EU Sovereign Default Risk and Capital - A Bayesian Approach

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Abstract

We analyze EU sovereign real-world default probabilities and correlations using a Bayesian GLMM and compare regulatory and economic capital requirements. The main findings are, first, that capital under the Basel Internal Ratings Based Approach (IRBA) is by a factor between 2.6 and 9.5 higher than under the Standardized Approach (SA), depending on the method for estimating the probability of default. This divergence is mainly driven by zero capital charges for highly rated securities under the SA. Second, under the Bayesian model, Basel IRBA capital is roughly equivalent to economic capital using the Expected Shortfall at a 97.5% confidence level. Third, the results suggest that the zero risk weights under the SA are not consistent with economic risk and offer opportunities for regulatory arbitrage.

Keywords: Low Default, Sovereigns, PSPP, Regulatory Arbitrage, Probability of Default, Risk Capital.

JEL classification: C11, C51, G01, G28

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1 Introduction

The risk of sovereign debt has been an important issue for academics, practitioners and politicians, particularly due to the emergence of the European sovereign debt crisis. As a consequence, politicians and regulators have recently paid attention to monitor and regulate sovereign debt holdings more closely. The Basel Committee on Banking Supervision initiated a review of the regulatory rules of sovereign risk in their working program for 2015-16, see Basel Committee on Banking Supervision (2016), which resulted in a discussion paper in 2017, see Basel Committee on Banking Supervision (2017).

Particularly, the regulatory standardized approach (SA) has been criticized, because banks holding highly rated sovereign debt require zero capital for these holdings under the SA[1], which can be viewed as a subsidy of government debt and a form of 'financial repression', see Hannoun (2013). The recent European debt crisis has given rise to the conjecture that these risk weights are not consistent with real risk and sovereign risk should be handled with care. This is even more important as a divergence between regulatory capital and real economic risk might be exploited by banks via regulatory capital arbitrage, see Jones (2000), thereby putting a bank into a riskier position while, at the same time, lowering her regulatory capital. An alternative to the SA in Basel II/III is the Internal Ratings Based Approach (IRBA). Under the IRBA banks have to estimate the probability of default (PD) for all debtors. This, however, requires a proper risk analysis and sound statistical estimation methods which are challenging because historical samples of defaulted sovereigns are scarce. The aim of this paper is to provide such a risk analysis of EU sovereign debt and to compare regulatory and economic capital approaches.

Most papers which analyze sovereign debt risks focus on their (i) dynamics, (ii) their drivers, and (iii) their implied default probabilities using data from sovereign bond yields or Credit Default Swap (CDS) spreads. Among others, Attinasi et al. (2009) use a dynamic panel model for explaining the widening of bond yield spreads during the European sovereign debt crisis and relate them to fiscal fundamentals. Hilscher and Nosbusch (2010) examine emerging markets sovereign CDS spreads with respect to macroeconomic variables. They find significant country fundamentals, such as the volatility of terms of trade, as significant variables and estimate default probabilities which provide explanatory power out-of-sample. Beirne and Fratzscher (2013) analyze advanced and emerging markets sovereign CDS spreads and find fundamentals as drivers for

[1] The Basel III standards even allow banks under the SA to reduce the risk weight for sovereign debt to zero when the default risk is appropriately small or national supervisory authorities permit this for dominated and funded exposures in home currency (Basel Committee on Banking Supervision (2006), p.20).

the increase in spreads during the European sovereign debt crisis. A related stream of literature focus on correlation and contagion between sovereign CDS spreads. Longstaff et al. (2011) use sovereign debt CDS spreads for advanced and emerging countries and find high commonality in the spreads indicating that sovereign credit risk is mainly driven by global factors and that these are more linked to the default risk component than the premium component. Beirne and Fratzscher (2013) find contagion effects for CDS spreads among a few markets.

These and other papers in this area have in common that their analyses are based on sovereign bond yields or CDS spreads. Yields and spreads are market based measures for risk and therefore contain investors' premia for risk aversion. In other words, risk measures, such as probabilities of default (PDs) which are estimated from spreads include risk premia (and possibly liquidity premia, see Nashikkar et al. (2011)) and are therefore based on the *risk-neutral* probability measure. While they offer valuable information about the risk structure as perceived and priced by the market, the risk-neutral measures might be far off from the real-world measures which are needed in practice for computing real-world risk, e.g. for purposes of bank regulation where banks' regulatory capital is calculated using real-world PDs. Studies which analyze real-world probabilities are, for example, due to Shumway (2001), Campbell et al. (2008) and Hilscher and Wilson (2017) using hazard rate or logistic regression models. However, these papers analyze real-world corporate default risk rather than sovereign default risk.

Hence, our paper differentiates from earlier literature in that we analyze EU sovereign debt risks, but using the real-world rather than the risk-neutral measure. The reason why literature on real-world default probabilities of sovereigns is scarce is simply because there is not much historical data, particularly sovereign defaults, which are required for estimating a statistical model, such as a hazard rate or logistic regression model. However, there are already approaches existing for estimating default probabilities of such 'low-default portfolios', namely the 'most prudent estimation principle' and the Bayesian approach. Pluto and Tasche (2005) suggest a 'most prudent' methodology to estimate PDs as the frontier of a likelihood band, assuming that the rating grades have monotonically increasing PDs. Comparable approaches were suggested by Forrest (2005) and Benjamin et al. (2006). The goal of these approaches is to estimate PDs even when no or only little defaults are in the sample at hand. Other approaches which use not only the default information but also migration events like Hanson and Schuermann (2006) or Fei et al. (2012) need at least a few defaults to obtain valid PDs.

Another strand in the literature attempts to fix the problem of low defaults using additional external information outside the estimation sample. Kiefer

(2011) shows that using the likelihood of the data for estimating PDs can yield implausible and misleading results in low default portfolios. Therefore he suggests linking observable data to external information given by a prior distribution via Bayesian statistics. In a low default portfolio the specification of the prior distribution is critical because the information in the estimation sample, the likelihood, is not very informative. Kiefer (2011) suggests extracting the prior distribution from expert judgment. Dwyer (2007) uses uninformative and Kazianka (2016) Jeffreys-style prior densities. In contrast to the former, Orth (2013) estimates the prior distribution from an additional data set in order to reduce the personal impact of a researcher or bank manager. As pointed out by Tasche (2013) the upper confidence bound for the PD of the most prudent approach could be represented as a special quantile of a Bayesian analysis. Hence, the most prudent and the Bayesian estimator are consistent with each other.

Our paper makes use of Bayesian statistics and estimates PDs and correlations for EU sovereigns by a Bayesian Generalized Linear Mixed Model (GLMM), see McNeil and Wendin (2007). The model allows a comparison of regulatory risk capital approaches with real-world economic risk and capital of sovereigns. Using our PD and correlation estimates we compare the regulatory SA with the IRBA, and both approaches with economic capital measured by the Expected Shortfall (ES). We find that the SA yields considerably lower risk capital than the IRBA and the economic approaches. This is mainly due to the zero risk weights for highly rated sovereigns in the SA. In addition to the lower amount of capital, the correlations between the SA and the IRBA capital, as well as between the SA and the economic capital, respectively, are low, which might create incentives for regulatory capital arbitrage. For banks abandoning the SA and using the IRBA instead, we find that IRBA capital is sensitive to the way how PDs are estimated (most prudent vs. Bayesian) and how capital is computed under the Bayesian method. IRBA capital is roughly consistent in size as well as rank order to economic capital measured by the ES on a 97.5% level. The results suggest that the SA should be treated with care - if not abandoned at all - and that Bayesian methods for PD estimating in low default portfolios, such as sovereign debt, might be promising.

The rest of the paper is organized as follows. Section 2 describes the data. Section 3 provides the empirical estimation approach and section 4 shows the estimation results. In section 5 we analyze regulatory and economic capital approaches. Section 6 summarizes the results and discusses policy implications.

2 Data

Our empirical analysis is based on long-term issuer foreign ratings of all EURO zone members from Moody's Corporate Bond Default Database. Our estimation sample contains ratings from 1999, the start of the EURO zone, up to 2015. We start with 11 sovereign ratings in 1999 and consider 19 in 2015. The analysis is performed on a yearly basis. Figure 1 shows the annual number of sovereigns and default rates. There are only two default events, one is Greece in 2012 with a rating grade of Ca and another is Cyprus in 2013 with a rating grade of $B3$.[2] Due to the low number of sovereigns, the default rate which has been 0% before 2012, jumps to 5.88% in 2012 and 6.25% in 2013.[3] Thus, in contrast to typical corporate portfolios with hundreds or thousands of borrowers, the volatility of the default rate is very large for sovereigns. Figure 2 shows the distribution of rating grades which is highly skewed to the right. That is, most ratings are Aaa, followed by Aa and A. At the beginning of the EURO zone in 1999 all members had a rating grade of $Aa3$ or better. As to the start of the European debt crises in 2009, Moody's substantially downgraded many of the EURO zone members. Greece was downgraded to $Ba1$ in 2010 and was the first member state with a non-investment grade rating.

3 Empirical Strategy

As we are interested in the real-world risk measures rather than the risk-neutral ones, we want to make use of the historical data, i.e., rating grades and defaults. This imposes at least three critical issues. First, the number of observations per year is very small as the maximum number of rated sovereigns per year is only 19 in 2015. Second, the number of defaults is very low (only two defaults, one in 2012 and one in 2013, respectively), that is, the event upon which we would like to base our estimation, is very rare. In other words, we have a low-default portfolio. Third, the time series is short, as we only have yearly data from 1999 to 2015, i.e., 17 yearly observations. It is well known that classical statistical methods for estimating probabilities using Maximum Likelihood, such as logistic regression, yield biased parameter estimates in small samples and sharply underestimate the probabilities for these rare events, see King and Zeng (2001). This is even worse if not only probabilities are estimated but also correlations between the events, see Gordy and Heitfield (2009). Thus, more

[2] Greece defaulted twice in 2012, March and December. As a consequence of our yearly approach we count one default for Greece only.

