Is the New Basel Accord Incentive Compatible?

by
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Abstract

No.

This paper develops a simple equilibrium model of a bank that benefits from subsidized insured deposits and endogenously selects the characteristics of its credit risk exposure. The model is used to analyze the traits of a bank's optimal loan portfolio under the regulatory capital requirements of the 1988 Basle Accord and the alternative regulatory capital rules proposed in the New Basel Accord (NBA) Consultative Document. The analysis shows that, while the proposed changes reduce the value of the deposit insurance subsidy relative to levels attainable under the 1988 Accord, the proposals create unintended consequences that are not aligned with regulatory interests. For example, banks using either of the proposed Internal Ratings Based (IRB) approaches will face incentives to construct loan portfolios that generate large losses should a bank default. Such incentives are at odds with the objective of least cost resolution mandated in FDICIA. Moreover, because they create conditions that will foster the development of stable banking clienteles in which banks using the Advanced IRB approach will choose to hold the safest loan portfolios and banks using the Standardized approach the riskiest portfolios, the proposals abandon the objective of establishing "a level playing field." The NBA capital proposals do not encourage diversification across the business cycle. Instead they create financial incentives that encourage banks to concentrate lending to those creditors most likely to default in economic recessions and may thereby create economic stability issues beyond those recognized in the procyclical provisioning discussion in the Consultative Document. In moving from the Accord through the progression of capital approaches in the NBA, the more complex capital schemes reduce the probability of bank default. In contrast to the NBA's stated objective, voluntary evolution from the Standardized, to the Foundation, and to the Advanced IRB approach is unlikely as the *ex ante* value of the deposit insurance subsidy is shown to be significantly smaller under each step in the regulatory progression.

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Is the New Basel Accord Incentive Compatible?

I. Introduction

In the Consultative Document, "The New Basel Accord," the Basel Committee on Banking Supervision (BCBS) provides the rational for a proposed revision to the 1988 Basel Accord (the Accord). The proposed revisions are, "intended to align capital adequacy assessment more closely with key elements of banking risks…and to secure the objective of prudentially sound, incentive-compatible and risk sensitive capital requirements."[2]

The New Basel Accord (NBA) proposal departs from the format of the Accord, and specifies credit risk weights that are linked either to internal loan classification schemes, as in the Internal Rating Based (IRB) approaches, or to external credit ratings, as in the Standardized approach. Both approaches set credit risk weights according to a credit's anticipated probability of default and are, at least in part, designed to mimic the techniques used internally by banks. The decision to base regulatory capital on credit risk measurement processes that are designed to be consistent with banks' internal risk measurement processes is a deliberate attempt to harmonize regulatory capital requirements with the best practices of internationally active banks.[3]

The BCBS believes that the proposed NBA will "provide incentives for banks to enhance their risk measurement and management capabilities…."[4] In particular, the Committee reports that the capital proposals include incentives that are intended to encourage banks to evolve from the Standardized, through the Foundation IRB, and finally towards to the Advanced IRB approach for calculating capital. The BCBS's stated objective is to place " a greater emphasis on banks' own assessment of the risks to which they are exposed in the calculation of regulatory capital charges."[5] This objective reflects the Committee's view that "ultimate responsibility for managing risks and ensuring that capital is held at a level consistent with a bank's risk profile remains with that bank's management."[6] It is in this context that the Basel Committee characterizes its proposed NBA as an incentive-compatible approach for bank regulation.

The NBA may be designed to be compatible with banks' internal credit risk measurement practices, but is it really an incentive-compatible approach for bank regulation? Underlying this question is a deeper unresolved issue concerning whether or not it is even possible to design an effective regulatory capital measure that is based upon risk measures that banks themselves design for their own internal management purposes. If the need for bank regulation is based in part on the existence of externalities, it is important to understand if (and how) these externalities can be measured and controlled using the internal processes that banks have designed for their own profit maximization objectives. While the goal of harmonizing regulatory capital guidelines with those used by banks in their internal risk management processes is appealing by virtue

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[2] Basel Committee on Banking Supervisions, "Overview of The New Basel Capital Accord," January 2001, paragraphs 2, and 35.
[3] *Ibid*., paragraphs 99 and 100.
[4] *Ibid.,* paragraph 2.
[5] *Ibid.,* paragraph 5.
[6] *Ibid.,* paragraph 30.

of the implicit promise of reduced regulatory burden, *a priori* it is far from clear how such an approach will control the externalities that mandate bank regulation. While BCBS consultative documents fully embrace the goal of harmonizing capital regulation with bank internal processes, it is troubling that the BCBS fail to discuss the nature of the externalities that mandate capital regulation or provide any analysis that supports the claim that bank's internal processes can be harnessed to control the underlying market failure(s).

While skirting the deeper bank internal models issue, this paper provides a formal analysis of the NBA's regulatory capital alternatives in regard to their abilities to control the externalities generated by underpriced deposit guarantees. It provides a detailed analysis of the incentives that are created by the specific capital regulations that are proposed in the NBA in the context of a simple but powerful equilibrium model of a bank that benefits from subsidized insured deposits and endogenously selects its optimal level of credit risk exposure.

The analysis shows that, while all the NBA approaches make regulatory capital requirements more sensitive to credit risk, low quality credits remain the most valuable to banks. The adoption of a Standardized approach promises to have almost no effect on bank behavior regarding what loans banks choose to securitize and what loans banks choose to retain on their balance sheets. None of the proposed approaches for regulatory capital creates an incentive for banks to select loans with minimal expected loss given default. Even the Advanced IRB approach does not encourage a bank to try to increase loan recovery rates. This feature of the NBA is particularly troubling as the loss given default characteristics of a bank's loans are the primary factor determining an insurer's cost of resolving a failed bank.[7] Moreover, in contrast to the BCBS's stated intentions, the analysis finds no economic incentive in the NBA that will encourage banks to evolve from the Standardized to the Advanced IRB capital approach. Consequently there is little reason to expect banks to voluntarily evolve toward model-based capital regulations.

The findings in this study suggest that if only some banks are required by their national supervisors to adopt the IRB approaches, it is likely that natural banking clienteles will emerge from the incentives created by the NBA's alternative capital schemes. Differences in the regulatory capital treatment will allow Standardized approach banks to increase their share values by competing away lower quality credit business from IRB banks. Large IRB banks have a regulatory capital induced competitive advantage in attracting relatively high quality credits that may allow them to successfully attract these borrowers away from banks that use the Standardized approach. The IRB granularity adjustment will make small IRB banks completely uneconomic. Among IRB banks, there is scope for additional market segmentation as large Advanced IRB banks have a competitive advantage in attracting high quality borrowers with above average expected recovery rates. The analysis suggests that the NBA will encourage segmentation in the credit qualities of internationally active banks and consolidation in IRB banks, as banks respond to the risk taking incentives

[7] The regulatory objective of least cost resolution is the basis for FDICIA's prompt corrective action supervisory guidelines (12 U.S.C. §1831o) and the guidelines that govern the U.S. FDIC's actions in insurance related activities (12 U.S.C. §1823c(4)).

that arise under the alternative capital rules. Under the NBA, the "level playing field" objective of the 1988 Accord is abandoned.

The portfolio realignments encouraged under the NBA will likely strengthen the prudential solvency standards of sophisticated (IRB) money center banks. Financial market stability, however, may not be enhanced as poor quality credits will be concentrated in Standardized approach banks (of which there are expected to be many).

A final interesting result is the finding that all of the NBA's proposed capital schemes create incentives that will encourage banks to concentrate lending to credits that are expected to default in recessions. Capital requirements under the Accord do not depend on physical probabilities of loan default. Once regulatory capital is explicitly conditioned on a credit's physical probability of default, banks face new incentives that encourage them to discriminate among credits on another basis. Among credits with a given probability of default, if investors are risk averse, the credits that default in recessions offer the largest risk premiums.[8] When capital requirements constrain bank insurance values according to a credit's expected physical probability of default and loss given default, an important aspect of credit risk that remains unregulated is the timing of default. By choosing credits that are expected to default in recessions, shareholders benefit from the larger credit risk premium in nonrecession periods and thereby enhance the value of their deposit insurance guarantee. Because there are many ways to influence the deposit insurance value under the Accord, the capital requirements do not favor a particular phase of the business cycle when considering the timing of loan defaults.

The limitations of this study should be recognized at the outset. The analysis is based on a single period model with competitive lending markets and no corporate taxes. In this setting, optimal loan selection is driven by the objective of maximizing deposit insurance values. If capital requirements are set sufficiently high so that the deposit insurance guarantee is valueless, the incentives discussed in the paper will no longer exist. In a multiperiod setting that recognizes market power and taxes, bank franchise values and interest tax shields will become important determinants of bank behavior. While significant bank franchise values will, other things equal, lower insurance values from those calculated in this analysis, interest deductibility under corporate taxes will offset this effect. Alternatively, when deposit insurance is valueless, if regulatory capital requirements affect investment decisions at all it is because they have indirect effects limiting tax shields or perhaps by limiting the terms of contracts that can be used to enhance operating efficiencies. When deposit insurance is valuable, regulatory capital requirements have a direct effect on a bank's investment decision when the selection of an investment is not neutral with respect to the *ex ante* insurance values that are generated under the regulatory capital scheme. It is the direct effects of regulatory capital requirements that are the focus of this study.

An outline of the paper follows. Under the assumption of investor risk neutrality, Section II develops an equilibrium model in which banks fund themselves with insured deposits and endogenously select the risk characteristics of the single risky loan in which they invest. The single loan and risk neutrality assumptions greatly simplify the analysis with little cost as a subsequent section establishes the generality of the results. The regulatory capital requirements under the Accord and the proposed alternative regulatory capital requirements of the NBA are discussed in Section III. The subsequent section analyzes optimal bank behavior when bank investment and financing decisions are constrained by the alternative regulatory capital regimes. Section V discusses the potential market equilibrium implications of the NBA. Section VI extends the simple risk neutral banking model to a general equilibrium setting in which risk averse investors are able to transact in complete Arrow-Debreu contingent claims markets. This extension establishes a link between optimal bank credit risk allocation and the macroeconomic environment. A subsequent section analyzes optimal bank behavior when investors are risk averse and banks are constrained by the alternative regulatory capital schemes. Section VIII extends the single loan model to an analysis of the characteristics of an optimal bank portfolio. It is demonstrated that the results of the single loan analysis immediately generalize once the credit risk parameters in the single loan case are reinterpreted as measures of the insured bank's credit risk---the bank's probability of default and the loss given default on its loan portfolio. The penultimate section discusses the incentives created by the IRB "granularity adjustment" and a final section concludes the paper.

II. The Risk Neutral Model

For simplicity, we assume a two state distribution of cash payoffs on the bank's loan: it either makes its entire payment of principle and interest, P, or it defaults. If the bond defaults, the bank's loss is assumed to be a fraction, LGD, of the promised principle and interest payment, P.[9] In this binomial setting, the bank selects the level of insured deposits to issue and the characteristics of its loan portfolio (the loan's probability of default and its loss given default) to maximize the *ex ante* wealth of the bank's shareholders. The analysis does not consider information asymmetries that may arise in the context of the valuation of bank shares and assumes that the value of bank assets are transparent to equity market investors.[10]

Initially, the analysis assumes that investors behave as if they are risk neutral so that financial assets are valued as the discounted value of their expected future cash flows, where discounting takes place at the risk free rate. A subsequent section relaxes this assumption and analyzes the bank incentive that arise when investors are risk averse.

Bank Loan Valuation

Under the risk neutral valuation assumption, the present value of the bank's loan is given by,

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[8] More formally, credits that default when the marginal utility of consumption is high (a recession, roughly speaking) offer the largest *ex ante* risk premia.

[9] In the analysis that follows, the par value of the loan, P, is fixed. The qualitative aspects of the analysis do not depend on this normalization.

[10] We assume away information issues not because they are unimportant, but because the simplification allows for an analysis of the underlying operational incentives created by the proposed credit risks capital requirements under the NBA.

$$L(pr_i, LGD_i) = \frac{P(1 - pr_i \, LGD_i)}{1 + r_f}, \tag{1}$$

where the index i is used to indicate that the loan characteristics selected by the bank, and r_f is the risk free rate of interest. When a bank makes a loan, it lends the full fair present market value of the loan. The bank is not assumed to have any market power in lending markets.

The Valuation of the Claims of Bank Stakeholders

We assume the existence of a government agentcy that insures the value of banks' transactions deposits at a fixed *ex ante* premium rate normalized to 0, and consider the present value of the claims of three bank stakeholders: equity, insured debt, and the deposit insurance authority.[11] All fixed income claims are modeled as discount instruments in this one period, two date model.

