Disclosure Risk and Price Drift

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ABSTRACT

Disclosures play an apparently critical role in the empirical regularity of the short-run momentum and long-run reversal in stock returns. Motivated by this evidence, this paper integrates an analysis of disclosures within an asset pricing model to arrive at a framework in which disclosures and asset returns are jointly determined. Disclosures resolve uncertainty, but the increased information flow also raises the risks during the disclosure period. When disclosures and asset returns are modeled jointly, apparently good news is associated with the upward revision of future disclosure risks. The model generates predictions that have the outward appearance of short-run momentum and long-run reversal.

1. Introduction

For most stocks, the dominant channel for the resolution of uncertainty is the regular cycle of earnings announcements. The risks associated with such disclosures form the backdrop for the pricing of these stocks. However, unlike many other sources of public information, such as macroeconomic announcements, earnings disclosures are distinguished by the fact that the information is provided by interested parties (the managers of the firm) who have a material interest in the way that the recipients of the news react. The resolution of uncertainty in such contexts takes on distinctive features that are quite unlike the resolution of uncertainty associated with exogenous

public signals. These distinctive features leave their mark on the evolution of stock returns. Taking account of such distinctive features is important when interpreting empirical properties of stock returns.

An enduring empirical regularity that has received much attention is the combination of the short-run momentum and the long-run reversal of individual stock returns. Over horizons of perhaps 1 to 12 months, stock prices apparently *underreact* to news. Following good news, there is a predictable outperformance of the stock over a 12-month horizon, and this phenomenon is particularly well documented following positive earnings surprises. Following Ball and Brown [1968], a very large empirical literature has documented this phenomenon.[1] The term "post-earnings announcement drift" was coined to describe such apparent underreaction to earnings announcements. However, over longer horizons of perhaps three to five years, stock prices seem to *overreact* to news in the sense that stocks that have had a long record of good news tend to become over-extended and have low subsequent average returns.[2]

Importantly, these empirical regularities appear to be intimately tied to disclosures. Indeed, the term "drift" seems to be something of a misnomer. Much of the upward adjustment of a stock's performance following good news is delayed until subsequent earnings announcements (Freeman and Tse [1989], Bernard and Thomas [1990]). In other words, following a positive earnings surprise, the stock price adjusts by jumps at subsequent earnings announcements.

These regularities pose a challenge for conventional asset pricing models, although some have cautioned against neglecting shifts in riskiness. For instance, Ball, Kothari, and Watts [1993] find evidence of shifts in beta across yearly intervals that go some way towards explaining the evidence, and argue that earnings and asset returns are both reflections of the riskiness of the underlying projects undertaken by the firm. Fama and French [1996] argue that their three-factor model can account for the overreaction, although the underreaction is not well accounted for. More recent work has focused on behavioral explanations (Barberis, Shleifer, and Vishny [1998], Hong and Stein [1999]).

On the face of it, attempting to give an explanation of the empirical evidence in terms of rational, risk-averse investors seems to pose some formidable challenges. In order to explain the evidence, one would have to argue that a stock becomes more risky over short horizons following good news, but at the same time it becomes less risky over long horizons. Bernard and Thomas [1990] pose the following stark challenge to any rationalization of the evidence in terms of changing risks.

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[1] See Foster, Olsen, and Shevlin [1984], Freeman and Tse [1989], and Bernard and Thomas [1989].

[2] See Cutler, Poterba, and Summers [1991], De Bondt and Thaler [1985], Chopra, Lakonishok, and Ritter [1992], Fama and French [1992], Lakonishok, Shleifer, and Vishny [1994], and La Porta [1996].

> Firms announcing good (bad) news at quarter *t* would need to experience a temporary upward (downward) shift in risk that occurs three months later, six months later, and nine months later, and then a downward (upward) shift in risk twelve months later. In addition to requiring risk changes in opposite directions for the same portfolios, this explanation also requires that the changes occur over short periods that coincide with an earnings announcement date. (Bernard and Thomas [1990, p. 334])

This challenge would be difficult to meet in an asset pricing model in which disclosures fall outside the scope of the model. However, such an approach seems unnecessarily restrictive. Since earnings are perhaps the most important source of public information for a stock, the distribution of future earnings plays an important role in the determination of asset prices today. To see the full picture, it seems essential to have an asset pricing framework that incorporates a model of disclosures as an integral part of the overall framework. Treating disclosures as exogenous places an unfair burden on the asset pricing part of the framework, which unsurprisingly, will struggle under the strain. We know from earlier work on time series of earnings (e.g. Foster [1977]) that earnings display complex time series patterns, and any theory that does not address both the asset returns and the earnings disclosures together may risk missing some critical ingredient.

This paper is motivated by the need for an asset pricing framework that incorporates disclosures as an integral part of the framework. Disclosures convey information on a firm's activities and performance, and hence resolve some of the uncertainty surrounding the firm's true value, and likely future course of events. However, the increased flow of information is also associated with increased instantaneous risks during the announcement window. If earnings information is cross-correlated across firms, the covariance with the market portfolio would also increase during the disclosure window, and hence the increased instantaneous risk is likely to be priced. Ball and Kothari [1991] have examined how the greater expected return during the announcement window can be decomposed into the compensation for increased risks associated with disclosures and the abnormal return that cannot be accounted for in this way, and Stapleton and Subrahmanyam [1979], Epstein and Turnbull [1980], and Holthausen and Verrecchia [1988] have analyzed the issue of the sequential resolution of uncertainty over time.

I propose a framework for asset pricing that accommodates endogenously generated disclosures and the risks associated with such disclosures. The task is to model the dynamic interaction between an informed manager of a firm and sophisticated market traders. The manager aims to maximize the market price of the firm, but the market's inference pegs the price of the firm to its fair value given the manager's disclosure strategy and the market's capacity to take on risk. The manager has more information about the true value of the firm and is mandated to make reports to the market through periodic earnings reports. The market's response is to discount any disclosure appropriately given the estimated asymmetry of information and the actual disclosure received. The uncertainty facing the market is therefore

the residual uncertainty remaining after the self-interested disclosure by the interested party has been discounted by the market.

The framework delivers an attractively tractable model for the joint determination of disclosures and asset returns. One of the key messages that emanates from the analysis is that asset returns rest on the expectations of future disclosures. Other things being equal, a firm that has more private information will tend to have (in equilibrium) more informative announcements, and more informative announcements are inherently more volatile. At the same time, however, the motives of the managers in disclosing news must also be taken into account. Since the manager's objective is to maximize the market price of the firm, the disclosures are tailored to further the manager's goals. For firms with a greater degree of private information, the market will therefore exercise greater scepticism, treating the reports with more circumspection.

In such an environment, good news in the form of higher earnings tells the market more than simply what the outcome is today. It tells the market that future earnings are more likely to reflect the greater information flow, and hence future earnings are more likely to be volatile. Prices must adjust to account for the changed risk. In particular, if there is some uncertainty concerning how much better informed the managers are relative to the market, then good news and bad news convey rather different information about the future volatility of earnings. Good news increases the likelihood that the flow of information to the firm is higher than was previously supposed. Hence, expected future volatility of earnings goes up, implying higher short-run expected returns. Lackluster earnings suggest that the market may have overestimated the information flow to the managers. Expected future short-run volatility goes down, implying lower short-run expected returns. Taken together, this generates asset returns that look like short-run momentum. At the same time, for any given set of fundamental projects of the firm, an increase in the short-run volatility must be accompanied by a decline in the long run volatility of outcomes. This is because there is only so much fundamental uncertainty associated with the projects, and early resolution of uncertainty implies that there is less uncertainty to be resolved later on. Thus, over the long run, asset returns are low. This looks like long-run reversal.

The paper develops the theoretical model in three stages. It builds on the binomial model of disclosures introduced in Shin [2003], and then extends it to a Poisson context, which has several advantages in terms of tractability in a dynamic context. I outline the theoretical features of the model and discuss the connections with related issues in the empirical literature, such as the "leverage effect" and the volatility feedback hypothesis. In section 5, I introduce uncertainty concerning the rate of information flow to the firm in the Poisson disclosure model. I illustrate, through both numerical examples and a closed form solution of the model, that the model is capable of generating short-run momentum and long-run reversals. I conclude with

a summary of empirical implications of my framework, including some new hypotheses that could potentially be used to test my theory.