[3] Note that Greece is not part of our estimation sample in 2013 because it was in default at 2013/1/1.

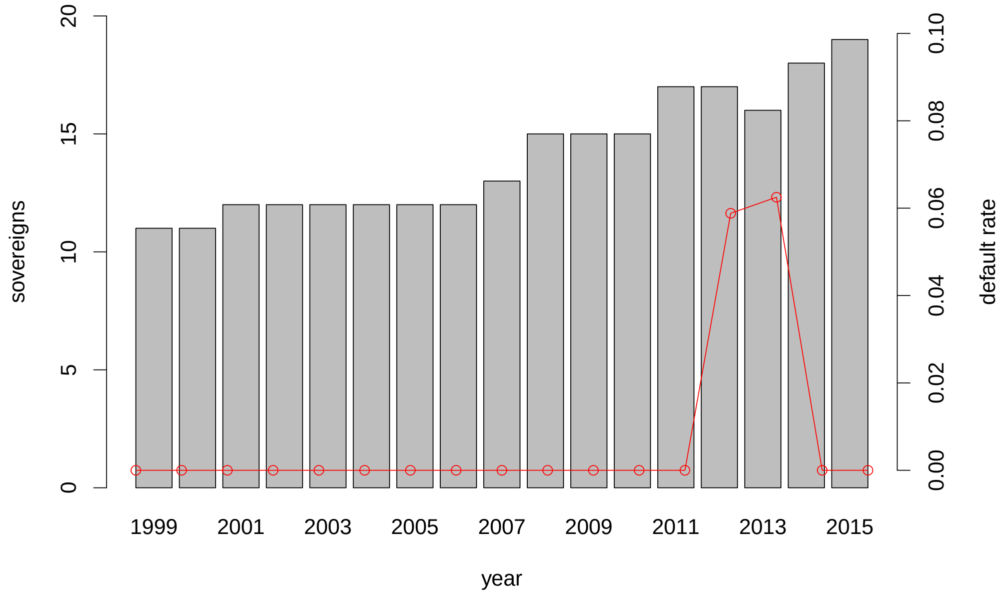

Figure 1: Number of sovereigns and annual default rate This figure shows the number of sovereigns (bars) and the default rate (dots and line) per year.

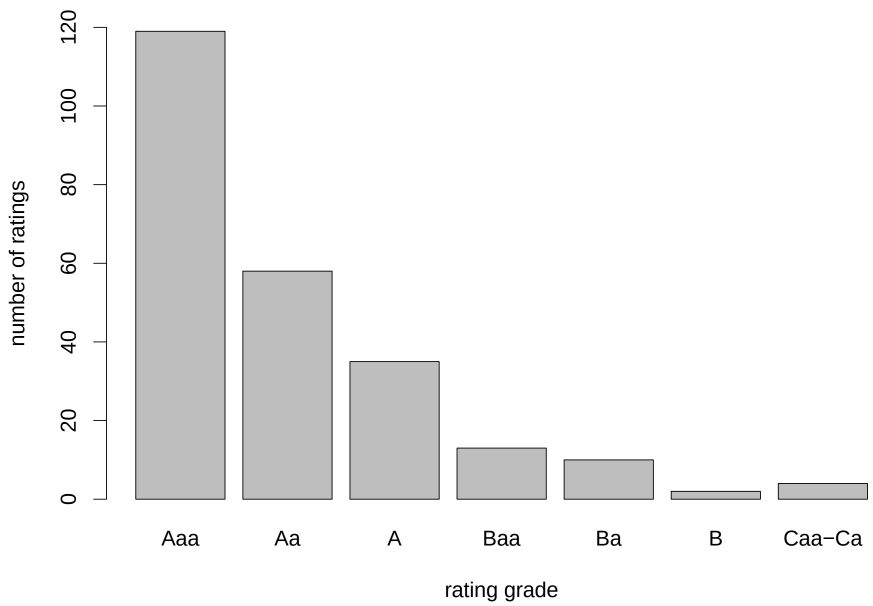

Figure 2: Distribution of rating grades This figure shows the distribution of rating grades (full letter ratings) for the entire time period from 1999 to 2015.

information beyond the observed sample data is eligible. This prior information can be linked to the sparse sample data using the Bayesian approach.

3.1 Bayesian Approach

Let $\boldsymbol{\beta}$ be a vector of unknown parameters and $\mathbf{x}$ be all relevant data. The primary goal of Bayesian analysis is the derivation of a posterior density $f(\boldsymbol{\beta}|\mathbf{x})$ for the unknown parameters given all relevant data $\mathbf{x}$ using a prior density $f(\boldsymbol{\beta})$ of the unknown parameters. The prior information and the data are linked via Bayes' Theorem:

$$f(\boldsymbol{\beta}|\mathbf{x}) = \frac{f(\mathbf{x}|\boldsymbol{\beta}) \cdot f(\boldsymbol{\beta})}{\int f(\mathbf{x}|\boldsymbol{\beta}) \cdot f(\boldsymbol{\beta}) \cdot \mathbf{d}\beta} \tag{1}$$

where $f(\mathbf{x}|\boldsymbol{\beta})$ is the Likelihood function from the data. The constant integral in the denominator ensures that the posterior $f(\boldsymbol{\beta}|\mathbf{x})$ is a probability density function. We first derive the likelihood and then show how the prior is specified.

3.2 Modeling the Likelihood

For modeling the likelihood we use the class of *generalized linear models* (GLM), see Nelder and Wedderburn (1972) and McNeil and Wendin (2007) for applications in credit risk. Consider a dependent variable y to be modeled by a vector of explanatory variables or covariates $\boldsymbol{x} = (1, x_1, ..., x_p)$ including a constant. For variable y we have independent observations $y_1, ..., y_n$, where y_i is considered as realization of a random variable Y_i $(i = 1, ..., n)$, and we have a deterministic covariate vector $\boldsymbol{x}_i = (1, x_{i1}, ..., x_{ip})$ for each observation. Generalized linear models then consist of two specifications:

1. *Distribution*: The Y_i are independent (given $\boldsymbol{x}_i$) and their distribution belongs to the exponential family.
2. *Link Function*: The mean $E(Y_i) = \mu_i$ is modeled by a link function g as $\mu_i = g^{-1}(\boldsymbol{x}_i\boldsymbol{\beta})$.

In our case the dependent variable is a binary event (default or non-default of a sovereign) and an appropriate distribution is the Bernoulli distribution. Define the binary default event as D_i, then

$$D_i = \begin{cases} 1 & \text{sovereign } i \text{ defaults} \\ 0 & \text{otherwise} \end{cases}$$

follows a Bernoulli distribution where $E(D_i|\boldsymbol{x}_i) = P(D_i|\boldsymbol{x}_i) = \pi_i$ is the *probability of default* (PD) and $\text{Var}(D_i|\boldsymbol{x}_i) = \pi_i(1-\pi_i)$.

Widely used link functions are the logistic function or the probit function. As both usually perform similarly in empirical applications and because the latter is consistent with a state-of-the-art credit risk model as it is implemented in Pillar 1 of the Basel Capital Accord, see Basel Committee on Banking Supervision (2006), Gordy (2000) and Gordy (2003), we decide to use the probit function given by

$$E(D_i|\boldsymbol{x}_i) = \pi_i = \Phi(\boldsymbol{x}_i\boldsymbol{\beta})$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution. The model can also be re-formulated in terms of a threshold model where a latent default propensity or credit quality index $R_i = -\boldsymbol{x}_i\boldsymbol{\beta} + \varepsilon_i$ is introduced where $\varepsilon_i \sim N(0,1)$ i.i.d for all i. A default happens when this latent variable crosses a threshold. As R_i is not observable in the GLM, the probit specification implies a normal distribution and the threshold can be standardized to the value of zero as it is absorbed by the constant of the model, that is,

$$R_i < 0 \Leftrightarrow D_i = 1$$

Under the normal distribution assumptions the analogy to structural credit risk models becomes clear, as R_i can be thought of as return on assets according to the model due to Merton (1974). Note that the probability of default is $P(R_i < 0) = P(-\boldsymbol{x}_i\boldsymbol{\beta} + \varepsilon_i < 0) = P(\varepsilon_i < \boldsymbol{x}_i\boldsymbol{\beta}) = \Phi(\boldsymbol{x}_i\boldsymbol{\beta})$.

The GLM is then extended into two directions: (i) A time-series dimension is included, and (ii) correlation between sovereign defaults is allowed. The time-series dimension is included by modeling D_{it} by deterministic covariates $\boldsymbol{x}_{it}$ ($i \in N_t$, where N_t is the set of sovereigns in the portfolio at the start of period t; $t = 1, 2, ..., T$), and D_{it} are assumed to be conditionally independent in the cross-section and across time.

