Value-at-Risk vs. Building Block Regulation in Banking *

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Abstract

Existing regulatory capital requirements are often criticized for being only loosely linked to the economic risk of the banks' assets. In view of the attempts of international regulators to introduce more risk sensitivene capital requirements, we theoretically examine the effect of specific regulatory capital requirements on the risk taking behavior of banks. More precisely, we develop a continuous time framework where the banks' choice of asset risk is endogenously determined. We compare regulation based on the Basel I Building Block approach to Value-at-Risk or 'internal model' based capital requirements with respect to risk taking behavior, deposit insurance liability, and shareholder value. The main findings are (i) Value-at-Risk based capital regulation creates a stronger incentive to reduce asset risk when banks are solvent, (ii) solvent banks that reduce their asset risk reduce the current value of the deposit insurance liability significantly, (iii) under Value-at-Risk regulation the risk reduction behavior of banks is less sensitive to changes in their investment opportunity set, and (iv) banks' equityholders can benefit from risk based capital requirements.

JEL classification: G21; G28; G12

1 Introduction

> [W]e have no choice but to continue to plan for a successor to the simple risk-weighting approach to capital requirements embodied within the current regulatory standard. While it is unclear at present exactly what that successor might be, it seems clear that adding more and more layers of arbitrary regulation would be counter productive. We should, rather, look for ways to harness market tools and market-like incentives wherever possible, by using banks' own policies, behaviors, and technologies in improving the supervisory process.

Greenspan (1998)

The impact of bank regulation on risk-taking behavior has been a major focus during periods of severe financial crises, such as the 1999 Asian experience. While there is still an ongoing debate whether regulation is beneficial at all,[1] regulation is an evolving process and a number of regulatory guidelines have been issued by the Basel Committee on Banking Supervision and by national regulators over time.

One of the milestones in banking regulation is the 1988 Basel Accord,[2] (also called Basel I) where regulators set up minimum capital requirements for banks. The idea is to oblige banks to hold capital as a safety cushion to ensure bank solvency. Banks holding riskier assets must hold more capital as they have a higher probability of failure. To link the required capital to the riskiness of a banks' assets, the accord assigns assets to different risk buckets,[3] and specifies bucket-specific equity requirements (risk weights). Whereas capital requirements are homogenous within each of these buckets, the economic

[1] See e.g. Freixas and Rochet (1997), pp. 257 for a survey.

[2] See Basel Committee on Banking Supervision (1988).

[3] All assets are assigned to one of four buckets. These buckets coarsely classify the riskiness of the respective contract, e.g., loans to OECD governments, loans to OECD banks and other OECD public sector entities, residential mortgage loans, loans to the private sector. For a more detailed description see, e.g., Jorion (2000).

risk of assets assigned to the same risk bucket may vary substantially (e.g., all corporate loans have to be backed by 8% of capital regardless of the companies' rating).[4] This fact gives rise to criticism of the Basel I Accord since it opens the opportunity for 'regulatory capital arbitrage' by 'intra-bucket' risk shifting, i.e. increasing the risk of the bank's assets without increasing the capital requirements. Several regulatory agencies therefore have proposed linking minimum capital requirements to economic risk more closely.[5]

Regulators have recognized this problem and there have been two important steps towards enhanced risk sensitiveness of capital requirements since the release of Basel I. An amendment to the Basel I Accord[6] incorporates the market risk of the trading book into the international banking regulation framework. It offers banks the opportunity to compute minimum capital requirements for proprietary trading activities using a Value-at-Risk approach. Recently, the Basel Committee released the second proposal for the New Basel Capital Accord[7] (also called Basel II). The newly proposed Internal Ratings Based Approach, while still a bucket building method, shows greater risk sensitiveness due to a finer granularity of the risk buckets and a dynamic assignment of loans to buckets based on the internal rating of the loan contracts.[8]

The aim of this paper is to theoretically examine the effect of different regulatory capital requirements on the risk taking behavior of banks. We set up a continuous time framework allowing banks to choose between two different asset portfolios that are

[4]Dimson and Marsh (1995) analyze the relationship between economic risk and capital requirements for different regulatory frameworks using trading book positions of UK securities firms. They find that the building block approach leads only to modest correlation between capital requirements and total risk.

[5]see Santos (2000) or Meyer (1998), who notes for example: "[C]apital arbitrage also undermines the effectiveness of our capital rules and creates some economic distortions".

[6]See Basel Committee on Banking Supervision (1996).

[7]See Basel Committee on Banking Supervision (2001), the final version is to be published in 2002, with implementation planned in 2005.

[8]In their analysis of the Basel II Accord Altman and Saunders (2001) and Linnell (2001) criticize that the granularity of the buckets still remains to coarse and propose risk weights that 'will bring regulatory capital closer to economic capital estimates'.

characterized by different levels of risk. We study the optimal risk taking behavior of banks when capital requirements have different risk sensitivity. Specifically we compare (i) a simple Basel I Building Block (BB) approach and (ii) a Value-at-Risk (VaR) based approach as two genuine examples, recognizing that current regulations including Basel II lie between these polar cases. We also examine the effect on equity value, on the fair upfront deposit insurance premium, and derive policy implications for prudent bank regulation.

We find that both, the Building Block and the Value-at-Risk approach, generally do not prevent banks from switching to the high-risk portfolio when they are in financial distress. However, Value-at-Risk based capital requirements provide a stronger incentive for well capitalized banks to reduce asset risk by rewarding low-risk banks with lower capital requirements. Furthermore, we show that the current value of the deposit insurance liability decreases significantly when solvent banks chose the low risk portfolio. Finally, we point out that auditing intensity and regulatory capital requirements have to be carefully attuned to each other in order to provide the risk-reduction incentive. In a comparative static analysis we solve for the minimum level of auditing, that regulators have to provide in order to create an incentive for risk reduction at least in some states. We find that under Value-at-Risk based regulation less auditing has to be performed and that the corresponding audit intensity is less sensitive to changes in the banks' investment opportunity set. Thus, our findings support the Basel Committee's recognition of capital requirements and auditing policy as equally important pillars of the new capital accord.

There are two branches of literature related to our approach. The first addresses the issue of bank regulation in a continuous time framework. Merton (1977) derives the insurance premium of a fixed length deposit contract applying the Black and Scholes (1973) option pricing framework. Merton (1978) introduces random audits by the regulator and derives the fair up-front price of deposit insurance under the assumption of a constant volatility of the bank's assets. Pennacchi (1987) considers risk taking incentives by banks, where he defines risk in terms of financial leverage. He also points out the importance of regulatory response to a bank failure and compares direct payments to depositors to merging a failed bank. Fries, Mella-Barral, and Perraudin (1997) consider optimal bank closure rules balancing social bankruptcy costs against future auditing costs. They find incentives for managers to take risk, where risk is defined as the volatility of the underlying state variable and not as leverage, and they derive subsidy policies and equity support schemes that eliminate these risk taking incentives by linearizing the equityholders' value function. Finally, Bhattacharya, Plank, Strobl, and Zechner (2001) derive optimal closure rules that eliminate risk taking incentives for managers—at least in the region where the bank is adequately capitalized. All these models above assume that the volatility of the underlying state variable is constant. The existence of a risk taking incentive is deduced solely from the convexity of the equityholders' value function, however, the process of risk shifting is not explicitly treated.

The second branch of literature examines risk shifting in a continuous time corporate finance setting. Ericsson (1997) and Leland (1998) introduce models where equityholders are allowed to switch from one risk level to another. Their goal is to price corporate securities and to derive the optimal capital structure policy of firms in the presence of agency costs arising from the asset substitution opportunity. While the modeling technique of these papers is similar to our approach, the economic context in banking is substantially different. Due to deposit insurance, debt can be raised at the riskless rate. Consequently, a conflict of interest evolves between equityholders and the deposit insurer. To prevent the exploitation of the insurance system, banks have to satisfy regulatory constraints which are enforced by an auditing mechanism. Our paper explores the incentives of these regulatory rules on risk taking as well as the optimal auditing policy.

The paper is composed as follows: Section 2 describes the model, Section 3 derives the general solution for claims on the banks assets, Section 4 compares Building Block and Value-at-Risk regulation and explores the risk taking incentives created by these mechanisms. Section 5 derives comparative statics, considers welfare effects and gives some policy implications for prudent regulation, and in Section 6 we conclude.

2 Model

As in Merton (1974), the value of the banks assets V is assumed to follow a geometric Brownian motion. However, we extend this framework allowing the bank's management to choose between two asset portfolios with different risk. More precisely, there is a 'low-risk' portfolio available whose dynamics are geometric Brownian with volatility σ_L and drift $\mu(\sigma_L)$ as well as a 'high-risk' portfolio, characterized by σ_H and $\mu(\sigma_H)$, with $\sigma_H > \sigma_L$. At any instant in time the management[9] has the freedom to substitute the current asset portfolio with the alternative portfolio thereby changing the risk level of the underlying assets. Thus, our model explicitly allows for asset substitution, however, this substitution is costly such that a certain small fraction k of the asset value V is lost at any switch between portfolios. We assume that the bank's portfolio includes a major proportion of loans and other assets for which a shift in the risk structure is opaque for regulators. As the regulatory agency has no information on the bank's investment choice, it has to perform audits in order to get a detailed status of the portfolio's risk.

[9]In line with most of the previous literature, the management's interests are assumed to be perfectly aligned with the equityholders'. A recent contribution by John, Saunders, and Senbet (2000) explicitly considered the agency conflict between equityholders and management and examined the interesting idea of linking bank regulation to management compensation.

To keep the model feasible, the portfolio choice is restricted to a discrete choice, i.e., the bank is either fully invested in the low-risk portfolio or in the high-risk portfolio. Formally, the asset value process of the bank can be written as

$$
\begin{aligned}
dV &= \begin{cases} (\mu(\sigma_L) - \delta_L)V\,dt + \sigma_L V\,dz_L & : \text{bank owns the low-risk portfolio,} \\ (\mu(\sigma_H) - \delta_H)V\,dt + \sigma_H V\,dz_H & : \text{bank owns the high-risk portfolio,} \\ -kV & : \text{on asset substitution,} \end{cases} \\
V(0) &= V_0 > 0,
\end{aligned}
\tag{1}
$$

where $\mu(\sigma_L)$ and $\mu(\sigma_H)$ are the total expected returns on the asset value V of the low-risk and of the high risk portfolio respectively. When holding the assets, the bank earns a profit flow which is a certain proportion $\delta \in \{\delta_L, \delta_H\}$ of the portfolio value V. The differentials dz_L and dz_H are the increments of (possibly correlated) standard Wiener processes representing the random shocks the two portfolio values are exposed to. Since a combination of the two portfolios is not permitted correlation has no effect on the choice, thus, the distinction between dz_L and dz_H is suppressed in the remainder of the paper. The instantaneous variance of the process V is $\sigma_L^2 V^2$ and $\sigma_H^2 V^2$ depending on the current risk level. Hence, the state of the bank is characterized by the location in the two dimensional state space $[0, \infty) \times \{\sigma_L, \sigma_H\}$ over the ranges of V and σ.

We assume that the bank has issued deposits with face value c/r (where r is the riskless rate of interest) requiring a continuous coupon flow c. These deposits are fully insured, thus, in case of bankruptcy the depositors receive the full face value. Equityholders have limited liability and are the residual claimholders of the bank's assets. If—on closure—the asset value V is not sufficiently high to cover the claim of the depositors, the difference is borne by the regulatory authority.

Modeling the value of the bank charter, we assume—in line with the previous literature—that coupon payments are tax deductible, i.e., as long as the bank serves its obligations it receives a tax benefit of the magnitude τc. Alternatively, the value of the bank charter could be motivated by access to cheap deposits, barriers to entry, or access to a screening or monitoring technology.

Equityholders as the residual claimants are responsible to maintain the obligations of the bank. Whenever the profit flow from holding the asset portfolio δV is less than the required interest payment $(1-\tau)c$, the equityholders have the choice to either inject money to guarantee solvency in order to keep the prospect of future benefits from running the bank or—alternatively—they may voluntarily close the bank. Thus, we focus on the bank's optimal investment decision, i.e., the optimal choice of the risk level and the closure level (see the discussion of the bank's strategy bellow).