2. Binomial Model of Disclosures

The model builds on the static, binomial model of disclosures discussed in Shin [2003]. A firm undertakes N independent and identical projects, where each project succeeds with probability r and fails with probability $1 - r$. The liquidation value of the firm given s successes and $N - s$ failures is given by

$$u^s \tag{1}$$

where $u > 1$. The notation follows the binomial tree model of Cox, Ross, and Rubinstein [1979]. Each success corresponds to an "up" move in a tree that raises the total liquidation value by a factor of u, while each failure corresponds to a "down" move that leaves the liquidation value unchanged. The ex ante value of the firm, denoted by V_0, is the expected liquidation value obtained from the binomial density with success probability r. Thus,

$$V_0 = \sum_{s=0}^{N} \binom{N}{s} (ru)^s (1 - r)^{N-s}$$

$$= (1 + r(u - 1))^N.$$

For illustrative purposes, it is helpful to begin with the assumption that there are just three dates—*initial*, *interim*, and *final*, labeled as dates 0, 1, and 2, respectively. At the initial date, nothing is known about the value of the firm other than the description above. As time progresses, projects begin to yield outcomes. At the interim date, not enough time has elapsed for the manager to know the outcomes of all the projects. However, the outcomes of *some* of the projects will have been realized. In particular, there is a probability θ that the outcome of a project is revealed to the manager by the interim date. This probability is identical across all projects, and whether the outcome is revealed is independent across projects. By the final date, all uncertainty is resolved. The outcomes of all projects become common knowledge. The firm is liquidated, and consumption takes place.

The focus is on the interim date. The manager is able to observe the success and failure of each project as it occurs, and hence knows the numbers of successes and failures at the interim date, but the rest of the market does not. Instead, the only information available to the market at the interim date is a mandated disclosure by the manager. The disclosure must be verifiable. That is, the manager is free to disclose some or all of what he knows, by actually exhibiting the outcomes of those projects whose outcomes have already been determined. However, he cannot concoct false evidence. If he knows that project j has failed, he cannot claim that it has succeeded. Grossman [1981] and Milgrom [1981] introduced disclosure games of this

sort. In choosing to analyze disclosures in terms of the verifiable reports framework rather than the alternative "cheap talk" framework popularized by Crawford and Sobel [1982], I attempt to capture the twin themes of the discretion that managers have in their disclosures and also the broad limits imposed by the accounting system on what is possible, with the ultimate sanction being the one against fraud. In this respect, I follow the standard approach in the accounting literature (see, e.g., Verrecchia [1983, 1990], Dye [1985a, b], and Jung and Kwon [1988]).[3]

More formally, the information available to the manager at the interim date can be summarized by the pair

$$(s, f)$$

where s is the number of successes observed, while f is the number of failures observed. The manager's disclosure strategy $m(\cdot)$ maps his information (s, f) to the pair (s', f'), giving the number of *disclosed* successes and failures, where the requirement of verifiability imposes the constraint that

$$s' \leq s \quad \text{and} \quad f' \leq f. \tag{2}$$

This constraint reflects the requirement that the disclosure takes the form of actually exhibiting a subset of the realized outcomes to the market.

We assume that the disclosure policy of the manager is motivated by the objective of maximizing the price of the firm. Since the initial and final prices of the firm are based on symmetric information, the focus of the analysis is on the interim price, V_1. The market, however, anticipates the manager's disclosure policy and prices the firm by discounting the manager's disclosures appropriately. This gives rise to a game of incomplete information. To begin with, we assume that traders are risk neutral. We model the "market" as a risk-neutral player in the game who sets the price of the firm to its fair value based on all the available evidence, taking into consideration the reporting strategy of the manager.

More formally, the market's strategy is the pricing function

$$(s', f') \mapsto V_1. \tag{3}$$

We ensure that the market sets the price of the firm to its expected liquidation value by assuming that its objective in the game is to minimize the squared loss function:

$$(V_1 - V_2)^2 \tag{4}$$

where V_2 is the (commonly known) liquidation value of the firm at the final date. The market then sets V_1 equal to the expected value of V_2 conditional on the disclosure of the manager, as generated by his disclosure strategy.

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[3] Bull and Watson [2004] revisit the issue of verifiable reports and develop a foundational approach with verifiability as the linchpin in their theory of evidence.

The manager, on the other hand, anticipates the optimal response of the market, and chooses the disclosure that maximizes V_1.

In this game, a policy of full disclosure by the manager turns out never to be an equilibrium strategy. To see this, suppose for the sake of argument that the manager always discloses fully, so that the disclosure strategy is the identity function:

$$m(s, f) = (s, f).$$

The best reply by the market is to set V_1 to be

$$V_1(s, f) = u^s (1 + r(u - 1))^{N-s-f},$$

since there are $N - s - f$ unresolved projects, and the expected value of the firm is

$$u^s \sum_{i=0}^{N-s-f} \binom{N - s - f}{i} (ru)^i (1 - r)^{N-s-f-i} = u^s (1 + r(u - 1))^{N-s-f}. \tag{5}$$

But then, the manager's disclosure policy is suboptimal, since the feasible disclosure $(s, 0)$ that suppresses all failures elicits the price:

$$u^s (1 + r(u - 1))^{N-s}, \tag{6}$$

which is strictly higher than (5) for positive f. Hence, we are led to a contradiction if we suppose that full disclosure can figure in an equilibrium of the disclosure game.

Instead, the natural equilibrium in this context is the opposite extreme, in which all successes are disclosed but none of the failures are disclosed. This is the strategy that maps (s, f) to $(s, 0)$, and I call it the *sanitization strategy* in that the disclosure is "sanitized" by removing the bad news but leaving all the good news. I show that this strategy can, indeed, be supported in equilibrium and derive the observable implications for equilibrium prices.

The literal interpretation of this strategy (of suppressing all failures) is somewhat stark, and runs counter to the principle of conservative accounting. However, I focus on this strategy as a metaphor for the way that managers use the discretion they have in presenting the best case to the market. Any disclosure strategy will be a mapping from some underlying state space to the message space, and an element of discretion will be present in any such strategy. The sanitization strategy is a caricature, but one that captures a crucial ingredient.

Although I do not devote much space to other types of equilibria, it should be borne in mind that examples can be constructed of more complex equilibria that do not fall under this category (see Shin [2003, appendix B]). The construction of these examples serves as a reminder of the difficulty of tying down beliefs in sequential games of incomplete information. However, they do not detract from the appeal of the simplicity and intuitive force of the sanitization strategy, especially for the purpose of drawing empirical

hypotheses. For this reason, we can confine our attention for the rest of the paper to equilibria in which the manager uses the sanitization strategy. Since failures are never disclosed under the sanitization strategy, the interim prices that occur with positive probability are of the form $V_1(s, 0)$, which I denote simply by $V_1(s)$.

The partial revelation of information that arises from the manager's disclosure at the interim date suggests a parallel between the problem of pricing the firm and the pricing of a compound lottery. The similarity lies in the fact that, in both cases, the uncertainty is resolved in two steps. At the interim date, the manager's disclosure still leaves some residual uncertainty in the true value of the firm, so that the firm at the initial date (date 0) is like a compound lottery in which the prizes at the interim date (date 1) are also lotteries over prizes at the final date (date 2). However, when the manager follows the sanitization strategy, the uncertainty from date 0 to date 1 takes a very simple form. We know that the probability density over the true number of successes is binomial with probability r. When the manager follows the sanitization strategy, the density over *disclosed* successes at date 1 is also binomial, with success probability θr. This is because the manager observes the success of any particular project with probability θr, and observations of successes are independent across projects.

Of interest is the residual uncertainty following the manager's disclosure at date 1. Consider the joint density over the *disclosed* successes at date 1 and the *realized* successes at date 2 when the manager follows the sanitization strategy. We can depict this density in tabular form:

		realized successes at $t = 2$				
		0	$\cdots$	k	$\cdots$	N
	0					
disclosed	$\vdots$					
successes	s			$h(s, k)$		
at $t = 1$	$\vdots$					
	N					

Since the number of disclosed successes cannot exceed the number of realized successes, all entries below the leading diagonal are zero. $h(s, k)$ is the probability that the manager discloses s successes at date 1 when the realized number of successes at date 2 turns out to be k. Denoting by

$$h(k \mid s)$$

the probability of k realized successes conditional on the disclosure of s successes, we have the following result.