Correlation is introduced by including a random effect along the time dimension, thereby resulting in a generalized linear mixed model (GLMM), see McNeil and Wendin (2007). The latent variable is then

$$\begin{aligned} R_{it} &= -\boldsymbol{x}_{it}\boldsymbol{\beta} + \varepsilon_{it} && (2) \\ &= -\boldsymbol{x}_{it}\boldsymbol{\beta} + \sqrt{\rho}\cdot\Psi_t + \sqrt{1-\rho}\cdot U_{it} && (3) \end{aligned}$$

where $\varepsilon_{it} = \sqrt{\rho}\cdot\Psi_t + \sqrt{1-\rho}\cdot U_{it}$ and $\Psi_t \sim N(0,1,)$, $U_{it} \sim N(0,1,)$ and independent from each other and over time. The parameter ρ gives the weight of the time-specific random effect Ψ_t and is often called asset correlation, because if R_{it} is the asset return in a framework due to Merton (1974), then $\rho = corr(R_{it}, R_{jt})$ for $i \neq j$. A higher correlation increases the probability for the occurrence of large simultaneous losses in a credit portfolio.

A sovereign i defaults at time t ($i \in N_t$, $t = 1, 2, ..., T$), if R_{it} crosses the default boundary:

$$R_{it} < 0 \Leftrightarrow D_{it} = 1$$

where D_{it} is the binary random event defined as

$$D_{it} = \begin{cases} 1 & \text{sovereign } i \text{ defaults in } t \\ 0 & \text{otherwise} \end{cases}$$

The *conditional probability of default* (conditional of the random effect $\Psi_t = \psi_t$ and the covariates) is given as

$$\begin{aligned} \lambda_{it}(\boldsymbol{x}_{it}, \psi_t) &= P(D_{it} = 1 | \boldsymbol{x}_{it}, \Psi_t = \psi_t) = P(R_{it} < 0 | \boldsymbol{x}_{it}, \Psi_t = \psi_t) && (4) \\ &= P\left(-\boldsymbol{x}_{it}\boldsymbol{\beta} + \sqrt{\rho} \cdot \psi_t + \sqrt{1-\rho} \cdot U_{it} < 0\right) && (5) \\ &= \Phi\left(\frac{\boldsymbol{x}_{it}\boldsymbol{\beta} - \sqrt{\rho} \cdot \psi_t}{\sqrt{1-\rho}}\right) \\ &= \Phi\left(\frac{\Phi^{-1}(\pi_{it}) - \sqrt{\rho} \cdot \psi_t}{\sqrt{1-\rho}}\right) \end{aligned}$$

where $\pi_{it} = \Phi(\boldsymbol{x}_{it}\boldsymbol{\beta})$ is the unconditional probability of default (PD) in the GLMM.

The ψ_t-conditioned likelihood at time t is then obtained as

$$L_t(\psi_t) = f_t(\boldsymbol{x}_{it} | \boldsymbol{\beta}, \psi_t) = \prod_{i \in N_t} \lambda_{it}(\boldsymbol{x}_{it}, \psi_t)^{d_{it}} \cdot (1 - \lambda_{it}(\boldsymbol{x}_{it}, \psi_t))^{1-d_{it}} \qquad (6)$$

and unconditionally by integrating over the distribution of the random effect Ψ_t as

$$L_t = f_t(\boldsymbol{x}_{it} | \boldsymbol{\beta}) = \int_{-\infty}^{\infty} \prod_{i \in N_t} \lambda_{it}(\boldsymbol{x}_{it}, \psi_t)^{d_{it}} \cdot (1 - \lambda_{it}(\boldsymbol{x}_{it}, \psi_t))^{1-d_{it}} \phi(\psi_t) d\psi_t \qquad (7)$$

where $\phi(\cdot)$ is the probability density function of the standard normal distribution. Finally, the likelihood is obtained as

$$L = f(\boldsymbol{x}_{it} | \boldsymbol{\beta}) = \prod_{t=1}^{T} \int_{-\infty}^{\infty} \prod_{i \in N_t} \lambda_{it}(\boldsymbol{x}_{it}, \psi_t)^{d_{it}} \cdot (1 - \lambda_{it}(\boldsymbol{x}_{it}, \psi_t))^{1-d_{it}} \phi(\psi_t) d\psi_t \qquad (8)$$

As covariates in the model we use the full letter rating grades by Moody's coded as dummy variables, that is we estimate one β-parameter per rating grade $j, j = 1, ..., J$ and omit the constant.

3.3 Prior Distribution

The specification of the multivariate prior density $f(\boldsymbol{\beta})$ of the parameters is very important. Especially, in low default portfolios as for sovereigns the likelihood contains only little information about the default probabilities and the correlation. With only little information from the likelihood, the prior distribution becomes more crucial.

In a classical Bayesian view, the prior density is the subjective belief of the researcher or bank manager about the density of parameters, before drawing a sample. Kiefer (2009) and Kiefer (2011) show how to obtain a prior density from expert information. This subjectivity might be problematic, when using the results for regulatory purposes because a regulator might have a more conservative view about the prior density than a bank manager. And even within a bank itself, the managers' beliefs can be quite different. Tasche (2013) suggests a continuous uniform density on $(0, 0.1)$ which leads to a conservative (mean) PD in a stylized low default portfolio with one default and 125 debtors. Dwyer (2007) uses a continuous uniform density on $(0, 1)$ for validating purposes of low default portfolios. Kazianka (2016) argues that all uniform densities are not uninformative and proposes a Jeffreys-style objective (reference) prior density which leads to a reduction of the mean PDs in contrast to the approach due to Tasche (2013). However, Kass and Wasserman (1996) point out that reference priors are very helpful with large samples but for small ones the choice of reference priors requires more serious attention due to limited information in the likelihood.

Another possibility to fix the prior density is to use the posterior density from former research results in the relevant field. Doing so, it is very important to ensure that the sample data of actual and former differ. Otherwise, the prior and the posterior would be derived from the same data. One possibility for sovereign default risk is to estimate risk-neutral default probabilities from CDS data and then convert them into real default probabilities such as Moody's sovereign EDFs. Nyambuu and Bernard (2015) show how to construct EDFs for sovereigns in a model like Moody's KMV using macroeconomic variables. However, the link between risk-neutral and real-world probabilities is ambiguous and requires strong assumptions as risk-neutral probabilities contain add-ons for (possibly time-varying) risk aversion and liquidity premia which are hard to disentangle.

In our modeling framework we want to specify the PD distributions for seven Moody's rating grades and a prior density for the asset-correlation ρ. We choose to follow Orth (2013) and estimate the prior of parameter β_j for each rating grade j, $j = 1, ..., J$ from the historical default rates of Moody's *corporates*, see Ou (2016), rather than from CDS spreads, because historical defaults rates should be closer to real-world default probabilities than CDS spread implied probabilities.

We use the same time period 1999 – 2015 as for the sovereign default data. We assume that all prior densities are independent from each other and the default probabilities $\pi_j = \Phi(\beta_j)$ for each grade j are beta distributed, that is, $\Phi\left(\beta_j\right) \sim b(a_j, b_j)$, $j = 1, ..., J$. The two relevant parameters a_j and b_j for each rating grade j are estimated with the Method of Moments as in Orth (2013), assuming issuer specific weighting. For Aaa rated corporates Moody's does not observe any defaults and as a consequence we cannot estimate the relevant parameters from the historical default rates. We set a_{Aaa} and b_{Aaa} so that the mean of the prior beta density is only half as large as for Aa and the variance of Aaa and Aa are the same. For the asset correlation ρ we suggest a beta distribution as prior with a mean of 20 % and a standard deviation of 5%, see Jacobs Jr and Kiefer (2010).

4 Estimation Results

Table 1 shows the Method of Moments estimates for the seven rating grades which are obtained from Moody's corporate default rate history. These serve as parameters of the beta distributions for each grade which we use as priors for the Bayesian model.

Table 1: Estimated parameters for the prior beta densities. Moody's corporate default rates from 1999-2015 are used to estimate the parameters of a beta distribution per rating grade with Method of Moments.

Rating j	a_j	b_j	Mean	St.Dev.
Aaa	0.054	213.882	0.025%	0.108%
Aa	0.216	428.547	0.050%	0.108%
A	0.874	1127.680	0.077%	0.083%
Baa	0.619	245.272	0.252%	0.319%
Ba	1.227	180.780	0.674%	0.605%
B	0.858	32.975	2.535%	2.663%
Caa-Ca	1.448	13.259	9.848%	7.518%

The evaluation of the posterior density is done with JAGS via Markov-Chain-Monte-Carlo (MCMC) with slice sampling, see Neal (2003). After the burn-in phase we simulate ten chains with ten million iterations and include only every tenth observation. Our final MCMC sample has ten million values for each parameter. We do not impose the restriction that the unconditional default probability for better rating grades have smaller values than for worse rating grades in each iteration step, thus in some iterations there might be a non-monotone behavior of rating grades. Table 2 presents the results from the MCMC simulation. Figure 3 shows the resulting marginal posterior distributions for each grade and the asset correlation. Figure 4 plots the trace plots for all parameters which indicate that only β_{Aaa} has some problems to converge. To overcome this, we simulate ten million observations.