Apart from voluntary closure there is the possibility of forced closure by the regulatory authorities if the bank is not in accordance with the regulatory mechanism implemented. We consider regulatory mechanisms $\langle \lambda, B(\sigma) \rangle$ characterized by (i) an auditing intensity λ and (ii) by a closure threshold $B(\sigma)$. In detail:

- Audits are assumed to occur randomly following a Poisson process with intensity λ. That means, we model an audit counter A defined by the stochastic differential equation

$$\begin{aligned} dA &= \begin{cases} 1 & : \quad \text{with probability } \lambda \, dt, \\ 0 & : \quad \text{with probability } 1 - \lambda \, dt, \end{cases} \\ A(0) &= 0, \end{aligned} \tag{2}$$

  which is incremented by one at any occurrence of an audit.

- The closure threshold $B(\sigma)$ determines the consequences of an audit by partitioning the state space of the bank into a 'closure region' ($V < B(\sigma)$) and a 'continuation region' ($V \geq B(\sigma)$). When an audit occurs and the bank's state is found to be in the closure region—i.e., it is not in accordance with the regulatory requirements—it is forced to close. Due to the fact that our model allows for two levels of asset risk (σ_L and σ_H) only the two critical thresholds $B(\sigma_L)$ and $B(\sigma_H)$ are relevant for the bank.

For a given regulatory mechanism $\langle\lambda, B(\sigma)\rangle$, bank management sets an optimal response in order to maximize equity value. The available choices are (i) *stick* to the current risk level, (ii) *switch* the level of asset risk, or (iii) *close* the bank. In particular, a strategy $\mathcal{S}$ fot the bank management is a mapping from the state space into the space of available choices,

$$\mathcal{S}: \ (V, \sigma) \rightarrow \{stick, switch, close\}.$$

In technical terms, switching and closure points are absorbing barriers to the asset value process. While the first hit of a closure point results in the default of the bank, the first hit of a switching point $(\hat{V}, \sigma_H)$ absorbs the high volatility process and creates a low volatility process at $((1-k)\hat{V}, \sigma_L)$, i.e., switching from the high risk asset portfolio to the low risk portfolio destroys a fraction k of the asset value due to trading costs. Analogously, a switching point at $(\hat{V}, \sigma_L)$ absorbs the low volatility process and creates one with high volatility at $((1-k)\hat{V}, \sigma_H)$. The decision to *stick* means to leave the current risk level unchanged.

Obviously, the possible structure of such a strategy could be very complex. However, an entirely disordered set of *stick*ing, *switch*ing and *clos*ing points cannot be the optimal response to a regulatory mechanism with simple structure as assumed above. As it is the management's task to find the optimal switching and closure thresholds, we study the class of strategies $\mathcal{S}$ where switching points and closure points are boundaries of intervals with constant volatility (or in other words, where for given volatility the partition of the state space with $\mathcal{S} = stick$ is the union of open intervals). Inside these intervals of stable volatility the asset value V follows a simple geometric Brownian motion (see (1)). Consequently, given a strategy $\mathcal{S}$ the value of any claim contingent on the bank's asset value can be obtained by standard contingent claims analysis when proper boundary conditions are applied at the respective switching and closure points (see Section 3).

Concluding this section we will summarize the different claims contingent on the state of the bank (V, σ) that will be used to analyze the model and give their characteristics.

- The *market value of deposits*—denoted as $D(V, \sigma)$—is the market value of the non-insured coupon flow provided by the bank. In contrast to the insured contract held by depositors which is always worth c/r, the claim D is exposed to default risk. Furthermore, the loss in asset value caused by the management's asset substitution strategy is regarded when evaluating D, i.e., the holders of D implicitly bear a certain proportion of the switching costs.

- The *value of the deposit insurance* is denoted as $DI(V, \sigma)$. This is the current value of possible future expenditures necessary to guarantee the full face value to depositors in case of bank closure. Obviously, the value of the deposit insurance is the difference between the insured value of deposits and the market value of the coupon flow, thus,
$$DI(V, \sigma) = \frac{c}{r} - D(V, \sigma). \tag{3}$$

- The *tax benefits*—denoted by $TB(V, \sigma)$—are the current value of the profit flow originating from the tax shield τc.

- The *equityholders' portion of the switching costs*—denoted by $SC(V, \sigma)$—summarize the current value of the losses for equityholders that arise from shifting the portfolio risk from σ_L to σ_H or vice versa. In other words, anticipating future portfolio restructuring the value of the asset portfolio to the equityholders is not V but only $V - SC$.

- The *value of equity*—denoted by $E(V, \sigma)$—is simply the residual value

$$E(V, \sigma) = V - SC(V, \sigma) + TB(V, \sigma) - D(V, \sigma). \tag{4}$$

3 Valuing a Claim Contingent on $(\mathbf{V}, \sigma)$

The issue in this section is the valuation of a claim contingent on the state of the bank (V, σ). The respective equations will be derived by investigating a general claim $F(V, \sigma)$ which covers all the claims involved in our model as special cases. The adaptation of the general results to the special claims D, TB, and SC is presented in Appendix B. DI and E can then be obtained using Equations (3) and (4).

Suppose $F(V)$ is a claim contingent on V and—for a given $\sigma \in \{\sigma_L, \sigma_H\}$—the thresholds V_1 and V_2 ($V_1 < V_2$) are boundaries of a stable regime (see Section 2). That means there are (i) no switching points and (ii) no closure points inside these boundaries and (iii) the interval (V_1, V_2) either belongs entirely to the 'closure' region ($V_2 \leq B(\sigma)$) or is entirely in the 'continuation' region ($B(\sigma) \leq V_1$). Furthermore, this claim provides (iv) a constant profit flow α as long as the process V is inside these boundaries, and (v) if the regulator closes the bank at some $\hat{V}$, the claim pays $\beta + \gamma \hat{V}$.

Deriving the valuation equations we assume that the two portfolios which span the bank's investment opportunities are traded.[10] Let r denote the constant riskless interest rate, then applying Itô calculus, we find that F has to satisfy the second order ordinary differential equation

$$rF = \frac{1}{2}\sigma^2 V^2 F_{VV} + (r - \delta) V F_V + \alpha + \mathbf{1}_{[0,B(\sigma))} \lambda(\beta + \gamma V - F) \tag{5}$$

[10] We make this assumption because we want to analyze how regulation affects risk shifting of banks abstracting from the effects driven by risk preferences of investors. However, we could alternatively assume that only the bank's equity is traded. Then the equity price process reveals the market price of risk which in turn determines the market price of any claim contingent on the banks assets (see, e.g., Björk (1998), Chapter 10). The model can be solved in a very similar way, e.g. Equation 5 will change to $rF = \frac{1}{2}\sigma^2 V^2 F_{VV} + (\mu - \kappa\sigma) V F_V + \alpha + \mathbf{1}_{[0,B(\sigma))} \lambda(\beta + \gamma V - F)$, where κ denotes the market price of risk. The results are qualitatively similar but partly driven by the parameterization of the model with respect to the market price of risk and the drift rates of the portfolios.

inside the interval (V_1, V_2), where $\mathbf{1}_{[0,B(\sigma))}$ denotes the indicator function over the interval $[0, B(\sigma))$ and F_V, F_{VV} are the first and second partial derivatives of the claim value with respect to V.

The general solution of this equation—in the case that V is in the closure region—is given by

$$F(V,\sigma) = \frac{\alpha}{r+\lambda} + \lambda \left(\frac{\beta}{r+\lambda} + \frac{\gamma}{\lambda+\delta} V \right) + A_1 V^{x_1(\sigma)} + A_2 V^{x_2(\sigma)}. \tag{6}$$

Outside this region the solution is

$$F(V,\sigma) = \frac{\alpha}{r} + A_1 V^{y_1(\sigma)} + A_2 V^{y_2(\sigma)}. \tag{7}$$

The constants $x_1(\sigma)$, $x_2(\sigma)$, $y_1(\sigma)$, $y_2(\sigma)$ are the negative and the positive root of the characteristic quadratic polynomial of the respective homogeneous differential equation

$$\begin{gathered} \tfrac{1}{2}\sigma^2 x(\sigma)[x(\sigma)-1] + [r-\delta]x(\sigma) - [r+\lambda], \\ \tfrac{1}{2}\sigma^2 y(\sigma)[y(\sigma)-1] + [r-\delta]y(\sigma) - r. \end{gathered} \tag{8}$$

Thus, inside an interval of stable regime the value of the claim F is entirely characterized by (6) and (7) respectively which are the analytical solutions of the Hamilton-Jacobi-Bellman equation (5). The only unknowns remaining are the two parameters A_1 and A_2 which must be determined by boundary conditions at the boundaries of this interval.

In our model the canonical boundaries which determine intervals of stability are

1. switching thresholds,
2. closure thresholds set by the bank's management,
3. the boundaries $B(\sigma_L)$ and $B(\sigma_H)$ of the closure region resulting from the regulatory mechanism $\langle \lambda, B(\sigma) \rangle$, and

4. the critical value $\frac{c}{r}$; at this threshold the functional form of the default payoff of the deposit insurance contract changes – below $\frac{c}{r}$ the default payoff is $V - \frac{c}{r} < 0$, since the deposit insurance has to bear the difference between the asset value and the face value of deposits. Above $\frac{c}{r}$ the default payoff to the deposit insurance claim is zero, since the bank's assets value is sufficiently high to cover deposits.

The boundary conditions are derived for the general claim F in Appendix A, and for the specific claims in Appendix B. For a given strategy of the bank management, that is for given switching and closing thresholds, the value of any claim is analytically determined as it is shown in Appendix C. In the next section, the conditions determining the optimal strategy are derived.

3.1 Optimality Conditions

The management's aim is to find the operational strategy which maximizes the equity value. As stated in Section 2, the choice variables are the switching points and the exit thresholds which have to be fixed simultaneously. The first order conditions for switching and closure points that are boundaries of intervals of stability imply smoothness at the respective boundaries (see Dixit (1993) for a discussion of the so called 'smooth pasting conditions').

- if $(\hat{V}, \sigma_c)$ is a switching point, substitution of the respective boundary condition for D, TB, and SC (see Appendix B) into (4) leads to

$$\lim_{V \to \hat{V}} E(V, \sigma_c) = E((1-k)\hat{V}, \sigma_{-c}), \tag{9}$$

stating that there is no jump in equity value when the asset portfolio is reorganized. Taking the first derivative of this boundary condition with respect to $\hat{V}$ leads to the optimality condition

$$\lim_{V \to \hat{V}} E_V(V, \sigma_c) = (1-k) E_V((1-k)\hat{V}, \sigma_{-c}). \tag{10}$$

- if $(\hat{V}, \sigma_c)$ is the point at which management decides to close the bank, the boundary condition for E is

$$\lim_{V \to \hat{V}} E(\hat{V}, \sigma_c) = 0, \tag{11}$$

  leading to the optimality condition

$$\lim_{V \to \hat{V}} E_V(\hat{V}, \sigma_c) = 0. \tag{12}$$

Since the optimality conditions (10) and (12) are non-linear, the determination of the optimal thresholds and the verification of the second order conditions has to be performed numerically.

4 BB versus VaR – Comparison of two Regulatory Approaches

Based on the framework developed in the last two sections we now consider two stylized regulatory systems – a Basel I Building Block (BB) approach and a genuine Value-at-Risk (VaR) based approach. We start with briefly outlining current regulation and then look at the main differences in capital requirements. Finally, we analyze the implications of these regulatory mechanisms on the optimal risk taking behavior of the bank management.