Let D be the terminal value of insured deposits. Assume $P > D$ to ensure that, in the high payoff state, deposits pay out their promised par value even in the absence of deposit insurance. Notice that deposit insurance is valuable provided, $D > P(1 - LGD_i)$, and default probability is positive, $pr_i > 0$.

If the bank issues insured deposits with a terminal value of *D*, their present fair market value is $D(1 + r_f)^{-1}$. Assuming that deposit insurance is valuable, the present market value of the deposit insurer's stake, $INS(D, pr_i, LGD_i)$, depends on the level of the deposits issued by the bank, the probability of default, and loss given default characteristics of the loan selected by the bank,

$$INS(D, pr_i, LGD_i) = \frac{[D - P(1 - LGD_i)] \, pr_i}{1 + r_f}. \tag{2}$$

The market value of the bank's equity claims, $EQ(D, pr_i, LGD_i)$, depend on the level of insured deposits it issues D, as well as on the probability of default, and loss given default risk characteristics selected by the bank,

$$EQ(D, pr_i, LGD_i) = \frac{Max[0, P(1 - LGD_i) - D] \, pr_i + (P - D)(1 - pr_i)}{1 + r_f}. \tag{3}$$

When deposit insurance is valuable, the present fair market value of equity simplifies,

$$EQ(D, pr)_i = \frac{(P - D)(1 - pr_i)}{1 + r_f}. \tag{4}$$

[11] As a point of comparison, it should be noted that the U.S. deposit insurance premium rate is currently 0 for well-capitalized banks.

Optimization and the Value of the Deposit Guarantee

Bank shareholder/managers decide on the level of insured deposits to be issued by the bank and select the credit risk characteristics (pr_i, LGD_i) of the bank's loan. Assuming that deposit insurance has value (i.e., that LGD_i is sufficiently large), the fair present market value of the profits that accrue to equity holders are given by expression (4). Given that the bank's deposits are subsidized, the initial investment that equity holders must commit, i.e., the shareholders paid in capital, is given by, $L(pr_i, LGD_i) - D(1+r_f)^{-1}$. The difference between the fair present market value of profits and shareholder's paid in capital $EQ(D, pr_i, LGD_i) - \left(L(pr_i, LGD_i) - \dfrac{D}{1+r_f} \right) = INS(D, pr_i, LGD_i),$ is the *ex ante* value of the deposit insurance guarantee. The *ex ante* value of the deposit insurance guarantee, a wealth transfer from the deposit insurer to the bank shareholders, is pure profit from the shareholders perspective. Bank shareholders maximize their *ex ante* wealth by maximizing the *ex ante* value of the deposit insurance guarantee.

III. Regulatory Capital Requirements

Regulatory capital requirements limit the degree to which a bank can use insured deposits to fund its loan portfolio. Under the Accord, a bank must have an amount of qualifying regulatory capital that is at least 8 percent of the value of its risk-weighted assets. A corporate or retail loan has a 100 percent risk weight. Qualifying regulatory capital includes Tier 1 capital that is composed of paid in shareholder equity capital (at least 4 percent of the loan's value), and Tier 2 capital that includes qualifying subordinated debt (limited to 4 percent of the loan's value) and a share of a bank's general loan loss provisions. For purposes of this analysis, qualifying capital is limited to paid in equity capital.[12] An 8 percent paid in equity capital requirement for a loan imposes a limit on insured deposit issuance,

$$D \leq .92\ (1+r_f)\ L(pr_i,\ LGD_i). \tag{5}$$

If deposit insurance is valuable, the bank will, in this model setting, always maximize the use of insured deposit funding and equation (5) will hold as an equality.

The NBA proposes that regulatory capital requirements for non-sovereign banking credits be determined according to one of three methods: the so-called Standardized approach, or either the Foundation or Advanced IRB approach. The Standardized approach itself is not a single approach but two alternative approaches. One approach sets capital requirements according to the credit's sovereign external credit rating. The second approach bases the capital requirement on an issuer-specific external credit ratings. The analysis that follows will consider only the second variant of the Standardized approach.

[12] The restriction is made not only to simplify the analysis, but because a reasonable treatment analyzing the incentives generated when subordinated debt is included as qualifying equity capital requires that taxes and debt tax shields be included in the model.

Table 1 reports the proposed risk weights and capital requirements under the Standardized approach assuming that the regulatory minimum risk-weighted capital ratio is 8 percent. We define the correspondence $Cap(rating_i)$ to be a rule that assigns a capital requirement according to a credit's Standard & Poor's (S&P) rating ($rating_i$) using the rule in Table 1. Under the assumption that only paid in equity qualifies as regulatory capital, the Standardized approach imposes an insured deposit limit,

$$D \leq \left(1 - Cap(rating_i)\right)(1 + r_f)\, L(pr_i, LGD_i)\,. \tag{6}$$

If deposit insurance is valuable, the bank will maximize the use of insured deposit funding and equation (6) will hold as an equality.

Standard & Poor's Rating	Standardized Risk Weight	Standardized Capital Requirement
AAA to AA-	20 percent	1.6 percent
AA+ to A-	50 percent	4 percent
BBB+ to BB-	100 percent	8 percent
Below BB-	150 percent	12 percent
Unrated	100 percent	8 percent

Table 1: Risk weights and capital requirements under the Standardized approach assume an 8 percent minimum regulatory risk weighted capital ratio.

Under the NBA's IRB proposals, regulatory capital requirements for loans will be determined by a risk weighting function that depends on the type of customer (corporate, retail, project finance) and on the *ex ante* risk characteristics of the credit. Under the Foundation IRB approach, the risk weight depends on the credit's *ex ante* probability of default. Under the Advanced IRB approach, the risk weight depends on the credit's *ex ante* probability of default and loss given default.[13]

If qualifying capital is limited to Tier 1 capital, the Foundation IRB approach requires that the paid in equity capital for a corporate loan be at least, $.08 \dfrac{Min\left[BRW_C(pr_i),\, 625\right]}{100} L(pr_i, LGD_i)$, where $BRW_C(pr_i)$, the regulatory risk weighting function for corporate credits is given by,

$$BRW_C(pr_i) = 976.5\left(1 + .0470\frac{1 - Max\left[pr_i,\, .0003\right]}{Max\left[pr_i,\, .0003\right]^{.44}}\right)\Phi\left(1.288 + 1.118\;\Phi^{-1}\left(Max\left[pr_i, .0003\right]\right)\right),$$

where $\Phi(.)$ represents the cumulative standard normal distributions function, and $\Phi^{-1}(.)$ represents the inverse of this function. Under the Foundation IRB capital requirement, insured bank deposits must satisfy the inequality,

$$D \leq (1 + r_f)\, L(pr_i, LGD_i)\left(1 - .08\frac{Min\left[BRW_C(pr_i),\, 625\right]}{100}\right) \tag{7}$$

[13] The Advanced IRB approach also will include a maturity adjustment. The maturity adjustment is ignored in this single period model analysis.

where the use of insured deposits will be maximized (the equality will hold) when deposit insurance is valuable.

Ignoring the maturity adjustment and restricting qualifying capital to Tier 1 capital, the minimum paid in equity capital requirement under the Advanced IRB Approach is

$.08\ Min\left[\frac{LGD_i}{.50}\ \frac{BRW_C(pr_i)}{100},\ 12.5 * LGD_i\right] L(pr_i, LGD_i)$, and insured deposits must satisfy,

$$D \leq (1+r_f)\ L(pr_i, LGD_i)\left(1-.08\ Min\left[\frac{LGD_i}{.50}\ \frac{BRW_C(pr_i)}{100},\ 12.5 * LGD_i\right]\right) \tag{8}$$

where the equality will hold at a shareholder optimum when deposit insurance is valuable.

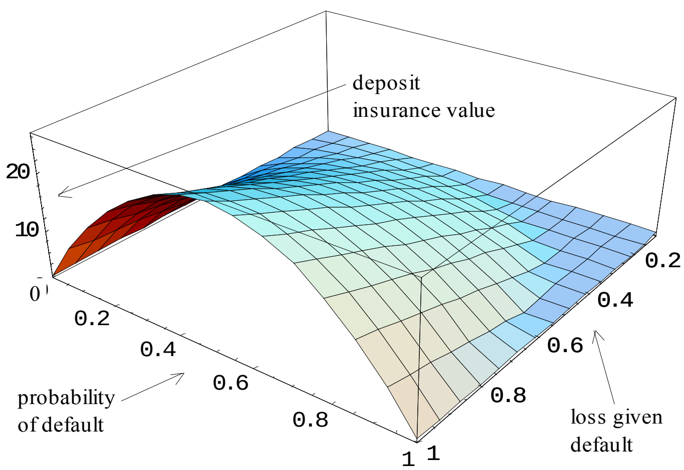

Figure 1: Deposit insurance value under the Accord

IV. Shareholder Value Maximization Under Risk Neutrality

Shareholder-managers select the level of insured deposits and the loan risk characteristics to maximize the value of the deposit insurance guarantee subject to any regulatory capital requirements that may constrain their admissible choice set. We consider bank optimization under the four alternative capital requirement regimes: the Accord, the Standardized approach and the Foundation and Advanced IRB capital regimes assuming that the entire 8 percent capital requirement must be met with paid in equity capital.

The 1988 Basel Accord

When paid in equity capital is constrained by the rules of the Accord, shareholders attempt to maximize the deposit insurance value [equation (2)] subject to the deposit issuance constraint in equation (5). Figure 1 plots the constrained deposit insurance surface generated for a loan with $P = 110$ using $r_f = .05$. Notice that under the Accord, the deposit insurance value is maximized by selecting loans with a high probability of default and large expected loss given default. Under the assumptions of this simple model, the loan characteristics $pr_i = .5, \quad LGD_i = 1$ provide the global optimal for shareholder wealth. If the value of LGD_i is constrained by some upper bound, $LGD^U < 1$, the optimal solution is to set the loss given default to its upper bound and select the default probability to satisfy, $pr_i^* = \dfrac{LGD^U - .08}{1.84\ LGD^U}$.

Standard & Poor's Rating	1 year historical probability of default (percent)	Insurance value under 1988 Accord	Insurance value under the Standardized approach	Insurance value under the Foundation IRB approach
AAA	0	0	0	0
AA+	0	0	0	0
AA	0	0	0	0
AA-	0.03	0.0132	0.0152	0.0149
AA+	0.04*	0.0176	0.0193	0.0198
A	0.05	0.0219	0.0241	0.0247
A-	0.05	0.0219	0.0241	0.0247
BB+	0.12	0.0527	0.0527	0.0578
BBB	0.22	0.0966	0.0966	0.1032
BBB-	0.35	0.1534	0.1534	0.1593
BB+	0.44	0.1927	0.1927	0.1965
BB	0.94	0.4093	0.4093	0.3826
BB-	1.33	0.5767	0.5767	0.5074
B+	2.91	1.2396	1.1194	0.8744
B	8.38	3.3488	3.0123	1.0311
B-	10.32	4.0276	3.6174	0.8040
CCC	21.94	7.3339	6.5154	0

Table 2: Deposit insurance values for selected Standard & Poor's ratings assuming a 1 year probability of default equal to the historic S&P average, and a 50 percent loss given default. The calculations are based upon the assumption that investors are risk neutral, and $P = 110,\ r_f = .05.$

*For credits rates AA+, the true historical 1 year default rate average is 0.02 percent. The calculations use 0.04 percent to retain a monotonic relationship between rating quality and the expected default rate.

The exact optimizing loan characteristics are less important than the qualitative prediction of the model. This simple model clearly indicates that, under the Accord, banks face a strong incentive to hold relatively risky loans. The existing capital requirement framework creates no incentive for the bank to retain high quality credits—credits with low probability of default and small expected losses given default. The model's predictions are consistent with the observed trend in bank behavior to securitize high quality credits.

The Standardized Approach

Table 2 reports the insurance values associated with alternative Standard & Poor's rating categories under the Accord and under the proposed Standardized approach for regulatory capital. The calculations in Table 2 use expression (6) (as an equality) to solve for D in expression (2), and further assume that $P = 110$, $r_f = .05$, and $LGD_i = .50 \; \forall i$. The 1 year rating-specific default probabilities are the Static Pools Average Cumulative Default Rates by Rating as reported by S&P, and the loss given default assumption corresponds closely with S&P's reported historical "all instruments" average recovery rate of 51.15%.[14] An S&P rating reportedly depends both on the probability of default and the loss given default. While there clearly are a range of *ex ante* default probabilities and recovery rates that could be consistent with a given S&P rating category, this information is not public and so attention is restricted to the historical averages associated with each S&P credit rating category.