Table 2: Summary statistics of parameters - MCMC sample (unconditional PDs and asset-correlation)

Parameter	Mean	St.Dev.	IQR	Min.	Median	Max.
Aaa	0.011%	0.062%	0.000%	0.000%	0.000%	1.261%
Aa	0.044%	0.095%	0.041%	0.000%	0.006%	2.616%
A	0.074%	0.080%	0.085%	0.000%	0.049%	1.251%
Baa	0.236%	0.300%	0.285%	0.000%	0.127%	6.193%
Ba	0.628%	0.565%	0.650%	0.000%	0.469%	8.269%
B	5.237%	3.673%	4.616%	0.000%	4.422%	45.257%
Caa-Ca	13.306%	7.697%	10.207%	0.016%	11.977%	67.420%
ρ	19.948%	4.985%	6.745%	2.721%	19.630%	99.999%

The mean and the standard deviation of unconditional PDs for the better rating grades are quite similar to those of the prior densities except for Aaa where PD and standard deviation are smaller in the posterior distribution. For the rating grades with default observations, B and Caa-C, we obtain distinctly higher means and standard deviations. This is due to the high default rates in these rating grades and the low number of observations. In rating grade B we notice the default of Cyprus with overall two observations and in rating grade Caa-Ca Greece defaults with overall four observations only. The posterior density of the asset correlation is nearly unchanged to the prior density. Lopez (2004) and Duellmann et al. (2010) find in their literature reviews values from 0.3% to 26% for asset correlations, dependent, inter alia, on rating, industry sector and estimation technique. In a Bayesian framework like ours, McNeil and Wendin (2007) estimate a mean value of the posterior density from 7.5% using Standard and Poor's data for U.S. corporate firms. Also Luo and Shevchenko (2013) obtain a mean of 8.15% for 1982-2010 in a Bayesian model setup which includes recovery risk. Estimates for risk-neutral asset correlations from equity or CDS data can even be much higher, see Friewald (2009) who obtains values for asset correlations from CDS spreads between 60% and 80%. Boudreault et al. (2015) get a value of 57.6% for companies which constitute the CDX North American Investment Grade Index (CDX.NA.IG.17).

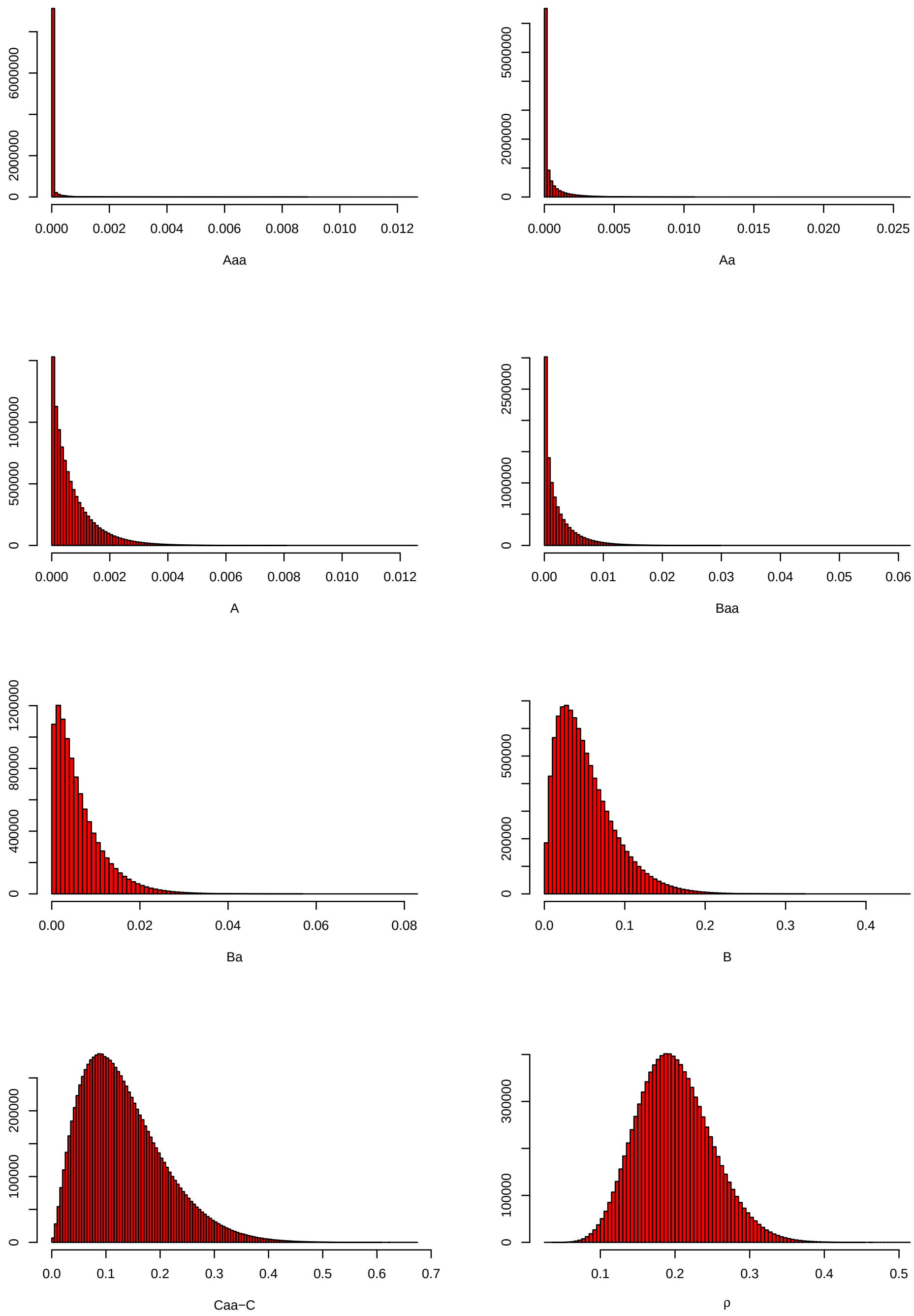

Figure 3: Marginal posterior distributions This figure graphs the marginal posterior distributions of unconditional PDs and asset correlation ρ.

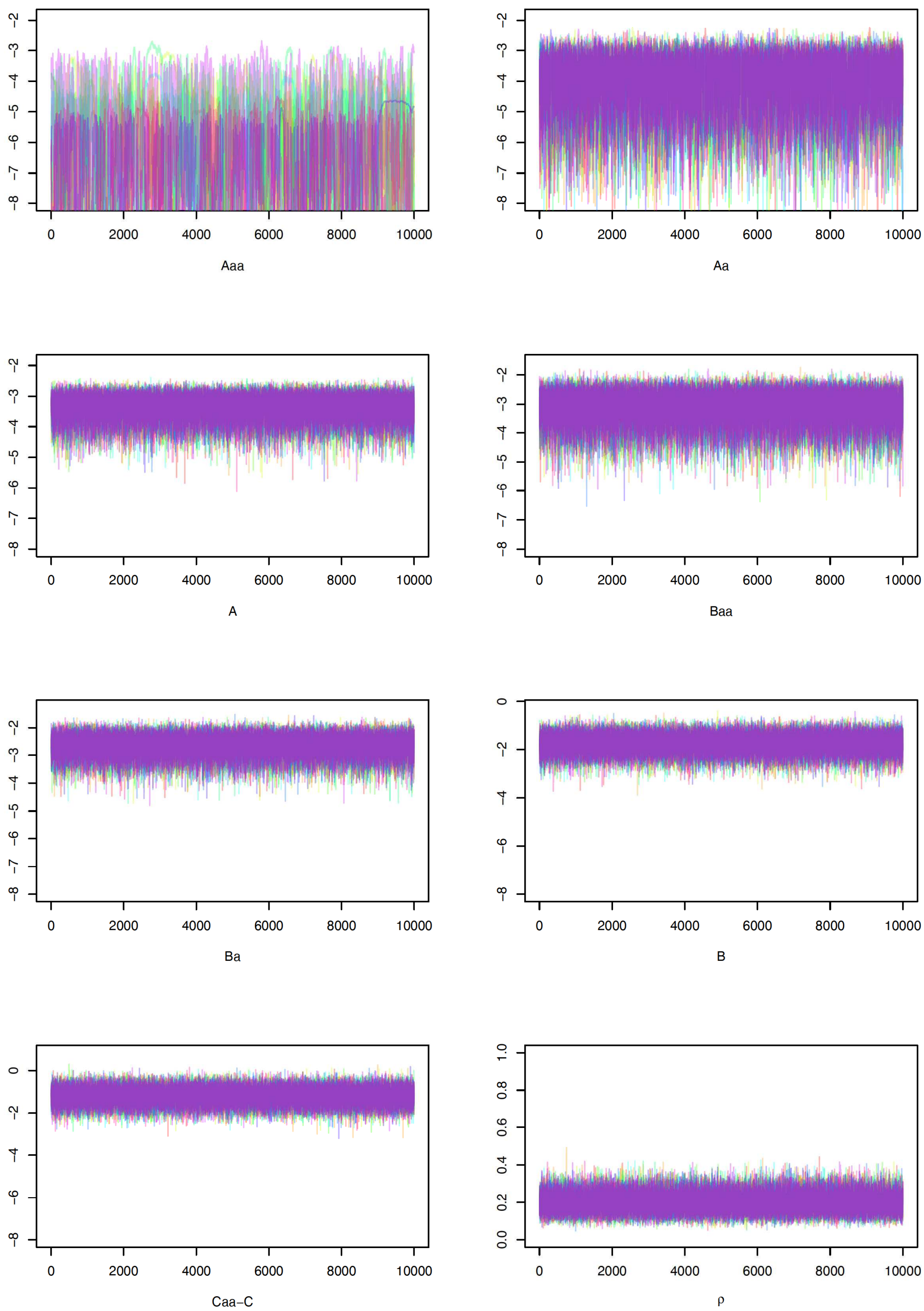

Figure 4: Trace plots This figure graphs trace plots for $\Phi^{-1}(\pi_{it})$ (linear predictor) for all rating classes and asset-correlation ρ (simulation steps $500,000 - 510,000$, ten different chains).