4.1 Capital Requirements

One of the main ideas of the 1988 Basel Accord (see Basel Committee on Banking Supervision (1988)) is to increase bank soundness by requiring banks to back up their assets with a pre-specified amount of equity capital.[11] In general the capital requirement which should cover credit risk is set to 8% but for asset classes that are considered less risky, like loans to the government and supranational organizations, OECD banks and asset backed residential mortgage loans, there are discounts on the capital requirement. In an amendment to the accord in 1996 the bank's assets are divided into the trading book, containing all positions intended for short-term resale, and the banking book, that comprises all other assets—especially the loan portfolio.[12] In the same document capital requirements are also specified for the market risk in the trading book. To fulfill these requirements, banks can either choose a building block method or use their internal Value-at-Risk models to compute the adequate capitalization. The most recent step in international bank regulation is the proposal on the New Basel Capital Accord of the Basel Committee on Banking Supervision (2001), which is intended to improve capital adequacy regulation for credit risk. Multiple options are available to the bank to compute the capital requirement for credit risk. The 'standardized approach' is again a building block method, whereas the most advanced method, the 'internal ratings based' (IRB) approach, additionally includes some refinements to adjust for portfolio diversification (called 'granularity adjustment'). While the latter mechanism is still based on building blocks it follows the overall goal of the new accord to link capital requirements more closely to credit risk.

[11] See e.g. Jorion (2000) or Dewatripont and Tirole (1994) for a comprehensive treatment of bank capital standards. E.g., corporate debt and real estate have a capital requirement of 8%, asset backed mortgage loans require 4%, claims on OECD banks and regulated securities firms require 1.6% and cash and claims on OECD central governments do not have to be backed up. The overall capital requirement of a bank is calculated as a weighted average.

[12] See Basel Committee on Banking Supervision (1996).

These international guidelines have been implemented by almost all countries with minor modifications. In our paper, however, we do not want to model a country specific implementation but rather theoretically analyze the rationale for the fast development in bank regulation towards more risk sensitive capital standards. To formalize this transition, we model two stylized approaches for setting capital requirements. As the starting point, with low risk sensitivity, we consider a simple Building Block approach, while we use a Value-at-Risk approach as a framework, where capital is directly linked to asset risk.

The Building Block approach, which is current practice in almost all countries, is easy to implement. First the assets are assigned to risk buckets and then capital requirements are computed using given weights. Once assigned to a bucket, the asset has the same capital requirement as all others in this bucket. Thus while banks are penalized by higher capital requirements for inter-bucket risk shifting, e.g. substituting government bonds with corporate loans, intra-bucket risk shifting is not captured.

We model the Building Block regulation in a stylized way by focusing our analysis on intra-bucket risk shifting. We assume, that the two asset portfolios available to the bank are formed such that the relative proportions of assets in the respective buckets and thus the capital requirement does not change when the bank shifts from one portfolio to the other. This assumption may seem very stringent at first, however, it is justified by the fact that Basel I provides only four buckets. Since all corporate loans are in one bucket, regardless of the borrowers rating, this opens a wide range of intra-bucket risk shifting possibilities. This highlights an important feature of any building block regulation. With a finite number of buckets and a continuum of financial assets' risk levels it is not possible to assign a capital requirement to each asset that is consistent with its economic risk. While there will always be some heterogeneity in the risk structure of the assets in a bucket, the differences are huge under capital regulations that are in the spirit of Basel I. Banks have recognized this weakness of regulation and are exploiting the mismatch. This regulatory capital arbitrage (see e.g., Jones (2000)) is seen as a threat for regulatory supervision by many authorities (see Meyer (1998)). In our analysis the principal results also hold when the two portfolios have different capital requirements under the Building Block approach as long as there is a discrepancy between regulatory capital requirement and capital necessary to cover economic risk.

Due to the assumed capital structure (see Section 2) the bank has a simple balance sheet. The assets with current market value V are on the asset side. The liabilities are represented by perpetual deposits with a constant instantaneous coupon of c and face value c/r (where r denotes the riskless interest rate) and by equity. The regulator's goal is to preserve a safety cushion, such that the value of the assets V is sufficient to satisfy the depositors' claims c/r. Under Building Block regulation the minimum cushion $V - c/r$ is determined by the risk weighted assets of the bank. Depending on the bank's borrowers this capital requirement will be a fraction ρ of the bank's assets. In the case of an audit, the bank will be allowed to continue operation only if the safety cushion is at least as large as the capital requirement:

$$V - \frac{c}{r} \geq \rho V \tag{13}$$

The main feature of the Building Block approach is that the exposure to a risk factor is limited, e.g., for a given amount of capital the notional value of loans a bank can give to the corporate sector is limited. Additional equity capital has to be raised before the bank can grant new loans. The variability of the risk factor is not included in the computation of necessary capital, e.g. for loans the default and recovery rates are not relevant for the capital requirements.[13] According to the assumption that the relative proportions assigned to the building blocks are identical for both portfolios, the fraction ρ does not change when asset substitution takes place. The bank can, e.g.,

[13] Another example is equity risk of the trading book. While the maximum amount invested in stocks is limited by the banks capital base, the volatility of the stocks in the banks portfolio is not considered.

lower the average rating of its corporate loan portfolio or grant mortgage loans to less credit worthy borrowers. The closure threshold $B(\sigma)$ under BB regulation is therefore constant

$$B(\sigma)^{BB} = \frac{1}{(1-\rho)}\frac{c}{r}, \quad \sigma \in \{\sigma_L, \sigma_H\}. \tag{14}$$

The Value-at-Risk approach is conceptually different from the building block approach, since it includes not only the exposure to risk factors but also the volatility of the risk factors. As we do not want to model the current regulation of a specific country but instead examine the incentive effects of different regulatory systems we look at capital requirements that are solely based on Value-at-Risk.[14]

VaR regulation demands that—in case of an audit—the bank's safety cushion $V - \frac{c}{r}$, the difference between asset value and the face value of debt, must be at least as high as the $p\%$ Value-at-Risk for a time horizon of T multiplied by a 'panic factor' ξ, which is set by the regulator.[15] The panic factor is set to three in most countries and is intended to cover model risk.[16]

Since the asset value of the bank V follows a geometric Brownian motion (see Equation (1)), the returns are normally distributed with mean $(\mu - \delta - \frac{1}{2}\sigma^2)T$ and a standard deviation of $\sigma\sqrt{T}$. The factor T scales the moments of the distribution, e.g., if μ, δ, and σ are measured with respect to the time unit of one year (= 250 trading days), we have $T = 10/250$ to capture the risk over the next ten days. After linearizing and neglecting the mean of the distribution, as it is done in most VaR implementations, the $p\%$ quantile of the loss distribution is given by $\Phi^{-1}(p)\sigma V\sqrt{T}$, where $\Phi^{-1}(p)$ is the $p\%$

[14]Comparing the pure building block approach to a combination of building block and Value-at-Risk capital requirements as it is current practice in most countries would give the same principal results but would weaken the effects. Another reason why we look at a pure Value-at-Risk regulation is that there is an active discussion, whether regulators should accept internal models to compute capital requirements for credit risk.

[15]Usually p is set to 99% and T is 10 days.

[16]As outlined in Basel Committee on Banking Supervision (1996) this multiplier is 'designed to account for potential weaknesses in the modeling process' such as fat tails in the distribution of risk factor returns, sudden changes in volatilities and correlations, intra day trading, event risk and model risk (especially with options).

quantile of the standard normal distribution. Or in other words, the bank is allowed to continue its operation if

$$V - \frac{c}{r} \geq \xi \; a \; \sigma \; V, \tag{15}$$

where $a = \Phi^{-1}(p)\sqrt{T}$. The closure threshold for VaR regulation is, thus, given by

$$B(\sigma)^{VaR} = \frac{1}{(1 - \xi \; a \; \sigma)} \frac{c}{r}. \tag{16}$$

Comparing equations (14) to (16) we can see that the main difference between the two regulatory regimes is that VaR regulation explicitly accounts for the risk of the portfolio by adjusting the capital requirements, whereas the BB regulation is independent of the volatility of the institution's assets if risk shifting occurs within buckets. This is important for the risk taking (shifting) incentives the management is exposed to, which will be discussed in the following subsection.

4.2 Risk Shifting Incentives

The risk taking incentive which leads bank managers to increase an institution's risk stems from the fact that the deposit insurance corporation gives the equityholders a put option on the bank's assets.[17] The value of this put option increases with the volatility of the underlying and, thus, makes higher risk more favorable to equityholders. The intuition is that high volatility creates an upside chance while the downside risk is bounded by limited liability. Since deposits are fully insured, the bank is able to raise deposits at the risk free rate r (it does not have to compensate depositors for default risk), thus, risk taking is at the expense of the insurance corporation. To mitigate this problem, different regulatory responses have been proposed, all of them focusing on resolving the convexity in the value function of equity. Fries, Mella-Barral, and Perraudin

[17] The strike of this option according to our model assumptions is $(1 - \tau)\frac{c}{r}$ which is the current value of tax adjusted coupon payments.

(1997) suggest state dependent subsidies and equity support schemes to make the equity function linear for troubled banks. Bhattacharya, Plank, Strobl, and Zechner (2001) choose the closure threshold and the auditing intensity such that the value function is linear for solvent banks (i.e., for banks whose asset values satisfy the minimum capital requirement).

The regulatory mechanisms we consider in this paper (Building Block approach as well as Value-at-Risk regulation) are of the form $\langle \lambda, B(\sigma) \rangle$, consisting of an audit intensity λ and a closure threshold $B(\sigma)$ (see Section 2). If one of the randomly occurring audits reveals an asset value below the closure threshold, the bank is forced to close. Hence, equityholders are not entirely free in setting the optimal closure point for the bank with the consequence that they cannot fully exploit the benefit of the put option. Depending on λ and $B(\sigma)$ the risk taking incentive is weakened or managers might even find it beneficial to reduce asset risk.

Under BB regulation the minimum capital requirement depends only on a coarse classification of the bank's assets and not on economic risk. Therefore, the auditor's toughness (i.e., choosing a high λ) is the key instrument for mitigating risk taking. Figure 1 illustrates the impact of different audit intensities on the equity value by means of an example. When λ is low, the convex shape of the simple put option prevails over the entire range of the underlying, which means that equityholders have a global incentive to take risk. However, under strict auditing, the curvature of the equity value changes its sign. When the asset value is significantly below the closure threshold, an audit will result in the immediate closure of the bank. Since higher asset volatility increases the chance that the bank recovers before the next audit takes place it is preferred to low volatility. Or in other words, if the bank is in real distress, the management has a strong incentive to gamble for resurrection—regardless of the audit intensity. When the capital requirement is met, there is still the positive effect of high volatility on the equity value that stems from exploiting the deposit insurance system. However,

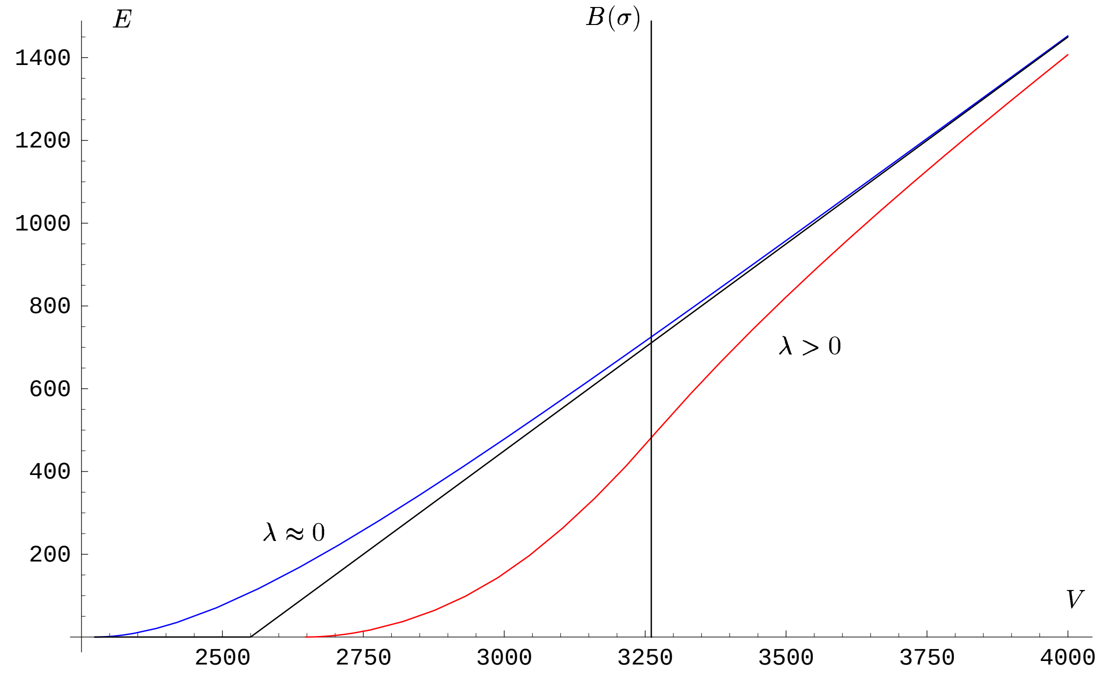

Figure 1: Bank equity value E under Basel I Building Block regulation as a function of the asset value V for high and low audit intensities plotted against the asymptote. The vertical line represents the minimum capital requirement. While convexity prevails for low audit intensities, high audit intensities create an incentive to reduce risk for the solvent bank. The face value of debt is assumed to be 3000.

high volatility increases the probability that the bank runs into distress (i.e. the asset value drops below the closure threshold) and that it will—due to auditing—be closed by the regulator. This harms the equityholders, who loose the charter value of future tax benefits. If λ is sufficiently high the negative effect of high volatility on equity dominates the positive one and the well capitalized bank prefers low risk to high risk. When well capitalized banks reduce their assets' risk, they essentially lower the deposit insurance corporation's liability (see Section 5 for a more detailed discussion of this feature). Nevertheless, whether the bank managers really switch the risk level and when they optimally do it also depends on the costs for rearranging the portfolio and can only be answered after analyzing the model with a particular parameterization (under full consideration of higher order effects, see Section 5).