LEMMA 1. *Let* $q = (r - \theta r)/(1 - \theta r)$. *When the manager follows the sanitization strategy,*

$$h(k \mid s) = \begin{cases} \dbinom{N-s}{k-s} q^{k-s}(1-q)^{N-k} & \textit{if } s \leq k \\ 0 & \textit{otherwise.} \end{cases} \tag{7}$$

In other words, the residual uncertainty can also be characterized by a binomial density in which the probability of success of an undisclosed project is given by $q = (r - \theta r)/(1 - \theta r)$. Pursuing the analogy with compound lotteries further, the lottery at date 0 is governed by a binomial density over prizes at date 1, where the prizes are themselves lotteries governed by binomial densities. The proof of Lemma 1 is given in appendix A.

The recursive nature of the uncertainty underlying our problem implied by Lemma 1 makes possible a convenient diagrammatic device to represent the resolution of uncertainty over time. Refer to figure 1 for an example of a firm with five projects.

The starting point at date 0 is indicated as the origin in figure 1. Between date 0 and date 1, uncertainty is resolved according to a binomial tree in which the probability of an up move is $r\theta$ while the probability of a down move is $1 - r\theta$. The point x represents the point reached at date 1 when the manager has disclosed two successes. Lemma 1 tells us that at point x, the residual uncertainty over the three undisclosed projects is governed by a binomial density with success probability q. This is represented by the binomial tree of length 3 with x at its apex, where an up move has probability q and a down move has probability $1 - q$. One possible outcome starting from x is for the eventual number of successes at date 2 to be exactly three. In terms of figure 1, this is represented by the fact that the terminal node u^3 forms part of the binomial tree with root x. Finally, when viewed from the origin at date 0, the final density over the terminal nodes at date 2 is given by the binomial density with success probability r.
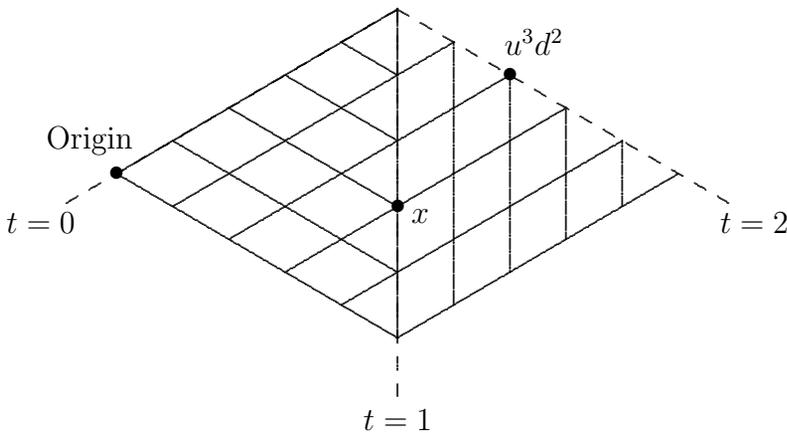

FIG. 1.—Concatenation of binomial trees.

Note that $q \to 0$ as $\theta \to 1$. This has a natural interpretation. When θ is large, the manager is well informed about the true number of successes, and the disclosure is informative. In the limit, the manager is fully informed, so that there is full revelation of the ex post number of successes. The market's response is to put all the weight on the worst possible outcome consistent with the manager's disclosure. This is the so-called "unraveling" argument discussed by Milgrom [1981], Grossman [1981], and Milgrom and Roberts [1986]. However, as long as $\theta < 1$, the manager is not always fully informed, and the market must make allowance for some pooling between the genuinely uninformed types of manager and those types that are fully informed but are withholding information.[4] The probability q reflects the degree of scepticism in the market. The higher θ is, the lower q is, and hence the lower is the market price of the firm for any given face value of the disclosure. This is because a high θ firm has more scope to suppress bad news, and the market exercises more scepticism to any disclosure by the firm by discounting the price more.[5]

We can characterize equilibrium prices at date 1. The solution concept is the notion of sequential equilibrium due to Kreps and Wilson [1982].

LEMMA 2.   *There is a sequential equilibrium in which the manager uses the sanitization strategy. Moreover, in any equilibrium in which the manager uses the sanitization strategy,*

$$V_1(s) = u^s (1 + q(u-1))^{N-s}, \tag{8}$$

where $q = (r - \theta r)/(1 - \theta r)$.

The second statement follows directly from Lemma 1, since

$$
\begin{aligned}
V_1(s) &= u^s \sum_{k=s}^{N} h(k \mid s) u^k \\
&= u^s \sum_{i=0}^{N-s} \binom{N-s}{i} (qu)^i (1-q)^{N-s-i} \\
&= u^s (1 + q(u-1))^{N-s}. \tag{9}
\end{aligned}
$$

To complete the proof, we need to construct an equilibrium of the disclosure game in which the manager follows the sanitization strategy. The details are presented in appendix B.

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[4] This type of pooling has been examined by Lewis and Sappington [1993], Austen-Smith [1994], and Shin [1994]. See also Jung and Kwon [1988], Verrecchia [1990], and Dye and Finn [2003], who develop these themes in the accounting context.

[5] See Dickhaut et al. [2003] for experimental evidence on the exercise of greater scepticism in the face of informed disclosures.

3. Poisson Model of Disclosures

In spite of its simplicity, the binomial model is still not tractable enough for our purpose of addressing the short-run momentum and long-run reversals in returns. Also, it is essentially a one-shot game, and must be extended to accommodate the dynamics of asset returns. For these reasons, I take a limiting case of the binomial model. I maintain, for the moment, the assumption that traders are risk neutral so that prices reflect expected liquidation values. I make the following extensions to the binomial model.

- First, I make the model dynamic by introducing several disclosure dates. Let the initial date be 0 and the terminal date be T, but there are multiple disclosure dates between 0 and T. Let θ_t be the probability that the outcome of any particular project becomes known to the manager by disclosure date t, and we assume that

$$\theta_t = t\theta \qquad (10)$$

for some constant θ. Thus, θ is the rate at which the manager's information improves from one disclosure date to the next. We may think of θ as the speed of information flow to the manager. In an asymmetric information setting, therefore, it also has the interpretation of the degree of asymmetry of information between the manager and the market as a whole.[6]
- Second, we take the Poisson limit of the binomial by taking $N \to \infty$, $r \to 0$, but where $rN \to \lambda$, where λ is a positive constant.

The issue of equilibrium selection is more difficult in the dynamic context than in the static model. However, for the same reasons of descriptive simplicity, I confine the discussions to outcomes in which the firm uses the sanitization strategy at each disclosure date, and the market makes inferences by pricing the firm at fair value given this strategy.[7] Among other things, I do not examine smoothing strategies by the manager. While recognizing this to be a limitation of the current model, I leave such developments to future work.

The ex ante value of the firm in the limit can be otained by substituting in $\lambda = rN$, and letting $N \to \infty$. The ex ante value of the firm is

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[6] Clearly, θ must be small relative to T to ensure that $\theta T \leq 1$, so that the probability of being informed is bounded by 1.

[7] One simple way to achieve this outcome as an equilibrium is to suppose that the manager's consumption at date t is proportional to the market price of the firm at date t, and that the manager maximizes a discounted sum of current and future consumption. As the discount rate becomes large, the manager's objective will be to maximize the current value of the firm. Other, more general formulations will also support the sanitization strategy in equilibrium.

$$V_0 = \lim_{N \to \infty} \left(1 + \frac{\lambda(u-1)}{N}\right)^N$$

$$= \exp\{\lambda(u-1)\}.$$

The Poisson limit has two key properties. The first concerns the residual uncertainty following a disclosure. In the concatenation of binomial pricing trees in figure 1, the residual uncertainty depends on the disclosure, since the size of the residual tree depends on the disclosed outcome. However, in the Poisson limit, I make all residual trees of infinite size and thus eliminate the dependence of the residual uncertainty on the disclosed outcome. Thus, in contrast to the binomial model (in which the residual uncertainty is larger for lower disclosures), the Poisson model is a limiting case in which the severity of the residual uncertainty is invariant to how good the disclosure is. I return to this issue later when discussing the relationship with the literature on the "leverage effect."