In general, these diverging results in the literature show that estimates for asset correlations are very noisy which can be accounted for in our Bayesian approach because rather than a single estimate we obtain a full posterior distribution. In our data set, the asset correlation should be higher than for corporates because the Euro zone members may have more pronounced dependencies than big U.S. companies from different industrial sectors which have no or little contractual relations. This is also consistent with Longstaff et al. (2011) who show that sovereigns have higher correlations than equity index returns for the same countries. Therefore, our posterior distribution for the asset-correlation with a mean around 20% is not implausible.

Figure 5 shows the means of the random effects per year. Positive values refer to adverse and negative values to good economic conditions. Before 2012 the means of the posterior densities are slightly negative. With the default of Greece in 2012, the mean of the random effect equals 0.36. It increases to 0.45 in 2013, the year of the default of Cyprus. The higher value in 2013 is due to the better rating grade of Cyprus in the year of default. After 2013, the means of the random effects are more negative than before 2012 due to rating downgrades of some Euro zone members. This effect makes the zero default rates per year more unlikely.

5 Regulatory and Economic Capital

5.1 The Public Sector Purchase Programme (PSPP) Portfolio

Using the posterior densities from the previous section we now compute regulatory (RCR) and economic capital requirements (ECR) for the PSPP. The PSPP is part of the expanded asset purchase programme (APP) of the European Central Bank (ECB). It allows the ECB resp. the national central banks to buy bonds from private resp. public sector up to 80,000 mill. EURO against central bank money. From January 2018 the monthly net asset purchases of the APP are limited to 30,000 mill. EURO. Main goal of the APP is to reduce the risk of deflation in the EURO zone and help to reach the target inflation level of 2%. The share of a sovereign on the PSPP is determined by its ECB's capital key.

We use the PSPP as of 2016/06/30 for our capital exercise. The only EURO member state who is not part of the program is Greece, because of their low rating grade. Table 3 shows the ratings and exposures at default (EADs) of all participating EURO member states. As of end of June 2016 the total PSPP holdings to these sovereigns were 782,122 mill. EURO where the main constituents were Germany, France, Italy and Spain.[4]

[4]We do not consider international and supranational institutions and agencies like the

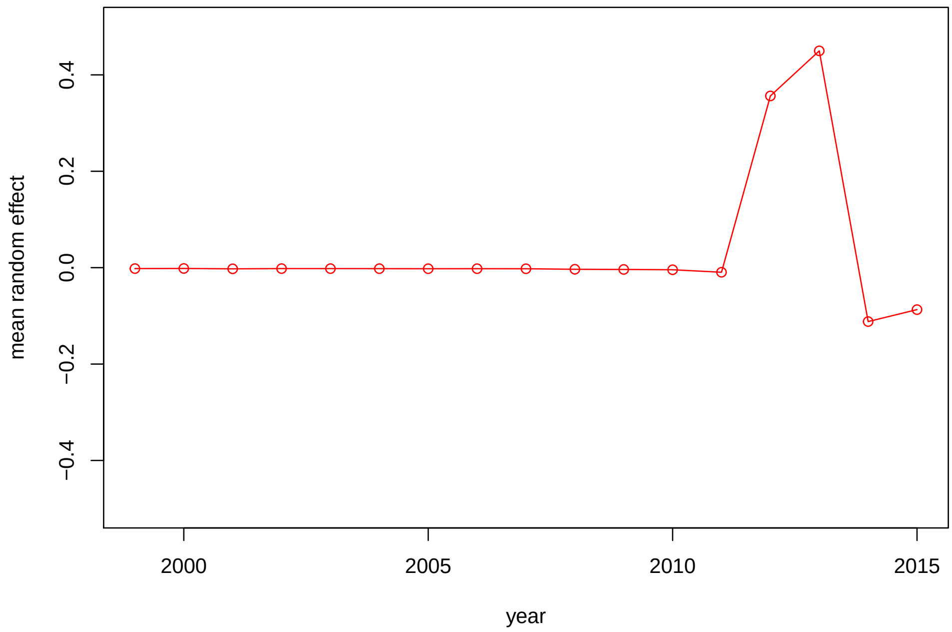

Figure 5: Mean random effect This figure shows the means of the posterior density for the random effects per year.

Table 3: PSPP-program as of 2016/06/30

Country	Rating	EAD_{it} (in mill. EURO)	Maturity
Austria	Aa1	22,750	8.92
Belgium	Aa3	28,670	9.65
Cyprus	B1	269	5.27
Estonia	A1	66	2.04
Finland	Aa1	14,615	7.53
France	Aa2	165,591	7.65
Germany	Aaa	208,269	7.44
Ireland	A3	13,251	9.38
Italy	Baa2	144,009	9.29
Latvia	A3	1,023	6.62
Lithuania	A3	1,772	6.62
Luxembourg	Aaa	1,615	6.37
Malta	A3	586	10.87
Netherlands	Aaa	46,365	7.57
Portugal	Ba1	19,138	10.14
Slovakia	A2	7,069	8.02
Slovenia	Baa3	3,729	8.27
Spain	Baa2	103,335	9.73
All		782,122	

5.2 Regulatory Capital

The foundation for regulatory minimum capital is Pillar I of the Basel II/III framework, see Basel Committee on Banking Supervision (2006) resp. Basel Committee on Banking Supervision (2011). Under this framework the regulatory capital for sovereigns can be computed using the Standardized (SA) or the Internal Ratings-based Approach (IRBA). Under the SA, estimates for the PDs are not eligible to calculate the regulatory capital. Rather, the relevant risk weights (RW_{it}^{SA}) for sovereigns are 100%, or, if a bank has nominated a rating agency, have to be obtained from Table 4 (see Basel Committee on Banking Supervision (2006), p. 19).[5]

Table 4: Risk weights - Standardized Approach

Rating	Aaa to Aa3	A1 to A3	Baa1 to Baa3	Ba1 to B3	Below B3
Risk weight	0%	20%	50%	100%	150%

The risk-weighted asset of sovereign i is then

$$RWA_{it}^{SA} = RW_{it}^{SA} \cdot EAD_{it} \cdot 12.5 \tag{9}$$

Regulatory minimum capital (regulatory capital requirement, RCR) for a bank at time t has to be calculated according to

$$RCR_t^{SA} = \sum_{i=1}^{n} RCR_{it}^{SA} = \sum_{i=1}^{n} RWA_{it}^{SA} \cdot 0.08 \tag{10}$$

Given the ratings and EADs we can straightforwardly computed the regulatory minimum capital for the PSPP according to the SA.

Another possibility for meeting the legal requirements for regulatory capital is the IRBA. Here, banks are allowed to use their own estimates for the relevant risk drivers PD, EAD, loss given default (LGD) and maturity. In this article we focus only on the first risk driver, the PD π_{it}, and do not estimate the others. The LGD value is fixed to 0.765, which is the market LGD for Greece, see Tudela (2015). The maturity M_{it} and EAD_{it} are from the PSPP, see Table 3. The RWAs

European Investment Bank. As of end of June 2016 the total PSPP holdings to these institutions are 102,426 mill. EURO.

[5]Under Basel II/III national supervisory authorities could allow banks to reduce the risk weight to zero (see Basel Committee on Banking Supervision (2006), p. 20). Currently, all Committee members use this discretion (see Basel Committee on Banking Supervision (2017), p. 19).

under the IRBA are computed as (Basel Committee on Banking Supervision (2006), p. 64):[6]

$$
\begin{aligned}
\rho_{IRBA}(\pi_{it}) =& 0.12 \cdot \frac{1-e^{-50\cdot\pi_{it}}}{1-e^{-50}} + 0.24 \cdot \frac{1-(1-e^{-50\cdot\pi_{it}})}{1-e^{-50}} \\
b(\pi_{it}) =& (0.11852 - 0.05478 \cdot \ln(\pi_{it}))^2 \\
RW_{it}^{IRBA}(\pi_{it}) =& (LGD_{it} \cdot \Phi\left(\frac{\Phi^{-1}(\pi_{it})}{\sqrt{1-\rho_{IRBA}(\pi_{it})}} + \sqrt{\frac{\rho_{IRBA}(\pi_{it})}{1-\rho_{IRBA}(\pi_{it})}} \cdot \Phi^{-1}(0.999)\right) \\
& - \pi_{it} \cdot LGD_{it}) \cdot \frac{1+(M_{it}-2.5)\cdot b(\pi_{it})}{1-1.5\cdot b(\pi_{it})} \\
RWA_{it}^{IRBA}(\pi_{it}) =& RW_{it}^{IRBA}(\pi_{it}) \cdot EAD_{it} \cdot 12.5
\end{aligned}
\tag{11}
$$

In our Bayesian approach we do not have a single estimated PD π_{it} but rather a whole posterior distribution. Kiefer (2011) suggests two approaches to obtain a single number from the entire distribution. The first approach is to use a point estimate from the PD distribution $\bar{\pi}(\boldsymbol{x}_{it})$, such as the mean,[7] and calculate the regulatory capital under the IRBA for this single value ($RCR_{it,mean}^{IRBA}$). The second is to calculate the regulatory capital $RCR_{it}^{IRBA}(\pi_{it})$ for all steps of our MCMC sample and then take the mean ($RCR_{it,sim}^{IRBA}$). If the PD is smaller than 1 bp we set $RCR_{it}^{IRBA}(\pi_{it})$ to zero. This is due to the behavior of the Basel II/III capital formula which produces meaningless values for very small PDs (see Kiefer (2011)). For countries with a Aaa resp. Aa rating this restriction is relevant in 79% resp. 35% of all cases of the second approach.