The calculations reported in Table 2 suggest that, for most ratings categories, the Standardized approach will have only a modest effect on insurance values relative to the values that can be attained under the Accord. For the group of intermediate quality credits rated between BB+ and BB□, the Standardized approach is identical to the Accord. For more highly rated credits, the lower risk weights under the Standardized approach increase slightly the value of deposit insurance for these credits, but the higher insurance values are still tiny relative to the insurance values that can be generated by lending to more risky credits. For the lower quality credits in the 150 percent risk weight category, the Standardized approach lowers insurance values relative to those attainable under the Accord, but not by much. Lower quality credits are by far still the most profitable investment alternative for a bank that can fund them with subsidized insured deposits. Among the loans considered in Table 2, the CCC rated credit maximizes shareholder wealth. Thus, notwithstanding increased sensitivity of regulatory capital requirements to credit risk, the Standardized approach promises to be completely ineffective at stemming the trend of securitizing high quality assets. It is unlikely to encourage banks to retain high quality credit on their balance sheets.

The Foundation IRB Approach

The value of the shareholder investment opportunity set under the Foundation IRB can be calculated using equation (7) [as an equality] to substitute for D in equation (2). Figure (2) plots the value of the deposit insurance surface under the Foundation IRB capital regime assuming $P = 110$, and $r_f = .05$. A visual comparison of Figures 1 and 2 conveys an impression that the Foundation IRB capital requirement lowers the value of the deposit insurance subsidy available to bank shareholders relative to the Accord, and indeed this is the case. Under the model assumptions, the shareholder's global optimum is achieved by selecting a loan with $pr_i = .295$, and $LGD_i = 1$. The Foundation IRB approach will induce the bank to shift toward retaining assets with lower probabilities of default, but it will not create any incentive for banks to hold loans with small expected losses in default.

[14] See, "Ratings Performance 2000", Standard & Poor's , p. 16 (default probabilities) and p. 82 (recovery rates).

While the Foundation IRB reduces the global maximum insurance value that can be generated relative to the Accord or the Standardized approached, it does not reduce insurance values for all possible sets of loan characteristics. The last column of Table 2 reports the value of the insurance guarantee under the Foundation IRB approach for alternative S&P rated credits under the assumptions $P = 110$, $r_f = .05$, and $LGD_i = .50 \; \forall \; i$. Compared to capital requirements under the Accord and the Standardized approach, the Foundation IRB offers higher insurance values for all credits in the Standardized approach's 50 percent risk bucket, and many in the 100 percent category (AA+ to BB+).

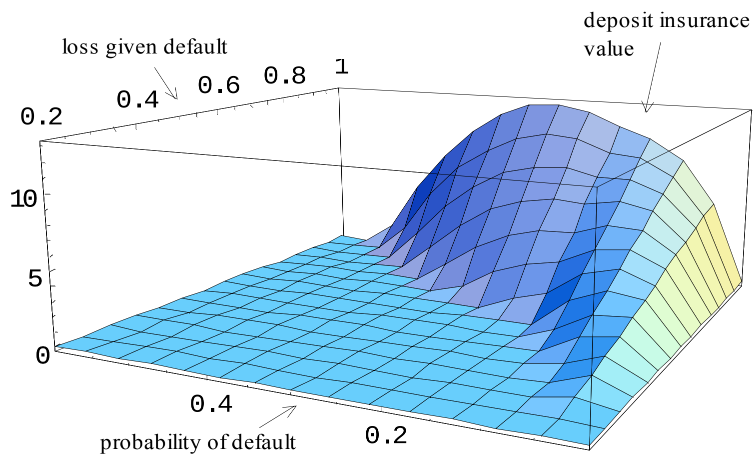

Figure 2: Deposit insurance value under the Foundation IRB capital requirement

The Advanced IRB Approach

The deposit insurance values attainable under the Advanced IRB approach are calculated using equation (8) [as an equality] to substitute for D in equation (2). Figure (3) plots the deposit insurance value surface under the Advanced IRB approach assuming $P = 110$, and $r_f = .05$. Relative to the Foundation IRB approach, the Advanced IRB approach lowers the maximum value of the deposit insurance subsidy that can be generated by shareholders. Somewhat surprisingly, however, insurance values are not reduced by encouraging firms to select loans with smaller expected losses given default. The global optimum loan under the Advanced IRB is a loan with $pr_i = 0.068$, and $LGD_i = 1$.

While the Advanced IRB approach lowers the global maximum insurance value relative to the Foundation IRB (and all other approaches), insurance values are not reduced for all sets of loan characteristics. Table 3 reports the optimal insurance values and corresponding loan default probabilities associated with alternative LGD assumptions under both the Foundation and Advanced IRB approaches. The Table shows that when $LGD < 0.5$ the optimal insurance values and corresponding optimal probabilities of loan defaults are greater under the Advanced IRB approach. When $LGD > 0.5$, the Foundation IRB generates the largest insurance values and the largest corresponding optimal probabilities of loan default.

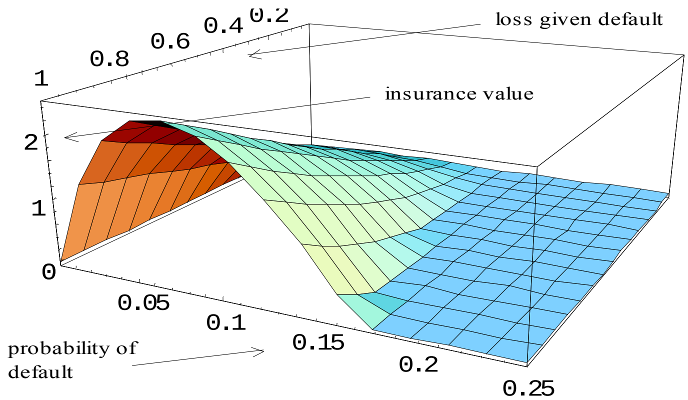

Figure 3: Deposit insurance value under the Advanced IRB regulatory capital requirement

LGD (percent)	Foundation IRB Capital: Optimal default probability (percent)	Foundation IRB Capital: Optimal insurance value	Advanced IRB Capital: Optimal default probability (percent)	Advanced IRB Capital: Optimal insurance value
10	0.45	0.02	5.74	0.22
20	1.34	0.11	5.83	0.45
30	2.58	0.31	5.93	0.68
40	4.16	0.65	6.03	0.92
50	6.13	1.16	6.13	1.16
60	8.57	1.89	6.25	1.41
70	11.60	2.88	6.37	1.66
80	15.52	4.18	6.50	1.92
90	20.92	5.91	6.64	2.19
100	29.46	8.24	6.80	2.46

Table 3: Optimal loan default probabilities and insurance values assuming $P = 110$, and $r_f = .05$.

V. Assessing the Implications of the New Basel Accord

The results that have been derived thus far are based upon the assumption of investor risk neutrality, but they are not are not dependent on this assumption. Sections VI and VII demonstrate that investor risk aversion raises additional issues of importance for the regulatory debate, but it will not reverse any of the results that have been derived thus far. Given the increased level of complexity required to discuss the implications of investor risk aversion, it is useful to summarize the results that are most clearly apparent in the simpler risk neutral setting.

A major goal of the NBA is to make regulatory capital requirements more sensitive to credit risk, in part at least, in order to remove incentives (under the Accord) that encourage banks to securitize high quality loans and retain low quality credits on their balance sheets. The analysis shows that, while all the NBA approaches make regulatory capital requirements more sensitive to credit risk, low quality credits remain the most valuable to banks. Indeed the results suggest that a regulatory change from the Accord to the Standardized approach will have almost no effect on bank behavior regarding what they loans they choose to securitize and what loans they choose to retain on their balance sheets.

It is interesting to observe that none of the proposed approaches for regulatory capital creates an incentive for banks to select loans with minimal expected losses given default. Even the Advanced IRB approach—the only regulatory capital approach that specifically takes *LGD* into account—does not reduce a bank's incentive to try to select loans that are expected to experience substantial default losses. This feature of the NBA is particularly troubling as the loss given default characteristics of a bank's loans are the primary factor determining the insurer's cost of resolving a failed bank.

Notwithstanding the BCBS's intentions, the NBA does not include any economic incentive that will encourage a bank to evolve from the Standardized, to the Foundation IRB, to the Advanced IRB capital approaches. If banks are free to choose their loan characteristics, the Standardized approach offers the largest deposit insurance value. Insurance values that are attainable under the Foundation IRB approach are significantly smaller, but remain larger than those that can be generated under the Advanced IRB approach. There is no reason to expect that banks will voluntarily evolve toward the more complex model-based capital regulations.

If some banks are required to adopt the IRB approaches, it is likely that natural banking clienteles will emerge from the incentives created by the NBA's alternative capital schemes. To add clarity, we focus discussions around the S&P ratings. If a bank is forced to adopt one of the IRB approaches, given complete freedom, the bank would choose the Foundation IRB approach and focus on retaining low quality loans (B- to B+, see Table 2) as these credits maximize the bank's insurance value. If however, other national regulators allow competitor banks to operate under the Standardized approach, the larger insurance values that can be generated by low quality credits would allow Standardized approach banks to bid away the lower quality credit business by underpricing their loans (subsidizing loan rates). Thus differences in the regulatory capital treatments will allow the Standardized approach banks to gain business at the expense of the Foundation IRB banks and the insurer of the banks that use the Standardized approach. Notice that, should some authorities allow their banks to continue using the 1988 Accord's capital requirements, other things equal, these banks would be able to dominate the market for low quality credits.

Foundation IRB banks do, however, have an advantage in retaining relatively high quality credits. Under the assumptions of Table 2, this is true for example, for those loans rated BB+ to AA+ by S&P. The small insurance value advantage could allow the Foundation IRB banks to slightly under-price these credits to bid them away from the banks using the Standardized approach, or indeed even banks remaining under the Accord's capital requirements.

The final segmentation of the market is related to sorting highly rated loans (BB+ to AA+) according to loss given default. Expected loss given default can vary widely among loans in a given S&P rating category. For any subset of loans with $LGD < .50$, Table 3 shows that a bank can increase its insurance value by moving from the Foundation to the Advanced IRB approach. Indeed some banks may choose to do so. These banks may use part of the gain in insurance value to attract the set of high quality credits that also have below average expected losses in default.

In the resulting banking market equilibrium, simple absolute advantage arguments suggest that banks using the Standardized approach will choose to compete for the lowest quality loans; banks using the Foundation IRB approach will choose to compete for loans similar to those that would receive an S&P rating of between BB+ and AA+ but have above average expected loss given default ($LGD > .50$); and banks using the Advanced IRB would compete for the loans that would receive an S&P rating of between BB+ and AA+ but also have below average expected loss given default ($LGD < .50$). In this equilibrium, the banks using the most sophisticated approach for internal credit risk measurement (a necessary condition to qualify for using the Advanced IRB approach) retain the highest quality credits whereas the banks least capable of quantifying their credit risk retain the lowest quality loans on their balance sheets. Thus the three option approach of the NBA will encourage segmentation in the credit qualities of internationally active banks according to their regulatory capital scheme. The "level playing field" objective of the Accord is abandoned.

While the BCBS has been studying the potential regulatory capital implications of the NBA by having cooperating banks estimate their potential capital requirements under the proposed IRB approaches, these estimates are based on banks' current portfolio compositions. Given the incentives created under the NBA, it is likely that the composition of banks' credit portfolios will change, perhaps markedly, following the implementation of the regulatory capital regime. Ultimately then, after banks rebalance their portfolios, it is unclear whether the NBA will improve the stability of the international banking system. While the prudential solvency standards of sophisticated money center banks may be strengthened, the incentives that encourage a concentration of lower quality credits in Standardized approach banks (of which there are expected to be many) may not result in enhanced financial market stability.

VI. Modeling Investor Risk Aversion

Introducing the assumption that shareholders are risk averse significantly enriches the analysis. In order to gauge the effects of risk aversion in this simple model setting, it is necessary to establish the theoretical link between the physical probability of default and the equivalent martingale (or risk-neutral) probability of default that is used by risk averse investors to value claims on future cash flows. This section will introduce an explicit general equilibrium model with risk averse investors that elicits a transparent and intuitive link between the physical and equivalent martingale probabilities of loan default.

Assume that the nominal value of aggregate output in the economy evolves according to a discrete probability distribution with S possible outcomes. Without any loss of generality, we index the possible states of aggregate output values in order of increasing magnitude and let y_i represent the physical probability that the *i*th output state is realized.

