Convertible bonds and bank risk-taking

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Abstract

We study the effect of going-concern contingent capital on bank risk choice. Optimal conversion ahead of default forces deleveraging in highly levered states, when risk incentives are worse. The equity infusion reduces endogenous risk shifting by diluting returns in high states. Interestingly, contingent capital may be less risky in equilibrium than traditional debt, as its lower priority is compensated by reduced endogenous risk. Its effectiveness in risk reduction depends critically on the informativeness of the trigger.

Keywords: Banks; Risk shifting; Financial Leverage; Contingent Capital

JEL Classifications: G13, G21, G28.

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1 Introduction

During the recent credit boom, bank capital had fallen at historical lows. In the subsequent crisis, banks could not absorb asset losses, leading to credit market disruption and spillovers to the real economy. Regulatory reform has called for more bank equity to ensure ex post risk absorption by shareholders, as well as to reduce ex ante incentives for excess risk.

Under Basel III rules, the new capital ratios may be satisfied only by common equity. Yet there is support for allowing contingent capital to count for extra buffers, such as those for SIFI. This form of long term debt (called also contingent convertible, or CoCo bonds) automatically converts to equity upon a trigger signaling reduced solvency. So called bail-in capital converts into equity only upon bank insolvency, when equity is worthless. This protects other lenders, but does not have an effect on asset risk in equilibrium. The more interesting version is ”going-concern” contingent capital, where debt may convert in a timely fashion, ahead of distress.

Originally proposed by Flannery (2002), the case for this form of contingent capital has been carefully outlined in Kashyap et al. (2008). A recent literature has discussed its design in terms of reducing financial distress costs and deposit insurance losses (Albul et al. (2010), McDonald (2011), Pennacchi (2011), Pennacchi et al. (2011)).

While most authors argue that contingent capital reduce risk shifting incentives (asset substitution), for tractability their models assume asset risk is exogenous and unaffected by the introduction of CoCo bonds, focusing on their ex post buffering effect.[1]

In our model, asset risk is a choice that reflects bankers' incentives, which deteriorate as leverage increases. Our basic result is that the chance of conversion in high leverage states reduces ex ante risk shifting. The intuition is that conversion dilutes high returns, discouraging gambling. CoCo effectiveness is shown to depend on the precision of the trigger, which optimally should deliver deleveraging when this is most valuable, namely when risk incentives deteriorate.

There are clear trade offs in CoCo design. A higher trigger and larger CoCo amount lead to more frequent and larger conversions respectively, and a higher equity content. We show that increasing the amount of CoCo ratio capital ultimately becomes counterproductive. Once a very large conversion at a fixed conversion ratio delivers a capital gain to equityholders, this increases their risk incentives. [2]

[1]A partial exception is Chen et al. (2013), who analyze endogenous strategic default and show that conversion reduces its frequency.

[2]Value transfers cannot be ruled out by varying the conversion ratio, unless the bonds may convert in an infinite amount of shares. Such a contractual feature would be impossible in reality, not least for legal reasons.

As a result of this tradeoff, there is an optimal design in terms of the trigger level and optimal amount of contingent capital, even in the absence of issuance or bankruptcy costs.

CoCos are incorrectly considered a package of conventional bonds and a short position in a put option on the value of assets. This neglects their risk-reducing effect, which reduces the value of their short put position. (It also ignores the fact that deposit insurance also bears some risk). We obtain the interesting result that optimally designed CoCo bonds may be in equilibrium safer than conventional bank bonds, because they reduce endogenous asset risk.

The model allows to measure how well contingent capital compares with straight equity. More CoCo debt may need to be issued to substitute for equity in terms of risk reduction. However, we show that this ratio declines as trigger precision improves.

In order to focus on their risk prevention effect, CoCo bonds are more stylized in our setting relative to other models in the literature (Albul et al. (2010), Bolton and Samama (2012), Glasserman and Nouri (2012), Hilscher and Raviv (2011), Koziol and Lawrenz (2012), Madan and Schoutens (2010)). Alternative approaches are offered in Duffie (2010) and McDonald (2011), where the case of conversion in a systemic crisis is examined. A specific design aimed at containing endogenous risk is sketched by Squam Lake Working Group, who propose banker compensation to be based on gradual vesting of contingent bonds.

Similarly to the existing literature, our analysis of the regulatory framework is limited, as we take initial bank leverage as exogenous for the sake of tractability. In principle, an optimal capital ratio already trades off some cost of bank equity capital against endogenous risk shifting. Deposit insurance risk is also not priced. (This is partially justified in our set up, where deposit insurance losses are a transfer among risk neutral agents, and would be zero in the absence of deliberate risk taking.) Changing this assumption would not alter our basic results, though for banks with very high leverage, for which even conversion cannot restore risk incentives, a different policy tool would be needed.

In general, CoCos remain less effective than equity at risk control, so they may be justified only as a cheaper solution for bank shareholders. Just as capital requirements, they are less effective at controlling deliberate exposure to tail risk (Perotti et al. (2011),Chen et al. (2013)).

Section 2 presents the basic model, and Section 3 shows how CoCo design affect the banker's risk taking incentives. Section 4 compares the risk-reducing effect of CoCos against equity and convertible debentures converted at will, which also have been proposed as a solution to risk-shifting (Green, 1984). Section 5 concludes. All proofs are in Appendix.

2 The Model

2.1 Setup

The bank has an exogenous amount of debt D, which may include CoCo bonds of amount C and deposits $D - C$. Issuing CoCo bonds substitute a part of deposits, which drop to $D - C$, so the initial leverage does not change. We assume that it is mandatory for the bank to issue CoCos.[3] The deposit rate is normalized to zero. Bank deposits are insured.

The bank is financed with $1 - D$ of equity invested at $t = 0$, so as to satisfy an exogenous capital requirements of $1 - D$. The assets are not risk-weighted. The initial assets value at $t = 0$ is 1, so there is no excess capital. Deposit interest rate is normalized to zero.

There is one active agent in the model: the banker/bank owner[4]. Borrowers are price-takers, so lending is represented as an asset choice by the banker. Depositors are insured and passive. Agents are risk-neutral and rational.

At $t = 1$, asset value is subject to an exogenous shock $V_1 = 1 + \zeta$, where ζ is uniformly distributed over $[-\delta, \delta]$. We denote the realization of interim asset value V_1 as $v \in [1 - \delta, 1 + \delta]$, observed only by banker. The interim leverage is then $\frac{D}{v}$.[5] .

The banker chooses whether to exert effort ($e = 1$) to control credit risk or not ($e = 0$). Effort is costless. The banker's payoff is the value of the original bank equity at $t = 2$.

Depending on her choice, asset values at $t = 2$ may have two outcomes, safe or risky. If the banker exercises risk control, assets at $t = 2$ produce a safe payoff with gross rate of return 1. Alternatively, without risk control, at $t = 2$ a risky credit strategy has payoff V_2, where V_2 follows a distribution $F(V_2)$ with density function $f(V_2)$, mean $\mathbf{E}(V_2) = v - z$, and standard deviation σ, where $z > 0$. Thus, the riskier strategy yields a lower mean payoff relative to the safer asset choice.[6]

After the risk choice is made, the value v is revealed with probability φ to all investors. A riskier strategy may enhance equity in high leverage states. To ensure bank solvency under a safe strategy, we assume that the maximum interim asset drop never fully wipes out equity, namely $1 - \delta - D \geq 0$. [7]

[3]Later we show that the banker never issues CoCos voluntarily at $t = 0$.

[4]We assume the bank manager is the sole shareholder, to focus on the interaction of the share price and risk-taking incentives, rather than on the agency conflict between the manager and the shareholder.

[5]We assume that no bank equity may be raised at time $t = 1$ if leverage turns out to be high.

[6]As a result, the distribution of asset return in the safe outcome has second-order dominance relative to risky outcome, though not first-order dominance.

[7]See Appendix for the discussion of the case when this assumption is relaxed (see Appendix).

CoCos are automatically converted into equity when the interim asset value v falls below a pre-specified trigger level v_T at $t = 1$. If $v > v_T$, conversion does not occur. Trigger value is initially set lower than the initial book value 1, else there is an immediate conversion at $t = 0$. To simplify the analysis, the interim coupon rate is normalized to zero.

The trigger value v_T is set by the regulator together with the required amount of CoCos C. Regulator's objective is to minimize ex ante risk, i.e probability of bank risk-taking.

The conversion ratio, modeled along existing CoCo bonds, is the ratio of nominal value over the trigger asset value minus debt: $d = \frac{C}{v_T - D}$.[8] After conversion, the amount of shares is $d + 1$. Note that the banker is never wiped out unless the value of CoCos is also zero. The payoff structure is presented in the Figure 1.

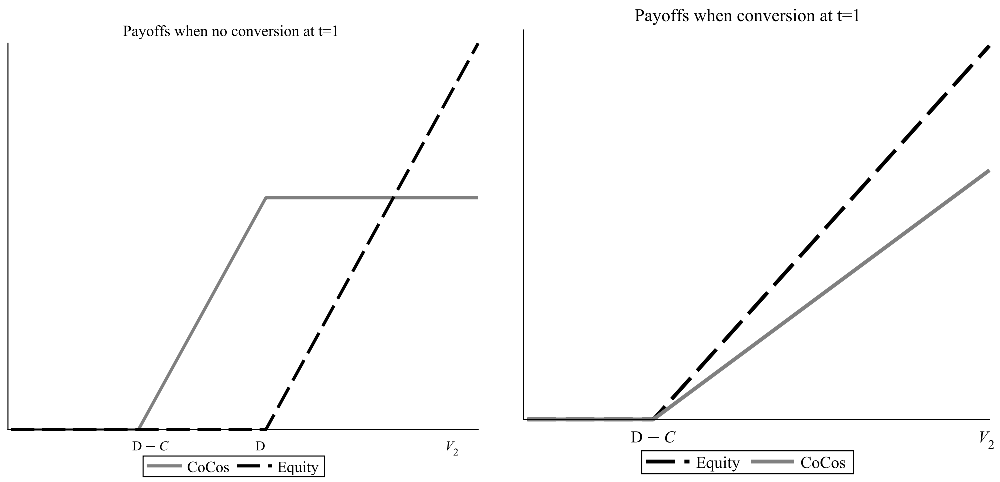

Figure 1: Payoff of bondholders and shareholders in case of no conversion and conversion at $t = 1$ ($d < 1$)

The sequence of events is presented in Figure 2.