As a reference for the PDs we use the 'most prudent' estimating methodology, see Pluto and Tasche (2011). This approach leads to very high probabilities and can be interpreted as upper limit for (mean) PDs in low default portfolios. Using this approach for our estimation samples with a confidence level of 50% leads to unrealistic high PDs π_{it}^{mp} especially for sovereigns with Aaa rating. The most prudent PD for these sovereigns are 1.108% in contrast to a mean PD of 0.011% under the Bayesian approach. As proposed by Pluto and Tasche (2011) we scale the overall most prudent PDs $\pi_{it}^{mp,scale}$ to 0.830%, the overall default rate of our estimation sample. Thus, these PD estimates become somewhat less conservative. Moreover, we set the PD of rating grade Caa-Ca to the PD of rating

[6]Large or unregulated financial institutions have to multiply $\rho_{IRBA}(\pi_{it})$ with 1.25, see Basel Committee on Banking Supervision (2011), pp. 39–40. We do not consider this adjustment when calculating the RWA.

[7]Taking the mean of the posterior is equivalent to minimizing a quadratic loss function, see Greenberg (2013).

grade B because the most prudent approach does not ensure that the PDs are increasing.

All PD estimates for the PSPP as of 2016/06/30 can be obtained from Table 5. The scaled most prudent PDs are still substantially higher than the mean PDs of the Bayesian approach except for rating grade Caa-Ca . However, for the PSPP this is irrelevant because no country falls into this rating grade.

Table 5: Unconditional PDs IRBA approach – PSPP as of 2016/06/30

Rating	$\bar{\pi}(\boldsymbol{x}_{it})$	$\pi_{it}^{mp,scale}$	π_{it}^{mp}
Aaa	0.011%	0.236%	1.108%
Aa	0.044%	0.466%	2.186%
A	0.074%	0.885%	4.156%
Baa	0.236%	1.941%	9.115%
Ba	0.628%	3.485%	16.365%
B	5.237%	8.975%	42.141%
Caa-Ca	13.306%	8.975%	42.141%
PSPP	0.402%	1.454%	6.825%

The regulatory capital requirements are reported in Table 6 where the first column shows the country and the second column its rating as of 2016/06/30. The third and fourth columns show the regulatory capital requirements under the SA (both absolute and relative); the remaining columns show the corresponding values for both Bayesian approaches and the scaled most prudent approach. According to the SA, all sovereigns with rating grades better than A1 have zero RWAs and therefore no regulatory capital requirements as a consequence. These countries have a proportion on the total EAD of 62.38%. The total risk weighted assets under the SA are 149,697 mill. EUR and, as a result, the regulatory capital is $149,697$ mill. EUR $\cdot\, 8\% = 11,976$ mill. EUR or 1.53% of the total volume under risk.

Looking at the regulatory capital requirements (RCR) under the IRBA approach, we see that they are more than 2.6 ($RCR_{t,sim}^{IRBA}$), 3.7 ($RCR_{t,mean}^{IRBA}$), resp. 9.5 ($RCR_{t,mp}^{IRBA}$) times higher than under the SA, depending on which PD estimate is used. Responsible for this divergence are mainly the sovereigns with a rating better than A1 (mainly Germany and France) because they have zero RWAs under the SA approach and as a consequence no capital requirements.

The two approaches using the Bayesian method lead to remarkably different results. The RCR for the PSPP as a whole increases by 42% when using only $\bar{\pi}(\boldsymbol{x}_{it})$ instead of taking the average of all simulated RCRs. When looking at the members of the program we see that especially the highly rated sovereigns

Table 6: Country-specific regulatory capital requirements - PSPP

Country	Rating	RCR_{it}^{SA}		$RCR_{it,mean}^{IRBA}$		$RCR_{it,sim}^{IRBA}$		$RCR_{it,mp}^{IRBA}$	
Austria	Aa1	0	0.00%	976	2.21%	613	1.97%	2,972	2.62%
Belgium	Aa3	0	0.00%	1,230	2.78%	772	2.49%	3,745	3.30%
Cyprus	B1	22	0.18%	67	0.15%	64	0.21%	78	0.07%
Estonia	A1	1	0.01%	2	0.00%	2	0.01%	7	0.01%
Finland	Aa1	0	0.00%	627	1.42%	394	1.27%	1,909	1.68%
France	Aa2	0	0.00%	7,106	16.07%	4,462	14.38%	21,631	19.07%
Germany	Aaa	0	0.00%	4,386	9.92%	1,335	4.30%	20,468	18.04%
Ireland	A3	212	1.77%	745	1.68%	643	2.07%	2,157	1.90%
Italy	Baa2	5,760	48.10%	14,160	32.03%	11,189	36.05%	28,544	25.16%
Latvia	A3	16	0.14%	57	0.13%	50	0.16%	167	0.15%
Lithuania	A3	28	0.24%	100	0.23%	86	0.28%	289	0.25%
Luxembourg	Aaa	0	0.00%	34	0.08%	10	0.03%	159	0.14%
Malta	A3	9	0.08%	33	0.07%	28	0.09%	95	0.08%
Netherlands	Aaa	0	0.00%	976	2.21%	297	0.96%	4,557	4.02%
Portugal	Ba1	1,531	12.78%	2,789	6.31%	2,430	7.83%	4,285	3.78%
Slovakia	A2	113	0.94%	397	0.90%	343	1.10%	1,151	1.01%
Slovenia	Baa3	149	1.25%	367	0.83%	290	0.93%	739	0.65%
Spain	Baa2	4,133	34.51%	10,161	22.98%	8,029	25.87%	20,482	18.06%
PSPP		11,976	100.00%	44,214	100.00%	31,037	100.00%	113,436	100.00%

(e.g., Germany and France) are responsible for this increase. Sovereigns with an Aaa rating are more than doubled in their RCRs. The main reason for these divergences is that the posterior distributions are highly skewed to the right and that the capital function is concave.

In most cases, Italy and Spain are the two countries with the highest RCRs because they have worse rating grades (high PDs) and large exposures. Only in the most prudent approach the relative risk exposure of France is slightly higher than that of Spain. This is due to the high exposure in the PSPP and the relative high PDs for Aa rated sovereigns under the most prudent approach. We have to stress that our most prudent approach is only moderately conservative, because we scaled the estimated overall PD to the overall default rate of the estimation sample.

Regarding regulatory capital we summarize the main results as follows:

- The regulatory capital requirements under the SA are quite small compared to all IRBA approaches.
- In particular, all IRBA approaches lead to non-zero risk weights, in contrast to the SA for sovereigns with rating better than A1.
- Depending on the method for estimating the PDs we find considerable differences between the IRBA approaches, and particularly the most prudent estimates - though scaled - yield the most conservative capital buffers.

5.3 Economic Capital

Besides minimum regulatory capital requirements under Pillar 1, each bank also has to fulfill the Internal Capital Adequacy Assessment Process (ICAAP) of Basel II/III (Pillar 2). For this purpose most banks compute some risk measures for future potential losses using a more or less sophisticated internal credit portfolio model.

The future losses from the PSPP for country i for time period 2016/07/01 to 2017/06/30 are modeled as

$$L_{it} = D_{it} \cdot LGD_{it} \cdot EAD_{it} \tag{12}$$

and the total future losses of the PSPP are

$$L_t = \sum_{i=1}^{18} L_{it} \tag{13}$$

where EAD_{it} is the exposure for sovereign i to the PSPP as of 2016/06/30 and LGD_{it} is set to 0.765, the market LGD for Greece for all countries, see Tudela (2015).

The two most popular risk measures are Value-at-Risk (VaR) and Expected Shortfall (ES).

As in Acerbi and Tasche (2002) we define the VaR at level α as the lowest value that meets the α-quantile definition. The ES of a portfolio is then given by

$$ES_t^{\alpha}(L_t) = \frac{1}{\alpha} \cdot E[L_t \cdot \mathbb{1}_{(L_t \geq VaR_t^{\alpha}(L_t))}] + \frac{\alpha - P(L_t \geq VaR_t^{\alpha}(L_t))}{\alpha} VaR_t^{\alpha}(L_t) \tag{14}$$

The individual contribution of a sovereign i to the portfolio ES at time period t is defined as

$$ES_{it}^{\alpha}(L_{it}) = \frac{1}{\alpha} \cdot E[L_{it} \cdot \mathbb{1}_{(L_t \geq VaR_t^{\alpha}(L_t))}] + \frac{\alpha - P(L_t \geq VaR_t^{\alpha}(L_t))}{\alpha} VaR_{it}^{\alpha}(L_t) \tag{15}$$

where $VaR_{it}^{\alpha}(L_t)$ is the loss of country i when $L_t = VaR_t^{\alpha}(L_t)$.

We use ES instead of VaR for the following analyses because the VaR is not subadditive and coherent, see Artzner et al. (1999).

The committee economic minimum capital (economic capital requirement, ECR) for a bank, for example using a confidence level of 97.5%, has to be calculated according to

$$ECR_t^{97.5\%} = \sum_{i=1}^{n} ECR_{it}^{97.5\%}(L_{it}) = \sum_{i=1}^{n} \left(ES_{it}^{97.5\%}(L_{it}) - E[L_{it}] \right) \tag{16}$$

As for regulatory minimum capital we have two possibilities to predict the loss distribution L_t from the posterior density of the parameters. First, we use the mean of each parameter and simulate the loss distribution ($L_{t,mean}$). In the second approach we account for the uncertainty of parameters. We follow Luo and Shevchenko (2013) and calculate the loss distribution for the PSPP from the full predictive loss density. For each run of our MCMC-sample we simulate one loss, only. At the end we obtain a loss distribution with 10 million entries ($L_{t,sim}$). Table 7 shows the results for the economic capital requirements (ECR) using both methods for confidence levels of $\alpha \in \{97.5\%, 99\%, 99.9\%\}$ where each row contains the country ECR contribution and the last row shows the portfolio ECR (in absolute numbers as mill. EUR and in percent).

