# Calculating Credit Exposure and Credit Loss: A Case Study

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We report a study of the estimation of credit exposure and credit loss of a portfolio of derivative transactions. The estimation is performed using a Monte Carlo simulation. The results are compared to the exposure and capital reserves obtained under the method recommended by the Bank for International Settlements (BIS). We show that the simulation method provides a much richer set of information for credit risk managers. Also, depending on the current exposure and the nature of the transactions, the BIS method can fail to account for potential exposure. In addition, depending on the level or prudence one requires in setting capital reserves, the BIS reserves can be either excessive or grossly insufficient.

The dramatic increase in over-the-counter derivative trading during the past ten years has made credit risk a major concern for banks. With increased competition and tighter spreads, banks must accurately quantify the credit risk they are facing so that they can appropriately price their products, set the proper level of capital reserves and manage their credit line efficiently. Currently, different banks employ different credit risk measurement techniques, ranging from applying a fixed percentage to the notionals of the transactions, to creating distributions of future credit exposure and credit loss.

This paper focuses on estimating counterparty credit risk for derivative portfolios. We accomplish three objectives: introduce the Monte Carlo Simulation method used to measure counterparty credit risk; highlight several significant aspects of counterparty exposure and loss profiles that result from Monte Carlo simulation using an example; and contrast these results to the Credit Equivalent Amounts and capital adequacy requirements stipulated by the Basle Accord (BIS 1988).

In the first section we define several credit risk terms used throughout this paper. Next, we outline the BIS and Monte Carlo-based methods used to estimate counterparty credit exposures and losses. We then compare the two methods, underscoring the limitations of the factor-based approach to measure capital and manage portfolios. The results are outlined followed by the conclusions we draw from using these two approaches.

## Credit risk definitions

In this paper, we define **credit risk** as the risk of loss that will be incurred in the event of default by a counterparty.[1] Default occurs if the counterparty fails to honor its contractual payments.

Calculating potential credit losses requires the modeling of three processes that define: (i) the counterparty credit exposure at the time of default; (ii) the probability that default will occur; and (iii) the amount that can be recovered after default occurs.

There are two types of credit risk: pre-settlement risk and settlement risk. Pre-settlement risk refers to losses arising from prematurely terminating a contract due to counterparty default. This loss depends on the replacement value of the contract at the time of default. In contrast, settlement risk refers to the losses realized when the contractual payments are not received on the settlement date. In this paper, we consider only pre-settlement risk.

### Credit exposure

**Credit exposure** is the cost of replacing or hedging the contract at the time of default. This is the maximum value that will be lost if the counterparty to that contract defaults. Since default is an uncertain event that can occur at any time during the life of the contract, we consider not only the contract's current credit exposure, but also potential changes in the exposure during the contract's life. This is particularly important for derivative contracts whose values can change substantially over time and according to the state of the market. For this reason, we introduce three measures of credit exposure: Actual Exposure, Total Exposure and Potential Exposure.

We define the **Actual Exposure** of contract $c$ at time $t$ as the maximum of zero and the value of the contract at that time:

$$AE(c,t) = max\{0, V(c,t)\} \quad (1)$$

where $V(c, t)$ is the value of contract $c$ at time $t$. As defined, $AE(c, t)$ is the maximum amount that will be lost if default occurs and if the contract is replaced at time $t$. If $t$ is the current time, the Actual Exposure then depends on the current marked-to-market value of the transaction. If $t$ is a future point in time, then $AE(c, t)$ is defined with respect to an assumed state of the market at time $t$.

The **Potential Exposure** of the contract at time $t$ is the maximum *additional* amount (over the Actual Exposure) that will be lost if default occurs not at time $t$, but at some time $\tau$, between $t$ and the maturity of the contract, $T$:

$$PE(c,t) = max\{0, max_{t < \tau \leq T}\{PV_t[V(c,\tau)] - V(c,t)\}\} \quad (2)$$

where $PV_t$ [*] is a function that transforms future values to their present values at time $t$. A **market scenario** is the path that the market takes during the period from $t$ to $T$. The Potential Exposure must be defined with respect to an assumed market scenario. This is because the value of the contract at any point in time during that period depends on the values that the underlying market risk factors take. As such, the Potential Exposure takes into account the aging of the portfolio and possible adverse movements in the underlying variables under that market scenario.[2]

The **Total Exposure** of the contract at time $t$ is the sum of the Actual Exposure and the Potential Exposure at that time:

$$TE(c,t) = AE(c,t) + PE(c,t) \quad (3)$$

Therefore, it is the maximum value that will be lost on the contract if the counterparty defaults at any point from time $t$ to the end of the contract's life, under an assumed market scenario.

As an example, consider a pay-fix/receive-float interest-rate swap whose current marked-to-market value is negative. In this case, the contract's current Actual Exposure is zero. However, if interest rates rise in the future, the contract can have a positive value. Although the contract has no current credit exposure, it carries potential additional exposure that presents credit risk to the holder. The Total Exposure captures this future positive value.

At the counterparty level, credit exposure depends on the netting arrangement and other credit mitigation provisions. For example, if full netting is allowed, contracts with positive values can be offset by contracts with negative values to reduce the net exposure. In this case, the Actual Exposure to counterparty $P$ at time $t$ is the maximum of zero and the sum of the values of all contracts with that counterparty:

$$AE(P,t) = max\left\{0, \sum_{c \in P} V(c,t)\right\} = max\{0, V(P,t)\} \quad (4)$$

The Potential Exposure to a counterparty under an assumed market scenario is the maximum additional exposure at some future time. It can be obtained by substituting the portfolio's value, $V(P,t)$, for the contract's value, $V(c,t)$, in Equation 2. The Total Exposure is the sum of the counterparty's Actual Exposure and its Potential Exposure.

On the other hand, if netting is not allowed, the Actual Exposure to a counterparty is simply the sum of the values of all positive-valued contracts with that counterparty. In other words, it is the sum of each contract's Actual Exposure:

$$AE(P,t) = \sum_{c \in P} max\{0, V(c,t)\} = \sum_{c \in P} AE(c,t) \qquad (5)$$

The Potential Exposure in this case is conservatively defined as the sum of each contract's Potential Exposure at that time:

$$PE(P,t) = \sum_{c \in P} PE(c,t) \qquad (6)$$

Note from Equation 2 that the Potential Exposure of a contract depends on the maximum value the contract will take during its life, and that different contracts may reach their maximum values at different points in time. Therefore, by defining the Potential Exposure to the counterparty in this manner, we allow for the conservative possibility that in the absence of a netting agreement, the counterparty may cherry-pick the timing of default of each contract.