Under VaR regulation the tradeoff between exploiting the deposit insurance and fearing closure due to regulatory enforcement is still valid. However, VaR regulation enhances the incentive for solvent banks to reduce risk by setting the minimum capital requirement according to the actual asset risk. Since higher asset volatility implies higher capital requirements (see Equation (16)), a bank can improve its capital ratio by reducing the asset risk. The effect of risk sensitive capital requirements is most evident in the case where the asset value is between the closure threshold for low risk $B(\sigma_L)$ and the closure threshold for high risk $B(\sigma_H)$. If an audit occurs and the bank is invested in the low-risk portfolio the audit confirms solvency (i.e., no negative consequences for the bank). If—in the same situation—the bank's portfolio consists of high-risk assets an audit results in bank closure. Due to the diffusion-nature of the asset value process, this effect creates an incentive for rational equityholders to reduce asset risk even for the well capitalized bank (i.e., $V > B(\sigma)$). Or in other words, by switching to the low-risk portfolio the bank can enhance its capital ratio and simultaneously reduce the probability of getting into financial distress.

Ignoring the possibility to switch the risk level, we know from existing literature (see e.g., Merton (1978)) that the bank's optimal strategy can be characterized solely by a lower exit level. Therefore, as long as the value V of the bank's assets is higher than this critical level, the bank maintains its obligations. When V hits this level for the first time, equityholders default from their obligations. According to the risk shifting incentives identified above we consider strategies that extend this base strategy allowing the distressed low-risk bank to respond to the risk taking incentive by switching to the high-risk portfolio. Furthermore, we give the well capitalized high-risk bank the opportunity to respond to the risk reduction incentive by switching to the low-risk portfolio. Whether switching is optimal or not has to be determined by comparing equity values. The locations of switching and closure points are determined by the optimality conditions given in Section 3.3.1. Figure 2 illustrates the general strategy and introduces the notation of the critical levels where the bank switches the asset risk

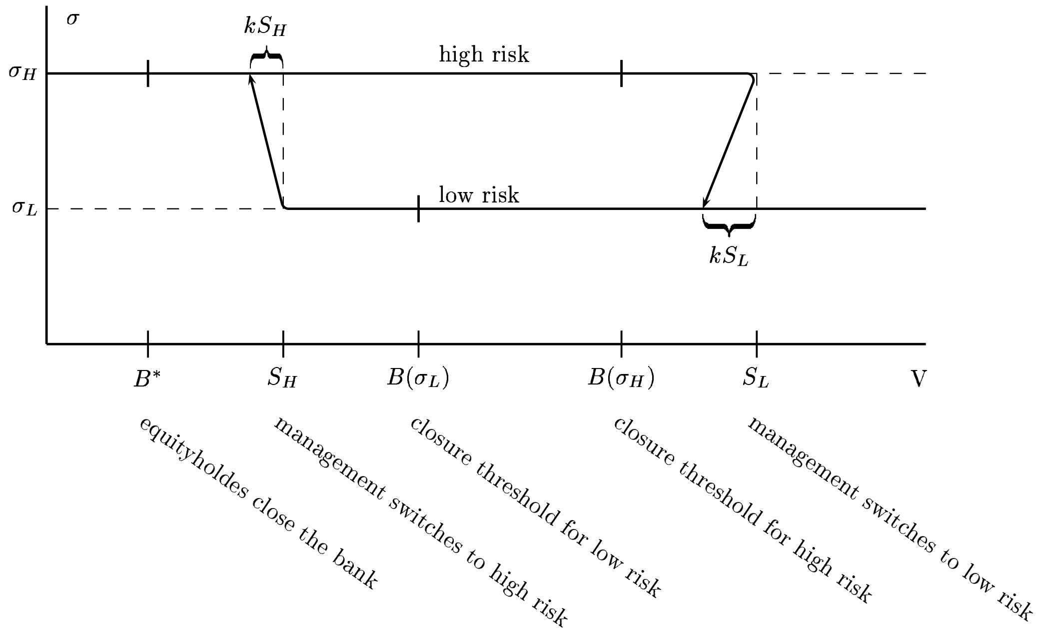

Figure 2: Management's risk taking behavior under Value-at-Risk regulation when the audit frequency is sufficiently high to induce a shift towards low risk when the bank is solvent. Due to switching costs – destroying a fraction k of the asset value at every switch – the possible states of the bank form a hysteresis.

or where it closes voluntarily. The switching costs (which form a dead weight loss) are responsible for the fact that the bank's possible states form a hysteresis. When the solvent bank's asset value deteriorates, the management will switch to high risk at a certain threshold S_H, thereby destroying asset value kS_H due to trading costs. If the asset value drops further equityholders will close the bank voluntarily at the level B^*. Recovering from distress, bank's management will wait until S_L to switch back to low risk (again losing asset value to cover trading costs). Between these switching thresholds there is a non-unique correspondence between asset value and asset risk.

Figure 3 shows the corresponding equity value as a function of the asset value. Despite the convexity of the high-risk value function, the VaR based capital requirements

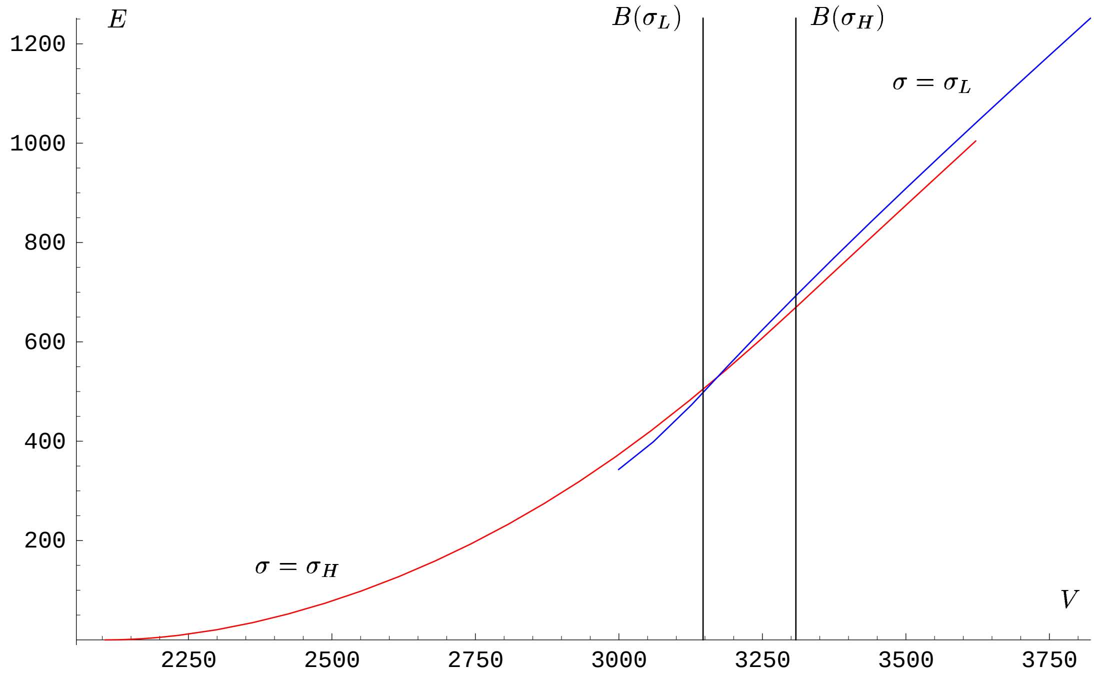

Figure 3: Bank equity value E under VaR regulation as a function of the asset value V. The vertical lines represent the closure thresholds for the low risk and the high risk portfolio. The two functions show the equity value for high risk ($\sigma = \sigma_H$) and low risk ($\sigma = \sigma_L$) respectively. While the bank prefers high risk when it is insolvent it reduces risk when sufficient solvency is regained.

(together with an appropriate λ) create enough incentive for the well capitalized bank to switch back to low risk.

5 Results and Comparative Statics

In this section we analyze the different incentives and potential benefits created by BB and VaR regulation by means of a numerical example. For this purpose we first take a closer look at the mechanics behind the optimal risk choice and derive some comparative static results. Secondly, we analyze these consequences of different risk taking behavior on the deposit insurance agency and the bank's equity holders. Finally, we derive some policy implications. Unless otherwise stated, we take the parameter values from Table 1. Note, that we chose the panic factor to be one for the base case. This is because the panic factor is intended to capture model risk, which is not existent in our model. Nevertheless, we examine the general effect of a panic factor greater than one on the risk shifting behavior of banks.

As argued in the previous section, the risk-reduction incentive under BB based capital requirements is weaker than under VaR regulation. To demonstrate this feature, the parameter set of the base case (Table 1) is chosen such that it is optimal for the solvent bank to reduce risk when it is VaR regulated, and to stick to high risk when it is BB regulated.[18] In the following comparative statics we will explore under what circumstances other strategies will be selected by the bank.

5.1 Comparative statics

In Figure 4 the locations of the critical thresholds for the Value-at-Risk regulated bank are plotted for different levels of the volatility of the risky portfolio (σ_H). We see that the closure threshold set by the regulator for the high-risk portfolio, $B(\sigma_H)$ increases with the portfolio's risk. Looking at the equityholders' optimal closure decision, we see that a higher volatility gives banks a greater value of gambling for resurrection by increasing the probability that the asset value will grow beyond the closure threshold again within the foreseeable future. As this effect is not compensated by the increase in the closure threshold $B(\sigma_H)$, which is approximately linear for small changes in the financial institution's risk, bank equityholders are willing to support the bank for a longer period of time (B^* decreases). The increased attractiveness of gambling also makes it more advantageous for low-risk banks to start gambling by switching to high risk at point S_H once the bank is under-capitalized (i.e., the asset value is lower than $B(\sigma_L)$). The

[18] To be accurate, if the bank is established as a well capitalized low-risk bank, then it will not switch to the high-risk portfolio immediately. However, once the BB regulated bank has reorganized its portfolio at S_H it will stay a high-risk bank.

Table 1: Parameter values for the numerical analysis and results of the base case scenario.
Panel A: parameter values

coupon of debt	c	150
riskless interest rate	r	0.05
face value of debt	c/r	3000
corporate tax rate	τ	0.15
audit frequency	λ	0.45
return-volatility of low risk portfolio	σ_L	0.1
return-volatility of high risk portfolio	σ_H	0.2
switching costs	k	0.01
cash flow rate	$\delta_L = \delta_H$	0.01
capital requirements - Basel regulation	ρ	8%
Value-at-Risk confidence level	p	99%
Value-at-Risk holding period	T	10 days
Panic Factor	ξ	1

Panel B: Regime switching points for the VaR regulated bank

Equityholders abandon Bank	B^*	2105.31
Closure Threshold - low risk	B_L	3146.63
Closure Threshold - high risk	B_H	3308.34
Managers switch to high risk	S_H	2995.94
Managers switch to low risk	S_L	3658.62

Panel C: Regime switching points for the bank with Building Block capital requirements.

