% higher market valuation translates into1.5-2% better return on defaulted debt. Finally in this group, the cash-flow measure FAR only appears in Model 3, with a PE of -0.24. If a defaulted firm has 10% greater cash generating ability by this measure, then holding other factors constant its RDD should return about 2-2.5% less.

Finally, the size of the firm relative to the market appears in only Models 2 and 3. The PE's of about -0.06 to -0.04%. As this is in logarithmic terms, we interpret this as if a defaulted firm doubles in relative market capitalization, then we should expect its RDD to be depressed by around 5%, all other factors being held constant.

In order to settle upon a "favored model", we performed an out-of-sample and out-of-time analysis. Fixing the explanatory variables in each model, we re-estimated the models for different sub-samples of the available data, starting from the middle of the data-set in year 1996. We then evaluate how the model predicts the realized RDD a year ahead. We employed a resampling procedure, sampling randomly with replacement from the development data-set (e.g., the period 1987-1996), and in each iteration re-estimating the model. Then from the year ahead, we resample with replacement (e.g., the 1997 cohort), and evaluate the goodness-of-fit for the model. This is performed 1000 time, then a year is added, and this is repeated until the sample is exhausted. At the end of the procedure, we collect the r-squared's, and study their distribution, for each of the 3 models. The results of this show that the mean out-of-ample r-squared in Model 1 is highest, at 21.2%, followed by Model 3 (17.8%) and Model 2 (12.1%). On the basis of the numerical standard errors (on the order 1-2%), we deem these to be significantly different. Given the best performance on this basis, in conjunction with other considerations, we decide that Model 1 is the best. The other reasons fro choosing Model 1 are is parsimony relative to Model 2, and that it contains a credit market variable (LGD), the latter we believe makes for a more compelling story.

8. Analysis of the Regulatory Capital Impact of the Discount Rate for LGD

In this section we discuss the results of an exercise in which we asses the effect upon regulatory capital of the discount rate choice with respect to workout recoveries. The results of this analysis appear in Table 10 and Figures 13.1 through 13.6. We compare three methods of discounting LGD: the contractual coupon rate (CCR), the RDD regression Model 1, and a punitive discount rate (PDR). The CCR is available to us in the MULGD database. In the case of the PDR, we somewhat arbitrarily pick a flat 25%, which is about the average of our full-sample mean RDD (29.2%) and the full-sample MLE estimate of the MLDR (22.4%). We believe this to be a conservative approach to implementing the PDR method.

We perform the exercise of treating our sub-set of the MULGD database as a hypothetical non-defaulted portfolio, for which we happen to know the post-default cash-flows, and need only discount those to form an estimate of LGD. The formula for regulatory capital (denoted by K^R) that we compute is a version of the published formula (Basel II U.S. Final Rule, page 69335):

$$K^R = \left(N\left(\frac{N^{-1}(PD) + \sqrt{R}N^{-1}(0.999)}{\sqrt{1-R}} \right) - PD \right) \times LGD^D \qquad (8.1)$$

Where PD is the estimated probability of default, R is the asset value correlation, and N (N^{-1}) is the Normal cumulative (inverse) distribution[29]. We estimate PD by the Moody's long-run default rates associated with each observation, according to it's rating at approximately one-year prior to default (see Table 10.1). We derive R from the regulatory formula[30]:

$$R = 0.12 + 0.18 \times e^{-50 \times PD} \qquad (8.2)$$

The "downturn" LGD, LGD^D, is derived from the supervisory mapping function:

$$LGD^D = 0.08 + 0.92 \times LGD \qquad (8.3)$$

Where LGD is calculated as the actual loss rate in the database, according to the different methods of discounting. We assume a unit EAD, so that capital is normalized to represent a fraction of par, and portfolio capital is simply the arithmetic average of loan-level capital.

The results in Table 10 show that discounting according to the RDD regression model results in higher estimates of discounted LGD, and higher regulatory capital requirements, as compared to either discounting at the contractual rate or a constant punitive rate. Among all three methods, discounting by the contractual coupon rate is least conservative.

[29] The main difference with the formula in the Final Rule is that we ignore the maturity adjustment for wholesale capital.
[30] Results are not materially different when using a flat asset correlation R = 6.12%, the MLE estimate from calibrating the 2-factor ASRF model to annual Moody's all-corporate default rates for the period 1987-2007.

First considering the distribution of LGD, we note from Figures 13.1 and 13.2 that while all three measures provide highly correlated estimates of LGD (r-squareds of 0.79 and 0.78 for RDD vs. contract and punitive, respectively), the RDD model provides estimates shifted considerably upward (respective intercept terms of 0.22 and 0.08). Furthermore, note that in the comparison of the RDD model to the contract rate all the cases where the latter would yield a zero LGD, yet by a risk sensitive discount we get a non-zero LGD (sometimes a very large LGD). From the summary statistics in Table 10 we see that under the RDD model is has a higher mean (median) LGD of 64.1% (72.1%), as compared to 59.0% (62.1%) under a punitive rate, and 52.1% (55.0%) under the contract rate. Examining the distributions of LGD in this portfolio in Figures 13.3 and 13.4, it is clear that under RDD model discounting there is a shift in probability mass to the right, compared with either the contract rate (Figure 13.3) or a punitive rate (Figure 13.4).

Second, considering the regulatory capital impact of discounting, we observe that under the RDD regression model capital is significantly higher than under either the contractual coupon rate or a punitive rate. Portfolio capital under our model is 8.04% (the mean of the distribution), 73 bps higher than under a punitive discount rate, where portfolio capital is 7.31%. Under the contractual discount rate it is even lower, 6.91%, a difference of 40 bps (113 bps) to a punitive rate (the RDD model). This is evident by examining the distributions of portfolio regulatory capital in Figures 12.5 and 13.6, where we see that the density mass is shifted right-ward under the RDD model relative to either the contract rate or the 25% punitive discount rate. However, we see that there is less peakedness and skewness in the distribution of capital under the RDD model as compared to contract or punitive rate discounting, so that most of the difference is coming in the body and not the tails of the distribution (although the standard deviation is higher).

We can interpret these results as follows. The model for RDD is discounting at a much higher rate types of loans that have larger recovery cash flows, in order to adjust for the increased recovery risk associated with those. This mechanism operates through the discounted LGD. Furthermore, one can argue that the market is impounding other material direct and indirect costs into this empirical measure, such as workout costs.

9. Benchmarking Analysis of Alternative Modeling Frameworks for the LGD Discount Rate

In this section we discuss the results of an exercise in which we investigate alternative means for deriving the appropriate discount rate for workout recoveries. This exercise is summarized in Table 11, where we divide the methodologies into 4 types: market or model based, purely empirical (ex-post returns based), model-free or and regulatory prescriptions. In the model based approaches, we specify in the second column if it is a regression model, or if it is the calibration of a structural model to default and loss data, both of which producing an estimate of the correlation of LGD to a systematic risk factor. In the case of the latter, there are 2 varieties that we consider: the single-factor model of Frye (2000), and the 2-factor model developed herein, both of which having systematic recovery risk. In the case of any of the model based approaches in the top panel, we employ a simplified version of the inter-temporal Capital Asset Pricing Model (CAPM) in equation (3.7), in which case we need to make further assumptions. We take the risk-free rate to be 5%, which is a commonly made assumption in practical modelling situations, when we wish to abstract from term-structure effects. In the case of the market risk premium (MRP) and the volatility of the market (σ_M), we take the average and standard deviation of the Fama-French return on the market factor. Finally, for firm-specific asset volatility (σ_i), out of convenience and for lack of data, we use the 32% estimate derived by Frye (2000).

The general conclusion that arises from this comparison is that the model based approaches that invoke a CAPM structure as in equation (3.7), whether structural calibration or regression, generate discount rate estimates significantly lower than purely empirical approaches (in the range of 7-11%, as compared to 14-43% for model-free approaches). We also see that in the model-based approaches, discount rate estimates for loans tend to be lower than for bonds, and this is reversed for empirical based approaches. Secondly, model-based approaches tend to produce lower estimates for loans than bonds, which is generally reversed for empirical approaches. Finally, in model-based approaches as implemented, there is not a great degree of sensitivity to the correlation estimate, as the other assumptions remain fixed and carry a lot of weight.

While the comparison to the model-free approaches is inconclusive, as it relies upon factors outside the scope of this analysis, we may assume that they lie closer to the implications of the model-based than the empirical-based approaches. Among the lower estimates we find 7.2% from Machlachlan's regression of 90 defaulted bond bid quotes on the S&P 500 return for the period 4/02-8/03. Clearly, the sample size and limited time period makes us uneasy about relying on that estimate, even as a lower bond. This is lower than other approaches correlating defaulted bond returns to broad stock indices. The highest estimate is Altman and Jha (2003), where regressing the Altman / Solomon Center defaulted bond index on the S&P 500 returns for the period 1986-2002, they come up with an 11.1% discount rate (based upon a 20.3% correlation estimate), the highest in this group. Regressing monthly RDD on the Fama-French market factor gives a discount rate of 6.8% (6.6%) for the period 1987-2007 (1995-2007), based upon a correlation estimate of 13.2% (11.8%), not far from Maclachlan's 7.2% distressed bond bi-ask quote estimate. Using alternative defaulted debt indices yield similar estimates, as illustrated by the Solomon Center Defaulted Debt and Defaulted Loan indices, compiled by Ed Altman. In the case of bonds (loans), a regression of the respective monthly index returns on the S&P 500 index returns for the period 1/99-9/08 comes up with implied discount rates of 10.6% (6.5%). If instead we use the Moody's trailing 12-month speculative grade default rate in lieu of an equity index as the systematic variable (which is arguably more appropriate) and our RDD measure for the same periods and frequencies, this yields a slightly higher discount rate of 8.7% (7.9%) for bonds (loans) in 1987-2007 (1995-2007), based upon a 13.2% (11.8%) correlation estimate.

Turning to the structural type models, we start to see some higher values, depending upon the factor structure. In the well known single-factor model of Frye (2000), where LGD is driven by the same systematic variable as PD, his

calibration on Moody's loss data 1982-1999 yields a discount rate estimate of 7.3%, as per the 17% LGD-PD correlation estimate that he derives.
However, we employ a 2-factor version of the structural model using Moody's data from 1987-2007 (encompassing 2 downturns), and find higher estimates: 8.0% (9.9%) for bank loans (senior secured bonds), with the difference being very slight for loans.

A well cited empirical model is the 15% results cited by J.P. Morgan (Araten, 2004), which is supposedly based upon ex-post realized returns on the Moody's Bankrupt Bond Index for the period 1988-1998 (Hamilton and Berthault, 2000), as well as a commonly cited rate of return demanded by vulture investors. This surprisingly close to the MLDR estimate that we get in this study for loans of 14.5%; however, the RDD estimate that we get for loans is 43.3%, far above that, as well as elevated far above the overall RDD (MLDR) estimates of 29.2% (21.3%). The overall MLDR estimate of Brady et al (2006), based upon the S&P LossStats database, is closer to this well-known figure, at 14%.

It is interesting to note that a rounded 10% figure, which many of these studies come close to (e.g., Frye (2000), Altman & Jha (2003)), is currently being cited by several banks as their choice of a discount rate for LGD based upon their "weighted average cost of capital", "cost of equity" or "average contract rate" in their portfolio.

Comparing the empirical estimates in the literature, whether model-based or purely empirical, to the model-free approaches, is largely inconclusive. Probably the closest to any of these methodologies discussed above would be using the market price of debt at default (Gupton and Stein, 2002), as that would embed a market consensus on an appropriate discount rate. However, the applicability of this would be conditional upon a bank having a policy of selling defaulted debt almost immediately, or otherwise hedging its recovery risk. The cost of equity (Eales and Bosworth, 1998) would only be comparable if the bank's business were entirely defaulted debt investing, clearly an unlikely scenario. Any of the approaches advocating some measure of the promised return on the debt – either the contract (Asarnow and Edward, 1995 or Carty et al, 1998) or coupon (Friedman and Sandow, 2003) rate – would have to be such that the rate is adjusted to a truly distressed level prior to default for any kind of comparability to obtain. Finally, the risk-free rate suggestion (Carey and Gordy, 2006) is the clear outlier, as it is at odds with our thesis advanced herein.

Finally, regarding the supervisory requirements of either Basel II in the U.S. (OCC et al, 2007) or the U.K (FSA, 2003), we conclude that something between a model-based and purely empirical approach, would best be in the spirit of this. We can think that the 10-15% implied by the former may be a lower bound, and the 20-40% implied from directly measuring defaulted debt performance in some manner forms an upper bound, on the discount rate in the spirit of the supervisory language. The extent to which a Bank is closer to the former vs. the latter depends on two institution-specific factors. First, the degree of systematic risk inherent in the loan portfolio in question, and second, the amount of non0-sytenmatic yet non-diversifiable risk contained in the said portfolio. It is the latter quantity that moves us toward the elevated discount rates as estimated herein, the empirical RDD and MLDR that we observe.

10. Conclusions and Directions for Future Research

In this paper, we have address questions surrounding the discount rate that should be applied to workout recoveries, for purposes of Basel II compliance as well as internal credit risk measurement purposes. To this end, we perform a comprehensive analysis of empirically derived discount rates for LGD, derived from market price at default and emergence prices of defaulted debt. We utilize the Moody's Ultimate Loss-Given-Default database in order to accomplish this. Alternative methodologies for estimating such empirically derived discount rates are examined, the *return on defaulted debt* (RDD) and the *most likely discount rate* (MLDR) measures. First, we examine the distributional properties of the discount rate measures across different segmentations in the dataset (e.g., default type, facility type, time period, seniority, industry). Second, we develop a multiple regression model for RDD in the generalized linear model (GLM) class. Having a model for assigning a proper, risk-adjusted discount rate to defaulted instruments, we quantify the effect of discounting on the distribution of economic LGD, and on estimated regulatory capital, for a hypothetical portfolio. Finally, we perform a benchmarking analysis, comparing the empirical RDD and MLDR methods developed herein to alternative techniques I the literature, including model or market based approaches that develop a risk premium over the risk-free rate.

We find that empirically derived discount rate measures vary significantly according to certain different factors. There is some evidence that discount rate metrics are elevated for loans having better collateral quality rank or better protected tranches within the capital structure; and for obligors rated higher at origination, more financially levered or having higher Cumulative Abnormal Returns on Equity (CARs) at default. However, discount rate measures are increasing in market implied loss severity at default. We also find evidence that LGD discount rates vary pro-cyclically, as they vary directly with industry default rates, but there tends to a lag in the relationship; further, they are inversely related to short-term interest rates. However, for other demographics results are inconclusive, such as the industry group of the obligor. Finally, we conduct an analysis of the impact of the discounting method upon the distribution of estimated LGD and regulatory capital. We find that a regression model based discounting, for a sub-sample of the MULGD database, results in a significantly higher capital charge than either discounting at a constant punitive rate or at the contractual coupon rate. Our empirically derived estimates of the appropriate discount rate for workout recoveries are to be significantly higher than what has been found in the previous literature, as well as what is used commonly in industry and for Basel 2 purposes. Further, discount rates implied from a theoretical model of credit risk incorporating systematic recovery risk in an asymptotic structural risk factor framework (Gordy, 2003) are found to be significantly lower than the RDD or MLDR. We conclude that this conservativeness of the risk-sensitive RDD model, as well as the evidence that the risk in recovery cash flows contain a significant non-diversifiable component, supports the appropriateness of this framework for regulatory capital calculations.

This research, as enlightening as we believe it to be, opens up further questions regarding which discount rate for workout recoveries is optimal in some sense, from either a supervisory or risk measurement perspective. A great challenge in this regard we see as somehow reconciling the results of this empirical exercise, the implications of structural credit models as well as common industry practice. We suspect that generalizations of the ASRF framework may hold promise in this regard, as well have seen that incorporating systematic LGD as well as a second factor specific to recovery risk has resulted in higher estimates than previously obtained. Fruitful avenues of extension could be incorporating stochastic duration of bankruptcy resolution, simultaneous calibration by rating and seniority class, or incorporating strategic bankruptcy. On the empirical side, it would be useful to quantify the undiversifiable and non-systematic component of recovery risk, as that would help us sharpen our upper bound on the appropriate discount rate for LGD, as we argue has been derived herein. Finally, with a view towards the evolution 0of supervisory requirements, an examination of the impact of this choice upon economic credit, or even integrated, risk capital.

11. Tables

Table 1 - Characteristics of RDD[1] and MLDR[2] Observations by Default and Instrument Type (Moody's Ultimate LGD Database 1987-2007)

			Bankruptcy					Out-of-Court					Total				
			Count	MLDR	MSE	Average	Standard Deviation	Count	MLDR	MSE	Average	Standard Deviation	Count	MLDR	MSE	Average	Standard Deviation
Sub-population of Moody's Recoveries Database Having Trading Price of Debt at Default	Bonds and Term Loans	RDD[1]		21.74%	128.51%	28.86%	121.85%		66.98%	108.26%	38.86%	135.78%		21.86%	120.83%	29.48%	122.71%
		LGD at Default[3]				59.62%	30.54%				40.73%	27.07%				58.43%	30.67%
		Discounted LGD[4]	1121			55.12%	37.01%	73			35.15%	24.76%	1194			53.90%	36.68%
		Time-to-Resolution[5]				1.7439	1.4336				17.90%	40.73%				1.6482	1.4422
		Principal at Default[6]				194,478	267,800				390,990	479,024				206,492	288,783
	Bonds	RDD[1]		23.88%	162.22%	23.27%	119.93%		66.93%	111.31%	39.96%	137.54%		24.02%	150.44%	24.50%	121.32%
		LGD at Default[2]				65.53%	28.31%				41.77%	26.64%				63.73%	28.86%
		Discounted LGD[3]	888			27.00%	31.10%	71			36.14%	24.37%	959			26.44%	31.01%
		Time-to-Resolution[4]				1.3205	1.1599				24.01%	58.26%				1.2931	1.1631
		Principal at Default[5]				261,431	417,060				397,859	483,936				257,588	413,525
	Revolvers	RDD[1]		14.14%	3.50%	31.95%	58.58%		N/A	N/A	0.01%	0.01%		15.24%	3.57%	31.28%	58.14%
		LGD at Default[2]				34.57%	27.42%				3.33%	4.04%				33.94%	27.50%
		Discounted LGD[3]	141			27.00%	31.10%	3			0.00%	0.00%	144			26.44%	31.01%
		Time-to-Resolution[4]				1.3205	1.1599				0.27%	0.00%				1.2931	1.1631
		Principal at Default[5]				261,431	417,060				76,933	30,490				257,588	413,525
	Loans	RDD[1]		14.45%	32.82%	43.31%	106.72%		N/A	N/A	0.010%	0.009%		14.45%	32.83%	42.74%	106.13%
		LGD at Default[2]				43.31%	106.72%				2.55%	3.15%				35.33%	35.33%
		Discounted LGD[3]	374			28.88%	31.70%	5			0.00%	0.00%	379			28.50%	31.66%
		Time-to-Resolution[4]				1.4130	1.2132				18.84%	54.73%				1.2513	1.2202
		Principal at Default[5]				204,251	284,142				105,020	57,797				202,942	282,547
	Total	RDD[1]		21.31%	114.15%	29.21%	116.49%		52.54%	104.03%	37.33%	133.25%		22.38%	107.83%	29.67%	117.46%
		LGD at Default[2]				56.79%	31.22%				39.32%	27.50%				55.78%	31.28%
		Discounted LGD[3]	1262			51.98%	37.45%	76			33.76%	25.21%	1338			50.94%	37.09%
		Time-to-Resolution[4]				1.6966	1.4116				0.1720	0.4006				1.6100	1.4187
		Principal at Default[5]				201,958	288,884				378,593	473,392				211,992	304,884
Entie Population of Moody's Recoveries Databas	Bonds and Term Loans	Discounted LGD[3]				52.70%	38.28%				20.48%	29.65%				48.68%	38.80%
		Time-to-Resolution[4]	2712			1.7029	1.3458	386			0.2371	0.5783	3098			1.5203	1.3644
		Principal at Default[5]				144,150	218,187				199,686	343,228				151,069	238,020
	Bonds	Discounted LGD[3]				52.70%	38.28%				20.48%	29.65%				48.68%	38.80%
		Time-to-Resolution[4]	2712			1.7029	1.3458	386			0.2371	0.5783	3098			1.5203	1.3644
		Principal at Default[5]				144,150	218,187				199,686	343,228				151,069	238,020
	Revolvers	Discounted LGD[3]				17.94%	28.73%				1.89%	7.32%				15.72%	27.37%
		Time-to-Resolution[4]	679			1.3906	1.1507	109			0.1578	0.5322	788			1.2200	1.1666
		Principal at Default[5]				132,497	286,104				87,536	177,706				126,278	274,032
	Loans	Discounted LGD[3]				21.74%	31.47%				3.35%	11.52%				19.32%	30.26%
		Time-to-Resolution[4]	1288			1.4130	1.2132	196			0.1884	0.5473	1484			1.2513	1.2202
		Principal at Default[5]				123,620	229,593				109,288	291,125				121,727	238,566
	Total	Discounted LGD[3]				45.74%	39.12%				16.39%	27.50%				42.00%	39.08%
		Time-to-Resolution[4]	3391			1.6404	1.3149	495			0.2196	0.5689	3886			1.4594	1.3320
		Principal at Default[5]				141,816	233,374				126,278	274,032				146,042	245,914

1 - "Return on Defualted Debt" : annnualized simple rate of return on defaulted debt from just after the time of default (1st trading date of debt) until the time of ultimate resolution.
2 - "Most Likely Discount Rate: maximum likelihood discount rate estimate that minimizes averages pricing error between price at default and ultimate resolution.
3 - One miunus the price of defaulted debt at the time of default.
4 - The ultimate dollar loss-given-default on the defaulted debt instrument = 1 - (total recovery at emergence from bankruptcy or time of final settlement)/(outstanding at default).
Alternatively, this can be expressed as (outstanding at default - total ultimate loss)/(outstanding at default)
5 - The total instrument outstanding at default.
6 - The time in years from the instrument default date to the time of ultimate recovery.

Table 1.1 - RDD[1] and MLDR[2] Observations by Default and Instrument Type (Moody's Ultimate LGD Database 1987-2007)

	Bankruptcy							Out-of-Court							Total						
		MLDR		RDD					MLDR		RDD					MLDR		RDD			
	Count	MLE Est.	MLE Std Err	Average	Standard Deviation	Minimum	Maximum	Count	MLE Est.	MLE Std Err	Average	Standard Deviation	Minimum	Maximum	Count	MLE Est.	MLE Std Err	Average	Standard Deviation	Minimum	Maximum
Bonds and Term Loans	1121	21.74%	128.51%	28.86%	121.85%	-100.00%	893.76%	73	52.54%	104.03%	38.86%	135.78%	-91.87%	846.73%	1194	21.86%	120.83%	29.48%	122.71%	-100.00%	893.76%
Bonds	888	23.88%	162.22%	23.27%	119.93%	-100.00%	893.76%	71	52.54%	104.03%	38.86%	135.78%	-91.87%	846.73%	959	24.02%	150.44%	24.50%	121.32%	-100.00%	893.76%
Revolvers	141	15.24%	3.57%	31.95%	58.58%	-100.00%	340.51%	3	N/A	N/A	0.01%	4.04%	0.00%	-0.03%	144	15.24%	3.57%	31.28%	58.14%	-100.00%	340.51%
Loans	374	14.45%	32.83%	43.31%	106.72%	-100.00%	853.84%	5	N/A	N/A	0.01%	0.01%	0.00%	0.03%	379	14.45%	32.83%	42.74%	106.13%	-100.00%	853.84%
Total	1262	21.31%	114.15%	29.21%	116.49%	-100.00%	893.76%	76	52.54%	104.03%	37.33%	133.25%	-91.87%	846.73%	1338	22.38%	107.83%	29.67%	117.46%	-100.00%	893.76%

1 - "Return on Defualted Debt" : annnualized simple rate of return on defaulted debt from just after the time of default (1st trading date of debt) until the time of ultimate resolution.
2 - "Most Likely Discount Rate: maximum likelihood discount rate estimate that minimizes averages pricing error between price at default and ultimate resolution.

