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23 July 2003

Basel Committee on Banking Supervision Bank for International Settlements CH-4002, Basel Switzerland

Dear Sirs

Comments on Third Consultative Document relating to the New Capital Accord

The Loan Market Association (LMA) was formed in 1996 and is now commonly regarded as the authoritative voice of the syndicated loan market in Europe. Its initial aim was to facilitate the constructive development of the secondary market for loans and, having achieved this aim, the LMA has expanded its area of activities to embrace all aspects of the primary and secondary loan markets, including addressing the key issues relating to active loan portfolio management.

We welcome the opportunity for a further chance to comment on the Committee's proposals for a New Capital Accord.

Our members are, in general terms, supportive of the New Capital Accord. The move towards a more risk sensitive approach to the calculation of capital requirements combined with a greater acceptance of a wider range of risk mitigation methodologies, including the eligibility of a much wider range of instruments as eligible collateral against credit risk in the banking book, are welcome developments to the syndicated loan market. Equally welcome is the move away from a "one size fits all" model of regulation of capital requirements against credit risk towards a model of regulation, in the case of more sophisticated banks, based on the internal ratings that the banks increasingly use for their own internal management of risk and allocation of capital to different business units.

As well as producing a regulatory capital regime that more closely tracks the true level of risk to the business, we also believe that the New Accord should incentivise active management by banks of their credit portfolios. The provisions on risk management techniques are generally helpful in that regard - particularly the recognition of a wider range of assets as eligible collateral. These steps also help to equate the treatment of different classes of assets which have a similar economic risk profile. For example, the eligibility of a greater range of collateral to cover credit risk in the banking book as proposed under the New Accord, will mean that there will be less incentive to treat secured lending activities as, for example, reverse repo transactions and thereby to arbitrage the differences in capital treatment between those structures where the assets "repo'd" do not constitute eligible collateral. Under the current Accord, due to the unrealistically narrow range of collateral that is eligible to be treated as collateral against credit risk, there is every incentive to structure collateralised loans in a form - such as a repo - where more generous credit is given for the collateral or collateral equivalent.

We do, however, continue to have concerns about several of the elements of the New Accord which, we consider, if implemented in their current form, could lead to disincentivisation of active management of credit portfolios.

On some of these issues we have already made representation to the Committee in respect of previous consultations on the proposals for the New Accord. Whilst we do not intend, in response to the current consultation, to repeat in detail the arguments that we have previously presented on these issues, nevertheless, we consider it appropriate at least to make clear where we still consider that issues remain to be dealt with.

Restructuring as a credit event in a credit derivative - We note the proposal in the current consultation that one of the credit events required for a credit derivative to be recognised as effective to reduce credit risk is restructuring - the exception to this being where the bank "...has complete control over the decision of whether or not there will be a restructuring of the underlying obligation" (see para 161). We also note that a footnote to this provision (footnote 47) states that the Committee will continue to consider how to deal with credit derivatives where restructuring is not included as a credit event. One of the approaches suggested in the footnote is to apply a discount to the capital relief available in respect of such credit derivatives.

We note that there are differing views on this issue across the industry. We would encourage the Committee to continue its work in this area, as suggested in the consultation paper. In doing so, the Committee should continue to canvass views and consult widely on this issue and to set the policy on this issue informed by such views.

Failure to take account of the impact of imperfect default correlation in relation to guaranteed obligations and credit derivatives. This is an issue on which we made representation under cover of the letter to you dated 29 May 2002. A copy of this letter and the detailed submission are enclosed. We note that, despite this and other representations from various industry bodies setting out detailed arguments on this issue, the current consultation still does not permit banks to incorporate the possible benefits of imperfect correlation between the credit risk of a borrower and that of a guarantor or protection provider when calculating the risk weighting for a fully guaranteed / covered credit exposure. Credit mitigation lies at the core of effective portfolio management and, in our view, the current proposal of a substitution approach is overly conservative and should be replaced by a more risk-sensitive basis for assessing capital requirements for hedged credit positions. We continue to view this as a serious omission from the proposals and would urge the Committee to consider this issue and to develop appropriate proposals, preferably in time for implementation of the New Accord at the end of 2006. Failure to do so would, in our view,

conflict with the overall, and laudable, aim of incentivising banks in the application of prudent risk management practices in combination with risk sensitive analysis techniques.

Treatment of securitisations - It is our view that securitisations represent an important means of risk diversification for the banking sector - representing a key tool to create liquidity from, and spread risk across, large pools of otherwise illiquid assets. We therefore consider that any regulatory measure which disincentivises securitisations is prima facie inappropriate.

Together with many other banking and finance industry representative bodies, we are concerned that the proposals for the New Accord, which have their fullest exposition to date in the current consultation, will have the effect of disincentivising some forms of securitisations - thereby potentially leading to less rather than more risk spreading and diversification across the financial sector.

We are aware that other trade bodies are presenting detailed expositions of their concerns in this area. In particular, the ESF are in the process, with others, of producing a detailed response on the proposals relating to securitisations. We do not propose to repeat arguments made by these organisations which specialise in dealing with issues relating to securitisation products. We would, however, make the following general comments on the current proposals:

The proposal for IRB banks which originate securitisations to deduct automatically positions below KIRB will act as a disincentive. This approach fails to recognise that one of the key beneficial effects of a securitisation is that it distributes the risk of a portfolio of assets that are otherwise concentrated with one institution - deduction of positions below KIRB fails to give credit for this. Also, first loss pieces of securitisations are structured so as to cover volatility of loss over the life of the securitisation - KIRB is only calculated to a 1 year time horizon. Consequently, it is our view that deduction of KIRB results in an inappropriately punitive treatment. We would propose that, instead of the automatic deduction of positions below KIRB, positions below KIRB which are rated BB- or better should be treated in accordance with the RBA matrix.

We continue to be concerned by some of the disparities between RBA weights applied to IRB banks and those applicable to Standardised banks - as well as by the huge disparities in relation to lower rated tranches between the treatment of securitisations and treatments of similarly rated corporate exposures. We are not persuaded that the risk profiles between assets that the rating agencies have rated as representing a similar level of risk are as disparate as the Committee's proposals suggest.