Assume that there are N representative investors, each with an initial wealth level W_0, who invest their entire wealth in a portfolio of S Arrow-Debreu securities with the objective of maximizing the value of a mean-variance expected utility function over end of period wealth, $E\left[U\left(\widetilde{W}\right)\right]=\alpha E\left(\widetilde{W}\right)-\Gamma Var\left(\widetilde{W}\right)$, where $E\left(\widetilde{W}\right)=\sum_{i=1}^{S} y_i x_i$, and $Var\left(\widetilde{W}\right)=\sum_{i=1}^{S} y_i x_i^2-\left(\sum_{i=1}^{S} y_i x_i\right)^2$, and x_i is the number of Arrow-Debreu securities held by the agent. Each of these securities pays 1 unit of value (henceforth referred to as a dollar) when state *i* is realized and nothing in any other state of nature. We assume that investors behave as if they are price takers.

If p_i represents the price of an Arrow-Debreu security that pays a dollar in state i, it is straightforward to show that the representative agent's utility maximizing share demands are given by,

$$x_i^* = \frac{1}{\sum_{i=1}^{S} p_i}\left[W_0+\frac{a}{2\Gamma}\left[\left(\sum_{i=1}^{S}\frac{p_i^2}{y_i}\right)-\frac{p_i}{y_i}\right]\right],\ i=1,2,3,.....,S. \tag{9}$$

Equilibrium market clearing prices are determined by setting aggregate supply equal to aggregate demand and solving for the individual Arrow-Debreu security prices. If X_i^Q represents aggregate output in state i, equilibrium requires, $N x_i^* = X_i^Q, \forall i,$ or in terms of per capital output, $x_i^* = \frac{X_i^Q}{N}, \forall i.$ The second condition implies that in this representative agent setting, N can be eliminated by solving for equilibrium in terms of per capita aggregate output. To simplify notation, reinterpret X_i^Q as per capita output in state i and drop any subsequent reference to the number of investors in the economy.

Define the expected value of per capita output, $E(\widetilde{X}^Q)=\sum_{i=1}^{S} y_i X_i^Q$, the variance of per capita output, $Var(\widetilde{X}^Q)=\sum_{i=1}^{S} y_i\left(X_i^Q\right)^2-\left(\sum_{i=1}^{S} y_i X_i^Q\right)^2$, and a constant, $K=\frac{aW_0}{a E(\widetilde{X}^Q)-2\Gamma Var(\widetilde{X}^Q)}$.

The equilibrium market clearing Arrow Debreu security prices can be written as,

$$p_i = y_i K\left[1+\frac{2\Gamma}{a}\left(E(\widetilde{X}^Q)-X_i^Q\right)\right],\quad i=1,2,3,...,S. \tag{10}$$

A fundamental risk free claim is a portfolio comprised of one Arrow-Debreu security from each state. The equilibrium price of a risk free claim, $\sum_{i=1}^{S} p_i = K,$ implies an equilibrium risk free rate of $r_f=\frac{1}{K}-1.$

Security Valuation and the Risk Neutral Probability Measure

In this model setting, it is well known that the equilibrium value of a security or contingent claim can be determined by valuing a portfolio of Arrow-Debreu securities that replicate the state payoffs on the contingent claim contract that is being priced. The equilibrium absence of arbitrage condition requires that the value of the claim must equal the equilibrium price of the replicating portfolio of Arrow-Debreu securities. In addition to the traditional Arrow-Debreu portfolio pricing solution, Harrison and Pliska (1981) establish that the absence of arbitrage in a state space model implies the existence of the risk neutral pseudo probability measure and the equivalent martingale market valuation condition. The pseudo probability measure is unique if markets are complete (as they are assumed to be in this model).

The equivalent martingale market valuation condition requires that the equilibrium price of a security equal the present value of the securities' expected future payoffs, where the expectation is taken with respect to the equivalent martingale (risk neutral) measure, and the present value discounting occurs at the risk free rate of interest. In the case of a simple Arrow-Debreu security, the equivalent martingale valuation condition requires,

$$p_i = \frac{y_i^E}{(1+r_f)}, \tag{11}$$

where y_i^E represents the equivalent martingale probability of a realization of state i. Using expressions (10), (11), and $(1+r_f) = \frac{1}{K}$, the physical and risk neutral probabilities relationship is given by,

$$y_i^E = y_i\left[1+\frac{2\Gamma}{a}\left(E(\tilde{X}^Q)-X_i^Q\right)\right], \text{ for } i=1,2,3,\ldots.,S. \tag{12}$$

Parameters "a" and "Γ" are both required to be positive. Equation (12) requires that the risk neutral probability associated with state i is greater than the state's physical probability if the level of per capita output in state i is below the expected value of per capital output. Conversely, the risk neutral probability associated with state i is less than the physical probability of state i if per capita output in state i exceeds average per capita output. Other things equal, the differences between the risk neutral and physical probabilities are greater the larger are investors' aversion to taking risk (the larger is Γ).

Loan Valuation Under the Risk Neutral Measure

Consistent with the simple two state model of a bank loan developed in section two, consider a loan that has two possible cash flow states: a good state in which it pays off its promised maturity value P, and a default state in which it pays of $P(1-LGD_i)$. Given P, and a bank selected LGD_i, when investors are risk averse, the fair market value of this bank loan depends not only on the probability that the bank defaults, but also on the economic states in which the bank defaults. Let Ω_i represent the set of states in which loan i defaults. Let $pr_i^E = \sum_{\forall i \in \Omega_i} y_i^E$ represent the probability of default under the equivalent martingale measure.

When investors are risk averse, the equilibrium value of the bank loan is given by,

$$L(LGD_i, pr_i^E) = \frac{P(1 - LGD\ pr_i^E)}{1 + r_f} \tag{13}$$

which is identical to expression (1) after replacing the physical probability of default with the equivalent martingale probability of default. Using expression (12), it is straight forward to show that,

$$pr_i^E = pr_i \left[1 + \frac{2\Gamma}{a} \left(E(\tilde{X}^Q) - \sum_{\forall i \in \Omega_i} \left(\frac{y_i}{pr_i} \right) X_i^Q \right) \right], \tag{14}$$

where, $pr_i = \sum_{\forall i \in \Omega_i} y_i$. Expression (14) shows that a loan's equivalent martingale probability of default will exceed its physical probability of default if the physical expected per capital output in loan default states is less than the physical unconditional expected per capital output. Thus the risk neutral probability of default exceeds the physical probability of default when the loan default occurs in states that have a conditional average GDP per capita that is below the unconditional expected GDP per capita for the economy. Conversely, if the average level of output per capita in default states exceeds the economy's unconditional expected output per capita, $pr_i^E < pr_i$.

The Value of Deposit Insurance Under Risk Aversion

The introduction of risk aversion complicates the expression for deposit insurance valuation because, while the loan pricing condition requires only the substitution of the risk neutral for the physical probability measure, the regulatory capital restrictions on the level of insured deposits may depend on both the physical and the risk neutral probability measures. Let

$$INS(pr_i, pr_i^E,\ LGD_i) = \frac{\left[D(pr_i, pr_i^E, LGD_i) - P(1 - LGD_i) \right] pr_i^E}{1 + r_f} \tag{15}$$

represent the generic expression for deposit insurance value where the notation $D(pr_i, pr_i^E, LGD_i)$ indicates that the level of insured deposits may be a function of a credit's physical probability of default, its risk neutral probability of default, and its loss given default.

Under the Accord, qualifying capital must be at least 8 percent of a loan's value. If qualifying capital is restricted to paid in equity capital, this condition requires that,

$$D \leq\ .92\ (1 + r_f)\ L(pr_i^E,\ LGD_i)\,. \tag{16}$$

Similarly, under the Standardized approach, the use of insured deposit funding must satisfy the inequality,

$$D \leq \left(1 - Cap(rating_i)\right)(1 + r_f)\ L(pr_i^E,\ LGD_i)\,. \tag{17}$$

Under the Foundation IRB, a credit's risk weight is set according to a loan's physical probability of default. The risk weight determines what proportion of the value of the loan must be financed with paid in equity capital, but the fair value of the loan itself is determined by the equivalent martingale probability of default. Thus, under the Foundation IRB, the regulatory capital requirement restricts insured deposits according to,

$$D \leq (1+r_f)\ L(pr_i^E, LGD_i)\left(1-.08\frac{BRW_C(pr_i)}{100}\right). \tag{18}$$

Similarly, under the Advanced IRB approach, insured deposit financing will be restricted by the relationship,

$$D \leq (1+r_f)\ L(pr_i^E, LGD_i)\left(1-.08\frac{LGD_i}{.50}\frac{BRW_C(pr_i)}{100}\right). \tag{19}$$

Under any of these capital rules, shareholders maximize the use of insured deposit financing (the equality will hold) when the insurance guarantee is valuable.

VII. Optimal Bank Behavior When Shareholders are Risk Averse

The risk characteristics of a bank's optimal loan portfolio depend on both the risk aversion of equity investors and the regulatory capital scheme under which banks operate. Under some of the regulatory capital schemes, deposit insurance values can be enhanced by concentrating bank loan defaults so that they occur in states of nature that are characterized by below average output per capita. The alternative capital regimes are considered in turn. Deposit insurance values are determined by substituting a regulatory capital requirement's insured deposit issuance restriction (as an equality) into expression (15).

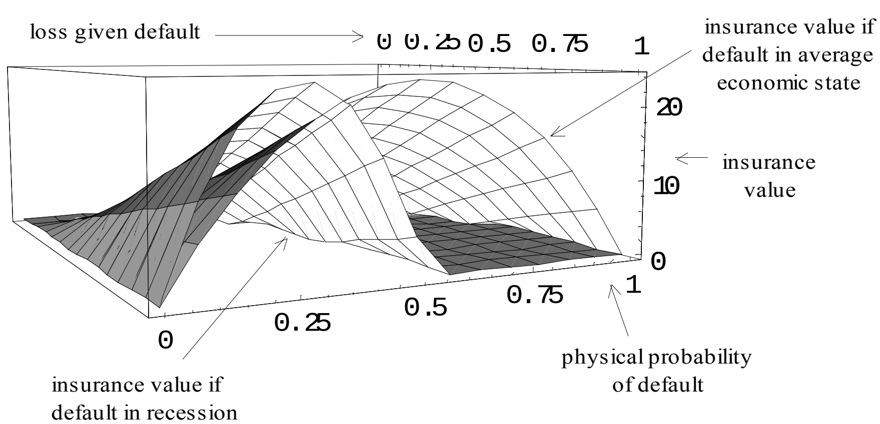

Figure 4: Deposit insurance value surface under the Accord when investors are risk averse.

Figure 4 plots the deposit insurance value surface under the Accord when investors are risk averse. The figure includes surfaces for alternative assumptions about per capita output in the states of nature in which a loan defaults. The surface in Figure 4 is generated under the assumptions: $a = 45, \Gamma = 1,$ and the representative investor's wealth, W_0, has been normalized to be consistent with $r_f = .05$ when the aggregate output per capita satisfies the implicit assumption $E(\widetilde{X}^{Q}) = 30, Var(\widetilde{X}^{Q}) = 60.5$.[15] The qualitative results are independent of the parameter values assumed.

If a loan is expected to default in a state in which aggregate per capita output is equal to the unconditional average aggregate output per capita, the deposit insurance value surface is identical to the surface that prevails when investors are risk neutral. In this instance, the global optimal corresponds to $pr_i = .50, LGD_i = 1$. If the loan defaults in states of nature in which aggregate per capita output is less than the unconditional output per capita, Figure 4 shows that the optimal insurance and loss given default values are unchanged, but the optimal physical probability of default is reduced from 50 percent. The converse is true if the loan is expected to default in states in which per capita output exceeds its unconditional average.

Table 4 (following references) provides additional details about the relationship between insurance values, physical probabilities of default, and recovery rates under the assumption that the loan defaults in states of nature in which output per capita deviates from its unconditional average value. The characteristics of an economy's aggregate output per capita distribution restricts a bank's ability to choose the probability of default and the state of default. Loans that have a very large physical probabilities of default cannot also default in states of nature which have a conditional expected output per capita that is significantly below average.[16] To account for this technical limitation without imposing any distributional restrictions that are otherwise unnecessary, default rates greater than 60 percent are arbitrarily considered to be infeasible for loans that default in states of nature in which conditional expected per capita output is significantly below average.

The results reported in Table 4 show that, under the Accord, provided that a bank is able to freely select a loan's physical probability of default, there is no incentive for the bank to prefer that a loan default in any particular state of nature. While the optimal physical probability of default will depend on the macroeconomic conditions that are expected to prevail when loan default occurs, there is nothing in the regulatory capital requirement that makes a bank prefer that default occur in any particular macroeconomic state. To the extent that supplemental supervisory actions (for example CAMEL bank ratings systems) may create incentives for banks to select loans with low physical probabilities of default, banks under the Accord may prefer loans that default under recessionary or slow growth conditions, but the capital regulations of the

[15] These specific parameter values underlie Figures 4 and 5, and Tables 4-7.