Table 2 - RDD[1] and MLDR[2] by Seniority Ranks and Collateral Types (Moody's Ultimate LGD Database 1987-2007)

		Revolving Credit / Term Loan					Senior Secured Bonds					Senior Unsecured Bonds					Senior Subordinated Bonds					Subordinated Bonds					Total Instrument				
	Collateral Type	Cnt	Avg of RDD	Std Dev of RDD	MLE Est MLDR	MLE Std Err of MLRD	Cnt	Avg of RDD	Std Dev of RDD	MLE Est MLDR	MLE Std Err of MLRD	Cnt	Avg of RDD	Std Dev of RDD	MLE Est MLDR	MLE Std Err of MLRD	Cnt	Avg of RDD	Std Dev of RDD	MLE Est MLDR	MLE Std Err of MLRD	Cnt	Avg of RDD	Std Dev of RDD	MLE Est MLDR	MLE Std Err of MLRD	Cnt	Avg of RDD	Std Dev of RDD	MLE Est MLDR	MLE Std Err of MLRD
Minor Collateral Category	Guarantees	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A
	Oil and Gas Properties	1	34.3%	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	1	34.3%	N/A	N/A	N/A
	Inventory and Accounts Receivable	3	20.3%	1.3%	19.9%	0.6%	1	-27.6%	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	4	8.3%	24.0%	N/A	N/A
	Accounts Receivable	1	97.5%	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	1	97.5%	N/A	N/A	N/A
	Cash	1	29.3%	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	1	29.3%	N/A	N/A	N/A
	Inventory	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A
	Most Assets	3	72.1%	0.0%	72.1%	0.0%	1	78.1%	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	1	72.1%	N/A	N/A	N/A	5	73.3%	2.7%	77.2%	0.7%
	Equipment	0	N/A	N/A	N/A	N/A	14	16.1%	21.1%	15.9%	8.1%	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	14	16.1%	21.1%	15.9%	8.1%
	All Assets	288	41.8%	113.9%	16.4%	3.7%	30	58.3%	163.3%	57.3%	4793.7%	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	318	43.4%	119.2%	24.1%	452.3%
	Real Estate	2	42.3%	0.7%	42.3%	0.3%	3	48.1%	114.2%	19.8%	39.0%	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	5	45.8%	80.8%	20.7%	24.1%
	All Non-current Assets	3	46.8%	44.6%	13.0%	5.1%	6	117.8%	225.0%	46.7%	43.5%	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	9	94.1%	182.7%	41.9%	29.3%
	Capital Stock	34	40.9%	58.9%	20.7%	13.8%	41	67.4%	111.8%	36.0%	15.5%	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	75	55.4%	92.1%	34.0%	9.7%
	PP&E	4	68.5%	82.3%	21.0%	18.4%	21	43.0%	102.7%	41.6%	56.7%	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	25	47.1%	98.6%	40.1%	17.9%
	Second Lien	11	105.6%	161.2%	19.7%	20.6%	16	41.5%	51.3%	31.3%	14.3%	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	1	-34.2%	N/A	N/A	N/A	28	64.0%	111.5%	27.1%	11.7%
	Other	0	N/A	N/A	N/A	N/A	1	-7.6%	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	1	-7.6%	N/A	N/A	N/A
	Unsecured	23	28.9%	55.3%	19.7%	14.1%	3	5.2%	19.1%	9.8%	9.2%	432	22.8%	105.0%	21.1%	22.2%	178	21.5%	155.9%	22.9%	16.4%	126	-5.5%	105.5%	22.9%	18.5%	762	17.9%	118.0%	18.3%	10.8%
	Third Lien	0	N/A	N/A	N/A	N/A	3	-26.2%	40.1%	0.0%	24.2%	5	0.3%	25.0%	9.2%	17.5%	1	439.4%	N/A	N/A	N/A	2	0.7%	28.4%	9.5%	14.6%	11	33.1%	137.7%	11.8%	27.9%
	Intellectual Property	0	N/A	N/A	N/A	N/A	2	28.6%	62.0%	23.2%	40.2%	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	2	28.6%	62.0%	23.2%	40.2%
	Intercompany Debt	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A
Major Collateral Category	Cash, Accounts Receivables & Guarantees	6	37.0%	30.2%	37.0%	13.4%	1	-27.6%	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	7	27.8%	36.9%	33.3%	13.4%
	Inventory, Most Assets & Equipment	3	72.1%	0.0%	N/A	N/A	15	20.2%	25.9%	19.6%	9.5%	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	1	72.1%	N/A	N/A	N/A	19	31.1%	31.5%	20.6%	7.8%
	All Assets & Real Estate	290	41.8%	113.5%	16.5%	3.7%	33	57.4%	158.1%	42.0%	4358.0%	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	323	43.4%	118.6%	30.4%	44.5%
	Non-Current Assets & Capital Stock	37	41.4%	57.4%	20.7%	12.7%	47	73.9%	129.1%	36.1%	14.6%	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	84	59.6%	104.5%	34.0%	9.2%
	PPE & Second Lien	15	95.7%	142.4%	20.1%	15.9%	38	41.1%	82.7%	36.6%	33.2%	0	N/A	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A	1	-34.2%	N/A	N/A	N/A	54	54.8%	104.3%	33.7%	10.4%
	Unsecured & Other Illiquid Collateral	23	28.9%	55.3%	9.9%	9.5%	8	-0.7%	40.6%	2.5%	15.1%	437	22.5%	104.5%	20.9%	22.0%	179	23.9%	158.6%	23.4%	16.4%	128	-5.4%	104.7%	16.7%	18.1%	775	18.2%	118.1%	18.2%	10.6%
	Total Secured	351	44.3%	109.2%	17.9%	3.4%	139	51.6%	117.5%	36.5%	1034.6%	5	0.3%	25.0%	21.1%	17.6%	1	439.4%	N/A	100.0%	210.3%	4	9.8%	47.6%	16.5%	12.8%	500	46.4%	118.0%	31.7%	28.8%
	Total Unsecured	23	28.9%	55.3%	9.8%	9.5%	3	5.2%	19.1%	9.8%	9.2%	432	22.8%	105.0%	9.2%	17.5%	178	21.5%	155.9%	21.3%	13.6%	126	-5.5%	105.5%	4.7%	12.8%	762	17.9%	112.0%	18.3%	10.8%
	Total Collateral	374	43.3%	106.7%	14.5%	32.8%	142	50.7%	116.4%	38.4%	101.3%	437	22.5%	104.5%	20.9%	17.4%	179	23.9%	158.6%	21.9%	13.7%	130	-5.0%	104.1%	16.5%	15.1%	1262	29.2%	116.5%	21.3%	114.2%

1 - Annualized "Return on Defaulted Debt" from the time of default until the time of ultimate resolution.

2 - "Most Likely Discount Rate: maximum likelihood discount rate estimate that minimizes averages pricing error between price at default and ultimate resolution.

Table 2.1 - RDD[1] and MLDR[2] by Seniority Ranks and Major Collateral Types (Moody's Ultimate LGD Database 1987-2007)

	Collateral Type	Cash, Accounts Receivables & Guarantees	Inventory, Most Assets & Equipment	All Assets & Real Estate	Non-Current Assets & Capital Stock	PPE & Second Lien	Unsecured & Other Illiquid Collateral	Total Secured	Total Unsecured	Total Collateral
Revolving Credit / Term Loan	Cnt of RDD	6	3	290	37	15	23	351	23	374
	Avg of RDD	37.0%	72.1%	41.8%	41.4%	95.7%	28.9%	44.3%	28.9%	43.3%
	Std Dev of RDD	30.2%	0.0%	113.5%	57.4%	142.4%	55.3%	109.2%	55.3%	106.7%
	MLE Est of MLDR	37.0%	N/A	16.5%	20.7%	20.1%	9.9%	17.9%	9.9%	14.5%
	MLE Std Err of MLDR	13.4%	N/A	3.7%	12.7%	15.9%	9.5%	3.4%	9.5%	32.8%
Senior Secured Bonds	Cnt of RDD	1	15	33	47	38	8	139	3	142
	Avg of RDD	-27.6%	20.2%	57.4%	73.9%	41.1%	-0.7%	51.6%	5.2%	50.7%
	Std Dev of RDD	N/A	25.9%	158.1%	129.1%	82.7%	40.6%	117.5%	19.1%	116.4%
	MLE Est of MLDR	N/A	19.6%	42.0%	36.1%	36.6%	2.5%	36.5%	9.8%	38.4%
	MLE Std Err of MLDR	N/A	9.5%	4358.0%	14.6%	33.2%	15.1%	1034.6%	9.2%	101.3%
Senior Unsecured Bonds	Cnt of RDD	0	0	0	0	0	437	5	432	437
	Avg of RDD	N/A	N/A	N/A	N/A	N/A	22.5%	0.3%	22.8%	22.5%
	Std Dev of RDD	N/A	N/A	N/A	N/A	N/A	104.5%	25.0%	105.0%	104.5%
	MLE Est of MLDR	N/A	N/A	N/A	N/A	N/A	20.9%	21.1%	9.2%	20.9%
	MLE Std Err of MLDR	N/A	N/A	N/A	N/A	N/A	22.0%	17.6%	17.5%	17.4%
Senior Subordinated Bonds	Cnt of RDD	0	0	0	0	0	179	1	178	179
	Avg of RDD	N/A	N/A	N/A	N/A	N/A	23.9%	439.4%	21.5%	23.9%
	Std Dev of RDD	N/A	N/A	N/A	N/A	N/A	158.6%	N/A	155.9%	158.6%
	MLE Est of MLDR	N/A	N/A	N/A	N/A	N/A	23.4%	100.0%	21.3%	21.9%
	MLE Std Err of MLDR	N/A	N/A	N/A	N/A	N/A	16.4%	210.3%	13.6%	13.7%
Subordinated Bonds	Cnt of RDD	0	1	0	0	1	128	4	126	130
	Avg of RDD	N/A	72.1%	N/A	N/A	-34.2%	-5.4%	9.8%	-5.5%	-5.0%
	Std Dev of RDD	N/A	N/A	N/A	N/A	N/A	104.7%	47.6%	105.5%	104.1%
	MLE Est of MLDR	N/A	N/A	N/A	N/A	N/A	16.7%	16.5%	4.7%	16.5%
	MLE Std Err of MLDR	N/A	N/A	N/A	N/A	N/A	18.1%	12.8%	12.8%	15.1%
Total Instrument	Cnt of RDD	7	19	323	84	54	775	500	762	1262
	Avg of RDD	27.8%	31.1%	43.4%	59.6%	54.8%	18.2%	46.4%	17.9%	29.2%
	Std Dev of RDD	36.9%	31.5%	118.6%	104.5%	104.3%	118.1%	118.0%	112.0%	116.5%
	MLE Est of MLDR	33.3%	20.6%	30.4%	34.0%	33.7%	18.2%	31.7%	18.3%	21.3%
	MLE Std Err of MLDR	13.4%	7.8%	44.5%	9.2%	10.4%	10.6%	28.8%	10.8%	114.2%

1 - "Annualized Return on Defaulted Debt" from the time of default until the time of ultimate resolution.

2 - Most "Likely Discount Rate": maximum likelihood discount rate estimate that minimizes averages pricing error between price at default and ultimate resolution.

Table 3 - RDD[1], MLDR[2], LGD[3], Default Rate[4], Dollar Loss[5] and Duration[6] of Defaulted Instruments by Cohort Year (Moody's Ultimate LGD Database 1987-2007)

Year	Count of RDD	Average of RDD	Std Dev of RDD	MLE Est of MLDR	MLE Std Err of MLDR	Average LGD at Default[2]	Average of Moody's Speculative Grade Default Rate	Average of Total Defaulted Amount ($MM)[4]	Average of Time-to-Resolution (Yrs.)[5]
1987	5	-5.03%	10.81%	-2.90%	202.18%	70.08%	4.71%	3,803	2.0379
1988	11	-1.34%	50.08%	9.07%	12.37%	63.80%	3.32%	3,697	2.0123
1989	14	29.07%	108.33%	35.47%	29.34%	74.55%	4.80%	7,915	2.2638
1990	64	46.40%	143.41%	17.58%	91.36%	75.63%	10.36%	26,148	1.6991
1991	92	43.75%	117.69%	39.44%	35.02%	57.15%	9.85%	25,252	1.6074
1992	27	32.54%	172.26%	5.33%	15.73%	58.89%	6.13%	6,340	1.7309
1993	10	1.32%	91.31%	13.38%	31.14%	51.10%	3.02%	3,912	1.1740
1994	6	-27.64%	40.67%	12.11%	20.54%	70.57%	2.42%	3,926	0.9574
1995	35	27.45%	107.80%	11.27%	26.85%	41.96%	3.19%	8,966	2.0764
1996	25	27.43%	73.00%	20.42%	19.46%	44.32%	2.18%	5,223	1.3572
1997	17	9.14%	61.15%	-1.27%	12.81%	46.65%	2.27%	4,386	1.3159
1998	33	-37.79%	49.62%	-25.43%	9.69%	57.05%	3.77%	8,837	1.3471
1999	92	6.83%	53.26%	10.74%	6.53%	65.01%	6.12%	28,296	1.3468
2000	105	-3.95%	67.96%	7.59%	60.25%	62.63%	7.93%	34,383	1.7394
2001	257	16.15%	101.98%	24.59%	10.97%	58.25%	11.39%	96,929	1.7314
2002	207	50.77%	147.80%	32.83%	17.41%	61.61%	7.89%	183,801	1.3021
2003	106	64.24%	161.24%	46.20%	15.92%	49.34%	5.83%	43,151	0.9826
2004	75	46.60%	88.50%	15.73%	8.63%	33.12%	3.31%	22,863	0.7811
2005	63	42.59%	118.00%	50.84%	53.22%	35.58%	2.33%	43,461	1.1160
2006	9	29.65%	88.81%	-6.25%	20.82%	32.50%	1.73%	2,355	0.5652
2007	9	9.98%	46.68%	-31.19%	8.24%	18.97%	1.26%	3,388	0.2189
Total	1,262	29.21%	116.49%	22.38%	107.83%	55.78%	7.14%	567,520	1.4594

1 - Annualized "Return on Defaulted Debt" from just after the time of default (1st trading date of debt) until the time of ultimate resolution.

2 - "Most Likely Discount Rate": maximum likelihood discount rate estimate that minimizes averages pricing error between price at default and ultimate resolution.

3 - One minus the price of defaulted debt at the time of default.

4 - Proportion of Moody's speculative rated companies defaulted during the year relative to those rated at the begining of the year.

5 - The total instrument or obligor outstanding at default.

6 - The time in years from the instrument or firm default date to the time of ultimate recovery.

Table 4 - RDD[1] and MLDR[2] of Defaulted Instruments by Quintiles of Time-to-Resolution[3] and Time-in-Distress[4] from Last Cash Pay to Default Date (Moody's Ultimate LGD Database 1987-2007)

			Quintiles of Time from Default to Resolution Date											
			1		2		3		4		5		Total	
			Avg. / MLE Est.	Std. Dev. / Std Err	Avg. / MLE Est.	Std. Dev. / Std Err	Avg. / MLE Est.	Std. Dev. / Std Err	Avg. / MLE Est.	Std. Dev. / Std Err	Avg. / MLE Est.	Std. Dev. / Std Err	Avg. / MLE Est.	Std. Dev. / Std Err
Quintiles of Time from Last Cash Pay to Default Date	1	RDD	97.22%	188.87%	74.94%	223.36%	16.03%	75.64%	32.95%	106.44%	-14.35%	45.77%	41.29%	145.59%
		MLDR	99.48%	23.93%	35.72%	5.42%	38.07%	4792.95%	15.23%	9.54%	0.96%	198.63%	30.79%	24.65%
	2	RDD	66.61%	95.13%	35.97%	136.49%	16.30%	89.65%	11.57%	54.76%	17.71%	36.03%	27.61%	94.49%
		MLDR	12.16%	19.73%	36.64%	15.14%	5.52%	6.48%	28.70%	13.10%	3.67%	10.77%	16.71%	6.63%
	3	RDD	53.22%	126.53%	18.64%	63.92%	45.01%	121.25%	37.67%	105.23%	12.36%	34.48%	30.98%	93.11%
		MLDR	30.13%	16.93%	1.55%	11.17%	58.58%	28.64%	8.37%	8.29%	11.05%	12.18%	25.94%	53.47%
	4	RDD	53.67%	167.12%	70.95%	212.53%	6.34%	74.61%	29.25%	68.66%	12.43%	45.65%	29.81%	120.09%
		MLDR	89.77%	76.07%	21.92%	14.81%	31.60%	59.62%	40.13%	14.19%	36.04%	26.79%	24.20%	5.52%
	5	RDD	97.77%	269.07%	20.44%	118.14%	10.03%	85.60%	14.74%	85.77%	-14.31%	55.86%	21.28%	132.97%
		MLDR	1.41%	14.48%	16.38%	18.71%	23.27%	11.53%	22.89%	11.41%	8.77%	15.18%	13.34%	26.89%
	Total	RDD	70.83%	178.49%	41.43%	156.65%	17.80%	91.31%	26.36%	84.70%	7.51%	45.03%	29.21%	116.49%
		MLDR	23.56%	96.60%	16.80%	5.74%	21.43%	7.57%	42.92%	18.27%	18.50%	6.67%	21.31%	114.15%

1 - Annualized "Return on Defaulted Debt" from just after the time of default (1st trading date of debt) until the time of ultimate resolution.
2 - "Most Likely Discount Rate": maximum likelihood discount rate estimate that minimizes averages pricing error between price at default and ultimate resolution.
3 - TTR: Duration in years from the date of default (bankruptcy filing or other default) to the date of resolution (emergence from bankruptcy or other settllement).
4 - TID: Duration in years from the date of the last interest payment to the date of default (bankruptcy filing or other default).

Table 5 - RDD[1] and MLDR[2] of Defaulted Instruments by Credit Rating at Origination (Moody's Ultimate LGD Database 1987-2007)

		Count	Average of RDD	Standard Deviation RDD	MLE Estimate of MLDR	MLE Standard Error of MLDR
Rating Groups	AA-A	130	26.43%	63.36%	20.67%	28.16%
	BBB	58	48.62%	110.94%	111.61%	51.45%
	BB	299	18.10%	91.67%	22.13%	7.18%
	B	497	32.06%	140.92%	13.24%	13.68%
	CC-CCC	89	19.58%	78.55%	18.30%	7.93%
	Investment Grade (BBB-A)	188	33.28%	80.75%	23.70%	25.89%
	Junk Grade (CC-BB)	885	26.09%	212.83%	18.48%	8.10%
	Total	1262	29.21%	116.49%	21.31%	114.15%

1 - Annnualized "Return on Defaulted Debt" from just after the time of default (1st trading date of debt) until the time of ultimate resolution.

2 - "Most Likely Discount Rate": maximum likelihood discount rate estimate that minimizes averages pricing error between price at default and ultimate resolution.

Table 6 - RDD[1] and MLDR[2] of Defaulted Instruments by Tranche Safety Index[3] (TSI) Quintiles and Categories (Moody's Ultimate LGD Database 1987-2007)

		Count	Average RDD	Standard Deviation RDD	MLDR	MLE Std Err MLDR
Debt Tranche Groups	1st Quintile TSI	154	33.03%	162.52%	33.84%	28.96%
	2nd Quintile TSI	324	9.55%	97.95%	21.29%	20.71%
	3rd Quintile TSI	372	26.55%	109.95%	14.38%	38.66%
	4th Quintile TSI	326	43.63%	116.28%	25.05%	3.22%
	5th Quintile TSI	86	53.23%	99.63%	29.62%	32.06%
	NDA / SDB[4]	427	44.16%	103.17%	25.29%	6.87%
	SDA / SDB[5]	232	25.08%	124.31%	35.39%	8.46%
	NDA / NDB[6]	154	26.30%	122.01%	12.03%	93.37%
	NDB / SDA[7]	449	18.12%	121.14%	19.72%	17.73%
	Total	1262	29.21%	116.49%	21.31%	114.15%

1 - Annnualized "Return on Defaulted Debt" from just after the time of default (1st trading date of debt) until the time of ultimate resolution.

2 - "Most Likely Discount Rate": maximum likelihood discount rate estimate that minimizes averages pricing error between price at default and ultimate resolution.

3 - An index of the tranche safety calculated as TTS = (% Debt Below - % Debt Above + 1) /

4 - No Debt Above & Some Debt Below

5 - Some Debt Above & Some Debt Below

6 - o Debt Above & No Debt Below

7 - No Debt Below & Some Debt Above

Table 7 - RDD[1] and MLDR[2] of Defaulted Instruments by Industry (Moody's Ultimate LGD Database 1987-2007)

		RDD		MLDR	
Industry Group	Count	Average	Standard Deviation	MLE Estimate	MLE Standard Error
Aerospace / Auto / Capital Goods / Equipment	156	41.30%	122.21%	22.29%	22.80%
Consumer / Service Sector	235	34.27%	121.28%	28.84%	61.19%
Energy / Natural Resources	183	26.56%	55.42%	20.35%	34.96%
Healthcare / Chemicals	93	24.03%	90.98%	17.99%	10.77%
High Technology / Telecommunications	225	40.98%	159.31%	17.92%	6.94%
Leisure Time / Media	114	36.58%	114.81%	26.42%	13.25%
Transportation	236	6.02%	99.95%	5.76%	16.04%
Forest / Building Products / Homebuilders	20	23.01%	128.62%	34.94%	21.06%
Grand Total	1,262	29.21%	116.49%	22.38%	107.83%

1 - Annnualized "Return on Defaulted Debt" from just after the time of default (1st trading date of debt) until the time of ultimate resolution.

2 - "Most Likely Discount Rate": maximum likelihood discount rate estimate that minimizes averages pricing error between price at default and ultimate resolution.

Tabble 8 - Summary Statistics on Selected Variables and Correlations with RDD (Moody's Ultimate LGD Database 1987-2007)