We are also aware that a number of trade associations are submitting comments in relation to the floor applied to the capital requirements attributed to super senior positions in securitisations. We understand that the industry has done quantitative assessment of historic losses in relation to these positions which show that the proposals in CP 3 lead to significantly greater capital requirements than would be justified by the historic PD and LGD figures. We would therefore support comments of the industry to the effect that these requirements should be reduced in the final Accord - both for originator and investor banks.

Inclusion of loans in the trading book - On 1 March 2000, we submitted a paper to the Secretary General of the Committee, under cover of a letter from our Chairman, Tim Ritchie,

setting out our view, in response to the first consultation paper in relation to the New Accord, that the Committee should take the opportunity of the amendment to the New Accord to make clear that the trading book includes positions in actively traded loans.

We note with approval that the proposed definition of the trading book contained in Part 2, paragraph VI (A) of the current consultation does not discriminate against any particular types of instruments - but allows the inclusion of any financial instrument, irrespective of its form, so long as, inter alia, it is actively managed as a trading position and it constitutes a financial asset of one entity and a financial liability of another entity. We also note that to be eligible for inclusion in the trading book, a financial instrument must either be free of any restrictive covenants on its tradability or be able to be hedged completely.

Our concern here is to ensure that, in implementing the New Accord, regulators take a consistent and non-restrictive attitude to these definitional provisions in a way which does not discriminate against particular types or classes of instrument. In particular, we are concerned to ensure that loans which are actively traded and managed as traded positions are generally accepted by regulators as eligible for inclusion in the trading book. We would hope, and anticipate, that, for example, where a loan is able to be subject to a hedging arrangement providing protection against loss in respect of the principal value of the loan that this will be seen as a position that is "...able to be hedged completely". We would hope that the Committee will use its influence to encourage regulators to implement and interpret these provisions in a pragmatic and consistent manner - allowing the inclusion of instruments such as loans where they clearly fulfil the spirit of the provisions in the New Accord.

We hope that you find these relatively brief comments of assistance and would be pleased to discuss any of the issues raised above with you further in the context of your final deliberations on the New Accord.

Yours faithfully.

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Clare Dawson Executive Director

29 May 2002

The Basel Committee on Banking Supervision C/o Bank for International Settlements Postfach CH-4002 Basel Switzerland

Dear Sirs,

Re Loan Market Association Submission to The Basel Committee on Banking Supervision

The Loan Market Association (LMA) would like to take the opportunity to comment further on the New Basel Capital Accord proposals. Our submission relates specifically to the incorporation of the correlation between borrower and guarantor in the derivation of credit risk weightings as applied to fully guaranteed credit exposures.

It is widely recognised by credit market participants that, for fully guaranteed credit exposures, except where there is a very close relationship between borrower and guarantor, the joint default probability will be less than default risk of either the individual borrower or guarantor. Nevertheless, the Accord, as it currently stands, does not allow for this. We consider this a serious omission and accordingly, we submit the attached paper which proposes a simple formula which could be used by banks for the calculation of the joint default probability, irrespective of their quantitative sophistication. The paper includes rigorous mathematical proof and also comments on the inequity of only permitting a degree of capital relief for corporate guarantors with a rating of A or better.

We thank you for this opportunity to submit the paper and hope that you will find our comments helpful and constructive. If you have any queries regarding our response, please feel free to contact me.

Yours faithfully.

Clare Dawson Executive Director

Loan Market Association Submission to the Basel Committee on the Banking Supervision's Consultative Document on the New Basel Capital Accord

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Executive Summary

The current proposals, as set out in the Consultative Document issued in January 2001, do not permit banks to incorporate the possible benefits of imperfect correlation between borrower and guarantor when calculating the risk weighting for a fully guaranteed credit exposure. The Loan Market Association ("the LMA") views this as a serious omission in that it removes an important loan portfolio management factor. One of the vital aspects of the update to the Accord is recognition of the need to incentivise banks in the application of prudent risk management practices in combination with risk sensitive analytical techniques, and non-recognition of correlation appears to conflict with this aim.

The attached paper sets out a proposal which clearly demonstrates the importance of including correlation effects when deriving the appropriate risk weightings for guaranteed credit positions. It also provides a simple formula which can be used to derive joint default probability, along with rigorous mathematical proof, and, if the proposal to adopt a fixed asset correlation were to be accepted, it would be possible for all banks, irrespective of the level of their quantitative sophistication, to use the formula by applying internally derived default rates or using default rates provided by rating agencies or by regulators.

An outline summary of the quantitative methodology is set out in the main body of the paper with the comparative results and detailed mathematical steps being provided in the Appendices.

Introduction

Further to the LMA's submission to the Committee in June 2001, the LMA has consulted further with its members on the specific issue of incorporating asset correlation into the calculation of risk weightings relating to guaranteed obligations. This is considered a critically important aspect, and the LMA has decided to make a further submission, concentrating on this, and directly related issues.

In considering the need for this further response, the LMA fully recognises the difficulties of agreeing a common methodology which is both simple and prudent. Nevertheless, the LMA takes the view that non-recognition of asset correlation is a major disincentive for banks in the development of their portfolio management methods, and the regulatory treatment of joint default risk is of crucial importance to its members. Correlation effects are widely recognised among banks, credit system providers, rating agencies and academics and the LMA believes it is vital that the methodology for calculating risk weightings incorporates the dynamics of asset correlation and joint default probabilities over economic and business cycles.

Accordingly, the LMA has decided to put forward three, complementary, proposals which it feels should become applicable to the Foundation and Advanced IRB approaches, and recommends that all three should be adopted.

Proposal 1

Where a credit risk has been fully guaranteed by an independent guarantor, it should be recognised that the probability of simultaneous default by both obligor and guarantor is less, and in most instances considerably less, than the probability of individual default by either obligor or guarantor. Accordingly, it is proposed that a basic equation should be adopted for deriving joint default probabilities which accommodates the benefits of introducing credit mitigation and which can be used by a wide range of banks, thus incentivising them to extend the use of credit mitigation techniques.