For the lower risk levels (97.5% resp. 99%) and both methods, Italy and Spain are the two countries with the highest contributions to the economic capital. This is due to the high PDs and exposures of both countries. At the 99.9%-level Italy and France are the two sovereigns with by far the highest economic capital requirements followed by Germany and Spain. Beside the PDs the main reasons

Table 7: Country-specific risk contributions - PSPP

Country	Rating	$ECR_{it}^{97.5\%}(L_{t,mean})$		$ECR_{it}^{99\%}(L_{t,mean})$		$ECR_{it}^{99.9\%}(L_{t,mean})$		$ECR_{it}^{97.5\%}(L_{t,sim})$		$ECR_{it}^{99\%}(L_{t,sim})$		$ECR_{it}^{99.9\%}(L_{t,sim})$	
Austria	Aa1	306	1,18%	777	1,29%	199	0,15%	297	1,14%	753	1,26%	220	0,17%
Belgium	Aa3	378	1,46%	959	1,60%	188	0,14%	374	1,44%	950	1,59%	216	0,17%
Cyprus	B1	66	0,26%	26	0,04%	69	0,05%	64	0,25%	23	0,04%	62	0,05%
Estonia	A1	0,3	0,00%	0,2	0,00%	0,3	0,00%	0,3	0,00%	0,2	0,00%	0,4	0,00%
Finland	Aa1	190	0,73%	34	0,06%	101	0,08%	194	0,75%	43	0,07%	122	0,09%
France	Aa2	2,200	8,48%	5,584	9,30%	56,351	43,37%	2,194	8,46%	5,568	9,30%	56,188	43,03%
Germany	Aaa	794	3,06%	2,014	3,36%	20,328	15,64%	712	2,75%	1,808	3,02%	18,240	13,97%
Ireland	A3	289	1,11%	46	0,08%	129	0,10%	297	1,14%	45	0,08%	125	0,10%
Italy	Baa2	10,201	39,32%	25,894	43,13%	47,677	36,69%	10,203	39,36%	25,901	43,25%	49,016	37,54%
Latvia	A3	23	0,09%	3	0,01%	10	0,01%	23	0,09%	3	0,01%	8	0,01%
Lithuania	A3	40	0,15%	6	0,01%	19	0,01%	40	0,15%	5	0,01%	17	0,01%
Luxembourg	Aaa	6	0,02%	1	0,00%	4	0,00%	5	0,02%	1	0,00%	4	0,00%
Malta	A3	13	0,05%	2	0,00%	6	0,00%	13	0,05%	2	0,00%	5	0,00%
Netherlands	Aaa	178	0,69%	452	0,75%	198	0,15%	150	0,58%	382	0,64%	127	0,10%
Portugal	Ba1	3,552	13,69%	5,667	9,44%	1,256	0,97%	3,599	13,88%	5,707	9,53%	997	0,76%
Slovakia	A2	154	0,59%	25	0,04%	76	0,06%	156	0,60%	24	0,04%	58	0,04%
Slovenia	Baa3	265	1,02%	35	0,06%	116	0,09%	264	1,02%	46	0,08%	193	0,15%
Spain	Baa2	7,289	28,10%	18,503	30,82%	3,212	2,47%	7,336	28,30%	18,621	31,10%	4,974	3,81%
		25,943	100,00%	60,032	100,00%	129,937	100,00%	25,921	100,00%	59,882	100,00%	130,577	100,00%

for the high economic capital requirements for France and Germany are that the EADs of the PSPP are very heterogeneous, our overall PD is relative low and the number of sovereigns in our portfolio is very small. The 99.9%-VaR occurs in the simulation only when Cyprus and Italy simultaneously default. So, every default of France and Germany is considered when calculating $ES_{it}^{99.9\%}$. The individual risk contributions at the 99.9%-level of France and Germany are ten times higher than to the 99%-level. The overall economic capital requirement increases by a factor of 2.2, only. Both approaches ($L_{t,sim}$ and $L_{t,mean}$) show very similar figures.

Due to the low number of sovereigns and the heterogeneity of EADs and PDs we observe an absolute risk reduction for some sovereigns when changing from 97.5%- to 99%-level resp. 99%- to 99.9%-level. Most affected from this anomaly is Spain when changing from 99%- to 99.9%-level. The 97.5%- resp. 99%-VaR occurs in the simulation when Cyprus resp. Portugal defaults, only. As a consequence, all countries with a higher exposure than Cyprus resp. Portugal have no impact on the amount of the Value-at-Risk, but have an even higher impact on the relevant Expected Shortfall. This effect is very similar for both approaches.

5.4 Economic vs. regulatory capital

Finally, we compare the individual regulatory and economic capital requirements. Table 8 shows Kendall's τ as a measure of rank order dependence among the various approaches. We see that regulatory capital under the SA has a different rank order (low Kendall's τ) compared to all other risk sensitive approaches. Here, the regulatory capital of the most prudent approach is more comparable to the Bayesian approaches than the SA with Kendall's τ reaching from 0.78 to 0.92. All Bayesian approaches have virtually the same rank order. Their Kendall's τ reaches from 0.80 to 1. Note that the absolute capital requirements of the different approaches and risk levels has no influence on the calculated dependency measures. However, a different rank order of capital across approaches indicates incentives for regulatory arbitrage, particularly across economic capital (i.e., real risk) and regulatory capital, as banks may exchange securities with high regulatory capital and low risk against low capital securities with high risk.

As can be seen from Table 9, the minimum capital requirements of the various approaches are quite different at first sight, depending on the risk (confidence) level. The risk level for the economic capital requirements has to be individually set by a bank in advance of their overall risk profile, target rating and controlling approach (going-concern resp. gone-concern).

However, for a risk level of 97.5% we have comparable figures for the economic capital as under $RCR_{t,sim}^{IRBA}$. The regulatory capital requirements for the most prudent approach, with calibration to the overall default rate of the estimation sample, lies between the risk levels of 99% and 99.9%. The economic capital for a risk-level of 99.9% is only 15% higher than the highest regulatory capital requirements under the most prudent approach.

Table 8: Kendall's τ - regulatory vs. economic capital requirements

	RCR_{it}^{SA}	$RCR_{it,mean}^{IRBA}$	$RCR_{it,sim}^{IRBA}$	$RCR_{it,mp}^{IRBA}$	$ECR_{it,sim}^{97.5\%}$	$ECR_{it,sim}^{99\%}$	$ECR_{it,sim}^{99.9\%}$	$ECR_{it,mean}^{97.5\%}$	$ECR_{it,mean}^{99\%}$	$ECR_{it,mean}^{99.9\%}$
RCR_{it}^{SA}	1									
$RCR_{it,mean}^{IRBA}$	0.20	1								
$RCR_{it,sim}^{IRBA}$	0.27	0.91	1							
$RCR_{it,mp}^{IRBA}$	0.13	0.92	0.83	1						
$ECR_{it,sim}^{97.5\%}$	0.30	0.86	0.92	0.78	1					
$ECR_{it,sim}^{99\%}$	0.24	0.90	0.88	0.82	0.93	1				
$ECR_{it,sim}^{99.9\%}$	0.17	0.84	0.80	0.79	0.86	0.90	1			
$ECR_{it,mean}^{97.5\%}$	0.28	0.87	0.91	0.79	0.99	0.95	0.87	1		
$ECR_{it,mean}^{99\%}$	0.24	0.90	0.88	0.82	0.93	0.97	0.90	0.95	1	
$ECR_{it,mean}^{99.9\%}$	0.18	0.87	0.83	0.84	0.86	0.90	0.95	0.87	0.90	1

Table 9: Minimum capital requirements for different approaches

Approach	Minimum capital requirements
RCR_t^{SA}	11,976
$RCR_{t,mean}^{IRBA}$	44,214
$RCR_{t,sim}^{IRBA}$	31,037
$RCR_{t,mp}^{IRBA}$	113,436
$ECR_{t,sim}^{97.5\%}$	25,921
$ECR_{t,sim}^{99\%}$	59,882
$ECR_{t,sim}^{99.9\%}$	130,577
$ECR_{t,mean}^{97.5\%}$	25,943
$ECR_{t,mean}^{99\%}$	60,032
$ECR_{t,mean}^{99.9\%}$	129,937

6 Summary and Policy Implications

This paper uses a Bayesian Generalized Linear Mixed Model (GLMM) to estimate real-world probabilities of default (PDs) and correlations for EU sovereigns. The priors for the PDs are derived from historical default rates of corporate bonds with the same rating grades. Using our estimates as well as 'most prudent' estimates we compare regulatory capital under the standardized Approach (SA) and the Internal Ratings Based Approach (IRBA) with economic capital computed by the Expected Shortfall (ES) for the European Central Bank's Public Sector Purchase Programme (PSPP). We obtain the following main findings:

- The IRBA yields capital requirements which are by factors 2.6 to 9.5 higher than those under the SA
- This is mainly driven by the zero risk weights under the SA for highly rated countries
- IRBA capital requirements may substantially differ, depending on which method for PD estimation is used (Bayes vs. 'most prudent') where the latter results in most conservative capital charges
- IRBA capital requirements are highly correlated (in terms of rank order) with economic capital, as computed by the risk measure ES, while SA capital is neither highly correlated to IRBA nor to economic capital
- Absolute IRBA charges are close in size to economic capital if an ES confidence level of 97.5% is used

Our results may have important policy implications. As SA capital is too low in size and only poorly correlated to real-world risk - mainly due to zero risk weights of highly rated securities - it should be revised or, better, completely avoided if possible. Using the SA is sometimes associated with 'financial repression', see Hannoun (2013), which means that governments benefit from zero risk weights that provide incentives for holding government debt. On the other hand, this incentive might be misaligned because real-world sovereign risk is far off from SA regulatory capital which implies a form of regulatory arbitrage as financial institutions may prefer exchanging low-risk high-capital securities by higher-risk lower-capital securities, thereby reducing capital and boosting risk.