Category	Variable	Count	Minimum	1st Percentile	25th Percentile	Median	Mean	75th Percentile	99th Percentile	Maximum	Standard Deviation	Skewness	Kurtosis	Correlation with RDD	P-Value of Correlation
Financial Statement and Market Valuation	Book Value Total Liabilities / Book Value Total Assets	1111	42.00%	51.10%	87.00%	117.00%	138.89%	152.00%	380.00%	380.00%	73.94%	1.5817	1.7579	17.18%	2.95E-04
	Book Value Total Liabilities / Market Value Total Assets	1111	25.00%	38.00%	89.00%	97.00%	91.23%	98.00%	99.00%	99.00%	12.02%	-2.6681	8.6235	-1.23%	1.53E-03
	Market Value of Total Assets	790	0.0000	0.6418	1.6912	2.0913	2.0691	2.4992	3.9333	4.0194	0.9462	-0.7762	2.3489	-8.15%	6.85E-04
	Net Sales	980	0.0000	0.5586	2.3572	2.8596	2.8035	3.2974	4.5685	4.5685	0.6744	-0.8539	4.0622	-2.69%	4.67E-04
	Book Value of Assets	983	0.3604	1.6386	2.7249	3.0659	3.0362	3.3615	5.0167	5.0167	0.5958	-0.3530	2.8015	-1.83%	3.48E-05
	Tobin's Q	735	0.05%	18.35%	57.57%	84.01%	100.69%	145.64%	281.20%	357.23%	61.21%	1.0144	0.9860	-0.12%	5.74E-04
	Market-to-Book (Market Value Assets / Book Value Assets)	1111	46.00%	65.00%	96.00%	127.00%	153.82%	175.50%	407.00%	549.00%	83.03%	1.7253	3.2514	18.50%	7.80E-04
	Intangibes Ratio (Book Value Intangibles / Book Value Assets)	773	0.00%	0.00%	0.00%	18.34%	21.02%	32.82%	87.85%	87.85%	20.98%	0.9874	0.6403	2.00%	6.18E-04
	Price / Earnings Ratio	791	-119.3577	-43.4082	-3.4833	-0.3084	-2.2438	-0.0622	25.8284	135.9592	14.8584	0.4390	26.8715	4.02%	1.14E-03
	Current Ratio	911	4.31%	9.14%	77.03%	113.41%	133.57%	196.70%	342.39%	434.65%	84.43%	0.6315	-0.2678	2.33%	3.41E-03
	Interest Coverage Ratio	982	-55.1870	-17.0393	-1.8780	0.0122	-1.4338	0.7895	2.1393	4.3066	4.5557	-5.7867	50.2722	-6.57%	1.68E-03
	Working Capital / Book Value of Total Assets	914	-210.85%	-138.47%	-7.60%	2.86%	-5.88%	15.67%	44.48%	50.35%	38.10%	-2.0045	4.0675	2.87%	7.59E-04
	Cash Flow to Current Liabilities	902	-374.91%	-374.91%	-22.10%	-0.06%	-31.79%	15.06%	110.70%	371.33%	122.08%	-1.7075	4.2163	-2.98%	7.93E-04
	Free Asset Ratio	881	-95.51%	-68.64%	-12.08%	13.30%	9.27%	34.48%	90.05%	95.86%	35.02%	-0.1009	-0.4035	-12.56%	3.41E-03
	Free Cash Flow / Book Value of Total Assets	946	-107.64%	-54.78%	-14.27%	0.06%	-8.02%	3.42%	26.52%	34.61%	20.07%	-1.5929	2.4374	-4.03%	1.05E-03
	Cash Flow from Operations / Book Value of Total Assets	954	(669.12)	(669.12)	(45.58)	0.47	102.76	43.56	7,778.00	7,778.00	988.72	5.9960	41.2103	1.26%	3.25E-04
	Net Income / Book Value of Total Assets	1111	-195.00%	-173.00%	-21.00%	-8.00%	-20.71%	-2.00%	2.00%	5.00%	38.64%	-2.8853	7.4787	-2.85%	6.82E-04
	Net Income / Market Value of Total Assets	1111	-85.00%	-82.00%	-15.00%	-4.00%	-11.67%	-1.00%	2.00%	5.00%	17.93%	-2.2659	4.5726	0.21%	1.33E-03
	Retained Earnings / Book Value of Total Assets	971	-757.97%	-570.38%	-88.67%	-22.57%	-55.95%	-1.25%	27.46%	56.32%	97.26%	-3.5022	16.4845	-6.69%	1.72E-03
	Return on Assets	971	-159.12%	-115.79%	-28.81%	-7.08%	-19.29%	-1.45%	11.31%	36.35%	27.95%	-1.6529	3.0797	-8.13%	2.09E-03
	Return on Equity	971	-2950.79%	-2156.55%	-36.99%	-1.85%	17.41%	47.65%	2779.89%	6492.67%	568.41%	3.3165	47.5613	-2.78%	7.13E-04
Equity Price Performance	1-Year Expected Return on Equity	1111	-132.00%	-126.00%	-100.00%	-81.00%	-73.93%	-66.00%	161.00%	161.00%	41.35%	3.0805	13.4970	-6.36%	1.53E-03
	1-Month Equity Price Volatility	1111	13.00%	42.00%	159.00%	206.00%	262.88%	293.00%	599.00%	6116.00%	410.89%	13.0467	183.4988	2.31%	5.53E-04
	Relative Size (Market Cap of Firm to the Market)	1111	-17.3400	-16.4700	-14.4900	-12.7400	-13.1512	-12.1200	-8.2500	-7.5000	1.9645	0.1415	0.0873	-8.61%	2.07E-03
	Relative Stock Price (Percentile Ranking to Market)	1111	0.47%	1.00%	5.50%	9.00%	13.04%	15.00%	68.00%	81.00%	14.16%	2.7860	8.5318	-5.40%	1.30E-03
	Stock Price Trading Range (Ratio of Current to 3 Yr High/Low)	1111	0.00%	0.00%	0.07%	0.54%	3.01%	3.00%	33.00%	88.00%	8.56%	6.4581	51.7666	-2.78%	6.66E-04
	Cumulative Abnormal Returns (90 Days to Default)	1111	-127.70%	-126.85%	0.00%	0.00%	-5.13%	0.00%	101.24%	147.14%	29.42%	-0.8873	8.8981	10.90%	2.63E-03
Capital Structure	Number of Instruments	3886	1.0000	1.0000	4.0000	6.0000	10.5252	10.0000	61.0000	80.0000	12.8458	2.4687	6.0337	-4.01%	5.14E-04
	Number of Creditor Classes	3886	1.0000	1.0000	2.0000	2.0000	2.5980	3.0000	6.0000	7.0000	1.1071	0.9869	1.2531	-3.17%	4.06E-04
	Percent Secured Debt	3886	0.00%	0.00%	13.79%	42.22%	43.43%	68.63%	100.00%	100.00%	32.63%	25.64%	-106.57%	9.21%	1.18E-03
	Percent Bank Debt	3886	0.00%	0.00%	13.19%	39.92%	40.78%	62.40%	100.00%	100.00%	30.84%	31.43%	-93.01%	7.32%	9.40E-04
	Percent Subordinated Debt	3886	0.00%	0.00%	11.25%	38.81%	40.34%	62.62%	100.00%	100.00%	31.23%	36.34%	-97.31%	5.60%	7.18E-04
Credit Quality / Credit Market	Altman Z-Score	733	-8.5422	-7.1053	-1.4286	0.5266	-0.1010	1.1896	3.6821	4.6276	2.2087	-0.9280	1.0222	-11.23%	3.33E-03
	Credit Spread	1262	0.00%	0.50%	4.06%	8.70%	8.11%	11.13%	15.60%	17.50%	3.95%	-0.1380	-1.0346	-5.66%	1.27E-03
	Contractual Coupon Rate	3886	0.00%	0.00%	6.62%	9.00%	8.64%	11.04%	16.79%	30.00%	3.89%	-0.3625	0.5296	-5.76%	7.39E-04
	LGD at Default	1375	-8.50%	-1.38%	30.00%	60.00%	55.78%	84.25%	99.40%	99.87%	31.28%	-0.2863	-1.2174	6.88%	1.48E-04
	Moody's Original Credit Rating Investment Grade Dummy	3178	0.0000	0.0000	0.0000	0.0000	0.1954	0.0000	1.0000	1.0000	0.3966	1.5371	0.3629	2.35%	3.33E-04
	Moody's Original Credit Rating (Major Code)	3178	1.0000	1.0000	3.0000	4.0000	3.3106	4.0000	5.0000	5.0000	1.0784	-0.8063	-0.0949	-0.03%	3.73E-06
	Moody's Original Credit Rating (Minor Code)	3178	3.0000	3.0000	10.0000	14.0000	12.5296	15.0000	20.0000	20.0000	3.4435	-0.5798	-0.1107	1.92%	2.72E-04
	Moody's Long Run Default Rate (Minor Code)	3178	0.0024%	0.0200%	0.3083%	2.4859%	3.3688%	4.1478%	29.0960%	29.0960%	4.6052%	2.7195	10.0277	0.29%	4.13E-05
Instrument / Contractual	Seniority Rank	3886	1.0000	1.0000	1.0000	1.0000	1.7123	2.0000	5.0000	7.0000	0.8953	1.4491	2.4882	-9.64%	1.24E-03
	Collateral Rank	3886	1.0000	1.0000	3.0000	6.0000	4.5844	6.0000	6.0000	6.0000	1.6206	-0.5951	-1.0651	-9.97%	1.28E-03
	Percent Debt Below	3886	0.00%	0.00%	0.00%	10.13%	25.82%	49.87%	95.63%	100.00%	30.19%	0.8172	-0.7078	10.51%	1.35E-03
	Percent Debt Above	3886	0.00%	0.00%	0.00%	0.00%	21.51%	40.92%	96.57%	100.00%	28.95%	1.1209	-0.0418	-6.51%	8.35E-04
	Tranche Safety Index	3886	0.00%	1.77%	32.73%	50.00%	52.16%	72.82%	97.81%	100.00%	25.44%	-0.0908	-0.8873	9.69%	1.25E-03
Macro / Cyclical	Moody's All-Corporate Quarterly Default Rate	1262	0.00%	1.31%	4.89%	7.05%	7.38%	9.85%	13.26%	13.26%	3.28%	-14.34%	-0.9222	5.72%	1.29E-03
	Moody's Speculative Quarterly Default Rate	1262	1.31%	1.31%	4.89%	7.05%	7.40%	9.85%	13.26%	13.26%	3.24%	-9.80%	-1.0016	5.43%	1.22E-03
	Moody's All-Corporate Quarterly Default Rate by Industry	1262	0.00%	0.00%	2.07%	3.78%	4.13%	6.05%	10.94%	12.68%	2.70%	64.39%	-0.1938	7.40%	1.67E-03
	Moody's Speculative Quarterly Default Rate by Industry	1262	0.00%	0.00%	3.49%	6.52%	7.03%	9.80%	17.50%	17.50%	4.19%	45.87%	-0.5255	6.66%	1.50E-03
	Fama-French Excess Return on Market Factor	3886	-1076.00%	-1032.00%	-241.00%	86.00%	33.35%	355.00%	818.00%	1030.00%	464.59%	-33.72%	-0.4057	-0.07%	9.16E-06
	Fama-French Relative Return on Small Stocks Factor	3886	-2218.00%	-1373.00%	-214.00%	31.00%	13.66%	270.00%	697.00%	843.00%	394.25%	-129.03%	5.3616	2.37%	3.03E-04
	Fama-French Excess Return on Value Stock Factor	3886	-912.00%	-701.00%	-155.00%	64.00%	81.98%	269.00%	1280.00%	1380.00%	373.74%	73.80%	1.7726	-3.62%	4.64E-04
	Short-Term Interest Rates (1-Month Treasury Yields)	1262	6.00%	7.00%	14.00%	32.00%	31.82%	44.00%	69.00%	79.00%	16.83%	11.14%	-1.0411	-10.41%	2.35E-03
	Long-Term Interest Rates (10-Month Treasury Yields)	1111	337.00%	371.00%	451.00%	535.00%	548.19%	603.00%	904.00%	904.00%	125.45%	105.57%	0.7194	-6.69%	1.61E-03
	Stock-Market Volatility (2-Year IDX)	1111	3.00%	4.10%	7.00%	9.00%	9.98%	11.00%	19.00%	19.00%	3.84%	94.25%	-0.0862	-0.36%	8.55E-05
Durations / Vintage	Time from Origination to Default	3365	0.2500	0.2500	1.6384	2.8849	4.0128	5.0027	19.6559	29.9534	3.7660	2.3676	7.4186	-0.67%	9.18E-05
	Time from First Rating to Default	3178	1.0000	1.0000	3.1288	5.7425	10.2523	14.5753	56.1260	56.9781	11.3994	2.1219	4.7533	-0.51%	7.23E-05
	Time from Last Cash-Pay Date to Default	3886	0.0000	0.0000	0.0986	0.2411	0.3907	0.4959	2.5049	4.3808	0.4849	2.7920	10.5645	0.22%	2.80E-05
	Time from Default to Resolution	3886	0.0027	0.0027	0.5507	1.1685	1.4594	1.9534	6.0740	9.3151	1.3320	1.8273	5.2410	-13.72%	1.77E-03
	Time from Origination to Maturity Date	3365	0.1000	0.1000	5.0027	7.8219	8.9032	10.0137	30.0219	50.0329	6.5084	1.6668	3.5430	-1.31%	1.80E-04

Table 8.1 - Summary Statistics on Financial Statement and Market Valuation Variables and Correlations with RDD (Moody's Ultimate LGD Database 1987-2007)

Variable	Cnt	Minimum	25th Prcntle	Median	Mean	75th Prcntle	Maximum	Std Dev	Skew	Kurt	Corr with RDD	P-Val of Corr
BVTL / BVTA	1111	42.00%	87.00%	117.00%	138.89%	152.00%	380.00%	73.94%	1.5817	1.7579	17.18%	2.95E-04
BVTL / MVTA	1111	25.00%	89.00%	97.00%	91.23%	98.00%	99.00%	12.02%	-2.6681	8.6235	-1.23%	1.53E-03
MVTA	790	0.0000	1.6912	2.0913	2.0691	2.4992	4.0194	0.9462	-0.7762	2.3489	-8.15%	6.85E-04
Net Sales	980	0.0000	2.3572	2.8596	2.8035	3.2974	4.5685	0.6744	-0.8539	4.0622	-2.69%	4.67E-04
BVA	983	0.3604	2.7249	3.0659	3.0362	3.3615	5.0167	0.5958	-0.3530	2.8015	-1.83%	3.48E-05
Tobin's Q	735	0.05%	57.57%	84.01%	100.69%	145.64%	357.23%	61.21%	1.0144	0.9860	-0.12%	5.74E-04
MVTA / BVTA	1111	46.00%	96.00%	127.00%	153.82%	175.50%	549.00%	83.03%	1.7253	3.2514	18.50%	7.80E-04
BVI / BVTA	773	0.00%	0.00%	18.34%	21.02%	32.82%	87.85%	20.98%	0.9874	0.6403	2.00%	6.18E-04
PE Ratio	791	-119.3577	-3.4833	-0.3084	-2.2438	-0.0622	135.9592	14.8584	0.4390	26.8715	4.02%	1.14E-03
CR	911	4.31%	77.03%	113.41%	133.57%	196.70%	434.65%	84.43%	0.6315	-0.2678	2.33%	3.41E-03
ICR	982	(55.19)	(1.88)	0.01	(1.43)	0.79	4.31	4.56	-5.7867	50.2722	-6.57%	1.68E-03
WC / BVTA	914	-210.85%	-7.60%	2.86%	-5.88%	15.67%	50.35%	38.10%	-2.0045	4.0675	2.87%	7.59E-04
CF / CL	902	-374.91%	-22.10%	-0.06%	-31.79%	15.06%	371.33%	122.08%	-1.7075	4.2163	-2.98%	7.93E-04
FAR	881	-95.51%	-12.08%	13.30%	9.27%	34.48%	95.86%	35.02%	-0.1009	-0.4035	-12.56%	3.41E-03
FCF / BVTA	946	-107.64%	-14.27%	0.06%	-8.02%	3.42%	34.61%	20.07%	-1.5929	2.4374	-4.03%	1.05E-03
CFO / BVTA	954	(669.12)	(45.58)	0.47	102.76	43.56	7,778.00	988.72	5.9960	41.2103	1.26%	3.25E-04
NI / BVTA	1111	-195.00%	-21.00%	-8.00%	-20.71%	-2.00%	5.00%	38.64%	-2.8853	7.4787	-2.85%	6.82E-04
NI / MVTA	1111	-85.00%	-15.00%	-4.00%	-11.67%	-1.00%	5.00%	17.93%	-2.2659	4.5726	0.21%	1.33E-03
RE / BVTA	971	-757.97%	-88.67%	-22.57%	-55.95%	-1.25%	56.32%	97.26%	-3.5022	16.4845	-6.69%	1.72E-03
ROA	971	-159.12%	-28.81%	-7.08%	-19.29%	-1.45%	36.35%	27.95%	-1.6529	3.0797	-8.13%	2.09E-03
ROE	971	-2950.79%	-36.99%	-1.85%	17.41%	47.65%	6492.67%	568.41%	3.3165	47.5613	-2.78%	7.13E-04

Table 8.2 - Summary Statistics on Equity Price Performance Variables and Correlations with RDD (Moody's Ultimate LGD Database 1987-2007)

Variable	Cnt	Minimum	1st Percentile	25th Prcntle	Median	Mean	75th Prcntle	99th Prcntle	Maximum	Std Dev	Skew	Kurt	Corr with RDD	P-Val of Corr
1-Yr Expected Equity Return	1111	-132.00%	-126.00%	-100.00%	-81.00%	-73.93%	-66.00%	161.00%	161.00%	41.35%	3.0805	13.4970	-6.36%	1.53E-03
1-Month Equity Return Volatility	1111	13.00%	42.00%	159.00%	206.00%	262.88%	293.00%	599.00%	6116.00%	410.89%	13.0467	183.4988	2.31%	5.53E-04
Market Cap to Relative to Market	1111	-17.3400	-16.4700	-14.4900	-12.7400	-13.1512	-12.1200	-8.2500	-7.5000	1.9645	0.1415	0.0873	-8.61%	2.07E-03
Stock Price to Relative to Market	1111	0.0047	0.0100	0.0550	9.00%	13.04%	15.00%	68.00%	81.00%	14.16%	2.7860	8.5318	-5.40%	1.30E-03
Stock Price Trading Range	1111	0.0000	0.0000	0.0007	0.54%	3.01%	3.00%	33.00%	88.00%	8.56%	6.4581	51.7666	-2.78%	6.66E-04
Cumulative Abnormal Returns	1111	-127.70%	-126.85%	0.00%	0.00%	-5.13%	0.00%	101.24%	147.14%	29.42%	-0.8873	8.8981	10.90%	2.63E-03

Table 8.3 - Summary Statistics on Capital Structure Variables and Correlations with RDD (Moody's Ultimate LGD Database 1987-2007)

Variable	Cnt	Min	25th Prcntle	Median	Mean	75th Prcntle	Max	Std Dev	Skew	Kurt	Corr with RDD	P-Val of Corr
Number of Instruments	3886	1.0000	4.0000	6.0000	10.5252	10.0000	80.0000	12.8458	2.4687	6.0337	-4.01%	5.14E-04
Number of Creditor Classes	3886	1.0000	2.0000	2.0000	2.5980	3.0000	7.0000	1.1071	0.9869	1.2531	-3.17%	4.06E-04
Percent Secured Debt	3886	0.00%	13.79%	42.22%	43.43%	68.63%	100.00%	32.63%	25.64%	-106.57%	9.21%	1.18E-03
Percent Bank Debt	3886	0.00%	13.19%	39.92%	40.78%	62.40%	100.00%	30.84%	31.43%	-93.01%	7.32%	9.40E-04
Percent Subordinated Debt	3886	0.00%	11.25%	38.81%	40.34%	62.62%	100.00%	31.23%	36.34%	-97.31%	5.60%	7.18E-04

Table 8.4 - Summary Statistics on Credit Quality / Credit Market Variables and Correlations with RDD (Moody's Ultimate LGD Database 1987-2007)

Variable	Cnt	Min	1st Percentile	25th Prcntle	Median	Mean	75th Prcntle	99th Prcntle	Max	Std Dev	Skew	Kurt	Corr with RDD	P-Val of Corr
Altman Z-Score	733	-8.5422	-7.1053	-1.4286	0.5266	-0.1010	1.1896	3.6821	4.6276	2.2087	-0.9280	1.0222	-11.23%	3.33E-03
Credit Spread	1262	0.0000	0.0050	0.0406	8.70%	8.11%	11.13%	15.60%	17.50%	3.95%	-0.1380	-1.0346	-5.66%	1.27E-03
Contractual Coupon Rate	3886	0.00%	0.00%	6.62%	9.00%	8.64%	11.04%	16.79%	30.00%	3.89%	-36.25%	52.96%	-5.76%	7.39E-04
LGD at Default	1375	-8.50%	-1.38%	30.00%	60.00%	55.78%	84.25%	99.40%	99.87%	31.28%	-28.63%	-121.74%	6.88%	1.48E-04
Moody's Original Credit Rating Investment Grade	3178	0.0000	0.0000	0.0000	0.0000	0.1954	0.0000	1.0000	1.0000	0.3966	1.5371	0.3629	2.35%	3.33E-04
Moody's Original Credit Rating (Major Code)	3178	1.0000	1.0000	3.0000	4.0000	3.3106	4.0000	5.0000	5.0000	1.0784	-0.8063	-0.0949	-0.03%	3.73E-06
Moody's Original Credit Rating (Minor Code)	3178	3.0000	3.0000	10.0000	14.0000	12.5296	15.0000	20.0000	20.0000	3.4435	-0.5798	-0.1107	1.92%	2.72E-04
Moody's Long Run Default Rate (Minor Code)	3178	0.0000	0.0002	0.0031	0.0249	0.0337	0.0415	0.2910	0.2910	0.0461	2.7195	10.0277	0.29%	4.13E-05

Table 8.5 - Summary Statistics on Instrument / Contractual Variables and Correlations with RDD (Moody's Ultimate LGD Database 1987-2007)

Variable	Cnt	Min	25th Prcntle	Median	Mean	75th Prcntle	Max	Std Dev	Skew	Kurt	Corr with RDD	P-Val of Corr
Seniority Rank	3886	1.0000	1.0000	1.0000	1.7123	2.0000	7.0000	0.8953	1.4491	2.4882	-9.64%	1.24E-03
Collateral Rank	3886	1.0000	3.0000	6.0000	4.5844	6.0000	6.0000	1.6206	-0.5951	-1.0651	-9.97%	1.28E-03
Percent Debt Below	3886	0.00%	0.00%	10.13%	25.82%	49.87%	100.00%	30.19%	81.72%	-70.78%	10.51%	1.35E-03
Percent Debt Above	3886	0.00%	0.00%	0.00%	21.51%	40.92%	100.00%	28.95%	112.09%	-4.18%	-6.51%	8.35E-04
Tranche Safety Index	3886	0.00%	32.73%	50.00%	52.16%	72.82%	100.00%	25.44%	-9.08%	-88.73%	9.69%	1.25E-03

Table 8.6 - Summary Statistics on Macroenonomic and Cyclical Variables and Correlations with RDD (Moody's Ultimate LGD Database 1987-2007)

Variable	Cnt	Minimum	25th Prcntle	Median	Mean	75th Prcntle	Maximum	Std Dev	Skew	Kurt	Corr with RDD	P-Val of Corr
Moody's All-Corporate Quarterly Default Rate	1262	0.00%	4.89%	7.05%	7.38%	9.85%	13.26%	3.28%	-0.1434	-0.9222	5.72%	1.29E-03
Moody's Speculative Quarterly Default Rate	1262	1.31%	4.89%	7.05%	7.40%	9.85%	13.26%	3.24%	-0.0980	-1.0016	5.43%	1.22E-03
Moody's All-Corporate Quarterly Default Rate by Industry	1262	0.00%	2.07%	3.78%	4.13%	6.05%	12.68%	2.70%	0.6439	-0.1938	7.40%	1.67E-03
Moody's Speculative Quarterly Default Rate by Industry	1262	0.00%	3.49%	6.52%	7.03%	9.80%	17.50%	4.19%	0.4587	-0.5255	6.66%	1.50E-03
Fama-French Excess Return on Market Factor	3886	-1076.00%	-241.00%	86.00%	33.35%	355.00%	1030.00%	464.59%	-0.3372	-0.4057	-0.07%	9.16E-06
Fama-French Relative Return on Small Stocks Factor	3886	-2218.00%	-214.00%	31.00%	13.66%	270.00%	843.00%	394.25%	-1.2903	5.3616	2.37%	3.03E-04
Fama-French Excess Return on Value Stock Factor	3886	-912.00%	-155.00%	64.00%	81.98%	269.00%	1380.00%	373.74%	0.7380	1.7726	-3.62%	4.64E-04
Short-Term Interest Rates (1-Month Treasury Yields)	1262	6.00%	14.00%	32.00%	31.82%	44.00%	79.00%	16.83%	0.1114	-1.0411	-10.41%	2.35E-03
Long-Term Interest Rates (10-Month Treasury Yields)	1111	337.00%	451.00%	535.00%	548.19%	603.00%	904.00%	125.45%	1.0557	0.7194	-6.69%	1.61E-03
Stock-Market Volatility (2-Year IDX)	1111	3.00%	7.00%	9.00%	9.98%	11.00%	19.00%	3.84%	0.9425	-0.0862	-0.36%	8.55E-05

Table 8.7 - Summary Statistics on Duration / Vintage Variables and Correlations with RDD (Moody's Ultimate LGD Database 1987-2007)

Variable	Cnt	Min	25th Prcntle	Median	Mean	75th Prcntle	Max	Std Dev	Skew	Kurt	Corr with RDD	P-Val of Corr
Time from Origination to Default	3365	0.2500	1.6384	2.8849	4.0128	5.0027	29.9534	3.7660	2.3676	7.4186	-0.67%	9.18E-05
Time from First Rating to Default	3178	1.0000	3.1288	5.7425	10.2523	14.5753	56.9781	11.3994	2.1219	4.7533	-0.51%	7.23E-05
Time from Last Cash-Pay Date to Default	3886	0.0000	0.0986	0.2411	0.3907	0.4959	4.3808	0.4849	2.7920	10.5645	0.22%	2.80E-05
Time from Default to Resolution	3886	0.0027	0.5507	1.1685	1.4594	1.9534	9.3151	1.3320	1.8273	5.2410	-13.72%	1.77E-03
Time from Origination to Maturity Date	3365	0.1000	5.0027	7.8219	8.9032	10.0137	50.0329	6.5084	1.6668	3.5430	-1.31%	1.80E-04

Table 9 - Beta-Link Generalized Linear Model for Annualized Returns on Defaulted Debt (Moody's Ultimate LGD Database 1987-2007)

	Model 1		Model 2		Model 3	
Variables	Partial Effect	P-Value	Partial Effect	P-Value	Partial Effect	P-Value
Intercept	0.3094	1.42E-03	0.51005	9.35E-04	0.4342	6.87E-03
Moody's 12 Month Lagging Speculative Grade Default Rate by Industry	2.0501	1.22E-02	2.2538	6.94E-03	2.1828	1.36E-02
Collateral Rank Secured	0.2554	7.21E-03	0.2330	1.25E-02	0.2704	9.36E-04
Tranche Safety Index	0.4548	3.03E-02	0.4339	3.75E-02		
Loss Given Default	0.3273	1.44E-02	0.2751	3.88E-02		
Cumulative Abnormal Returns on Equity Prior to Default	0.3669	1.51E-03	0.3843	1.00E-03	0.4010	9.39E-04
Total Liabilities to Total Assets	0.2653	5.22E-08				
Moody's Original Rating Investment Grade	0.2118	2.80E-02	0.2422	6.84E-03	0.1561	6.25E-02
1-Month Treasury Yield	-0.4298	3.04E-02	-0.3659	1.01E-02	-0.4901	3.36E-02
Size Relative to the Market			-0.0366	4.76E-02	-0.0648	3.41E-03
Market Value to Book Value			0.1925	2.64E-05	0.1422	5.63E-03
Free-Asset Ratio					-0.2429	2.25E-02
Degrees of Freedom	959		958		783	
Log-Likelihood	-592.30		-594.71		-503.99	
McFadden Pseudo R-Squared (In-Sample)	32.48%		38.80%		41.73%	
McFadden Pseudo R-Squared (Out-Of-Sample) - Bootstrap Mean	21.23%		12.11%		17.77%	
McFadden Pseudo R-Squared (Out-Of-Sample) - Bootstrap Standard Error	2.28%		1.16%		1.70%	

Table 10 - Summary Statistics on Discounted LGD and Regulatory Capital for Different Discounting Methodologies (Moody's Ultimate LGD Database 1987-2007)

	Count	Minimum	1st Percentile	25th Percentile	Median	Mean	75th Percentile	99th Percentile	Maximum	Standard Deviation	Skewness	Kurtosis
Discounted LGD - Contractual Coupon Rate[2]	960	5.00%	5.00%	13.68%	54.97%	52.07%	88.17%	100.00%	100.00%	35.86%	-0.0569	-1.5467
Discounted LGD - RDD Regression Model[3]	960	5.00%	5.00%	33.29%	72.10%	64.05%	95.07%	105.00%	105.00%	33.03%	-0.3122	-1.2998
Discounted LGD - Punitive Discount Rate[4]	960	5.00%	5.00%	33.08%	62.12%	59.03%	89.62%	100.00%	100.00%	31.53%	-0.2133	-1.3288
Regulatory Capital - Contractual Coupon Rate	960	0.00%	0.03%	0.78%	2.66%	6.91%	9.91%	37.78%	57.15%	9.18%	1.9366	4.3220
Regulatory Capital - RDD Regression Model	960	0.00%	0.03%	1.00%	3.72%	8.04%	10.87%	39.96%	60.03%	9.86%	1.7132	3.3883
Regulatory Capital - Punitive Discount Rate	960	0.00%	0.07%	0.94%	3.14%	7.31%	10.14%	38.19%	57.22%	9.26%	1.8544	3.9928

1 - Basel II capital formula (Final Rule, 2007) based upon Asymptotic Single Risk Factor (ASRF) model (Gordy, 2003): $K^R = N([N^{-1}(PD)+N^{-1}(0.999)*R^{.5}]/[(1-R)^{.5})]-PD)*(0.08+.92*LGD)$, where K^R denotes regulatory capital, LGD is discounted LGD in the MULGD database, PD is the average default rate according to the Moody's rating, and the asset correlation is the Final Rule prescribed R = 0.12 + 0.18*exp(-50*PD) for wholesale exposures. Unit LEQ is assumed on each loan, so thatportfolio capital is the mean of the

2 - Contract rate on instrument prevailing just before defult.