Proposal 2

It is recognised that when there is a close relationship between the obligor and guarantor (e.g. legal ownership), correlation tends towards 100%, and direct substitution would be the appropriate approach with regard to guaranteed exposures. However, in situations where there is no close direct relationship between the obligor and the guarantor, it is proposed to use a fixed correlation of 50% in the formula proposed for the derivation of the joint default probability for fully guaranteed positions.

Proposal 3

For corporate guarantors, the Accord only permits any degree of capital relief when the guarantor is rated A or better. Quantitative investigation suggests that relief should be granted irrespective of the guarantor's rating.

Discussion of Quantitative Methods

There are a number of different methods widely used in the financial markets to calculate default probabilities. The one selected for this quantitative study is the so-called "Merton approach" which utilises an option basis for the calculation with the basic premise that a company will default when its market asset value falls below the face value of its obligations. The approach utilises option pricing theory whereby both the market value of the assets and the volatility of this asset value can be derived i.e. asset value variance can be seen as a function of market value variance.

In turn, it can be demonstrated that changes in market value are partly driven by overall market factors and partly by internal factors, such as management. The extent to which the price volatility depends on overall market factors can be mathematically derived, and is referred to as the *systematic portion*, R^2 . Research conducted by KMV demonstrates that, for all reasonable practical applications, R^2 varies between 10% and 65%. It should be recognised that the above is expressed in an oversimplified way in that R^2 values will depend on a number of variable factors such as country, industry etc. However, even after allowing for multiple and variable factors in the simulations, the R^2 range effectively remains at 10% to 65%.

The introduction of a guarantor requires consideration of the probability of joint default. What is the probability of the borrower and the guarantor (or provider of credit default protection) both defaulting simultaneously? It is too simple to express this joint default probability solely in terms of their respective default probabilities and the reason for this is that there is likely to be *correlation* between the borrower and the guarantor. If the borrower and guarantor are closely related e.g. if there is a legal relationship between them, then the correlation between their respective default probabilities will tend towards 100%. However, what is much more likely, and actually observed in the markets, is that there will be some degree of positive correlation between the two. It is widely recognised that the mathematical relationship between the two default probabilities can be expressed as:

$$PD^* = BN[N^{1}(PD_B), N^{1}(PD_G), AC]$$

where

PD*	= the joint default probability
BN	= the bivariate normal distribution function
N^{-1}	= the inverse normal distribution function
PD _B	= the Borrower's default probability
PD_G	= the Guarantor's default probability
AC	= the asset correlation

The equation is proposed as a good approximation for both borrower and guarantor defaulting at the same time.

In order to derive the correlation between two assets, it is necessary to compare their R^2 values after taking into account all the relevant factors. As stated previously, the R^2 range is 10% to 65%. By using standard covariance equations and inserting the upper range limit of 65%,

assuming the variances are both independent and identically distributed, the worst-case correlation is equal to 65%.

$$AC_{jk} = \sqrt{R_j^2} \cdot R_k^2$$
 (standard covariance equation)

In the worst-case scenario, both R^2 are equal to 0.65

Therefore,

$$AC_{jk} = \sqrt{0.65 \cdot 0.65} = 0.65$$

The initial consultative document introduced the so-called w-factor into the formula for deriving the risk weightings for fully guaranteed exposures. This was subsequently withdrawn and, as yet, no replacement formula has been proposed. On the basis of the statement made in the original consultative document whereby, "As a general rule, no claim on which the credit protection has been purchased should receive a higher capital requirement than an otherwise identical claim on which there is no credit protection" (Pillar 1: Section II: B4: Page 25) and the withdrawal of the w-factor, for the purposes of this discussion paper, the implied methodology assumed is direct substitution i.e. the guarantor's PD, if lower than the borrower's PD, will be applied to the guaranteed exposure. If the borrower's PD is the lower of the two, the borrower's PD will be retained for calculation purposes. In terms of the original formula, this is mathematically equivalent to w = zero, and possible correlation benefits are ignored by assuming 100% correlation.

It is recognised that this assumed methodology for derivation of the risk weightings is simple and, therefore, possibly, there is little meaningful difference in the derived weightings using direct substitution and the option approach. This was investigated and the resulting matrices are set out in Appendix 1.

Notes:

1. The option pricing matrices have been compiled using the most recent S&P published default probabilities adjusted to accommodate the proposed AAA/AA default floor of 0.03%.

2. Tables 2, 4, 6 and 8 are all identical and have been repeated to make comparisons with other tables easier.

Comparison of the matrices, which have been compiled to illustrate different levels of borrower/guarantor correlation, shows that the weightings derived using the direct substitution methodology are significantly higher than those derived using the alternative option based method.

The results show that, at the higher ratings level, the difference reflects incorporation of the AAA/AA default floor. However, at lower ratings levels, the differences between the results become significant, as illustrated by the sample results table below.

Borrower	Guarantor	Correlation	Option approach	Substitution approach
AA	AAA	Very high	0.00	0.03
		High	0.00	0.03
		Medium	0.00	0.03
		Low	0.00	0.03
BB	BBB	Very high	0.10	0.27
		High	0.06	0.27
		Medium	0.03	0.27
		Low	0.01	0.27
В	BB	Very high	0.76	1.29
		High	0.52	1.29
		Medium	0.34	1.29
		Low	0.14	1.29

This comparison strongly supports the need to incorporate correlation into the methodology for calculating risk weightings for fully guaranteed positions if prudent risk management practices are to be incentivised.

A further table of sample results was extracted from Appendix 1 (see next page) which illustrates the impact of introducing a guarantor, and comparing the option approach methodology with the substitution method. The samples have been selected from the Very High correlation and Low correlation calculations, and they clearly indicate that introducing guarantors with ratings lower than A result in the joint default probability being equal to, or lower than, the default probability assigned to the single borrower. This statement applies equally to all the calculations listed in Appendix 1 and clearly demonstrates the flaw in the proposal to restrict capital relief with regard to corporate guarantors to those with ratings of A and higher.