[16] Assume that the states of nature are ranked in order of increasing output per capita. For any given probability of default, assume default occurs in the states of nature of nature with the smallest output per capita (this is an optimal ordering under all approaches other than the Accord, and it is equivalent to any other optimal ordering under the Accord) As the physical probability of default increases, the conditional expected output per capita is always less than the unconditional expected output per capita, but the difference between the expectations must converge to zero as the probability of default approaches unity. For any given probability of default, the difference between the conditional and unconditional expected output per capita will depend on the specific characteristics of the output per capita probability density function.

Accord itself do not create a bank preference for loans that are expected to default when macroeconomic output is below average.

While the capital regulations of the Accord may not create incentives for banks to concentrate their lending to counterparties that are expected to default when per capita output is below average, all of the approaches proposed in the NBA include this feature. Table 5 (following references) reports the optimal equivalent martingale probability of default that is associated with each risk weight category of the Standardized approach assuming a 50 percent loss given default. Recall that each rating class has an associated (publicly) unknown expected probability of default that is determined by S&P. Historical default rate data by S&P rating suggest that the physical default rates associated with each regulatory "bucket" in the Standardized approach are significantly less than the optimal risk neutral default rates associated that bucket (reported in Table 5). Banks using the Standardized approach face an incentive to choose loans with risk neutral default rates that exceed (likely by as much as possible) the physical default rate S&P uses to determine a rating grade. Banks accomplish this by selecting among credits with a given S&P rating those credits that are expected to default when aggregate per capita output is the smallest.[17]

Similar to the Standardized approach, when investors are risk averse, both IRB approaches create an incentive for a bank to prefer loans that are expected to default when per capita output is below average. Figure 5 provides a visual guide to the implications of risk aversion for the IRB approaches. Figure 5 plots the deposit insurance value surface under the Advanced IRB approach when investors are risk averse and the bank can select the macroeconomic conditions that prevail when its loan defaults. Figure 5 shows that the introduction of investor risk aversion does not affect the optimal loss given default setting (it remains 100 percent), but the bank can, however, increase the value of its deposit insurance by selecting a loan that is expected to default when output per capita is below average. Similar effects are generated under the Foundation IRB approach (not pictured).

Tables 6 and 7 (following references) provide more detail regarding the implications of investor risk aversion for the IRB regulatory capital approaches. Table 6 reports optimal physical probabilities of default and corresponding insurance values for alternative combinations of assumptions regarding loss given default and the average per capita output in default states under the Foundation IRB approach. Table 7 repeats the analysis for the Advanced IRB approach. The results show that for any loss given default assumption, deposit insurance values under both IRB approaches increase as the average value of per capita output in default states decreases. In other words, *ex ante* insurance values are enhanced if loan defaults are expected to occur when macroeconomic activity is depressed.

The results in Tables 6 and 7 show that under either IRB approach, for any loss given default, the optimal physical probability of default is a decreasing function of the conditional average per capital output in default states. Thus, to the extent that investor risk aversion creates incentives for banks to select loans with smaller physical probabilities of default, it is because banks can identify loans that are expected to default in states of nature where aggregate output is below average. In such an instance, even though the bank loans appear to be safer when measured according to their physical probability of default, the banks' deposit insurance guarantee will actually have greater value.

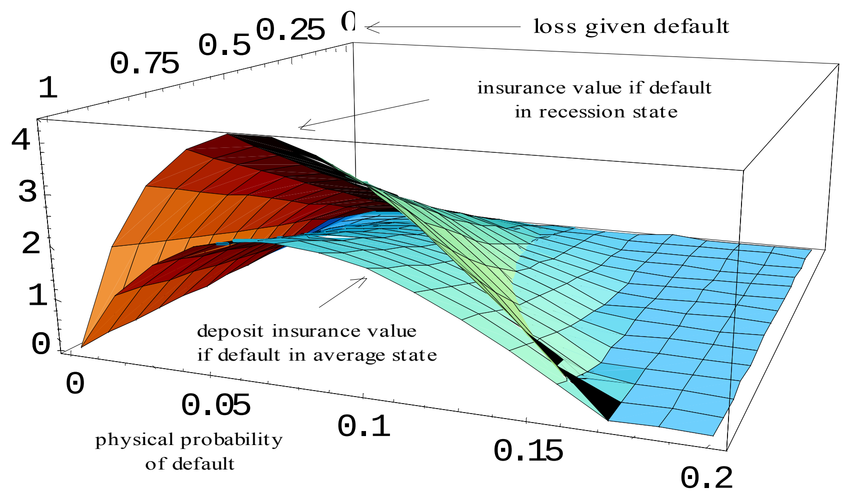

Figure 5: Illustration of the implications of risk aversion for the deposit insurance value function under the proposed Advanced IRB capital requirement.

VIII. Loan Portfolios

Thus far, the discussion has focused on a bank's choice of the risk characteristics of a single loan and has excluded consideration of issues related to credit risk diversification and the construction and characteristics of optimal bank loan portfolios. This section considers the characteristics of an optimal credit portfolio when deposit insurance is valuable. It develops a formal argument that justifies the emphasis on analyzing the profitability of a single loan investment. The discussion establishes that, should a bank be maximizing the value of its insurance guarantee, only loans that have a positive *ex ante* insurance values will be included in an optimal bank loan portfolio.

In the single loan setting , if deposit insurance is valuable, when the bank's loan defaults, the bank defaults on its insured deposits. When the bank has a portfolio of loans, this one-to-one default correspondence no longer holds. In a one-period model, the bank defaults on its deposits when the end-of-period value of its loan portfolio falls short of the value of its insured deposits.

Let P_{Bnk} represent the promised terminal payoff on a bank's entire loan portfolio. Let pr_{Bnk} and pr^{E}_{Bnk} represent the physical and risk neutral probabilities that the bank defaults on its loan portfolio, and LGD_{Bnk} represent the fractional loss on the bank's loan portfolio that is expected to occur if the bank

[17] In the real world setting, this is accomplished for example by selecting among credit with a given rating, those that offer the greatest interest margins.

defaults on its deposits. Loss given default is measured relative to the loan portfolio's promised payoff. Let D_{Bnk} represents the promised terminal payment on the bank's entire base of insured deposits. D_{Bnk} is restricted in magnitude if the bank is under a regulatory capital constraint. The value of the bank's deposit insurance guarantee can be written,

$$INS(D_{Bnk}, P_{Bnk}, LGD_{Bnk}, pr^{E}_{Bnk}) = \frac{\left(D_{Bnk} - P_{Bnk}(1 - LGD_{Bnk})\right) pr^{E}_{Bnk}}{1 + r_f} . \tag{20}$$

The similarities between the expression for value of the deposit guarantee in the portfolio case and the value of the guarantee in the context of a single loan (expression (15)) are transparent. In a portfolio context, the bank will select loans so that the implied values for P_{Bnk}, D_{Bnk}, LGD_{Bnk}, and pr^{E}_{Bnk}, maximize the *ex ante* value of the bank's insurance guarantee subject to any constraints imposed by regulatory capital requirements.

Section 1 in the Appendix derives the relationship between individual loan characteristics and P_{Bnk}, D_{Bnk}, LGD_{Bnk}, and pr^{E}_{Bnk} in binomial insurance valuation expression (20) for a two loan portfolio under regulatory capital requirements specified by the Accord. Arguments similar to those in the Appendix can be used to derive a binimial insurance valuation expression for any bank portfolio under any of the alternative capital regimes.[18]

Optimal Portfolio Construction

While expression (20) is a general expression for calculating the *ex ante* value of a bank deposit insurance guarantee, the expression itself is not very revealing as to the characteristics of the loans that are included in an optimal bank loan portfolio. This section addresses this issue.

Consider a bank that is considering adding an additional loan, loan i, to an existing portfolio that generates a positive *ex ante* insurance value for the bank. Let the insurance value of the existing portfolio be represented by $\frac{\left[D_{Bnk} - P_{Bnk}(1 - LGD_{Bnk})\right] \; pr^{E}_{Bnk}}{1 + r_f}$, where the magnitude of D_{Bnk} depends on the regulatory capital scheme in force as well as the characteristics of the individual loans in the banks portfolio.[19] If loans are fairly priced and so the bank's objective is to maximize the value of its insurance guarantee, section 2 in the Appendix proves the following:

Theorem 1: *If a bank is attempting to maximize the value of its deposit insurance guarantee, a loan must have positive insurance value when it is evaluated as a stand alone investment if it is to be included in a bank's optimal loan portfolio.*

[18] The granularity adjustment is treated separately below.

A necessary condition for a loan to be included in an optimal bank loan portfolio is that the loan have a positive *ex ante* deposit insurance value when it is evaluated on a stand alone basis investment.[20] If the loan does not have a positive insurance value as a stand alone investment, the addition of the loan to the portfolio will reduce the maximum attainable deposit insurance value that can be generated by the bank. This theorem provides a justification for focusing attention single loan model of a bank that has guided the analysis of the alternative NBA capital proposals.

Under any of the capital proposals, profit maximizing banks will only consider loans that generate positive insurance values, and they will select the combination of loans that generate implied values for P_{Bnk}, D_{Bnk}, LGD_{Bnk}, and pr_{Bnk}^{E} that maximize expression (20). If banks attempt to maximize the value of their insurance guarantee, they will select loans to achieve target values for P_{Bnk}, LGD_{Bnk}, and pr_{Bnk}^{E} that depend on the regulatory capital scheme in force.

Excepting banks under the IRB regulatory capital rules, banks do not face any incentive to follow a diversification strategy when constructing their loan portfolios. Indeed it can be shown that bank insurance values are enhanced when a bank's loans are choosen so that they default, as nearly as is possible, in identical states of nature. The so-called "granularity adjustment" included in the proposed IRB approaches is an attempt to create a regulatory incentive to mandate diversification. The granularity adjustment complicates the analysis of the IRB capital schemes. The next section considers these complications in more detail.

IX. The IRB Granularity Adjustment

The regulatory capital requirements that apply under the Accord, the Standardized approach, and indeed even the IRB approaches are implemented using individual loan risk weights that are invariant with respected to the characteristics of a bank's loan portfolio. While regulatory capital requirements under the Accord and the proposed Standardized approaches are completely determined by the characteristics of the bank's individual credits, capital requirements under the proposed IRB approaches are modified to reflect the overall level of diversification or "granularity" in a bank's loan portfolio. The so-called "granularity adjustment" constructs a specific regulatory measure of the diversification in a bank's portfolio, and then uses this measure to augment baseline IRB regulatory capital requirements.[21]

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[19] In this section, we ignore any complications associated with the granularity adjustment that applies under the IRB regulatory capital approaches.

[20] A complete characterization of the construction of an optimal loan portfolio in instances when a bank is attempting to maximize the *ex ante* value of its deposit insurance guarantee is tedious and not required to establish the arguments of this paper. Such a characterization is however available upon request from the author.

[21] The proposed regulatory measure of diversification and associated regulatory capital adjustment is a theoretical construct based upon the assumptions that credit VaR model estimates provide accurate prudential capital guidelines and that individual credit risk exposure profiles can be accurately represented using a single

The baseline IRB capital requirement used in the granularity calculations is 4 percent of the sum of the IRB risk-weighted loans. Should the regulatory diversification measure indicate that a portfolio is very well diversified, the granularity adjustment reduces regulatory capital from baseline IRB required capital levels. If a bank's portfolio is determined to be poorly diversified according to the regulatory measure, baseline IRB required capital is increased by the granularity adjustment.

The details of the regulatory diversification measure and the granularity adjustment to capital are given in section 3 of the Appendix. An intuitive explanation of the practical implications of the granularity adjustment are provided through a series of portfolio simulations. Abstracting away from the rules regarding customer categorizations (sovereign, corporate, retail, project finance) and focusing on corporate exposures, the qualitative preconditions for use of the IRB include a requirement that the bank's performing loans be classified into at least 6 credit rating grades where grade are differentiated according to their probabilities of default and no more than 30 percent of the bank's loan counterparties can be categorized in any single grade.

To illustrate the properties of the granularity adjustment, we consider alternative loan portfolios that are modifications of a baseline set of loans that represent 6 credit rating grades. Baseline loan characteristics are given in Table 8. All loans are assumed to have a maturity value of 110, and the risk free rate is assumed to be 5 percent. Loan equivalent martingale probabilities of default (used in valuation) are arbitrarily set to be five times a loan's physical probability of default.