We consider now what CoCo design improves banker's risk incentives. Intuitively, the trigger should induce CoCos conversion when bank interim leverage is high enough to create poor risk control incentives, but conversion is unnecessary in well capitalized banks.

[8]The fixed conversion ratio produces value redistribution at conversion as soon as v is strictly below v_T.

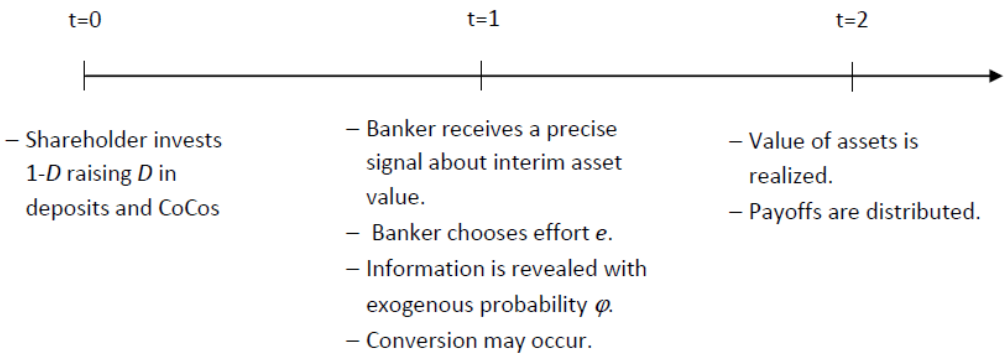

Figure 2: The sequence of events

2.2 Results

2.2.1 The risk taking incentive

The banker bases her risk decision on the expected payoff, conditional on being solvent. For very low realizations of asset values the bank will default, wiping out also CoCo holders and forcing a payment by the deposit insurance fund.

The expected banker payoff from a risky asset choice is:

$$(1 - F(D)) \cdot \mathbf{E}(V_2 - D | V_2 > D) = \int_D^{\infty} (V_2 - D) f(V_2) dV_2 \tag{1}$$

which can be presented as the sum of its unconditional mean $\mathbf{E}(V_2 - D) = v - z - D$ (which may be negative) and a measure of the right tail return in solvent states:

$$\begin{aligned} \int_D^{\infty} (V_2 - D) f(V_2) dV_2 &= v - z - D - \int_{-\infty}^{D} (V_2 - D) f(V_2) dV_2 && (2) \\ &= v - z - D + \Delta(v, D, \sigma) && (3) \end{aligned}$$

Here $\Delta(v, D, \sigma) = -\int_{-\infty}^{D} (V_2 - D) f(V_2) dV_2$ is the value of the put option (also called Merton's put) enjoyed by shareholders under limited liability. It also affects the temptation of the banker to shift risk, defined as the return difference between a risky and safe strategy for the banker:

$$(1 - F(D)) \cdot \mathbf{E}(V_2 - D | V_2 > D) - (v - D) = -z + \Delta(v, D, \sigma)$$

From now on we refer to the return $\Delta(v, D, \sigma)$ as $\Delta(v)$, the measure of bank risk shifting incentives. Next, we characterize how its value depends on the specific distribution of asset risk.

Note that if the risky payoff is normally or uniformly distributed, risk shifting incentives $\Delta(v)$ are monotonically increasing and convex in leverage. Moreover, risk shifting incentives increase with a higher volatility of risky asset σ. (See Appendix for details.)

Without any specific assumption on $f(V_2)$, we assume that the risk incentive function has a similar structure as under normal or uniform distribution.

Assumption 1. *Risk shifting incentives $\Delta(v)$ are an increasing and convex function of leverage $\frac{D}{v}$: $\Delta'(v) \leq 0, \Delta''(v) \geq 0$. Also $\Delta(v)$ are increasing with σ: $\Delta'(\sigma) \geq 0$.*

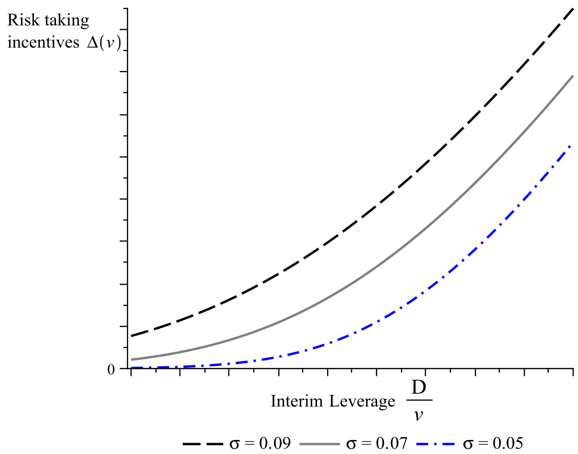

Figure 3: Risk incentives under Gaussian risk distribution

For a normal distribution, risk shifting incentives are given by:

$$\Delta(v) = (v - D - z) \cdot \left[\Phi\left(\frac{v - D - z}{\sigma}\right) - 1\right] + \sigma \cdot \phi\left(\frac{v - D - z}{\sigma}\right) \tag{4}$$

2.2.2 Bank risk without convertible bonds

First, we consider the risk choice of the banker in the absence of convertible bonds $C = 0$.

The banker compares the payoff from the risky and the safe asset. The banker's program is:

$$\begin{aligned} &\max_e e \cdot (v - D) + (1 - e) \cdot (v - z - D + \Delta(v)) \\ &\text{s.t. } e \in \{0, 1\} \end{aligned} \tag{5}$$

Under the Assumption 1, the optimal effort choice by the banker takes the form:

$$e = \begin{cases} 1 & \text{if } v \geq \Delta^{-1}(z) \equiv v^* \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

We denote as $v^* \equiv \Delta^{-1}(z)$ the cut-off interim asset value, above which the banker exerts effort without conversion. At $v = v^*$ the net present value of the the banker's choice of a risky lending strategy is zero.

For normal distribution function the cut-off interim asset value v^* is given implicitly by:

$$(v - D - z) \cdot \left[\Phi \left(\frac{v - D - z}{\sigma} \right) - 1 \right] + \sigma \cdot \phi \left(\frac{v - D - z}{\sigma} \right) = z \tag{7}$$

Lemma 1. *If at the interim period leverage is low ($v \geq v^*$), the banker exerts effort in order to control risk. If $v < v^*$, she does not. Moreover, the ex ante probability that the banker will choose at $t = 1$ to control risk ($\frac{1-\delta-v^*}{2\delta}$) decreases with the volatility of risky asset σ.*

Note that the asset value revelation of v does not have any effect on the banker's risk incentives, as disclosure does not change leverage.

3 Optimal CoCo design

This section considers the bank which has CoCos. It studies how the banker's incentives change if the bank issues convertible bonds, finds their optimal trigger level v_T and the amount of CoCo debt C.

3.1 Trigger value

First, we look at the banker's risk decision under a given amount of CoCos C with a given trigger v_T.

From Lemma 1, setting a trigger asset value higher than v^* does not change risk incentives for low leverage banks (with $v \geq v^*$). This enables us to restrict the range of trigger values to the interval $v_T < \min[v^*; 1]$.[9]

[9] We later show that this is efficient, as dilution which does not affect risk incentive may be counterproductive.

Next, consider the banker's program:

$$
\begin{aligned}
&\max_e e \cdot [\underbrace{(v-D)\cdot(I(v \geq v_T) + (1-\varphi)\cdot I(v < v_T))}_{\text{equity value if no conversion and } e=1} + \\
&\underbrace{\frac{v-D+C}{d+1}\cdot\varphi\cdot I(v < v_T)}_{\text{equity value if conversion and } e=1}] + \\
&(1-e)\cdot[\underbrace{(v-z-D+\Delta(v))\cdot(I(v \geq v_T)+(1-\varphi)\cdot I(v<v_T))}_{\text{equity value if no conversion and } e=0} + \\
&\underbrace{\frac{v-z-D+C+\Delta(v+C)}{d+1}\cdot\varphi\cdot I(v<v_T)}_{\text{equity value if conversion and } e=0}] \\
&\text{s.t. } e \in \{0,1\}
\end{aligned}
\tag{8}
$$

where $I(\cdot)$ is an indicator function, $d = \frac{C}{v_T - D}$ is the conversion ratio, and $\Delta(v+C) = -\int_{-\infty}^{D-C}(V_2 - D + C)f(V_2)dV_2$ is risk taking incentives under CoCos conversion. .

Figure 4 shows that the effort choice may not be monotonic in the interim asset value.

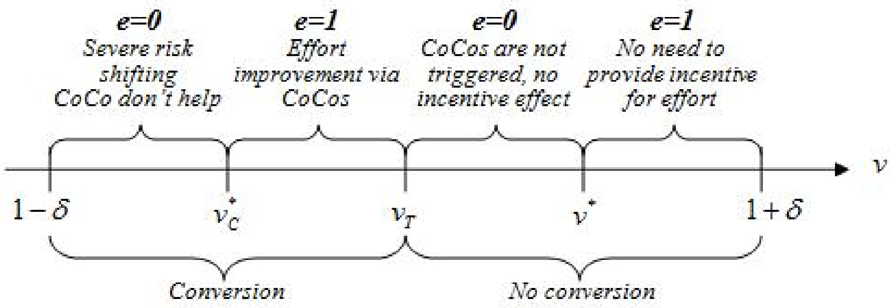

Figure 4: Risk incentives

There are two critical interim asset values. The first is v^*, the threshold for effort even when no conversion takes place. The second is v_C^*, the value of interim assets above which the introduction of CoCos improves effort.

Intuitively, risk incentives may improve with CoCos only if $\varphi > 0$, that is, if the trigger is informative about poor incentives and forces recapitalization in the right states.

Lemma 2. *The introduction of CoCos improves effort for banks with $v_C^* \leq v \leq v_T$. Banks with extremely high leverage $v < v_C^*$ do not change their effort choice since their risk-shifting return is too large. Banks with $v > v_T$ are not affected.*

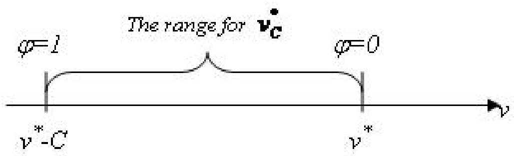

Figure 5: Cut-off value v_C^*

A bank with $v < v_C^*$ has such high leverage that CoCos can not improve its risk-shifting incentives[10].