Borrower	Guarantor	Substitutio	Joint	Borrower	Guarantor	Substitutio	Joint
Rating	Rating	n	PD	Rating	Rating	n	PD
		PD				PD	
	AAA	0.03%	0.00%		AAA	0.03%	0.00%
AA	AA	0.03%	0.00%		AA	0.03%	0.00%
	А	0.03%	0.00%	А	А	0.05%	0.01%
	BBB	0.03%	0.01%		BBB	0.05%	0.01%
	BB	0.03%	0.02%		BB	0.05%	0.03%
	В	0.03%	0.03%		В	0.05%	0.04%
	С	0.03%	0.03%		С	0.05%	0.05%

Very High Correlation (AC = 65%)

Borrower	Guarantor	Substitutio	Joint	Borrower	Guarantor	Substitutio	Joint
Rating	Rating	n	PD	Rating	Rating	n	PD
		PD				PD	
	AAA	0.03%	0.01%		AAA	0.03%	0.02%
	AA	0.03%	0.01%		AA	0.03%	0.02%
	А	0.05%	0.01%		А	0.05%	0.03%
BBB	BBB	0.27%	0.04%		BBB	0.27%	0.10%
	BB	0.27%	0.10%	BB	BB	1.29%	0.31%
	В	0.27%	0.20%		В	1.29%	0.76%
	С	0.27%	0.26%		С	1.29%	1.19%

Low Correlation (AC = 10%)

Borrower	Guarantor	Substitutio	Joint	Borrower	Guarantor	Substitutio	Joint
Rating	Rating	n	PD	Rating	Rating	n	PD
		PD				PD	
	AAA	0.03%	0.00%		AAA	0.03%	0.00%
AA	AA	0.03%	0.00%		AA	0.03%	0.00%
	А	0.03%	0.00%	А	А	0.05%	0.00%
	BBB	0.03%	0.00%		BBB	0.05%	0.00%
	BB	0.03%	0.00%		BB	0.05%	0.00%
	В	0.03%	0.00%		В	0.05%	0.01
	С	0.03%	0.01%		С	0.05%	0.02%

Borrower	Guarantor	Substitutio	Joint	Borrower	Guarantor	Substitutio	Joint
Rating	Rating	n	PD	Rating	Rating	n	PD
		PD				PD	
	AAA	0.03%	0.00%		AAA	0.03%	0.00%
	AA	0.03%	0.00%		AA	0.03%	0.00%
	А	0.05%	0.00%		А	0.05%	0.00%
BBB	BBB	0.27%	0.00%		BBB	0.27%	0.01%
	BB	0.27%	0.01%	BB	BB	1.29%	0.03%
	В	0.27%	0.03%		В	1.29%	0.14%

	С	0.27%	0.11%	С	1.29%	0.49%
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<u>Summary</u>

The application of the option approach method clearly enables good approximations of joint default probabilities to be derived, whereas the direct substitution method fails to incorporate correlation. This is considered a fundamental flaw and therefore, adoption of the proposed equation for the calculation of the appropriate joint default probabilities is recommended, namely:

 $PD^* = BN[N^{-1}(PD_B), N^{-1}(PD_G), AC]$

where

PD*	= the joint default probability
BN	= the bivariate normal distribution function
N^{-1}	= the inverse normal distribution function
PD_B	= the Borrower's default probability
PD_G	= the Guarantor's default probability
AC	= the asset correlation

The LMA considers it critically important that the applicable methodology captures the true dynamics of default probabilities across all stages of credit cycles, which is adequately accommodated by the proposed change. The benefits are clear, but, in order to establish a basic formula which will eliminate the requirement for banks to have the ability to apply relatively complex mathematical techniques, it is proposed that the asset correlation should be fixed at 50%. This retains simplicity and the proposed figure is close to the upper level of the derived asset correlation range.

In situations where there is a close economic relationship between borrower and guarantor (e.g. legal ownership), it is proposed that direct substitution would be the appropriate method rather than using the above formula.

In recognising the benefits of asset correlation, it is inconsistent, given the overall objectives of the Basel Committee, to disallow relief when corporate guarantors are rated lower than A.

Appendix 1

			Guaranto	r					
	S&P	Rating	AAA	AA	A	BBB	BB	В	С
		PD %	0.03%	0.03%	0.05%	0.27%	1.29%	6.71%	28.76%
Borrower	AAA	0.03%	0.00%	0.00%	0.00%	0.01%	0.02%	0.03%	0.03%
	AA	0.03%	0.00%	0.00%	0.00%	0.01%	0.02%	0.03%	0.03%
	A	0.05%	0.00%	0.00%	0.01%	0.01%	0.03%	0.04%	0.05%
	BBB	0.27%	0.01%	0.01%	0.01%	0.04%	0.10%	0.20%	0.26%
	BB	1.29%	0.02%	0.02%	0.03%	0.10%	0.31%	0.76%	1.19%
	В	6.71%	0.03%	0.03%	0.04%	0.20%	0.76%	2.55%	5.43%
	С	28.76%	0.03%	0.03%	0.05%	0.26%	1.19%	5.43%	17.14%

Very High Asset Correlation (AC=65%)

 Table 1 Credit risk matrix derived from the option pricing approach (AC=65%)

			Guarantor							
	S&P	Rating	AAA	AA	A	BBB	BB	В	С	
		PD %	0.03%	0.03%	0.05%	0.27%	1.29%	6.71%	28.76%	
Borrower	AAA	0.03%	0.03%	0.03%	0.03%	0.03%	0.03%	0.03%	0.03%	
	AA	0.03%	0.03%	0.03%	0.03%	0.03%	0.03%	0.03%	0.03%	
	A	0.05%	0.03%	0.03%	0.05%	0.05%	0.05%	0.05%	0.05%	
	BBB	0.27%	0.03%	0.03%	0.05%	0.27%	0.27%	0.27%	0.27%	
	BB	1.29%	0.03%	0.03%	0.05%	0.27%	1.29%	1.29%	1.29%	
	В	6.71%	0.03%	0.03%	0.05%	0.27%	1.29%	6.71%	6.71%	
	С	28.76%	0.03%	0.03%	0.05%	0.27%	1.29%	6.71%	28.76%	

Table 2 Credit risk matrix based on the direct substitution approach (which is equivalent to w = zero) and assuming that no guaranteed claim can attract a higher capital requirement than an otherwise identical claim which is not guaranteed.