Equityholders abandon Bank	B^*	2100.66
Closure Threshold	B_L	3260.87
Managers switch to high risk	S_H	3191.50

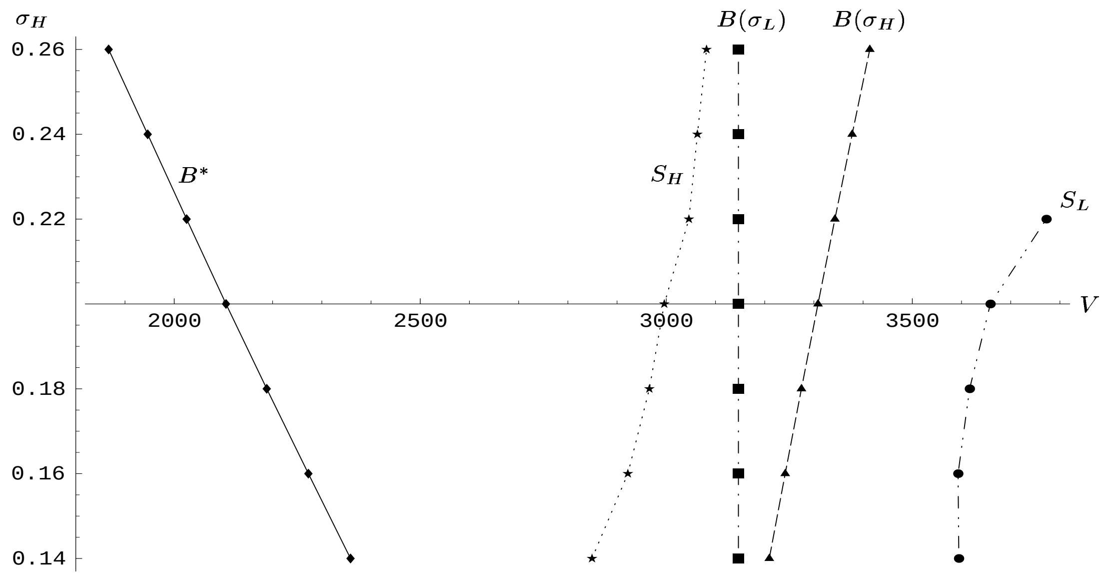

Figure 4: Locations of the critical thresholds for different values of the high risk technology's volatility σ_H. Higher risk increases the chances of an insolvent bank to regain solvency and makes equityholders keep the bank alive for a longer period of time (B^* decreases). Higher risk also makes banks switch to high risk earlier at S_H. And it makes managers switch back to low risk (S_L) later, as the option value of the deposit insurance decreases with volatility. If $\sigma_H \gtrapprox 0.23$, equityholders will not switch back to low risk any more.

switching point S_L, where high-risk banks switch to low risk again, is substantially increasing with σ_H. The value of the deposit insurance put option increases with volatility. This effect dominates the gain from reduced insolvency risk when switching to low risk and the reward in form of lower capital requirements. It is interesting to see that beyond a certain level (which is ≈ 0.23 in our example) equityholders see no reason to switch back to low risk any more. Lower capital requirements cannot offset the high value of the deposit insurance option. Under the current parameter set the BB regulated bank has no incentive to reduce risk, independent of the investment opportunity σ_H. That is, it finds it optimal to stick to high risk even if it is well capitalized.

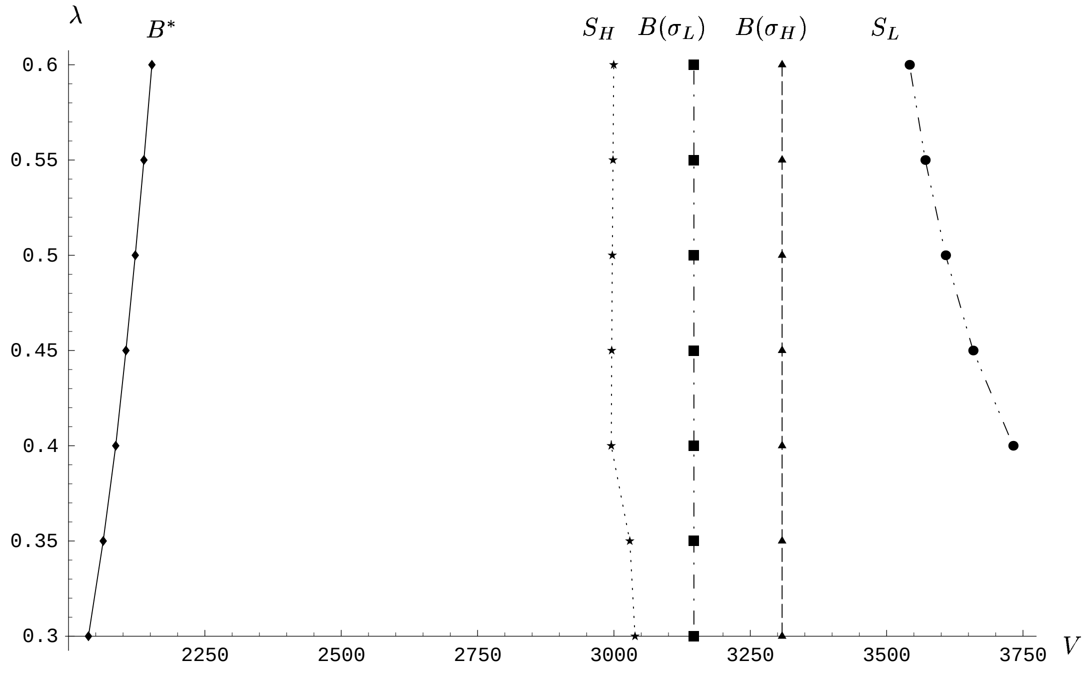

Figure 5: Locations of the critical thresholds for the VaR regulated bank for different values of audit frequency λ. While the closure thresholds ($B(.)$) are not affected by the auditing intensity, a tougher regulator makes it less attractive for shareholders to support an ailing bank (B^* increases). For high audit intensities, the high risk portfolio also becomes less attractive, managers switch earlier back to low risk (S_L decreases). If auditing is too relaxed (here below $\lambda \approx 0.37$), bank managers do not have an incentive to switch back to low risk any more.

We see that the investment opportunities of the bank have substantial impact on the risk taking incentives. If banks can increase their risk substantially, the incentive for solvent banks to reduce risk is destroyed. The only way for the regulator to maintain the risk reduction incentive also for higher volatility levels is to increase the auditing intensity, which will be explored next.

Figure 5 illustrates the impact of the auditing frequency λ. The closure thresholds $B(\sigma_L)$ and $B(\sigma_H)$ are not affected by the auditing policy of the regulator. The regime switching points show intuitive behavior. Equityholders are less willing to support an insolvent bank as the probability of an audit increases. The critical asset value B^* at which the equityholders will close the bank therefore increases with the audit frequency. The switching point S_H, where equityholders switch to high risk and start to gamble is determined by two offsetting effects. If the value of the banks assets is below the closure threshold $B(\sigma_L)$, a higher probability of an audit makes it more likely to get caught in the closure region. A more stringent auditing policy therefore puts additional pressure on management to start gambling. But, once switched to higher risk, capital requirements increase, and these capital requirements are harder to meet before the next audit. This effect together with dead weight switching costs[19] determines the location of the switching threshold S_H. In this example the two effects approximately offset each other.[20] A similar trade-off determines the location of the point S_L, where the manager switches from high risk back to low risk. The switch reduces the value of the deposit insurance option, resulting in a value gain from switching late (i.e., switching at high V). On the other hand, switching allows the manger to be more relaxed, since the capital requirements are lower, i.e., the distance to the closure region increases and the probability of getting into trouble decreases. As the regulator becomes tougher (λ increases), the latter effect dominates and managers have an incentive to switch at lower values of V. If the auditor reduces λ below a certain level $\lambda_{\min}$ (which is ≈ 0.37 in our example) an audit is so unlikely that banks will focus on exploiting the deposit insurance option instead of switching back to low risk. Under BB regulation the minimum audit intensity that creates a sufficient risk-reduction incentive is significantly higher ($\lambda_{\min} \approx 0.51$).

[19]Since switching costs are assumed to be proportional to the asset value there is a general incentive to switch at low asset values. However, to reduce the switching frequency, decision makers try to increase the distance between the switching points S_L and S_H.

[20]While in this section the intuition is explained using only first order effects, actually all future switching decisions and all future switching costs are incorporated in the equityholders' optimization.

5.2 Deposit insurance and equity value

Once the regulator has specified the regulatory mechanism, i.e., closure thresholds and an auditing intensity, the bank's equityholders will respond by choosing an optimal risk taking policy. The bank's strategy will be crucial for evaluating the liability of the deposit insurance corporation. As the deposit insurance guarantees the face value c/r to the bank's depositors, the current value of the potential future liability (DI) of the deposit insurance corporation is given by the difference between the face value and the current market value of deposits. Figure 6 shows the regulator's liability as a function of V for two banks following different strategies. The VaR regulated bank switches to the low-risk asset portfolio when it is well capitalized, while under BB regulation the bank finds it optimal to always stick to high risk. The bank which adopts the switching strategy reduces the risk in the banking sector, and thus, lowers the liability of the deposit insurance for all asset values. The chosen regulatory mechanism has an impact on the deposit insurance system, as it influences the risk shifting strategies adopted by the financial institutions. Independent of V the regulator's liability is lower when implementing a regulatory mechanism that encourages risk reduction. The regulator can either keep the surplus or, when she is interested in providing a fairly priced deposit insurance, can also significantly lower the insurance premia which will directly benefit the bank's equityholders.[21] Troubled banks, however, are still a threat for the regulator under both regulatory regimes as the regulator's liability increases sharply, when banks come into financial distress.

Concerning the impact of the regulatory regime on the equityholders' claim, one expects that the reduction in the deposit insurance liability due to risk reduction is—when not adjusting the insurance provision—at the expense of the equityholders. However, Figure 7 shows the equity value for the solvent bank ($V = 4000$) under both, VaR regu-

[21] We have not included an insurance premium in our model, but if the regulator charges the bank an upfront premium, the fair value is DI, which is lower under VaR regulation. The same intuition applies for a continuously paid premium.

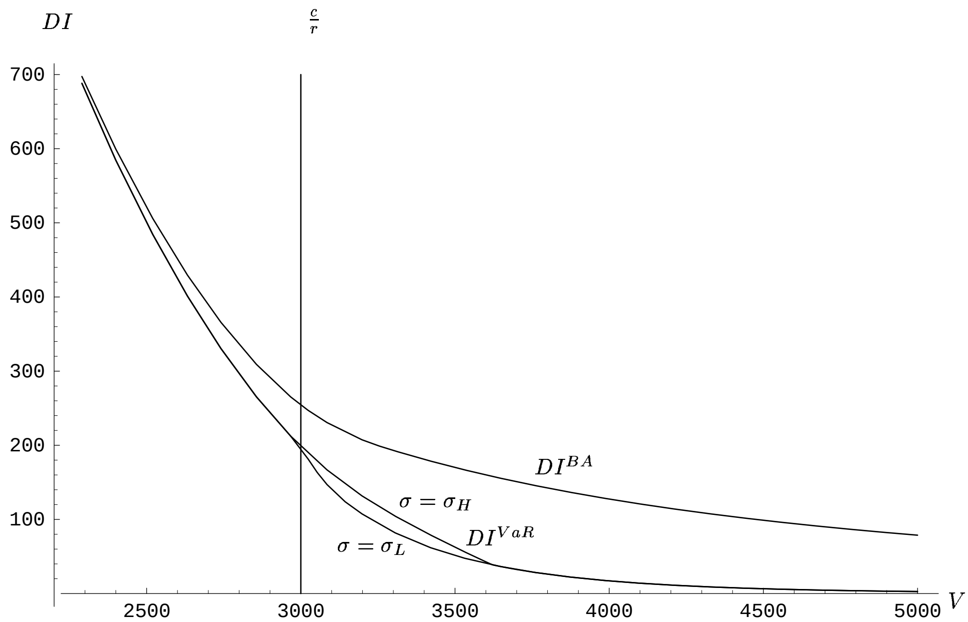

Figure 6: The current value of the potential liability of the deposit insurance corporation for different levels of V under Building Block and VaR regulation. For both choices of asset risk the regulator's liability under VaR regulation is below the one under BB regulation. Under Value-at-risk regulation the deposit insurance claim forms a hysteresis, i.e., depending on history bank managers either choose high risk σ_H or low risk σ_L.

lation (= risk reduction as long as $\sigma_H \lessapprox 0.23$) and BB regulation (= no risk reduction) for equal audit intensity ($\lambda = 0.45$). Exploiting the deposit insurance system is not that attractive for well capitalized banks, since the put option is far out of the money. In this situation the tax benefits contribute most to the shareholders' wealth. Under Value-at-Risk regulation, banks optimally choose the low-risk portfolio, hence, their default probability is significantly lower which increases the present value of the expected future tax benefits. Solvent banks and regulators, thus, may have common interests. Banks want to reduce risk to increase the value of their charter while the regulator desires a sound banking system. Value-at-Risk regulation makes it easier to bring these incentives in line, by rewarding low risk banks with lower capital requirements.