Finally, from Equations 3, 5 and 6, the Total Exposure to the counterparty in the absence of a netting agreement is simply the sum of each contract's Total Exposure:

$$TE(P,t) = \sum_{c \in P} AE(c,t) + \sum_{c \in P} PE(c,t) = \sum_{c \in P} TE(c,t) \qquad (7)$$

### Credit losses

While credit exposure is the maximum amount that will be lost if the counterparty defaults, the credit losses take into account the amount that can be recovered after the default occurs. That is, credit loss on contract $c$, if default occurs at time $\tau$ under some assumed market scenario is:

$$L(c,\tau) = AE(c,\tau) \times [1 - R(c,\tau)]$$

where the recovery rate, $R(c,\tau)$, is the percentage of the value that will be recovered on the contract if default occurs at time $\tau$. Usually, $R(c,\tau)$ depends on the seniority of the contract. $R(c,\tau)$ can also depend on the state of the market when default occurs.

If netting is allowed, counterparty credit losses are a function of Actual Exposure, which in turn depends on the netting provisions stipulated in the master agreement. The credit loss on a counterparty portfolio if default occurs at time $\tau$ is:

$$L(P,\tau) = AE(P,\tau) \times [1 - R(P,\tau)] \qquad (8)$$

When netting is allowed, a single recovery rate is applied to all transactions. This is because it is reasonable to assume that the contracts for which netting is allowed have the same seniority class.

If, on the other hand, netting is not allowed, credit losses from a counterparty portfolio (if default occurs at time $\tau$) are conservatively defined as a function of the portfolio's Total Exposure:

$$L(P,\tau) = \sum_{c \in P} L(c,\tau) = \sum_{c \in P} TE(c,\tau)[1 - R(c,\tau)] \qquad (9)$$

The Total Exposure is used because, in the absence of a netting agreement, the institution cannot offset the exposure immediately after default occurs on the first contract. Equation 9 allows for the possibility that the counterparty may choose to default on the contracts when each of them reaches its maximum exposure. In this case, it is possible to have contracts with different recovery rates.

In practice, master agreements can be quite complex. For example, each counterparty may hold several different master agreements that permit netting across certain instruments, but not others. In addition, netting may not be applicable in certain jurisdictions at the time of default. An accurate estimation of counterparty exposures requires that netting agreements be accurately modeled in a flexible netting hierarchy. Furthermore, given the strong dependence of exposures to the netting provisions, stress testing these provisions is very important.

Netting is perhaps the most popular mitigation technique. Other mitigation techniques include the posting of collateral, marked-to-market caps, re-couponing and early termination clauses (Wakeman 1997). Accurate exposure calculations require a thorough modeling of these provisions.

## Credit risk measurement methodologies

Current credit risk measurement methodologies differ from one another in their assumptions of (i) credit exposure, (ii) default probability, and (iii) recovery rate. A simple methodology may assume that the contract's credit exposure is equal to its notional. Potential loss is then estimated by applying a fixed percentage to the notional, where, presumably, this percentage encompasses both the default possibility and the recovery rate. A more sophisticated methodology creates distributions of future exposures, default rates and recoveries before calculating the losses through simulation.

In this section, we introduce two methodologies. The first is the method put forth by the Bank for International Settlements. It is currently the only method financial institutions are allowed to use to calculate their capital reserves. The second method is a simulation-based method in which Monte Carlo simulations are employed to create profiles of exposures and losses.

### The BIS methodology

Concerns about credit risk have prompted the Bank for International Settlements to introduce capital adequacy requirements for financial institutions that deal in derivative securities. This requirement, commonly referred to as the Basle Accord (BIS 1988), specifies the method that banks must use to calculate credit exposure and is intended to establish a minimum level of capital reserves. Though the methodology prescribed by the BIS follows the framework described in the previous section, some characteristics of the methodology are noteworthy.

First, the Actual Exposure, and thus the Potential and the Total Exposure, are defined only at the current time, $t=0$. At the contract level, the Total Exposure of a derivative position, known as the Credit Equivalent Amount (*CEA*), consists of two parts, the Actual Exposure and the Potential Exposure. The Actual Exposure is defined according to Equation 1. The Potential Exposure, on the other hand, is calculated by multiplying the notional of the transaction, *N*, by a pre-defined credit conversion factor, *CF*:

$$PE = CF \times N \tag{10}$$

This factor depends on the broadly classified types of the securities underlying the derivative contracts and the time to maturity of the contracts. Table 1 summarizes the factor by the maturity date and the type of underlying.

The credit conversion factor is intended to account for the possibility that future exposures may exceed current exposures. The Potential Exposure, calculated according to Equation 10, is known as the "static add-on for Potential (Future) Exposure" because it is a fixed amount of the notional, independent of passing time, that is added to the Actual Exposure. Thus, the *CEA*:

$$CEA = AE(c, 0) + PE(c, 0) \tag{11}$$

is defined only at the current time.

| Residual Maturity | Interest Rate | Exchange Rate & Gold | Equity | Precious Metal | Commodities |
|---|---|---|---|---|---|
| < 1 year | 0% | 1% | 6% | 7% | 10% |
| 1 - 5 years | 0.5% | 5% | 8% | 7% | 12% |
| > 5 years | 1.5% | 7.5% | 10% | 8% | 15% |

**Table 1:** BIS credit conversion factors, *CF* (BIS 1997)

If netting is allowed, the *CEA* is equal to the sum of the netted Actual Exposure (Equation 4) and the sum of each transaction's Potential Exposure (Equation 10) adjusted by a netting factor, *NF*:

$$CEA = AE(P, 0) + NF \times PE(P, 0) \qquad (12)$$

The netting factor, *NF*, is defined as:

$$NF = (0.4 + 0.6*NGR)$$

where the Net-to-Gross Ratio, *NGR*, is the ratio of the Actual Exposure with netting (Equation 4) to the Actual Exposure without netting (Equation 5). Note that if netting is not permitted, $NGR=1$, $NF=1$ and the *CEA* for the portfolio is the sum of each transaction's *CEA*.