High Asset Correlation (AC = 50%)

			Guarantor								
	S&P	Rating	AAA	AA	A	BBB	BB	В	С		
		PD %	0.03%	0.03%	0.05%	0.27%	1.29%	6.71%	28.76%		
Borrower	AAA	0.03%	0.00%	0.00%	0.00%	0.00%	0.01%	0.02%	0.03%		
	AA	0.03%	0.00%	0.00%	0.00%	0.00%	0.01%	0.02%	0.03%		
	А	0.05%	0.00%	0.00%	0.00%	0.01%	0.02%	0.03%	0.05%		
	BBB	0.27%	0.00%	0.00%	0.01%	0.02%	0.06%	0.14%	0.23%		
	BB	1.29%	0.01%	0.01%	0.02%	0.06%	0.18%	0.52%	1.03%		
	В	6.71%	0.02%	0.02%	0.03%	0.14%	0.52%	1.84%	4.54%		
	С	28.76%	0.03%	0.03%	0.05%	0.23%	1.03%	4.54%	14.74%		

 Table 1 Credit risk matrix derived from the option pricing approach (AC=50%)

			Guarantor								
	S&P Ratin	Rating	AAA	AA	А	BBB	BB	В	С		
		PD %	0.03%	0.03%	0.05%	0.27%	1.29%	6.71%	28.76%		
Borrower	AAA	0.03%	0.03%	0.03%	0.03%	0.03%	0.03%	0.03%	0.03		
	AA	0.03%	0.03%	0.03%	0.03%	0.03%	0.03%	0.03%	0.03%		
	A	0.05%	0.03%	0.03%	0.05%	0.05%	0.05%	0.05%	0.05%		
	BBB	0.27%	0.03%	0.03%	0.05%	0.27%	0.27%	0.27%	0.27%		
	BB	1.29%	0.03%	0.03%	0.05%	0.27%	1.29%	1.29%	1.29%		
	В	6.71%	0.03%	0.03%	0.05%	0.27%	1.29%	6.71%	6.71%		
	С	28.76%	0.03%	0.03%	0.05%	0.27%	1.29%	6.71%	28.76%		

Table 2 Credit risk matrix based on the direct substitution approach (which is equivalent tow = zero) and assuming that no guaranteed claim can attract a higher capital requirement thanan otherwise identical claim which is not guaranteed.

Medium Asset Correlation (AC=35%)

			Guarantors								
	S&P	Rating	AAA	AA	А	BBB	BB	В	С		
		PD %	0.03%	0.03%	0.05%	0.27%	1.29%	6.71%	28.76%		
Borrower	AAA	0.03%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.02%		
	AA	0.03%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.02%		
	A	0.05%	0.00%	0.00%	0.00%	0.00%	0.01%	0.02%	0.04%		
	BBB	0.27%	0.00%	0.00%	0.00%	0.01%	0.03%	0.09%	0.19%		
	BB	1.29%	0.00%	0.00%	0.01%	0.03%	0.10%	0.34%	0.83%		
	В	6.71%	0.01%	0.01%	0.02%	0.09%	0.34%	1.30%	3.68%		
	С	28.76%	0.02%	0.02%	0.04%	0.19%	0.83%	3.68%	12.62%		

Table 5 Credit risk matrix derived from the option pricing approach (AC = 35%)

			Guarantors								
	S&P	P Rating	AAA	AA	A	BBB	BB	В	С		
		PD %	0.03%	0.03%	0.05%	0.27%	1.29%	6.71%	28.76%		
Borrower	AAA	0.03%	0.03%	0.03%	0.03%	0.03%	0.03%	0.03%	0.03		
	AA	0.03%	0.03%	0.03%	0.03%	0.03%	0.03%	0.03%	0.03%		
	A	0.05%	0.03%	0.03%	0.05%	0.05%	0.05%	0.05%	0.05%		
	BBB	0.27%	0.03%	0.03%	0.05%	0.27%	0.27%	0.27%	0.27%		
	BB	1.29%	0.03%	0.03%	0.05%	0.27%	1.29%	1.29%	1.29%		
	В	6.71%	0.03%	0.03%	0.05%	0.27%	1.29%	6.71%	6.71%		
	С	28.76%	0.03%	0.03%	0.05%	0.27%	1.29%	6.71%	28.76%		

Table 6 Credit risk matrix based on the direct substitution approach (equivalent to w = zero) and assuming that no guaranteed claim can attract a higher capital requirement than an otherwise identical claim which is not guaranteed.

Low Asset Correlation (AC =10%)

			Guarantors								
	S&P	Rating	AAA	AA	A	BBB	BB	В	С		
		PD %	0.03%	0.03%	0.05%	0.27%	1.29%	6.71%	28.76%		
Borrower	AAA	0.03%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%		
	AA	0.03%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%		
	A	0.05%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.02%		
	BBB	0.27%	0.00%	0.00%	0.00%	0.00%	0.01%	0.03%	0.11%		
	BB	1.29%	0.00%	0.00%	0.00%	0.01%	0.03%	0.14%	0.49%		
	В	6.71%	0.00%	0.00%	0.01%	0.03%	0.14%	0.64%	2.39%		
	С	28.76%	0.01%	0.01%	0.02%	0.11%	0.49%	2.39%	9.45%		

Table 7 Credit risk matrix derived from the option pricing approach (AC = 10%)