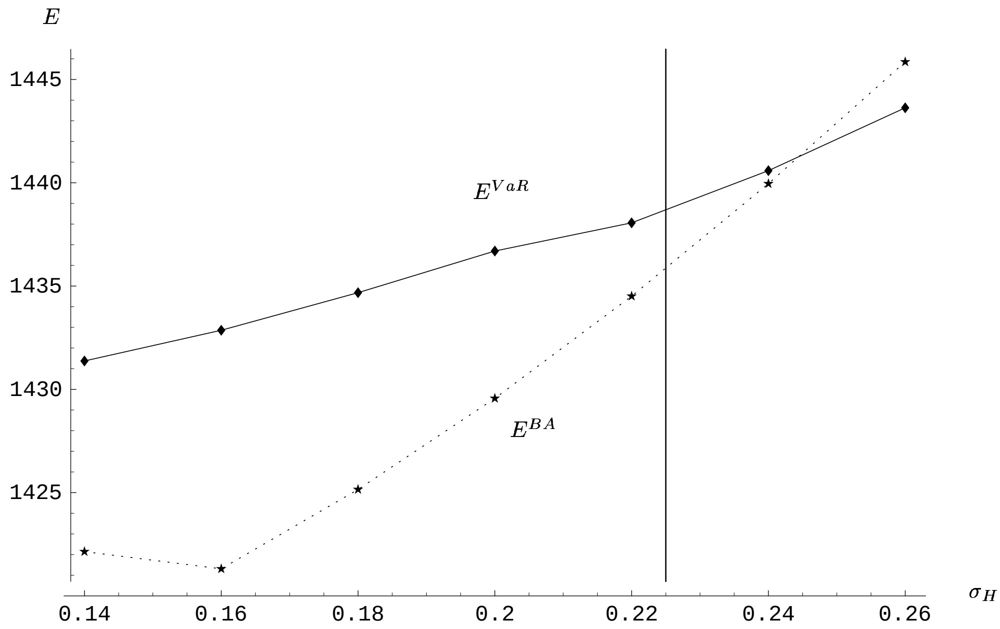

Figure 7: Equity value of the solvent bank ($V = 4000$) as a function σ_H of the volatility of the high risk portfolio. Value-at-Risk regulation gives solvent banks a higher equity value than regulation according to the Building Block approach as long as they have an incentive to reduce risk (left of the vertical line). When sticking to high risk is optimal, banks may be better off with Building Block regulation, because of lower capital requirements.

For high values of σ_H, ceteris paribus, equityholders will vote against VaR for two reasons. First the benefits of risk reduction are reduced as the higher volatility makes it more attractive for banks to exploit the deposit insurance put option. Second, while the BB regulated bank faces a uniform capital requirement, the closure threshold $B(\sigma_H)$ increases significantly in σ_H, which increases the 'average capital requirement' for the VaR regulated bank. Many regulators (e.g., in the EU countries) allow banks to choose the regulatory framework for the trading book. According to our model, we should find that solvent banks will vote in favor of VaR while troubled banks will stick to BB regulation.

5.3 Prudent Regulation

In our paper we focus on the optimal response of the bank to a given regulatory mechanism, where we assume that the banks decision makers follow the objective to maximize equity value. Knowing the optimal reaction of a bank to a given regulation, it would be very interesting to derive an optimal regulatory framework, modeling the entire game between policymakers and banks. This, however requires a specification of the regulator's value function. To do so, one must include the social value of the banking system (e.g., value of the payment system, welfare increasing projects that would not be founded by capital markets, ...) and balance it with the social costs of bank supervision (deposit insurance, auditing) and the social costs of bankruptcy (e.g., direct bankruptcy costs, systemic risk considerations, loss of confidence in the banking system, ...). To quantify these effects is beyond the scope of this paper and is therefore omitted. Nevertheless our analysis allows us to explore the influence of the regulatory mechanism on certain components of social welfare.

In the previous section it is demonstrated that implementing a regulatory mechanism which gives solvent banks sufficiently large incentive to lower their asset risk leads to significantly reduced deposit insurance liabilities DI, compared to the case where banks—once having switched to the high-risk portfolio—remain 'high-risk' banks forever. When charging banks a fair premium the insurance system can be kept at a significantly smaller scale if more banks follow the risk reduction strategy. If there are dead weight losses in the deposit insurance system, e.g., a certain fraction of the premium paid has to be used to cover administrative expenses, the smaller insurance system is more efficient.[22] Or alternatively, suppose that the social costs entailing bankruptcy are proportional to the shortfall upon liquidation, i.e., the amount by which the liabilities exceed the value of the assets. Since the deposit insurance claim DI denotes the present

[22]E.g., the FDIC's budget for administrative expenses in 2000 was 1.18 billion dollars.

value of the future shortfall, social costs are proportional to DI and can be reduced when providing a sufficient risk-reduction incentive.

Hence, given the evidence that it is beneficial for society that solvent banks reduce their asset risk, we focus on the question, whether a specific regulatory mechanism induces this behavior or not. From Section 5 we know that—for both, BB and VaR regulation—a certain minimum audit intensity $\lambda_{\min}$ is required to provide this incentive. Figure 8 compares the minimum level of auditing that has to be performed in order to give solvent banks an incentive to reduce risk for different investment opportunity sets σ_H. According to the discussion in Section 4 the weaker risk-reduction incentive provided by BB regulation transforms into a higher minimum audit frequency $\lambda_{\min}$ compared to VaR regulation. Since auditing costs form dead weight losses, a reduction in the required audit intensity reduces undesirable externalities and increases social welfare. From a social planner's perspective, we might again favor capital requirements which are based on VaR rather than on a building block approach as they require less auditing..

We can also see from Figure 8 that $\lambda_{\min}$ depends on the bank's investment opportunity set. For high values of σ_H exploiting the deposit insurance option is very tempting for the equityholders, resulting in a positive slope of $\lambda_{\min}$. That means, to maintain the risk-reduction incentive the regulator has to apply a higher audit intensity when banks have the possibility to invest into very risky portfolios. Because of the lack of sensitivity to economic risk, this increase in $\lambda_{\min}$ is more pronounced under BB than under VaR regulation.[23] Thus when the supervisory authorities are not informed about the banks investment opportunities ex ante, they may either apply too much auditing thereby wasting resources or apply insufficient auditing in order to maintain the switching incentive. For example, applying FDIC's current auditing policy with inspections every 12-18 months to the base case scenario under BB regulation gives well capitalized banks

[23]Interestingly, we also see higher minimum audit intensities when σ_H is low. This simply stems from the fact that the gain from reducing risk decreases as the difference of the two portfolio-volatilities gets smaller, whereas the switching costs are assumed to be constant with respect to volatility. Again, this effect is more evident under BB than under VaR regulation.

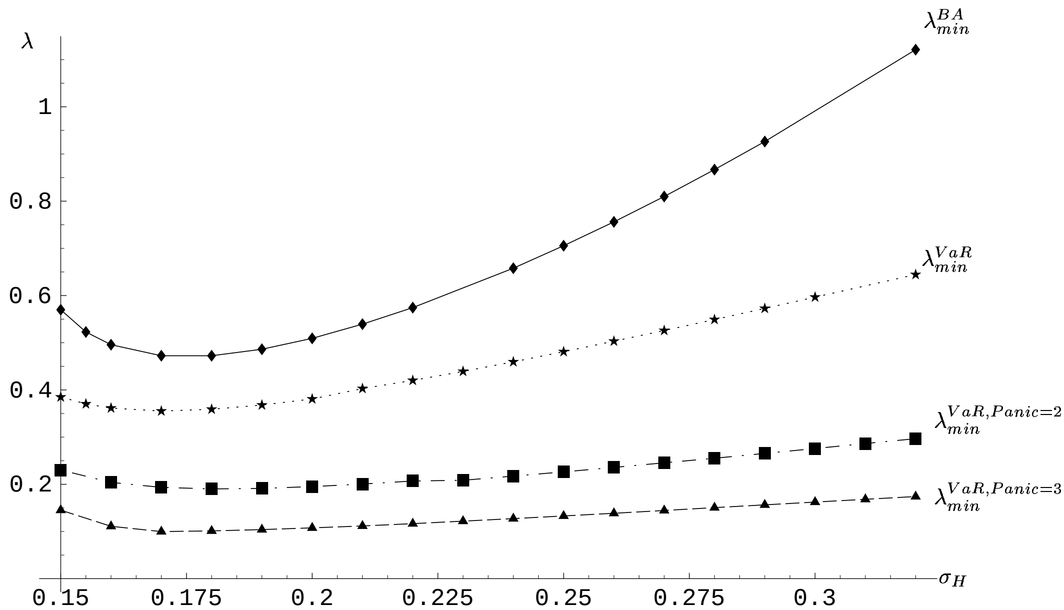

Figure 8: Minimum audit intensity $\lambda_{\min}$ required to maintain the switching incentive under different regulatory systems for different volatilities σ_H of the high risk technology. Under Basel Accord regulation the necessary auditing level increases sharply with the volatility of the high risk technology. Under Value-at-Risk regulation the regulator's awareness is less sensitive to the investment opportunity set of the bank, especially if a panic factor is included.

an incentive to reduce risk only if the volatility of their high-risk portfolio is less than 25%-30%. Our findings on the necessity of a prudent auditing policy support the decision of the Basel Committee to recognize auditing as one of the main pillars of the new accord. In terms of robustness and to facilitate the calibration of the regulatory mechanism, it is favorable to specify capital requirements that allow the regulator to apply a uniform audit intensity for all banks independent of specific investment possibilities. This can be achieved by introducing a panic factor $\xi > 1$ for VaR regulation. The capital requirements are affected in two ways. First, they jointly increase, and second, they become more risk sensitive (because $\frac{\partial^2 B(\sigma)}{\partial\sigma\partial\xi} > 0$). In Figure 8 we see two effects when moving from BB to VaR regulation: a general drop in the minimum audit intensity and a reduced sensitivity to changes in the investment opportunity set (especially for $\xi > 1$). As pointed out before the official rationale for the panic factor is to cover model risk. However, our analysis demonstrates that a further benefit of the panic factor greater one is an increased risk sensitiveness of the regulatory framework which makes Value-at-Risk regulation more robust in the sense that the auditing behavior of the supervisor does not have to be tuned so carefully to the bank's risk shifting possibilities.

6 Conclusion

The proposal on the New Capital Accord of the Basel Committee on Banking Supervision (2001) is the most recent important step in an ongoing regime change in international bank regulation. Simple rules of capital adequacy are replaced in order to make required capital more sensitive to the financial institution's risk, thereby trying to close the gap between regulatory and economic capital. In our paper we want to provide a theoretical justification for this trend in bank supervision and rigorously analyze the impact of risk sensitive capital requirements on banks' optimal risk taking behavior. We choose a modeling approach, where banks are allowed to switch between two asset portfolios with different volatility. This explicit treatment of the risk shifting process allows the comparison of regulatory mechanisms that are based on asset value—like the Basel I Building Block block approach—and risk-contingent regulations—like Value-at-Risk based capital requirements.