On the other hand, abandoning the SA requires data and methods for estimating real-world sovereign PDs which is challenging due to their low default properties. Bayesian approaches, as applied in this paper, might be a way forward to alleviate this issue.

References

Acerbi, Carlo and Dirk Tasche (2002), 'On the coherence of expected shortfall', *Journal of Banking & Finance* **26**(7), 1487–1503.

Artzner, Philippe, Freddy Delbaen, Jean-Marc Eber and David Heath (1999), 'Coherent measures of risk', *Mathematical Finance* **9**(3), 203–228.

Attinasi, Maria-Grazia, Cristina Checherita and Christiane Nickel (2009), 'What explains the surge in euro area sovereign spreads during the financial crisis of 2007-09?', *ECB Working Paper Series* **1131**.

Basel Committee on Banking Supervision (2006), *International Convergence of Capital Measurement and Capital Standards: A Revised Framework, Comprehensive Version*, Bank for International Settlements.

Basel Committee on Banking Supervision (2011), *Basel III: A global regulatory framework for more resilient banks and banking systems*, Bank for International Settlements.

Basel Committee on Banking Supervision (2016), 'The Basel Committee's work programme for 2015 and 2016'.
URL: *http://www.bis.org/bcbs/about/work_programme.htm (2016/09/06)*

Basel Committee on Banking Supervision (2017), *The regulatory treatment of sovereign exposures*, Discussion paper, Bank for International Settlements.

Beirne, John and Marcel Fratzscher (2013), 'The pricing of sovereign risk and contagion during the European sovereign debt crisis', *ECB Working Paper Series* **1625**.

Benjamin, Nathanaël, Alan Cathcart and Kevain Ryan (2006), Low Default Portfolios: A Proposal for Conservative Estimation of Default Probabilities, Discussion paper, Financial Services Authority.

Boudreault, Mathieu, Geneviève Gauthier and Tommy Thomassin (2015), 'Estimation of correlations in portfolio credit risk models based on noisy security prices', *Journal of Economic Dynamics and Control* **61**, 334–349.

Campbell, John Y., Jens Hilscher and Jan Szilagyi (2008), 'In Search of Distress Risk', *Journal of Finance* **63**(6), 2899–2939.

Duellmann, Klaus, Jonathan Küll and Michael Kunisch (2010), 'Estimating asset correlations from stock prices or default rates—Which method is superior?', *Journal of Economic Dynamics and Control* **34**(11), 2341–2357.

Dwyer, Douglas W. (2007), 'The distribution of defaults and Bayesian model validation', *Journal of Risk Model Validation* **1**(1), 23–53.

Fei, Fei, Ana-Maria Fuertes and Elena Kalotychou (2012), 'Credit Rating Migration Risk and Business Cycles', *Journal of Business Finance & Accounting* **39**(1-2), 229–263.

Forrest, Alan (2005), Likelihood Approaches to Low Default Portfolios, Discussion paper, Dunfermline Building Society.

Friewald, Nils (2009), 'Estimating Asset Correlation from CDS Spreads: An Analysis of the Hedge Effectiveness of FTD Baskets', *Working Paper, Vienna University of Economics and Business* .

Gordy, Michael B. (2000), 'A comparative anatomy of credit risk models', *Journal of Banking & Finance* **24**(1), 119–149.

Gordy, Michael B. (2003), 'A risk-factor model foundation for ratings-based bank capital rules', *Journal of Financial Intermediation* **12**(3), 199–232.

Gordy, Michael B. and Erik Heitfield (2009), Small-Sample Estimation of Models of Portfolio Credit Risk, *in* M.Kijima, C.Hara, K.Tanaka and Y.Muromachi, eds, 'Recent Advances in Financial Engineering 2009', World Scientific Publishing, Singapore, pp. 43–66.

Greenberg, Edward (2013), *Introduction to Bayesian Econometrics*, 2. edn, Cambridge University Press, Cambridge; New York.

Hannoun, Herve (2013), Sovereign risk in bank regulation and supervision: Where do we stand?, *in* Bank für Internationalen Zahlungsausgleich, ed., 'Sovereign risk: a world without risk-free assets?', number 72 *in* 'BIS papers', Bank for International Settlements, Basel.

Hanson, Samuel and Til Schuermann (2006), 'Confidence intervals for probabilities of default', *Journal of Banking & Finance* **30**(8), 2281–2301.

Hilscher, Jens and Mungo Wilson (2017), 'Credit Ratings and Credit Risk: Is One Measure Enough?', *Management Science* **63**(10), 3414–3437.

Hilscher, Jens and Yves Nosbusch (2010), 'Determinants of Sovereign Risk: Macroeconomic Fundamentals and the Pricing of Sovereign Debt', *Review of Finance* **14**(2), 235–262.

Jacobs Jr, Michael and Nicholas M. Kiefer (2010), The Bayesian approach to default risk: A guide, *in* K.Böcker, ed., 'Rethinking risk measurement and reporting', Vol. 2, Risk Books, London, pp. 319–334.

Jones, David (2000), 'Emerging problems with the Basel Capital Accord: Regulatory capital arbitrage and related issues', *Journal of Banking & Finance* **24**(1-2), 35–58.

Kass, Robert E. and Larry Wasserman (1996), 'The Selection of Prior Distributions by Formal Rules', *Journal of the American Statistical Association* **91**(435), 1343–1370.

Kazianka, Hannes (2016), 'Objective Bayesian estimation of the probability of default', *Journal of the Royal Statistical Society: Series C (Applied Statistics)* **65**(1), 1–27.

Kiefer, Nicholas M. (2009), 'Default estimation for low-default portfolios', *Journal of Empirical Finance* **16**(1), 164–173.

Kiefer, Nicholas M. (2011), 'Default estimation, correlated defaults, and expert information', *Journal of Applied Econometrics* **26**(2), 173–192.

King, Gary and Langche Zeng (2001), 'Logistic Regression in Rare Event Data', *Political Analysis* **9**(2), 137–163.

Longstaff, Francis A., Jun Pan, Lasse H. Pedersen and Kenneth J. Singleton (2011), 'How Sovereign Is Sovereign Credit Risk?', *American Economic Journal: Macroeconomics* **3**(2), 75–103.

Lopez, Jose A. (2004), 'The empirical relationship between average asset correlation, firm probability of default, and asset size', *Journal of Financial Intermediation* **13**(2), 265–283.

Luo, Xiaolin and Pavel V. Shevchenko (2013), 'Markov chain Monte Carlo estimation of default and recovery: dependent via the latent systematic factor', *Journal of Credit Risk* **9**(3), 41–76.

McNeil, Alexander J. and Jonathan P. Wendin (2007), 'Bayesian inference for generalized linear mixed models of portfolio credit risk', *Journal of Empirical Finance* **14**(2), 131–149.

Merton, Robert C. (1974), 'On the Pricing of Corporate Debt: The Risk Structure of Interest Rates', *Journal of Finance* **29**(2), 449–470.

Nashikkar, Amrut, Marti G. Subrahmanyam and Sriketan Mahanti (2011), 'Liquidity and Arbitrage in the Market for Credit Risk', *Journal of Financial and Quantitative Analysis* **46**(3), 627–656.

Neal, Radford M. (2003), 'Slice Sampling', *Annals of Statistics* **31**(3), 705–767.

Nelder, John A. and Robert W. Wedderburn (1972), 'Generalized Linear Models', *Journal of the Royal Statistical Society. Series A (General)* **135**(3), 370–384.

Nyambuu, Unurjargal and Lucas Bernard (2015), 'A Quantitative Approach to Assessing Sovereign Default Risk in Resource-Rich Emerging Economies: sovereign default risk in resource-rich emerging economies', *International Journal of Finance & Economics* **20**(3), 220–241.

Orth, Walter (2013), 'Default probability estimation in small samples – with an application to sovereign bonds', *Quantitative Finance* **13**(12), 1891–1902.

Ou, Sharon (2016), Corporate Default and Recovery Rates, 1920-2015, Technical report, Moody's Investors Service.

Pluto, Katja and Dirk Tasche (2005), 'Thinking positively', *Risk* **18**(8), 72–78.

Pluto, Katja and Dirk Tasche (2011), Estimating Probabilities of Default for Low Default Portfolios, *in* B.Engelmann and R.Rauhmeier, eds, 'The Basel II Risk Parameters', Springer, Berlin, Heidelberg, pp. 75–101.

Shumway, Tyler (2001), 'Forecasting Bankruptcy More Accurately: A Simple Hazard Model', *Journal of Business* **74**(1), 101–124.

Tasche, Dirk (2013), 'Bayesian estimation of probabilities of default for low default portfolios', *Journal of Risk Management in Financial Institutions* **6**(3), 302–326.

Tudela, Merxe (2015), Sovereign Default and Recovery Rates, 1983-2014, Technical report, Moody's Investors Service.