	S&P Rating	Guarantors								
		Rating	AAA	AA	A	BBB	BB	В	С	
		PD %	0.03%	0.03%	0.05%	0.27%	1.29%	6.71%	28.76%	
Borrower	AAA	0.03%	0.03%	0.03%	0.03%	0.03%	0.03%	0.03%	0.03	
	AA	0.03%	0.03%	0.03%	0.03%	0.03%	0.03%	0.03%	0.03%	
	A	0.05%	0.03%	0.03%	0.05%	0.05%	0.05%	0.05%	0.05%	
	BBB	0.27%	0.03%	0.03%	0.05%	0.27%	0.27%	0.27%	0.27%	
	BB	1.29%	0.03%	0.03%	0.05%	0.27%	1.29%	1.29%	1.29%	
	В	6.71%	0.03%	0.03%	0.05%	0.27%	1.29%	6.71%	6.71%	
	С	28.76%	0.03%	0.03%	0.05%	0.27%	1.29%	6.71%	28.76%	

Table 8 Credit risk matrix based on the direct substitution approach (equivalent tow=zero) and assuming that no guaranteed claim can attract a higher capital requirement than anotherwise identical claim which is not guaranteed.

Appendix 2

Option Pricing Approach to the Quantification of Credit Risk

The academic research by Black, Scholes and Merton laid the foundations for the now well established techniques whereby option pricing techniques can be applied in the credit evaluation of a company. If a company has assets with a market value of A, equity of E and debt of D maturing at time T, the company will default if, at time T, A is less than D. Quite simply, the assets have insufficient market value to meet the debt obligations as illustrated in Figure 1.



Figure 1 : Default from a balance sheet perspective

Effectively, if the asset value falls below the value of the debt, the company defaults, with this boundary being referred to as the *default point*. It defines the probability of default and can be expressed in the following equation.

$$PD_{k} = \Pr(Asset_{i} \le Debt_{k}) \tag{1}$$

The equation states that the probability of default of company k, (PD_k) , increases if the distance to the default point falls, and decreases if the distance to the default point rises. This distance is called the *distance to default* and is standardised by the volatility of the asset value and is expressed as



Figure 2 - Measuring the default probability using an option pricing approach (Source: KMV LLC)

Effectively, the equity of the company is a call option on the company's assets with the exercise price and maturity represented by the face value and maturity of the debt. If the company performs poorly, the value of the assets will be insufficient to repay debt. By application of put/call parity theory, the optionality can be restated as, the equity holders own the assets, have borrowed the debt but also own a put option whereby they can sell the assets to the debt holders for the face value of the debt. As a result, the credit analysis can focus on estimating the value of this put option and the probability of it being exercised i.e. the company defaults.

Clearly, neither the current value of the assets, nor its volatility, can be directly observed. However, the movement in the company's stock price can be directly observed, and its volatility derived, and a methodology has been devised whereby the link between stock price volatility and asset value volatility can be quantified.

Note: the methodology outlined in this study relates directly to companies which are publicly quoted, allowing observation of stock price movements. However, it can be applied to non-quoted companies and other borrowers by utilising credit mapping techniques, which enables credit equivalence to be established. In this way, the method can be applied to all borrowers.

The Black-Scholes formula for valuing a put option is:

 $P = Xe^{-rt} N(-d2) - SN(-d1)$

The formula has 5 variables namely the stock price (S), the option strike price (X), the risk free rate (r), the time to expiry (t) and the volatility of the stock price movements (derived from d1 and d2).

In applying the formula to determining the value of the put option owned by the equity holders, these variables will be the value of assets, the value of liabilities, the risk free rate, the time to maturity and the volatility of asset value. The formula can be expressed in a new format which is:

$$Equity_{k} = Asset_{k}N(d_{1}) - Debt_{k}e^{-rT}N(d_{2})$$

where r is the risk-free and constant rate of growth in the risk-free model, N is the standard normal distribution function, d_1 and d_2 are defined as follows:

$$d_{1} = \frac{\ln\left(\frac{Asset}{Debt}\right) + \left(r + \frac{1}{2}\sigma_{k}^{2}\right)T}{\sigma_{k}\sqrt{T}}; \quad d_{2} = d_{1} - \sigma_{k}\sqrt{T}$$

There are 2 unknown factors namely, the market value of the assets and its volatility. However, the stock price movements and related volatility can be observed/derived directly from the markets and these in turn can be mathematically linked to asset value changes and related volatility.

From the stochastic differential equation for the change of the asset value (where W_t denotes the standardised *Brownian motion* or the so-called *Wiener process*) we derive

$$\frac{dAsset_{t_k}}{Asset_{t_k}} = rdt + \sigma_k dW_t$$
(2)

After the application of the Itô formula, equation (2) leads us to the stochastic differential equation for the change of the equity value:

$$\frac{dEquity_{t_k}}{Equity_{t_k}} = rdt + \frac{N(d_1(t))Asset_{t_k}}{Equity_{t_k}}\sigma_k dW_t$$

Now we can recognise the direct relation between the volatility of the equity and the volatility of the asset:

$$\sigma_{k} = \frac{Equity_{k}\sigma_{Equity}}{N(d_{1})Asset_{k}}$$
(3)

Combining equations (1), (2) and (3), we get the equation for the default probability: $\begin{pmatrix} 4 \text{ sset} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$PD_{k} = \Pr(Asset_{T_{k}} \le Debt) = \Pr\left(W_{T} \le -\frac{\ln\left(\frac{Asset_{k}}{Debt_{k}}\right) + \left(r - \frac{1}{2}\sigma_{k}^{2}\right)T}{\sigma_{k}\sqrt{T}}\right) = N(-d_{2})$$

Thus, it can be demonstrated that asset value volatility can be tracked in terms of stock price volatility.

Asset Variance

It is widely accepted in finance theory that stock price movements are partly driven by overall market factors and partly by internal factors, and that this is equally valid for both the volatilities and variances of stock prices. Therefore, asset variance can be seen as a function of market value variance and, by application of linear regression techniques, the degree of dependency on overall market factors can be derived.