We find that both regulatory mechanisms generally do not prevent banks from switching to high risk when they are in distress. Under Value-at-Risk regulation well capitalized banks have a stronger incentive to reduce asset risk than Building Block regulated banks. This is driven by the reward in form of lower capital requirements for low-risk banks.

This reduction of risk decreases the current value of the deposit insurance liability while it increases the current value of the bank charter, which is advantageous to the shareholders. Thus, shifting from the Basel Accord Building Block approach to the risk based Value-at-Risk regulation may benefit both, the regulatory authority and the equityholders of banks.

As Value-at-Risk based regulation gives stronger incentives to the bank's management, it requires less auditing effort to maintain the risk reduction behavior. Furthermore, under Value-at-Risk regulation this risk reduction behavior is less sensitive to a change in the bank's investment opportunity set.

Our findings provide support for the ongoing tendency towards more risk sensitive capital requirements and they encourage regulators to complete this transition. We also highlight the interaction between capital requirements and auditing as it is recognized in the Basel II proposal by specifying these two aspects as equally important pillars of the new regulation.

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A Boundary conditions

This section derives the boundary conditions for the general claim F. As an abbreviation for Equations (6) and (7) we write

$$F = F(V, \sigma; A_1, A_2). \tag{17}$$

1. Switching threshold: Suppose $\sigma_c \in \{\sigma_L, \sigma_H\}$ denotes the volatility at the current risk level and σ_{-c} is the volatility at the alternative risk level. Furthermore, let V_i be a switching threshold set by the bank's management at which the assets are reorganized into a portfolio with volatility σ_{-c}. Let $F(V, \sigma_c; A_1, A_2)$ denote the market value of the claim prior to the volatility shift at V_i and $F(V, \sigma_{-c}; A_1', A_2')$ the claim value subsequent to the volatility shift in a neighborhood of $(1-k)V_i$ (according to the convention (17)).
   Market equilibrium requires

$$\begin{aligned} \lim_{V \to V_i} F(V, \sigma_c; A_1, A_2) &= F((1-k)V_i, \sigma_{-c}; A_1', A_2') - kV_i \quad &: \text{for claim SC}, \\ \lim_{V \to V_i} F(V, \sigma_c; A_1, A_2) &= F((1-k)V_i, \sigma_{-c}; A_1', A_2') \quad &: \text{for other claims}, \end{aligned} \tag{18}$$

   where the limit is the left hand side or the right hand side limit, depending on whether V_i is the upper or the lower bound of the interval of stable regime. This results in an equation which is linear in the four unknowns A_1, A_2, A_1', A_2' and therefore allows eliminating one of these parameters.

2. closure by bank management: Suppose V_i is a trigger at which the bank's management decides to default, i.e., V_i is an absorbing barrier to the process V. Again, depending on the state (V, σ) of the bank the market value of the claim prior to

default can be written as $F(V,\sigma;A_1,A_2)$. Since the claim pays $\beta+\gamma V_i$ in case of closure, market equilibrium requires

$$\lim_{V\to V_i} F(V,\sigma;A_1,A_2)=\beta+\gamma V_i, \tag{19}$$

which eliminates one of the unknown parameters A_1, A_2.

3. closure by regulators: Suppose V_i is the bound of the closure region corresponding to the current asset volatility σ, i.e., $V_i=B(\sigma)$. In contrast to the boundaries discussed in the previous two points, V_i is not an absorbing barrier now, but instead the process V can freely enter and leave the closure region. Thus, according to the results of Feynman and Kac (see Björk (1998) or on a more formal level Karatzas and Shreve (1988)), market equilibrium requires that the value function of the claim is continuous and smooth at the boundary of the closure region:

$$\begin{aligned}\lim\nolimits_{V\to V_i^-} F(V,\sigma;A_1,A_2) &= \lim\nolimits_{V\to V_i^+} F(V,\sigma;A_1',A_2'),\\ \lim\nolimits_{V\to V_i^-} F_V(V,\sigma;A_1,A_2) &= \lim\nolimits_{V\to V_i^+} F_V(V,\sigma;A_1',A_2').\end{aligned} \tag{20}$$

This condition yields two equations linear in A_1, A_2, A_1', A_2' which eliminate two of these parameters.

4. Suppose $V_i=c/r$ and the functional form of the claim's default payoff changes at c/r. Again, V_i is not an absorbing barrier, thus, boundary condition (20) has to be satisfied at c/r. Note, the functional form changes at c/r only for deposits and via (3) and (4) for deposit insurance and equity value respectively. For tax benefits and switching costs condition (20) leads to $A_1=A_1'$ and $A_2=A_2'$.

5. The last case we consider are boundary conditions for the situation where the interval of stable regime is unbounded—either from above or from below. Let $F(V,\sigma;A_1,A_2)$ denote the market value of the claim and, first, suppose $V_2=\infty$, i.e., the interval of stability is unbounded from above. With higher asset values V a switch of the regime of stability in the foreseeable future becomes less likely.

Thus, for growing V the market value of the claim has to converge to the market value which the profit flow has if the current regime prevails forever. Excluding speculative bubbles we get boundary conditions that characterize the asymptotic behavior of F for finite capital requirements ($B(\sigma) < \infty$) as follows,

$$\lim_{V \to \infty} F(V, \sigma; A_1, A_2) = \frac{\alpha}{r} \tag{21}$$

Second, suppose $V_1 = 0$, i.e., the interval of stability is unbounded from below. Regarding that $V = 0$ is a fixed point of the process (1) we can determine the market value of the claim at $V = 0$. Market equilibrium for positive capital requirements ($B(\sigma) > 0$) requires that

$$\lim_{V \to 0} F(V, \sigma; A_1, A_2) = \frac{\alpha}{r + \lambda} + \lambda \left(\frac{\beta}{r + \lambda} \right) \tag{22}$$

In both cases the respective boundary condition eliminates one of the unknowns A_1 and A_2.

B Valuing a Claim Contingent on $(\mathbf{V}, \sigma)$

B.1 The Market Value of Deposits

As long as the bank is alive depositholders receive a constant coupon flow c. In case of closure the value of the claim is $\min\{V, c/r\}$. In terms of the general claim F (which we use in Section 2) the market value of deposits determines the parameters α, β and γ to

$$\begin{aligned} \alpha &= c, \\ \beta &= \mathbf{1}_{[c/r, \infty)} c/r, \\ \gamma &= \mathbf{1}_{[0, c/r)} 1. \end{aligned} \tag{23}$$

The market value of debt in an interval of stable regime can be written as

$$D(V,\sigma;A_1,A_2)=\begin{cases}\begin{array}{c}\frac{c}{r+\lambda}+\lambda\left(\mathbf{1}_{[c/r,\infty)}\frac{c/r}{r+\lambda}+\mathbf{1}_{[0,c/r)}\frac{1}{\lambda+\delta}V\right)\\ +A_1V^{x_1(\sigma)}+A_2V^{x_2(\sigma)}\end{array} & : \quad V\le B(\sigma)\\ \\ \frac{c}{r}+A_1V^{y_1(\sigma)}+A_2V^{y_2(\sigma)} & : \quad V>B(\sigma)\end{cases} \tag{24}$$

The boundary conditions at the different bounds of stability are

1. If (V_i,σ_c) is a switching threshold:

$$\lim_{V\to V_i} D(V,\sigma_c;A_1,A_2)=D((1-k)V_i,\sigma_{-c};A_1',A_2'). \tag{25}$$

2. If V_i is a bankruptcy trigger:

$$\lim_{V\to V_i} D(V,\sigma;A_1,A_2)=\min\{V,\frac{c}{r}\}. \tag{26}$$

3. If V_i is the bound of the closure region, i.e., $V_i=B(\sigma)$:

$$\begin{aligned}\lim_{V\to V_i^-} D(V,\sigma;A_1,A_2) &= \lim_{V\to V_i^+} D(V,\sigma;A_1',A_2'),\\ \lim_{V\to V_i^-} D_V(V,\sigma;A_1,A_2) &= \lim_{V\to V_i^+} D_V(V,\sigma;A_1',A_2').\end{aligned} \tag{27}$$

4. If $V_i=c/r$:

$$\begin{aligned}\lim_{V\to V_i^-} D(V,\sigma;A_1,A_2) &= \lim_{V\to V_i^+} D(V,\sigma;A_1',A_2'),\\ \lim_{V\to V_i^-} D_V(V,\sigma;A_1,A_2) &= \lim_{V\to V_i^+} D_V(V,\sigma;A_1',A_2').\end{aligned} \tag{28}$$

5. If the current regime is unbounded from above:

$$\lim_{V\to\infty} D(V,\sigma;A_1,A_2)=\begin{cases}\frac{c}{r+\lambda}+\lambda\left(\frac{c/r}{r+\lambda}\right) & : \quad B(\sigma)=\infty\\ \\ \frac{c}{r} & : \quad B(\sigma)\ne\infty\end{cases} \tag{29}$$

If it is unbounded from below:

$$\lim_{V \to 0} D(V, \sigma; A_1, A_2) = \begin{cases} \frac{c}{r+\lambda} & : \quad B(\sigma) \neq 0 \\ \frac{c}{r} & : \quad B(\sigma) = 0 \end{cases} \tag{30}$$

B.2 The Value of Tax Benefits

The advantage of debt is that coupon payments to debtholders are tax deductible, i.e., as long as the bank is alive there is a profit flow to the banks equityholders of the magnitude τc. In the case of bankruptcy, this tax shield is irretrievably lost. Therefore, the parameters α, β, and γ which characterize this claim are

$$\begin{aligned} \alpha &= \tau c, \\ \beta &= 0, \\ \gamma &= 0. \end{aligned} \tag{31}$$

The market value of tax benefits in an interval of stable regime can be written as

$$TB(V, \sigma; A_1, A_2) = \begin{cases} \frac{\tau c}{r+\lambda} + A_1 V^{x_1(\sigma)} + A_2 V^{x_2(\sigma)} & : \quad V \leq B(\sigma) \\ \frac{\tau c}{r} + A_1 V^{y_1(\sigma)} + A_2 V^{y_2(\sigma)} & : \quad V > B(\sigma) \end{cases} \tag{32}$$

The boundary conditions at the different bounds of stability are

1. If (V_i, σ_c) is a switching threshold:

$$\lim_{V \to V_i} TB(V, \sigma_c; A_1, A_2) = TB((1-k)V_i, \sigma_{-c}; A_1', A_2'). \tag{33}$$

2. If V_i is a bankruptcy trigger:

$$\lim_{V \to V_i} TB(V, \sigma; A_1, A_2) = 0, \tag{34}$$

3. If V_i is the bound of the closure region, i.e., $V_i = B(\sigma)$:

$$\begin{aligned} \lim_{V \to V_i^-} TB(V, \sigma; A_1, A_2) &= \lim_{V \to V_i^+} TB(V, \sigma; A_1', A_2'), \\ \lim_{V \to V_i^-} TB_V(V, \sigma; A_1, A_2) &= \lim_{V \to V_i^+} TB_V(V, \sigma; A_1', A_2'). \end{aligned} \tag{35}$$

4. The value c/r does not change the functional form of the payoff one receives in case of closure, thus it is not a bound of stable regime.