The second property of the Poisson limit is a closure property in which the weighted average of Poisson densities with Poisson weights is itself a Poisson density. This closure property allows a recursive analysis of the dynamic asset pricing problem. To state the closure property, define the *a-Poisson matrix* as the doubly infinite array:

$$P(a) \equiv e^{-a} \begin{bmatrix} 1 & a & \dfrac{a^2}{2!} & \dfrac{a^3}{3!} & \cdots \\ 0 & 1 & a & \dfrac{a^2}{2!} & \cdots \\ 0 & 0 & 1 & a & \cdots \\ 0 & 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}.$$

The sth row of $P(a)$ starts with a sequence of zeros until it reaches the main diagonal. Thereafter, the entries follow the Poisson density with parameter a. Poisson matrices are triangular, and hence their multiplication is well defined due to the fact that each entry in the product is a finite sum. We have the following result.

LEMMA 3. $P(a)\,P(b) = P(a+b)$

Proof.   The (i, j)th entry of $P(a)P(b)$ is given by

$$\sum_{k=i}^{j} e^{-a} \frac{a^{k-i}}{(k-i)!} e^{-b} \frac{b^{j-k}}{(j-k)!} = \frac{e^{-(a+b)}}{(j-i)!} \sum_{k=i}^{j} \binom{j-i}{k-i} a^{k-i} b^{j-k}$$

$$= e^{-(a+b)} \frac{(a+b)^{j-i}}{(j-i)!}$$

which is the (i, j)th entry of $P(a+b)$.

Lemma 3 states that a compound lottery in which the prizes are drawn according to a Poisson density and in which the prizes themselves are Poisson lotteries is equivalent to a simple Poisson lottery. This closure property gives the dynamic asset pricing problem a recursive structure that we can usefully exploit.

The residual uncertainty over final outcomes following the disclosure at date t can be obtained taking limits of the binomial model. From Lemma 1, the binomial probability q in the dynamic context is given by

$$q_t = \frac{r(1 - t\theta)}{1 - rt\theta}.$$

Then, substituting $r = \lambda/N$, and taking the limit as $N \to \infty$ and $r \to 0$, the probability of $s + j$ realized successes at the terminal date T conditional on s disclosed successes at disclosure date t is Poisson with parameter $\lambda(1 - t\theta)$. In other words, conditional on s disclosed successes at date t, the probability of $s + j$ realized successes at the terminal date is given by

$$\begin{cases} e^{-\lambda(1-t\theta)} \frac{(\lambda(1-t\theta))^j}{j!} & \text{if } j \geq 0 \\ 0 & \text{otherwise.} \end{cases} \tag{11}$$

We can write down an exactly analogous expression for the residual uncertainty following the disclosure at date $t + 1$, namely, the Poisson density with parameter $\lambda(1 - (t + 1)\theta)$. Then, we can appeal to Lemma 3 to derive the probability density over possible disclosures at date $t + 1$ conditional on being at date t. Thus, appealing to Lemma 3 and letting

$$\lambda(1 - t\theta) = a + b$$

$$\lambda(1 - (t + 1)\theta) = b$$

I conclude that the probability density over possible disclosures at date $t + 1$ must also be Poisson with parameter $a = \lambda\theta$. I have thus proved the following:

THEOREM 4.   *When the firm has announced* s *successes at date* t, *the probability that the firm will announce* s $+$ j *successes at date* t $+ 1$ *is given by*

$$\begin{cases} e^{-\lambda\theta} \frac{(\lambda\theta)^j}{j!} & \text{if } j \geq 0 \\ 0 & \text{otherwise.} \end{cases} \tag{12}$$

This is a key property of my model, which also generates a rich set of empirical implications that I return to later in the paper. Recall that the mean of a Poisson random variable is also its variance. Thus, when the expected outcome increases, so does the volatility of that outcome. The fact that the mean and the variance move in the same direction arises naturally from the strategic nature of disclosures, and it is no accident that our disclosure model yields Poisson densities over disclosures and outcomes. I comment

below on why such a feature differs from alternative scenarios in which news arrives exogenously.

When traders are risk neutral, the price of the firm at date t conditional on s disclosed successes can be obtained as the expected liquidation value of the firm using the probability density (11).[8]

$$V_t(s) = u^s \sum_{j=0}^{\infty} e^{-\lambda(1-t\theta)} \frac{(\lambda(1-t\theta)u)^j}{j!}$$
$$= u^s \exp\{\lambda(1-t\theta)(u-1)\}. \tag{13}$$

As noted already, Theorem 4 implies that firms with high θ will have more volatile disclosures. The economic interpretation is quite natural. A firm with a greater short-run flow of information will (in equilibrium) have more informative disclosures. More informative disclosures are more volatile. The volatility of disclosures is crucial, since the short-run volatility of returns is determined, in part, by how volatile the diclosures are. The short-run return from date t to date $t+1$, conditional on s disclosed successes at date t, is the random variable:

$$R_{t,t+1} = \frac{V_{t+1}}{V_t}.$$

The long-run return is the return from date t to the terminal date T, namely, $R_{t,T} = V_T/V_t$.

Note that the short-run return is defined as being from one disclosure date to the next. If earnings reports are made on a quarterly calendar, then the short-run return would be a quarterly return. The fact that short-run returns in our model are from one disclosure date to another means that some caution must be exercised when interpreting the results. I elaborate on some potential pitfalls below in section 5. With this qualification in mind, consider the volatility of the short- and long-run returns in our model. In appendix C, I show that the date t variance of short-run return does not depend on disclosed successes s or date t, and is given by

$$\text{Var}_t(R_{t,t+1}) = \exp\{\lambda\theta(u-1)^2\} - 1 \tag{14}$$

while the variance of the long-run return is given by

$$\text{Var}_t(R_{t,T}) = \exp\{\lambda(1-t\theta)(u-1)^2\} - 1. \tag{15}$$

From (14) and (15), we see that the volatility of short-run return is *increasing* in θ, but the volatility of long-run return is *decreasing* in the same parameter θ. The divergent behaviour of short- and long-run volatility is a distinctive feature of the framework.

---

[8] We can verify that the same expression for $V_t(s)$ can be obtained by taking the limit of (9) from the binomial model, where $V_t(s) = \lim_{N\to\infty} u^s (1 + \frac{\lambda(1-\theta_t)(u-1)}{N(1-r\theta_t)})^{N-s}$.

To understand the greater volatility of the short-run return for the high θ firms, it is helpful to consider the probability density over possible disclosures in the immediate future. When viewed from date t, the immediate uncertainty concerns the possible realizations of V_{t+1}, which in turn depends on the disclosure at date $t + 1$. When the possible disclosures at $t + 1$ vary widely, then the short-run return volatility will be high. However, we know from Theorem 4 that high θ firms have greater volatility of disclosures in the immediate future. This greater volatility translates into greater volatility of short-run returns. This greater volatility also reflects the greater information value of the high θ firms' announcements. Thus, high θ firms have a more front-loaded flow of information, and hence an earlier resolution of uncertainty than low θ firms.

4. Price under Risk Aversion

So far we have confined our attention to a risk-neutral world in which the market price of the firm is its expected liquidation value. Let us now consider the risk-averse world where asset prices reflect the degree of risk. In principle, we must address the prior question of whether the uncertainty associated with the firm's disclosures is priced in equilibrium. A full answer to this question would depend on the modeling of aggregate risk and the extent to which investors can hedge the risks associated with firms' disclosures. A strong presumption would be that not all of the risks associated with disclosures could be hedged costlessly (e.g., by diversification). Ball and Kothari [1991] note that earnings information is cross-correlated across firms over the compressed reporting window, so that the instantaneous risks in the reporting window will have pricing implications. Indeed, Ball and Brown [1968, fn 40] argue that if the risks that arise during the disclosure window are purely idiosyncratic, then the market portfolio itself will have a vanishingly small risk itself. Even for purely idiosyncratic risks, some authors argue that such risks are priced.[9]

For these reasons, I proceed on the assumption that the risks from disclosures are priced. I assume a homogeneous population of investors who consume only at the terminal date T, each with the constant relative risk aversion utility function

$$\frac{c^{1-\alpha}}{1-\alpha}$$

where $\alpha > 0$. With risk aversion, the marginal rate of substitution across states is given by the ratio of probabilities each weighted by the respective marginal utility of consumption. From this, we can derive the following result, whose proof is given in appendix D.