The regression function is composed of a systematic portion (driven by overall market factors) and a non-systematic portion (driven by internal factors such as management) and can be expressed in the following formula:

$$\sigma_k^2 = \beta_k^2 \cdot \sigma_{\phi_k}^2 + \varepsilon_k^2 = \text{Asset Variance of k}$$
⁽⁵⁾

where

$$\beta_k^2 \cdot \sigma_{\phi_k}^2$$
 = systematic portion, that is, slope of the linear function multiplied by the market variance

 ε_k = unsystematic portion (firm specific)

The slope, or the beta of company k, is related to R^2 . R^2 not only represents the regression quality, but also the systematic portion denoted in percent:

$$\beta_k^2 = R_k^2 \frac{\sigma_k^2}{\sigma_{\phi_k}^2} \tag{6}$$

From (5) and (6) we get:

$$\sigma_k^2 = R_k^2 \cdot \sigma_k^2 + \varepsilon_k^2$$

The systematic portion is commonly referred to as R^2 and multiple simulations result in R^2 having a range of values between 10% and 65%. It must be recognised that there are multiple factors which affect the systematic portion (e.g. country, industry sector etc.) and these have to be derived. Their values have 3 dominant influences which have different levels of impacts: the asset value or turnover (high impact), the industry (medium impact) and the country (low impact). Consequently, the value of R^2 is primarily determined by the size of the firm, represented by total assets for financial institutions, or revenue for all other types of companies. This, of course, is as expected as the larger the company, the more inter-related it will be with the economy.

As stated above, markets are defined by countries and industries and, consequently, market variance can be expressed by a composition of country and industry variances:

$$\sigma_{\phi_k}^2 = \sum_{c=1}^m w_{kc}^2 \sigma_c^2 + \sum_{i=1}^n w_{ki}^2 \sigma_i^2$$
(7)

where $\sigma^2_{(c,i)}$ = specific country and industry variances

w = weights. For countries or industries they sum up to 100% respectively.

Example:

General Electric is not only active locally in the US but also globally. However, most of its operations are based in America and so, in order to analyse the firm's activities, specific market weights are needed.

If we assume *m* countries with index *c* for *country* and *n* industries with index *i* for *industry*, the equation for asset variance can be derived as:

$$\sigma_k^2 = \beta_k^2 \cdot \left(\sum_{c=1}^m w_{kc}^2 \sigma_c^2 + \sum_{i=1}^n w_{ki}^2 \sigma_i^2\right) + \varepsilon_k^2$$
(8)

Joint Default Between Borrower and Guarantor

Note: the expression "guarantor" is intended to include the provider of credit default protection.

Joint default probability, in terms of this study, is the probability of both borrower and guarantor defaulting simultaneously. It is a function of 3 factors namely, the default probability of the borrower, the default probability of the guarantor and the *asset correlation* between the borrower and guarantor. The impact of the asset correlation on credit risk can be illustrated by the following case study:

Case 1:

A bank lends money to BASF (PD=0.08%) with Heineken (PD=0.04%) as guarantor.

Case 2:

A bank lends money to BASF (PD=0.08%) with Bayer (PD=0.04%) as guarantor.

where PD = probability of default

The question arises whether the resulting credit risk is the same for both cases.

If we only recognise the default probabilities of the borrowers and guarantors, the credit risk would be the same in both examples. However, this does not accurately represent the actual circumstances, and the credit risk, in reality, is higher in Case 2. The reason for this is the degree of interdependency of the companies' asset values. Both Bayer and BASF operate in the same country and industry whereas BASF and Heineken are based in different countries and operate in different industries. If the asset value of BASF falls short of the default point because of a national crisis, it is highly likely that the same thing would happen to Bayer. Industry downturns would have a similar impact. This simple example clarifies the importance of asset correlation in credit evaluation. This example can be illustrated diagrammatically in the following way. Assuming normal distributions for the market value of assets, the stochastic relationship of the default probability of the borrower (PD_B), guarantor (PD_G) and the asset correlation can be illustrated by the following graphs:





The graphs above show that both PDs are random variables which span a *probability distribution mountain*. The circles represent the mountain's contour lines, with the likelihood of joint default increasing towards the centre. Mathematically, this is called a *bivariate normal distribution* (BN), and can be interpreted in the following way.

Given three events:

A. Bayer is performing very well (high level of the asset value) while BASF is performing relatively poorly (low level of the asset value).

B. Bayer and BASF are both performing at the expected level (both firms' asset values are medium).

C. Bayer and BASF both have major problems (both firms' asset values are close to the default level).

From Graph 1 we can form the following conclusions: Events A and C are very unlikely with Event B being most likely. However, if the shape of the contour lines change to the shape shown in Graph 2, we observe that Event A becomes much more unlikely, while Event C's probability rises markedly. However, the probability of Event B doesn't change at all. This demonstrates the important impact of asset correlation on the joint default probability and that the joint default probability is dynamic.

Mathematically the relationship of PD_B , PD_G and the asset correlation (AC) can be expressed as follows:

$$JDP = BN(N^{-1}(PD_B), N^{-1}(PD_B), AC)$$

The JDP in the formula above is computed by means of a bivariate normal distribution function (BN), where the three arguments are the standard normal quantiles.

This equation is widely accepted and represents a good approximation for the probability of both borrower and guarantor defaulting at the same time. Consequently, the LMA recommends that the adoption of the equation for the calculation of joint default probabilities for fully secured positions.

Calculation of Asset Correlation

Asset correlation measures the tendency of two asset values moving simultaneously over time and arriving at the default point coevally. The measurement of this tendency can be achieved by comparing the systematic portions (i.e. R^2) of two companies. If those portions have the same country and industry factor composition, and the companies operate in the same market, then the asset correlation relationship rises. However, it is not only the factor composition which impacts on the asset correlation. The range of the systematic portion, represented by R^2 , is also an important factor. As previously explained, R^2 expresses the dependency on the overall market, and its range lies between 10% and 65%. It follows therefore, that the asset correlation can only range between these two parameters.