The Capital Reserves, *CR*, are then calculated by multiplying the *CEA* first by a risk-weight factor, *RF*, which depends on the type of the counterparty to the contract, and then multiplying by an 8% capital-to-exposure ratio:

$$CR = RF \times CEA \times 0.08 \qquad (13)$$

The BIS guideline for the values of the risk-weight factors are summarized in Table 2.

| Type of Counterparty | Risk Weight |
|---|---|
| Organization for Economic Cooperation and Development (OECD) governments | 0% |
| OECD banks and public-sector entities | 20% |
| Corporate and other counterparties | 50% |

**Table 2:** Risk weights for off-balance-sheet transactions, *RF* (BIS 1997)

The foremost conclusion of the ISDA evaluation of credit risk and regulatory capital requirements (ISDA 1998) is that there is an urgent need for reform. The report promotes adoption of a models-based approach as an alternative to the current standardized rules. A models-based approach, such as the MC approach described below, is more consistent with internal risk management practise and is more conducive to prudent credit risk management. They note a number of shortcomings in the current regime, including

- limited differentiation of credit risk among broad categories of credit
- static measures of default risk based on an 8% capital requirement adjusted for broad categories of risk
- no recognition of the term structure of credit risk, and thus no recognition of the evolution of risk factors and the greater probability of default associated with longer exposures
- simplified potential future risk calculation, leading to limited and inexact recognition of netting and of moneyness of the position
- lack of recognition of portfolio diversification effects and thus encouragement for judicious diversification.

While the BIS assumes a static add-on for Potential Exposure, the simulation-based method computes the contract's value through time under each scenario. Moreover, the probability of default and recovery rate are implicitly taken into account by the BIS through the risk-weight factors and the capital-to-exposure ratio, whereas they are explicit under the simulation-based approach. In the next section, we consider the simulation-based method.

### The Monte Carlo-based method

Monte Carlo simulation is a comprehensive method for estimating credit exposures and losses for derivative portfolios. Although Monte Carlo simulation is generally difficult to implement and computationally intensive, it can realistically incorporate the impact of all sources of risk, offsetting correlations between various positions and counterparties, as well as netting and mitigation provisions.

In its most general form, during a Monte Carlo simulation a large number of joint scenarios are generated based on (i) the market risk factors affecting the value of the portfolio, (ii) credit events such as default and credit migration, and (iii) recovery rates for each contract upon default. Every scenario is a path over time of tens, and possibly hundreds, of risk factors covering market and credit events. Typically, the simulation is performed over the entire life of the transactions in the portfolio. Under each scenario and time step, the portfolio is revalued and recoveries are applied when default occurs. Finally, the results of the simulation are aggregated and various statistics are computed from the distributions of the exposures over time and from the distribution of losses.

A naive simulation is not computationally practical. A prohibitively large number of scenarios is required since credit events, especially default, occur with low probabilities.

This limitation can be overcome by exploiting some basic properties of the problem and by making some simplifying assumptions. In particular, one can speed up the computation dramatically by applying a simple "decomposition" of the problem. We have defined credit exposure as the cost of replacing or hedging a contract when default occurs. Therefore, exposures depend only on the state of the market at that time, and not on the credit state of the counterparty.

As a first stage, one can run a simulation on the whole portfolio to compute a "table of counterparty exposures". The table of exposures summarizes the Actual Exposure (Equation 4 or 5) and Total Exposure (Equation 3 or 7) of every counterparty, under each market scenario and time step. This is generally the most computationally expensive step in the simulation since it requires the valuation of all the instruments in the portfolio under each scenario. Then, to compute credit losses one runs a new simulation with joint random draws from the table of exposures, counterparty defaults and recoveries. A large number of scenarios can be drawn in this case, since every time a default occurs in a scenario, the losses are simply computed by reading the table and applying the recovery rates; no expensive portfolio valuations are necessary.

The simulation of credit events can be eliminated altogether when computing counterparty credit losses if we further assume that (i) the probabilities of default and recovery rates are deterministic and (ii) market and credit events are independent.

The first assumption is quite common in many pricing models (Jarrow and Turnbull 1995, Hull and White 1995, and Jarrow et al. 1997). Das and Tuffano's model (1997) relaxes the assumption of deterministic recovery rates in Jarrow et al. and correlates recoveries to interest rates. Credit risk models such as Wilson (1997) and CreditMetrics (JPMorgan 1997) have also traditionally assumed deterministic defaults and recoveries, although this assumption can be relaxed. The CreditRisk+ model (Credit Suisse 1997) explicitly incorporates stochastic probabilities of default.

The assumption of market and credit independence, while not always realistic, greatly simplifies the calculations. Hence, it has been frequently employed; for example, Hull and White (1995) and Jarrow and Turnbull (1995) use it to price derivative securities, while Jamshidian and Shu (1997) use it in their credit risk calculations. Hull and White argue that the assumption is not unreasonable when the counterparties are large, well-diversified financial institutions whose books are unlikely to be sensitive to movements in a single market factor. The assumption is less realistic, however, when the counterparties are production firms that engage in derivative transactions to hedge their positions or to speculate in the areas in which they have special knowledge. Duffee (1996) demonstrates by example that in these cases, the assumption significantly impacts the results.

If one makes assumptions (i) and (ii) above, the loss distribution can be obtained explicitly, without further simulation, from the table of exposures, the recoveries and the cumulative default probabilities of the counterparty. This is depicted in Figure 1 for a single counterparty.

Consider a set of $n$ market scenarios, $\omega_j$, each with probability $p_j$, and assume that default can occur at $m$ discrete points in time, $t_i$. In this figure, $\lambda(i,j)$ is the conditional loss if default occurs at time $t_i$ and scenario $\omega_j$, i.e. $\lambda(i,j)=L(P,t_i)$, as given by Equation 8 or 9, assuming scenario $\omega_j$ occurred. These are the entries of the table of exposures adjusted by the recovery rates. Because of the independence assumption, the probability of a loss of $\lambda(i,j)$ is simply $p_j \times D_i$, the product of the probability of market scenario $\omega_j$ occurring and the probability of default at time $t_i$. The full counterparty loss distribution is simply obtained by ordering these losses in ascending order.