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[9] See Goyal and Santa-Clara [2003].

THEOREM 5.   *When investors have relative risk aversion coefficient α, the price of the firm at date* t *given* s *disclosed successes is*

$$V_t(s) = u^s \exp\left\{\frac{\lambda}{u^\alpha}(1 - t\theta)(u - 1)\right\}. \tag{16}$$

The effect of introducing risk aversion is thus to replace the parameter λ by the smaller parameter λ/u^α. Thus, in a risk-averse world, it is as if the probability of success of the firm is lower. The more risk averse are the traders (high α), the greater is the price discount.

Using the density over disclosures given by (12), the short-run expected return from t to $t + 1$ conditional on s disclosed successes at date t is given by

$$
\begin{aligned}
E_t(R_{t,t+1} \mid s) &= \frac{E_t(V_{t+1})}{V_t(s)} \\
&= \frac{u^s \exp\left\{\frac{\lambda}{u^\alpha}(1 - (t+1)\theta)(u - 1)\right\} \sum_{j=0}^{\infty} e^{-\lambda\theta} \frac{(\lambda\theta u)^j}{j!}}{u^s \exp\left\{\frac{\lambda}{u^\alpha}(1 - t\theta)(u - 1)\right\}} \\
&= \exp\left\{\lambda\theta(u - 1)\left(1 - \frac{1}{u^\alpha}\right)\right\}. \tag{17}
\end{aligned}
$$

The long-run expected return from t to the terminal date T is given by

$$
\begin{aligned}
E_t(R_{tT} \mid s) &= \frac{E_t(V_T)}{V_t(s)} \\
&= \frac{u^s e^{-\lambda(1 - t\theta)} \sum_{j=0}^{\infty} \frac{(\lambda(1 - t\theta)u)^j}{j!}}{u^s e^{\lambda(1 - t\theta)(u-1)/u^\alpha}} \\
&= \exp\left\{\lambda(1 - t\theta)(u - 1)\left(1 - \frac{1}{u^\alpha}\right)\right\}. \tag{18}
\end{aligned}
$$

Based on these expressions for short-run and long-run expected returns, we can summarize our findings in terms of the following theorem.

THEOREM 6.   *The short-run expected return is increasing in θ, but the long-run expected return is decreasing in θ.*

The intuition for Theorem 6 is exactly analogous to the discussion in the previous section on why the volatility of short-run returns is increasing in θ, but the volatility of long-run returns is decreasing in θ. High θ firms have a greater variability of disclosures in the immediate future, reflecting the greater front-loading of the information flow. However, this also means that the high θ firms have an earlier resolution of uncertainty, implying lower variability of the final liquidation outcome. In short, for firms with a greater degree of information asymmetry between the managers and the market as

a whole, returns are high in the early stages of the life of the firm, but only at the expense of a reduction in returns near the end of the firm's life.

5. *Short-Run Momentum and Long-Run Reversal*

We are now ready to draw all the strands of the discussion together to address the issue of short-run momentum and long-run reversals in stock returns. As well as assuming risk aversion in the Poisson disclosure model, I now also assume that the parameter θ is itself the subject of some uncertainty. The market starts with some prior beliefs on the possible values of θ that a firm can have, and updates its belief concerning θ by observing actual disclosures over time. The introduction of uncertainty over θ introduces yet more difficulties in tying down the equilibrium of the game. Indeed, the sanitization strategy cannot be guaranteed to be part of an equilibrium in the general case with uncertainty over θ. I return to this issue in the next section, since I cannot do justice to the equilibrium selection issue in passing. It will also be illuminating to delve further into the strategic incentives in this type of game for directions for future work. For now, let us note that when θ is drawn from a density with sufficiently small support, the continuity of equilibrium entails that sanitization would still be an equilibrium disclosure strategy for the manager. So, it would be possible to construct equilibria with sanitization provided that the support of θ is small. In the rest of this section, I pursue the equilibrium pricing consequences of the sanitization strategy when θ is uncertain.

We know from Theorem 4 that different values of θ imply different probability distributions over disclosures. Hence, observing actual disclosures is informative to the market concerning the true value of θ, and hence affects both the market value of the firm and the beliefs on subsequent returns and disclosures.

From Theorem 4 conditional on s disclosed successes at date t, the probability of $s + j$ disclosed successes at date $t + 1$ is given by

$$e^{-\lambda\theta}\frac{(\lambda\theta)^j}{j!}. \tag{19}$$

Higher values of θ imply that higher disclosed successes are more likely. When the value of θ itself is uncertain, a higher number of disclosed successes is more likely to emanate from a firm with a higher θ than from a firm with a lower θ. Thus, good news increases the posterior value of the firm's θ. From Theorem 6, this inference will raise the short-run returns, but at the expense of lowering the long-run returns. Thus, good news is followed by an increase in short-run returns, but a fall in long-run returns. In this sense, there is short-run momentum in the firm's stock return, but it comes at the expense of a reversal in the firm's returns in the long run.

I begin by illustrating the short-run momentum and long-run reversal by means of a numerical example. I then go on to present a

closed form solution for equilibrium returns that exhibit momentum and reversal.

5.1 NUMERICAL EXAMPLE

Let us consider the case in which there are two possible values of θ, given by $\{\theta_L, \theta_H\}$, where $\theta_L < \theta_H$. Suppose that the prior probability weight on θ_H is given by p. Then, from (19), the posterior probability weight on θ_H following j additional disclosed successes is given by

$$
p_j = \frac{pe^{-\lambda\theta_H}\frac{(\lambda\theta_H)^j}{j!}}{pe^{-\lambda\theta_H}\frac{(\lambda\theta_H)^j}{j!} + (1-p)e^{-\lambda\theta_L}\frac{(\lambda\theta_L)^j}{j!}}
$$

$$
= \frac{1}{1 + e^{\lambda(\theta_H-\theta_L)}\left(\frac{\theta_L}{\theta_H}\right)^j\left(\frac{1-p}{p}\right)}
$$

which is an increasing function of the disclosure j. Thus, good news raises the posterior belief on the higher value of θ. Let us denote by $f(j, p)$ the probability that the firm announces j additional successes when the prior probability weight on θ_H is given by p. Then

$$
f(j, p) = pe^{-\lambda\theta_H}\frac{(\lambda\theta_H)^j}{j!} + (1-p)e^{-\lambda\theta_L}\frac{(\lambda\theta_L)^j}{j!}.
$$

Also, we denote by $V_t(p)$ the price of the firm at date t when the market believes with probability p that $\theta = \theta_H$. Then, the expected short-run return from date t to $t+1$ is given by

$$
E_t(R_{t,t+1} \mid p) = \frac{\sum_{j=0}^{\infty} f(j, p)u^j V_{t+1}(p_j)}{V_t(p)}. \tag{20}
$$

If this expression is an increasing function of p, we have short-run momentum in returns. A high disclosed success (good news) raises p, and hence raises the subsequent expected short-run return. Figure 2 plots short-run return as a function of p for parameter values $\lambda = 1$, $\theta_H = 0.1$, $\theta_L = 0.05$, $t = 8$, $T = 10$, $u = 2$, and $\alpha = 1.2$. As we can see, it is an increasing function of p, indicating short-run momentum in expected returns.