Note that the difference $\frac{v_T - v_C^*}{2\delta}$ measures the expected improvement in risk incentive $\mathbf{E}(\Delta e)$ induced by CoCos. It is strictly decreasing in v_C^*. It is easy to see that v_C^* is in the range $[v^* - C, v^*]$ and decreases with the probability of information revelation φ (see Figure 5).

Proposition 1. *The trigger value is optimally set at $v_T = v^*$, which maximizes the expected effort $\frac{v_T - v_C^*}{2\delta}$ for a given amount of CoCos C.*

Figure 4 shows that unless the trigger v_T is chosen optimally, risk incentives are not necessarily monotonic in v. If the trigger is too high (above v^*), CoCos will not affect effort. But if it is too low (below v^*), there will be no conversion for an intermediate range of highly levered banks. This is clearly inefficient. As it is easier to induce effort for higher v, so it cannot be efficient to allow effort to fall as v increases.

As a result, setting the trigger to $v_T = v^*$ guarantees the monotonicity of incentives with respect to leverage, as shown in Figure 6.

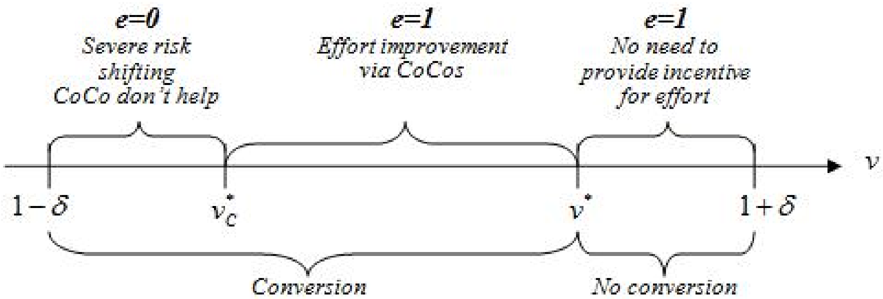

Figure 6: Risk incentives with restricted trigger asset value $v_T = v^*$

The optimal trigger value v^* depends on the risky opportunities available to the banker. A higher asset volatility increases the risk shifting return, which becomes attractive to the banker

[10]If CoCos are large enough ($v_C^* < 1 - \delta$), this range does not arise, and all banks with $v < v_T$ have incentives to contain asset risk.

for a larger range of interim values v. Intuitively, the trigger value should be raised to adjust incentives when asset values are riskier in a mean-preserving sense.

Lemma 3. *A higher asset volatility requires that the trigger value be raised to maintain risk-shifting incentives.*

3.2 Optimal amount of Contingent Capital

Having set v_T, we now seek to optimize risk incentives by varying the amount of CoCos.

Convertible bonds have two effects on the banker's effort for low interim asset values $v \leq v^*$. We can separate two effects: an equity dilution and a CoCo dilution effect.

Proposition 2. *The potential reduction in the banker's equity due to CoCo increases effort incentives when risk-shifting is most severe. The value transfer from CoCo to equity may discourage effort.*

The equity dilution effect arises because the chance of conversion reduces the banker's share of high payoffs, reducing the return to risk shifting.[11] This effect is more pronounced for highly levered banks.

Second, conversion leads to a value transfer from CoCo to equity due to the fixed conversion ratio. This may reduce effort. Figure 7 illustrates two effects.

When the amount of CoCos is so large that conversion exceeds what would be required to eliminate all risk shifting incentives, CoCo dilution effect is excessive. Recall that effort is both risk-reducing and value increasing. Thus, the disincentivizing CoCo dilution effect is strongest for low levered banks $v \geq v^* - C$, for which the risk shifting effect is limited.

This suggests there is an optimal amount of CoCo funding, which trades off reducing risk shifting while maintaining incentives for value enhancement.

Expected effort $\mathbf{E}(e)$ ireflects the range of states v when the banker exerts effort, and equals $\frac{1+\delta-v_C^*}{2\delta}$. In the Appendix we show the effect of an increase in the amount of CoCos, disentangling equity dilution and CoCo dilution effects.

[11]Note that this result match the intuition in Green's (1984) model of convertible debt. However, here conversion is automatic and occurs earlier, before risk is fully realized.

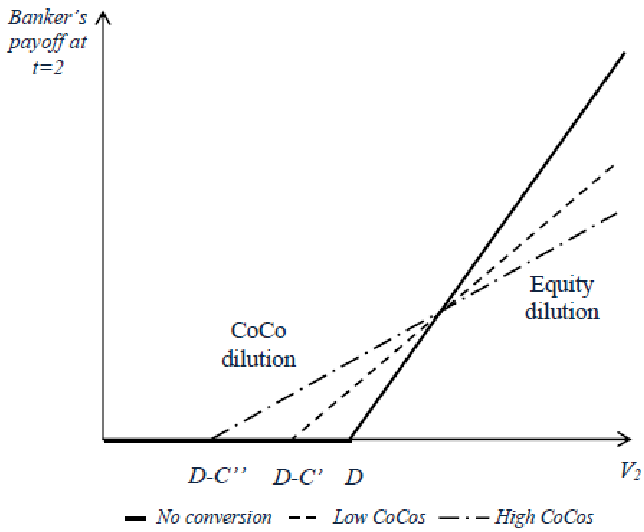

Figure 7: Equity and CoCo dilution effects

Proposition 3. *Expected effort increases with the amount of CoCos up to a threshold C^*, and then declines. Thus, there exists an optimal amount of CoCos in terms of effort improvement.*

$$\Delta'_C(v + C^*)(C^* + v_T - D) - \Delta(v + C^*) + z = 0 \tag{9}$$

Figure 8 shows effort improvement under the uniform distribution[12].

Lemma 4. *The amount of CoCos and trigger value act as substitutes in reducing risk.*

Thus a lower trigger value can be compensated by a higher amount of CoCos to achieve the same risk incentives. Intuitively, a less frequent conversion can be compensated by a larger dilution.

We next look at how key parameters on the economic environment (risky asset volatility σ, probability of information revelation φ) affect the expected improvement in effort.

Proposition 4. *For an exogenously given trigger value, the expected effort improvement $\frac{v_T - v_C^*}{2\delta}$ decreases in the volatility of risky asset (σ), since the risk shifting incentives grow with σ.*

Lemma 5. *Higher σ implies a higher optimal trigger value:* $\frac{\partial v^*}{\partial \sigma} \geq 0$.

[12]The graph uses the parameter values $D = 0.93, z = 0.04, \delta = 0.07, \varphi = 0.8$.

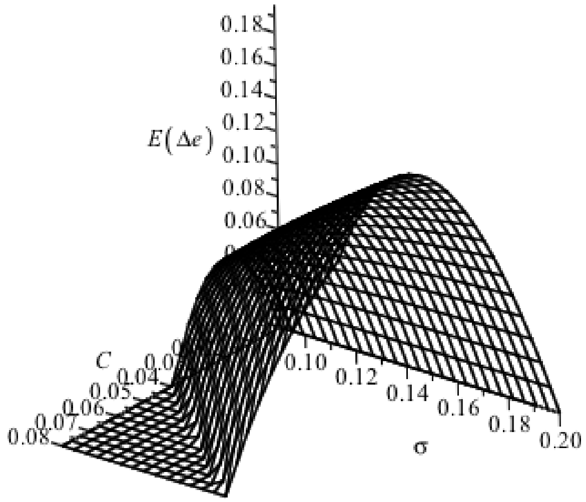

Figure 8: Effort improvement for different amount of CoCos

Lemma 6. *A higher probability of information revelation increases the expected effort improvement* $\frac{v^*-v_C^*}{2\delta}$.

Clearly, if the state is never revealed $\varphi = 0$, convertible bonds never convert and thus do not change risk incentives. An increasing chance of conversion upon revelation of high leverage triggers conversion precisely when incentives are poor.

4 Extensions

4.1 Private choice to issue CoCo bonds

It is easy to show that banks will not be willing to issue CoCos voluntarily. Since deposits are guaranteed by the deposit insurance fund, they can be issued at par, whereas CoCos are risky.[13] Moreover, CoCos force the banker to choose a safer strategy than she would prefer in some cases. This decreases the banker's return for a range of intermediate value states.

Suppose the banker may choose between the issuing CoCos of amount C at $t = 0$ or deposits of amount C. Consider the payoff of the CoCo holders. If the interim asset value v is not

[13]This result would not hold if deposit insurance fees (which we set to zero) were risk sensitive and properly priced. In our approach, such pricing is not easy, as risk is endogenous.

revealed, this is similar to conventional bondholders. If $v \geq v_C^*$, CoCo holders get the face value of the bond C, since the bank invests in the safe strategy. If $v < v_C^*$, CoCo holders face the risk that bank won't repay the value of the bond fully. As the risk is not borne by deposit insurance, it is fully priced.

It is easy to show that on average for $v < v^*$, CoCo holders get less than the face value of the bond [14], although post conversion they may enjoy a capital gain as shareholders.

Figure 1 show the payoff of the CoCo holders in highly leveraged banks ($v < v_C^*$).

As a result, CoCos are sold at the discount on their face value. Their price equals to:

$$
\begin{aligned}
P_C = \quad & \varphi \cdot \overbrace{[\underbrace{Prob(v > v^*) \cdot C}_{\text{safe strategy, no conversion}} + \underbrace{Prob(v_C^* < v \leq v^*) \cdot \frac{d}{d+1} \cdot \mathbf{E}(v - D + C | v_C^* < v \leq v^*)}_{\text{safe strategy, conversion}} +}^{\text{if information is revealed}} \\
& \overbrace{\underbrace{Prob(v \leq v_C^*) \cdot \frac{d}{d+1} \cdot Prob(V_2 > D - C) \cdot \mathbf{E}(V_2 - D + C | V_2 > D - C, v \leq v_C^*)] +}_{\text{risky strategy, conversion}}}^{\text{if information is revealed}} \\
& (1-\varphi)[\overbrace{\underbrace{Prob(v \geq v_C^*) \cdot C}_{\text{safe strategy}} + \underbrace{Prob(v < v_C^*) \cdot \mathbf{E}(B | v < v_C^*)}_{\text{risky strategy}}}^{\text{if information is not revealed}} \qquad (10)
\end{aligned}
$$

where B is the value of a traditional bond of face value C for a risky bank:

$$B = Prob(V_2 \geq D, v) \cdot C + Prob(D - C \leq V_2 < D, v) \cdot \mathbf{E}(V_2 - D + C | D - C \leq V_2 \leq D, v) \quad (11)$$

Figure 9 shows that the discount is at minimum when the CoCo amount is optimal. The intuition is that at that point, the risk reduction is maximized, and the discount increases with the asset risk.