Loan grade	Physical probability of default (percent)	Equivalent martingale probability of default (percent)	Loss given Default (percent)	Market value
1	.03	.15	50	104.68
2	.08	.4	50	104.55
3	.12	.6	50	104.45
4	.2	1	50	104.24
5	.3	1.5	50	103.98
6	.45	2.1	50	103.66

Table 8: Baseline loan characteristics for granularity adjustment simulations. All loans have a par value of 110, and thee risk free rate is assumed to be 5 percent.

Figure 6 plots granularity-adjusted IRB capital requirements for loan portfolios that constructed using the baseline loan characteristics reported in Table 8. Alternative loan portfolios capital requirements are constructed assuming an equal number of identical loans in each risk grade, and then varying the number of loans per grade. Each new loan represents an exposure to a new counterparty.[22] The relationship labeled "baseline" includes the loans in Table 8. The relationship labeled "2 x baseline" plots the regulatory capital requirements for a loan portfolio with an equal number of "names" in each credit grade where each loan grade has twice the probability of default (physical and equivalent martingale) of the corresponding baseline loan grade in Table 8 and a market value that is adjusted appropriately. The relationships labeled "5 x baseline"

common return factor and individual (uncorrelated) idiosyncratic return components. See the Consultative Document: The Internal Ratings Based Approach, Chapter 8 for additional details.

[22] In the granularity adjustment calculations, multiple loans to a single counterparty are aggregated and count as a single loan (or "name") with appropriately modified exposure measures.

and “10 x baseline” are constructed analogously with five (and ten) times the default probabilities of the corresponding baseline loans with appropriately reduced market values.

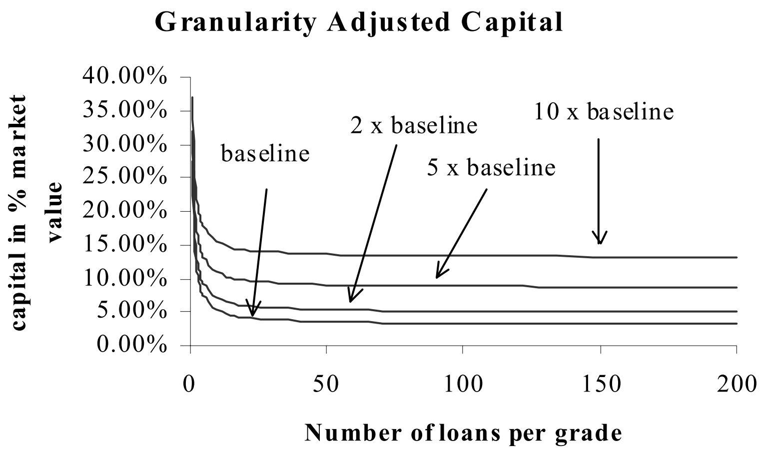

Figure 6: Granularity adjusted IRB capital requirements for alternative loan portfolios.

Figure 6 shows that the granularity adjustment imposes a significant capital penalty if the number of counterparties in the bank portfolio is small. As the number of counterparties in a balanced loan portfolio increases, the regulatory capital requirement—measured as a percentage of the market value of the portfolio—declines and approaches an asymptote that is marginally lower than the portfolio’s unadjusted capital requirement.

Figure 7 plots the granularity capital adjustment for these same portfolios where the adjustment is measured in basis points of the portfolio’s market value. When the number of “names” in each credit risk grade are small, capital requirements are elevated significantly above baseline IRB capital requirements. Figure 7 shows that the potential reductions in capital are larger, the greater is the level of risk in a bank’s balanced loan portfolio.

Simulations (not reported) of portfolios in which the highest risk grade bucket contains 30 percent of the portfolio’s loans and the remaining loans are uniformly distributed across the other 5 loan grades exhibit capital requirements as a function of size (number of names) that are qualitatively similar to the relationships illustrated in Figures 6 and 7. Capital requirements are exceptionally large when the number of counterparties in the loan portfolio is small. Capital requirements decrease rapidly until the loan portfolio has roughly 300 names; beyond 300 names, as the number of loans are increased, capital requirements decline more slowly toward an asymptote that offers modest capital relief over the unadjusted IRB capital requirement.

As a practical matter, given the minimum number of credit grades that are required to qualify for the IRB approach and the limits on loan concentrations among these grades, the granularity adjustment becomes a requirement that a bank have a reasonably large number of names (more than 300) before it would even consider migrating to an IRB approach. Earlier analysis demonstrated that banks under the IRB have a slight advantage over Standardized approach banks in offering relatively high quality credits (Table 2: B+ – AA+). This advantage (the only advantage under the IRB) will be removed if IRB capital requirements are raised by the granularity adjustment. Thus, unless banks have a significant number of counterparties, the granularity adjustment will make the IRB regulatory capital option unattractive relative to the Standardized approach. If a bank is to retain any insurance value under the IRB, it must have enough counterparties to benefit from (or at least mitigate significantly) the granularity adjustment. If forced into adopting an IRB approach, banks will face strong incentives to increase the number of counterparties in their loan portfolio perhaps through merger if internal expansion is infeasible or insufficient to achieve the valuable economy of scale in regulatory capital.

Other things equal, it is clear that the granularity adjustment will create incentives for a bank to lend to as many counterparties as possible. It is, however, far from clear that this requirement will ensure that a bank is adequately diversified and thereby attenuate the value of its insurance guarantee. While diversification may be assured under the assumptions that were used to derive the granularity adjustment, these assumptions are very restrictive and will not guarantee diversification in a general setting.[23]

The NBA granularity adjustment is derived under a one-factor Capital Asset Pricing Model of uncertainty under which all assets' returns can be decomposed into a market wide (systematic risk) component and a residual idiosyncratic risk component. Under this stochastic representation of returns, unless the so-called beta coefficient (sensitivity to the systematic factor) is unity, conditional on a realized value of the systematic factor, default remains random as the idiosyncratic component of returns can experience a large realization that offsets the market factor and either promotes or forestalls (depending on sign) default.

In the Arrow-Debreu model of equilibrium used in this paper, there is single state variable that determines an asset's payoff. In this model, credit defaults are completely determined by the state variable, and given its realized value, there is no idiosyncratic risk component that can be used to diversify losses in a portfolio. In the Arrow-Debreu setting, the bank can choose the states of nature in which its loans default. The only uncertainty regarding default relates to which state materializes. In other words, diversification benefits assumed in the derivation of the granularity assumption do not hold. In theory at least, a bank can have a large portfolio of "names" without diluting its insurance value by selecting the names so that defaults are coordinated in the same Arrow-Debreu states.

[23] The NBA Consultative Document does not provide proof of the veracity of the granularity adjustment even under the restrictive assumptions used to derive it.

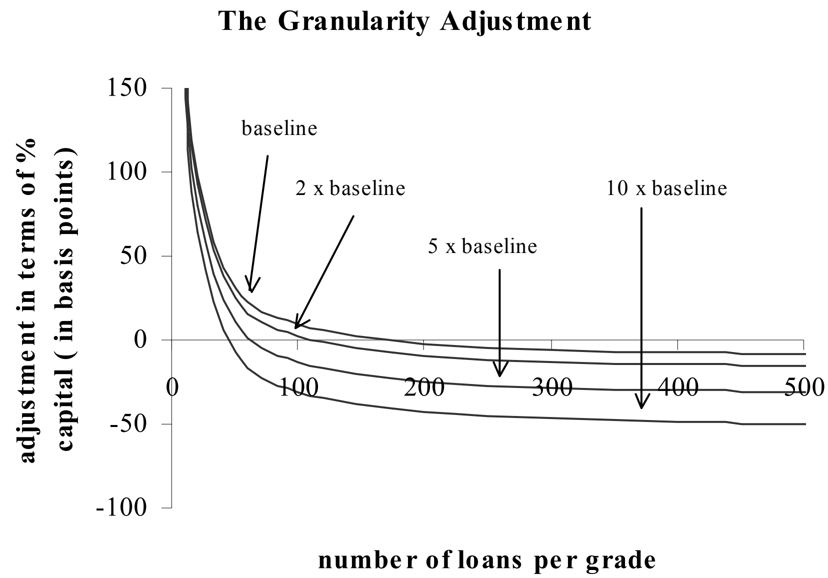

Figure 7: The Granularity adjustment measured in basis points of portfolio market value.

In reality, neither of these models of uncertainty is an accurate representation of reality. The Arrow-Debreu analysis, while clearly simplistic, is however instructive in clarifying the shortcomings of the granularity adjustment's assumptions. The Arrow-Debreu analysis highlights the importance of the equivalent martingale probability of default in determining insurance values. Regardless of the pricing model or characterization of asset market equilibrium, if a bank can choose to focus on loans that have equivalent martingales probabilities of default that are significantly larger than their physical expected default rates without any ramifications for its regulatory capital requirements, banks will choose loans that enhance their insurance value by choosing loans that are expected to default when the marginal utility of consumption is high. Even if banks do not overtly attempt to coordinate the timing of loan defaults, if the capital regulation's focus on the physical probability of default leads banks to favor loans with elevated equivalent martingale default probabilities, the Arrow-Debreu analysis implies that defaults will be temporally clustered.

A similar situation could also hold, for example, if returns were generated by two independent systematic factors and an idiosyncratic factor (e.g. a common equity factor, a common interest rate or industry factor, and an idiosyncratic factor). If the granularity assumption relies on a one-factor return representation, banks could appear to be well-diversified, but load up on assets that have large sensitivities to the second (unmeasured) factor assuming that it carries a positive risk premium. In this situation, loan defaults would be clustered around low realizations of the second common factor despite the fact that the bank's portfolio might appear to be well diversified relative to a single factor. The upshot is, if the granularity assumption does not properly adjust capital requirements for all relevant aspects of diversification, it cannot properly ensure that a credit portfolio is well diversified. That the granularity assumption does not ensure diversification in the Arrow-Debreu equilibrium setting is transparent. A demonstration of its weaknesses in a higher order risk factor model will, however, remain intuitive as a formal proof is beyond the interests of this (already long) paper.

X. Conclusion

The proposals in pillar 1 of the NBA will create regulatory capital requirements that are sensitive to credit risk. Banks following the Standardized approach will face reduced insurance values for credits in the 150 percent risk category, however this category will remain the most attractive category for bank lending. Bank's that are forced to adopt an IRB approach for capital stand to bear significant reductions in the values of their deposit insurance guarantees should they continue to hold the loan portfolios that were optimal under the 1988 Accord. There is, however, little likelihood that the latter condition will prevail. The NBA proposals will generate strong incentives for banks to modify their existing loan portfolios. The proposals will likely encourage the formation of stable bank clienteles in which a bank's customer base and risk profile will be determined by the bank's regulatory capital regime.

While the NBA may reduce deposit insurance values in some institutions, the incentives it engenders ultimately are not compatible with existing bank regulatory objectives or the stated goals of the BCBS. The Standardized approach promises to have little effect on bank securitization activities. The proposals do not foster incentives that will encourage banks to voluntarily evolve from the Standardized to the Advanced IRB approach. The proposals create natural economies of scale in IRB banks and will encourage market segmentation among internationally active banks that is completely at odds with the "level playing field" objective of the 1988 Accord. None of the proposed approaches for regulatory capital creates an incentive for banks to control their potential loss given default and indeed even the "sophisticated" IRB approaches encourage bank behavior that is at odds with the regulatory objective of least cost resolution.

A final issues related to macroeconomic stability. In contrast to the Accord, all of the NBA's proposed capital schemes contain incentives that may encourage banks to purposely concentrate lending to credits that are expected to default in recessions. When investors are risk averse and capital requirements focus on the physical probability of default as they do under the NBA, banks may be able to select the timing of default (relative to the business cycle) to enhance the value of their deposit insurance guarantee. While the IRB granularity assumption may imperfectly control for this feature, any control that it provides is achieved at the cost of introducing economies of scale in the regulatory capital scheme for IRB banks. Larger IRB banks will face lower capital requirements. Whether or not increased scale will increase bank safety is an open issue, but almost certainly the granularity adjustment will encourage consolidation among IRB banks. Taken to the extreme, reduced competition and associated "too big too fail" issues may effectively extend safety net related externalities beyond those analyzed in this paper.

Appendix

1. Derivation of Expression (20)

This appendix illustrates the mechanics of expressing a bank's deposit insurance value in a form equivalent to expression (20). To simplify the analysis, assume that the promised terminal payment on all loans are normalized to P. Loans differ according to their expected macroeconomic state of default, their expected loss given default, and their probability of default. To facilitate the analysis, we utilize the Arrow-Debreu state space representation of capital market equilibrium. To add clarity, we illustrate the derivation of expression (20) under the capital requirements of the Accord.