3- Model 1 of Table 9: RDD = 0.31 + 2.05*(Moody's Speculative Grade Default Rate) + 0.26*(Collateral Flag Secured) + 0.45*T.I.S. + 0.33*LGD + 0.37*C.A.R. + 0.27*TL/TA + 0.21*(Moody's Original Rating Investment Grade) - 0.43*(1-Month T-Bill Yield)

4 - 25% per annum.

Table 10.1: Moody's Long Run Default Rates (Annual Cohorts 1982-2007)

Rating	Default Rate
Aaa	0.020%
Aa1	0.020%
Aa	0.020%
Aa3	0.016%
A1	0.002%
A2	0.024%
A3	0.032%
Baa1	0.141%
Baa2	0.141%
Baa3	0.308%
Ba1	0.662%
Ba2	0.756%
Ba3	1.733%
B1	2.486%
B2	4.148%
B3	8.118%
Caa1	9.913%
Caa2	17.359%
Caa3	23.715%
Ca	29.096%
C	32.164%

Table 11: Benchmark Comparison of Alternative Methodologies for Deriving the Discount Rate for Workout Recoveries

	Data	Model for LGD Correlation to Systematic Factor	Source / Reference	Discount Rate[1]	LGD Correlation $(q_{i,m})$[2]	Asset Value Volatility (σ_i)[3]	Market Volatility (sig_m)[4]	LGD Beta[5]	MRP[6]	Risk-Free Rate (R_f)[7]
Market / Structural Based Models	Sample of Bid Quotes on 90 Defualted Bonds & S&P 500 Index Returns (4/02-8/03)	Linear Regression	Machlachan (2004)	7.23%	N/A	N/A	N/A	37.10%	6.00%	5.00%
	Monthly Altman Defaulted Bond Index & S&P 500 Index Returns (1986-2002)	Linear Regression	Altman & Jha (2003), Machlachan (2004)	11.05%	20.30%	32.00%	18.74%	76.92%	7.87%	5.00%
	Default Rates and Market Implied LGD (Moody's DRS Databases 1982-1999)	1-Factor Structural Model MLE Calibration	Frye (2000), Machlachan (2004)	7.28%	17.00%	32.00%	18.74%	29.02%	7.87%	5.00%
	Default Rates and Market Implied LGD 1987-2007 (Moody's DRS & MULGD Databases) - Loans[7]	2-Factor Structural Model MLE Calibration	Jacobs (2008)	7.96%	22.03%	32.00%	18.74%	37.61%	7.87%	5.00%
	Bonds[7]	2-Factor Structural Model MLE Calibration	Jacobs (2008)	9.92%	36.64%	32.00%	18.74%	62.56%	7.87%	5.00%
	Monthly RDD (Moody's DRS & MULGD Databases 1987-2007) and Fama-French Market Factor - Bonds	Linear Regression	Jacobs (2008)	6.78%	13.23%	32.00%	18.74%	22.59%	7.87%	5.00%
	Monthly RDD (Moody's DRS & MULGD Databases 1995-2007) and Fama-French Market Factor - Loans	Linear Regression	Jacobs (2008)	6.58%	11.76%	32.00%	18.74%	20.07%	7.87%	5.00%
	2007) - Bonds	Linear Regression	Jacobs (2008)	8.85%	28.66%	32.00%	18.74%	48.93%	7.87%	5.00%
	2007) - Loans	Linear Regression	Jacobs (2008)	7.89%	21.50%	32.00%	18.74%	36.70%	7.87%	5.00%
	Monthly Altman Public Bond Index & S&P 500 Return (1/99-9/08)	Linear Regression	Jacobs (2008)	10.50%	40.93%	32.00%	18.74%	69.87%	7.87%	5.00%
	Monthly Altman Bank Loan Index & S&P 500 Return (1/99-9/08)	Linear Regression	Jacobs (2008)	6.53%	11.41%	32.00%	18.74%	19.48%	7.87%	5.00%
Empirical Models	Most Likely Discount Rate (S&P LossStats & CreditPro Databases 1985-2004)	N/A	Brady et al (2006)	14.00%	N/A	N/A	N/A	N/A	N/A	N/A
	Ex Post Realized Returns (Moody's Bankrupt Bond Index 1988-1998)	N/A	Hamilton & Berthault (2000), Araten (2004)	15.00%	N/A	N/A	N/A	N/A	N/A	N/A
	Return on Defaulted Debt (Moody's DRS & MULGD Databases 1987-2007)	N/A	Jacobs (2008)	29.20%	N/A	N/A	N/A	N/A	N/A	N/A
	Return on Defaulted Debt (Moody's DRS & MULGD Databases 1987-2007) - Loans	N/A	Jacobs (2008)	43.30%	N/A	N/A	N/A	N/A	N/A	N/A
	Most Likely Discount Rate (Moody's DRS & MULGD Databases 1987-2007)	N/A	Jacobs (2008)	21.30%	N/A	N/A	N/A	N/A	N/A	N/A
	Most Likely Discount Rate (Moody's DRS & MULGD Databases 1987-2007) - Loans	N/A	Jacobs (2008)	14.50%	N/A	N/A	N/A	N/A	N/A	N/A
Model-Free	Contractual Rate (incluyding penalty)	N/A	Asarnow & Edwards (1995)	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	Lender's Cost of Equity	N/A	Eales & Bosworth (1998)	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	Coupon Rate	N/A	Friedman & Sandow (2003)	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	Risk-free rate of Return	N/A	Carey & Gordy (2006)	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	Price of Traded Debt On-Month post Default Implied	N/A	Gupton & Stein (2002)	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	Contactual Loan Rate	N/A	Carty et al (1998)	N/A	N/A	N/A	N/A	N/A	N/A	N/A
Regulatory	"Where positive or negative cash flows on a wholesale exposure to a defaulted obligor or a defaulted retail exposure … occur after the date of default, the economic loss must reflect the net present value of cash flows as of the default date using a discount rate appropriate to the risk of the defaulted exposure."	N/A	Basel II Final Rule in the U.S. (OCC et al, 2007, Page 450)	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	"Firms should use the same rate as that used for an asset of similar risk. They should not use the risk-free rate or the firms hurdle rate (unless the firm only invests in risky assets such as defaulted debt instruments)"	N/A	FSA (2003, page 68, Annex 3)	N/A	N/A	N/A	N/A	N/A	N/A	N/A
	"Effective original loan rate (the rate that exactly discounts expected future cash payments or receipts through the expected life of the financial instrument)"	N/A	IAS 39 (2003)	N/A	N/A	N/A	N/A	N/A	N/A	N/A

1 - In the case of market / structural model approaches, derived from equation 3.7.
2 - Estimated by MLE or linear regression in the case of market / structural models.
3 - Sourced from Frye (2000), Table A.
4 - Standard deviation of Fama-French return on market factor (7/26-3/2008)
5 - Average of Fama-French return on market factor (7/26-3/2008)
6 - Assumed.
7 - Reported herewith in Table 11.1.

Table 11.1: Simultaneous Full-Information Maximum Likelihood Estimation of 2-Factor Structural Credit Model

Moody's DRS Annual Speculative-Grade Default Rates and MULGD Market Implied Loss-Given-Default (1987-2007)

Asset Value Process for Rating Class r: $A_{t,r} = \rho_r X_t + (1-\rho_r^2)^{.5} Z_{t,r}$, Idiosyncratic PD Variable: $Z_{t,r} \sim NID(0,1)$, Systematic PD Variable: X_t , Asset Value Factor Loading (Correlation): ρ_r (ρ_r^2)

Loss Rate Process for Seniority Class s: $L_{t,s} = \rho_s Y_t + (1-\rho_s^2)^{.5} Z_{t,s}$, Idiosyncratic LGD Variable: $Z_{t,s} \sim NID(0,1)$, Systematic LGD Variable: Y_t , Recovery Value Factor Loading (Correlation):ρ_s (ρ_s^2)

Conditional Default Rate: $R(X_t \mid PD_r,\rho_r) = \Phi[(\Phi^{-1}[PD_r]-\rho_r X_t)/(1-\rho_r^2)^{.5}]$, PD_r: Long-Run (Expected) Probability of Default for Rating Class r

Conditional Loss Rate: $L(Y_t \mid LGD_s,\rho_s) = \Phi[(\Phi^{-1}[LGD_s]-\rho_s Y_t)/(1-\rho_s^2)^{.5}]$, LGDs: Long-Run (Expected) Loss-Given-Default for Seniority Classs s

$Z_{t,r}$, $Z_{t,s} \sim NID(0,1)$; $(X_t, Y_t) \sim N2 ([0,0]^T, [(1,r_{X,Y})^T,(r_{X,Y},1)^T]$

	Parameter	MLE Estimate	Standard Error
	Asset Value Correlation (ρ_r)	8.01%	4.66%
	Long-Run Probability of Default (PD_r)	4.96%	3.01%
Bank Loans	Recovery Value Correlation for Loans (ρ_l^2)	22.03%	4.79%
Bank Loans	Long-Run Loss-Given-Default for Loans (LGD_l)	28.90%	13.23%
Senior Secured Bonds	Recovery Value Correlation for Bonds (ρ_b^2)	36.64%	11.10%
Senior Secured Bonds	Long-Run LGD for Bonds (LGD_b)	44.61%	26.06%
	Correlation between Systematic Factors in Default and Loss Rate (PD-LGD) Processes (r_{xy})	64.42%	18.83%

12. Figures

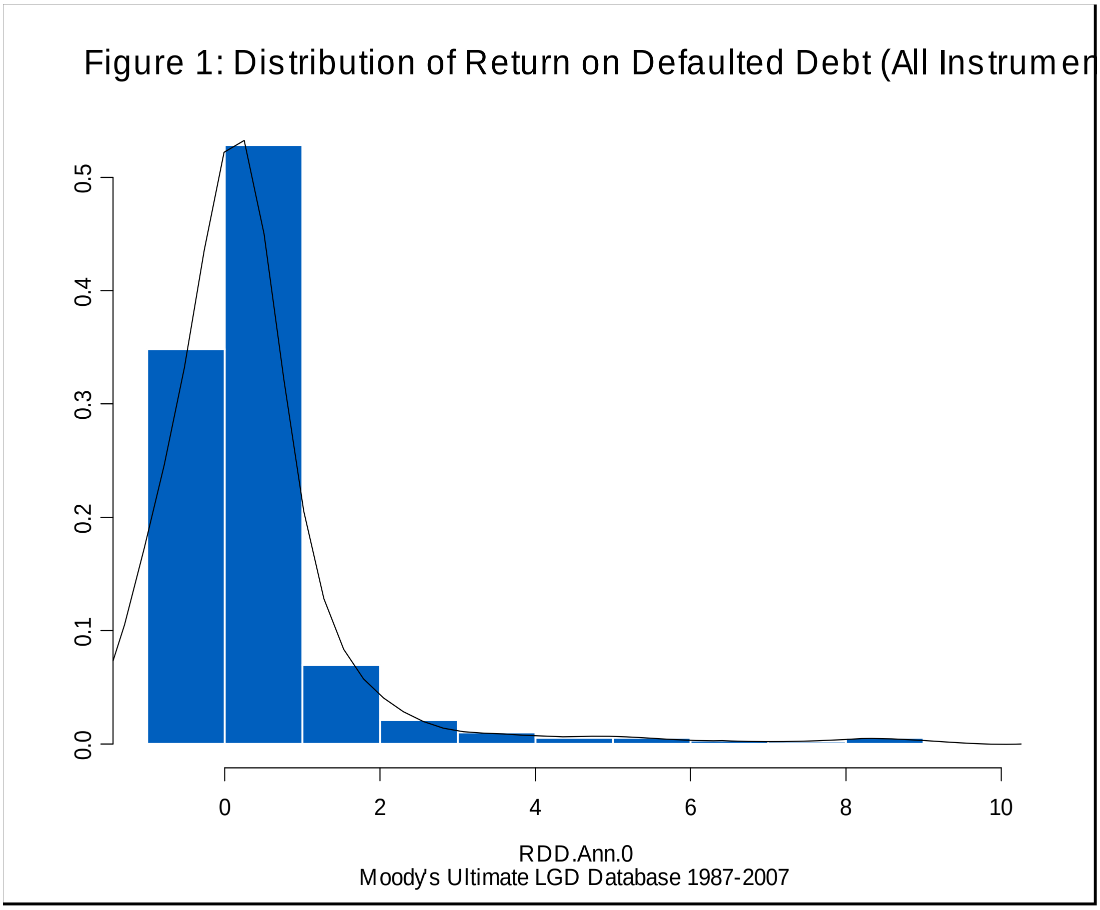


Figure 1: Distribution of Return on Defaulted Debt (All Instrumen

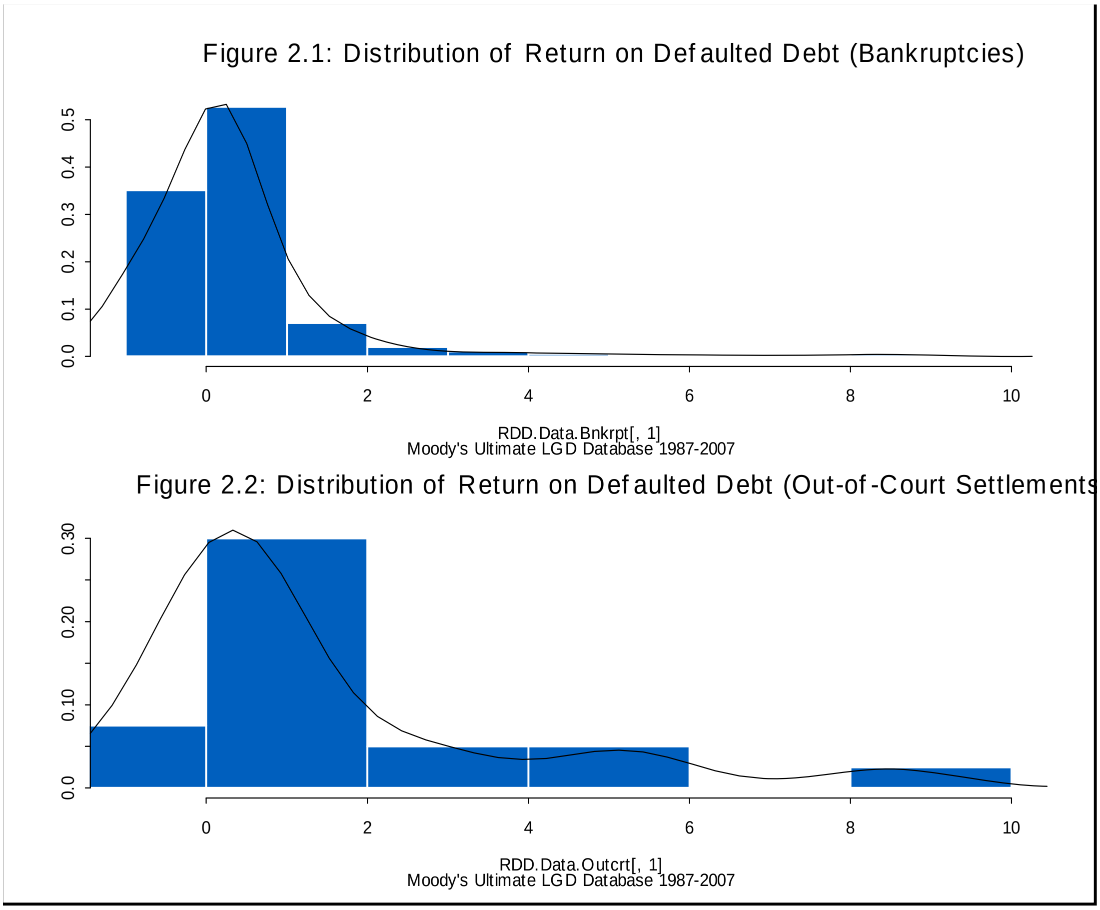


Figure 3.1: Distribution of Return on Defaulted Debt (Bonds)

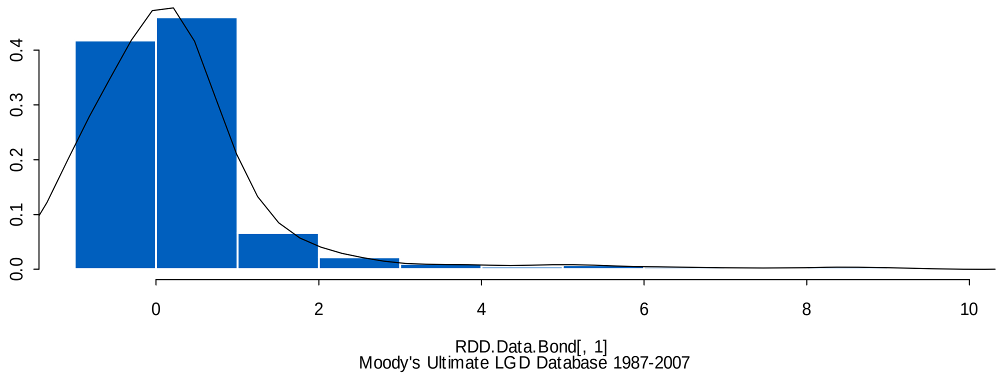
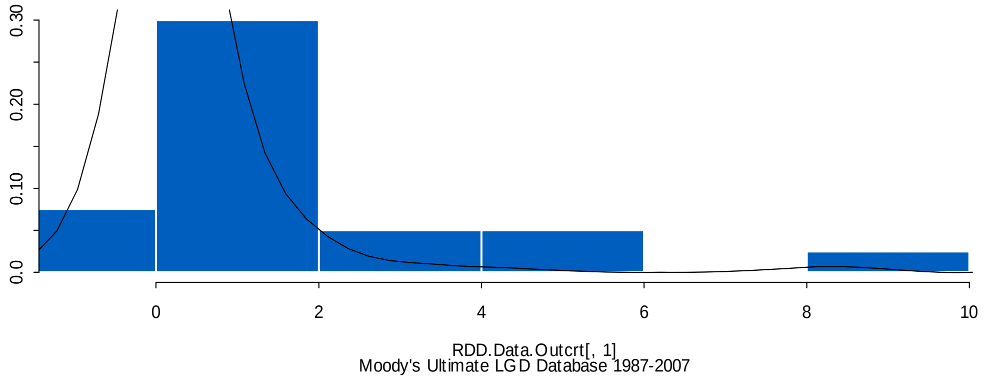


Figure 3.2: Distribution of Return on Defaulted Debt (Loans)



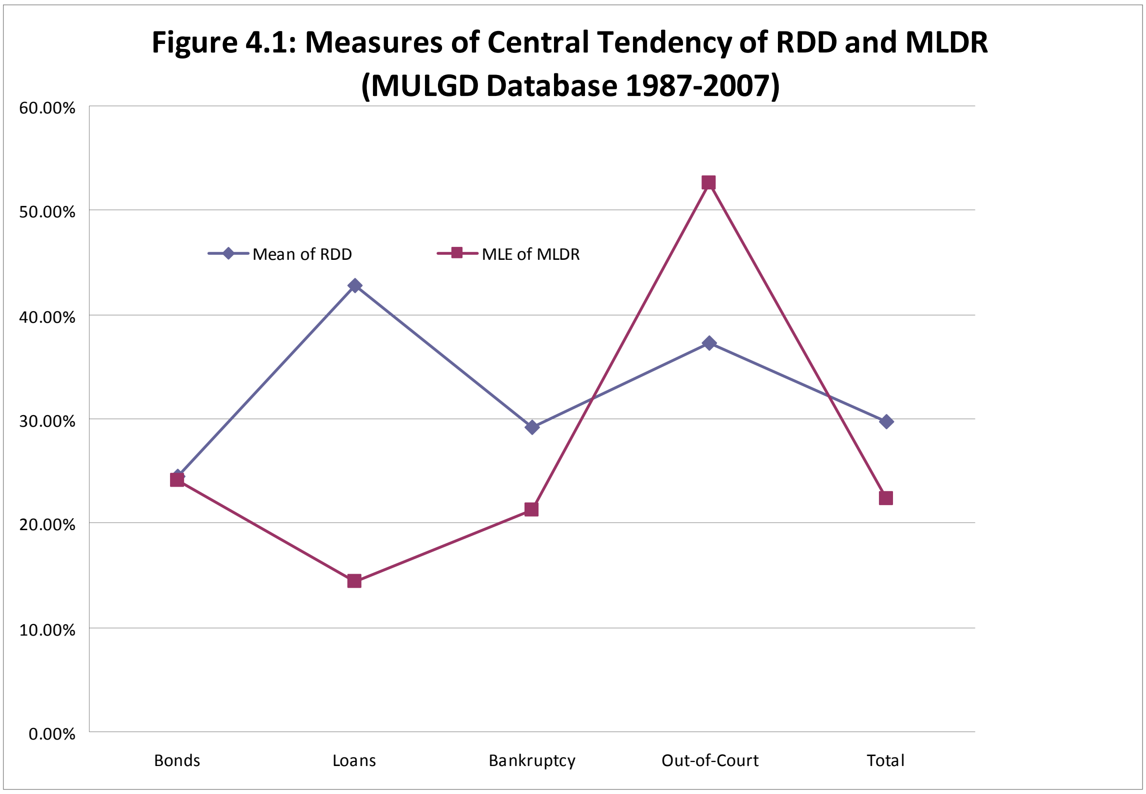


Figure 4.1: Measures of Central Tendency of RDD and MLDR (MULGD Database 1987-2007)
60.00%
50.00%
40.00%
30.00%
20.00%
10.00%
0.00%
Mean of RDD
MLE of MLDR
Bonds
Loans
Bankruptcy
Out-of-Court
Total

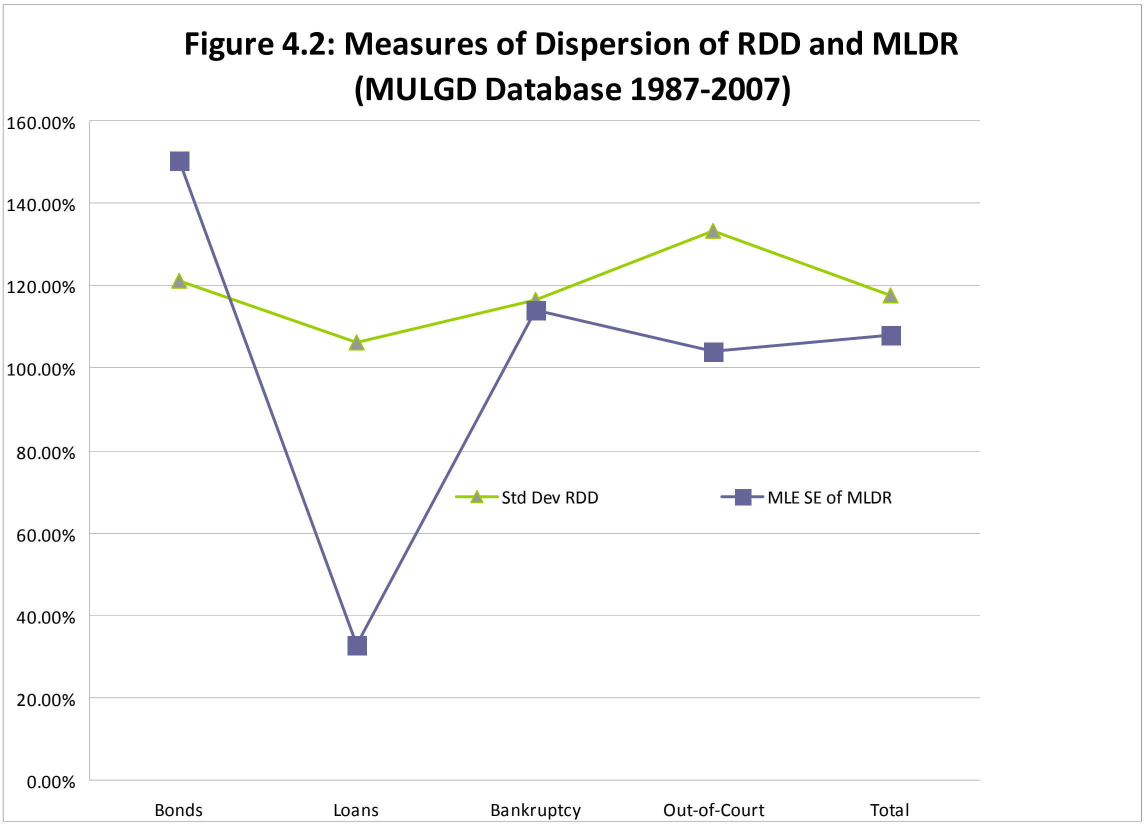


Figure 4.2: Measures of Dispersion of RDD and MLDR (MULGD Database 1987-2007)
160.00%
140.00%
120.00%
100.00%
80.00%
60.00%
40.00%
20.00%
0.00%
Std Dev RDD
MLE SE of MLDR
Bonds
Loans
Bankruptcy
Out-of-Court
Total