In order to illustrate the effect of the extent and composure of the systematic portion, the following table sets out characteristic scenarios for the asset correlation:

Scenario	Industry _j	Country _j	R ² _j	Industry _k	Country _k	\mathbf{R}^{2}_{k}	AC
А	Machinery	Singapore	10%	Software	USA	10%	3%
В	Software	USA	10%	Software	USA	10%	10%
С	Finance	Great Britain	65%	Utilities, Gas	Great Britain	10%	17%
D	Finance	Great Britain	65%	Finance	Great Britain	10%	26%
Е	Publishing	Switzerland	65%	Automotive	Germany	65%	42%
F	Automotive	Germany	65%	Automotive	Germany	65%	65%

 Table 9 Impact of R² and the country/industry composition re the asset correlation

We can observe that the asset correlation never exceeds the highest level in the R^2 range. Therefore, both R^2 , and the asset correlation, have a maximum value of 65%.

The mathematical proof of this is set out below.

The equation for asset correlation is simply derived from the usual correlation equation, that is, the covariance divided by the two volatilities:

$$AC_{jk} = \frac{\sigma_{jk}^2}{\sqrt{\sigma_j^2} \cdot \sqrt{\sigma_k^2}} = \frac{\text{Covariance}_{jk}}{\sqrt{\text{Asset Variance}_j} \cdot \sqrt{\text{Asset Variance}_k}}$$

$$= \frac{\beta_{j}\beta_{k}\left(\sum_{c=1}^{m} w_{jc}w_{kc}\sigma_{c}^{2} + \sum_{i=1}^{n} w_{ji}w_{ki}\sigma_{i}^{2}\right)}{\text{Asset Volatility}_{i} \cdot \text{Asset Volatility}_{k}}$$
(10)

From (5), (8) and (10) we get:

$$AC_{jk} = \sqrt{\frac{R_{j}^{2} \cdot R_{k}^{2}}{\sigma_{\phi_{j}}^{2} \cdot \sigma_{\phi_{k}}^{2}}} \left(\sum_{c=1}^{m} w_{jc} w_{kc} \sigma_{c}^{2} + \sum_{i=1}^{n} w_{ji} w_{ki} \sigma_{i}^{2} \right)$$
$$AC_{jk} = \left[\sqrt{\frac{R_{j}^{2} \cdot R_{k}^{2}}{\left(\sum_{c=1}^{m} w_{jc}^{2} \sigma_{c}^{2} + \sum_{i=1}^{n} w_{ji}^{2} \sigma_{i}^{2}\right)} \cdot \left(\sum_{c=1}^{m} w_{kc}^{2} \sigma_{c}^{2} + \sum_{i=1}^{n} w_{ki}^{2} \sigma_{i}^{2}\right)} \\ \cdot \left(\sum_{c=1}^{m} w_{jc} w_{kc} \sigma_{c}^{2} + \sum_{i=1}^{n} w_{ji} w_{ki} \sigma_{i}^{2}\right)} \right]$$

If we suppose that the variances are identically distributed and independent (i.i.d.) we can derive the value for the worst-case asset correlation from the equation above. As previously stated, the worst-case scenario occurs if the two companies depend on the market to a high degree, and if their country and industry factor composition is the same. Mathematically we write $R^2=65\%$, $w_{ic}=w_{kc}$ and $w_{ji}=w_{ki}$:

$$AC_{jk} = \left[\sqrt{\frac{R_{j}^{2} \cdot R_{k}^{2}}{\left(\sum_{c=1}^{m} w_{kc}^{2} \sigma_{c}^{2} + \sum_{i=1}^{n} w_{ki}^{2} \sigma_{i}^{2}\right) \cdot \left(\sum_{c=1}^{m} w_{kc}^{2} \sigma_{c}^{2} + \sum_{i=1}^{n} w_{ki}^{2} \sigma_{i}^{2}\right)} \\ \cdot \left(\sum_{c=1}^{m} w_{kc} w_{kc} \sigma_{c}^{2} + \sum_{i=1}^{n} w_{ki} w_{ki} \sigma_{i}^{2}\right)} \right]$$
$$AC_{jk} = \sqrt{\frac{R_{j}^{2} \cdot R_{k}^{2}}{\sigma_{\phi_{k}}^{2} \cdot \sigma_{\phi_{k}}^{2}}} \cdot \sigma_{\phi_{k}}^{2} = \sqrt{R_{j}^{2} \cdot R_{k}^{2}}$$
(11)

For $R_j^2 = R_k^2 = R_{Worst-Case}^2 = 65\%$ we get:

$$AC_{jk_{\text{Worst-Case}}} = \sqrt{R_j^2 \cdot R_k^2} = \sqrt{0.65 \cdot 0.65} = 0.65$$

Note: In reality, country and industry volatilities are <u>not</u> i.i.d. but they can be broken down into factors with these properties. If we do so, the same value for the worst-case asset correlation is derived.

Comparison of Current Basel Approach and the Proposed New Approach

As previously stated, direct substitution has been the assumed methodology post w-factor. The tables in Appendix 1 illustrate the marked differences in results when comparing the two methodologies, with many of the results derived using the direct substitution approach being a multiple of the equivalent results derived using the option pricing approach.

These tables clearly emphasise the vital need to incorporate correlation into the calculations.

Conclusion

The investigative study clearly illustrates the vital requirement to incorporate asset correlations into the proposed method for deriving risk weightings. By doing so, the dynamic features of the real credit world are captured. This requirement can be met by replacing the direct substitution methodology for calculating risk weightings with one which incorporates asset correlation into the calculation of the joint default probability, namely:

 $PD^* = BN[N^{-1}(PD_B), N^{-1}(PD_G), AC]$

where

In recognition of the comparative increased complexity of the replacement formula, it is proposed that the asset correlation be fixed at 0.50. This proposed figure is conservatively within the range derived in the simulation exercise and maintains a degree of commonality and simplicity.

The formula would then become

 $PD^* = BN[N^{-1}(PD_B), N^{-1}(PD_G), 0.50]$

In addition, the study demonstrates that the proposal, with regard to corporate guarantors, to grant capital relief only for guaranteed positions where the guarantor is rated A, or better, is inequitable and should be eliminated.

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