Under the capital requirements of the Accord, the general expression for deposit insurance value for a portfolio of two loans is given by, $\frac{\left[D_{Bnk}^{A} - P_{Bnk}(1 - LGD_{Bnk})\right]\ pr_{Bnk}^{E}}{1 + r_f}$, where $P_{Bnk} = 2P$, $D_{Bnk}^{A} = \left[.92\left(P(2 - LGD_i\ pr_i^{E} - LGD_j\ pr_j^{E})\right)\right]$, and pr_{Bnk}^{E} and LGD_{Bnk} depend on the characteristics of the loans included in the portfolio which, aside from the extent to which the bank uses insured deposit funding, are the only choice variables of the bank. The superscript *A* has been appended to D_{Bnk} denote that it is constrained by capital requirements under the Accord.

As a general matter, we remark that the deposit insurance values associated with individual loans are not additive. That is, unless specific conditions are satisfied,

$$\frac{\left[D_{Bnk}^{A} - P_{Bnk}(1 - LGD_{Bnk})\right]\ pr_{Bnk}^{E}}{1 + r_f} \neq \frac{\left[D_i^{A} - P(1 - LGD_i)\right]\ pr_i^{E}}{1 + r_f} + \frac{\left[D_j^{A} - P_j(1 - LGD_j)\right]\ pr_j^{E}}{1 + r_f}$$

where $D_i^{A} = .92\left(P(1 - LGD_i\ pr_i^{E})\right)$, $D_j^{A} = .92\left(P(1 - LGD_j\ pr_j^{E})\right)$.

Recall that pr_i and pr_j represent the physical probabilities that loan i and j default. Let Ω_i and Ω_j represent the set of economic states in which these respective loans default. Let $\Omega_{ij} = \Omega_i \cap \Omega_j$, represent the intersection of the set of states in which loan i and loan j default, and $pr(\Omega_{ij})$ [$pr^{E}(\Omega_{ij})$] represent, respectively, the physical [equivalent martingale] probability associated with the intersection of default states.[24]

In the two-loan portfolio case, we consider four alternative conditions under which the bank may default. In all cases, the prior definitions of D_{Bnk}^{A} and P_{Bnk} apply. It is straightforward to show that the bank will default if a single loan, loan i defaults, provided,

[24] $pr^{E}(\Omega_{ij}) > 0 \Leftrightarrow pr(\Omega_{ij}) > 0.$

$$LGD_i > \left[\frac{.16 - 1.84\,(1+r_f) + .92\,(1+r_f)\; LGD_j \;\; pr_j^E}{1 - .92\,(1+r_f)\; pr_i^E} \right]$$

In this case, if the bank maximizes the use of insured deposits, the deposit bank's deposit insurance value under the Accord is given by, $\frac{\left[D_{Bnk}^A - P_{Bnk}\,(1 - LGD_{Bnk})\right]\; pr_{Bnk}^E}{1+r_f}$, where, $LGD_{Bnk} = \frac{LGD_i}{2}$, and $pr_{Bnk}^E = pr_i^E$.

Consider as the second alternative, the case in which bank defaults if and only if both of its loans default. In this case, the physical [equivalent martingale] probability that both loans default is $pr(\Omega_{ij})$ [$pr^E(\Omega_{ij})$]. If a bank maximizes its use of insured deposits, the *ex ante* value of the bank's deposit insurance guarantee can be written in terms of loan specific characteristics as,

$$\left[D_{Bnk}^A - P_{Bnk}\,(1 - LGD_{Bnk})\right]\; pr_{Bnk}^E,$$

where, $LGD_{Bnk} = \frac{LGD_i + LGD_j}{2}$, and $pr_{Bnk}^E = pr^E(\Omega_{ij})$.

A third alternative is the case in which the bank will default if: (i) loan i defaults; or (ii) if both loans default. In this instance, the physical probability of default in this instance is $pr_{Bnk} = pr_i$, but the *ex ante* loss given default has two potential values depending on the states in which the bank defaults. In this case, the deposit insurance value can be written,

$$\begin{aligned}&\left[D_{Bnk}^A - P(2 - LGD_i)\right]\left(pr_i^E - pr^E(\Omega_{ij})\right) + \left[D_{Bnk}^A - P(2 - LGD_i - LGD_j)\right]\left(pr^E(\Omega_{ij})\right)\\ &\quad = \left[D_{Bnk}^A - P_{Bnk}\,(1 - LGD_{Bnk})\right]\; pr_{Bnk}^E\end{aligned}$$

where $LGD_{Bnk} = \frac{1}{2\,pr_i^E}\left(pr_i^E\; LGD_i + pr^E(\Omega_{ij})\, LGD_j\right)$, and $pr_{Bnk}^E = pr_i^E$.

A final alternative is the case in which the bank will default if: (i) loan i defaults; (ii) loan j defaults, or (iii) if both of its loans defaults. In this instance, the equivalent martingale probability of default is $pr_{Bnk}^E = pr_i^E + pr_j^E - pr^E(\Omega_{ij})$, and the *ex ante* loss given default has three potential values. If the bank maximizes the use of insured deposits, its insurance value can be written,

$$\begin{aligned}&\left[D_{Bnk}^A - P(2 - LGD_i)\right]\left(pr_i^E - pr^E(\Omega_{ij})\right) + \left[D_B^A - P(2 - LGD_j)\right]\left(pr_j^E - pr^E(\Omega_{ij})\right)\\ &\quad + \left[D_{Bnk}^A - P(2 - LGD_i - LGD_j)\right]\left(pr^E(\Omega_{ij})\right)\\ &\qquad = \left[D_{Bnk}^A - P_{Bnk}\,(1 - LGD\;\;)\right]\; pr_{Bnk}^E\end{aligned}$$

where $LGD_{Bnk} = \frac{1}{2\left(pr_i^E + pr_j^E - pr^E(\Omega_{ij})\right)}\left(pr_i^E\; LGD_i + pr_j^E\; LGD_j\right)$.

This two loan example formally establishes the link between the risk characteristics of the individual loans in a bank's loan portfolio and the probability of bank default $\left(pr_{Bnk}, pr_{Bnk}^{E}\right)$, and loss given bank default (LGD_{Bnk}) values that determine a bank's insurance value in expression (20). While the algebra gets complicated as the number of loans in portfolio increases, the same algorithm can be used to express the *ex ante* value of the bank's deposit insurance guarantee in terms of individual loan characteristics in these cases as well. Similar arguments can be used to the construct the insurance value expression for the bank in terms of the portfolio's individual loan characteristics under any of the regulatory capital rules. In the case of the IRB approaches, there is a so-called "granularity adjustment" that applies at the portfolio level that complicates the analysis by, in some cases, limiting D_{Bnk} to a value less than the sum of the individual loan related maximum deposit values. The granularity adjustment is considered in a separate section of the paper.

2. Proof of Theorem 1

Recall that loan i has a zero insurance value when funded using the maximum permitted share of insured deposits, D_i, if $D_i < P(1-LGD_i)$. Let Ω_{Bnk} represent the set of states of nature in which the bank defaults on its deposits given its exiting loan portfolio. Let $\Omega_{Bnk,i} = \Omega_{Bnk} \cap \Omega_i$. The objective is to prove that, should a bank add a loan that has 0 insurance value to an existing loan portfolio with positive insurance value, the addition of the loan will reduce the bank's exiting insurance value.

Part 1

In the first part of the proof we consider the case when the new loan is small relative to the bank's existing portfolio. In particular, we assume, $D_{Bnk} > P_{Bnk}(1-LGD_{Bnk}) - (P - D_i)$, so that even if the new loan does not default, the bank will still default on its insured deposit obligations. In this case, the addition of the new loan does not change the probability that the bank defaults, it only changes the bank's default severity.

Case 1: The bank portfolio and the additional loan default in exactly the same states, $\Omega_{Bnk,i} = \Omega_{Bnk}$. Define $P(1-LGD_i) - D_i = \varepsilon$, and note that $\varepsilon \geq 0$ when the new loan has a non-positive insurance value. After the addition of the new loan, the bank's new deposit insurance value can be written,

$$\frac{\left[D_{Bnk} + D_i - P_{Bnk}(1-LGD_{Bnk}) - P(1-LGD_i)\right] pr_{Bnk}^{E}}{1+r_f} =$$

$$\frac{\left[D_{Bnk} - P_{Bnk}(1-LGD_{Bnk}) - \varepsilon\right] pr_{Bnk}^{E}}{1+r_f} < \frac{\left[D_{Bnk} - P_{Bnk}(1-LGD_{Bnk})\right] pr_{Bnk}^{E}}{1+r_f}$$

Case 2: The intersection of the set of default states for the new loan and the bank under the existing portfolio of loans is the null set, $\Omega_{Bnk,i} = \varnothing$. In this case, in the states when the bank defaults on its existing portfolio, the new loan is worth $P - D_i > 0$. In this instance, the bank's new deposit insurance value can be written,

$$\frac{\left[D_{Bnk} + D_i - P_{Bnk}(1 - LGD_{Bnk}) - P\right] pr^E_{Bnk}}{1 + r_f} =$$

$$\frac{\left[D_{Bnk} - P_{Bnk}(1 - LGD_{Bnk}) - (P - D_i)\right] pr^E_{Bnk}}{1 + r_f} < \frac{\left[D_{Bnk} - P_{Bnk}(1 - LGD_{Bnk})\right] pr^E_{Bnk}}{1 + r_f}$$

Case 3: The intersection between the set of states in which the loan defaults and the bank defaults under its existing portfolio is non empty, but not identical, $0 < pr\left(\Omega_{Bnk,i}\right) < pr\left(\Omega_{Bnk}\right)$. Under these conditions, the new portfolio's deposit insurance value can be written,

$$\frac{\left[D_{Bnk} + D_i - P_{Bnk}(1 - LGD_{Bnk}) - P(1 - LGD_i)\right] pr^E(\Omega_{Bnk,i})}{1 + r_f} +$$

$$\frac{\left[D_{Bnk} - P_{Bnk}(1 - LGD_{Bnk}) - (P - D_i)\right]\left(pr^E(\Omega_{Bnk}) - pr^E(\Omega_{Bnk,i})\right)}{1 + r_f}$$

which can be written,

$$\frac{\left[D_{Bnk} - P_{Bnk}(1 - LGD_{Bnk})\right] pr^E(\Omega_{Bnk})}{1 + r_f} - \frac{\left[D_{Bnk} - P_{Bnk}(1 - LGD_{Bnk} - \frac{P}{P_B} LGD_i)\right] pr^E(\Omega_{Bnk,i})}{1 + r_f}$$

where $\dfrac{\left[D_{Bnk} - P_{Bnk}(1 - LGD_{Bnk} - \frac{P}{P_{Bnk}} LGD_i)\right] pr^E(\Omega_{Bnk,i})}{1 + r_f} > 0.$

Clearly, if the loan is small relative to the bank's existing portfolio, the addition of the loan to the bank's existing portfolio will reduce the *ex ante* value of the bank's insurance guarantee unless the loan has positive insurance value as a stand alone investment.

Part 2

Consider the case when the new loan is large relative to the bank's exiting loan portfolio. A trivial case occurs when the new loan has 0 insurance value and is very large relative to the bank's existing portfolio $\left(P(1 - LGD_i) - D_i\right) - \left(D_{Bnk} - P_{Bnk}(1 - LGD_{Bnk})\right) > 0$. In this case the loan is sufficiently large so that, even if the loan defaults the proceeds it generates (in excess of the maximum insured deposits used to fund the loan) are large enough to ensure that the bank can repay all of the insured deposits used to fund its existing portfolio. In this instance, the *ex ante* value of the insurance guarantee is 0.