The long-run expected return is a considerably simpler expression, since the terminal values are simple powers of u itself. The long-run expected return is given by

$$
E_t(R_{t,T} \mid p) = \frac{1}{V_t(p)} \sum_{j=0}^{\infty} \left(pe^{-\lambda(1-t\theta_H)}\frac{(\lambda(1-t\theta_H))^j}{j!} \right.
$$

$$
\left. + (1-p)e^{-\lambda(1-t\theta_L)}\frac{(\lambda(1-t\theta_L))^j}{j!} \right) u^j.
$$

Figure 3 plots long-run expected return as a function of p. We can see that it is a decreasing function of p, indicating long-run reversal of returns.
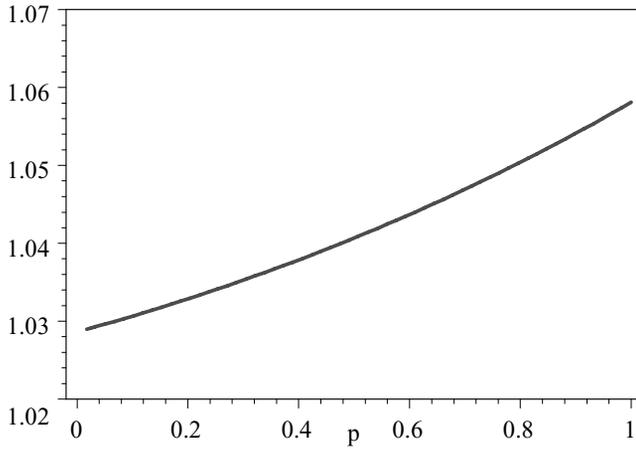
FIG. 2.—Short-run expected return as a function of p.

5.2 APPROXIMATE CLOSED FORM SOLUTION

So far, we have only illustrated the possibility of short-run momentum and long-run reversal using a numerical example. It is also possible to derive a closed form solution to our model with uncertainty over θ that exhibits short-run momentum and long-run reversal. We present one such case here.

The closed form solution relies on the uncertainty over θ following a specific density—the gamma density—and then exploiting the fact that the Poisson density and the gamma density are conjugates (De Groot [1970, ch. 9]). Suppose that the parameter $\xi \equiv \lambda\theta$ is uncertain and has prior density

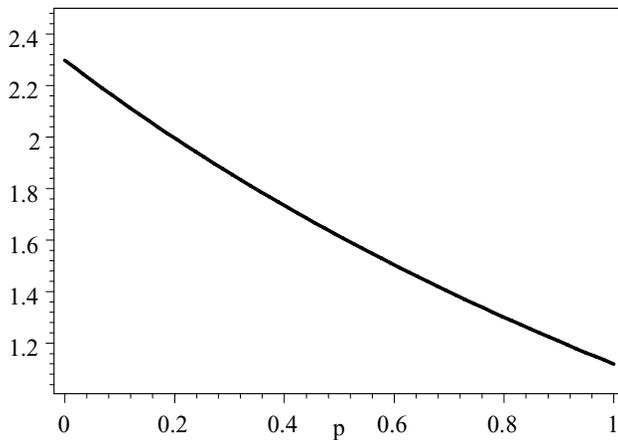$$f(\xi) = \frac{1}{K}\xi^{\sigma} e^{-\tau\xi} \tag{21}$$

FIG. 3.—Long-run expected return as a function of p.

over the interval $[0, \lambda]$. The constant K is such that the integral of this density is equal to one. Then, following s (cumulative) disclosed successes at date t, the posterior density over ξ is given by

$$g(\xi \mid s, t) = \frac{1}{\hat{K}} \xi^{\sigma+s+1} e^{-\xi(\tau+t)} \tag{22}$$

where the constant $\hat{K}$ is such that the integral is one (see De Groot [1970, section 9.4]). Using this formula for updating beliefs on ξ, we can derive approximations for short- and long-run returns, which are summarized as follows.

THEOREM 7.   *Let $\xi \equiv \lambda\theta$, and suppose that the prior density over ξ is given by (21). Then, the approximate expected short-run return is*

$$E_t(R_{t,t+1} \mid s) \simeq \left(\frac{\tau + t\left(1 + \frac{u-1}{u^\alpha}\right)}{\tau + t\left(1 + \frac{u-1}{u^\alpha}\right) - (u-1)\left(1 - \frac{1}{u^\alpha}\right)} \right)^{1+\sigma+s}.$$

The approximate expected long-run return is

$$E_t(R_{t,T} \mid s) \simeq e^{\lambda(u-1)\left(1 - \frac{1}{u^\alpha}\right)} \cdot \left(\frac{\tau + t\left(u - (u-1)\left(1 - \frac{1}{u^\alpha}\right)\right)}{\tau + tu} \right)^{1+\sigma+s}.$$

These approximations are exact in the limit as $\lambda \to \infty$.

The proof of Theorem 7 is presented in appendix E. Note that the short-run expected return is *increasing* in s, while simultaneously, the long-run expected return is *decreasing* in s. Thus, the better the news, the higher the expected short-run return, but the lower the expected long-run return. The approximations become exact in the limit as λ becomes large. It is noteworthy that the short-run return is well-defined in this limit, even though the asset price itself would be increasing without bound. The long-run return goes to infinity as $\lambda \to \infty$, but the ratio $E_t(R_{t,T} \mid s)/e^{\lambda(u-1)\left(1 - \frac{1}{u^\alpha}\right)}$ is well defined in the limit.

6. Discussion

It was noted earlier that the sanitization strategy may not be an equilibrium strategy for the manager when θ is uncertain, and it is now time to address some of the equilibrium selection issues for our game with a view to future work. The drift phenomenon arises in our model due to the positive association between good news and an increased estimate of the underlying θ parameter. In this paper, this association is predicated on the sanitization strategy by the manager, since good news is associated with the disclosure of a large number of successes, which in turn is more likely when θ is high. Thus, the drift phenomenon and sanitization are two sides of the same coin.

Problems arise when the manager chooses not to use the sanitization strategy in equilibrium. Shin [2003, appendix B] shows that even when

θ is common knowledge, there are equilibria in which the manager does not sanitize. The intuition is that the market associates certain disclosures with "bad" outcomes, and punishes the manager who announces them with a low price. Given this, the manager who can avoid the disclosure will do so, leaving only those managers who have no choice but to announce it. Knowing this, the market's belief becomes self-fulfilling.

The issue when θ is uncertain is whether it is possible to construct an equilibrium in which sanitization is used at all. The possibility arises that when every other type of manager uses the sanitization strategy, the manager of a particular type may find it optimal to deviate. For instance, when there is a wide range of possible θ in the population, a manager with a low θ may attempt to signal to the market that his firm is a firm with a low θ, and hence be spared from the price discount for high θ firms. The signaling gains may be large enough relative to the gains from exaggeration for the low θ firms to deviate from sanitization. If the signaling gains are large enough, it is also conceivable that a manager may choose not to disclose some known successes. Such an action would be tantamount to "smoothing" by the manager over time. Other types of managers may attempt to mimic the low θ types, in which case mixed strategy equilibria may arise as a consequence. It is an open question whether a counterexample to sanitization can be constructed, and would be an interesting task for future research.

As well as the equilibrium selection issue, it is also worth considering to what extent our results are attributable to strategic disclosures, and whether an alternative mechanism delivers similar predictions. One candidate would be a stochastic volatility model in which the mean and the variance of short-run return undergo (exogenous) random changes over time. Following a very good outcome, the updated return variance would be higher, leading to higher subsequent expected returns. This would be similar to the predictions of our model. However, the similarity breaks down following bad news. In the stochastic volatility model, very bad news also raises the updated return variance, thereby raising expected returns. This is the opposite of the prediction from our strategic disclosures model.

The stochastic volatility model is related to the "leverage effect" on stock returns. Fisher Black notes how low stock returns are associated with a subsequent increase in the return volatilities of the stock (Black [1976]). Campbell and Hentschel [1992] propose a "volatility feedback hypothesis" to explain the leverage effect. In their model, high current volatility increases expectations of *future* volatility, and prices must adjust down to raise expected returns. So, while good news is tempered by the prospect of increased future volatility, bad news is *amplified* by the prospect of increased future volatility.