Proposition 5. *The banker never chooses to issue CoCos instead of deposits, since CoCos are not insured and have a higher funding cost.*[15]

Therefore, CoCos will be issued only if required by regulators. Note that CoCos provide higher welfare, since the value of assets increases. The social welfare gain due to CoCos equals

[14] While CoCo holder gets less than face value at conversion because of the fixed conversion ratio, this loss is fully priced ex ante.

[15] When initial capital is very high, CoCos may actually be riskless, if they always improve risk incentives ($v_C^* \leq 1 - \delta$).

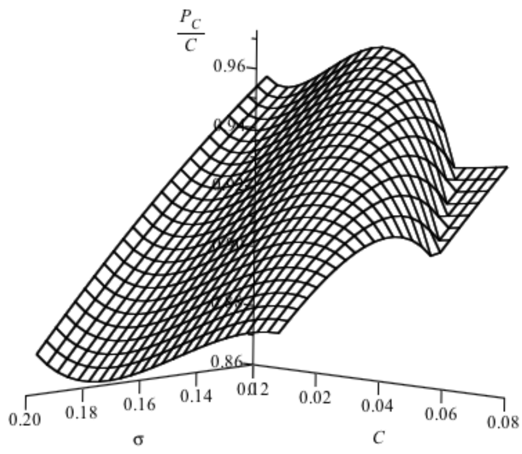

Figure 9: Price of CoCos as a percentage of face value

the range of states on which the inefficient risk outcome (which has an average cost z) is avoided:

$$\frac{v^* - v_C^*}{2\delta} \cdot z \tag{12}$$

4.2 Convertible bonds versus Debt

Are CoCos cheaper than ordinary uninsured bond?
There are two effects. CoCo bonds face less protection when converted than traditional debt, but they induce safer asset choices. We are able to show that an optimal amount of CoCos under some parameter values represent a less risky security than traditional bank debt.

The difference in payoffs is shown in the Figure 10.

The value of a traditional bond with face value C is:

$$P_B = Prob(v \geq v^*) \cdot C + Prob(v < v^*) \cdot \mathbf{E}(B|v < v_C^*) \tag{13}$$

The price of CoCos may be higher than for a traditional bond, when asset risk and trigger precision are high and the amount of CoCos is chosen optimally (Figure 11)[16].

Note that when the asset risk increases, the optimal trigger price on CoCo bonds should be raised to adjust incentives. Traditional bond holders instead will passively bear the increased risk.

[16] We use parameter values: $D = 0.93, z = 0.04, \delta = 0.07, \varphi = 0.8, \sigma = 0.14$

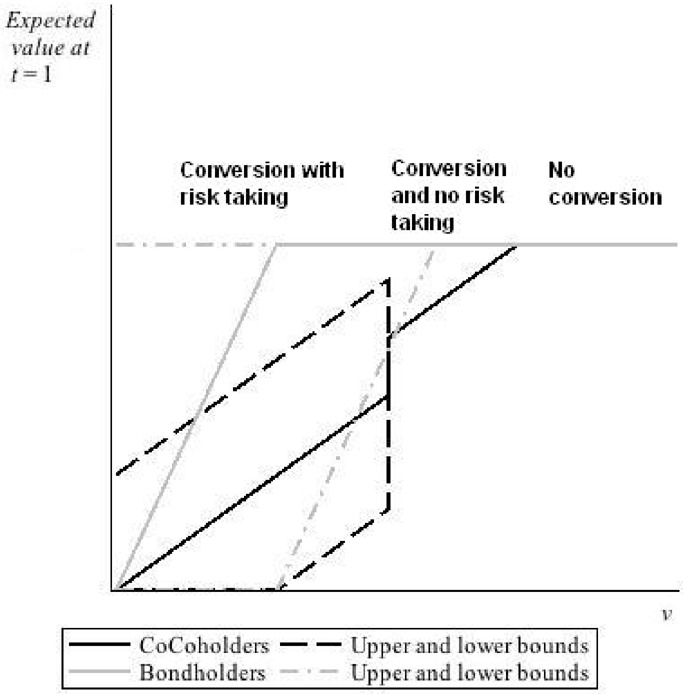

Figure 10: Expected CoCo and debt value and bounds

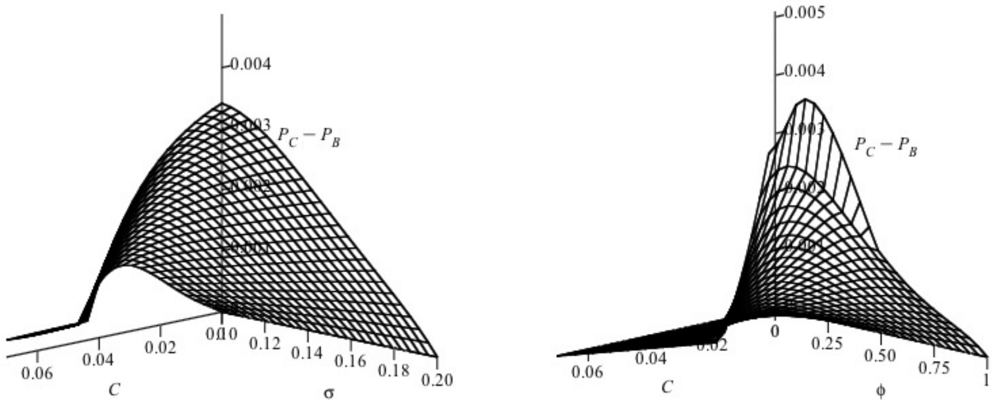

Figure 11: CoCo price minus bank debt around C^*

4.3 Contingent Capital versus Equity

What amount of contingent capital is required to substitute equity, to provide the same effort incentives?

Suppose the bank substitutes one unit of deposits by an extra amount of equity ϵ, or by an amount $k\epsilon$ of CoCos. We solve for the level of k which guarantees an equivalent improvement in risk incentives as with equity.[17]

The banker chooses effort according to the schedule:

$$e = \begin{cases} 1 & \text{if } v \geq v^* - \epsilon \\ 0 & \text{if } v < v^* - \epsilon \end{cases} \tag{14}$$

The expected improvement in effort compared to basic model (19) is $\frac{\epsilon}{2\delta}$, which reflects the increased range of asset values for which there are improved risk incentives. From earlier results, the improvement in effort achieved by CoCos is $\frac{v^* - v_C^*}{2\delta}$.

So the condition $v^* - v_C^* = \epsilon$ guarantees that the expected improvement in effort from introducing extra equity ϵ and CoCos $k\epsilon$ is the same.[18]

Proposition 6. *The effect of CoCos on effort is in general weaker than of equity, unless the trigger is perfectly informative ($\varphi = 1$).*

Lemma 7. *The substitution ratio k between extra equity and CoCos k decreases in a convex way with the probability of information revelation φ. It reaches its minimum in the fully informative trigger ($\varphi = 1$), when CoCos and equity are equivalent.*

Figure 12 shows the equivalence ratio is very sensitive to φ. As φ approaches zero, the substitution ratio becomes infinite.[19] The substitution ratio increases with asset risk (for a given v_T).

The key efficiency factor for CoCos depends on the precision of the trigger to signal a state where incentives are poor, relatively to equity which is always risk bearing. When the trigger is less precise, conversion takes less often when required. As a result, a larger amount of CoCos must be used.

[17] Note that after adding extra equity ϵ, the bank has debt $D - \epsilon$, so the amount of equity in the interim stage is $v - D + \epsilon$. The bank operates with lower leverage.

[18] As before, we set the trigger value to insure monotonic incentives in v, so $d = \frac{C}{v_T - D} = \frac{k\epsilon}{v^* - D}$.

[19] The graph assumes an uniform distribution and parameters $D = 0.93, z = 0.04, \delta = 0.07, \epsilon = 0.001$.

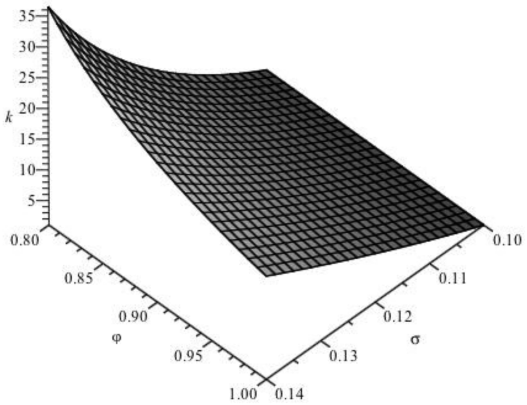

Figure 12: Substitution ratio between CoCos and equity for exogenous trigger price v_T

4.4 Debt with warrants

In this section we compare the overall risk incentive of automatic conversion against the convertible bonds proposed by Green (1984) as a solution to risk shifting.

Convertible bonds, freely convertible in shares at maturity, dilute higher risk-shifting payoffs, as investors always convert when asset value is high at maturity. This reduces the attractiveness of high risk strategies.[20]

There are three differences between CoCos and convertibles. First, conversion is not automatic. Bondholders have an option to convert into some amount of shares w. Second, the risky payoff in Green's model reflect a mean preserving spread.[21] Finally, conversion there occurs, if at all, only at the final stage $t = 2$.

We compare their effectiveness in containing risk choices and compute an equivalence ratio with CoCos. Consider a bank with a face value ϵ of convertibles outstanding, and deposits $D - \epsilon$. Bondholders will convert into w shares at $t = 2$ only if they are worth more than ϵ, namely when $V_2 > D + \frac{\epsilon}{w}$.

As in Section 2, the banker chooses to control risk according to the schedule shown in Figure 13.

[20]However, it relies on the counterintuitive notion of increasing bank equity in the best states, as opposed to the worse states. Voluntary conversion bonds also do not protect depositors, once the bank defaults.

[21]This could be easily introduced in our setting, provided we also add a (realistic) cost of bankruptcy.