Figure 5.1: Central Tendency Measures of RDD and MLDR Observations by Seniority Rank (MULGD Database 1987-2007)

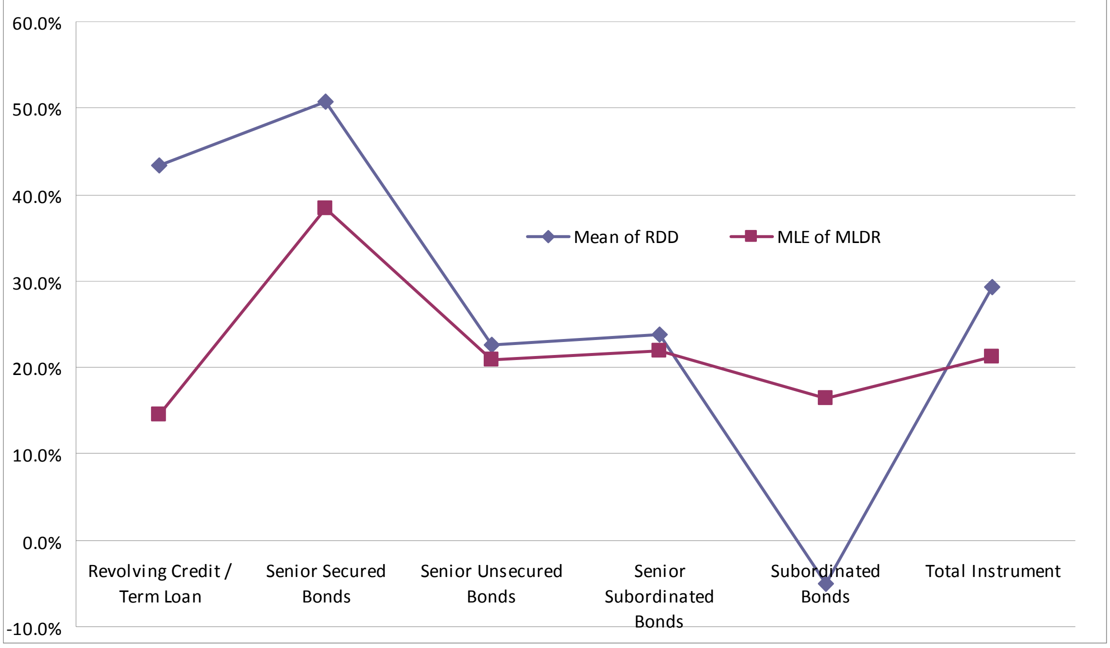
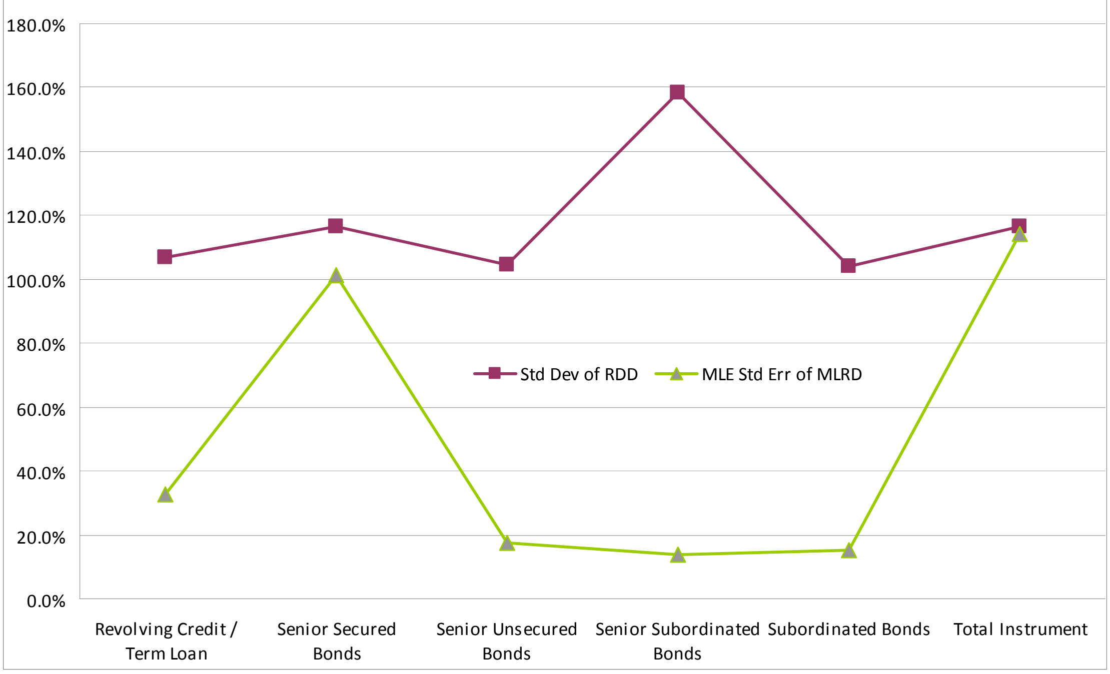


Figure 5.2: Dispersion Measures of RDD and MLDR Observations by Seniority Rank (MULGD Database 1987-2007)



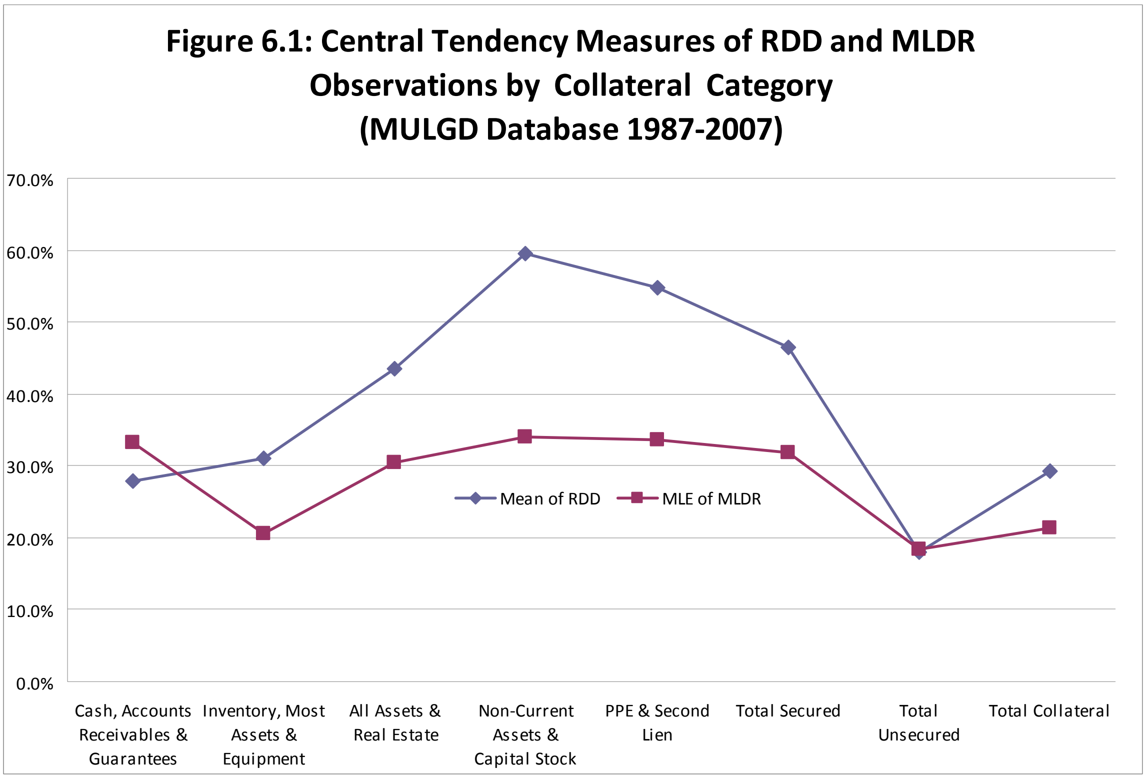


Figure 6.1: Central Tendency Measures of RDD and MLDR
Observations by Collateral Category
(MULGD Database 1987-2007)
70.0%
60.0%
50.0%
40.0%
30.0%
20.0%
10.0%
0.0%
Mean of RDD
MLE of MLDR
Cash, Accounts Receivables & Guarantees
Inventory, Most Assets & Equipment
All Assets & Real Estate
Non-Current Assets & Capital Stock
PPE & Second Lien
Total Secured
Total Unsecured
Total Collateral

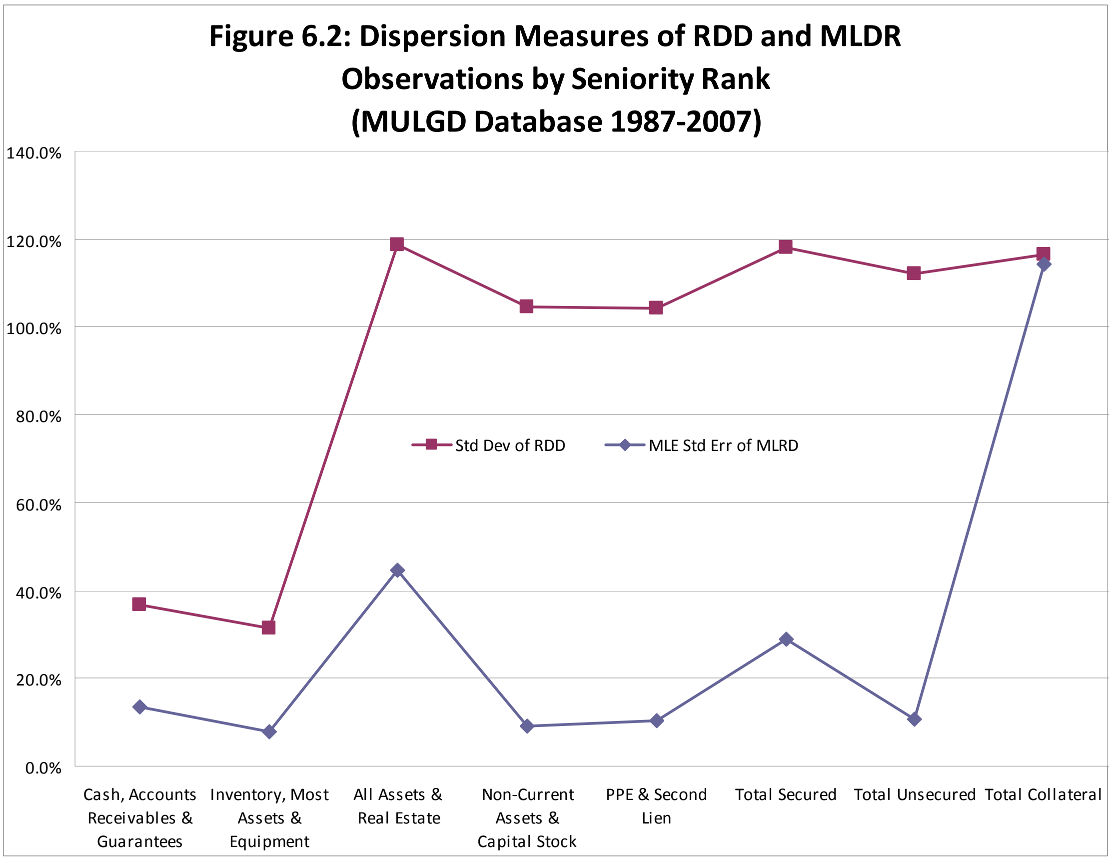


Figure 6.2: Dispersion Measures of RDD and MLDR
Observations by Seniority Rank
(MULGD Database 1987-2007)
140.0%
120.0%
100.0%
80.0%
60.0%
40.0%
20.0%
0.0%
Std Dev of RDD
MLE Std Err of MLRD
Cash, Accounts Receivables & Guarantees
Inventory, Most Assets & Equipment
All Assets & Real Estate
Non-Current Assets & Capital Stock
PPE & Second Lien
Total Secured
Total Unsecured
Total Collateral

Figure 7.1: Central Tendency Measures of RDD and MLDR Observations by Year of Default (MULGD Database 1987-2007)

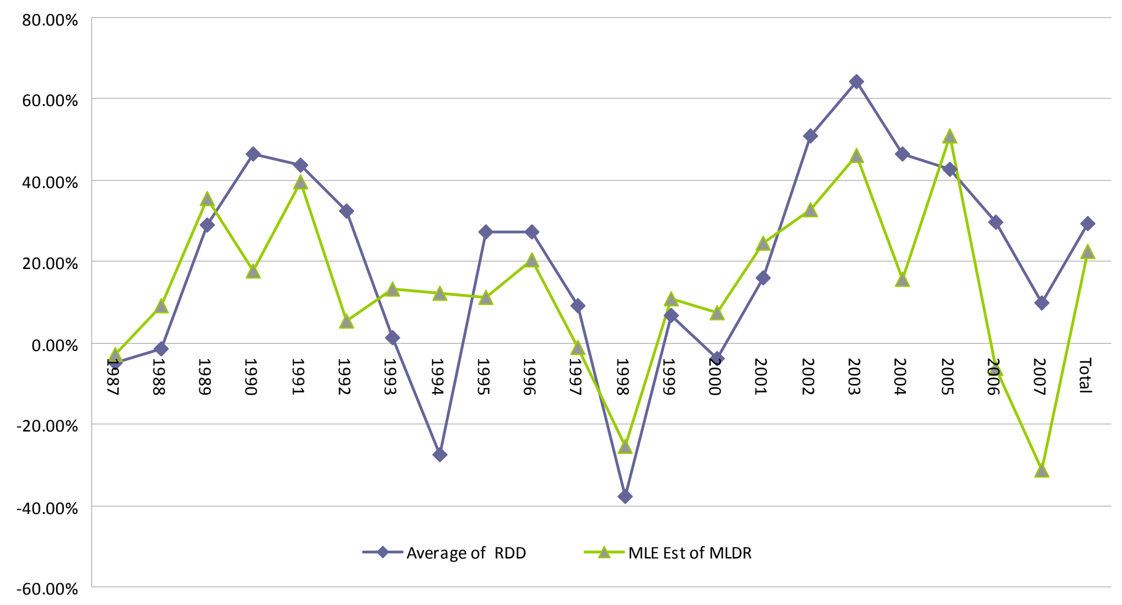
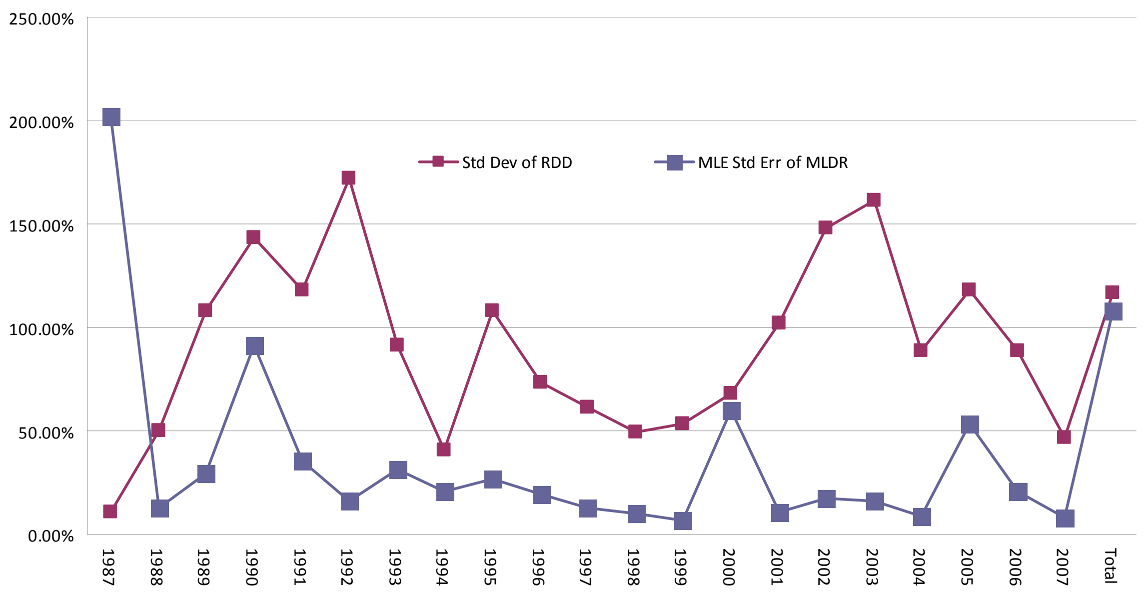


Figure 7.2: Dispersion Measures of RDD and MLDR Observations by Year of Default (MULGD Database 1987-2007)



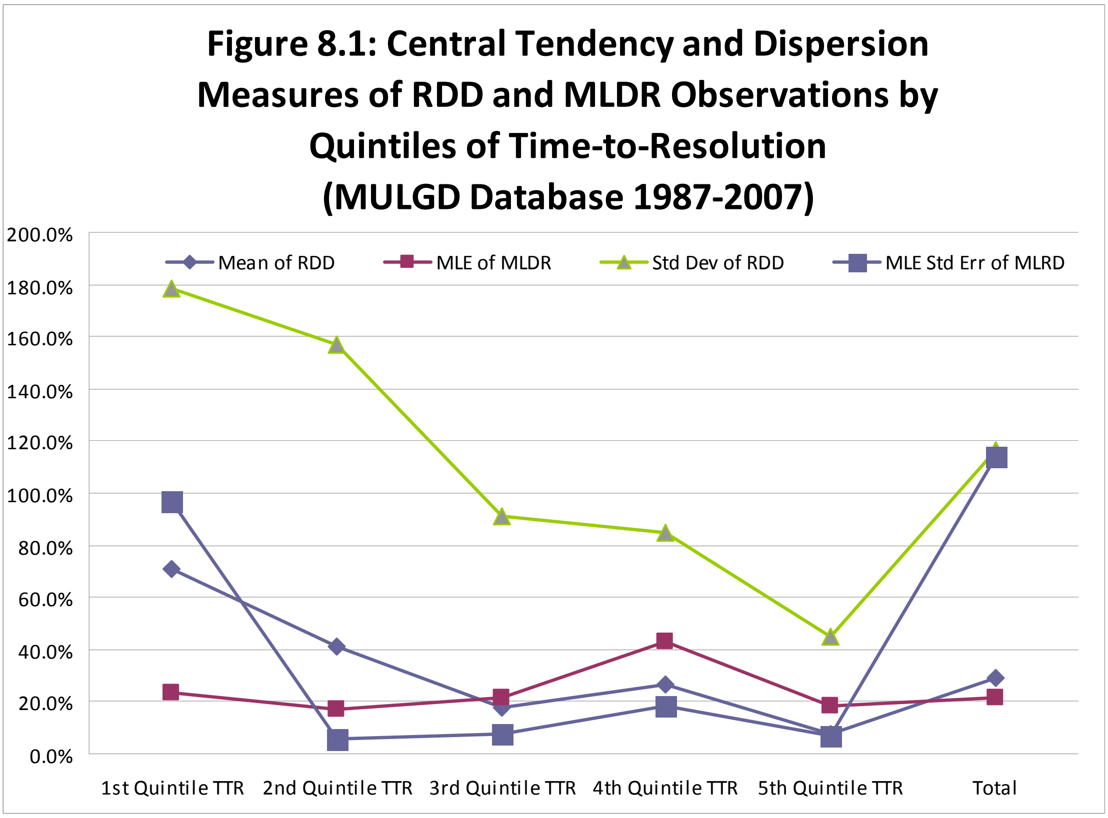


Figure 8.1: Central Tendency and Dispersion Measures of RDD and MLDR Observations by Quintiles of Time-to-Resolution (MULGD Database 1987-2007)
Mean of RDD
MLE of MLDR
Std Dev of RDD
MLE Std Err of MLRD
200.0%
180.0%
160.0%
140.0%
120.0%
100.0%
80.0%
60.0%
40.0%
20.0%
0.0%
1st Quintile TTR
2nd Quintile TTR
3rd Quintile TTR
4th Quintile TTR
5th Quintile TTR
Total

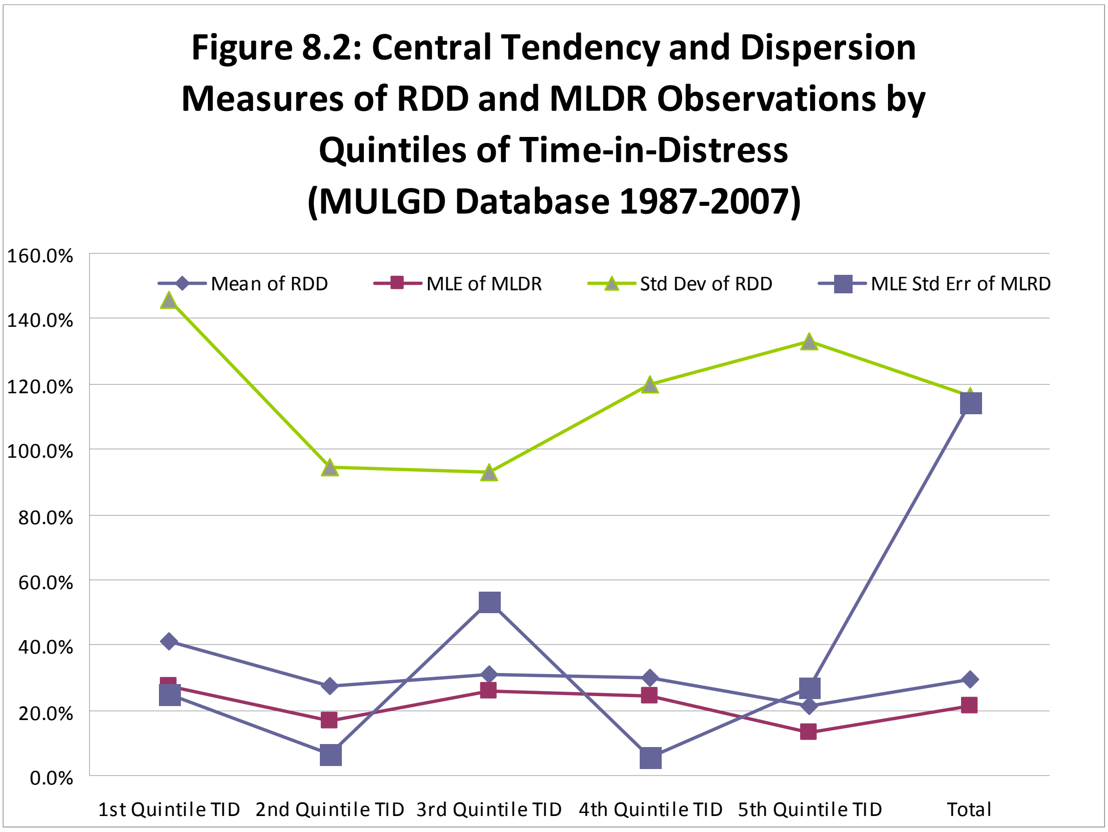


Figure 8.2: Central Tendency and Dispersion Measures of RDD and MLDR Observations by Quintiles of Time-in-Distress (MULGD Database 1987-2007)
Mean of RDD
MLE of MLDR
Std Dev of RDD
MLE Std Err of MLRD
160.0%
140.0%
120.0%
100.0%
80.0%
60.0%
40.0%
20.0%
0.0%
1st Quintile TID
2nd Quintile TID
3rd Quintile TID
4th Quintile TID
5th Quintile TID
Total

Figure 9.1: Central Tendency Measures of RDD and MLDR Observations by Credit Rating at Origination (MULGD Database 1987-2007)