Alternatively, if

$$\left(P(1-LGD_i)-D_i\right)-\left(D_{Bnk}-P_{Bnk}(1-LGD_{Bnk})\right)<0<\left(P-D_i\right)-\left(D_{Bnk}-P_{Bnk}(1-LGD_{Bnk})\right),$$

three cases must be considered. Case 1: If $\Omega_{Bnk,i}=\Omega_{Bnk}$, the details are analogous to case 1 above, and the addition of the new loan unambiguously lowers the bank's insurance value. Case 2: If $\Omega_{Bnk,i}=\varnothing$, the new loan always performs when the bank would otherwise default given its existing loan portfolio and the proceeds from the new loan are more than sufficient to forestall a bank default. In this case then, the *ex ante* value of the deposit insurance guarantee is 0 if the new loan is added to the bank's existing loan portfolio. Case3: The final case is when there is some overlap in the states in which the new loan and the bank's existing loan portfolio generate a default, $\varnothing < pr(\Omega_{Bnk,i}) < pr(\Omega_{Bnk})$. When the new loan is included, the bank's new deposit insurance value can be written,

$$\frac{\left[D_{Bnk}+D_i-P_{Bnk}(1-LGD_{Bnk})-P(1-LGD_i)\right]pr^E(\Omega_{Bnk,i})}{1+r_f}$$
$$+\frac{\left[Max\left(D_{Bnk}-P_{Bnk}(1-LGD_{Bnk})-(P-D_i),0\right)\right]\left(pr^E(\Omega_{Bnk})-pr^E(\Omega_{Bnk,i})\right)}{1+r_f}.$$

$$\left(P(1-LGD_i)-D_i\right)-\left(D_{Bnk}-P_{Bnk}(1-LGD_{Bnk})\right)<0<\left(P-D_i\right)-\left(D_{Bnk}-P_{Bnk}(1-LGD_{Bnk})\right),$$

implies that the second term is zero. Using the notation, $P(1-LGD_i)-D_i=\varepsilon\geq 0,$ the bank's new deposit insurance value can be written

$$\frac{\left[D_{Bnk}+D_i-P_{Bnk}(1-LGD_{Bnk})-\varepsilon\right]pr^E(\Omega_{Bnk,i})}{1+r_f} < \frac{\left[D_{Bnk}+D_i-P_{Bnk}(1-LGD_{Bnk})\right]pr^E(\Omega_{Bnk})}{1+r_f}$$

Q.E.D.

3. The Granularity Adjustment

Under the IRB approaches, a bank must sort its loans into a minimum number of internal ratings categories, each of which has an associated physical probability of default pr_k, and loss given default LGD_k. Let s_k represent the proportion of the bank's loan portfolio in internal ratings category *k.* Define the bank's "aggregate probability of default" as $PD_{AG}=\sum_{\forall i} s_k\ PD_k$. Define the bank's "aggregate loss given default" as, $LDG_{AG}=\dfrac{\sum_{\forall k} s_k\ PD_k\ LGD_k}{\sum_{\forall k} s_k\ PD_k}$, where LGD_k is the weighted-average loss given default for internal rating grade k.

For each internal rating grade, calculate $F_k = \Phi(1.118\ \Phi^{-1}(PD_k) + 1.288) - PD_k$. Calculate $F_{AG} = \sum_{\forall k} s_k\ F_k$.

Define EAD_i to be exposure at default for loan *i.* Under the assumptions of the model in this paper, $EAD_i = P\ \forall i.$ The granularity measure of exposure concentration within grade *k* is given by,

$H_k = \dfrac{\sum_{\forall i \in k} EAD_i^{\ 2}}{\left(\sum_{\forall i \in k} EAD_i\right)^2}$ which, under the assumptions of this models, simplifies to $\dfrac{1}{n_k}$, where n_k is the number of loans in ratings category *k*.

Calculate, A_k, $A_k = \dfrac{LGD_k^2\left(PD_k(1-PD_k) - .033F_k^2\right) + \frac{1}{4}PD_k\ LGD_k(1-LGD_k)}{LGD_{AG}^2\left(PD_{AG}(1-PD_{AG}) - .033F_{AG}^2\right) + \frac{1}{4}PD_{AG}\ LGD_{AG}(1-LGD_{AG})}$.

Calculate n^*, $n^* = \dfrac{1}{\sum_{\forall k} A_k H_k s_k^2}$, which, under the assumptions of this model simplifies to,

$n^* = \dfrac{1}{\sum_{\forall k}\left(\dfrac{A_k s_k^2}{n_k}\right)}$. Define the granularity scaling factor,

$GSF = (.6 + 1.8\,LGD_{AG})\left(9.5 + 13.75\dfrac{PD_{AG}}{F_{AG}}\right)$. If *TNRE* is defined to be total non-retain exposures, and $\sum_{non-retail} RWA$ is defined to be the sum of the non retail IRB risk weighted loans, the granularity adjustment to non-retail risk weighted assets is given by,

$Adj = TNRE\left(\dfrac{GSF}{n^*}\right) - .04 \sum_{non-retail} RWA$. If *Adj* is positive (negative), this amount is added to (subtracted from) the sum of non-retail IRB risk-weighted loans to arrive at an adjusted IRB risk weighted asset value to which the 8 percent capital ratio is applied.

References

Arrow, K. J., (1953), “Le rôle des valeurs boursières pour la répartition la meilleure des risques,” *Econométrie,* Paris, Centre National de la Recherche Scientifique, 41-48.

Debreu, G. (1959). *Theory of Value*, Cowles Foundation Monograph 17. New Haven: Yale University Press.

Bank for International Settlements, 1988, *International Convergence of Capital Measurement and Capital Standards,* Basel Committee on Banking Supervision, Basel.

———, 1999b, *A New Capital Adequacy Framework*, Basel Committee on Banking Supervision, Basel.

———, 2001, *The New Basel Accord*, Basel Committee on Banking Supervision, Basel.

Harrison, J. and S. Pliska, (1981). “Martingales and stochastic integrals in the theory of continuous trading,” *Stochastic Process and Their Applications*, Vol. 11, pp. 215-260.

Standard & Poor’s, 2001, “Ratings Performance 2000: Default Transition, Recovery, and Spreads,” (January).

	$E(X^S)-\sum_{\forall i\in D}\left(\frac{y_i}{pr_i}\right)X_i^S=-20$		$E(X^S)-\sum_{\forall i\in D}\left(\frac{y_i}{pr_i}\right)X_i^S=-10$		$E(X^S)-\sum_{\forall i\in D}\left(\frac{y_i}{pr_i}\right)X_i^S=0$		$E(X^S)-\sum_{\forall i\in D}\left(\frac{y_i}{pr_i}\right)X_i^S=10$		$E(X^S)-\sum_{\forall i\in D}\left(\frac{y_i}{pr_i}\right)X_i^S=20$	
LGD_i	pr_i^* percent	$INS(pr_i^*, LGD_i)$	pr_i^* percent	$INS(pr_i^*, LGD_i)$	pr_i^* percent	$INS(pr_i^*, LGD_i)$	pr_i^* percent	$INS(pr_i^*, LGD_i)$	pr_i^* percent	$INS(pr_i^*, LGD_i)$
0.1	> 60	-	19.56	.11	10.87	0.11	7.52	0.11	5.75	0.11
0.2	> 60	-	58.69	2.05	32.61	2.05	22.58	2.05	17.26	2.05
0.3	> 60	-	> 60	-	39.86	4.59	27.59	4.59	21.10	4.59
0.4	> 60	-	> 60	-	43.47	7.29	30.10	7.29	23.02	7.29
0.5	> 60	-	> 60	-	45.65	10.04	31.61	10.04	24.17	10.04
0.6	> 60	-	> 60	-	47.10	12.83	32.61	12.83	24.94	12.83
0.7	> 60	-	> 60	-	48.13	15.63	33.32	15.63	25.48	15.63
0.8	> 60	-	> 60	-	48.91	18.45	33.86	18.45	25.90	18.45
0.9	> 60	-	> 60	-	49.52	21.27	34.28	21.27	26.21	21.27
1.0	> 60	-	> 60	-	50.00	24.10	34.62	24.10	26.47	24.10

Table 4: Optimal physical probability of default and insurance values under the Accord. The entries correspond to optimal loan default probabilities and a corresponding insurance value conditional on a loss given default value and an assumption about the conditional expected output per capita in default states. The calculations are based upon the assumptions $a = 45$, $\Gamma = 1$, $E(X^Q) = 30$, $Var(X^Q) = 60.5$, and $r_f = .05$.

Standardized approach capital requirement (percent)	LGD	Approximate average S&P physical default probability (percent)	Optimal risk neutral default probability (percent)
1.6	0.5	0.03	49.19
4.0	0.5	0.05	47.97
8.0	0.5	0.50	45.65
12.0	0.5	10.00	43.18

Table 5: Insurance value-maximizing risk neutral probabilities of default for the Standardized approach risk buckets under representative loss given default and physical probabilities of default for each risk weight category.

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	$E(X^S)-\sum_{\forall i\in D}\left(\frac{y_i}{pr_i}\right)X_i^S=-20$		$E(X^S)-\sum_{\forall i\in D}\left(\frac{y_i}{pr_i}\right)X_i^S=-10$		$E(X^S)-\sum_{\forall i\in D}\left(\frac{y_i}{pr_i}\right)X_i^S=0$		$E(X^S)-\sum_{\forall i\in D}\left(\frac{y_i}{pr_i}\right)X_i^S=10$		$E(X^S)-\sum_{\forall i\in D}\left(\frac{y_i}{pr_i}\right)X_i^S=20$	
LGD_i	pr_i^* percent	$INS(pr_i^*, LGD_i)$	pr_i^* percent	$INS(pr_i^*, LGD_i)$	pr_i^* percent	$INS(pr_i^*, LGD_i)$	pr_i^* percent	$INS(pr_i^*, LGD_i)$	pr_i^* percent	$INS(pr_i^*, LGD_i)$
0.1	0.46	0.005	0.46	0.01	0.45	0.02	0.45	0.03	0.45	0.035
0.2	1.39	0.01	1.37	0.06	1.34	0.11	1.30	0.16	1.30	0.20
0.3	2.74	0.04	2.66	0.18	2.58	0.31	2.50	0.43	2.43	0.55
0.4	4.58	0.08	4.36	0.38	4.16	0.65	3.97	0.90	3.81	1.14
0.5	7.06	0.14	6.56	0.68	6.13	1.16	5.76	1.60	5.43	1.99
0.6	10.39	0.24	9.39	1.13	8.57	1.89	7.88	2.55	7.29	3.14
0.7	15.08	0.39	13.11	1.76	11.60	2.88	10.41	3.82	9.44	4.62
0.8	22.41	0.60	18.31	2.62	15.52	4.18	13.49	5.44	11.92	6.47
0.9	40.26	0.94	26.60	3.83	20.92	5.91	17.36	7.49	14.87	8.47
1.0	> 60	-	53.37	5.75	29.46	8.24	22.54	10.08	18.46	11.50

Table 6: Optimal physical probability of default and insurance values under the proposed Foundation IRB approach. The entries correspond to optimal loan default probabilities and a corresponding insurance value conditional on a loss given default value and an assumption about the conditional expected output per capita in default states. The calculations are based upon the assumptions $a = 45$, $\Gamma = 1$, $E(X^Q) = 30, Var(X^Q) = 60.5,$ and $r_f = .05.$

	$E(X^S)-\sum_{\forall i\in D}\left(\frac{y_i}{pr_i}\right)X_i^S=-20$		$E(X^S)-\sum_{\forall i\in D}\left(\frac{y_i}{pr_i}\right)X_i^S=-10$		$E(X^S)-\sum_{\forall i\in D}\left(\frac{y_i}{pr_i}\right)X_i^S=0$		$E(X^S)-\sum_{\forall i\in D}\left(\frac{y_i}{pr_i}\right)X_i^S=10$		$E(X^S)-\sum_{\forall i\in D}\left(\frac{y_i}{pr_i}\right)X_i^S=20$	
LGD_i	pr_i^* percent	$INS(pr_i^*, LGD_i)$	pr_i^* percent	$INS(pr_i^*, LGD_i)$	pr_i^* percent	$INS(pr_i^*, LGD_i)$	pr_i^* percent	$INS(pr_i^*, LGD_i)$	pr_i^* percent	$INS(pr_i^*, LGD_i)$
0.1	6.98	0.03	6.29	0.13	5.74	0.22	5.30	0.30	4.94	0.38
0.2	7.00	0.06	6.35	0.27	5.83	0.45	5.40	0.62	5.04	0.76
0.3	7.02	0.09	6.42	0.40	5.93	0.68	5.51	0.93	5.16	1.16
0.4	7.03	0.12	6.49	0.54	6.03	0.92	5.63	1.26	5.29	1.57
0.5	7.06	0.14	6.56	0.68	6.13	1.16	5.76	1.60	5.43	1.99
0.6	7.07	0.17	6.63	0.82	6.25	1.41	5.90	1.94	5.58	2.43
0.7	7.09	0.20	6.72	0.97	6.37	1.66	6.05	2.30	5.75	2.88
0.8	7.11	0.23	6.80	1.11	6.50	1.92	6.22	2.67	5.95	3.35
0.9	7.13	0.26	6.89	1.26	6.64	2.19	6.40	3.05	6.17	3.84
1.0	7.15	0.29	6.98	1.41	6.80	2.46	6.62	3.44	6.43	4.36

Table 7: Optimal physical probability of default and insurance values under the proposed Advanced IRB approach. The entries correspond to optimal loan default probabilities and a corresponding insurance value conditional on a loss given default value and an assumption about the conditional expected output per capita in default states. The calculations are based upon the assumptions $a = 45$, $\Gamma = 1$, $E(X^S) = 30, Var(X^S) = 60.5,$ and $r_f = .05$.