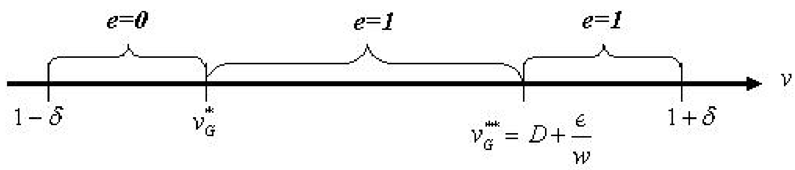

Figure 13: Risk incentives of bank with Green's convertibles

v_G^* and v_G^{**} are defined as:

$$\Delta(v_G^*) - \frac{w}{w+1} \cdot \gamma(v_G^* + \epsilon) - z = 0 \quad (15)$$

$$w \cdot (v_G^{**} - D - z + \Delta(v_G^{**}) - \gamma(v_G^{**} + \epsilon)) + \epsilon + z - \Delta(v_G^{**}) = 0 \quad (16)$$

The conversion ratio w is set optimally to ensure monotonicity of bank incentives, such that $D + \frac{\epsilon}{w} = v_G^{**}$. As in the basic model, by assumption we ensure the monotonicity of effort incentives in v.

Proposition 7. *The effect from CoCos on effort is stronger than from Green's convertibles for a sufficiently informative trigger, and certainly when $\varphi = 1$, as a lower amount is required to provide the same incentives. The substitution ratio k increases in a convex fashion with a lower trigger precision, and may become higher than* 1.

5 Discussion and conclusions

The paper assesses the optimal design of bank contingent capital. The literature so far has relied on models where the asset choice is exogenous, so CoCos have no effect on risk incentives. Pennacchi (2011) and Chen et al. (2013) study how CoCos terms affect credit yields. While not treating formally endogenous asset risk creation, their comparative statics analysis shows how conversion decreases shareholder returns in higher risk banks. Our contribution is to study explicitly contingent capital's effect on bank risk choices, a necessary feature for its optimal design and pricing.

We show that its effectiveness in controlling risk incentives and bankruptcy losses depends on the precision of its trigger in converting into equity in the worse incentive states, when leverage is very high. The intuition is that conversion contains risk shifting, as it dilutes high returns.

Our approach establishes how the optimal amount of CoCo and their trigger level trade off a risk reduction versus a dilution effect. It enables to assess what amount of CoCo produces an equivalent risk reduction as common equity, as well as freely convertible bonds. It helps clarify a key difference between bail in bonds, which convert in equity only in default, and going concern contingent capital which restore equity while the bank is still solvent. A one for one exchange ratio of CoCo for equity is equivalent in terms of loss absorption upon default. But once the risk prevention effect is taken into account, even optimally designed contingent capital is much less efficient than equity because of limited trigger precision, which does not ensure recapitalization in all states of excessive leverage.

Future research should focus on better understanding the effect of CoCo on share pricing, which is distorted by risk shifting. Share prices increase with bank risk when leverage is high, which may explain why Lehman shares peaked just a year before its default. For this reason, shareholder returns drop on conversion, creating multiple equilibria. This discontinuity, driven by the tendency of the share price to fall towards the trigger level once it comes in its neighborhood, is inappropriately named ”death spiral”. Yet it comes from the corrective effect of CoCo conversion on an underlying distortion (i.e, risk shifting), not from a distortion it introduces.

An open issue is whether potential CoCo conversion helps increase share pricing precision when leverage is excessive. Once CoCos are issued, the possibility of conversion may create downside risk. Were this to produce higher equity volatility, it would also enhance investor incentives to monitor bank risk.

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6 Appendix

Relaxing the initial capital constraint

In our model we assume that for any interim asset value v, book equity is non-negative. In this case the choice of the safe asset always provides the banker with a positive return, equal to $v - D$. It is equivalent to the condition $1 - \delta - D \geq 0$.

However, if initial capital is low (the banker observes interim asset value $v < D$) and this condition does not hold, the banker's return to the safe asset changes and the banker has different incentives to exert effort. .

In case if conversion is not triggered $v \geq v_T = v^*$, the banker's return from the safe strategy is zero, and then chooses $e = 0$. If conversion is triggered $v \leq v_T = v^*$, the choice of the banker depends on v. If $v < D - C$, the banker's payoff from the safe asset is zero. If $v \geq D - C$, the banker's payoff is positive and equal to $\frac{v-D+C}{d+1}$.

The banker's program becomes:

$$
\begin{aligned}
&\max_e e \cdot \{I(v \geq D) \cdot [\underbrace{(v - D) \cdot (I(v \geq v_T) + (1 - \varphi) \cdot I(v < v_T))}_{\text{equity value for } v \geq D \text{ if no conversion and } e=1} + \\
&\underbrace{\frac{v - D + C}{d+1} \cdot \varphi \cdot I(v < v_T)]}_{\text{equity value for } v \geq D \text{ if conversion and } e=1} \; + \\
&I(v < D) \cdot \underbrace{\frac{v - D + C}{d+1} \cdot \varphi \cdot I(D - C < v < v_T)\}}_{\text{equity value for } v<D \text{ if conversion and } e=1} \\
&(1 - e) \cdot [\underbrace{(v - z - D + \Delta(v)) \cdot (I(v \geq v_T) + (1 - \varphi) \cdot I(v < v_T))}_{\text{equity value if no conversion and } e=0} + \\
&\underbrace{\frac{v - z - D + C + \Delta(v + C)}{d+1} \cdot \varphi \cdot I(v < v_T)]}_{\text{equity value if conversion and } e=0} \\
&\text{s.t. } e \in \{0, 1\}
\end{aligned}
\tag{17}
$$

We solve the problem assuming that $v_T = v^*$.

The banker's incentives change when either two conditions hold: (1) $v^* < D$ and (2) $v^*_C < D - C$.

If we don't impose any condition on $v - D$ and the conditions defined above hold, the banker's effort choice is:

$$e = \begin{cases} 1 & \text{if } D < v \leq 1 + \delta \\ 0 & \text{if } v^* < v \leq D \\ 1 & \text{if } D - C < v \leq v^* \\ 0 & \text{if } 1 - \delta < v \leq D - C \end{cases} \tag{18}$$

As in the basic model, it is best to ensure monotonicity of e in v. In order to incentivize the banker to exert effort when $v^* < v \leq D$, the trigger value must be set as $v_T = D$.

As a result, when v may be below D, but $\forall v : v \geq D - C$, the banker's incentives don't change if the trigger value is set optimally: $v_T = D$. However, for all interim asset values v below $D - C$, risk incentives for bank with $v < D - C$ can not be improved.

Thus, our results will be valid for the weaker restriction of $v \geq D - C$. This leaves open the possibility of losses for depositors as V_2 may be below $D - C$.

Proof of Statement about Convex Risk Incentives

We consider two possible distribution of the asset value: normal and uniform.

In the first case let $x = V_2 - D$ be normally distributed with mean is $v - D - z$ and variance σ^2. We refer to x as the difference between the value of assets and debt.

In the second case let $x = V_2 - D$ be uniformly distributed with support $[v - D - z - \sigma\sqrt{3}, v - D - z + \sigma\sqrt{3}]$, so that mean is $v - D - z$ and variance is σ^2. We assume that the highest possible equity value when the bank takes the risky asset is positive, $v - D - z + \sigma\sqrt{3} \geq 0$. Otherwise, risky asset is never chosen. Moreover, the lowest possible capital value is negative $v - D - z - \sigma\sqrt{3} \leq 0$, else no risk shifting takes place.

The expected value of bank equity is the expected value of assets minus debt conditional on being solvent, multiplied by the probability of being solvent.

$$(1 - F(0, v)) \cdot \mathbf{E}(x | x > 0, v)$$

For a normal distribution, it is:

$$
\begin{aligned}
&\left(1-\Phi\left(\frac{-(v-D-z)}{\sigma}\right)\right)\cdot\frac{\int_0^\infty x\cdot\frac{1}{\sigma}\cdot\phi(\frac{x-(v-D-z)}{\sigma})dx}{1-\Phi(\frac{-(v-D-z)}{\sigma})}=\\
&\int_0^\infty x\cdot\frac{1}{\sigma}\cdot\phi(\frac{x-(v-D-z)}{\sigma})dx=\\
&(v-D-z)\cdot\Phi\left(\frac{v-D-z}{\sigma}\right)+\sigma\cdot\phi\left(\frac{(v-D-z)}{\sigma}\right)
\end{aligned}
$$

For a uniform distribution:

$$
(1-F(0,v))\cdot\mathbf{E}(x|x>0,v)=\int_0^\infty x\cdot\frac{1}{2\sigma\sqrt{3}}dx=\frac{(v-D-z+\sigma\sqrt{3})^2}{4\sigma\sqrt{3}}
$$

The expected value of equity in the case of risky asset is by definition the sum of unconditional mean of the value of asset minus debt $v-D-z$ and the risk taking incentives $\Delta(v)$ (the put option enjoyed by shareholders). Normal distribution:

$$
\begin{aligned}
\Delta(v)= \quad & (1-F(0,v))\cdot\mathbf{E}(x|x>0,v)-(v-D-z)=\\
& (v-D-z)\cdot\left[\Phi\left(\frac{v-D-z}{\sigma}\right)-1\right]+\sigma\cdot\phi\left(\frac{(v-D-z)}{\sigma}\right)
\end{aligned}
$$

Uniform distribution:

$$
\Delta(v)=(1-F(0,v))\cdot\mathbf{E}(x|x>0,v)-(v-D-z)=\frac{(v-D-z-\sigma\sqrt{3})^2}{4\sigma\sqrt{3}}
$$

Consider now how the risk shifting incentive changes with interim asset value v. It is easy to show that under these distributions the derivative of the risk shifting incentive function with respect to v is negative. Normal distribution:

$$
\frac{\partial\Delta(v)}{\partial v}= \quad \Phi\left(\frac{v-D-z}{\sigma}\right)-1\leq 0
$$

Uniform distribution:

$$
\frac{\partial\Delta(v)}{\partial v}=\frac{2(v-D-z-\sigma\sqrt{3})}{4\sigma\sqrt{3}}\leq 0
$$

Thus, the risk shifting incentive decrease with asset value v, or capital $v-D$.

The second derivative of function $\Delta(v)$ with respect to v is positive. Normal distribution:

$$\frac{\partial^2 \Delta(v)}{\partial v^2} = \quad \phi\left(\frac{(v-D-z)}{\sigma}\right) \cdot \frac{1}{\sigma} \geq 0$$

Uniform distribution:

$$\frac{\partial^2 \Delta(v)}{\partial v^2} = \frac{1}{2\sigma\sqrt{3}} \geq 0$$

Thus, risk shifting incentives fall in a convex fashion with bank capital $v - D$.