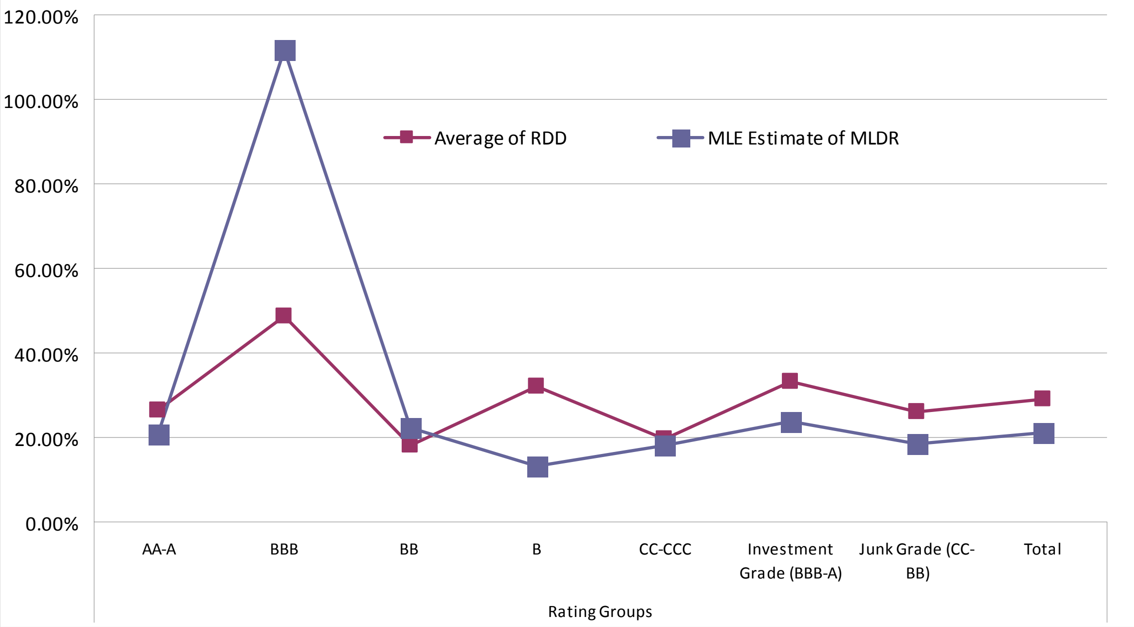
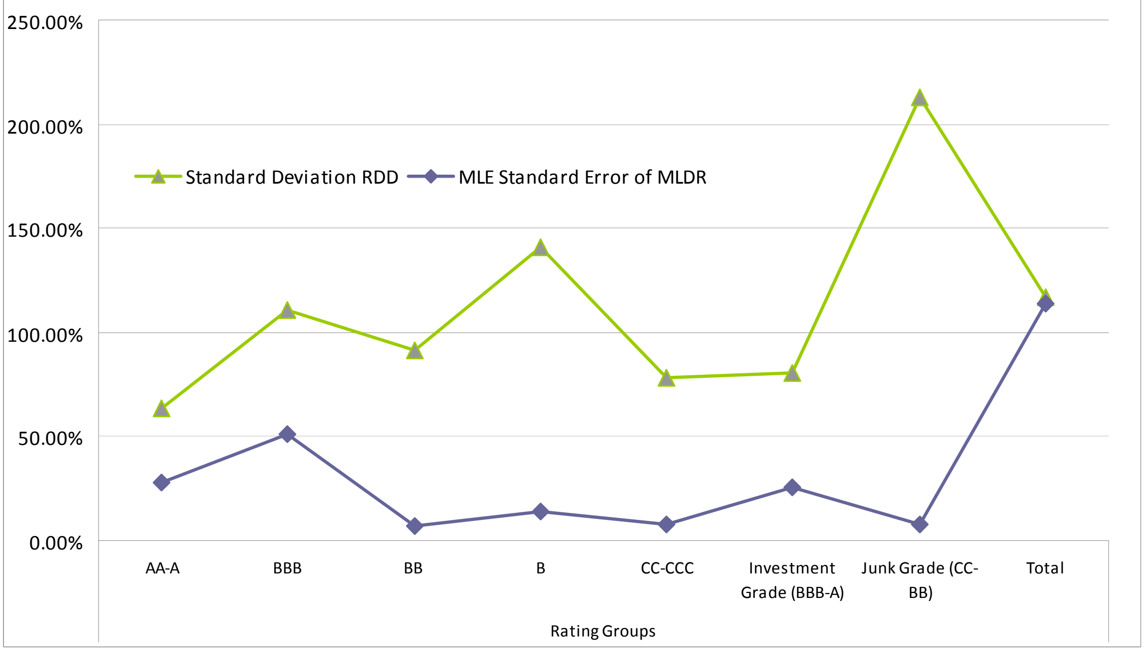


Figure 9.2: Dispersion Measures of RDD and MLDR Observations by Credit Rating at Origination (MULGD Database 1987-2007)



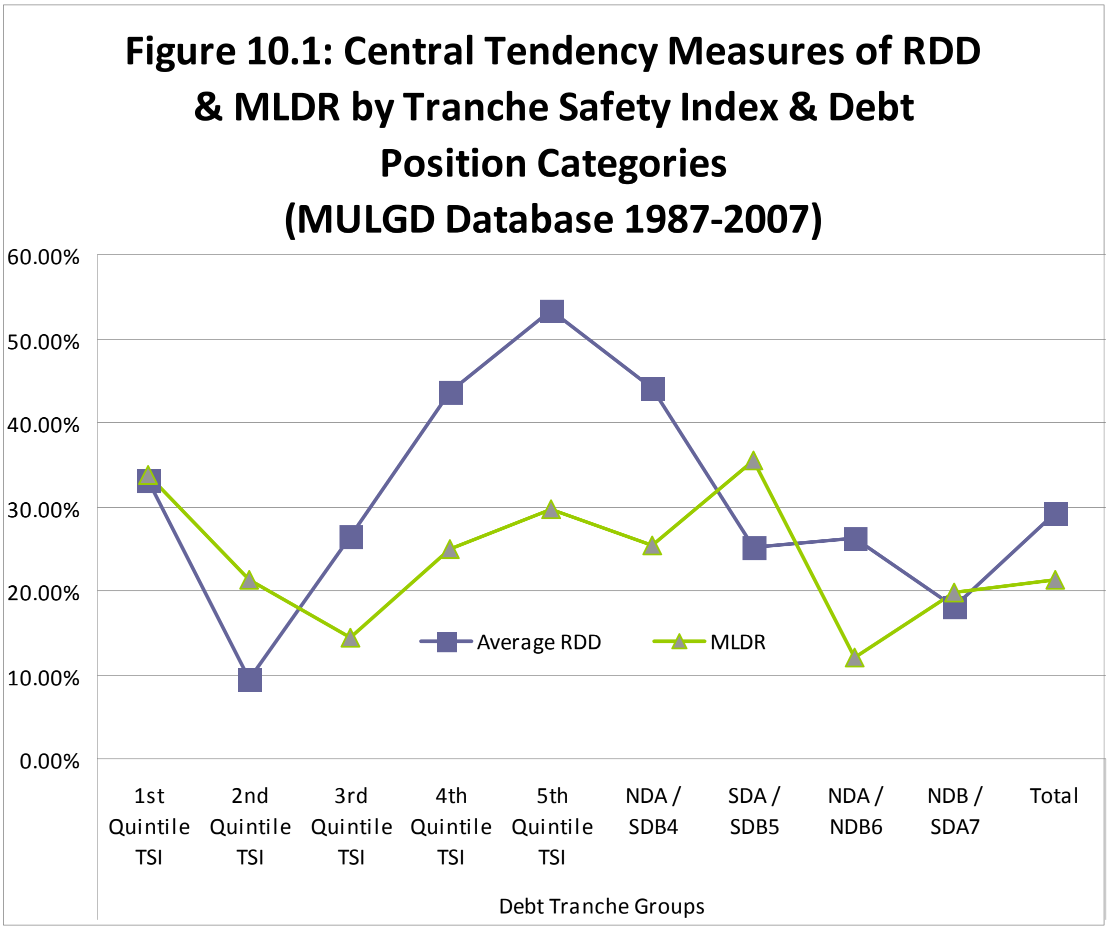


Figure 10.1: Central Tendency Measures of RDD & MLDR by Tranche Safety Index & Debt Position Categories
(MULGD Database 1987-2007)
60.00%
50.00%
40.00%
30.00%
20.00%
10.00%
0.00%
Average RDD
MLDR
1st Quintile TSI
2nd Quintile TSI
3rd Quintile TSI
4th Quintile TSI
5th Quintile TSI
NDA / SDB4
SDA / SDB5
NDA / NDB6
NDB / SDA7
Total
Debt Tranche Groups

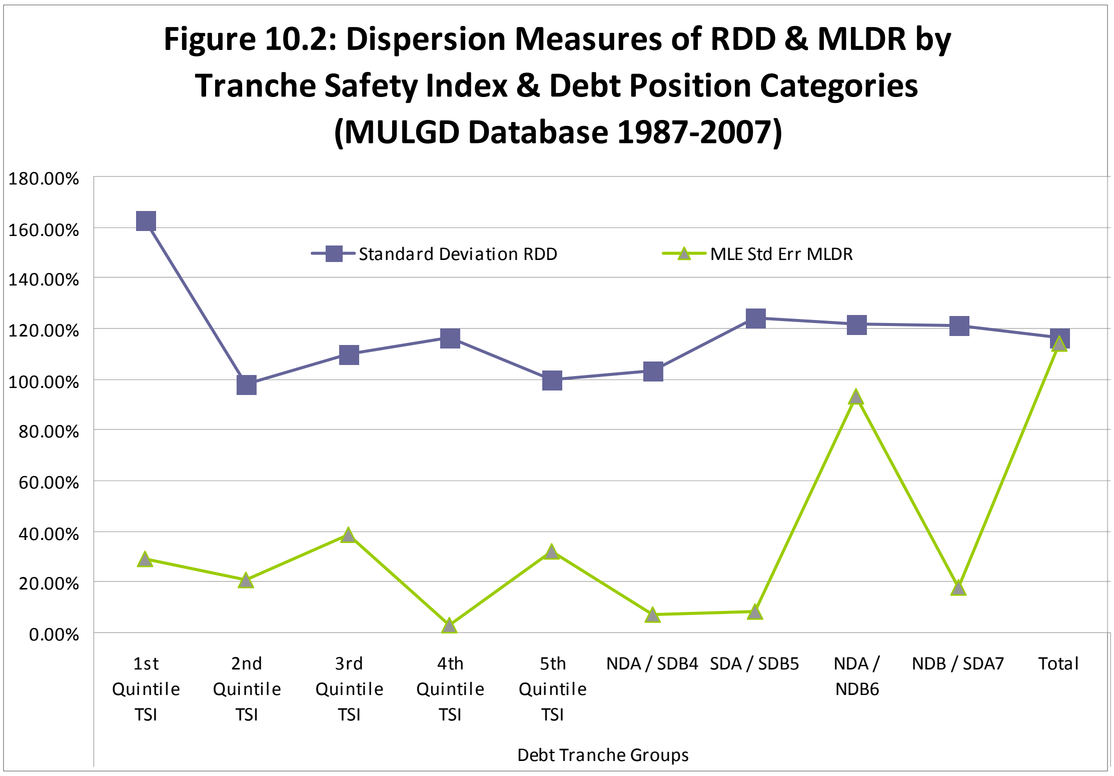


Figure 10.2: Dispersion Measures of RDD & MLDR by Tranche Safety Index & Debt Position Categories
(MULGD Database 1987-2007)
180.00%
160.00%
140.00%
120.00%
100.00%
80.00%
60.00%
40.00%
20.00%
0.00%
Standard Deviation RDD
MLE Std Err MLDR
1st Quintile TSI
2nd Quintile TSI
3rd Quintile TSI
4th Quintile TSI
5th Quintile TSI
NDA / SDB4
SDA / SDB5
NDA / NDB6
NDB / SDA7
Total
Debt Tranche Groups

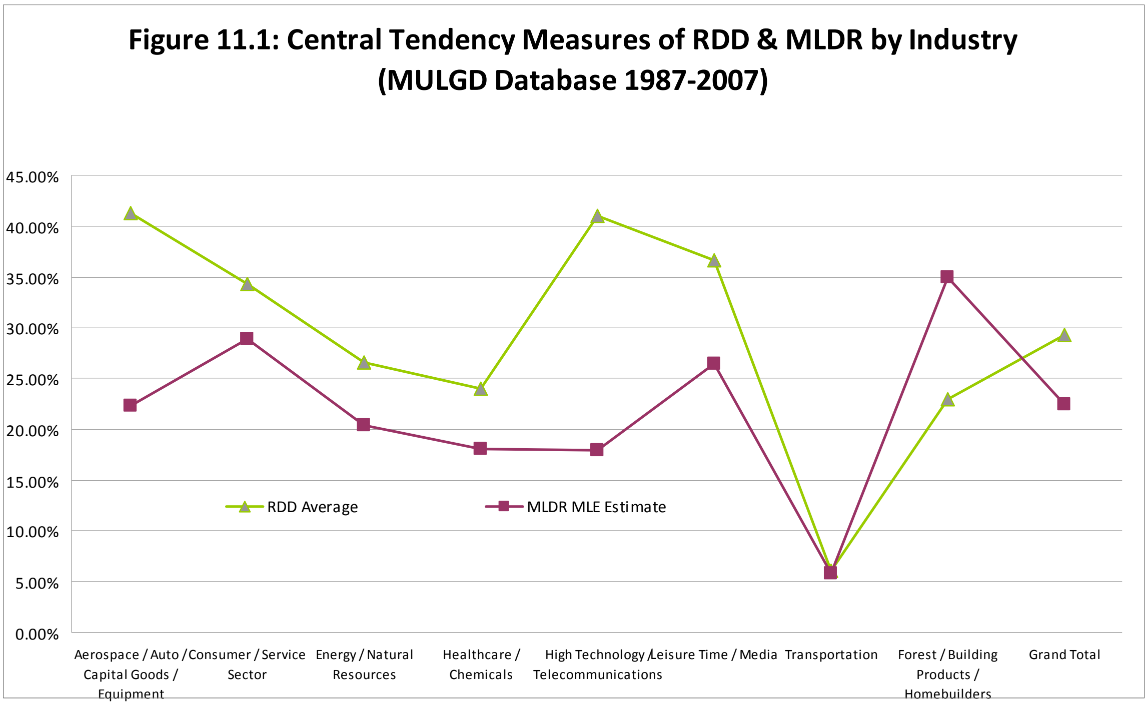


Figure 11.1: Central Tendency Measures of RDD & MLDR by Industry
(MULGD Database 1987-2007)
45.00%
40.00%
35.00%
30.00%
25.00%
20.00%
15.00%
10.00%
5.00%
0.00%
RDD Average
MLDR MLE Estimate
Aerospace / Auto / Capital Goods / Equipment
Consumer / Service Sector
Energy / Natural Resources
Healthcare / Chemicals
High Technology / Telecommunications
Leisure Time / Media
Transportation
Forest / Building Products / Homebuilders
Grand Total

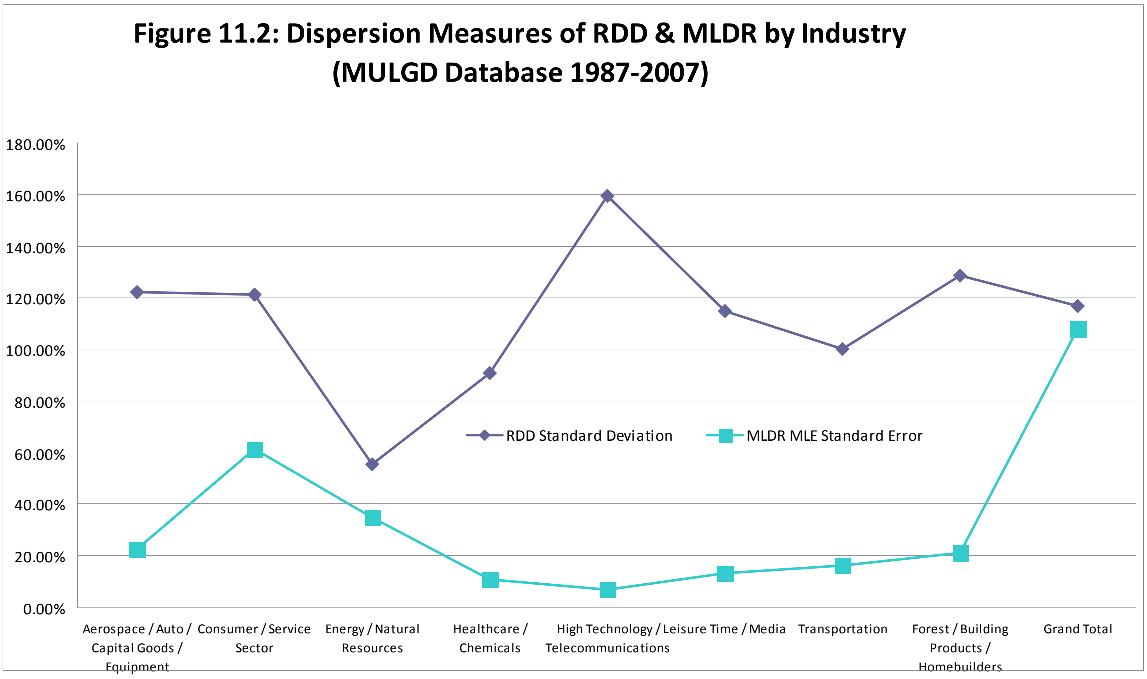


Figure 11.2: Dispersion Measures of RDD & MLDR by Industry
(MULGD Database 1987-2007)
180.00%
160.00%
140.00%
120.00%
100.00%
80.00%
60.00%
40.00%
20.00%
0.00%
RDD Standard Deviation
MLDR MLE Standard Error
Aerospace / Auto / Capital Goods / Equipment
Consumer / Service Sector
Energy / Natural Resources
Healthcare / Chemicals
High Technology / Telecommunications
Leisure Time / Media
Transportation
Forest / Building Products / Homebuilders
Grand Total

Figure 12.1: Annualized Return on Defaulted Debt vs. Market-to-Book Value (MULGD Database 1987-2007)

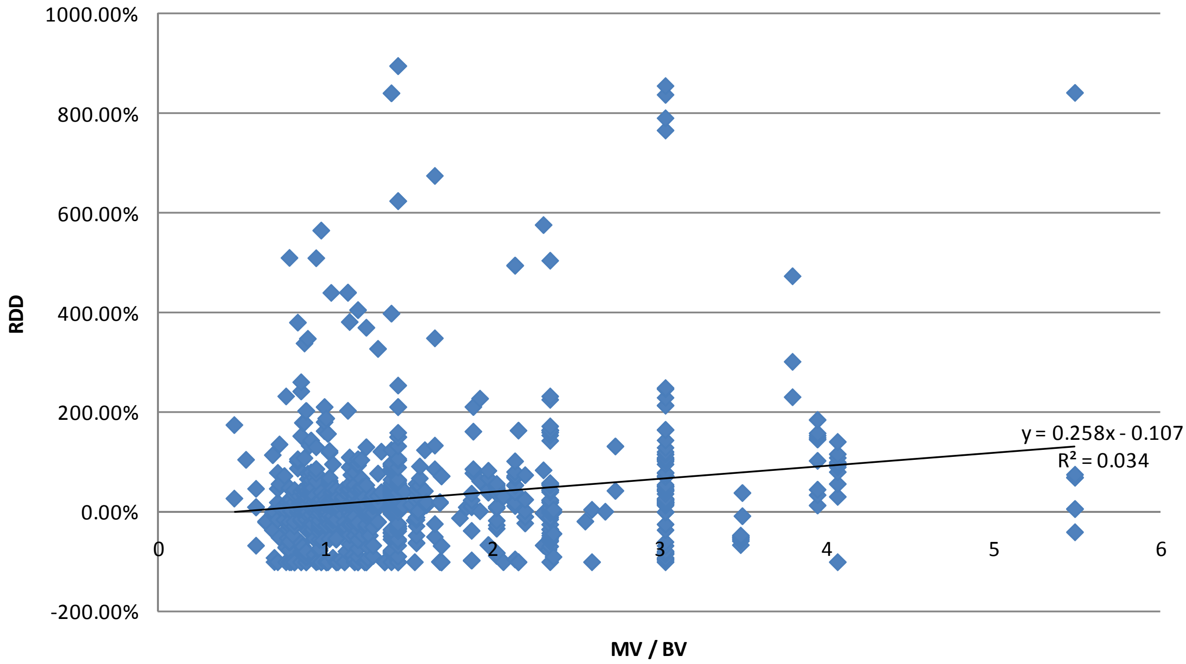


Figure 12.2: Annualized Return on Defaulted Debt vs. Total Liabilities to Book Value of Assets (MULGD Database 1987-2007)

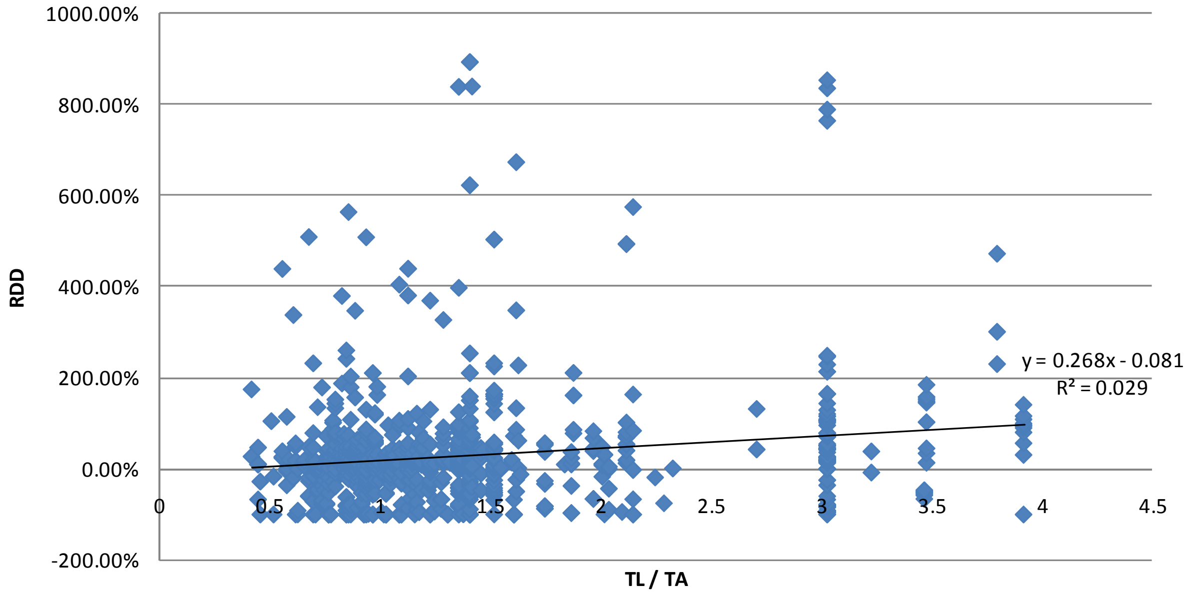


Figure 12.3: Annualized Return on Defaulted Debt vs. Free Asset Ratio (MULGD Database 1987-2007)

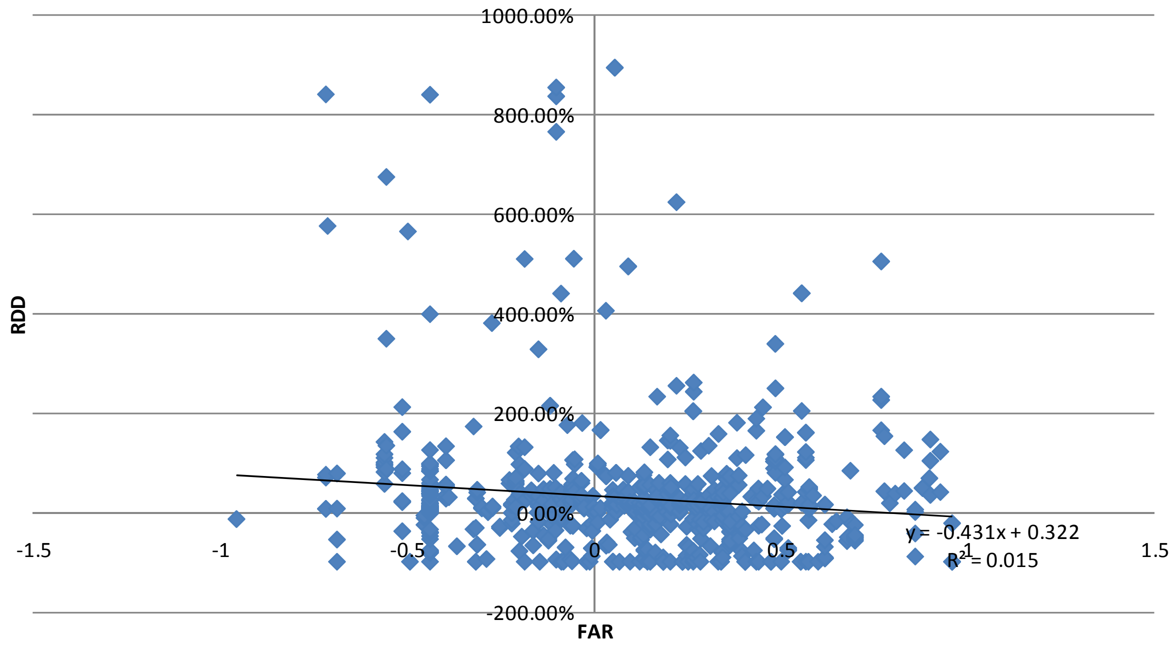


Figure 12.4: Annualized Return on Defaulted Debt vs. Cumulative Abnormal Equity Returns (MULGD Database 1987-2007)

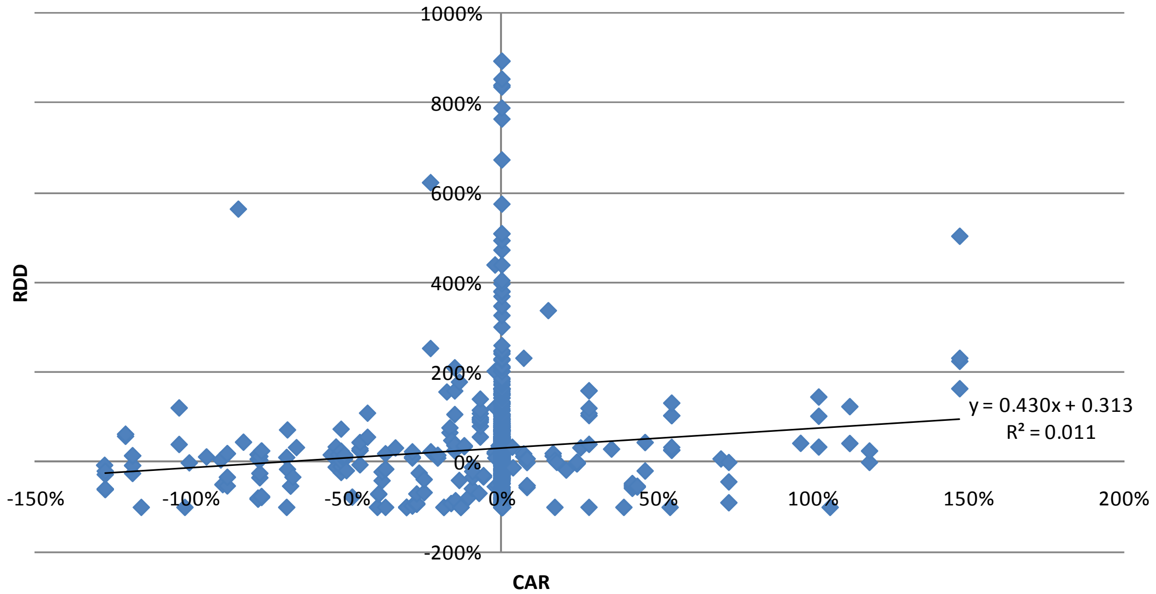


Figure 12.5: Annualized Return on Defaulted Debt vs. Percent Bank Debt (MULGD Database 1987-2007)