The leverage effect leads to predictions that run counter to short-run momentum and long-run reversals. Under the leverage effect, expected return is high following a bad outcome. However, a more careful interpretation of our model and results suggests a reconciliation between short-run momentum and the leverage effect. Bear in mind that the short-run return in our

strategic disclosures model refers to returns between one disclosure date and the next. Momentum arises in such a case because good news suggests a high information flow to the firm, and hence raises the probability of an informative disclosure at the next disclosure date. This result does not rule out the leverage effect at the level of the fundamentals. Indeed, the leverage effect over fundamentals is a natural consequence of the binomial disclosure model. This can be seen clearly from the concatenation of binomial trees in figure 1. Following bad news, the residual uncertainty is larger, reflecting the larger residual binomial tree. However, from one disclosure to the next, the dominant influence on returns is the volatility of *future* disclosures. To this extent, the leverage effect and momentum may be perfectly consistent.

Having said all this, note that in the Poisson model of disclosures, the residual uncertainty is invariant to whether the news is good or bad. To this extent, the Poisson model is likely to be missing a piece of the puzzle. Recently, Rogers, Schrand, and Verrecchia [2003] have found evidence that the leverage effect can, indeed, be attributed to some degree to strategic disclosures. For future work, the development of more general models that can accommodate the short-run leverage effect in a dyanamic context seems desirable in order to accommodate these features.

My model also has implications for the time series of "earnings surprises," i.e., the difference between the expected disclosure and the actual disclosure. One of the key empirical findings reported in the literatrue[10] is that earnings surprises are positively autocorrelated for adjacent quarters, and indeed for up to three quarters. On the surface, such evidence seems to point to the market failing to adjust to the surprise in one quarter's earnings by adjusting beliefs sufficiently for the following quarter. However, my model provides an alternative explanation for the positive autocorrelation in earnings surprises. When the market is uncertain about the true value for θ for a firm, a positive surprise (high disclosed success) leads to an upward revision in the expected value of θ, but as long as the prior has some weight, the revised value will not equal the true value of θ. For a firm whose true value of θ is high, it will continue to surprise the market on the upside as long as the market's Bayes estimate of θ has fully to catch up with the true value. Meanwhile, the firm whose value of θ is lower than the Bayes estimate of θ will continue to surprise on the downside as long as the market's estimate of θ has fully to catch up with the true value. Thus, there will be a tendency for earnings surprises to be persistent. This persistence, of course, is entirely rational and represents the Bayes updating of an underlying parameter in the market.

7. Concluding Remarks

In this paper, I investigate the properties of an asset pricing model that incorporates a fully fledged model of disclosures. Given the critical role played

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[10] See Bernard and Thomas [1990, table 1].

by disclosures in the empirical literature of short- and long-run returns, the incorporation of disclosures into an asset pricing model is an essential step for a better understanding of the issues.

The Poisson disclosure model with risk aversion, and with uncertainty over θ, is presented in section 5. The model has several empirical predictions. I list them below. Some of them have already been alluded to, but there are some predictions that are novel, and potentially provide avenues for testing the validity of the conclusions reached in this paper.

- The Poisson disclosure model with risk aversion and uncertainty over θ is capable of generating time series of returns that are consistent with short-run momentum and long-run reversal in returns. Good news raises the market's expectations of immediate volatility of returns, but lowers the market's expectations of long-run volatility. This combination of belief shifts raises short-run returns, but lowers long-run returns.
- The model is also consistent with the positive autocorrelation of earnings surprises at adjacent quarters. When there is uncertainty about θ, firms' disclosures reveal new information about θ that is incorporated into the market's updated estimates. However, as long as some uncertainty persists, firms with high θ will continue to surprise on the upside, while firms with low θ will continue to surprise on the low side. On the surface, this would be indistinguishable from a market not being able to adjust its beliefs sufficiently.
- My final empirical hypothesis follows from theorem 4. There, I showed that when viewed from date t, the probability density over earnings at the next disclosure date is Poisson with parameter $\lambda\theta$. Thus, both the mean earnings and variance of earnings are increasing in θ. For high θ firms, not only will they tend to announce high earnings they will also have volatile earnings. Since θ is directly involved with upward drift in the stock price, we have the following hypothesis. Firms that have had a positive surprise in earnings have more volatile subsequent earnings than firms that have had a negative surprise in earnings.

There are inevitable limitations of a simple model such as this in addressing such an important and large empirical literature. Clearly, more refined models are necessary to further develop the ideas in this paper.

APPENDIX A

Proof of Lemma 1. When $h(s, k)$ is positive, it is the product of two numbers—the probability that the *realized* number of successes is k and the probability that s of those successes is observed at the interim date. Thus, for $s \leq k$,

$$h(s, k) = \binom{N}{k} r^k (1 - r)^{N-k} \cdot \binom{k}{s} \theta^s (1 - \theta)^{k-s}$$

$$= \frac{N!}{(N - k)! s! (k - s)!} r^k (1 - r)^{N-k} \theta^s (1 - \theta)^{k-s}$$

Then,

$$\frac{h(s, k)}{h(s, k - 1)} = \frac{\frac{N!}{(N-k)! s! (k-s)!} \cdot r^k (1-r)^{N-k} \theta^s (1-\theta)^{k-s}}{\frac{N!}{(N-k+1)! s! (k-s-1)!} \cdot r^{k-1} (1-r)^{N-k+1} \theta^s (1-\theta)^{k-s-1}}$$

$$= \frac{N - k + 1}{k - s} \cdot \frac{r(1 - \theta)}{1 - r}$$

$$= \frac{\binom{N-s}{k-s} (r(1-\theta))^{k-s} (1-r)^{N-k}}{\binom{N-s}{k-s-1} (r(1-\theta))^{k-s-1} (1-r)^{N-k+1}}$$

$$= \frac{\binom{N-s}{k-s} \left[\frac{r(1-\theta)}{r(1-\theta) + (1-r)} \right]^{k-s} \left[\frac{1-r}{r(1-\theta) + (1-r)} \right]^{N-k}}{\binom{N-s}{k-s-1} \left[\frac{r(1-\theta)}{r(1-\theta) + (1-r)} \right]^{k-s-1} \left[\frac{1-r}{r(1-\theta) + (1-r)} \right]^{N-k+1}}$$

$$= \frac{\binom{N-s}{k-s} q^{k-s} (1-q)^{N-k}}{\binom{N-s}{k-s-1} q^{k-s-1} (1-q)^{N-k+1}}$$

where

$$q = \frac{r(1 - \theta)}{(1 - r) + r(1 - \theta)} = \frac{r - r\theta}{1 - r\theta}$$

This proves Lemma 1.

APPENDIX B

I construct an equilibrium with sanitization, thereby proving Lemma 2. The solution concept is the notion of sequential equilibrium proposed by Kreps and Wilson [1982]. The equilibrium pricing rule must specify a price for all feasible reports (s, f), not simply those that receive positive probability in equilibrium. In addition, the beliefs given these out-of-equilibrium disclosures must be consistent with the rules of the game and be obtained as the limit of a sequence in which each feasible disclosure receives positive probability from some type that "trembles" and discloses the out-of-equilibrium report by mistake.

The following pair of strategies are supported in a sequential equilibrium. The manager follows the sanitization strategy, while the market's pricing rule is given by

$$V_1(s, f) = \begin{cases} u^s (1 + q(u - 1))^{N-s} & \text{if } f = 0 \\ u^s & \text{if } f > 0. \end{cases}$$

The off-equilibrium prices are supported by the belief that any disclosed failures emanate from the manager who knows the true value of the firm perfectly, and for whom the number of successes is exactly the number reported by him (so that all other projects are known to have failed). The sanitization strategy is clearly the best reply against this pricing strategy, and the pricing strategy is the best reply to the sanitization strategy. This concludes the argument.