5. If the current regime is unbounded from above:

$$\lim_{V \to \infty} TB(V, \sigma; A_1, A_2) = \begin{cases} \frac{\tau c}{r+\lambda} & : \quad B(\sigma) = \infty \\ \frac{\tau c}{r} & : \quad B(\sigma) \neq \infty \end{cases} \tag{36}$$

   If it is unbounded from below:

$$\lim_{V \to 0} TB(V, \sigma; A_1, A_2) = \begin{cases} \frac{\tau c}{r+\lambda} & : \quad B(\sigma) \neq 0 \\ \frac{\tau c}{r} & : \quad B(\sigma) = 0 \end{cases} \tag{37}$$

B.3 The Current Value of Switching Costs

The claim SC denotes the current value of future switching costs, i.e., in the case of a switch at a threshold V_i the immediate expenditure of kV_i is required. The remaining characteristics of this claim are

$$\begin{aligned} \alpha &= 0, \\ \beta &= 0, \\ \gamma &= 0. \end{aligned} \tag{38}$$

The market value of the switching-cost claim in an interval of stable regime can be written as

$$SC(V,\sigma;A_1,A_2)=\begin{cases} A_1V^{x_1(\sigma)}+A_2V^{x_2(\sigma)} & : \quad V\le B(\sigma) \\ A_1V^{y_1(\sigma)}+A_2V^{y_2(\sigma)} & : \quad V> B(\sigma) \end{cases} \tag{39}$$

The boundary conditions at the different bounds of stability are

1. If (V_i,σ_c) is a switching threshold:

$$\lim_{V\to V_i} SC(V,\sigma_c;A_1,A_2)=SC((1-k)V_i,\sigma_{-c};A_1',A_2')+kV_i. \tag{40}$$

2. If V_i is a bankruptcy trigger:

$$\lim_{V\to V_i} SC(V,\sigma;A_1,A_2)=0, \tag{41}$$

3. If V_i is the bound of the closure region, i.e., $V_i=B(\sigma)$:

$$\begin{aligned} \lim\nolimits_{V\to V_i^-} SC(V,\sigma;A_1,A_2) &= \lim\nolimits_{V\to V_i^+} SC(V,\sigma;A_1',A_2'), \\ \lim\nolimits_{V\to V_i^-} SC_V(V,\sigma;A_1,A_2) &= \lim\nolimits_{V\to V_i^+} SC_V(V,\sigma;A_1',A_2'). \end{aligned} \tag{42}$$

4. The value c/r does not change the functional form of the payoff one receives in case of closure, thus it is not a bound of stable regime.

5. If the current regime is unbounded from above:

$$\lim_{V\to\infty} SC(V,\sigma;A_1,A_2)=0 \tag{43}$$

   If it is unbounded from below:

$$\lim_{V\to 0} TB(V,\sigma;A_1,A_2)=0 \tag{44}$$

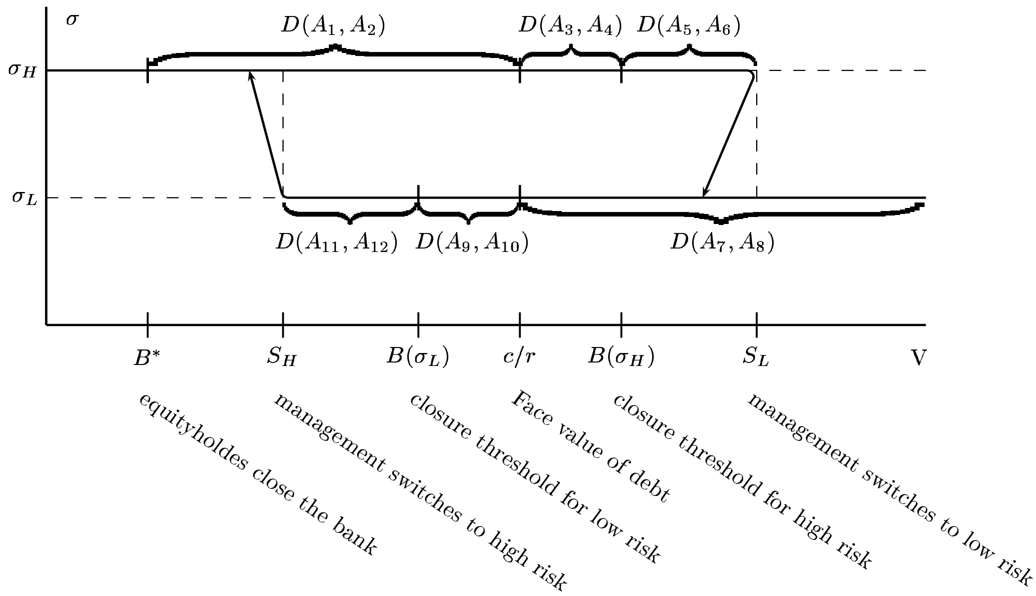

Figure 9: The value function of the banks assets consists of six functions that are defined over intervals of stable regime and linked by the respective boundary conditions.

C Determining the functional form of a claim

To show how the boundary conditions determine the constants A in the valuation equations for the claims involved in the analysis, we demonstrate one particular case. We explicitly derive the linear system that determines the value of debt D under the assumption that the critical thresholds are ordered in the manner: $B^* < S_H < \frac{c}{r} < B(\sigma_L) < B(\sigma_H) < S_L$. This assumptions corresponds to the risk shifting behavior illustrated in Figure 2. The value function of debt is constructed by linking six functions of the form $D(V, \sigma, A_i, A_j)$ as defined in Equation 24. As illustrated in Figure 9, each of these six functions is defined over an interval of stable regime and they are linked by the following boundary conditions:

At B^* the bank equityholders defaults (while running the high risk portfolio). The boundary condition follows from Equation 26 and the specific assumption of the closure thresholds:

$$D(B^*, \sigma_H, A_1, A_2) = B^* \tag{45}$$

At $\frac{c}{r}$ the functional form of the valuation equation changes, using equations 28 the corresponding boundary conditions are for the low risk bank

$$\begin{aligned} D(c/r, \sigma_H, A_1, A_2) &= D(c/r, \sigma_H, A_3, A_4) \\ D_V(c/r, \sigma_H, A_1, A_2) &= D_V(c/r, \sigma_H, A_3, A_4) \end{aligned} \tag{46}$$

The threshold $B(\sigma_H)$ determines the border of the closure threshold of the high risk bank. Using Equation 27 the corresponding condition is

$$\begin{aligned} D(B(\sigma_H), \sigma_H, A_3, A_4) &= D(B(\sigma_H), \sigma_H, A_5, A_6) \\ D_V(B(\sigma_H), \sigma_H, A_3, A_4) &= D_V(B(\sigma_H), \sigma_H, A_5, A_6) \end{aligned} \tag{47}$$

At S_L the bank switches to low risk and by using Equation 25 we find that

$$D(S_L, \sigma_H, A_5, A_6) = D((1-k)S_L, \sigma_L, A_7, A_8) \tag{48}$$

The interval of stable regime for the low risk bank is unbounded from above. Applying Equation 29 yields,

$$\lim_{V \to \infty} D(V, \sigma_L, A_7, A_8) = \frac{c}{r} \tag{49}$$

Using Equation 24 and the fact that $x_2(\sigma) > 0$ we can see that A_8 must be equal to zero.

The threshold $B(\sigma_L)$ determines the border of the closure threshold of the low risk bank, following Equation 27 we find

$$\begin{aligned} D(B(\sigma_L), \sigma_L, A_7, A_8) &= D(B(\sigma_L), \sigma_L, A_9, A_{10}) \\ D_V(B(\sigma_L), \sigma_L, A_7, A_8) &= D_V(B(\sigma_L), \sigma_L, A_9, A_{10}) \end{aligned} \tag{50}$$

At $\frac{c}{r}$ the functional form of the valuation equation changes again. The corresponding boundary conditions for the high risk bank are:

$$\begin{aligned} D(c/r, \sigma_L, A_9, A_{10}) &= D(c/r, \sigma_L, A_{11}, A_{12}) \\ D_V(c/r, \sigma_L, A_9, A_{10}) &= D_V(c/r, \sigma_L, A_{11}, A_{12}) \end{aligned} \tag{51}$$

At S_H the bank switches to high risk,

$$D(S_H, \sigma_L, A_{11}, A_{12}) = D((1-k)S_H, \sigma_H, A_1, A_2) \tag{52}$$

Setting $A_8 = 0$, these equations define a 11-dimensional linear system

$$\begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ A_7 \\ A_9 \\ A_{10} \\ A_{11} \\ A_{12} \end{pmatrix} = M^{-1} \begin{pmatrix} B^* - \frac{B^*\lambda}{\delta+\lambda} - \frac{c}{\lambda+r} \\ -\frac{c\lambda}{(\delta+\lambda)r} + \frac{c\lambda}{r(\lambda+r)} \\ -\frac{\lambda}{\delta+\lambda} \\ \frac{c}{r} - \frac{c}{\lambda+r} - \frac{c\lambda}{r(\lambda+r)} \\ 0 \\ 0 \\ -\frac{c}{r} + \frac{c}{\lambda+r} + \frac{c\lambda}{r(\lambda+r)} \\ 0 \\ \frac{c\lambda}{(\delta+\lambda)r} - \frac{c\lambda}{r(\lambda+r)} \\ \frac{\lambda}{\delta+\lambda} \\ \frac{\lambda S_H}{\delta+\lambda} - \frac{(1-k)\lambda S_H}{\delta+\lambda} \end{pmatrix} \tag{53}$$

where the Matrix M is defined in equation (54).

If the critical thresholds (B^*, S_H, c/r, $B(\sigma_L)$, $B(\sigma_H)$, S_L) are ordered in a different way, a similar procedure has to be applied. The solution of the other claims (tax benefits, deposit insurance, switching costs, and equity) is analogous. For given B^*, S_H, S_L the value functions of all claims are well defined. Maximizing the value of equity (see Section 3.3.1) these managerial decision variables are determined numerically.

$$M = \begin{bmatrix}
B^{*x_1(\sigma_H)} & B^{*x_2(\sigma_H)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\left(\frac{c}{r}\right)^{x_1(\sigma_H)} & \left(\frac{c}{r}\right)^{x_2(\sigma_H)} & -\left(\frac{c}{r}\right)^{x_1(\sigma_H)} & -\left(\frac{c}{r}\right)^{x_2(\sigma_H)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
x_1(\sigma_H)\left(\frac{c}{r}\right)^{x_1(\sigma_H)-1} & x_2(\sigma_H)\left(\frac{c}{r}\right)^{x_2(\sigma_H)-1} & -x_1(\sigma_H)\left(\frac{c}{r}\right)^{x_1(\sigma_H)-1} & -x_2(\sigma_H)\left(\frac{c}{r}\right)^{x_2(\sigma_H)-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & B(\sigma_H)^{x_1(\sigma_H)} & B(\sigma_H)^{x_2(\sigma_H)} & -B(\sigma_H)^{y_1(\sigma_H)} & -B(\sigma_H)^{y_2(\sigma_H)} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & x_1(\sigma_H)B(\sigma_H)^{x_1(\sigma_H)-1} & x_2(\sigma_H)B(\sigma_H)^{x_2(\sigma_H)-1} & -y_1(\sigma_H)B(\sigma_H)^{y_1(\sigma_H)-1} & -y_2(\sigma_H)-1B(\sigma_H)^{y_2(\sigma_H)-1} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & S_L^{y_1(\sigma_H)} & S_L^{y_2(\sigma_H)} & -((1-k)S_L)^{y_1(\sigma_L)} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & B(\sigma_L)^{y_1(\sigma_L)} & -B(\sigma_L)^{x_1(\sigma_L)} & -B(\sigma_L)^{x_2(\sigma_L)} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & y_1(\sigma_L)B(\sigma_L)^{y_1(\sigma_L)-1} & -x_1(\sigma_L)B(\sigma_L)^{x_1(\sigma_L)-1} & -x_2(\sigma_L)B(\sigma_L)^{x_2(\sigma_L)-1} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \left(\frac{c}{r}\right)^{x_1(\sigma_L)} & \left(\frac{c}{r}\right)^{x_2(\sigma_L)} & -\left(\frac{c}{r}\right)^{x_1(\sigma_L)} & -\left(\frac{c}{r}\right)^{x_2(\sigma_L)} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & x_1(\sigma_L)\left(\frac{c}{r}\right)^{x_1(\sigma_L)-1} & x_2(\sigma_L)\left(\frac{c}{r}\right)^{x_2(\sigma_L)-1} & -x_1(\sigma_L)\left(\frac{c}{r}\right)^{x_1(\sigma_L)-1} & -x_2(\sigma_L)\left(\frac{c}{r}\right)^{x_2(\sigma_L)-1} \\
((1-k)S_H)^{x_1(\sigma_H)} & ((1-k)S_H)^{x_2(\sigma_H)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -S_H^{x_1(\sigma_L)} & -S_H^{x_2(\sigma_L)}
\end{bmatrix} \tag{54}$$