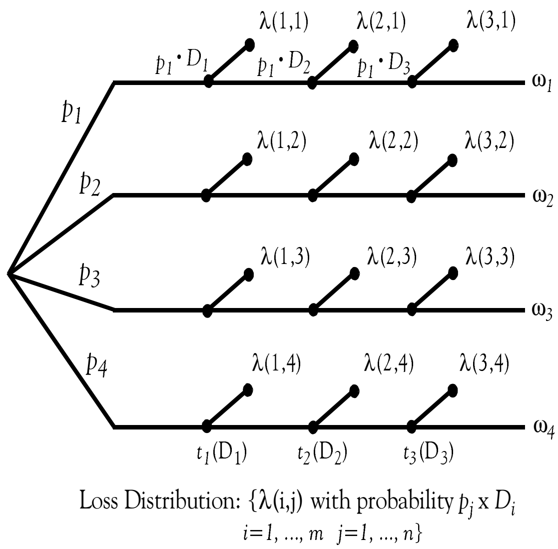


**Figure 1:** Calculation of losses in MC simulation

Figure 1 illustrates a small example with three points in time and four market scenarios, resulting in 13 possible loss values. The last value corresponds to the case of no default (and thus zero loss) occurring before the last time point.

Based on these loss numbers and their associated probabilities, various statistics can be obtained. We define three measures of credit loss: the Expected Loss, and two measures defined as the maximum losses that can be expected to occur at some level of confidence, $\alpha$.

- Expected Loss, *EL*

  The Expected Loss from a given time, $t_k$, (across scenarios and time):

$$E_{D,\omega|\tau > t_k}[L(P,\tau)] = \sum_{j=1}^{n} \sum_{i=k+1}^{m} \lambda(i,j) \times p_j \times D_i \qquad (14)$$

  where $E_{D,\omega|\tau > t_k}[\bullet]$ is the expectation function with respect to both the scenario and default probabilities, conditional on default not occurring before $t_k$.

- Maximum Scenario Loss, $MSL(\alpha)$

  The **scenario credit loss** is the expected loss from a given time, $t_k$, to the contract's maturity, *T*, under an assumed market scenario *j*:

$$E_{D|\tau > t_k,\omega_j}[L(P,\tau)] = \sum_{i=k+1}^{m} \lambda(i,j) \times D_i \qquad (15)$$

  where $E_{D|\tau > t_k,\omega_j}[\bullet]$ is the expectation function, in the interval $(t_k, T)$, with respect to the default probability, conditional upon market scenario $\omega_j$, and default not occurring before time $t_k$.

  The Maximum Scenario Loss at a level of confidence level $\alpha$, $MSL(\alpha)$, is obtained by ranking the expected value of the losses under each scenario (Equation 15) and choosing the maximum (at level $\alpha$):

$$Pr\{E_{D|\tau > t_k,\omega_j}[L(P,\tau)] \le MSL(\alpha)\} = \alpha \qquad (16)$$

- Maximum Loss, $ML(\alpha)$

  The Maximum Loss, across time and scenarios with respect to a confidence level, $\alpha$, $ML(\alpha)$, can be obtained directly from the distribution of the losses at all possible combinations of time points and market scenarios, ranking them and choosing the one that corresponds to the value at which the chance of losses higher than $ML(\alpha)$ does not exeed $1-\alpha$:

$$Pr\{\lambda(i,j) \le ML(\alpha)\} = \alpha \qquad (17)$$

In the case study in the next section, we assume that the probabilities of default and recovery are deterministic, and that counterparty defaults are independent of the market risk factors. The table of counterparty exposures is computed from a multi-step Monte Carlo simulation, where scenarios for the market risk factors (interest and foreign exchange rates) follow a general multi-curve, multi-factor model with mean reversion.

Mean reversion prevents the volatilities of the underlying processes from becoming unduly large even though a credit risk analysis covers a long simulation time horizon.

Each node on an interest rate curve is treated as a separate risk factor. The scenarios are then generated based on a historical estimation of the volatilities and correlations of the risk factors. These volatilities and correlations are assumed to be constant over time.

## A case study

In the previous section we introduce the MC method. In this section we accomplish the remaining two objectives of this paper. First, we present the details of a test portfolio for which we produce a credit risk report based on a Monte Carlo credit analysis. We examine this report for the information that might be derived by a credit risk manager and highlight several significant aspects of the counterparty exposure and loss profiles. Finally, we contrast these results with those derived from the BIS approach.

### The sample portfolio

The sample portfolio of the Spadina Bank is constructed such that it is representative of derivative portfolios held by major North American banks. It contains portfolios for six counterparties with four types of derivative transactions - foreign exchange options, foreign exchange forwards, currency swaps and interest-rate swaps. Five of the counterparties are OECD-incorporated banks with credit ratings of A or higher. The other counterparty is a BB-rated corporation. The counterparties, together with their credit ratings and outstanding contracts as of June 4, 1997, are summarized in Table 3.

The marked-to-market values of the transactions are calculated using the data on interest rates and foreign exchange rates as of June 4, 1997. These data are obtained from Reuters on-line services. In total, there are sixteen currencies (including USD) in the portfolio.

The Spadina Bank has a netting agreement with every counterparty. All agreements allow cross-product netting, except for the agreement with Ruby which allows all the foreign exchange contracts (i.e., FX forwards, FX options and currency swaps) to be netted together, and all interest-rate contracts (IR swaps) to be netted together, but does not permit netting between the two types.

### Monte Carlo simulation

To generate the distributions of credit exposures and credit losses at future points in time, we create 1,000 scenarios on the 16 relevant interest rate (zero) curves and 15 foreign exchange rates using a multi-factor, multi-step Monte Carlo method with mean reversion.

Because the volatilities of some Asian interest rates and currencies are extremely high, they are adjusted such that none of them exceeds 8%. This is to ensure that the simulation does not generate values of risk factors that are unrealistically high. We believe that this adjustment is justified based on past behaviors of interest rates. For interest rates that have low volatilities such as those of the G7 countries, a mean reversion coefficient of 0.01 is used. For other interest rates, we use a coefficient of 0.1.