APPENDIX C

In this appendix, I derive the expressions for the variance of short- and long-run returns for the Poisson disclosure model with risk-neutral traders, given by (14) and (15). The short-run variance of return conditional on s disclosed successes at date t is given by

$$
\begin{aligned}
\mathrm{Var}(R_{t,t+1} \mid s) &= E_t\big(R_{t,t+1}^2 \mid s\big) - (E_t(R_{t,t+1} \mid s))^2 \\
&= E_t\big(R_{t,t+1}^2 \mid s\big) - 1 \\
&= \frac{E_t\big(V_{t+1}^2\big)}{V_t^2} - 1 \\
&= \frac{u^{2s} \exp\{2\lambda(1 - \theta_{t+1})(u - 1)\} \sum_{j=0}^{\infty} e^{-\lambda\theta} \frac{(\lambda\theta u^2)^j}{j!}}{u^{2s} \exp\{2\lambda(1 - \theta_t)(u - 1)\}} - 1 \\
&= \exp\{\lambda\theta(u - 1)^2\} - 1. \tag{23}
\end{aligned}
$$

The long-run variance of return conditional on s disclosed successes at date t is given by

$$
\begin{aligned}
\mathrm{Var}(R_{t,T} \mid s) &= \frac{E_t\big(V_T^2\big)}{V_t(s)^2} - 1 \\
&= \frac{u^{2s} \sum_{j=0}^{\infty} e^{-\lambda(1-t\theta)} \frac{(\lambda(1 - t\theta)u^2)^j}{j!}}{u^{2s} e^{2\lambda(1-t\theta)(u-1)}} \\
&= \exp\{\lambda(1 - t\theta)(u - 1)^2\}. \tag{24}
\end{aligned}
$$

APPENDIX D

This appendix derives the formula for the price of the firm for risk-averse investors given in Theorem 5. In a competitive equilibrium, the marginal rate of substitution across states is given by the ratio of probabilities weighted by the respective marginal utilities of consumption. The marginal utility of consumption is $c^{-\alpha}$, so the marginal utility when there are j successes is $u^{-\alpha j}$. Conditional on s disclosed successes at date t, the residual uncertainty over terminal values is given by the Poisson density with parameter $\lambda(1 - t\theta)$. Hence, the product of the marginal utility and the conditional probability of $s + j$ successes is given by

$$K \frac{\left(\frac{\lambda}{u^\alpha}(1-t\theta)\right)^j}{j!} \tag{25}$$

where K is a constant. The pricing density is obtained by choosing K so that (25) sums to one. Thus, conditional on s disclosed successes at date t, the weight attached to the liquidation value being $s + j$ is

$$e^{-\frac{\lambda}{u^\alpha}(1-t\theta)} \frac{\left(\frac{\lambda}{u^\alpha}(1-t\theta)\right)^j}{j!}.$$

The price of the firm is then the expectation of the realized number of outcomes with respect to this density. Thus,

$$V_t(s) = u^s \sum_{j=0}^{\infty} e^{-\frac{\lambda}{u^\alpha}(1-t\theta)} \frac{\left(\frac{\lambda}{u^\alpha}(1-t\theta)u\right)^j}{j!}$$

$$= u^s \exp\left\{\frac{\lambda}{u^\alpha}(1-t\theta)(u-1)\right\}.$$

This proves Theorem 5.

APPENDIX E

In this appendix, I prove Theorem 7. Recall that the gamma density is defined on the half-line $[0, \infty)$ and is given by

$$\frac{\tau^{\sigma+1}}{\Gamma(\sigma+1)} \xi^\sigma e^{-\tau\xi}$$

where $\Gamma(.)$ is the gamma function. Hence, $\int_0^\infty \xi^\sigma e^{-\tau\xi} d\xi = \Gamma(\sigma+1)/\tau^{\sigma+1}$. Suppose we approximate the density over ξ by the gamma density. Then,

$$V_t(s) \simeq \frac{u^s}{K} \int_0^\infty \exp\left(\frac{\lambda(u-1)(1-t\theta)}{u^\alpha}\right) \xi^{\sigma+s} e^{-\xi(\tau+t)} d\xi$$

$$= u^s \exp\left(\frac{\lambda(u-1)}{u^\alpha}\right) \frac{1}{K} \int_0^\infty \exp\left(\frac{-\xi t(u-1)}{u^\alpha}\right) \xi^{\sigma+s} e^{-\xi(\tau+t)} d\xi$$

where $K = \frac{\Gamma(\sigma+s+1)}{\tau^{\sigma+s+1}}$. Simplifying,

$$V_t(s) \simeq u^s \exp\left(\frac{\lambda(u-1)}{u^\alpha}\right) \frac{1}{K} \int_0^\infty \xi^{\sigma+s} \exp\left(-\xi\left(t\left(\frac{u-1}{u^\alpha}\right)+\tau\right)\right) d\xi$$

$$= u^s \exp\left(\frac{\lambda(u-1)}{u^\alpha}\right) \frac{(\tau+t)^{\sigma+s+1}}{\Gamma(\sigma+s+1)} \frac{\Gamma(\sigma+s+1)}{\left(t\left(1+\frac{u-1}{u^\alpha}\right)+\tau\right)^{\sigma+s+1}}$$

$$= u^s \exp\left(\frac{\lambda(u-1)}{u^\alpha}\right) \left(\frac{\tau+t}{\tau+t\left(1+\frac{u-1}{u^\alpha}\right)}\right)^{\sigma+s+1}.$$

Then,

$$\frac{V_{t+1}(s+j)}{V_t(s)} = \frac{u^j \left(\frac{\tau+t+1}{\tau+(t+1)\left(1+\frac{u-1}{u^\alpha}\right)}\right)^{1+\sigma+s+j}}{\left(\frac{\tau+t}{\tau+t\left(1+\frac{u-1}{u^\alpha}\right)}\right)^{1+\sigma+s}}$$

$$= u^j A^{1+\sigma+s} B^j$$

where

$$A = \frac{\tau+t+1}{\tau+(t+1)\left(1+\frac{u-1}{u^\alpha}\right)} \frac{\tau+t\left(1+\frac{u-1}{u^\alpha}\right)}{\tau+t}$$

$$B = \frac{\tau+t+1}{\tau+(t+1)\left(1+\frac{u-1}{u^\alpha}\right)}.$$

Hence,

$$E_t(R_{t,t+1}\mid s) \simeq A^{1+\sigma+s} \frac{(\tau+t)^{1+\sigma+s}}{\Gamma(1+\sigma+s)} \int_0^\infty \xi^{\sigma+s} \exp\{-\xi(\tau+t+1-uB)\}\, d\xi$$

$$= A^{1+\sigma+s} \frac{(\tau+t)^{1+\sigma+s}}{\Gamma(1+\sigma+s)} \frac{\Gamma(1+\sigma+s)}{(\tau+t+1-uB)^{1+\sigma+s}}$$

$$= \left(\frac{\tau+t\left(1+\frac{u-1}{u^\alpha}\right)}{\tau+t\left(1+\frac{u-1}{u^\alpha}\right)-(u-1)\left(1-\frac{1}{u^\alpha}\right)}\right)^{1+\sigma+s}$$

as claimed. The long-run expected return is given by

$$E_t(R_{t,T}\mid s) \simeq \frac{\exp\{\lambda(u-1)\}\, E_t(\exp\{-\xi t(u-1)\})}{\exp\left(\frac{\lambda(u-1)}{u^\alpha}\right)\left(\frac{\tau+t}{\tau+t\left(1+\frac{u-1}{u^\alpha}\right)}\right)^{1+\sigma+s}}$$

$$= \frac{\exp\{\lambda(u-1)\left(1-\frac{1}{u^\alpha}\right)\}}{\left(\frac{\tau+t}{\tau+t\left(1+\frac{u-1}{u^\alpha}\right)}\right)^{1+\sigma+s}} \frac{(\tau+t)^{1+\sigma+s}}{\Gamma(1+\sigma+s)} \int_0^\infty \xi^{\sigma+s} e^{-\xi(\tau+tu)}\, d\xi$$

$$= \frac{\exp\{\lambda(u-1)\left(1-\frac{1}{u^\alpha}\right)\}}{\left(\frac{\tau+t}{\tau+t\left(1+\frac{u-1}{u^\alpha}\right)}\right)^{1+\sigma+s}} \frac{(\tau+t)^{1+\sigma+s}}{\Gamma(1+\sigma+s)} \frac{\Gamma(1+\sigma+s)}{(\tau+tu)^{1+\sigma+s}}$$

$$= e^{\lambda(u-1)\left(1-\frac{1}{u^\alpha}\right)} \left(\frac{\tau+t\left(1+\frac{u-1}{u^\alpha}\right)}{\tau+tu}\right)^{1+\sigma+s}$$

which reduces to the expression in Theorem 7. These approximations become exact in the limit as $\lambda \to \infty$, since then the support of the density over ξ tends to the half-line $[0, \infty)$.

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