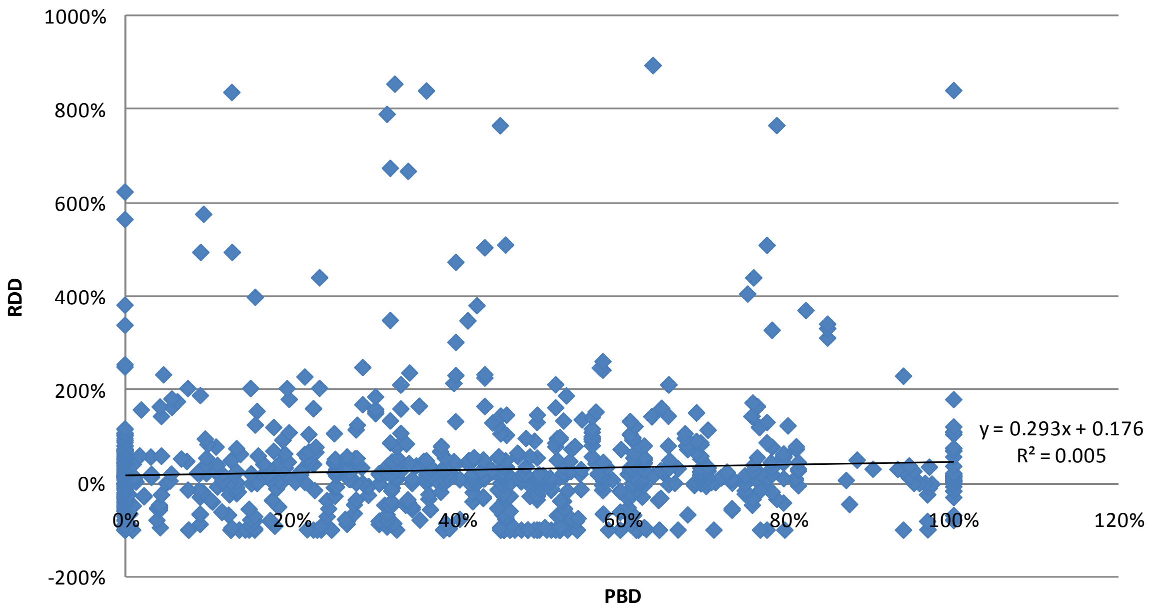
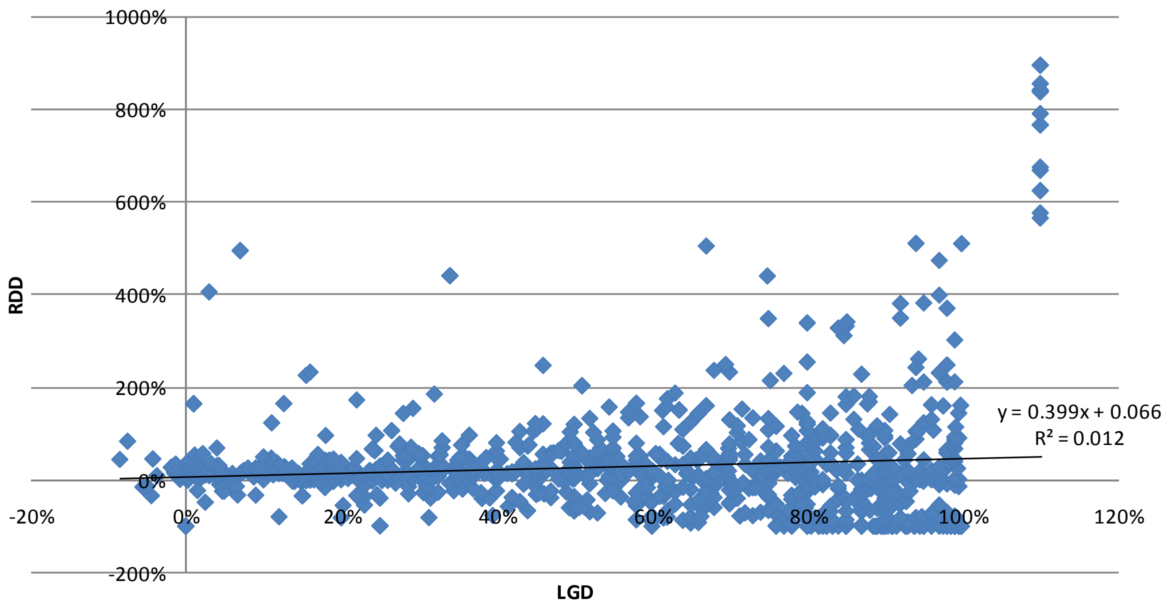


Figure 12.6: Annualized Return on Defaulted Debt vs. Loss-Given-Default (MULGD Database 1987-2007)



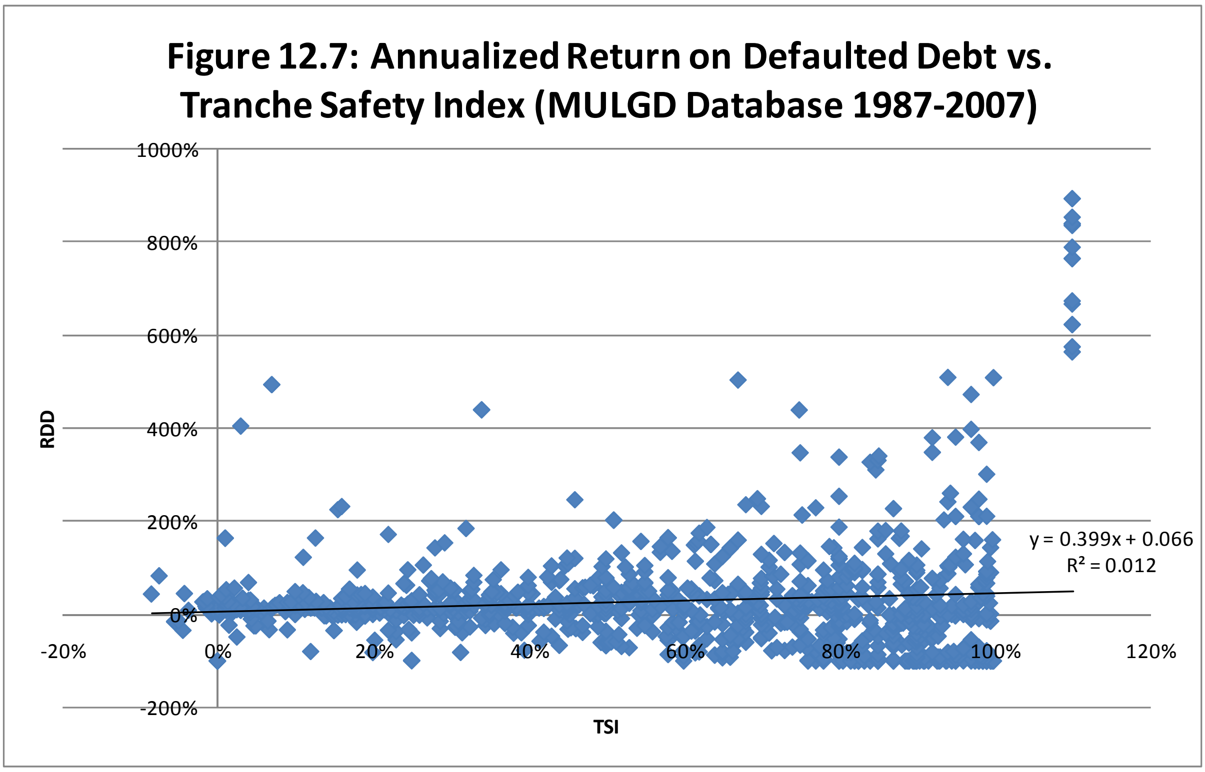


Figure 12.7: Annualized Return on Defaulted Debt vs. Tranche Safety Index (MULGD Database 1987-2007)
1000%
800%
600%
400%
200%
0%
-200%
RDD
-20%
0%
20%
40%
60%
80%
100%
120%
TSI
y = 0.399x + 0.066
R² = 0.012

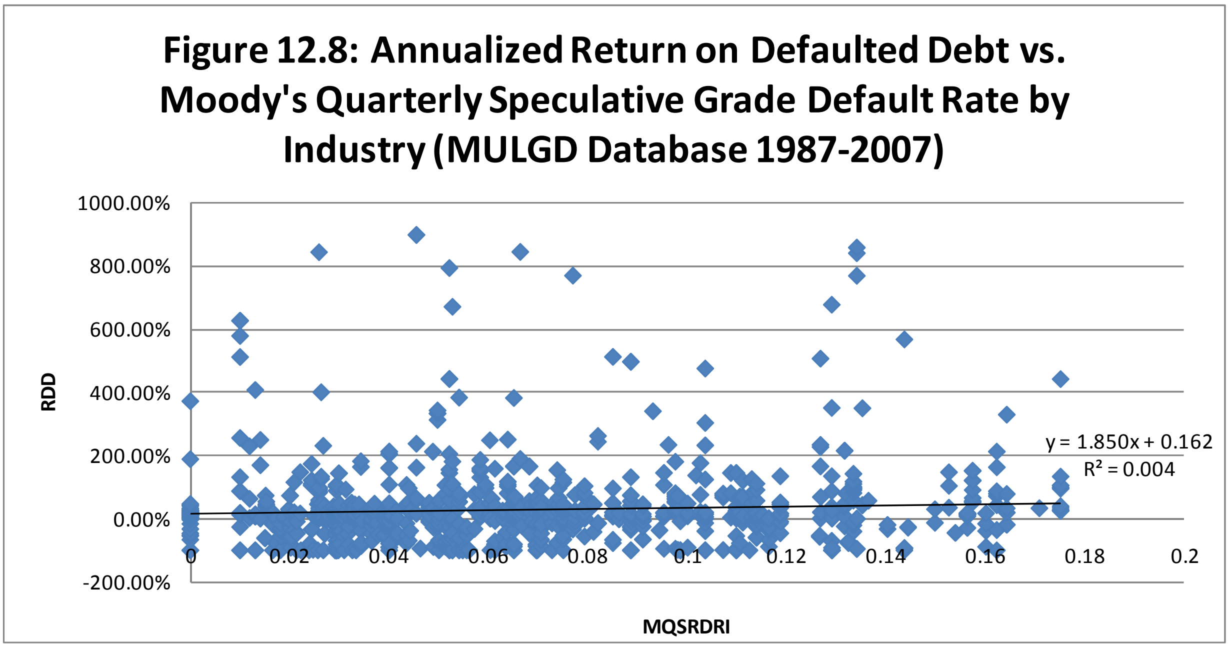


Figure 12.8: Annualized Return on Defaulted Debt vs. Moody's Quarterly Speculative Grade Default Rate by Industry (MULGD Database 1987-2007)
1000.00%
800.00%
600.00%
400.00%
200.00%
0.00%
-200.00%
RDD
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
MQSRDRI
y = 1.850x + 0.162
R² = 0.004

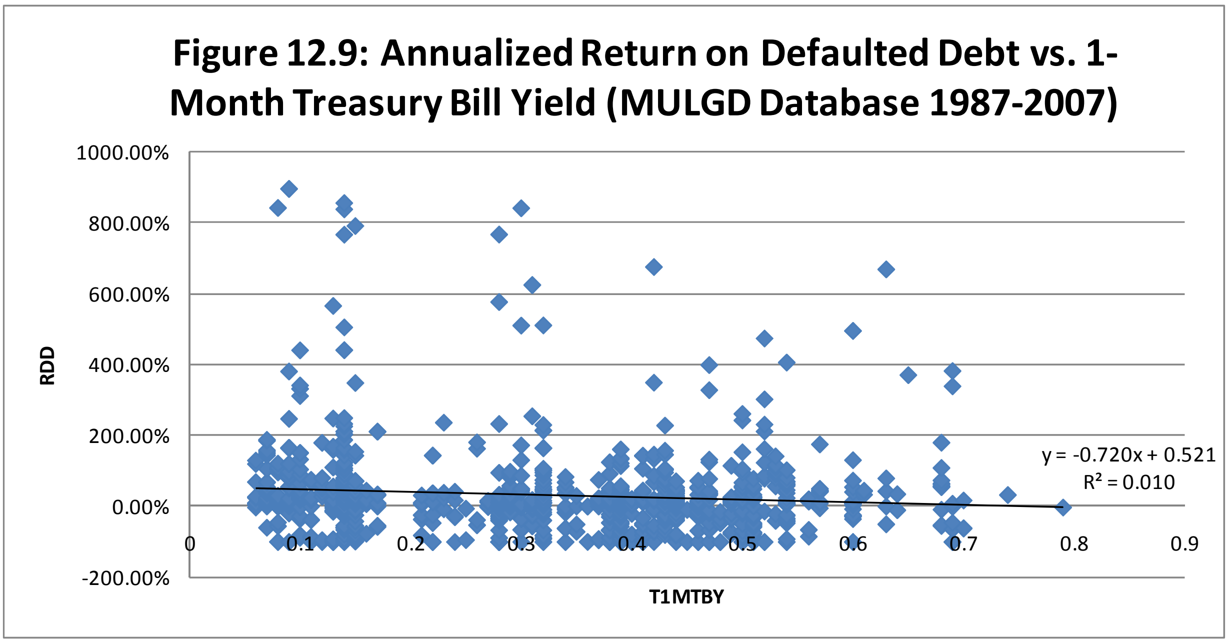


Figure 12.9: Annualized Return on Defaulted Debt vs. 1-Month Treasury Bill Yield (MULGD Database 1987-2007)
1000.00%
800.00%
600.00%
400.00%
200.00%
0.00%
-200.00%
RDD
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
T1MTBY
y = -0.720x + 0.521
R² = 0.010

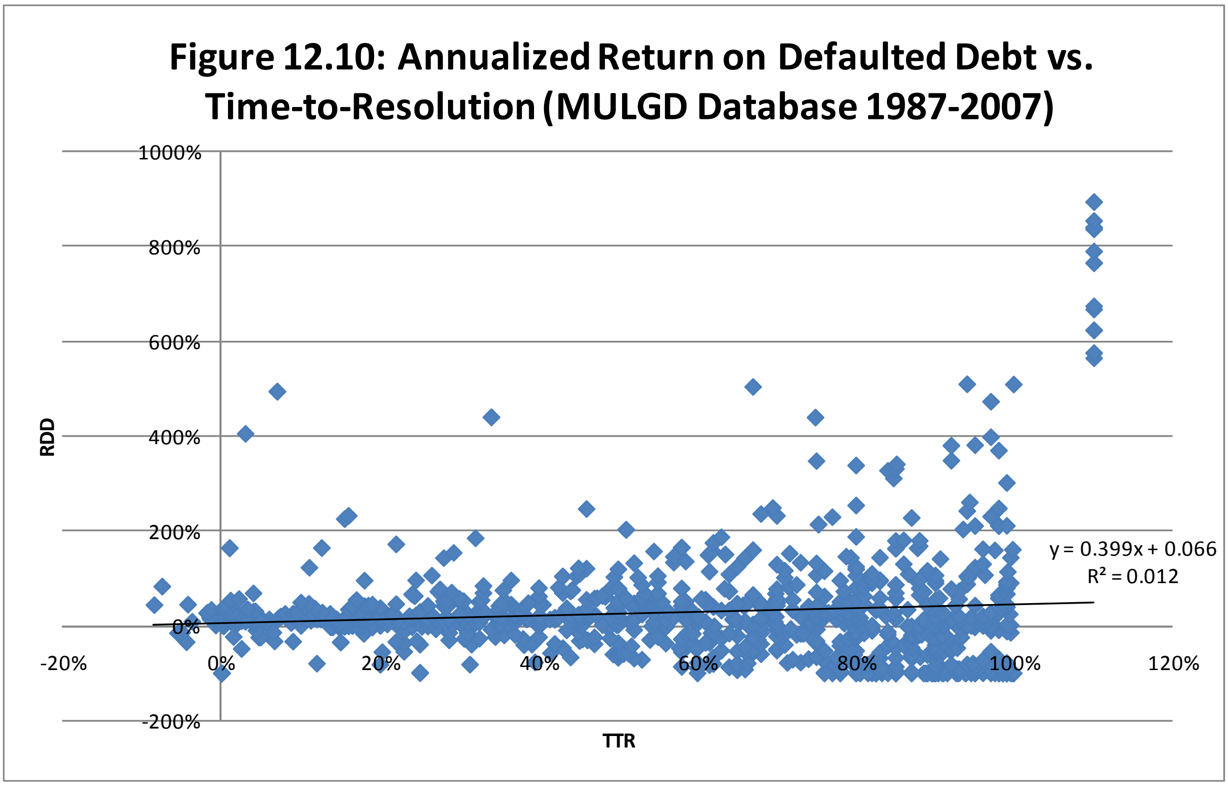


Figure 12.10: Annualized Return on Defaulted Debt vs. Time-to-Resolution (MULGD Database 1987-2007)
1000%
800%
600%
400%
200%
0%
-200%
RDD
-20%
0%
20%
40%
60%
80%
100%
120%
TTR
y = 0.399x + 0.066
R² = 0.012

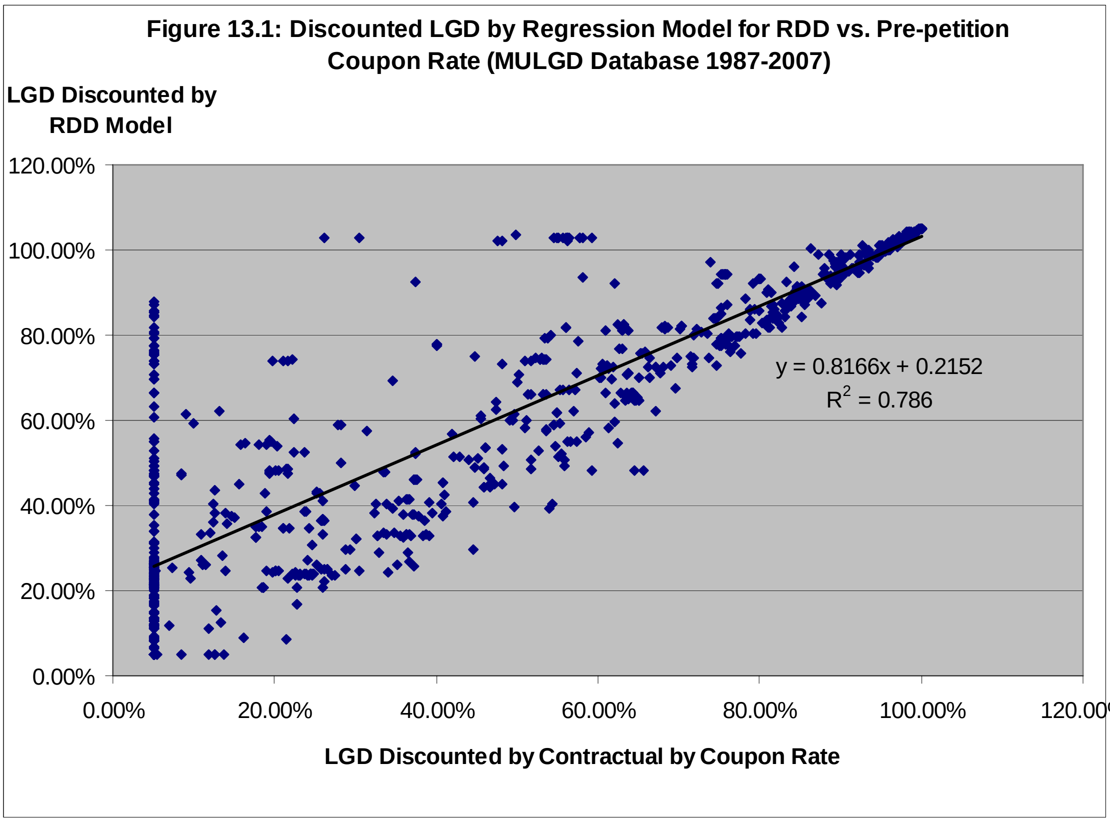


Figure 13.1: Discounted LGD by Regression Model for RDD vs. Pre-petition Coupon Rate (MULGD Database 1987-2007)
LGD Discounted by RDD Model
120.00%
100.00%
80.00%
60.00%
40.00%
20.00%
0.00%
y = 0.8166x + 0.2152
R2 = 0.786
0.00%
20.00%
40.00%
60.00%
80.00%
100.00%
120.00%
LGD Discounted by Contractual by Coupon Rate

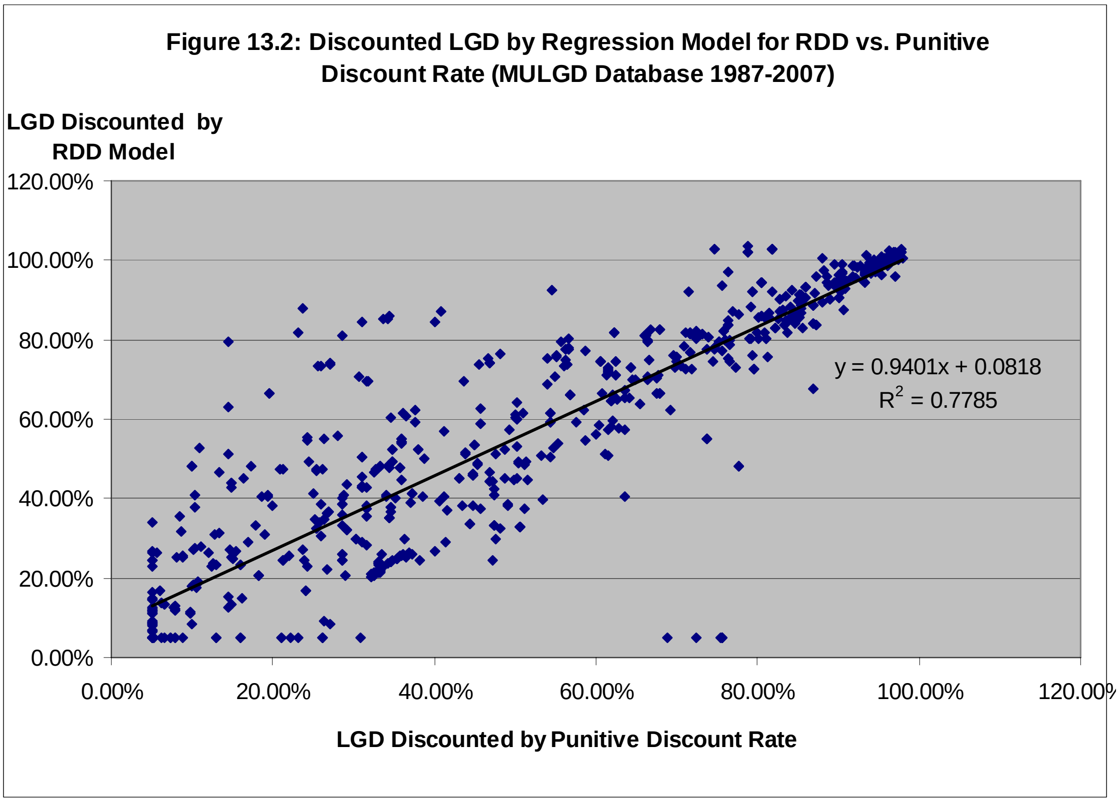


Figure 13.2: Discounted LGD by Regression Model for RDD vs. Punitive Discount Rate (MULGD Database 1987-2007)
LGD Discounted by RDD Model
120.00%
100.00%
80.00%
60.00%
40.00%
20.00%
0.00%
y = 0.9401x + 0.0818
R2 = 0.7785
0.00%
20.00%
40.00%
60.00%
80.00%
100.00%
120.00%
LGD Discounted by Punitive Discount Rate