### Probabilities of default and recovery rates

To calculate credit losses, we require the probabilities of default, $D_i$, and the recovery rates, $R(c,t)$, should default occur. The default probabilities are calculated based on average one-year transition matrices for banks and for corporations (Standard & Poor's 1997). These transition matrices contain the historically average probabilities that a company (bank or corporation) with a certain credit rating migrates to another rating or default by the end of one year. The cumulative probability that a company will default *by* the end of $n$ years is obtained by multiplying the appropriate transition matrix by itself $n$ times. These cumulative probabilities are summarized in Table 4.

The probability of default in a future period can be determined from Table 4. For example, the probability that a AA bank will default between year 3 and year 5 *and* not default before year 3 is 0.06% (= 0.08% – 0.02%).

| Counterparty (Type, Credit Rating) | No. of Transactions | Mark-to-Market Value (millions USD) | % of Value | Median Maturity (years) | Maximum Maturity (years) | Notional of Transactions (millions USD) |
|---|---|---|---|---|---|---|
| Diamond (Bank, A) | | | | | | |
| FX Forward | 525 | -12.177 | 5% | 1.8 | 4.8 | 5,482.403 |
| FX Options | 298 | 6.348 | -3% | 1.7 | 5.0 | 23.750 |
| IR Swaps | 286 | -233.334 | 98% | 5.7 | 19.1 | 11,990.292 |
| Total | 1,109 | -239.163 | | 2.1 | 19.1 | 17,496.445 |
| Ruby (Bank, AAA) | | | | | | |
| FX Forward | 16 | -0.271 | 0% | 0.9 | 3.4 | 165.679 |
| Currency Swaps | 46 | 432.047 | 83% | 0.4 | 15.4 | 4,106.855 |
| IR Swaps | 93 | 91.310 | 17% | 5.3 | 15.9 | 3,404.842 |
| Total | 155 | 523.086 | | 4.7 | 15.9 | 7,677,365 |
| Sapphire (Bank, AA) | | | | | | |
| FX Forward | 535 | 44.193 | -70% | 1.8 | 4.5 | 5,594.322 |
| IR Swaps | 594 | -89.991 | 170% | 4.9 | 16.0 | 17,897.094 |
| Total | 1,146 | -45.799 | | 3.2 | 16.0 | 23,491.416 |
| Topaz (Bank, A) | | | | | | |
| FX Forward | 12 | -8.534 | -6% | 1.4 | 3.9 | 133.581 |
| IR Swaps | 21 | 147.396 | 106% | 5.1 | 9.9 | 3,042.508 |
| Total | 33 | 138.862 | | 4.0 | 9.9 | 3,176.089 |
| Emerald (Corp, BB) | | | | | | |
| FX Forward | 117 | -10.272 | 100% | 1.4 | 3.9 | 1,248.488 |
| Total | 117 | -10.272 | | 1.4 | 3.9 | 1,248.488 |
| Turquoise (Bank, AA) | 455 | -65.495 | 121% | 2.0 | 3.9 | 4,710.904 |
| FX Forward | 26 | 9.062 | -21% | 6.1 | 15.9 | 956.849 |
| IR Swaps | 481 | -56.433 | | 2.1 | 15.9 | 5,667.753 |
| Total | | | | | | |
| Total Portfolio | 3,024 | 310.281 | | 2.4 | 19.1 | 58,757,566 |

The notionals of the swaps are equal to the swap principals. The notionals of the foreign exchange forwards and foreign exchange options are obtained by multiplying the contract size by one unit of the underlying currency, and then converting it into USD using the exchange rates on June 4, 1997.

**Table 3:** Sample portfolio by counterparties

| | Default Probability (in %) | | | | | | |
|---|---|---|---|---|---|---|---|
| Credit Rating | 1y | 3y | 5y | 8y | 12y | 16y | 20y |
| AAA - Bank | 0.00 | 0.00 | 0.02 | 0.09 | 0.36 | 0.91 | 1.81 |
| AA - Bank | 0.00 | 0.02 | 0.08 | 0.33 | 1.10 | 2.43 | 4.25 |
| A - Bank | 0.04 | 0.17 | 0.48 | 1.38 | 3.31 | 5.80 | 8.60 |
| BB - Corporation | 1.10 | 4.62 | 9.02 | 15.88 | 24.22 | 31.19 | 36.92 |

**Table 4:** Cumulative probabilities of default

Note that our calculation of default probabilities assumes that the default processes are Markovian. That is, we assume that the probability of migration depends solely on the counterparty's current credit rating, and not on its credit rating history. A conservative recovery rate of zero is assumed for all contracts.

## Results

Based on the implementation of a MC simulation described above, we produce a credit risk report which we examine for information that might be derived by a credit risk manager. We highlight several significant aspects of the counterparty exposure and loss profiles and contrast these results with those derived from the BIS approach.

### MC-based credit exposures and losses

Figure 2 is an example of a credit risk report for the Spadina Bank that summarizes the results of an analysis based on the MC approach. It comprises two tables. The first contains statistics on credit exposure:

- Actual Exposure, *AE*, (Equation 1)
- Expected Total Exposure, *ETE*
- Maximum Total Exposure at a 99% confidence level, *MTE(99%)*.

The Total Exposure to a counterparty under a scenario is calculated using Equation 3 or 7, depending on the provisions for netting.[3] The Maximum Total Exposure, *MTE(99%)*, is then obtained by ranking the Total Exposures under all 1,000 scenarios and choosing the one for which there is a less than 1% chance that Total Exposure in other scenarios will be higher than *MTE(99%)*:

$$Pr\{TE(P,t) \le MTE(99\%)\} = 99\%$$

The second table summarizes the results of the loss calculations:

- Expected Loss, *EL*, (Equation 14)
- Maximum Scenario Loss at a 99% confidence level, *MSL(99%)*, (Equation 16)
- Maximum Loss at a 99% confidence level, *ML(99%)*, (Equation 17)
- Maximum Loss at a 99.9% confidence level, *ML(99.9%)*, (Equation 17).