Proof of Statement about risk incentives and exogenous risk

We look at how risk incentives change when volatility of risky asset grows. The derivative of risk shifting function with respect to σ is positive.

Normal distribution:

$$\frac{\partial \Delta(v)}{\partial \sigma} = \phi\left(\frac{(v-D-z)}{\sigma}\right) \geq 0$$

Uniform distribution:

$$\frac{\partial \Delta(v)}{\partial \sigma} = -\frac{(v-D-z-\sigma\sqrt{3}) \cdot (v-D-z+\sigma\sqrt{3})}{4\sqrt{3} \cdot \sigma^2} \geq 0$$

Thus, the risk shifting incentives increase with volatility of the risky asset.

And finally, we find the effect of difference in means of payoffs from safe and risky assets z on the risk shifting incentives. The derivative of risk shifting function with respect to z is:
Normal distribution:

$$\frac{\partial \Delta(v)}{\partial z} = -\left[\Phi\left(\frac{v-D-z}{\sigma}\right) - 1\right] \geq 0$$

Uniform distribution:

$$\frac{\partial \Delta(v)}{\partial z} = -\frac{2(v-D-z-\sigma\sqrt{3})}{4\sigma\sqrt{3}} \geq 0$$

So, higher z leads to higher risk shifting incentives.

Proof of Lemma 1

First, we show that indeed the banker with interim leverage $v > v^*$ exerts effort. The banker solves the problem (5). Her decision depends on the whether the risk shifting incentive is higher or lower than the difference in means from safe and risky payoff z. If $\Delta(v) \leq z$, the banker exerts effort. According to the Assumption 1, the risk shifting incentive function $\Delta(v)$ is decreasing in v. Then our condition $\Delta(v) \leq z$ implies that the banker with interim asset value $v \geq \Delta^{-1}(z) \equiv v^*$ exerts effort.

Next we show that the probability that the banker controls risk ($\frac{1-\delta-v^*}{2\delta}$) decreases with the volatility of risky asset σ. The probability of risk control negatively depends on the magnitude v^*. Remember that v^* is derived from the condition $G(v, z, \sigma) \equiv \Delta(v) - z = 0$. We find the effect of σ on the critical value v^* using the implicit function theorem and computing

$$\frac{\partial v}{\partial \sigma} = -\frac{\partial G/\partial \sigma}{\partial G/\partial v}$$

where

$$\partial G/\partial \sigma = \Delta'_\sigma(v) \geq 0$$

and

$$\partial G/\partial v = \Delta'_v(v) \leq 0$$

As a result the derivative is $\frac{\partial v}{\partial \sigma} \geq 0$. The critical asset value v^* becomes higher if σ increases, since higher higher volatility provides larger risk-shifting benefits. Thus, the probability that the banker controls risk diminishes with σ.

Finally, note that the revelation of information does not have any effect on the banker's incentives. The reason is that information revelation makes market participants informed about the interim asset v, but does not change the incentives of the banker, since market does not have an instrument to affect the banker's payoff in case of high or low risk choice.

Proof of Lemma 2

In general, the effort choice is the solution to (8) and is:

$$
\text{If } v_C^* < v_T, e = \begin{cases} 1 & \text{if } v^* \leq v \leq 1+\delta \\ 0 & \text{if } v_T < v < v^* \\ 1 & \text{if } v_C^* \leq v \leq v_T \\ 0 & \text{if } 1-\delta \leq v < v_C^* \end{cases}
$$

$$
\text{If } v_C^* \geq v_T, e = \begin{cases} 1 & \text{if } v \geq v^* \\ 0 & \text{otherwise} \end{cases} \tag{19}
$$

where equation (20) defines the critical value v_C^*:

$$
\frac{\varphi}{d+1} \cdot \Delta(v+C) + (1-\varphi) \cdot \Delta(v) - z\left(1 - \varphi + \frac{\varphi}{d+1}\right) = 0 \tag{20}
$$

Proof of Proposition 3

We show how asset volatility affects the chosen trigger value schedule $v_T = v^*$. Remember that in the Proof of Lemma 1, we already demonstrated that critical value v^* increases with the volatility of the risky asset σ. This result implies that the trigger value $v_T = v^*$ should be raised when volatility grows in order to avoid increased risk-shifting incentives.

Proof of Proposition 2

We demonstrate the effect of the amount of CoCos on the risk choice. First, we define the risk improvement effect as a difference between banker's payoff from safe and risky strategies, i.e $\frac{\text{Safe payoff-Risky payoff}}{d+1}$, where $d = \frac{C}{v^*-D}$ is a fixed conversion ratio. The difference is then:

$$
\begin{aligned}
&\frac{v-D+C}{d+1} - \frac{v-D-z+C+\Delta(v+C)}{d+1} = \\
&\frac{(v-D+C)\cdot(v^*-D)}{v_D^*+C} - \frac{(v-D-z+C+\Delta(v+C))\cdot(v^*-D)}{v^*-D+C} = \\
&\frac{(z-\Delta(v+C))\cdot(v^*-D)}{v^*-D+C}
\end{aligned} \tag{21}
$$

The effect of CoCos on risk improvement is:

$$\frac{\partial \frac{\text{(Safe-Risky)}}{d+1}}{\partial C} = \frac{(v^* - D)[-\Delta'_C(v+C) + \Delta(v+C) - z]}{(v^* - D + C)^2} = \begin{cases} \geq 0 & \text{if } \Delta'_C(v+C) \leq \frac{\Delta(v+C)-z}{(v^*-D+C)^2} \\ < 0 & \text{otherwise} \end{cases} \tag{22}$$

Thus, the effect of CoCos on the risk incentives may be positive or negative. It increases as interim asset value goes down:

$$\frac{\partial \frac{\text{(Safe-Risky)}}{d+1}^2}{\partial C \partial v} = \frac{(v^* - D)[-\Delta''_{Cv}(v+C) \cdot (v^* - D + C) + \Delta'_v(v+C)]}{(v^* - D + C)^2} \leq 0 \tag{23}$$

Moreover, effect is always positive if $\Delta(v + C) - z \geq 0$, i.e $v < v^* - C$ (since $\Delta'_C(v + C)$ is negative, inequality $\Delta'_C(v + C) \leq \frac{\Delta(v+C)-z}{(v^*-D+C)^2}$ always holds if the right hand side is positive).

To disentangle the risk reducing effect, we assume no value transfer between equity and CoCos. It is achieved only if dilution ratio depends on the asset value, i.e $d_C = \frac{C}{\mathbf{E}(V_2-D+C|V_2>D-C)}$:

$$d_C = \begin{cases} \frac{C}{v-D} & \text{if safe strategy} \\ \frac{C}{v-D-z+\Delta(V+C)} & \text{if risky strategy} \end{cases}$$

Given these dilution ratios, we demonstrate that the effect of CoCos on risk improvement is always positive if there is no value transfer:

$$\frac{\text{Safe -Risky}}{d_C + 1} = v - D - (v - D - z + \Delta(v + C)) \tag{24}$$

$$\frac{\partial \frac{\text{(Safe-Risky)}}{d_C+1}}{\partial C} = -\Delta'_C(v + C) \geq 0 \tag{25}$$

Indeed, without value transfer the effect of CoCos on the risk incentives is always positive. It decreases as the interim asset value grows:

$$\frac{\partial \frac{\text{(Safe-Risky)}}{d_C+1}^2}{\partial C \partial v} = -\Delta''_{Cv}(v + C) \leq 0 \tag{26}$$

Only value transfer from CoCo to equity produces negative effect which is more pronounced for low levered banks (CoCo dilution effect is larger for banks with higher asset value):

$$\frac{\partial[\frac{\text{(Safe-Risky)}}{d+1} - \frac{\text{(Safe-Risky)}}{d_C+1}]}{\partial C} = \frac{C\Delta'_C(v+C)}{v^* - D + C} + \frac{(\Delta(v+C) - z)\cdot(v^* - D)}{(v^* - D + C)^2} \tag{27}$$

If v is high enough, i.e $v > v^* - C$ ($\Delta(v+C) - z < 0$), the effect is negative.

Equity dilution and CoCo dilution effects: Numerical example

Consider a bank with debt $D = 95$ and initial assets $V_0 = 100$. The risky asset return V_2 follows the binomial distribution:

$$V_2 = \begin{cases} v+5 & \text{with prob } \frac{1}{2} \\ v-10 & \text{with prob } \frac{1}{2} \end{cases}$$

Model parameters take values: $\varphi = 0.5$, $\delta = 5$, $z = 2.5$, $\sigma = 7.5$.

In the absence of CoCos bank with $v < v^* = 100$ does not control risk, i.e bank controls risk with probability 0.5.

Next we introduce CoCos of amount $C_L = 2.5$, and then show how the banker's incentives change if the amount of CoCos increases up to $C_H = 5$. The trigger value is $v^* = 100$.

First, consider the case of $C_L = 2.5$. The conversion ratio is $d_L = 0.5$. Payoff from $e = 1$ is:

$$\varphi \cdot \frac{v - D + C_L}{d_L + 1} + (1-\varphi)\cdot(v - D) = 0.5 \cdot \frac{v - 92.5}{1.5} + 0.5\cdot(v - 95)$$

Payoff from $e = 0$ is:

$$\varphi \cdot \frac{1}{2} \cdot \frac{v + 5 - D + C_L}{d_L + 1} + (1-\varphi)\cdot\frac{1}{2}\cdot(v + 5 - D) = 0.25 \cdot \frac{v - 87.5}{1.5} + 0.25\cdot(v - 90)$$

Bank with $v > v^*_C = 99$ chooses to control risk in the presence of CoCos $C_L = 2.5$, i.e bank controls risk with probability 0.6.

Second, consider the case of $C_H = 5$. The conversion ratio is $d_H = \frac{1}{2}$. Then the payoffs from safe and risky strategies are respectively:
Payoff from $e = 1$ is:

$$\varphi \cdot \frac{v - D + C_L}{d_L + 1} + (1-\varphi)\cdot(v - D) = 0.5 \cdot \frac{v - 90}{2} + 0.5\cdot(v - 95)$$

Payoff from $e = 0$ is:

$$\varphi \cdot \frac{1}{2} \cdot \frac{v + 5 - D + C_L}{d_L + 1} + (1 - \varphi) \cdot \frac{1}{2} \cdot (v + 5 - D) = 0.25 \cdot \frac{v - 85}{2} + 0.25 \cdot (v - 90)$$

Bank with $v > v_C^* = 98.33$ chooses to control risk when CoCos is $C_H = 5$.