Figure 13.3: Densities of LGD Discounted by RDD Model vs. Contract Rate

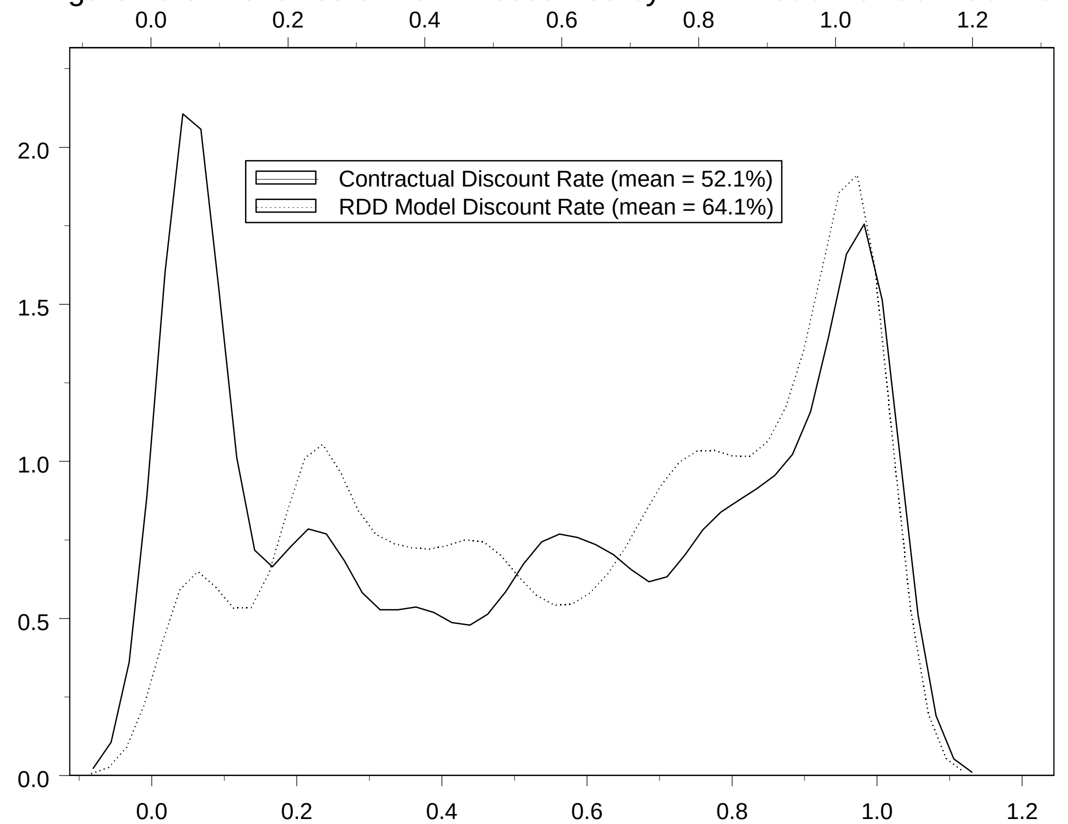
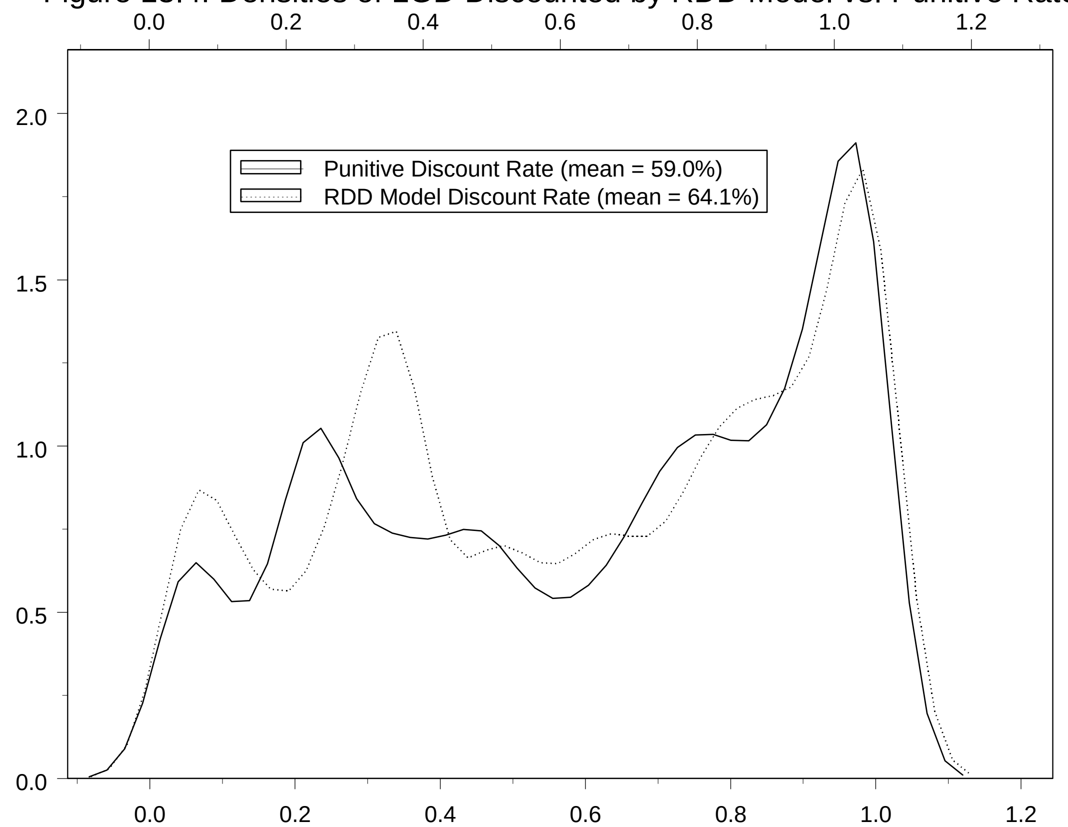


Figure 13.4: Densities of LGD Discounted by RDD Model vs. Punitive Rate



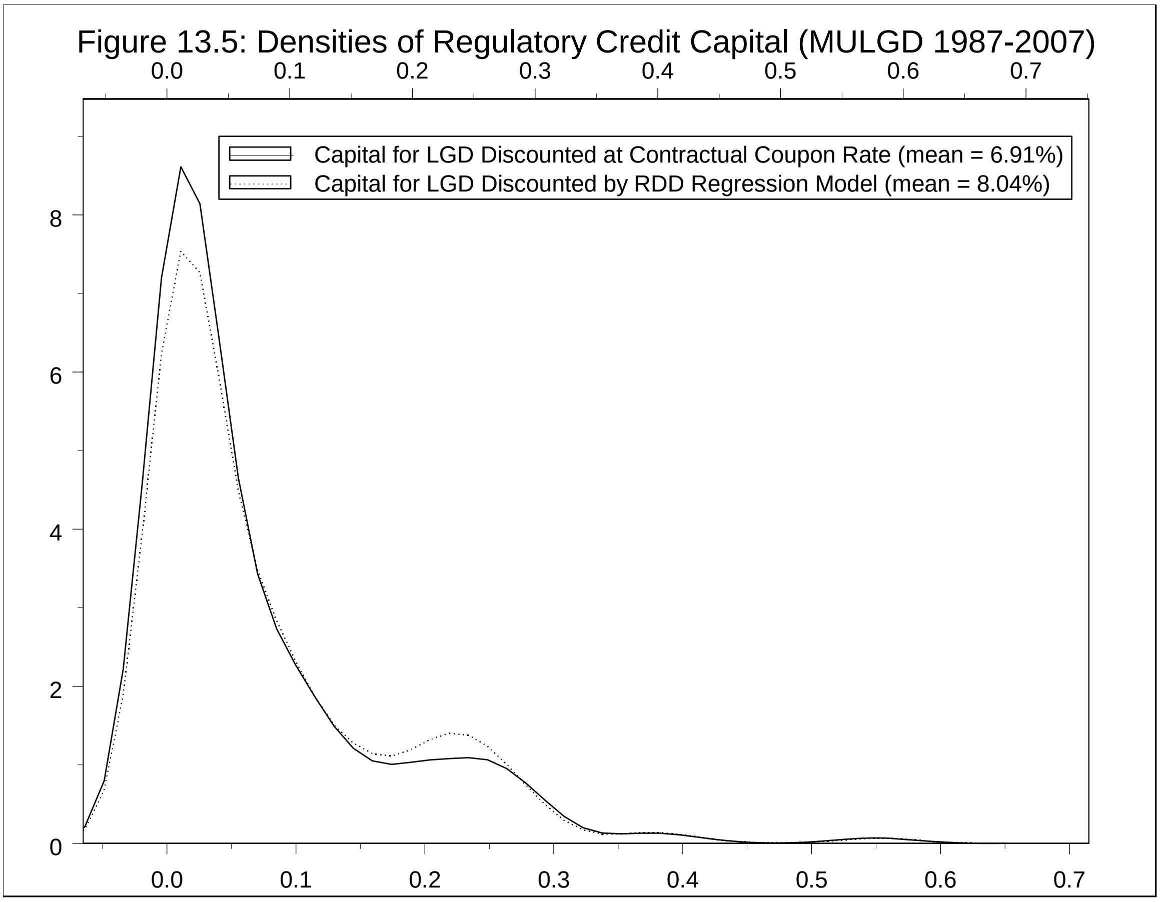


Figure 13.5: Densities of Regulatory Credit Capital (MULGD 1987-2007)
Capital for LGD Discounted at Contractual Coupon Rate (mean = 6.91%)
Capital for LGD Discounted by RDD Regression Model (mean = 8.04%)
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0
2
4
6
8

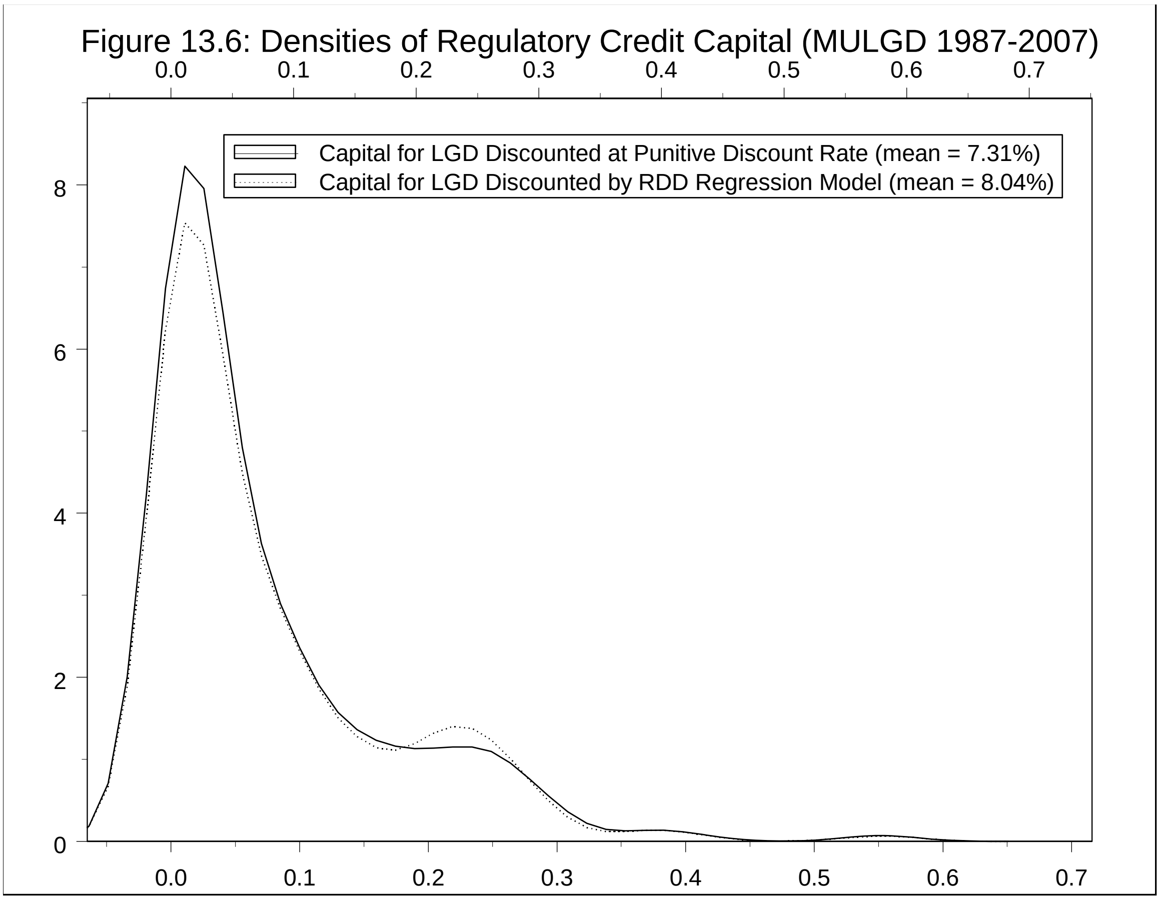


Figure 13.6: Densities of Regulatory Credit Capital (MULGD 1987-2007)
Capital for LGD Discounted at Punitive Discount Rate (mean = 7.31%)
Capital for LGD Discounted by RDD Regression Model (mean = 8.04%)
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0
2
4
6
8

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14. Appendix 1: Beta-Link Generalized Linear Model for RDD

Let the i[th] observation of dependent (or response) variable of interest, some measure of the LGD discount rate be denoted ε_i (e.g., the RDD), be observed independently. The vector of independent (or stimulus) variables corresponding to bounded random variable ε_i is denoted by $\mathbf{x}_i = \left(x_{1i},..x_{pi}\right)^{\mathrm{T}}$. We assume that the conditional expectation of $\varepsilon_i \in (l,u); l,u \in R$ depends upon a linear function $\eta : R^P \times R^P \to R$ of the $\mathbf{x}_i$ only through a smooth, invertible function $m : R \to R$:

$$E_P\left[\varepsilon_i \mid \mathbf{x}_i\right] = \mu = \int_l^u p\left(\varepsilon_i \mid \mathbf{x}_i\right)\varepsilon_i d\upsilon\left(\varepsilon_i\right) = m(\eta) \tag{14.1}$$

$$\eta = \boldsymbol{\beta}^T \mathbf{x}_i = m^{-1}(\mu) \tag{14.2}$$

Where $m^{-1}(.) \equiv \Lambda : R \to R$ is defined as the *link function* that maps from the conditional mean of the response variable μ to the linear function η. In this framework, the distribution of ε_i resides in the *exponential family*, membership in which implies a probability distribution function of the following form:

$$\mathrm{p}\left(\varepsilon_i \mid \mathbf{x}_i, \boldsymbol{\beta}, A_i, \zeta\right) = \exp\left[\frac{A_i}{\zeta}\left\{\varepsilon_i \theta\left(\mathbf{x}_i \mid \boldsymbol{\beta}\right) - \gamma\left(\mathbf{x}_i \mid \boldsymbol{\beta}\right)\right\} + \tau\left(\varepsilon_i, \frac{\zeta}{A_i}\right)\right] \tag{14.3}$$

Where $\gamma : R \to R$ and $\tau : R \to R$ are smooth functions (satisfying certain regularity conditions), A_i is a known *prior weight*, $\varsigma \in R^+$ is a *scale parameter* (possibly known), and the *location function* $\theta : R \to R$ is related to the linear predictor according to:

$$\theta\left(\mathbf{x}_i \mid \boldsymbol{\beta}\right) = (\gamma')^{-1}\left(\mu\left(\mathbf{x}_i\right)\right) = (\gamma')^{-1}\left(m\left(\boldsymbol{\beta}^T \mathbf{x}_i\right)\right) \tag{14.4}$$

This framework subsumes many of the models in the literature on the classical linear regression and limited /qualitative dependent variables framework.

We consider the case most relevant for RDD estimation, and that least pursued in the GLM literature. In this context, we are dealing with a random variable in a bounded region, with no loss of generality we assume to be the unit interval. This most conveniently modelled through employing a *beta distribution*, in which case if we denote the response percent RDDD rate as $\varepsilon_i \in [0,1]$, then the density is given by:

$$\mathrm{p}\left(\varepsilon_i \mid \mathbf{x}_i, \boldsymbol{\beta}\right) = \frac{\varepsilon_i^{\alpha\left(\boldsymbol{\beta}^T \mathbf{x}_i\right)-1}\left(1-\varepsilon_i\right)^{\beta\left(\boldsymbol{\beta}^T \mathbf{x}_i\right)-1}}{B\left[\alpha\left(\boldsymbol{\beta}^T \mathbf{x}_i\right), \beta\left(\boldsymbol{\beta}^T \mathbf{x}_i\right)\right]} \qquad \varepsilon_i \in [0,1]; \alpha, \beta : \mathbb{R} \to \mathbb{R}^{++} \tag{14.7}$$

Where $B[x,y]=\frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}=\int_0^1 u^{x-i}(1-u)^{y-1}du$ in (4.7) is the *standard beta function* , $\Gamma(x)=x!$ is the *gamma function*, and we allow the generalization in which the constant parameters governing the distribution are replaced by smooth, invertible functions $\alpha(.),\beta(.)$ of the linear predictors from the real-line to the positive half-plane $\mathbb{R}\to\mathbb{R}^{++}\times\mathbb{R}^{++}$. See Jacobs et al (2007) for a derivation of the canonical form (4.3) in this case. We follow Mallick and Gelfand (1994), in which the link function is taken as a mixture of cumulative beta distributions, the precise mixture of which being taking the form:

$$\theta(\mathbf{x}_i\,|\,\boldsymbol{\beta},\boldsymbol{\varphi},\mathbf{a},\mathbf{b})=\boldsymbol{\beta}^T\mathbf{x}_i=\sum_{j=1}^{k}\phi_j\int_{u=0}^{l_i}\frac{u^{a_j-1}(1-u)^{b_j-1}}{B\left[a_j,b_j\right]}du \quad (14.8)$$

Where the parameters $\boldsymbol{\varphi},\mathbf{a},\mathbf{b}$ are chosen to match features of the data. In this application, a mixture of 2 beta distributions was found to be sufficient. While in most cases we will not have a closed-form or even an analytic solution, we may always estimate the underlying parameters $\boldsymbol{\beta}$ consistently and efficiently by maximizing the log-likelihood function:

$$l\left(\theta(\boldsymbol{\beta}\,|\,\mathbf{x}_i),\zeta(\boldsymbol{\beta}\,|\,\mathbf{x}_i),\boldsymbol{\beta}\,|\,\mathbf{x}_i,\varepsilon_i,A_i\right)=\sum_{i=1}^{n}\left[\frac{A_i}{\zeta(\boldsymbol{\beta}\,|\,\mathbf{x}_i)}\left\{\varepsilon_i\theta_i(\boldsymbol{\beta}\,|\,\mathbf{x}_i)-\gamma(\boldsymbol{\beta}\,|\,\mathbf{x}_i)\right\}+\tau\left(\varepsilon_i,\frac{\zeta[\boldsymbol{\beta}\,|\,\mathbf{x}_i]}{A_i}\right)\right] \quad (14.9)$$

15. Appendix 2: Two-Factor Structural Credit Risk Model

In this appendix we outline the development of the likelihood function for a 2-factor extension of the asymptotic single risk factor model (Gordy, 2003), which underlies the Basel 2 framework for regulatory credit risk capital, incorporating systematic recovery risk. Let us denote asset value for the i^{th} segment of firms (these can be rating classes) by:

$$A_{t,r}=\rho_r X_t+\sqrt{1-\rho_r^2}Z_{t,r} \quad (15.1)$$

where $A_{t,r}$ is the asset value of the representative borrower in rating class r at time t, $Z_{t,r}\sim N(0,1)$ is the corresponding idiosyncratic risk factor, $X_t\sim N(0,1)$ (and independent of $Z_{t,r}$) is the systematic risk factor governing aggregate default rates at time t and the non-negative parameter ρ_r is the sensitivity (or loading) of assets in class r to the systematic risk factor (and ρ_r^2 is referred to as the asset value correlation). It follows that the conditional probability-of-default (PD) is given by the ubiquitous Vasicek (1987) formula:

$$R(X_t \mid PD_r, \rho_r) = \Pr\left(Z_{t,r} < \frac{\Theta^{-1}(PD_r) - \rho_r X_t}{\sqrt{1-\rho_r^2}} \right) = \Theta\left(\frac{\Theta^{-1}(PD_r) - \rho_r X_t}{\sqrt{1-\rho_r^2}} \right) \qquad (15.2)$$

where $R(X_t \mid PD_r, \rho_r)$ denotes the PD conditional as a function of the systematic risk factor, PD_r is the unconditional (or long-run) probability-of-default parameter for the r[th] rating and $\Phi(z) = \Pr(Z \le z) = \frac{1}{2\pi}\int_{-\infty}^{z} e^{-\frac{1}{2}u^2} du$ is the cumulative distribution function for a standard normal random variable and $\Phi^{-1}(p) = \inf_z \Pr(p \le \Phi(z))$ is the inverse of the distribution (or the quantile function). We can derive the distribution of the default rate (the realization of the conditional PD) in year t for rating class r, $dr_{t,r} \equiv L(x_t \mid PD_r, \rho_r)$, by a change-of-variables technique, as (15.2) is invertible. The systematic risk factor is:

$$X_t = \frac{\Phi^{-1}(PD_r) - \sqrt{1-\rho_r^2}\,\Phi^{-1}(dr_{t,r})}{\rho_r} \qquad (15.2)$$

Then, according to the formula

$$X \sim f_X(x), y = g(x) \to Y \sim f_Y(y) = \left| \frac{dg^{-1}(y)}{dy} \right| f_X\left(g^{-1}(y)\right) \qquad (15.3)$$

The distribution of $dr_{t,r}$:

$$f_{dr}\left(dr_{t,r} \mid PD_r, \rho_r\right) = \frac{\sqrt{1-\rho_r^2}}{\rho_r \phi\left(dr_{t,r}\right)} \phi\left(\frac{\Phi^{-1}(PD_r) - \sqrt{1-\rho_r^2}\,\Phi^{-1}(dr_{t,r})}{\rho_r} \right) \qquad (15.4)$$

Where $\varphi(z) = \frac{1}{2\pi} e^{-\frac{1}{2}z^2}$ is the normal density function.

We model the recovery side analogously, starting with a "loss process" $L_{t,s}$ at time t for seniority class s:

$$L_{t,s} = \rho_s Y_t + \sqrt{1-\rho_s^2}\, Z_{t,s} \qquad (15.5)$$

where $Z_{t,s} \sim N(0,1)$ is the corresponding idiosyncratic risk factor, $Y_t \sim N(0,1)$ (and independent of $Z_{t,C}$) is the systematic risk factor governing aggregate default rates at time t and the non-negative parameter ρ_s is the sensitivity (or loading) of assets in seniority class s to the systematic risk factor (and ρ_s^2 is the "loss correlation"). It follows that the conditional loss-given-default (LGD) is given by the ubiquitous Vasicek (1987) formula:

$$L(Y_t \mid LGD_s, \rho_s) = \Pr\left(Z_{t,s} < \frac{\Theta^{-1}(LGD_s) - \rho_s Y_t}{\sqrt{1-\rho_s^2}} \right) = \Theta\left(\frac{\Theta^{-1}(LGD_s) - \rho_s Y_t}{\sqrt{1-\rho_s^2}} \right) \qquad (15.6)$$

where $L(y_t \mid LGD_s, \rho_s)$ denotes the LGD conditional as a function of the systematic risk factor Y, LGD_c is the unconditional (or long-run) loss-given-default parameter for the s[th] seniority class. We can derive the distribution of the loss rate (the realization of the conditional LGD) in year t for seniority class S, $lr_{t,s} \equiv L(x_t \mid PD_r, \rho_r)$, by a change-of-variables technique, as (15.6) is invertible. The systematic risk factor is:

$$Y_t = \frac{\Phi^{-1}\left(LGD_s\right) - \sqrt{1-\rho_s^2}\,\Phi^{-1}\left(lr_{t,s}\right)}{\rho_s} \qquad (15.7)$$

Then, according to the (15.3) distribution of $lr_{t,s}$:

$$f_{lr}\left(lr_{t,s} \mid LGD_s, \rho_s\right) = \frac{\sqrt{1-\rho_s^2}}{\rho_s \phi\left(lr_{t,s}\right)} \phi\left(\frac{\Phi^{-1}\left(LGD_s\right) - \sqrt{1-\rho_s^2}\,\Phi^{-1}\left(lr_{t,s}\right)}{\rho_s}\right) \qquad (15.8)$$

We now derive the likelihood function for the model parameters. We assume that the systematic risk factors on the PD and LGD sides, X_t and Y_t, are bivariate normal with correlation r

$$\left(X_t, Y_t\right)^T \sim \Phi_2\left(\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}1 & r_{XY}\\ r_{XY} & 1\end{pmatrix}\right) \qquad (15.9)$$

where the $\Phi_2\left(x, y \mid \rho\right) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(x^2 - 2\rho xy + y^2\right)$ is bivariate standardized normal distribution is for zero-mean and unit variance random variables x, y with correlation ρ. The likelihood contribution for a representative instrument in r[th] rating and s[th] seniority class in year t has the integral form:

$$l\left(dr_{t,r}, dr_{t,s} \mid PD_r, \rho_r, LGD_s, \rho_s, r_{XY}\right) = \qquad (15.10)$$

$$\int_0^1\int_0^1 \frac{\sqrt{1-\rho_r^2}}{\rho_r \phi(u)} \phi\left(\frac{\Phi^{-1}\left(PD_r\right) - \sqrt{1-\rho_r^2}\,\Phi^{-1}(u)}{\rho_r}\right) \frac{\sqrt{1-\rho_s^2}}{\rho_s \phi(v)} \phi\left(\frac{\Phi^{-1}\left(LGD_s\right) - \sqrt{1-\rho_s^2}\,\Phi^{-1}(v)}{\rho_s}\right) \Phi_2\left(u, v \mid r_{XY}\right) du dv$$

Assuming independence across ratings, seniorities and years, the full log-likelihood is given by:

$$LogL\left(\left\{dr_{t,r}, dr_{t,s}\right\}_{t=,..,T;r=1,..,R;s=1,..,S} \mid \left\{PD_r, \rho_r, LGD_s, \rho_s\right\}_{t=,..,T;r=1,..,R;s=1,..,S}, r_{XY}\right) = \qquad (15.11)$$

$$= \sum_{t=1}^{T}\sum_{r=1}^{R}\sum_{s=1}^{S} \log\left[l\left(dr_{t,r}, dr_{t,s} \mid PD_r, \rho_r, LGD_s, \rho_s, r_{XY}\right)\right]$$