As well, the report includes a set of graphs that illustrates the evolution of exposures over time for each counterparty. Each of the figures contains three different profiles of Actual Exposure through time:

- Expected Actual Exposure[4]
- Maximum Actual Exposure at a 95% confidence level, *MAE(95%)*
- Maximum Actual Exposure at a 99% confidence level, *MAE(99%)*.

The report is dated June 4, 1997. We review the information that a credit risk manager might extract from this summary credit report, beginning with an examination of the credit exposures.

Only two counterparties – Ruby and Topaz – have positive current Actual Exposures. The Total AE represents the portfolio's replacement cost should both counterparties default today. The portfolio's current value of 310 million USD is about half of its current Actual Exposure (662 million USD) because netting across counterparties is not permitted.

The tabulated Expected Total Exposure and Maximum Total Exposure by counterparty suggest that future exposures are greater than current exposure in each case. This suggestion is confirmed by the counterparty exposure profiles. Each of the three exposure profiles for each counterparty increases from the current value and remains positive for some period during the simulation. At the portfolio level, Expected and Maximum Total Exposures, 1,047 million USD and 1,894 million USD respectively, suggest that future credit exposure is, on average, 58% higher than the current exposure. The Maximum Scenario Loss, *MSL(99%)*, is almost three times higher.

The exposure profiles identify the periods in which counterparty default would be most financially damaging. Consider the exposure

# The Spadina Bank
# Credit Report
# June 4, 1997

## Credit Exposures in millions of USD

| Counterparty | Mark-to-Market Value, V | Actual Exposure, AE | Expected Total Exposure, ETE | Maximum Total Exposure, MTE(99%) |
|---|---|---|---|---|
| Diamond (A) | -239.16 | 0.00 | 24.51 | 183.46 |
| Ruby (AAA) | 523.08 | 523.08 | 791.05 | 1,233.93 |
| Sapphire (AA) | -45.79 | 0.00 | 22.07 | 92.70 |
| Topaz (A) | 138.86 | 138.86 | 185.44 | 298.54 |
| Emerald (BB) | -10.27 | 0.00 | 0.41 | 10.14 |
| Turquoise (AA) | -56.43 | 0.00 | 23.06 | 75.63 |
| Total | 310.28 | 661.94 | 1,046.56 | 1,894.42 |

## Credit losses in millions of USD

| Counterparty | Expected Loss, EL | Max. Scenario Loss at 99% Confidence Level, MSL (99%) | Max. Loss at 99% Confidence Level, ML (99%) | Max. Loss at 99.9% Confidence Level, ML (99.9%) |
|---|---|---|---|---|
| Diamond (A) | 0.18 | 1.80 | 1.88 | 38.61 |
| Ruby (AAA) | 0.34 | 0.70 | 0.00 | 76.68 |
| Sapphire (AA) | 0.08 | 0.48 | 0.00 | 25.62 |
| Topaz (A) | 0.49 | 1.00 | 0.00 | 134.31 |
| Emerald (BB) | 0.01 | 0.04 | 0.00 | 0.00 |
| Turquoise (AA) | 0.05 | 0.29 | 0.00 | 14.99 |