Next, we disentangle equity dilution effect from the overall effect of CoCos increase. We make the conversion ratio such that it ensures no value transfer in order to disentangle equity dilution and CoCo dilution effects. Instead of $d_H = 1$ we use:

$$d_C = \begin{cases} \frac{C}{v-D} = \frac{5}{v-95} & \text{if safe strategy} \\ \frac{C}{\mathbf{E}(V_2 - D + C | V_2 > D - C) - C} = \frac{10}{v-95} & \text{if risky strategy} \end{cases}$$

Payoff from $e = 1$ is:

$$\varphi \cdot \frac{(v - D + C_H) \cdot (v - D)}{v - D + C_H} + (1 - \varphi) \cdot (v - D) = v - 95$$

Payoff from $e = 0$ is:

$$\varphi \cdot \frac{1}{2} \cdot \frac{(v + 5 - D + C_H)}{d_C + 1} + (1 - \varphi) \cdot \frac{1}{2} \cdot (v + 5 - D) = 0.25 \cdot (v - 95) + 0.25 \cdot (v - 90)$$

If the there were no value transfer to shareholders, bank with $v > v_C^* = 97.5$ would choose to control risk in the presence of CoCos $C_H = 5$. The effort improvement would be 0.15 due to the increase in CoCos from 2.5 to 5. This risk reduction arises because equity dilution reduces attractiveness of the risky payoff. We refer to this effect as equity dilution effect.

Because of lower conversion ratio d_H, the dilution is at the disadvantage of CoCo holders and advantage of shareholders. The effort improvement becomes lower.

CoCo dilution effect disincentives the banker to control risk. This effect is measured as the reduction in effort improvement of 0.083.

Thus, equity dilution effect raises the probability of bank controlling risk to 0.75 (effort improvement of 0.25), whereas CoCo dilution effect reduces this probability to 0.667 (effort decrease by -0.083). Overall effect from increasing the amount of CoCos from 2.5 to 5 is the expected effort increase by 0.067.

Proof of Proposition 3

The maximum improvement in effort is achieved when threshold for bank with CoCos v_C^* reaches its minimum. The condition for optimal amount of CoCos generating minimum v_C^* (this increases the probability of bank exerting higher effort, and thus expected effort improvement) is:

$$\frac{\partial v_C^*}{\partial C} = 0$$

We use the implicit function theorem to compute this derivative:

$$\frac{\partial v_C^*}{\partial C} = -\frac{\partial F/\partial C}{\partial F/\partial v_C^*}$$

where[22]

$$\frac{\partial F}{\partial C} = \frac{\varphi(v_T - D)\left(\Delta_C'(v + C) \cdot (C + v_T - D) - \Delta(v + C) + z\right)}{(C + v_T - D)^2}$$

$$\frac{\partial F}{\partial v} = \frac{\varphi(v_T - D)}{C + v_T - D} \cdot \Delta_v'(v + C) + (1 - \varphi) \cdot \Delta'(v)$$

The resulting condition is then:

$$\frac{\partial v_C^*}{\partial C} = -\frac{\varphi(v_T - D)\left(\Delta_C'(v + C) \cdot (C + v_T - D) - \Delta(v + C) + z\right)}{(C + v_T - D)^2 \cdot \left(\frac{\varphi(v_T - D)}{C + v_T - D} \cdot \Delta_v'(v + C) + (1 - \varphi) \cdot \Delta'(v)\right)} = 0$$

From here the condition for the amount of CoCos C guaranteeing the minimum v_C^* is:

$$\Delta_C'(v + C^*)(C^* + v_T - D) - \Delta(v + C^*) + z = 0$$

where $v_T = v^*$. Note that we treat the solution of this equation as the amount of CoCos providing the minimum v_C^*, since we know that at $C = 0$, the function $v_C^*(C)$ is decreasing, and at $C = \infty$ it is constant. This suggests the existence of at least one minimum point.

$$\lim_{C \to 0} \frac{\partial v_C^*}{\partial C} = -\frac{\varphi\left(\Delta_C'(v) \cdot (v_T - D) - \Delta(v) + z\right)}{(v_T - D) \cdot \Delta_v'(v)} \leq 0$$

[22] Further we use just v instead of v_C^*.

$$\lim_{C\to+\infty}\frac{\partial v_C^*}{\partial C}=0$$

Proof of Lemma 4

The marginal rate of substitution between the optimal amount of CoCos and the optimal is positive:

$$\frac{\partial C^*}{\partial v_T}=-\frac{\Delta_C'(v+C)}{\Delta_C''(v+C)(C+v_T-D)}>0 \tag{28}$$

Proof of Proposition 4

We show the effect of volatility σ on the effort improvement $\frac{v^*-v_C^*}{2\delta}$ upon assumption that the trigger value v_T is exogenous. We need to find the effect of σ on the critical value v_C^*, i.e. compute $\frac{\partial v_C^*}{\partial\sigma}$. Using the implicit function theorem, we define it as:

$$\frac{\partial v_C^*}{\partial\sigma}=-\frac{\partial F/\partial\sigma}{\partial F/\partial v}$$

where we use the result $\frac{\partial F}{\partial v}\leq 0$ from the proof of proposition 3, and

$$\frac{\partial F}{\partial\sigma}=\frac{\varphi}{d+1}\cdot\Delta_\sigma'(v+C)+(1-\varphi)\cdot\Delta_\sigma'(v)\geq 0$$

where we exploit the assumption 1 that $\Delta_\sigma'\geq 0$. Thus, $\frac{\partial v_C^*}{\partial\sigma}\geq 0$.

Consequently, the effect of the asset volatility on the expected effort improvement $\frac{v_T-v_C^*}{2\delta}$ is negative given that the trigger value is exogenously given.

Proof of Lemma 5

Next we examine the marginal effect from setting trigger value $v_T=v^*$ on effort. This effect consists of two: the effect on v^* as an upper bound of interim asset value for which the conversion takes place, and the effect of v^* on the v_C^* via dilution ratio d.

First effect is positive, as we already established in the proof of Lemma 1, that critical value v^* increases with the volatility of the risky asset σ.

Second effect is also positive for the expected effort:

$$\frac{\partial v_C^*}{\partial \sigma} = \frac{\partial v_C^*}{\partial d}\cdot\frac{\partial d}{\partial \sigma} = -\frac{\frac{\partial F}{\partial d}}{\frac{\partial F}{\partial v}}\cdot\frac{\partial d}{\partial \sigma} = -\underbrace{\frac{-\frac{\varphi(\Delta(v_C^*+C)-z))}{(d+1)^2}}{\frac{\varphi}{d+1}\cdot\Delta_v'(v+C)+(1-\varphi)\cdot\Delta'(v)}}_{\geq 0}\cdot\underbrace{\frac{-Cv_\sigma^{*'}}{(v^*-D)^2}}_{\leq 0}\leq 0$$

Since volatility increases trigger value v^*, the dilution ratio diminishes. This leads to the lower critical value v_C^*.

Thus, the marginal effect is positive, and setting trigger value to be v^* reduces the negative effect of volatility on the expected effort (achieved with exogenous trigger price).

However, the sign of the overall effect is undefined and depends on the parameters:

$$\frac{\partial\frac{v^*-v_C^*}{2\delta}}{\partial\sigma} = \frac{1}{2\delta}\cdot\left[\underbrace{v_\sigma^{*'}\cdot\left(1+\frac{\varphi d(\Delta(v_C^*+C)-z)}{\frac{\partial F}{\partial v}(v^*-D)(d+1)^2}\right)}_{\geq 0}+\underbrace{\frac{\frac{\varphi\cdot\Delta_\sigma'(v_C^*+C)}{d+1}+(1-\varphi)\Delta_\sigma'(v)}{\frac{\partial F}{\partial v}}}_{\leq 0}\right]$$

As a result, the overall effect of σ on effort may also become positive.

Proof of Lemma 6

Here we look at the effect of the higher trigger precision on the expected effort $\frac{v^*-v_C^*}{2\delta}$. The sign of the effect is opposite to the sign of the derivative $\frac{\partial v_C^*}{\partial\varphi} = -\frac{\frac{\partial F}{\partial\varphi}}{\frac{\partial F}{\partial v_C^*}}$, where $\frac{\partial F}{\partial v_C^*}\leq 0$

$$\frac{\partial F}{\partial\varphi} = \underbrace{\frac{\Delta(v+C)-z}{d+1}}_{\leq 0}+\underbrace{(z-\Delta(v))}_{\leq 0}\leq 0$$

The derivative $\frac{\partial v_C^*}{\partial\varphi}$ is negative, therefore the effect of probability of information revelation on the expected effort is positive. Note that critical asset value v^* is not affected by φ according to the results of Lemma 1.

Proof of Proposition 5

In order to show that banker never chooses to issue CoCos voluntarily instead of deposits, we compare the price of CoCos P_C and the price of deposits. Thus, we show that funding with

CoCos (face value C) is more expensive than with deposits of the same face value.

Price of deposits is equal to their face value C, since depositors get their money back with certainty and deposit rate is zero.

We show that priced of CoCos is lower than C, i.e $P_C \leq C$.

$$
\begin{aligned}
P_C = \quad & \varphi \cdot [\overbrace{\underbrace{Prob(v > v^*) \cdot C}_{\text{safe strategy, no conversion}} + \underbrace{Prob(v_C^* < v \leq v^*) \cdot \frac{d}{d+1} \cdot \mathbf{E}(v - D + C | v_C^* < v \leq v^*)}_{\text{safe strategy, conversion}}}^{\text{if information is revealed}} + \\
& \overbrace{\underbrace{Prob(v \leq v_C^*) \cdot \frac{d}{d+1} \cdot Prob(V_2 > D - C) \cdot \mathbf{E}(V_2 - D + C | V_2 > D - C, v \leq v_C^*)}_{\text{risky strategy, conversion}}}^{\text{if information is revealed}}] + \\
& (1 - \varphi)[\overbrace{\underbrace{Prob(v \geq v_C^*) \cdot C}_{\text{safe strategy}} + \underbrace{Prob(v < v_C^*) \cdot \mathbf{E}(B | v < v_C^*)}_{\text{risky strategy}}}^{\text{if information is not revealed}} \qquad (29)
\end{aligned}
$$

First, if information is not revealed, CoCos gets not higher than the face value, since in case of $D - C \leq V_2 \leq D$, they may get the value of the bond B, which is lower than face value C:

$$B = Prob(V_2 \geq D, v) \cdot C + Prob(D - C \leq V_2 < D, v) \cdot \mathbf{E}(V_2 - D + C | D - C \leq V_2 \leq D, v)$$

The reason is that $\mathbf{E}(V_2 - D + C | D - C \leq V_2 \leq D, v) \leq C$.

Second, if information is revealed and there is no conversion, CoCos receive the face value. If there is a conversion, and safe strategy is chosen by the banker, CoCos get for $v_C^* < v \leq v^*$

$$\frac{d}{d+1} \cdot (v - D + C) = \frac{\frac{C}{v^* - D}}{\frac{C}{v^* - D} + 1} \cdot (v - D + C) = C \cdot \frac{v - D + C}{v^* - D + C} \leq C \qquad (30)$$

If banker chooses risky strategy and conversion occurs ($v \leq v_C^*$), CoCo's payoff is:

$$
\begin{aligned}
& \frac{d}{d+1} \cdot Prob(V_2 > D - C) \cdot \mathbf{E}(V_2 - D + C | V_2 > D - C, v \leq v_C^*) = \\
& \frac{C}{v^* - D + C} \cdot (v - D - z + \Delta(v + C) + C) \leq C \qquad (31)
\end{aligned}
$$

since $v^* - D + C \geq v - D - z + \Delta(v + C) + C$ due to $\Delta(v + C) - z \leq v^* - v$.

Thus, in any possible case the value of CoCos does not exceed their face value C, and banker considers it more expensive funding option than deposits.

Proof of Proposition 6

The banker's program with extra equity is:

$$\max_{e} e \cdot (v - D + \epsilon) + (1 - e) \cdot (v - D + \epsilon - z + \Delta(v + \epsilon))$$
$$\text{s.t. } e \in \{0, 1\} \tag{32}$$

In order to compute the substitution ratio k, we use the condition for finding v^*_C:

$$G(v^* - \epsilon | k\epsilon, d) = 0$$

or equivalently

$$\frac{\varphi}{d+1} \cdot \Delta[v^* + \epsilon(k-1)] + (1-\varphi) \cdot \Delta[v^* - \epsilon] - z \left(1 - \varphi + \frac{\varphi}{d+1}\right) = 0 \tag{33}$$

Here we prove that $k \geq 1$. The proof is by contradiction. Assume that $k < 1$. We can rewrite condition (33) as

$$\frac{\varphi}{d+1} \cdot (\Delta[v^* + \epsilon(k-1)] - z) + (1-\varphi) \cdot (\Delta[v^* - \epsilon] - z) = 0$$

Note that $\Delta[v^* - \epsilon] \geq z$, since banker with $v < v^*$ does not exert effort. Since the whole expression is equal to zero, and the second term is non-negative, the first term should be non-positive. Hence,

$$\frac{\varphi}{d+1} \cdot (\Delta[v^* + \epsilon(k-1)] - z) \leq 0$$

The expression above is non-positive only if

$$\Delta[v^* + \epsilon(k-1)] - z \leq 0$$

The risk shifting incentive is smaller than or equal to z only if $v \geq v^*$. And if $k < 1$, then $v^* + \epsilon(k-1) < v^*$. This is a contradiction. Consequently, it always holds that $k \geq 1$, and higher amount of CoCos is required to provide the same effect as equity.

Proof of Lemma 7

In order to show the effect of the probability of information revelation on the substitution ratio k, we compute first and second derivatives of k with respect to φ: $\frac{\partial k}{\partial \varphi}$ and $\frac{\partial^2 k}{\partial \varphi^2}$. We apply the implicit function theorem to the condition $G(v^* - \epsilon | k\epsilon, d) = 0$. We rewrite it using the fact that $d = \frac{k\epsilon}{v^* - D}$:

$$\frac{\varphi \cdot (v^* - D)}{k\epsilon + v^* - D} \cdot (\Delta[v^* + \epsilon(k-1)] - z) + (1-\varphi) \cdot (\Delta[v^* - \epsilon] - z) = 0$$

According to the implicit function theorem:

$$\frac{\partial k}{\partial \varphi} = -\frac{\partial G(v^* - \epsilon | k\epsilon, d)/\partial \varphi}{\partial G(v^* - \epsilon | k\epsilon, d)/\partial k}$$

where $\frac{\partial G(v^* - \epsilon | k\epsilon, d)}{\partial k}$ equals to

$$\frac{\varphi \cdot (v^* - D) \cdot \epsilon}{(k\epsilon + v^* - D)^2} \cdot \left(\underbrace{(k\epsilon + v^* - D) \cdot \Delta'_k[v^* + \epsilon(k-1)]}_{\leq 0} - \underbrace{(\Delta[v^* + \epsilon(k-1)] - z))}_{\leq 0} \right)$$

which is non-positive for infinitesimal ϵ.

$$\frac{\partial G(v^* - \epsilon | k\epsilon, d)}{\partial \varphi} = \underbrace{\frac{(v^* - D)}{k\epsilon + v^* - D} \cdot (\Delta[v^* + \epsilon(k-1)] - z)}_{\leq 0} + \underbrace{(z - \Delta[v^* - \epsilon])}_{\leq 0} \leq 0$$

Thus, the substitution ratio falls as probability of revelation rises $\frac{\partial k}{\partial \varphi} \leq 0$.

Next, consider the second derivative of substitution ratio with respect to φ:

$$\frac{\partial^2 k}{\partial \varphi^2} = -\frac{(-1) \cdot \frac{\partial G}{\partial \varphi}}{\frac{(v^* - D) \cdot \epsilon}{(k\epsilon + v^* - D)^2} \cdot (\Delta'_k[v^* + \epsilon(k-1)] \cdot (k\epsilon + v^* - D) - (\Delta[v^* + \epsilon(k-1)] - z))} \geq 0$$

This result implies that the substitution ratio k is decreasing and convex function of the probability of information revelation φ.

Proof of Proposition 7

The banker's payoff from the risky strategy is lower than in the case of non-convertible debt by the value of the call option held by the bondholders, which we denote as $\frac{w}{w+1} \cdot \gamma(v+\epsilon)$:

$$v - z - D + \Delta(v) - \frac{w}{w+1} \cdot \gamma(v+\epsilon) \tag{34}$$

where the value of the call option:

$$\frac{w}{w+1} \cdot \gamma(v+\epsilon) = \frac{w}{w+1} \cdot (1 - F(D + \frac{\epsilon}{w} - v)) \cdot \mathbf{E}(V_2 - D | V_2 - D > \frac{\epsilon}{w}) \tag{35}$$

is positive and increasing in v.

If the interim asset value is high ($v > D + \frac{\epsilon}{w}$), shareholders will choose to convert at the final date. The banker's return from the safe strategy becomes then $\frac{v-D+\epsilon}{w+1}$. If $v \leq D + \frac{\epsilon}{w}$, the banker's payoff from the safe strategy is the same as in the case of non-convertible debt $v - D$.

The banker's problem is:

$$\begin{aligned}
&\max_e e \cdot [\underbrace{(v - D) \cdot I(v \leq D + \frac{\epsilon}{w})}_{\text{equity value if } e=1\text{, no conversion}} + \underbrace{\frac{v - D + \epsilon}{w+1} \cdot I(v > D + \frac{\epsilon}{w})}_{\text{equity value if } e=1\text{, conversion occurs}}] + \\
&(1-e) \cdot [\underbrace{v - z - D + \Delta(v) - \frac{w}{w+1} \cdot \gamma(v+\epsilon)}_{\text{equity value if } e=0}] \\
&\text{s.t. } e \in \{0, 1\}
\end{aligned} \tag{36}$$

The banker chooses effort according to the schedule:

$$\begin{aligned}
&\text{If } v_G^{**} > D + \frac{\epsilon}{w}, e = \begin{cases} 1 & \text{if } v \geq v_G^{**} \\ 0 & \text{if } D + \frac{\epsilon}{w} \leq v < v_G^{**} \\ 1 & \text{if } v_G^{*} \leq v < D + \frac{\epsilon}{w} \\ 0 & \text{if } v < v_G^{*} \end{cases} \\
&\text{If } v_G^{**} < D + \frac{\epsilon}{w}, e = \begin{cases} 1 & \text{if } v \geq D + \frac{\epsilon}{w} \\ 0 & \text{if otherwise} \end{cases}
\end{aligned} \tag{37}$$

We show here that the equivalence ratio between CoCos and Green's convertible bonds is lower than 1 ($k \leq 1$), which implies stronger effect on effort is produced by CoCos.

The condition for the equivalent effect from CoCos and Green's convertibles is:

$$G(D + \frac{\epsilon}{w} | k\epsilon, d) = 0 \tag{38}$$

or equivalently,

$$\frac{\varphi}{d+1} \cdot (\Delta[D + \epsilon(k + \frac{1}{w})] - z) + (1 - \varphi) \cdot (\Delta \left[D + \frac{\epsilon}{w}\right] - z) = 0 \tag{39}$$

Note that $D + \epsilon(k + \frac{1}{w}) \geq D + \frac{\epsilon}{w}$, when $k \geq 0$. For the equality (39) to hold, we need one part of the equation to be positive and another negative. $\Delta(v) - z$ is positive for $v < v^*$, and negative for $v > v^*$. This implies that $D + \epsilon(k + \frac{1}{w}) > v^*$ and $D + \frac{\epsilon}{w} < v^*$. This implies that for $\varphi = 1$, the equivalence condition is:

$$\begin{aligned} v^* - k\epsilon &= D + \frac{\epsilon}{w} \\ k &\geq \frac{v^* - D}{\epsilon} - \frac{1}{w} \end{aligned} \tag{40}$$

We proof by contradiction. Assume that $k > 1$. Then it implies that $\frac{v^* - D}{\epsilon} - \frac{1}{w} > 1$, which is equivalent to $w(v^* - D - 1) < \epsilon$. This is contradiction, since $v^* < 1$ ($v_T < 1$), $v^* - D - 1 < 0$, but $\epsilon > 0$ by construction. As a result, $k \leq 1$ for $\varphi = 1$.