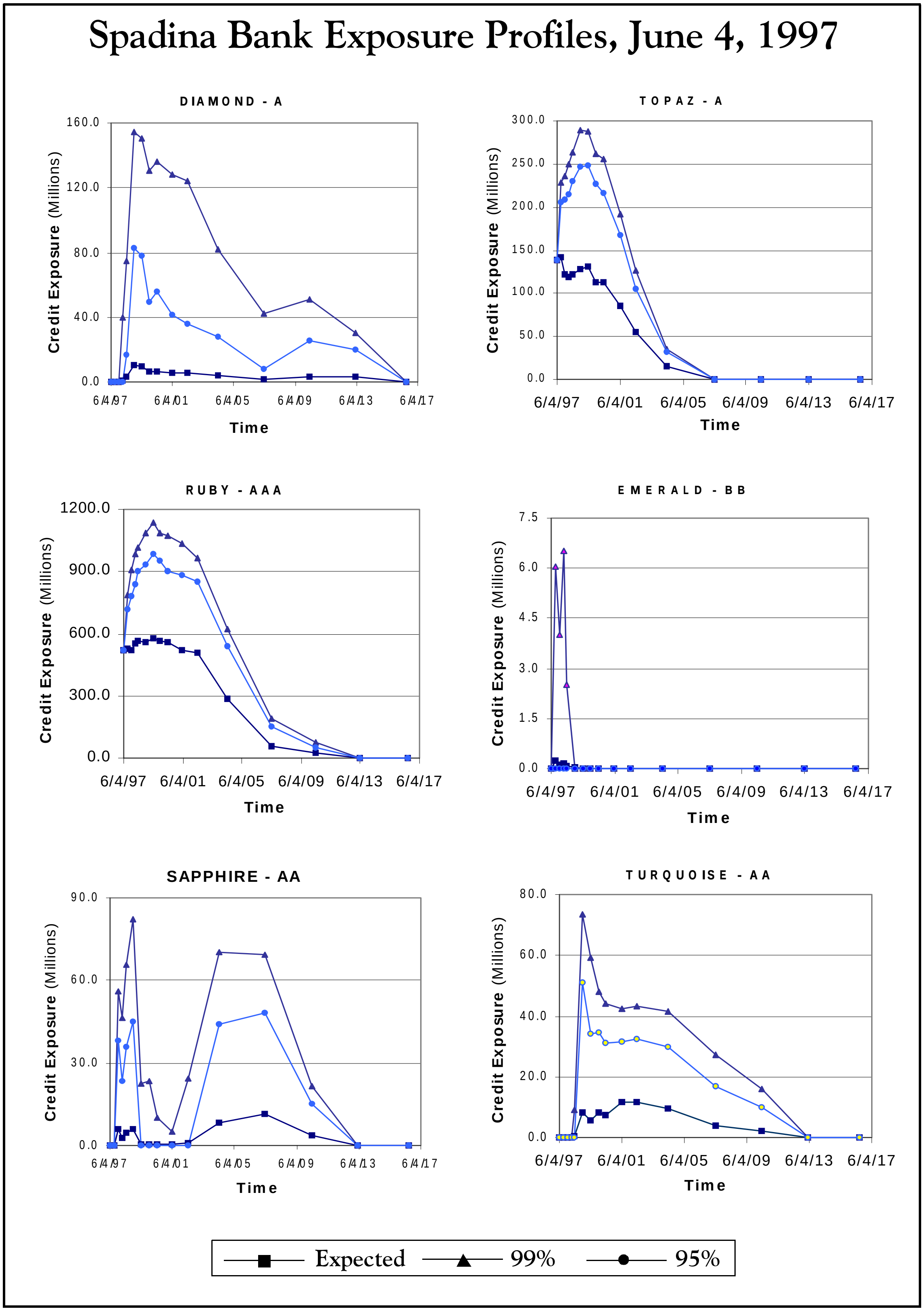


**Figure 2:** Spadina Bank credit report and exposure profiles

profiles of Diamond. The Maximum Actual Exposures, *MAE(95%)* and *MAE(99%)*, are zero during the first six months, and then surge to peaks of about 82 million USD and 150 million USD, respectively. Using the simulation infrastructure to investigate the causes of this increase, the credit manager discovers that it is primarily the result of foreign-exchange forward and option contracts, whose values drastically increase in a number of scenarios due to the volatilities of their underlying currencies. The behavior of the Expected Actual Exposure, however, suggests that there are other scenarios where the values of these contracts decrease; otherwise, the mean exposure would sharply increase as well.

Next we consider the report on credit losses, beginning with a comparison of the maximum credit losses at 99% and 99.9% confidence levels. The *ML(99%)* is non-zero only for Diamond. Default is such a low-probability event that (i) for *each* of the five counterparties, there is a less than 1% chance that it will default *when* the exposure to it is positive, and (ii) there is a less than 1% chance that Diamond will default when its exposure is greater than 1.88 million USD.

At an increased level of confidence of 99.9%, *ML(99.9%)*, we note that all counterparties except Emerald have positive losses, and that the maximum credit loss for Diamond is no longer the largest. Let's analyze the losses of these two counterparties.

Although Emerald is a BB-rated corporation, its *ML(99.9%)* is zero for two reasons. First, all of its contracts expire within four years, half of them in two years. Secondly, its contracts are collectively out-of-the-money (i.e., the current aggregated marked-to-market value is negative). As a result, the simulated exposure profiles over the lives of its contracts are zero at most time points and under most scenarios. Thus, its *MSL(99%)* and its Expected Loss are low.

The Maximum Scenario Loss for Diamond is the highest even though its exposures are much lower than those of Ruby and Topaz. Compared to Ruby, which is a AAA bank, Diamond has a higher probability of default because it has a lower credit rating. This causes Diamond's *MSL(99%)* to be higher. Compared to Topaz's, Diamond's contracts have much longer maturities. As a result, the calculation of Diamond's *MSL(99%)* (Equation 16) spans a much longer period. The longer period is especially important because, from Table 4, the marginal probabilities of default are higher in later years. Therefore, even though Diamond and Topaz have the same credit rating, there are a few scenarios under which Diamond has a higher Maximum Scenario Loss as the effect of Diamond's longer maturities dominates the effect of Topaz's higher credit exposures in the early years.

Although Diamond has the highest *MSL(99%)*, it does not have the highest Expected Loss. This is because its Expected Actual Exposure is only slightly positive through time. The Expected Losses of Ruby and Topaz are higher, which is to be expected since their exposure profiles indicate prolonged, significant, positive Expected Actual Exposures.

Though the exposures summarized in the report indicate that substantial losses can be incurred, the probabilities that those losses will occur, and thus the Expected Losses, are substantially smaller.

### Comparison of methods

In this section, we compare measures of credit exposure and loss under the MC approach to the Credit Equivalent Amounts, CEAs, and Capital Reserves, *CR*, under the BIS methodology.The issues raised by ISDA and noted above are well-illustrated by this case study.

The ratios of Maximum Total Exposures, *MTE(99%)*, to CEAs are depicted in Figure 3. They range from 0.56 for Emerald to 1.73 for Topaz, suggesting that the CEAs can be higher or lower than the simulated *MTE(99%)*, depending on the nature of the contracts with each counterparty. In general, it appears that the CEAs are higher when the counterparty's current Actual Exposures are zero (i.e., Sapphire, Emerald and Turquoise, but not Diamond). This is to be expected since the BIS approach applies the same add-on Potential Exposure regardless of the moneyness of the current position. As a result, unless the exposure increases substantially over time (which is the case with Diamond), the CEAs are likely to be higher than the simulated Maximum Total Exposure.

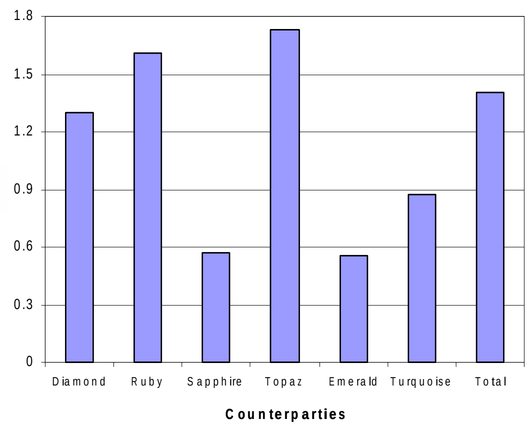


**Figure 3:** Ratio of MC/BIS exposure

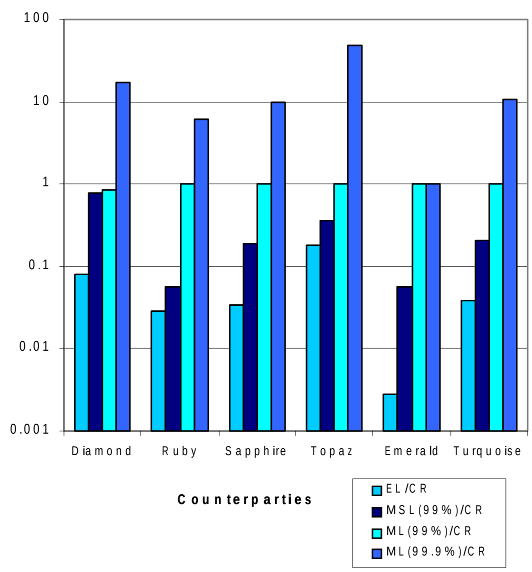


**Figure 4:** Ratios of MC/BIS losses

Because losses are a function of exposures, the discrepancy between the two methods is propagated through the loss calculation. Figure 4 shows the log ratios of four different MC measures of credit losses to the BIS capital reserves. The ratios of Expected Losses to capital reserves are very low, ranging from 0.003 for Emerald to 0.18 for Topaz. The ratios of *MSL(99%)* to the BIS capital reserves are all less than 1. The same is true for the ratios of *ML(99%)* to the BIS reserves. On the other hand, except for Emerald, the ratios of *ML(99.9%)* to the BIS reserves are very high, ranging from 6.25 for Ruby to 48.7 for Topaz.

The results indicate that credit reserves set to *ML(99%)* would be lower than the BIS capital reserves, while credit reserves set to *ML(99.9%)* would be much higher than the BIS capital reserves. Because the probability of a large credit loss for highly rated counterparties is very low, the first credit loss that is observed in a simulation typically falls between the 99% and 99.9% percentile. As a result, the decision taken on the appropriate confidence level significantly affects the amount of the reserve that should be set.

Next, we consider the capability of the methods to account for the impact of netting as a credit enhancement tool on credit exposures and losses. We compare the ratios of the Maximum Total Exposures with and and without netting to the ratios of CEAs with and without netting.

Figure 5 shows that Spadina Bank can benefit significantly from netting agreements with counterparties with zero current AE, as evidenced by the ratios for Diamond, Sapphire, Emerald and Turquoise. In addition, for these four counterparties, the CEA ratios consistently exceed the *MTE(99%)* ratios, implying that the BIS approach does not adequately account for the effect of netting. On the other hand, for Ruby and Topaz, whose current Actual Exposures are positive, the BIS approach over-estimates the benefit of netting.

This counter-intuitive outcome is a result of the non-forward looking nature of the BIS approach: the Net-to-Gross Ratio is calculated based on current exposures only and does not consider future exposures. It is possible that portfolios have exposures that benefit less from netting in the future than today. While a Monte Carlo simulation over time captures the changing characteristics of the portfolio and the reduced netting benefits, the static BIS approach does not recognize any changes in the exposure profile over time and overestimates the risk reduction due to netting.

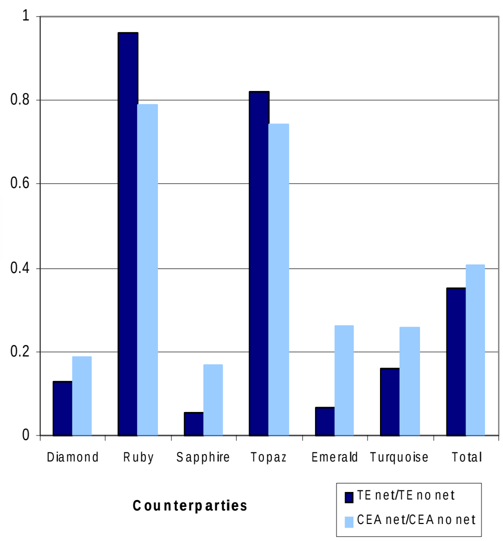


**Figure 5:** Netting/no netting exposure ratios

We have explored the merits of the Monte Carlo based analysis of credit exposure and losses. The MC approach differentiates among categories of credit risk, accounts for the moneyness of the position and the changes in the values of the underlying risk factors, and recognizes the impact of the term structure of credit risk and the benefits of netting and of portfolio diversification. As well as specifically accounting for each of the factors that impacts credit exposure and loss, the simulation underlying the MC approach provides an explanation of the outcomes of the credit analysis.

However, some caution must be exercised when applying statistical analyses to any problem, and this case study provides excellent examples of these dangers. We discuss one illustrated by exposure measures and one by the losses.

Emerald's exposure profiles show the Expected Actual Exposure lying between the Maximum Actual Exposure profiles of *MAE(95%)* and *MAE(99%)*. The reason for this counter-intuitive result is that Actual Exposures are positive in less than 5% of the scenarios. Therefore, *MAE(95%)* is zero over time, while the Expected Actual Exposures are slightly positive. This illustrates the potential danger of using a percentile measure (such as Value-at-Risk) to measure credit risk.

The significant difference between the Maximum Losses, *ML(99.9%)* and *ML(99%)*, for all counterparties also highlights the danger of the use of a percentile measure. The lower the percentile, the greater the chance that the measure will miss outcomes that have low probabilities but extremely severe impacts.

## Conclusions

The amount of credit risk capital reserve has been a subject of much debate. Currently, the BIS method is the only approach banks can use to set their reserves. The results of this case study are sufficient to support many of the observations made by the ISDA evaluation of credit risk and regulatory capital requirements (ISDA 1998) concerning the deficiencies of the regulatory standard. On the other hand, a models-based approach, such as the Monte Carlo method used in this study, addresses many of these concerns.

The MC method can explicitly account for probabilities of default and recovery, and thus differentiate levels of credit risk among broad categories of credit. It estimates future exposure based on a set of market scenarios over an appropriate simulation horizon, thus accounting for the term structure of credit risk and the evolution of risk factors. This more accurate estimation of potential exposure leads to a more precise evaluation of the impact of netting and moneyness of the position. Moreover, the Monte Carlo simulation infrastructure provides a much richer set of information which credit managers can use to explain the causes of the exposure.

Because the BIS approach does not take into account the evolution of the exposure through time, its resulting reserve can be either too low or too high, depending on the nature of the transactions and the level of prudence required. Under the Monte Carlo method, the capital reserves are set to cover a maximum loss calculated at some level of confidence. The level chosen will significantly affect the amount of reserve that should be set.

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## Endnotes

1. More generally, credit risk may also include the risk of loss that results from changes in expectations of the counterparty's likelihood to default, as reflected by an upgrade or downgrade of its credit rating.

2. Note that if the contract's value decreases monotonically with time, then Potential Exposure to that contract is zero.

3. Total Exposures are not discounted. As a result, the values are in the dollar value of the years that peak exposures occur.

4. The maximum values in the profiles of Expected Actual Exposures are not comparable to the tabulated values of the Maximum Total Exposure.